Welcome to QCD:

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} \]

\[ = \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G^a_\mu \bar{\psi}_i \gamma^\mu T^a_{ij} \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} \]
QCD is just like QED, only different...

\[ \mathcal{L} = \bar{\psi} \left( i \gamma^\mu D_\mu - m \right) \psi - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a, \]

\[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - 4 \delta_\mu_\nu F^{\nu\gamma}_a, \]

Length Scale

Perturbation theory at large distance is convergent

\[ \alpha(\infty) \sim \frac{1}{137}, \]

\[ \alpha(M_Z) \sim \frac{1}{128}, \]

\[ \alpha \text{ is good expansion parameter} \]
QCD is Non-Abelian SU(3) Symmetry; Quarks are Confined!!!

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}(i\gamma^\mu(D_{\mu})_\psi - m_\psi)\psi - \frac{1}{4}G^\mu_{\alpha\beta}G_{\mu\alpha\beta} \]

\[ G^\mu_{\alpha\beta} = \partial_\mu A^\nu_{\alpha\beta} - \partial_\nu A^\mu_{\alpha\beta} - g f^{\rho\alpha\beta} A^\rho_\mu A^\nu_\rho \]

\[ [T_a, T_b] = i \sum_{c=1}^{8} f_{abc} T_c \]

\[ \alpha_s(M_Z) \sim 0.118 \]

**Statement of the problem**

*Theorist #1:* The universe is completely described by the symmetry group SO(10)

*Theorist #2:* You're wrong; the correct answer is SuperSymmetric flipped SU(5)xU(1)

*Theorist #3:* You've flipped! The only rational choice is E8xE8 dictated by SuperString Theology.

*Experimentalist:* Enough of this speculative nonsense. I'm going to measure something to settle this question. What can you predict???

*Theorist #1:* We can predict the interactions between fundamental particles such as quarks and leptons.

*Experimentalist:* Great! Give me a beam of quarks and leptons, and I can settle this debate.

*Accelerator Operator:* Sorry, quarks only come in a 3-pack and we can't break a set!
We are going to look at the essence of what makes QCD so different from the other forces. As a consequence, we will need to be creative in how we study the properties, now we define observables, and interpret the results.

QCD has a history of more than 40 years, and we are still trying to fully understand its structure.

The goal of these lectures

Provide pictorial/graphical/heuristic explanations for everything that confused me as a student
### Lecture 1:
- Overview & essential features
- Nature of strong coupling constant & how it varies with scale
- Issues beyond LO and SM
- Renormalization Group Equation & Resummation
- Scaling and the proton Structure

### Lecture 2:
- The structure of the proton
- Deeply Inelastic Scattering (DIS)
- The Parton Model
- PDF's & Evolution
- Scaling and Scale Violation

### Lecture 3:
- Issues at NLO
- Collinear and Soft Singularities
- Mandelstam Variables
- An example from Freshman Physics
- Regularized Distributions
- Extension to higher orders

### Lecture 4:
- Drell-Yan and $e^+e^-$ Processes
- $W/Z/Higgs$ Production & Kinematics
- 3-body Phase Space & Dalitz Plots
- Sterman-Weinberg Jets
- Infrared Safe Observables
- Rapidity & Pseudo Rapidity
- Jet Definitions

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### Useful References
- Ellis, Stirling, Webber
- CTEQ Handbook
- Reviews of Modern Physics

### Homework:

**Physics is not a spectator sport**
More Useful References

The CTEQ Pedagogical Page
Linked from cteq.org

Everything you wanted to know about Lambda-QCD but were afraid to ask
Randall J. Scalise and Fredrick I. Olness

Regularization, Renormalization, and Dimensional Analysis:
Dimensional Regularization meets Freshman E&M
Fredrick Olness & Randall Scalise
e-Print: arXiv:0812.3578

Calculational Techniques in Perturbative QCD: The Drell-Yan Process.
Björn Pötter has prepared a writeup of the lecture given by Jack Smith.
This is a wonderful reference for those learning to do real 1-loop calculations.

