QCD in e^+e^- Interactions (Part 2)
– Measurement of the color factors
– Soft and semi-soft processes

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Outline

(I) Measurement of the color factors

(II) Differences between gluon & quark jets

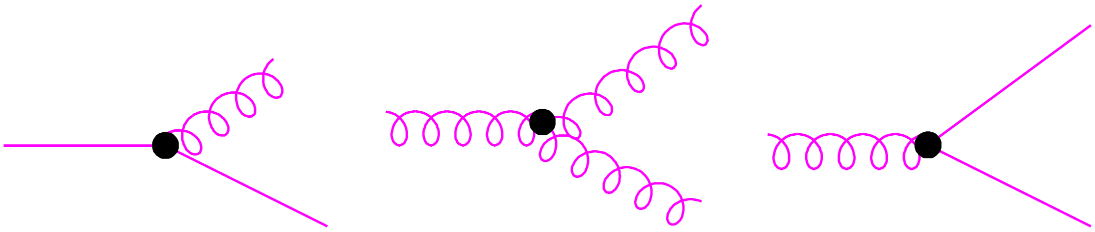
(III) Coherence and Local Parton Hadron Duality (LPHD)

(IV) Identified particles

(V) APPENDIX $\longrightarrow$ Overflow

(I) Measurement of the color factors

Effective interactions in e$^+$e$^-$ annihilations to $\mathcal{O}(\alpha_S^2)$



Gluon radiation	Triple gluon vertex	Gluon splitting
$q \longrightarrow qg$	$g \longrightarrow gg$	$g \longrightarrow q\overline{q}$
$\boxed{\mathrm{C_F}=\frac{4}{3}}$	$\boxed{\mathrm{C_A}=3}$	$\boxed{\mathrm{T_F}=\frac{1}{2}}$

The "color factors" $\mathrm{C_F}$, $\mathrm{C_A}$, $\mathrm{T_F}$ specify the relative probabilities of the three processes.

$t^{\mathrm{A}}(A=1,\cdots 8) \longrightarrow$ <u>generators</u> of SU(3)
(= $\frac{1}{2}$ the 3×3 Gell-Mann matrices)

$f^{\mathrm{ABC}} \longrightarrow$ <u>structure constants</u> of SU(3)
$\longrightarrow [t^{\mathrm{A}},t^{\mathrm{B}}] = i f^{\mathrm{ABC}} t^{\mathrm{C}}$

$$\sum_{\mathrm{A=1,8}} \sum_{\mathrm{b=1,3}} t^{\mathrm{A}}_{ab} t^{\mathrm{A}}_{bc} = \delta_{ac}\underline{\mathrm{C_F}} \qquad \sum_{\mathrm{A,B=1,8}} f^{\mathrm{ABC}} f^{\mathrm{ABD}} = \delta^{ac}\underline{\mathrm{C_A}}$$

$$\mathrm{Tr}\left(\mathrm{t^A_{ab} t^B_{ba}}\right) = \delta^{\mathrm{AB}}\underline{\mathrm{T_F}}$$

Ratios C_A/C_F, T_F/C_F

$\longrightarrow$ Can be determined with greater precision than individual color factors C_A, C_F or T_F

$\longrightarrow$ Sufficient to distinguish between gauge groups

Models with 3 color degrees of freedom for quarks:

	SU(3) (QCD)	SO(3)	$U(1)_3$ (Abelian gluon)
C_A/C_F	$\frac{9}{4}$	1	0
T_F/C_F	$\frac{3}{8}$	1	3

Techniques to measure the color factor ratios $\longrightarrow$

- Angular correlations in 4-jet events
- Angular correlations in 5-jet events (not discussed here)
- Event shapes (2- and 3-jet events, α_S)
- Differences between gluon and quark jets $\longrightarrow$ next section

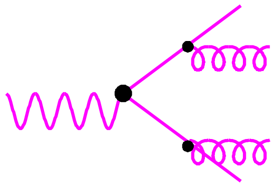
Angular correlations in 4-jet events

Expression for $\boxed{e^+e^- \longrightarrow 4\text{ jets}}$ to $\mathcal{O}(\alpha_S^2)$ (tree level)

ERT $\rightarrow$ R.K. Ellis, D.A. Ross, A.E. Terrano (1980)

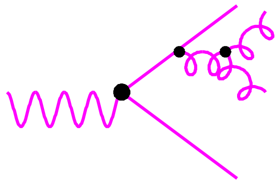
$$\frac{1}{\sigma_0}\frac{d\sigma}{dy} = \left(\frac{\alpha_S C_F}{\pi}\right)^2 \times$$

$$\sigma_A(y) \longrightarrow$$

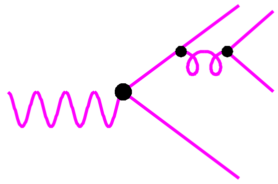


$$+ \left(1 - \frac{1}{2}\frac{C_A}{C_F}\right)\sigma_B(y)$$

$$+ \left(\frac{C_A}{C_F}\right)\sigma_C(y) \longrightarrow$$



$$+ \left(\frac{T_F}{C_F}n_f\right)\sigma_D(y) \longrightarrow$$



$$+ \left(1 - \frac{1}{2}\frac{C_A}{C_F}\right)\sigma_E(y)$$

$\sigma_A \cdots \sigma_E$ $\rightarrow$ kinematic terms, independent of the gauge group
($y \longrightarrow$ 2-jet invariant masses)

The angular correlations between the four jets differ for the three diagrams, cf. $g \rightarrow gg$ (spin 1 $\rightarrow$ 1 1) versus $g \rightarrow q\overline{q}$ (spin 1 $\rightarrow$ 1/2 1/2)

$\longrightarrow$ Allows the relative contributions of the coefficients σ_A, $(C_A/C_F)\,\sigma_C$, etc. to be distinguished experimentally, allowing a determination of the color factor ratios, with $\sigma_A \cdots \sigma_E$ taken from theory

Procedure

$\longrightarrow$ Select 4-jet events using a jet finder ($K_\perp$, JADE, etc.)

$\longrightarrow$ Order jets by energy $E_1 > E_2 > E_3 > E_4$
Jets 1,2 $\longrightarrow$ almost always quark jets

or else

(DELPHI) tag two of the jets as quark jets using b-tagging (jets 1,2= tagged jets; jets 3,4=untagged jets)

$\longrightarrow$ For simplicity, usually employ standard angular correlation variables rather than the 2-jet invariant masses y

- Bengtsson-Zerwas angle:

$$\cos\chi_{\mathrm{BZ}} = \left| \frac{(\vec{p}_1 \times \vec{p}_2)\cdot(\vec{p}_3 \times \vec{p}_4)}{|\vec{p}_1 \times \vec{p}_2|\,|\vec{p}_3 \times \vec{p}_4|} \right|$$

- (modified) Nachtmann-Reiter angle:

$$\cos\Theta_{\mathrm{NR}^*} = \left| \frac{(\vec{p}_1 - \vec{p}_2)\cdot(\vec{p}_3 - \vec{p}_4)}{|\vec{p}_1 - \vec{p}_2|\,|\vec{p}_3 - \vec{p}_4|} \right|$$

- Angle between jets 3 and 4:

$$\cos\alpha_{34} = \frac{\vec{p}_3 \cdot \vec{p}_4}{|\vec{p}_3|\,|\vec{p}_4|}$$

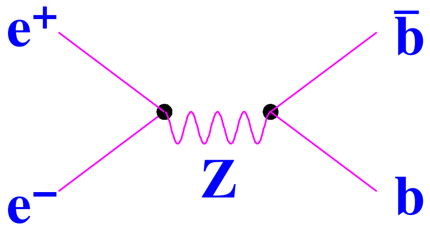
$\longrightarrow$ Account for hadronization using corrections from MC, fit to theoretical expression, using

$$\frac{\mathrm{C_A}}{\mathrm{C_F}},\ \frac{\mathrm{T_F}}{\mathrm{C_F}},\ \text{Overall normalization}$$

as the fitted parameters

B tagging of quark jets in e^+e^-

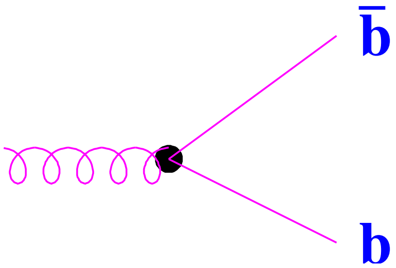
- Almost all b quarks in e^+e^- annihilations at present energies are produced at the electro-weak vertex



$$\frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})} \approx 0.21$$

i.e. about 1 in 5 events is a b event

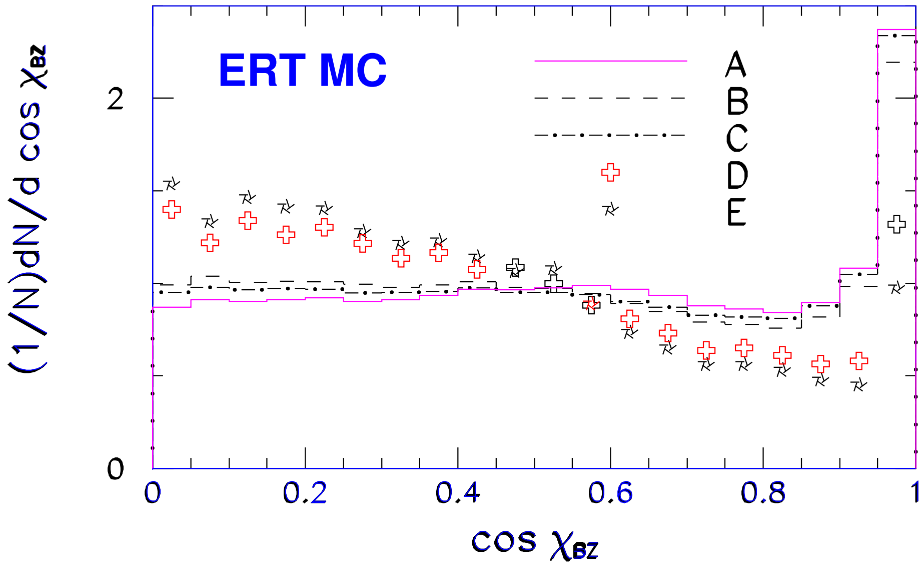
- In contrast, only about 1 in 300 $Z \to \text{hadrons}$ events contains a gluon splitting to $b\bar{b}$

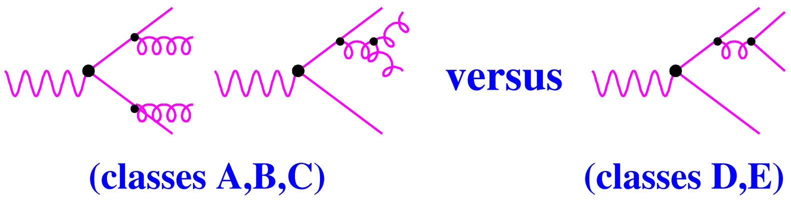


$\longrightarrow$ A jet with a b hadron is overwhelmingly likely to be a QUARK JET

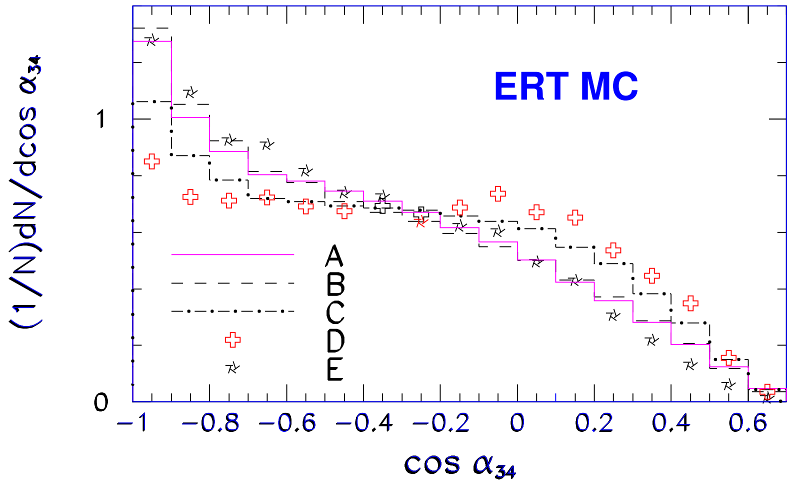
- This is in contrast to hadron colliders where $g \to b\bar{b}$ is the dominant production mechanism for b hadrons

Bengtsson-Zerwas angle →



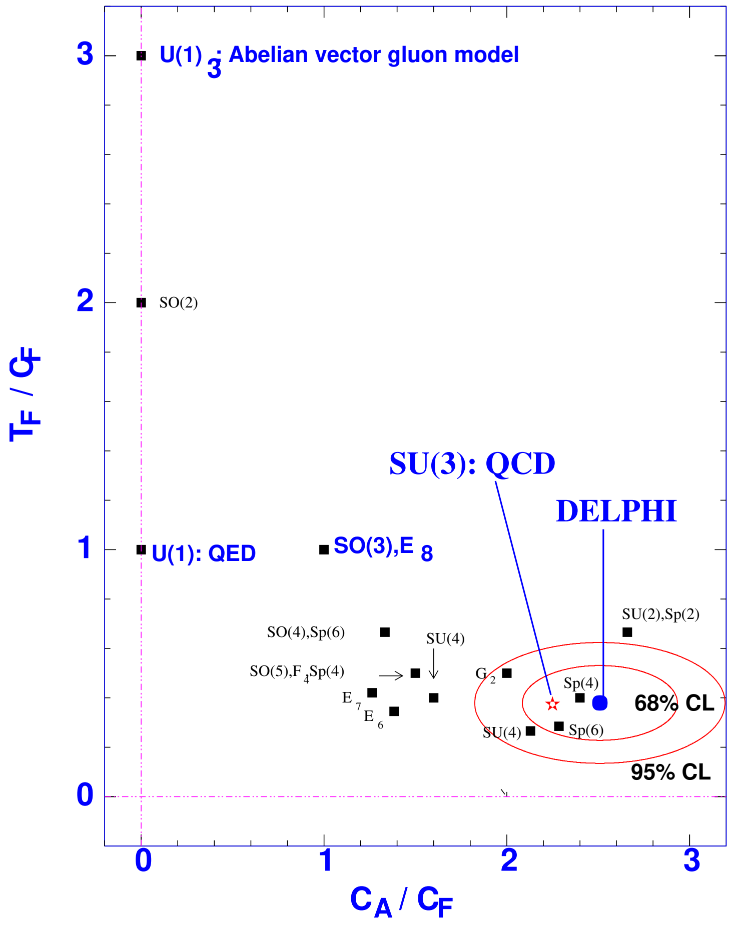


Angle between jets 3 and 4 (DELPHI 1991) →



Measured $\mathrm{T_F/C_F}$ versus $\mathrm{C_A/C_F}$

DELPHI at LEP-1 (1997) $\longrightarrow$ (b-tagging of jets 1,2)



Of the three gauge groups with three color degrees of freedom

SU(3), SO(3), U(1)$_3$

only SU(3) is consistent with the data

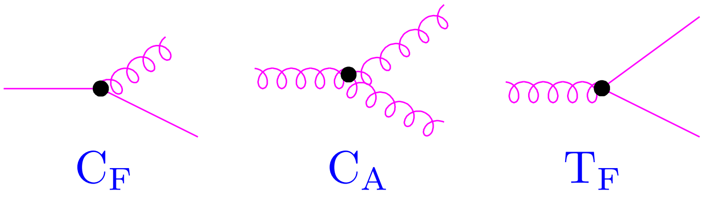
$$SU(3) \longrightarrow \mathrm{C_A/C_F} = 2.25,\ \mathrm{T_F/C_F} = 0.375$$

Three color degrees of freedom required by

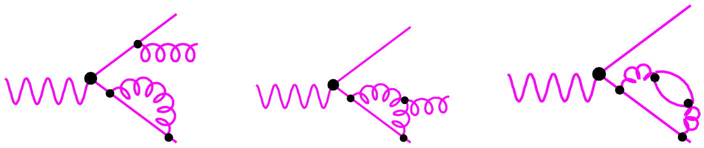
$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}, \text{ etc.}$$

Color factors from 2- and 3-jet events (event shapes: Thrust, etc.)

$\longrightarrow$ Virtual corrections to the $\geq \mathcal{O}(\alpha_S^2)$ 2- and 3-jet cross sections contain the same QCD vertices as the tree level 4-jet cross section:



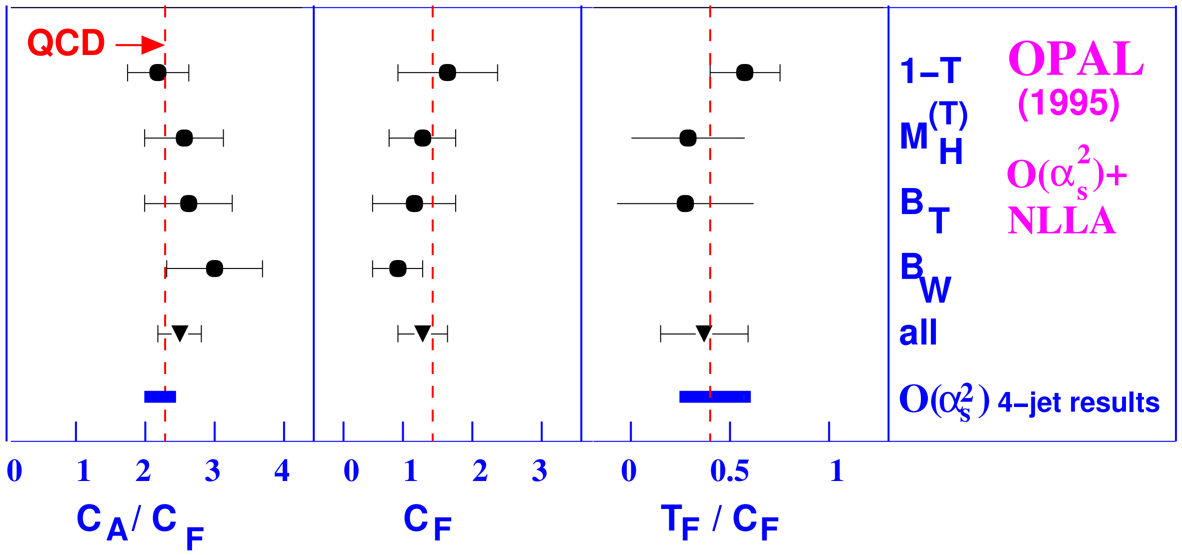
$\longrightarrow$



$\longrightarrow$ The $\mathcal{O}(\alpha_S^2)$+NLLA expressions for experimental observables like thrust can be decomposed into terms

$$\sim \; C_F^2, \quad C_A C_F, \quad T_F C_F$$

$\longrightarrow$ Extract measurements of the color factors using theory valid beyond leading order (complementary to the 4-jet results)



ALEPH at LEP-1 (1997)

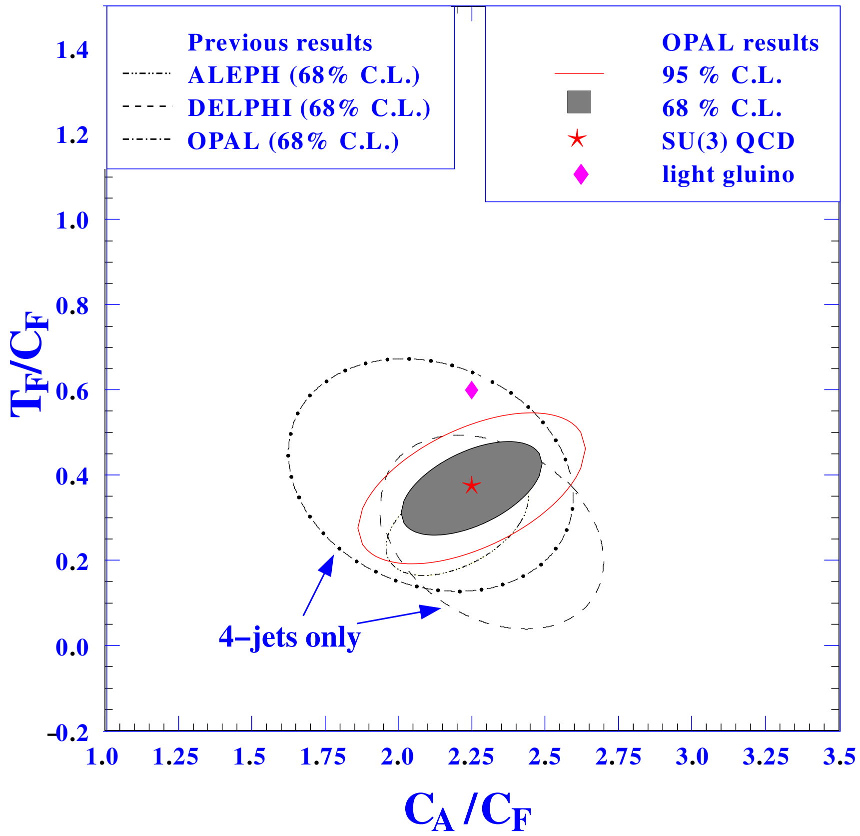
$\longrightarrow$ Fit color factor ratios simultaneously using 4-jet angular correlations AND event shapes

$\longrightarrow$ As the event shape, use the 2- to 3-jet transition variable D_2 (aka y_{23} or y_3) from the $k_\perp$ jet finder

OPAL at LEP-1 (2000)

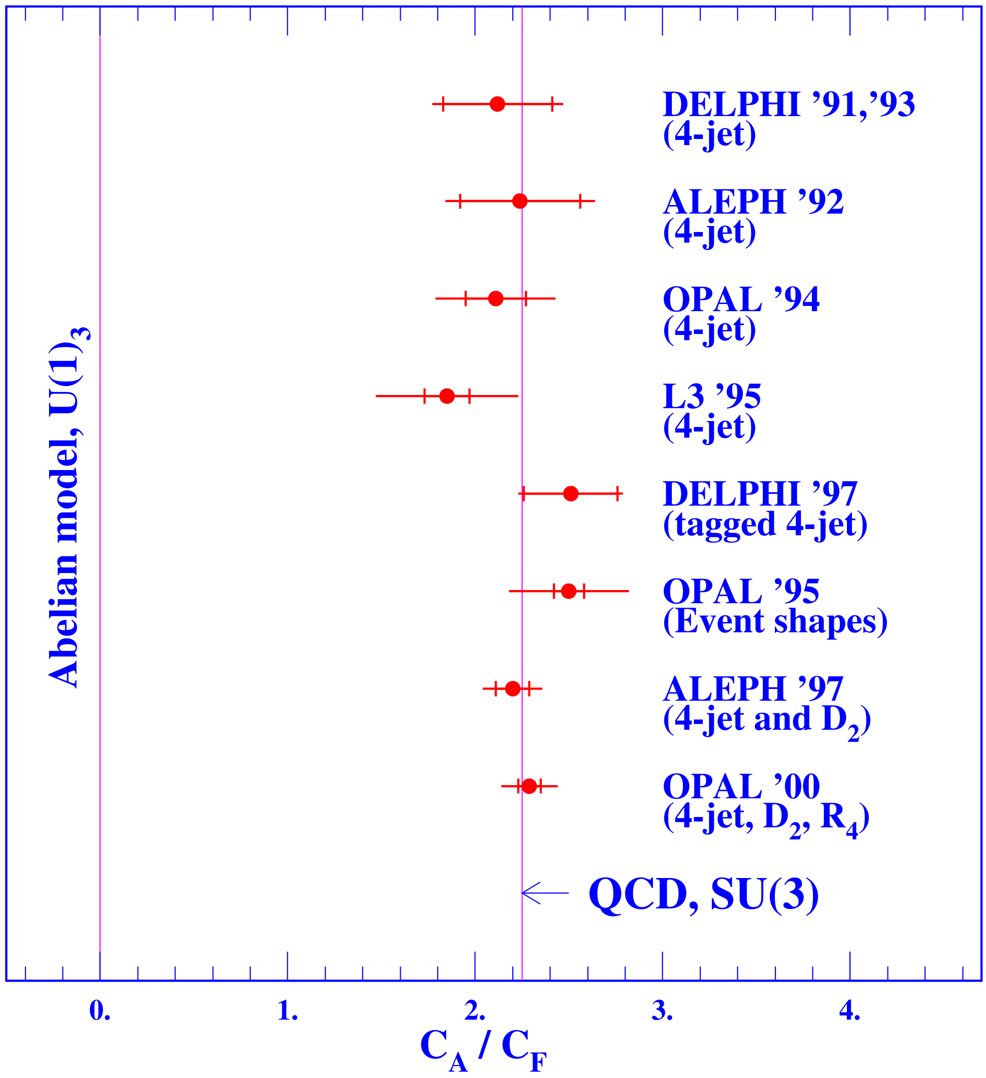
$\longrightarrow$ Same as ALEPH, and, in addition

- Include the 4-jet rate versus y_{cut}, defined with $k_\perp$
- Employ NLO calculations ($\mathcal{O}(\alpha_S^3)$) for the 4-jet angular correlations (1998)



$\longrightarrow$ Substantial reduction in the uncertainties compared to the earlier studies based on 4-jet angular correlations alone

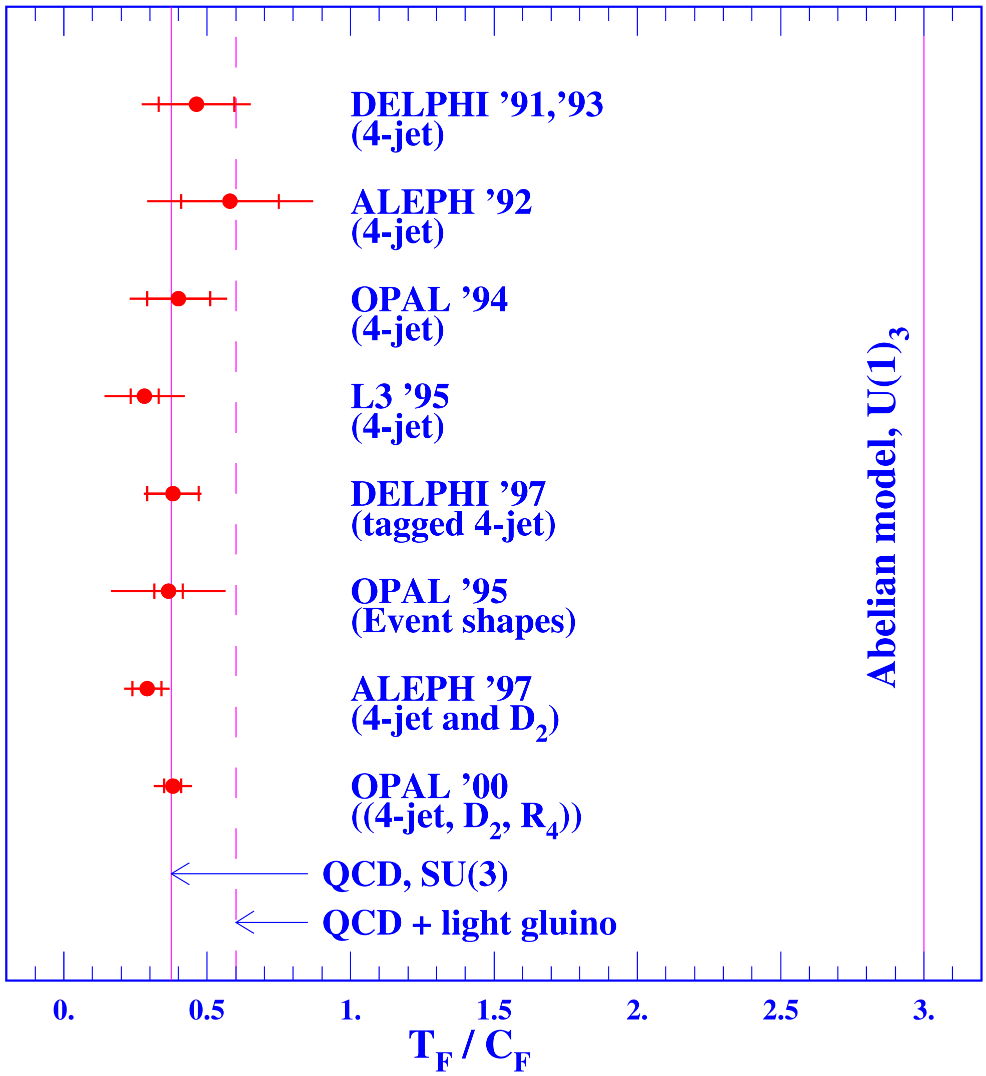
Color factor ratio C_A/C_F from event shapes & 4-jet angular correlations



→ Excellent agreement with the QCD value of 2.25

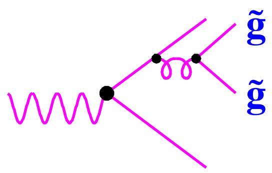
→ Abelian model (no triple gluon vertex) excluded by 15 standard deviations !