Thanks to:
Dave Soper, George Sterman,
John Collins, & Jeff Owens for ideas
borrowed from previous CTEQ
introductory lecturers

Thanks to Randy Scalise for the help on
the Dimensional Regularization.

Thanks to my friends at Grenoble who
helped with suggestions and corrections.

Thanks to Jeff Owens for help on
Drell-Yan and Resummation.

To the CTEQ and MCnet folks
for making all this possible.

and the many web pages where I borrowed my figures...
QCD is Non-Abelian SU(3) Symmetry; Quarks are Confined

The β-function: \( \beta(\alpha_s) = \alpha_s^2 \left( \frac{33}{12\pi} - \frac{2N_F}{12\pi} \right) + \ldots \)

Consider a physical observable: \( R(Q^2/\mu^2, \alpha_s) \)

\( Q \) is the characteristic energy scale of the problem
\( \mu \) is an artificial scale we introduce to regulate the calculation (more later)

The Renormalization Group Equation (RGE) is:

\[
\frac{d R}{d \mu^2} = 0
\]

Using the chain rule:

\[
\frac{d R}{d \mu^2} = \left[ \frac{d}{d \mu^2} \frac{\partial}{\partial \mu^2} \right] \left( \frac{Q^2}{\mu^2} \right) \alpha_s(\mu^2) = 0
\]

\( \beta(\alpha_s) \)

The Nobel Prize in Physics 2004
was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of
asymptotic freedom"
Solve for the running coupling and $\Lambda_{QCD}$

Let: $t = \ln \mu^2$

\[
\frac{1}{\alpha_S} \left[ \mu_1 \right] = b_0 \frac{1}{\mu_0} \left[ \mu_1 \right]
\]

\[
\frac{1}{\alpha_S(\mu_1)} - \frac{1}{\alpha_S(\mu_0)} = b_0 \ln(\mu_1/\mu_0)
\]

\[\alpha_S(\mu) = \frac{1}{b_0 \ln(\mu/\Lambda_{QCD})}\]

Energy Scale $t$

Landau pole

$\Lambda = \mu e^{-1/(b_0 \alpha_s(\mu))}$

$\Lambda_{QCD} \sim 200 \text{ MeV} \sim 1 \text{ fm}$

The Standard Model (SM) Running Couplings: U(1), SU(2), SU(3)

\[
b_1 = 0 + (2/3) N_F + \frac{4}{3} N_{Higgs} \\
b_2 = -22/3 + (2/3) N_F + 1 N_{Higgs} \\
b_3 = -11 + (2/3) N_F + 0
\]

Comparison with data

Caution: $\alpha_s$ is NOT a physical observable

Low Q points have more discriminating power
\[ \beta = -\alpha_s^2 \left[ \frac{33 - 2N_F}{12\pi} \right] + \ldots \]

At 1-loop and 2-loops, continuous at thresholds

\[ \alpha_S N_F + 1(\mu^2) = \alpha_S N_F(\mu^2) \begin{bmatrix} 1 \\ + \alpha_s^1 c_{10} + c_{11} L^1 \\ + \alpha_s^2 c_{20} + c_{21} L^1 + c_{22} L^2 \\ + \ldots \end{bmatrix} \]

This is zero

This is non-zero

\[ L = \ln(\mu^2/m^2) \]

\[ c_0 = 0 \]

\[ c_1 = -\frac{11}{72\pi} \neq 0 \]

\[ \alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2} \alpha_{(n_f-1)}^3(M) + O(\alpha_{(n_f-1)}^4) \]

Un-physical theoretical constructs:

(E.g., \(\alpha_s\), PDFs, ...)