Color factor ratio T_F/C_F from event shapes & 4-jet angular correlations



⟶ Agreement with QCD value of 0.375

⟶ Light gluino mimics 4-quark events
G. Farrar (1990)



⟶ Effective number of flavors increases from 5 to 8.

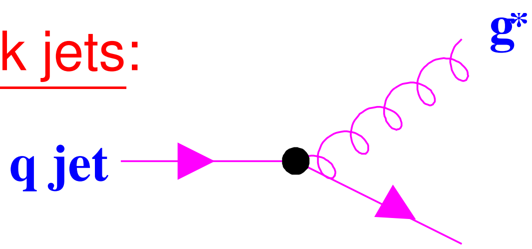
⟶ Light gluino hypothesis disfavored by the data (>3 s.d.)

(II) Differences between gluon & quark jets

QUARK and GLUON jets have different coupling strengths for gluon emission:

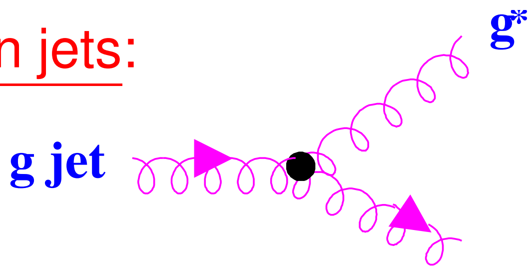
$\longrightarrow$ expressed by the color factors

<u>Quark jets:</u>



qg vertex, $\boxed{C_F = 4/3}$

<u>Gluon jets:</u>



ggg vertex, $\boxed{C_A = N_C = 3}$

The color charge of a gluon is

$$\frac{C_A}{C_F} = \frac{9}{4} = 2.25$$

<u>larger</u> than the color charge of a quark

Greatest theoretical interest in G/Q jet differences $\longrightarrow$ Multiplicity ratio $r_{g/q}$

- Fundamental prediction of QCD $\longrightarrow$

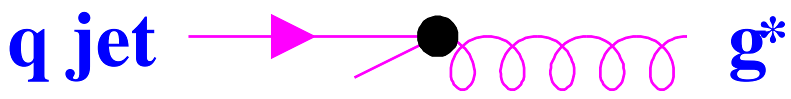
  The number of soft gluons emitted within a gluon jet should be $\sim$ twice that emitted within a quark jet:

$$r_{g/q} \equiv \frac{\langle n \rangle_{gluon}}{\langle n \rangle_{quark}} \approx \frac{\mathrm{C_A}}{\mathrm{C_F}} = 2.25$$

  This result is valid only for *soft* gluons:

  $\boxed{\mathrm{E}_{gluon} << \mathrm{E}_{\mathrm{jet}}} \longrightarrow$ asymptotic condition

- For *hard* gluon emission, the quark jet develops like a gluon jet $\longrightarrow$

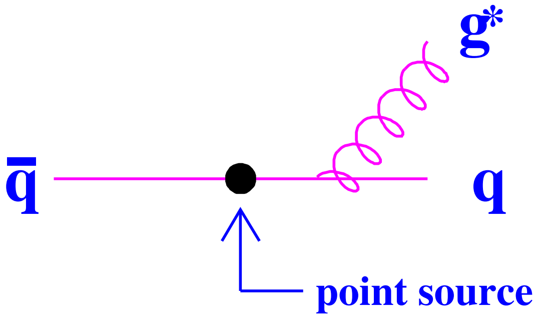


$\longrightarrow r_{g/q} \sim 1$

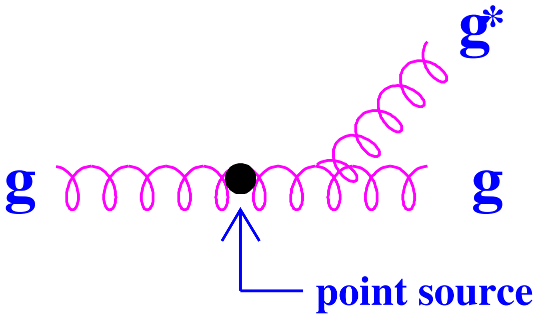
Quark & gluon jets in QCD calculations

⟶ G and Q jets are defined through pair production from a color singlet point source

Natural Occurrence



e^+e^- annihilations



$\Upsilon \rightarrow \gamma gg$ decays

Jet properties defined by an *inclusive sum* over the event or event hemispheres ⟶ UNBIASED JETS

⟶ No jet-finding algorithm

⟶ No ambiguity about which particles to associate with gluon or quark jet production

QCD prediction for $r_{g/q}$ (unbiased jets)

$$r_{g/q} = \frac{\langle n \rangle_{gg\, hemis.}}{\langle n \rangle_{q\bar{q}\, hemis.}}$$

$\boxed{= 2.25}$ Asymptotic: E $<<$ E$_{\rm jet}$

$\longrightarrow$ Full phase space (using ALL particles), E$_{\rm jet} \to \infty$

Brodsky & Gunion (1976)
Veneziano *et al.* (1978)

$\longrightarrow$ Limited phase space (using SOFT particles only), E$_{\rm jet}$ = finite, as applies to experiment

Khoze, Lupia & Ochs (1998)

$\boxed{\sim 1.5}$ Full phase space, finite E$_{\rm jet} \sim 40$ GeV (LEP-1)

$\longrightarrow$ α_S corrections up to n.n.l.o.: $r_{g/q} \approx 2.1$

Malaza & Webber (1984);
Gaffney & Mueller (1985)

$\longrightarrow$ Energy conservation up to n.n.l.o.: $r_{g/q} \approx 1.7$

Dremin (1994)

$\longrightarrow$ Numerical solutions $\to$ more accurate inclusion of energy conservation and phase space limits:

$r_{g/q} \approx 1.5$

Lupia & Ochs (1997)
Eden & Gustafson (1998)

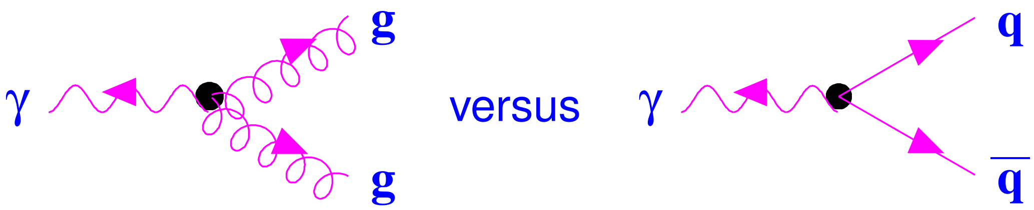
These calculations are based on massless quarks

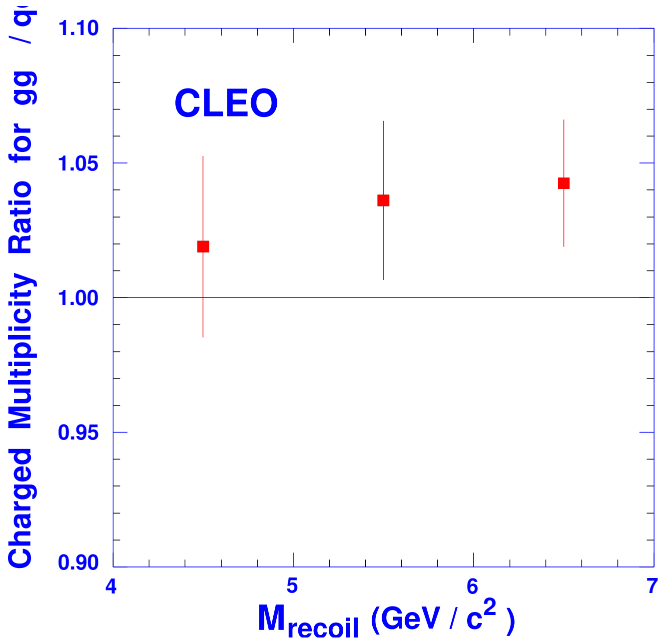
Unbiased gluon jets from Υ decays

CLEO at CESR, e^+e^- $E_{c.m.} \approx$ 10 GeV (1997)

$\longrightarrow$ Radiative Υ decays: $e^+e^- \to \Upsilon(1S) \to \gamma\, gg$

$\longrightarrow$ Compare with $e^+e^- \to \gamma q\bar{q}$ initial state radiation events with the same recoil mass, collected in the continuum.





$$\boxed{r_{g/q}^{hadron} = 1.04 \pm 0.05}$$

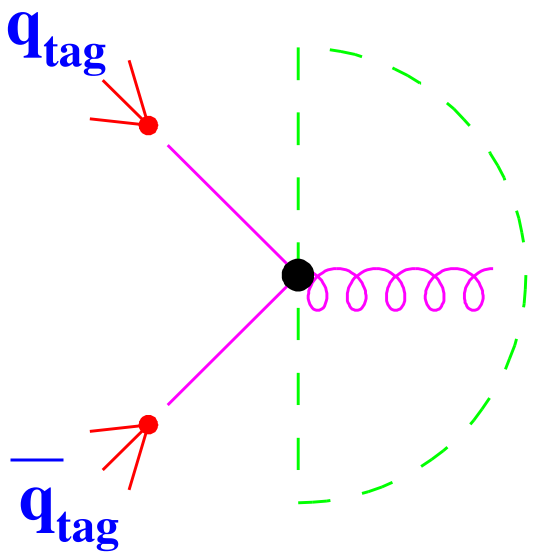
$\longrightarrow$ Jet energy ~ 5 GeV <u>too low</u> to observe a gluon-quark jet difference

$\longrightarrow$ Non-perturbative corrections are likely to be important at this scale

Unbiased gluon jets from Z^0 decays

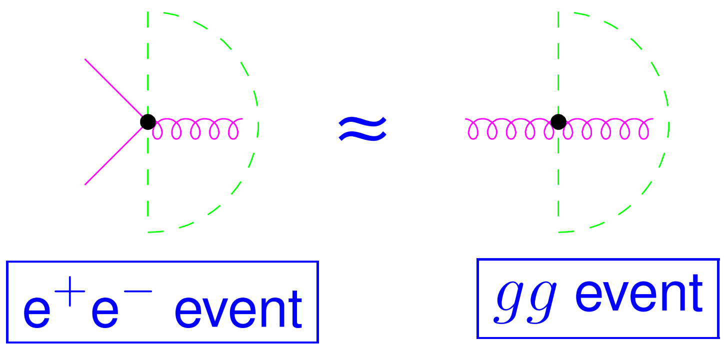
OPAL at LEP-1, e^+e^- $E_{c.m.} \approx$ 91 GeV (1996-1998)

$\longrightarrow$ Gluon jet hemispheres in $e^+e^- \rightarrow Z^0 \rightarrow q_{tag}\overline{q}_{tag}g_{incl.}$ events



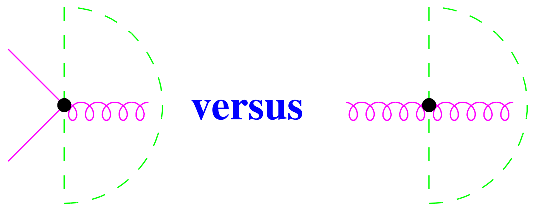
Gluon jet "$g_{incl.}$" defined by the particles in the hemisphere opposite to a hemisphere with two tagged quark jets (tagged quark jet is a b jet)

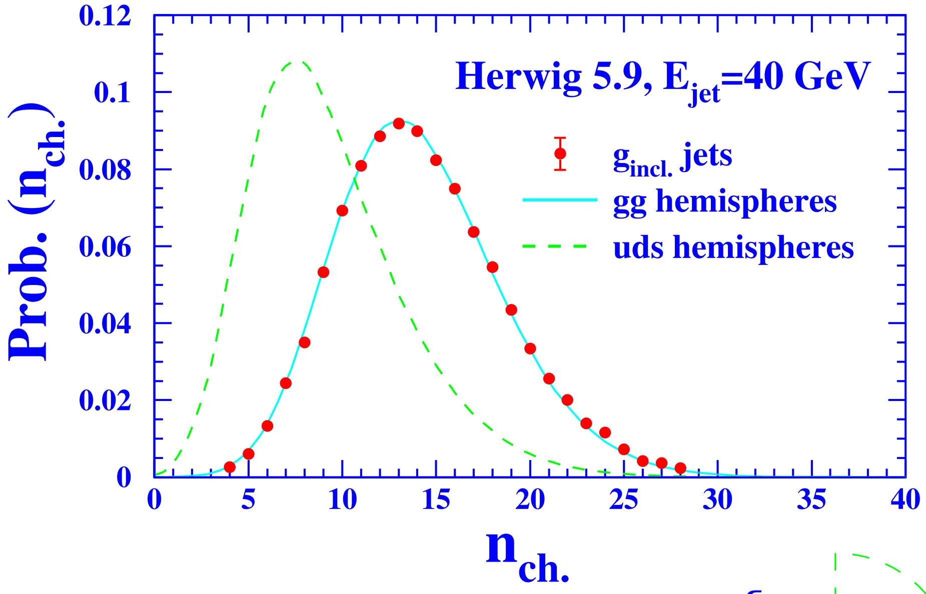
$\longrightarrow$ Invoke the equivalence of the $g_{incl.}$ and gg event hemispheres (exact in the limit of colinear q and $\overline{q}$)



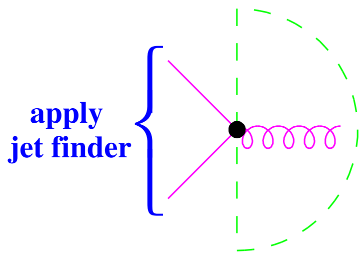
$\rightarrow$ Tested using the QCD Monte Carlo for which high energy gg production from a color singlet point source is possible !

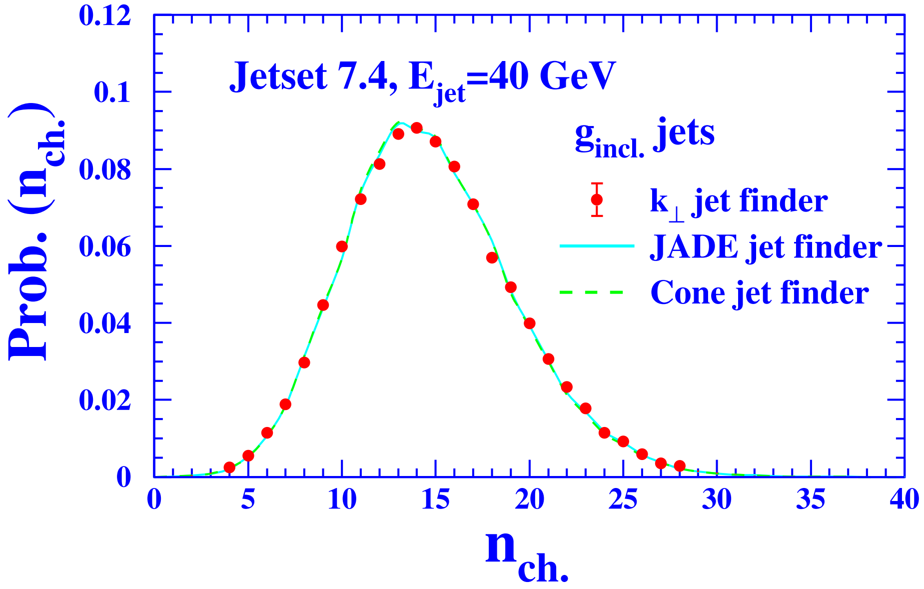
→ Properties of $g_{incl.}$ and gg hemispheres virtually identical · · ·





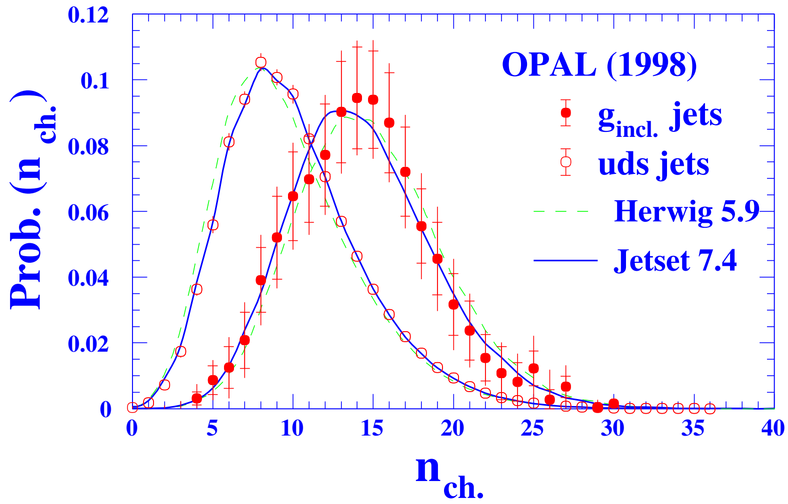
· · · and are independent of the choice of jet finder





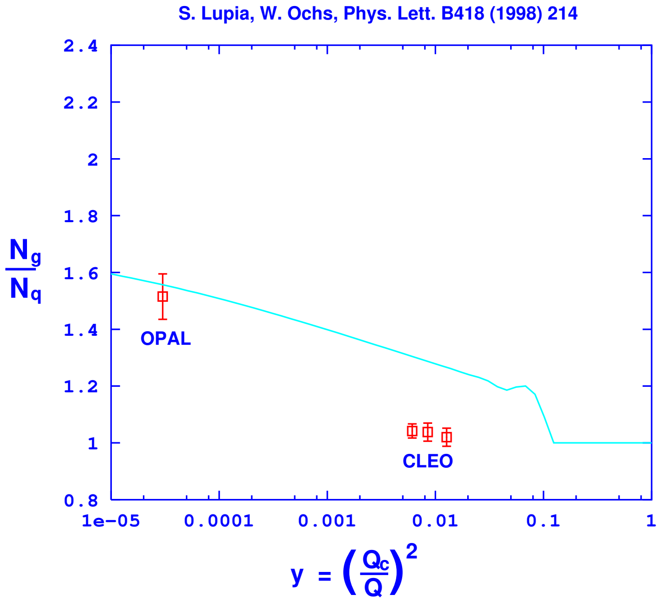
Multiplicity distribution $\longrightarrow g_{incl.}$ jets

4×10^6 Z^0 decays $\longrightarrow$ 440 selected $g_{incl.}$ jets (82% purity)



$$\boxed{r_{g/q}^{hadron}(40\,\mathrm{GeV}) = 1.514 \pm 0.039}$$

Compare the results of CLEO and OPAL to theory $\longrightarrow$



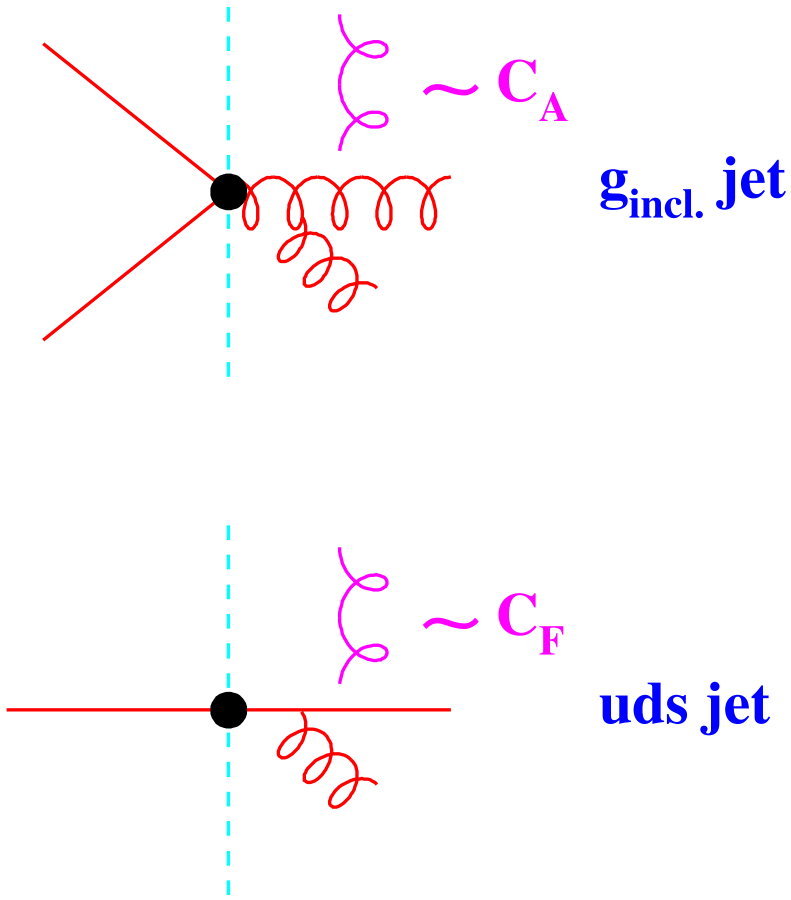
Measurement agrees with QCD prediction at the Z^0 scale

CLEO energies $\rightarrow$ non-perturbative effects are large

Multiplicity under the asymptotic condition for finite jet energies, $E_{particle} << E_{jet}$

$\longrightarrow$ Fulfilled by examining soft particles at large $p_\perp$ in the unbiased gluon and quark jets

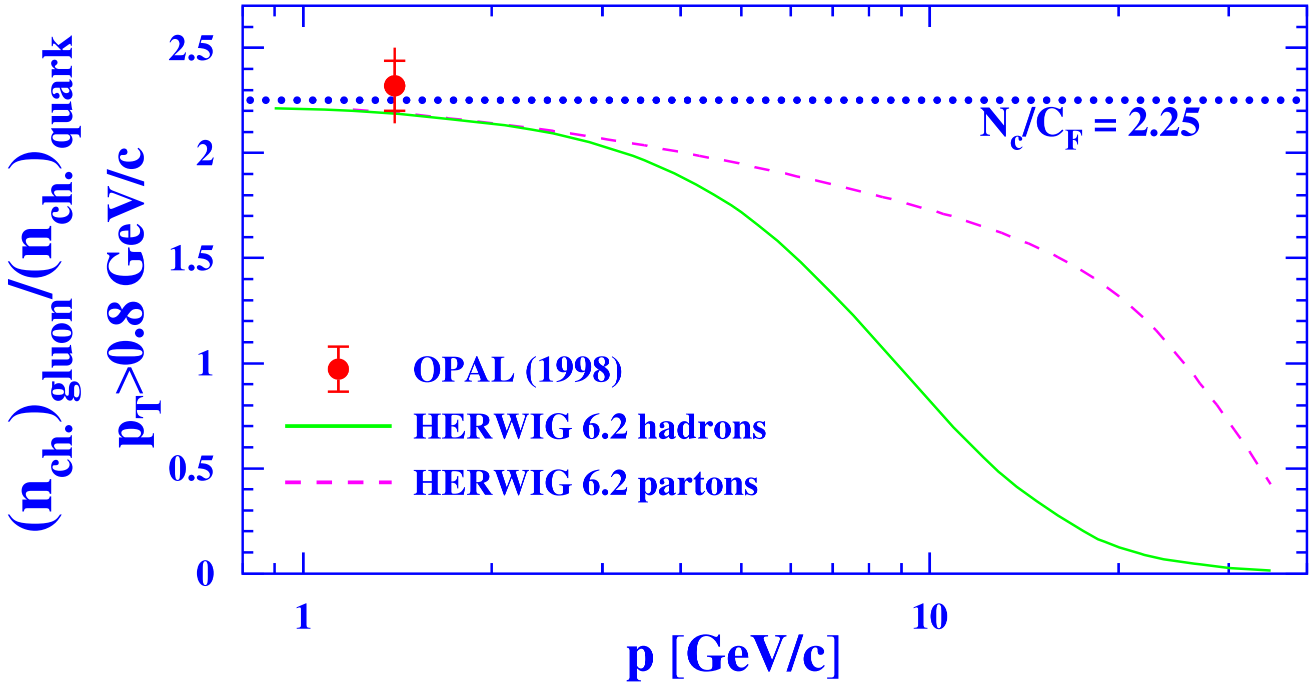
(V. Khoze, S. Lupia & W. Ochs, Eur. Phys. J. C5 (1998) 77)



Provides a means to measure C_A/C_F directly from a ratio of hadron multiplicities

$r_{g/q}$ for soft particles at large $p_\perp$

$p_\perp > 0.8$ GeV/c $\longrightarrow$ $p_\perp < 0.8$ GeV/c dominated by hadronization, decays



Data ($p < 2$ GeV/c)	$\boxed{2.32 \pm 0.18}$
Herwig hadrons, 91 GeV	2.21
Herwig partons, 91 GeV	2.23
Herwig hadrons, 10 TeV	2.24
Herwig partons, 10 TeV	2.25
Jetset partons, 91 GeV, $C_A = C_F$	1.00

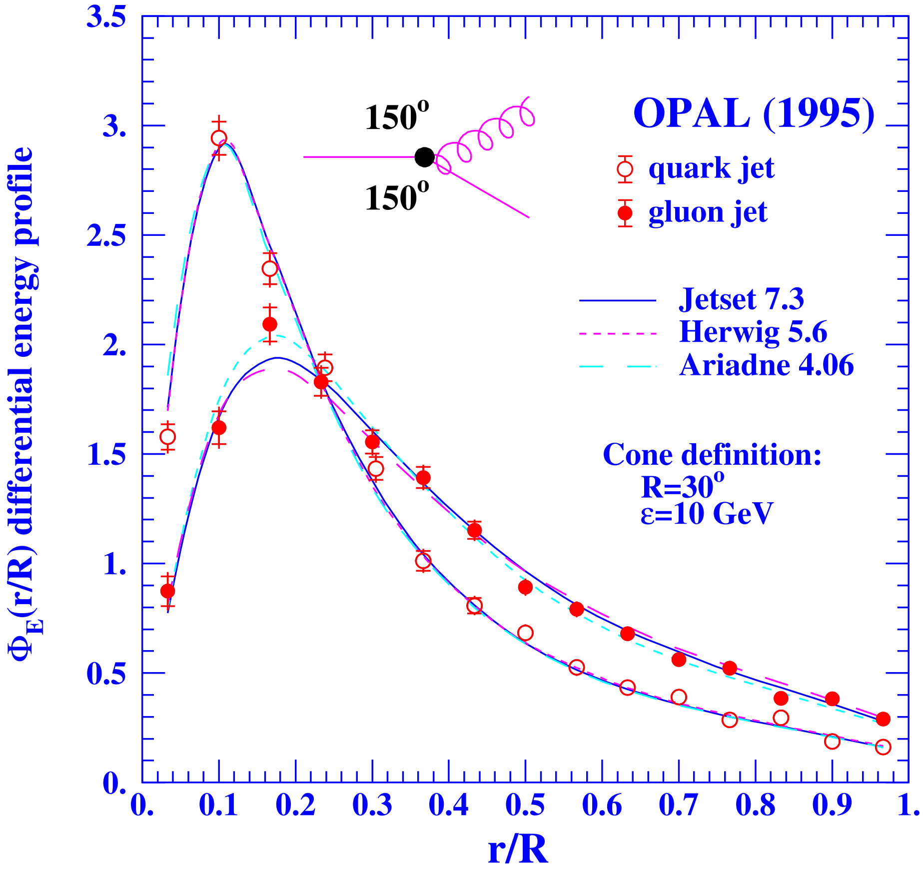
$\longrightarrow$ Quantitative verification of the QCD prediction from 1976 !