Cannot be measured directly

Depends on Schemes

Renormalization Schemes:

- MS, MS-Bar, DIS

Renormalization Scale \(m\)

Depends on Higher Orders

Physical Observables

Measure directly

Independent of Schemes/Definitions

Independent of Higher Orders
The Standard Model (SM) & SUSY Running Couplings


\[ b_1 = 0 + \frac{2}{3} N_F + \frac{1}{10} N_{Higgs} \]
\[ b_2 = -\frac{22}{3} + \frac{2}{3} N_F + \frac{1}{6} N_{Higgs} \]
\[ b_3 = -11 + \frac{2}{3} N_F + 0 \]

Can we do better???

We’ve only discovered half the particles

New particles effects evolution of \( \alpha_s(\mu) \)

Light particle thresholds

\[ \beta_0 = 11 - \frac{2}{3} \left( N_f + 3 N_b + \frac{1}{4} N_f \right) \]
Include a light gluino

\[ b_0 = \frac{1}{12 \pi} \{ 33 - 2 N_F - 6 N_g - \frac{1}{2} N_F \ldots \} \]

Siegfried Bethke
arXiv:1210.0325 [hep-ex]

CDF Collaboration, PRL 77, 438 (1996)

Indispensable for discovery of “new physics”

RESUMMATION

... over simplified

GOT QCD ???

Warm up: Dimensional Analysis: Pythagorean Theorem

GOAL: Pythagorean Theorem

METHOD: Dimensional Analysis

Two examples to come: 1) Resummation, and 2) Scaling
If we expand \( R \) in powers of \( \alpha_s \), and we know \( \beta \), we then know \( \mu \) dependence of \( R \).

Since \( \mu \) is arbitrary, choose \( \mu=Q \).

We just summed the logs.

**More Differential Quantities \( \Rightarrow \) More Mass Scales \( \Rightarrow \) More Logs!!!**

How do we resum logs? Use the Renormalization Group Equation

\[
\frac{dR}{d\mu} = 0
\]

Applied to boson transverse momentum

CSS: Collins, Soper, Sterman


Interesting reference:

Peskin/Schroeder Text

(Renomalization ala Ken Wilson)

Scaling, and the proton structure

Quarks confined, thus we must work with hadrons & mesons

*E.g., proton is a “minimal” unit*

Highest energy (smallest distance) accelerators involve hadrons

*E.g., HERA, TEV, LHC*

We’d better learn to work with proton.
What do we expect for a point like particle

Dimensional considerations

Structure Function

Is this a point like particle ???

We found the Higgs

Scaling, and the proton structure

Relative Sizes

Scale in m:

KeV

atom

MeV

nucleus

GeV

proton

TeV

quark

\[ d\sigma \sim \frac{4\pi \alpha^2}{Q^2} \times 1 \]

Relative Sizes

KeV \sim 10^{-9}

MeV \sim 10^{-12}

GeV \sim 10^{-15}

TeV \sim 10^{-18}

\[ \hbar c = 1 \simeq 0.2 \text{ GeV fm} \]

Going to smaller scale, we get simpler, more fundamental objects
### Structure of the Proton

\[ d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1 \]

\[ d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times F\left(\frac{Q^2}{\Lambda^2}\right) \]

\[ d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times \sum_i c_i^2 \]

\(\Lambda\) of order of the proton mass scale

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### The Scaling of the Proton Structure Function

Data lies along a universal curve. It is (relatively) independent of energy.

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### End of lecture 1: Recap

QCD is just like QED, ... only different

- QCD is non-Abelian, Quarks are confined,
- Running coupling \(\alpha_s(\mu)\) tells how interaction changes with distance

\[ \beta\text{-function: logarithmic derivative of } \alpha_s(\mu) \]

We can compute: Negative for QCD, positive for QED

\(\alpha_s(\mu)\) is not a physical quantity

- Discontinuous at NNLO

- New physics can influence \(\alpha_s(\mu)\)

- Unification of couplings at GUT scale

Running of \(\alpha_s(\mu)\) can help us “resum” perturbation theory

Scaling and Dimensional Analysis are useful tools.
END OF LECTURE 1