Width difference between gluon and quark jets

Gluon jets are predicted to be <u>less collimated</u> than quark jets

⟶ a consequence of the greater radiation of soft gluons in a gluon jet compared to a quark jet

⟶ the fraction of a jet's visible energy close to the jet axis is larger for quark jets than for gluon jets



⟶ The prediction is confirmed experimentally

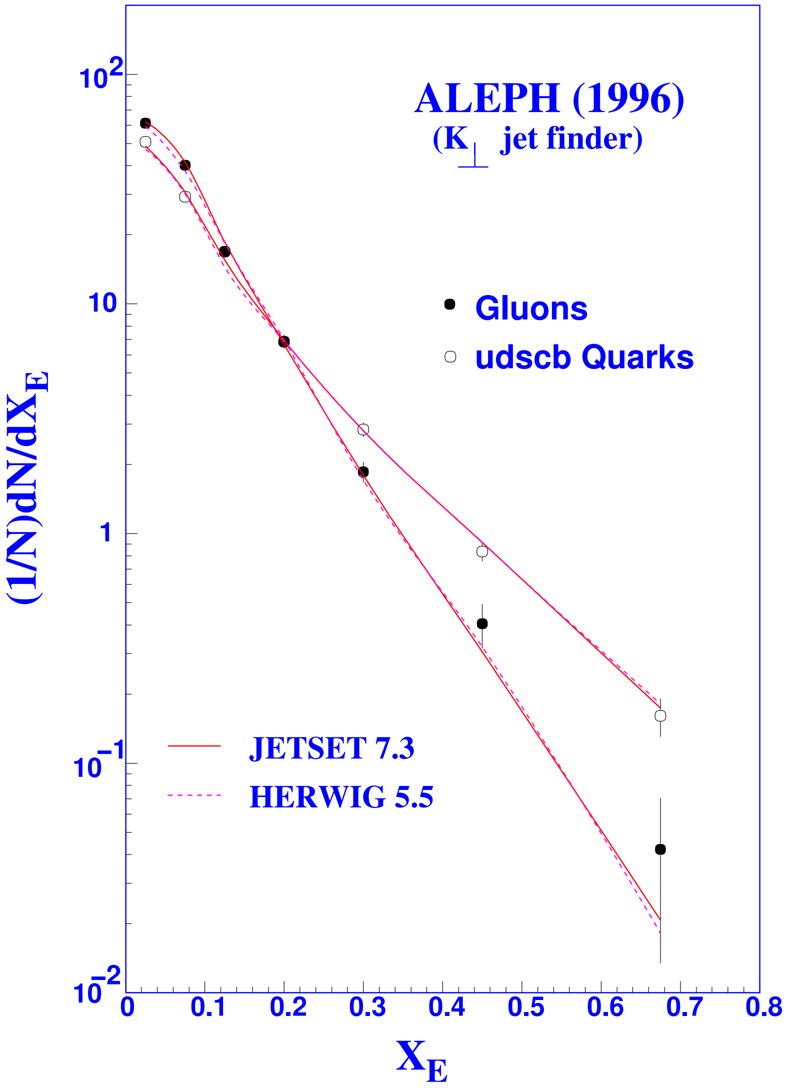
⟶ ∼30% of the quark jet's energy is within 4° of the jet axis compared to only $\sim 17\%$ for the gluon jets

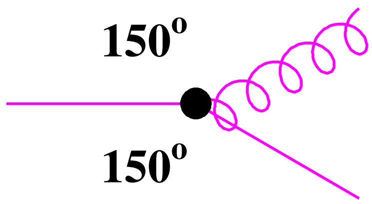
⟶ QCD Monte Carlos agree well with data

Fragmentation function difference $\longrightarrow$

$$\frac{1}{\sigma_{TOT}}\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,\mathrm{x_E}} \qquad x_E = \frac{2\,E_{particle}}{\sqrt{s}}$$

$\longrightarrow$ The larger multiplicity of gluon jets implies their fragmentation function is softer

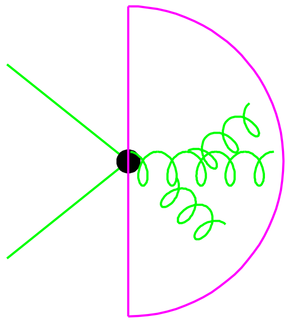




$\longrightarrow$ Effect confirmed experimentally

• Unbiased gluon jets from Z^0 decays

⟶ Provide a quantitative test of the QCD prediction for $r_{g/q}$



⟶ Fixed scale Q ≈ 40 GeV

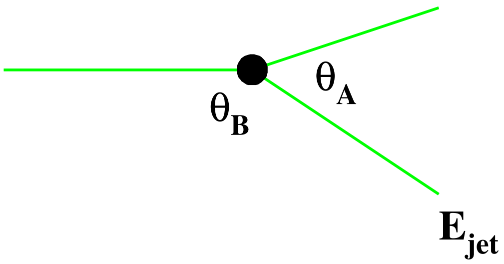
Can differences between gluon & quark jets be used for quantitative tests of QCD at other scales ??

Can we use biased jets from Z^0 decays ??

ANSWER ⟶ YES, if the appropriate scale is chosen for the jets

• QCD coherence

⟶ Evolution of parton cascade depends on a transverse momentum-like "hardness scale":

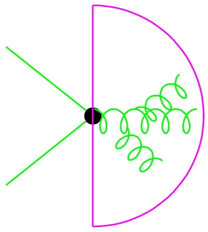


⟶ $$\boxed{\kappa = E_{jet} \sin\left(\frac{\theta_{min}}{2}\right)}$$

$$\theta_{min} = \min\,(\theta_A, \theta_B)$$

(Dokshitzer, Khoze, Ochs · · ·)

Corresponds to $\kappa = E_{jet}$ for $\theta \longrightarrow 180^\circ$



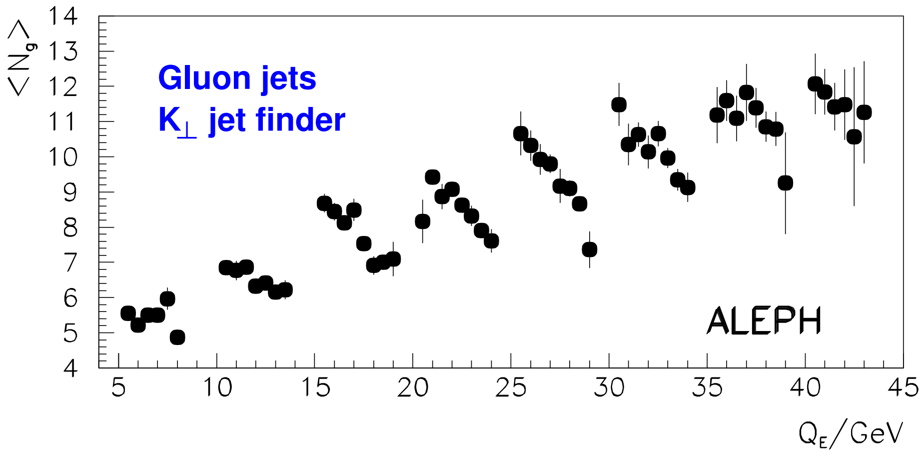
⟶ κ is more appropriate for jets in a general 3-jet topology than the jet energy

Gluon jet $\langle n_{ch.} \rangle \longrightarrow$ energy versus κ value

ALEPH at LEP-1 (1997) $\longrightarrow$

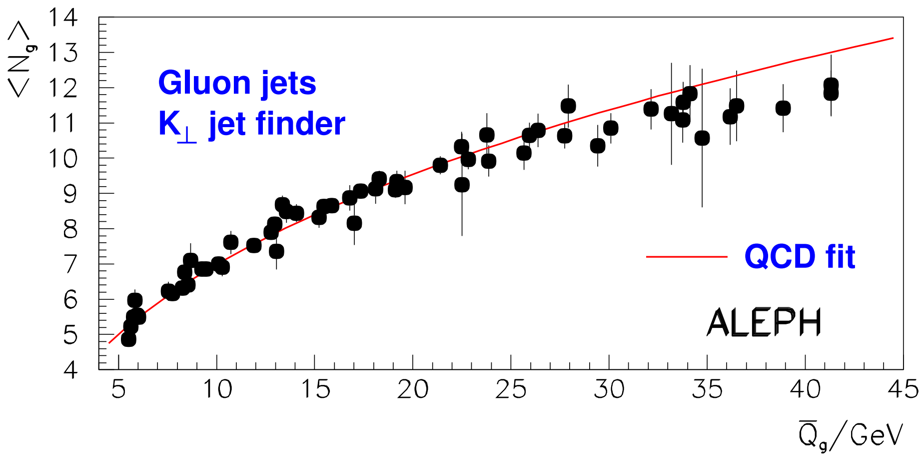
(1) Gluon multiplicity in general 3-jet events versus jet energy

$\longrightarrow$ Each of the eight bands corresponds to jets with the same energy but with a different angle to the nearest jet



$\longrightarrow$ The jet multiplicity in biased events depends critically on the event topology, not just the jet energy

(2) Gluon multiplicity in 3-jet events versus jet κ value

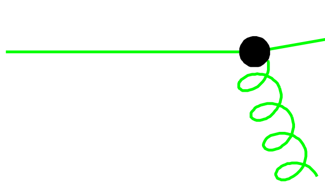


$\longrightarrow$ κ provides a much more meaningful scale for jets embedded in a general 3-jet event topology

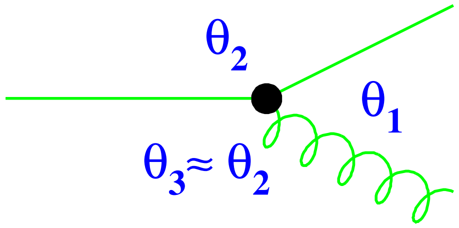
Scale dependence of gluon and quark jet fragmentation functions

DELPHI at LEP-1 (1999) $\longrightarrow$

- Select general 3-jet events:



and also 1-fold symmetric Y events:



using the $k_\perp$ and Cambridge jet finders

- Use b-tagging to identify *gluon jets* and light *udsc quark* events

- Measure the fragmentation functions

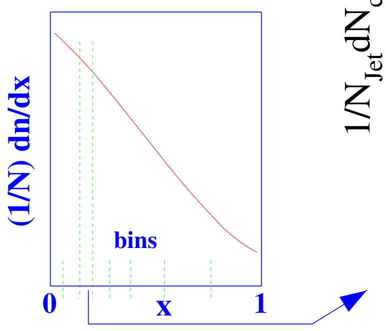
$$\longrightarrow \mathrm{D}(x_E,\mathrm{Q}) = \frac{1}{N}\frac{\mathrm{d\,n}}{\mathrm{d}x_E} \quad ; \quad x_E = \frac{2E}{\sqrt{s}}$$

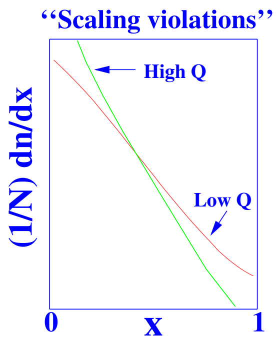
of the two lower energy jets (one gluon jet, one quark jet) versus the scale $\mathrm{Q} = \kappa$

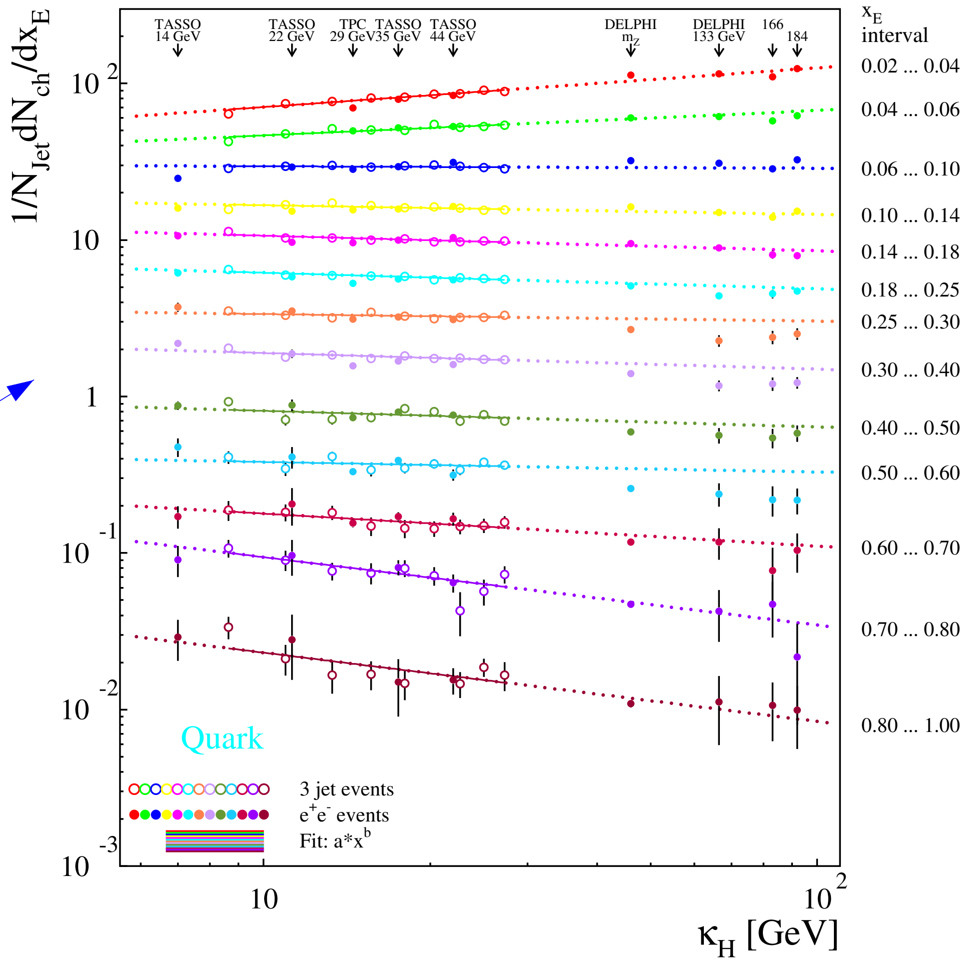
Quark jet fragmentation function versus κ

DELPHI (1999) $\longrightarrow$

Compare the quark jet ff from 3-jet events $(Q = \kappa)$ to 0.5 $\times$ the ff function from unbiased quark jets (hemispheres of e^+e^- events, $Q = E_{c.m.}/2$)





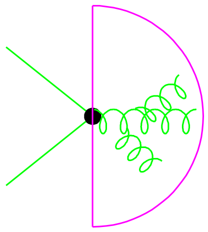


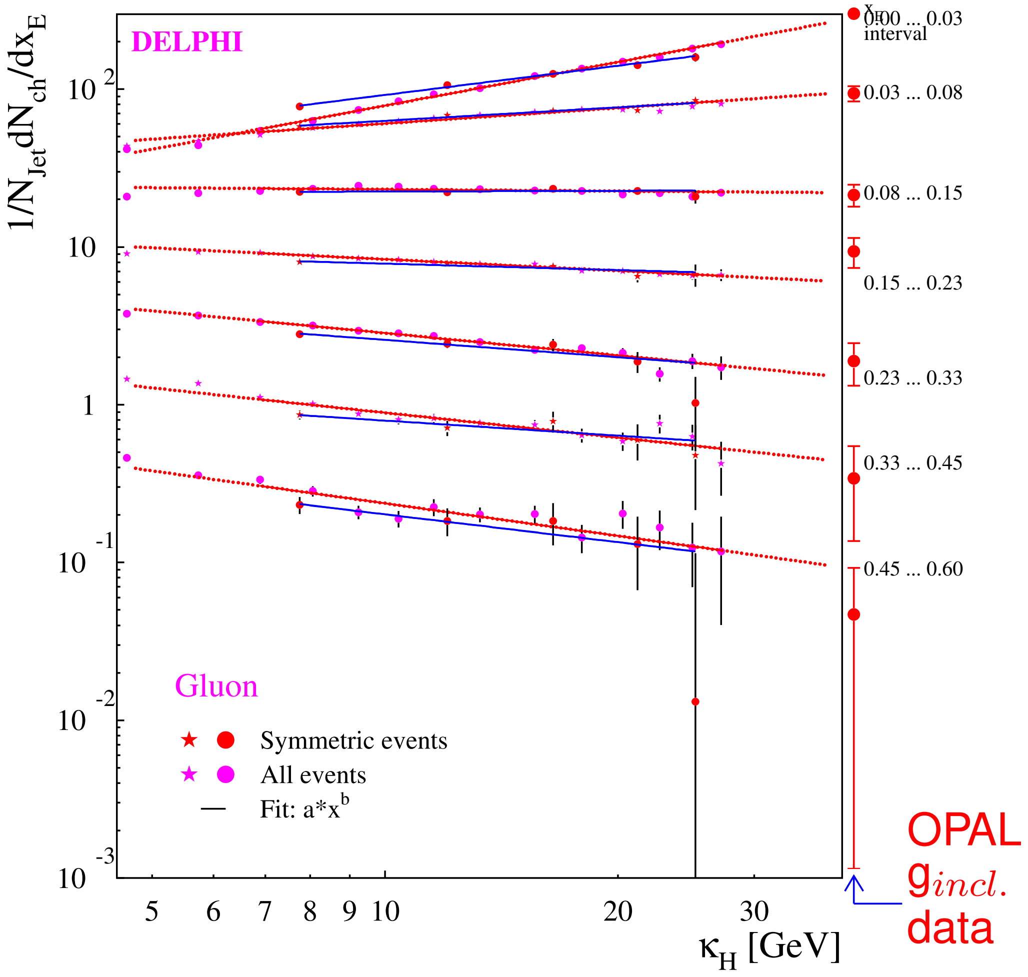
$\longrightarrow$ The quark jet results from 3-jet events correspond well with the results from the unbiased jets

$\longrightarrow$ Another good indication that κ is a meaningful scale for jets in a general 3-jet topology

Gluon jet fragmentation function versus κ

Compare the gluon jet ff from 3-jet events ($Q = \kappa$) to the ff function from the unbiased gluon jet hemispheres $Q \sim 40$ GeV





$\longrightarrow$ The correspondence of the results between gluon jets from 3-jet events and the unbiased gluon jets seems reasonable

$\boxed{\text{QCD}}$ $\longrightarrow$ Scale dependence of fragmentation functions D_g and D_q described by *DGLAP evolution equations*

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi

Leading order evolution $\longrightarrow$

$$\frac{dD_g(x_E;Q)}{d\ln Q} = \frac{\alpha_S(Q)}{2\pi}\int_{x_E}^{1}\frac{dz}{z}[P_{g\to gg}(z)D_g(\tfrac{x_E}{z};Q)+ P_{g\to q\bar{q}}(z)D_q(\tfrac{x_E}{z};Q)]$$

$$\frac{dD_q(x_E;Q)}{d\ln Q} = \frac{\alpha_S(Q)}{2\pi}\int_{x_E}^{1}\frac{dz}{z}[P_{q\to qg}(z)D_q(\tfrac{x_E}{z};Q)+ P_{q\to gq}(z)D_g(\tfrac{x_E}{z};Q)]$$

$\longrightarrow$ Splitting kernels $\boxed{P_{g\to gg}\sim \mathrm{C_A}}$, $\boxed{P_{q\to qg}\sim \mathrm{C_F}}$

$\longrightarrow$ Parametrize D_g and D_q at Q = κ = 5.5 GeV :

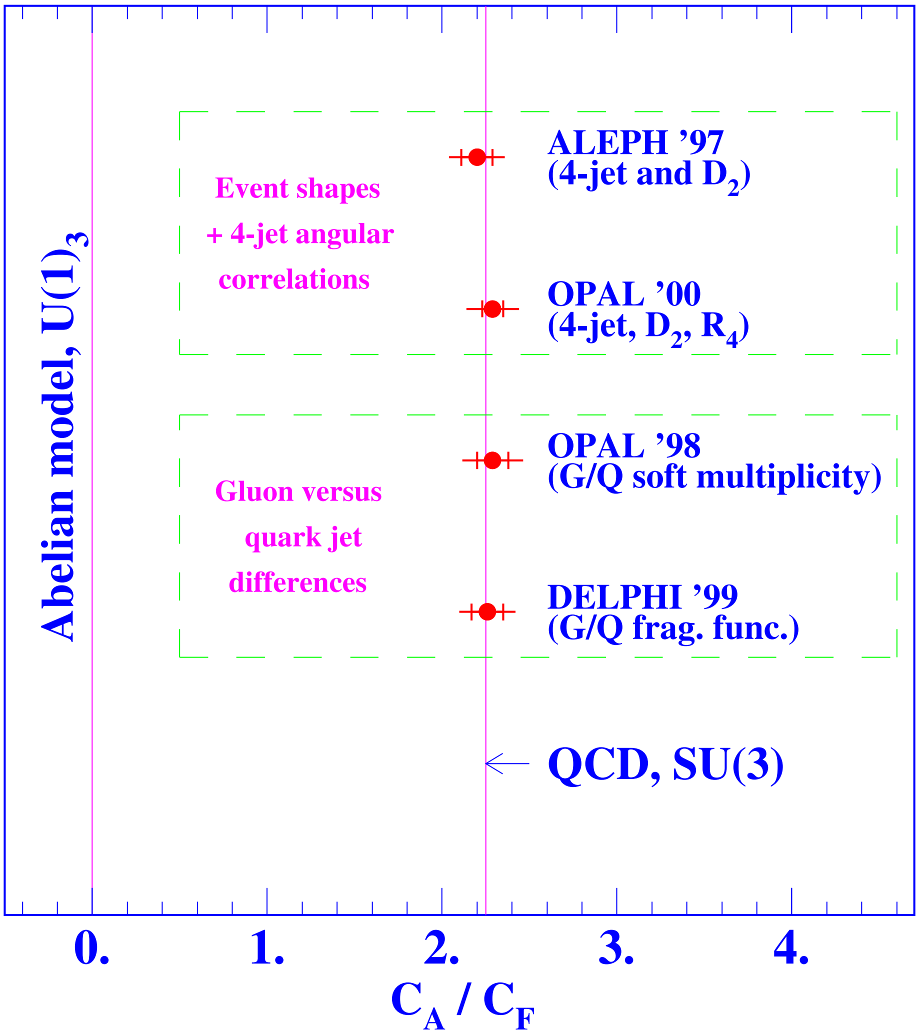
$$D(x_E) = a x_E^b (1-x_E)^c \exp(-d\ln^2 x_E)$$

$\longrightarrow$ Perform *simultaneous fit* of $(a,b,c,d)_p$; p = q,g, C_A and Λ_{QCD} using 1st order DGLAP evolution

$\longrightarrow$ C_A = 2.97 ± 0.12, Λ_{QCD} = 0.40 ± 0.11 GeV

$$\boxed{\frac{C_A}{C_F} = 2.26 \pm 0.16}$$

$C_A/C_F \longrightarrow$ Summary



$\longrightarrow$ Precision of the measurement of C_A/C_F from differences between gluon and quark jets is similar to that from the 4-jet angular correlations & event shapes

$\longrightarrow$ Precise measurements using very different techniques

$\longrightarrow$ Direct verification of the SU(3) gauge group underlying strong interactions !

(III) Coherence and LPHD

- Coherence $\longrightarrow$ QCD interference effects affecting the multiplicity and radiation pattern of soft gluons

  General feature $\longrightarrow$ Coherence reduces the multiplicity of soft gluons because of destructive interference

- LPHD $\longrightarrow$ Local Parton-Hadron Duality:
  The conjecture that the angular and energy distributions of soft hadrons are a direct reflection of the corresponding underlying distributions of soft partons

$\longrightarrow$ Three phenomenological parameters in this approach

(1) Λ_{QCD} (effective value of α_S),

(2) $Q_0 \sim m_\pi$ to terminate the perturbative shower

(3) An overall normalization constant K to relate the hadron and parton level distributions

$\longrightarrow$ LPHD was used implicitly for the comparison of $r_{g/q}$ in unbiased gluon and quark jet results with the analytic results, shown earlier

Coherence studies in e^+e^-

(1) Inter-jet multiplicities ("String effect")

(2) Event particle multiplicity versus $\sqrt{s}$

(3) Shape of the particle momentum distribution "Hump-backed spectra"

(4) Heavy versus light quark jet multiplicity

Due to lack of time I will not discuss:

(5) Two particle correlations (azimuthal, momentum, particle-particle correlations (PPC))

There is much additional circumstantial evidence for the existence of coherence effects and the validity of LPHD

$\longrightarrow$ Appropriateness of the $\kappa = E_{jet} \sin\left(\frac{\theta_{\min}}{2}\right)$ scale for jets

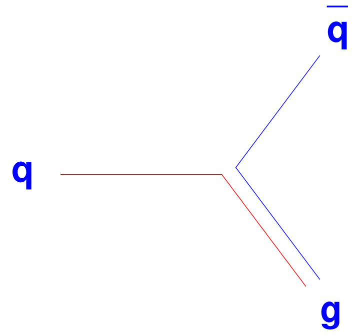
$\longrightarrow$ Near perfect agreement of parton level calculations with data for the ratio $r_{g/q}$ and higher moments of multiplicity in unbiased gluon and quark jets

The evidence taken together provides a rather convincing case for the existence of coherence effects, and by implication the relevance of LPHD for many distributions, providing a basic and comprehensive test of perturbative QCD in the soft domain

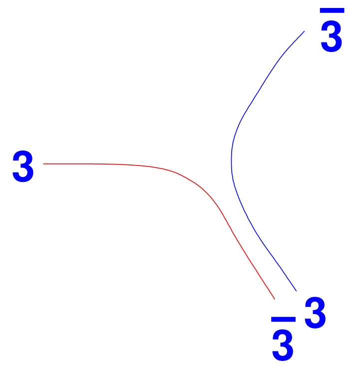
(1) Inter-jet multiplicities: The String effect

First predicted in the context of the LUND string model of hadronization (ca. 1979)

⟶ Color flow in 3-jet events connects the q and $\overline{q}$ with the g



⟶ Equivalent to two color dipoles



⟶ Radiation from one dipole interferes with that from the other

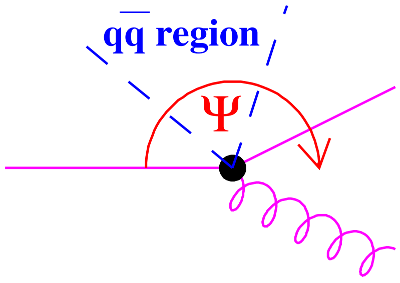
⟶ *suppression* of soft gluons in the region between the q and $\overline{q}$ relative to the region between the q (or $\overline{q}$) and g

⟶ A depletion of multiplicity in the $q\overline{q}$ region compared to the qg (or $\overline{q}g$) region

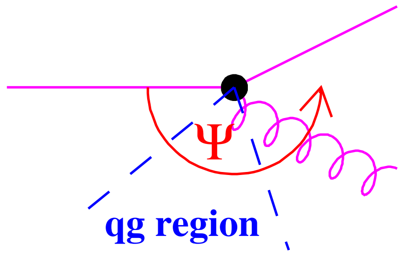
⟶ First studied experimentally by the JADE Collaboration at PETRA (1981)

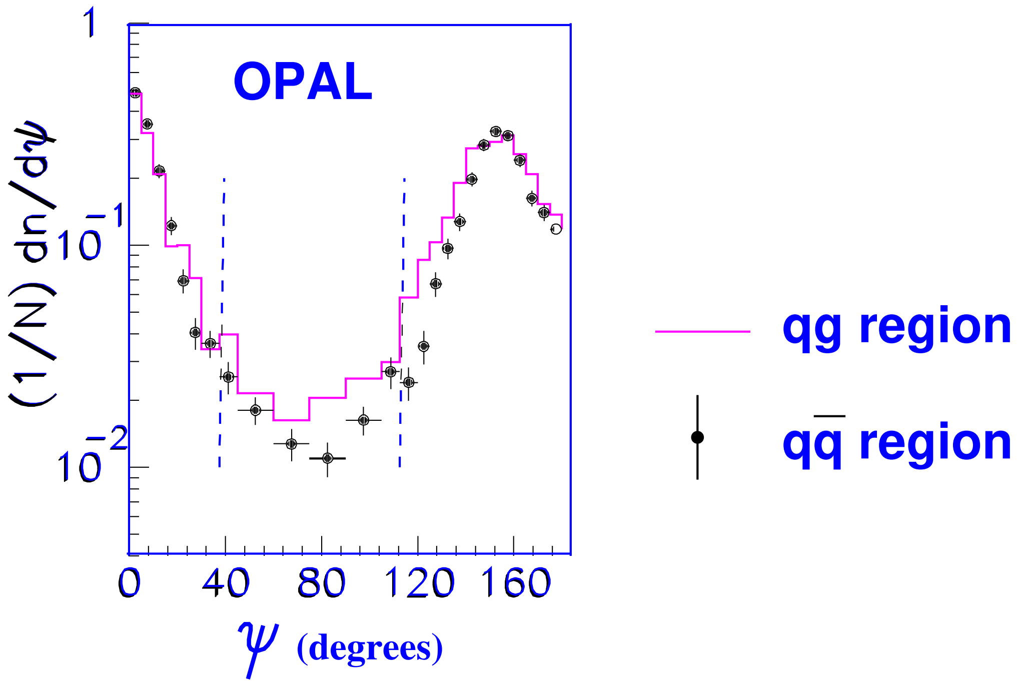
OPAL (1991) $\longrightarrow$ Charged particle multiplicity flow in Y events

(First study at LEP)



VERSUS





$\longrightarrow$ Clear depletion in $q\bar{q}$ region compared to qg region

$\longrightarrow$ Integrate over inter-jet region ($\sim$ 25-75% of the range between the peaks $\longrightarrow$ dashed vertical lines)

$\longrightarrow$ $n_{q\bar{q}}$ region $/ n_{qg}$ region = 1.66 ± 0.09 (data)

$= 1.54$ (JETSET $\rightarrow$ coherence & string hadronization)

$= 1.02$ (COJETS $\rightarrow$ no coherence & independent hadronization)

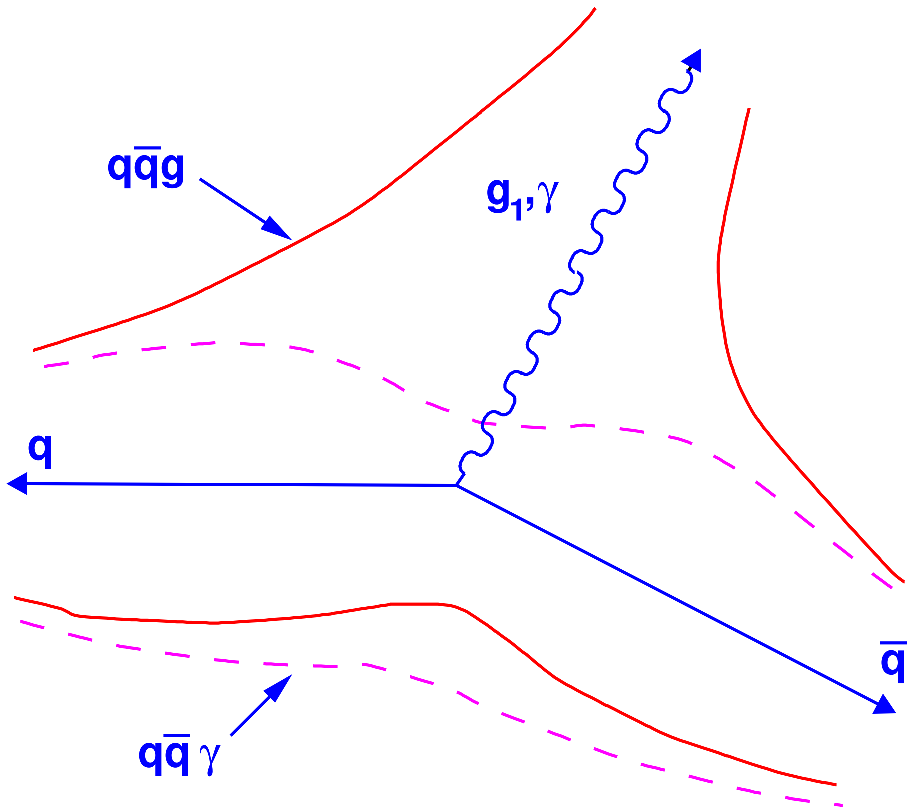
$\longrightarrow$ JETSET and COJETS both describe quark gluon jet differences well

$\longrightarrow$ The data provide evidence for coherence

String effect using $q\bar{q}\gamma$ versus $q\bar{q}g$ events

⟶ Azimov, Dokshitzer, Khoze & Troyan (1985)

⟶ Select 3-jet ($q\bar{q}g$) and radiative 2-jet $q\bar{q}\gamma$ events with similar kinematics



⟶ Examine particle multiplicity in the $q\bar{q}$ region

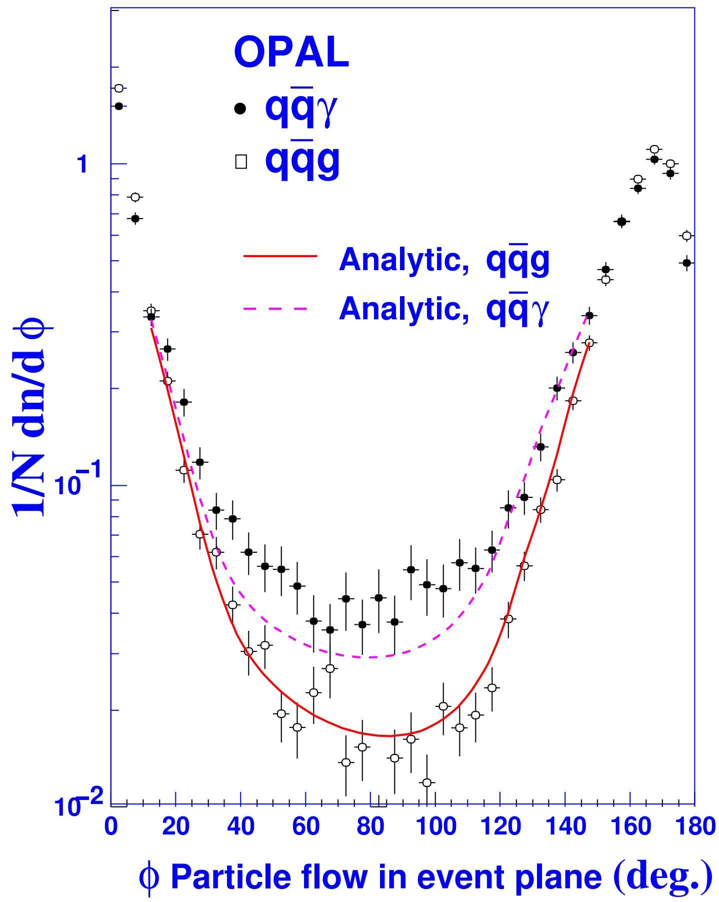
⟶ Coherence & LPHD predict a smaller particle density in this region for $q\bar{q}g$ than for $q\bar{q}\gamma$ due to soft gluon interference between the two color dipoles

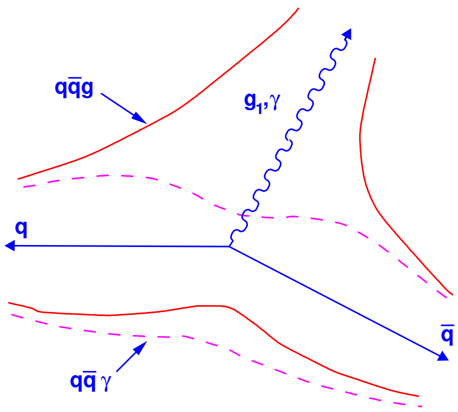
⟶ $q\bar{q}\gamma$ events have only one dipole ⟶ no interference

OPAL (1995) $\longrightarrow$

$\longrightarrow$ Measure particle flow in the $q\bar{q}$ region in $q\bar{q}g$ and $q\bar{q}\gamma$ events

$\longrightarrow$ Compare data to the leading order prediction for the soft gluon radiation pattern in the corresponding events





$\longrightarrow$ The predicted reduction in particle density for $q\bar{q}g$ events compared to $q\bar{q}\gamma$ events is observed

$\longrightarrow$ The magnitude of the measured effect is very similar to that predicted by the analytic prediction

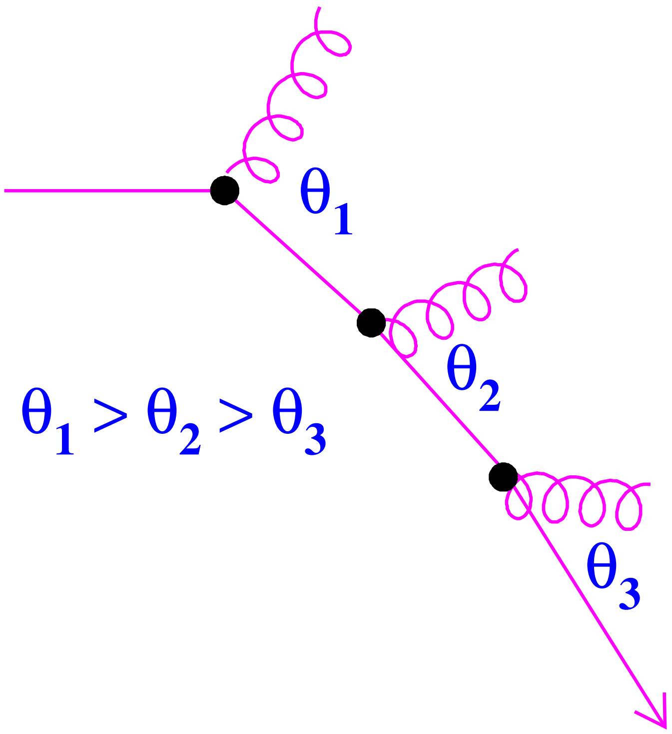
$\longrightarrow$ Provides more circumstantial evidence for coherence & LPHD as the origin of the string effect

Multiplicity within jets $\longrightarrow$ Intra-jet multiplicities & angular ordering

Coherence in the parton shower $\longrightarrow$

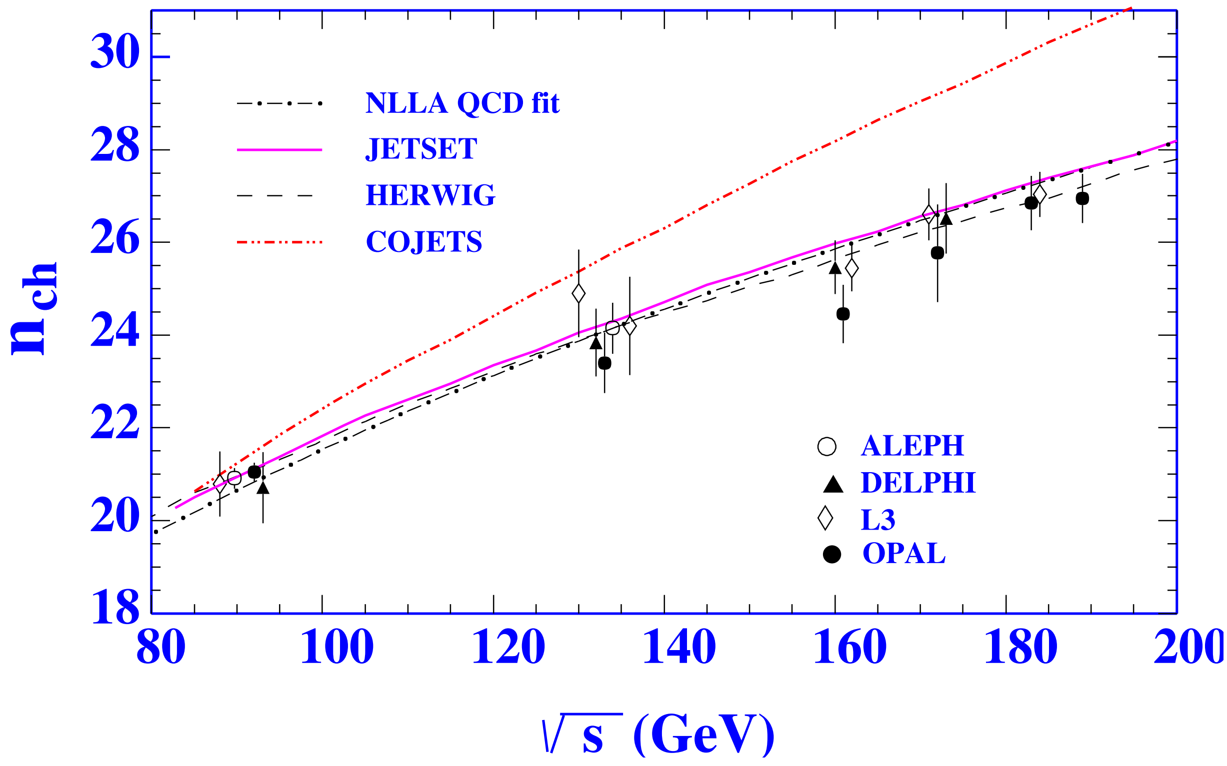
$\longrightarrow$ Destructive interference leads to a reduction in the phase space available for gluon emission

$\longrightarrow$ The principal manifestation is the angular ordering of parton emissions



$\longrightarrow$ This reduction in phase space corresponds to a smaller overall multiplicity and a smaller growth of multiplicity with $\sqrt{s}$ than in the absence of coherence

(2) Event particle multiplicity versus $\sqrt{s}$



Models with coherence (PYTHIA, HERWIG, ARIADNE)

$\longrightarrow$ Energy scaling is consistent with data

Model without coherence (COJETS)

$\longrightarrow$ Growth of multiplicity with $\sqrt{s}$ is too large, as expected from the lack of suppression of phase space

All four MCs are tuned to the 91 GeV data

$\longrightarrow$ They provide essentially equivalent descriptions for global event properties like multiplicity

(3) Suppression of soft particle multiplicity: the "Hump-backed" spectra

→ Angular ordering within jets suppresses the total event multiplicity (as discussed above)

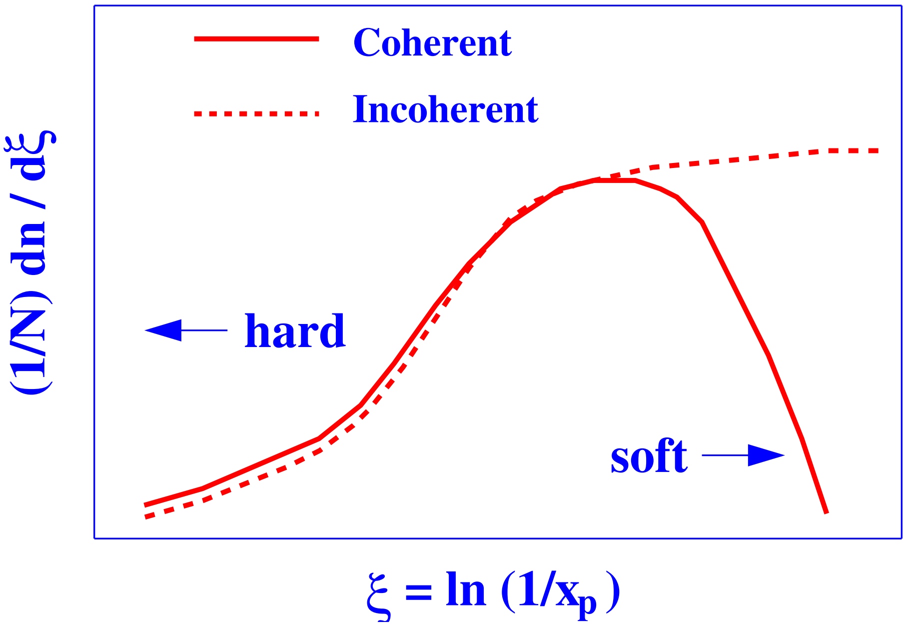
→ The suppression mostly affects soft particles

→ Examine the differential momentum spectra of particles, not just the mean value $\langle n \rangle$

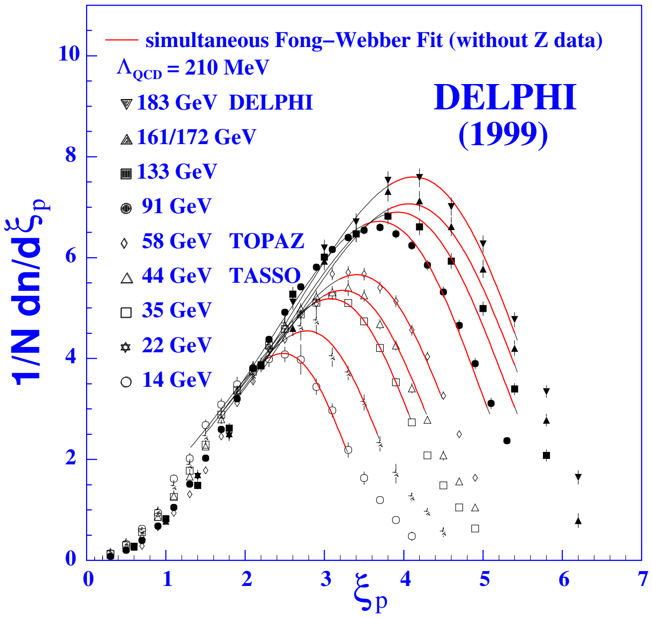
→ QCD predicts an approximately Gaussian shape for the differential particle momentum spectrum when expressed using the variable

$$\xi \equiv \ln\left(\frac{1}{x_p}\right) \quad \text{with} \quad x_p = \frac{2p}{\sqrt{s}}$$

Fong & Webber (1989); Dokshitzer, Khoze & Troyan (1992)

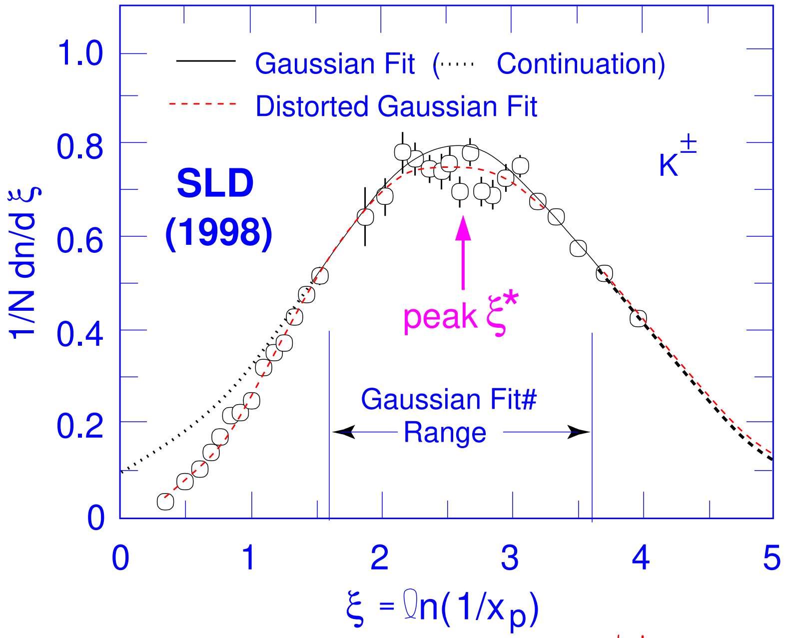


$\longrightarrow$ The data at the hadron level are indeed observed to follow a shape in close agreement with the parton level prediction of a "distorted Gaussian"



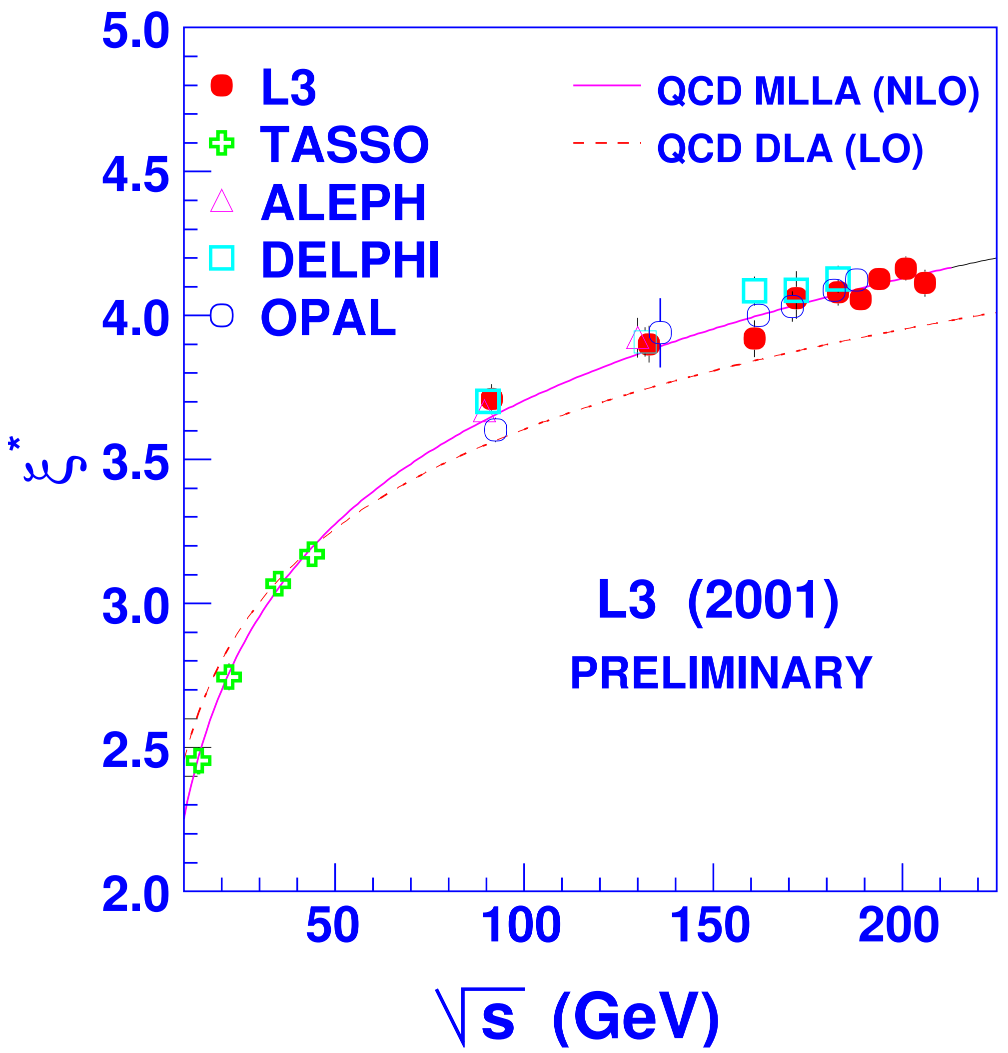
$\longrightarrow$ Two free parameters: $\Lambda_{\rm QCD}$ and normalization $K(\sqrt{s})$

($Q_0 = \Lambda_{\rm QCD} \longrightarrow$ "limited spectrum approximation")



$\longrightarrow$ The position of the peak, denoted ξ^*, is <u>independent of the normalization</u> and depends on $\Lambda_{\rm QCD}$ only

Energy evolution of the peak position: ξ^* versus $\sqrt{s}$



→ The one parameter NLO fit agrees well with the data

→ The agreement of the LO prediction with data is <u>much worse</u>, suggesting that the agreement of the NLO result is not "trivial"

→ Similar results have been obtained at HERA and the TEVATRON

(4) Heavy versus light quark jet multiplicity

- B quark jets at the hadron level
  - $\longrightarrow$ Larger multiplicity than light (uds) quark jets
  - $\longrightarrow$ B hadrons are heavy and yield a large decay multiplicity to stable particles
- For scale $E \approx m_Z$ (LEP/SLC:)

$$\langle n_{ch.} \rangle = 23.2 \pm 0.5 \text{ (b events)}$$

$$= 20.2 \pm 0.4 \text{ (uds events)}$$

$$\boxed{\delta_{b\ell}^{ch.}(E = m_Z) \equiv \langle n_{ch.} \rangle_b - \langle n_{ch.} \rangle_{uds} \approx 3}$$

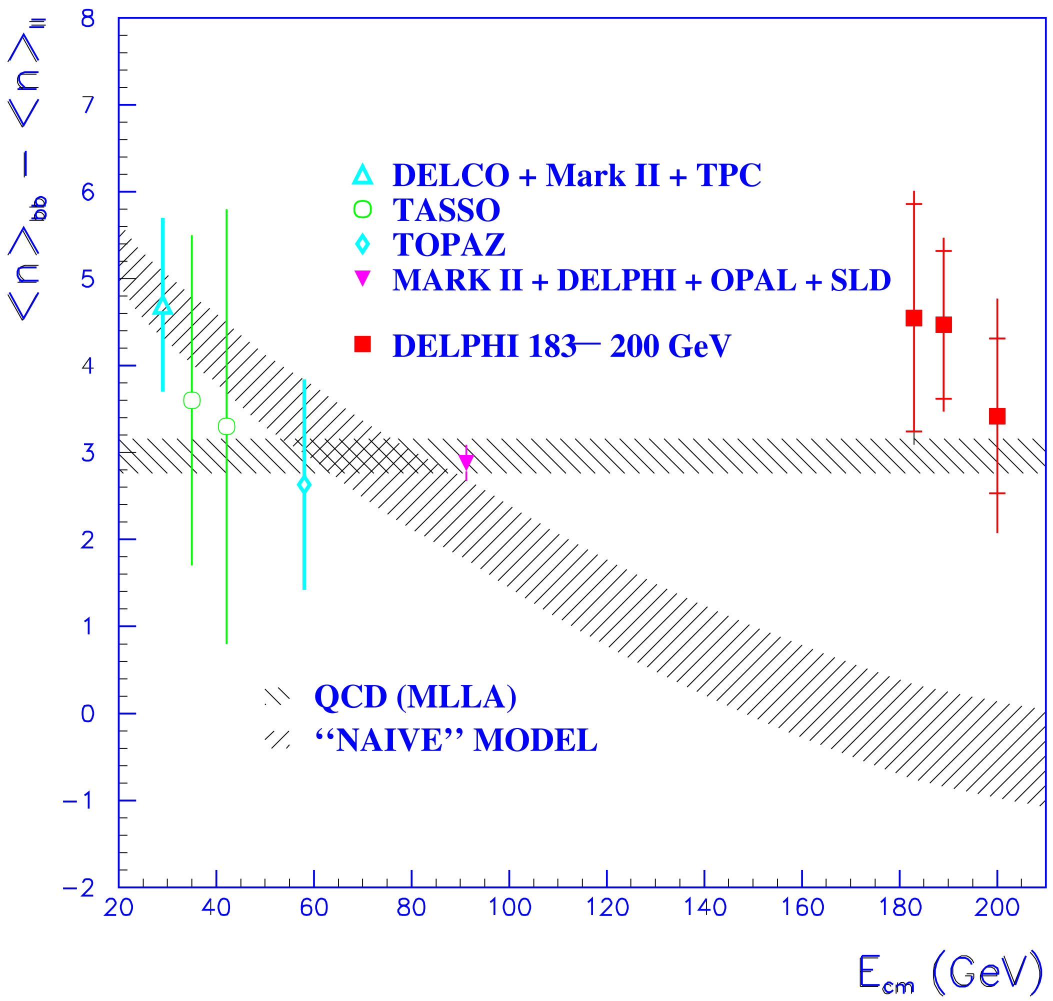
- As $\boxed{E \to \infty}$ a b quark jet should behave more and more like a light quark jet
  - $\longrightarrow$ The B hadron decay multiplicity will comprise a smaller component of total jet multiplicity
  - $\longrightarrow$ Naively, would expect

$$\underline{\delta_{b\ell}^{ch.}(E) \to 0} \quad \text{as} \quad \underline{E \to \infty}$$

However, this is NOT what is predicted by QCD !!
Because of differences in coherence effects between b and uds events, QCD predicts that $\delta_{b\ell}$ is

ENERGY INDEPENDENT

DELPHI (2000) $\longrightarrow$



$\longrightarrow$ $\delta_{b\ell}^{hadron}$ is observed to be approximately constant, in agreement with the QCD prediction and in contrast to the naive expectation

$\longrightarrow$ The result from LEP-1 $\boxed{\delta_{b\ell}^{hadron} \approx 3}$ is somewhat lower than the prediction of 3.7 (however the prediction is only approximate)

$\longrightarrow$ Yet more evidence for the relevance of coherence and LPHD !

(IV) Identified particles

$\longrightarrow$ Much work done at PEP & PETRA (E$_{\rm c.m.} \approx 30 - 35$ GeV)

$\longrightarrow$ LEP/SLC results more precise and more extensive due to the larger data samples ($\sim 4 \times 10^6$ events versus $\sim 10^5$)

Mesons with measured production rates at LEP/SLC:

Ang. mom.	Spin	J^{PC}	
L $= 0$	S $= 0$	0^{-+}	$\pi^\pm,\ \pi^0,\ \eta,\ \eta',\ K^0,\ K^\pm,$ $D^0,\ D^\pm, D^\pm_S$
L $= 0$	S $= 1$	1^{--}	$\rho^\pm,\ \rho^0,\ \omega,\ \phi,\ K^{*0},\ K^{*\pm},$ $D^{*\pm}, D^{*\pm}_S,\ J/\Psi,\ \Psi(2S),\ B^*$
L $= 1$	S $= 0$	1^{+-}	$\longrightarrow$ none observed
L $= 1$	S $= 1$	0^{++}	$f_0(980),\ a_0(980)$
		1^{++}	$\longrightarrow$ none observed
		2^{++}	$f_2(1270),\ f_2'(1525)$
L $= 1$			B^*_J (multiplet(s) not identified)

Baryons with measured production rates at LEP-1:

Ang. mom.	Spin	J^P	
L $= 0$	S $= 1/2$	$(1/2)^+$	p, Λ, Σ^+, Σ^-, Σ^0, Ξ^-, Λ_C^+
L $= 0$	S $= 3/2$	$(3/2)^+$	Δ^{++}, $\Sigma(1385)^\pm$, $\Xi(1530)^0$, Ω^-
L $= 1$	S $= 1/2$	$(3/2)^-$	$\Lambda(1520)$

The measured hadron production rates can be used to extract basic information on the hadronization process

- $\longrightarrow$ Probability to produce strangeness from the vacuum
- $\longrightarrow$ Probability to produce baryon number from the vacuum
- $\longrightarrow$ Probability to produce spin from the vacuum
- $\longrightarrow$ Differences between gluon & quark jet hadronization
- $\longrightarrow$ Tests of models for baryon production

In addition, these measurements provide basic input to tune of QCD Monte Carlo event generators, used for many other studies

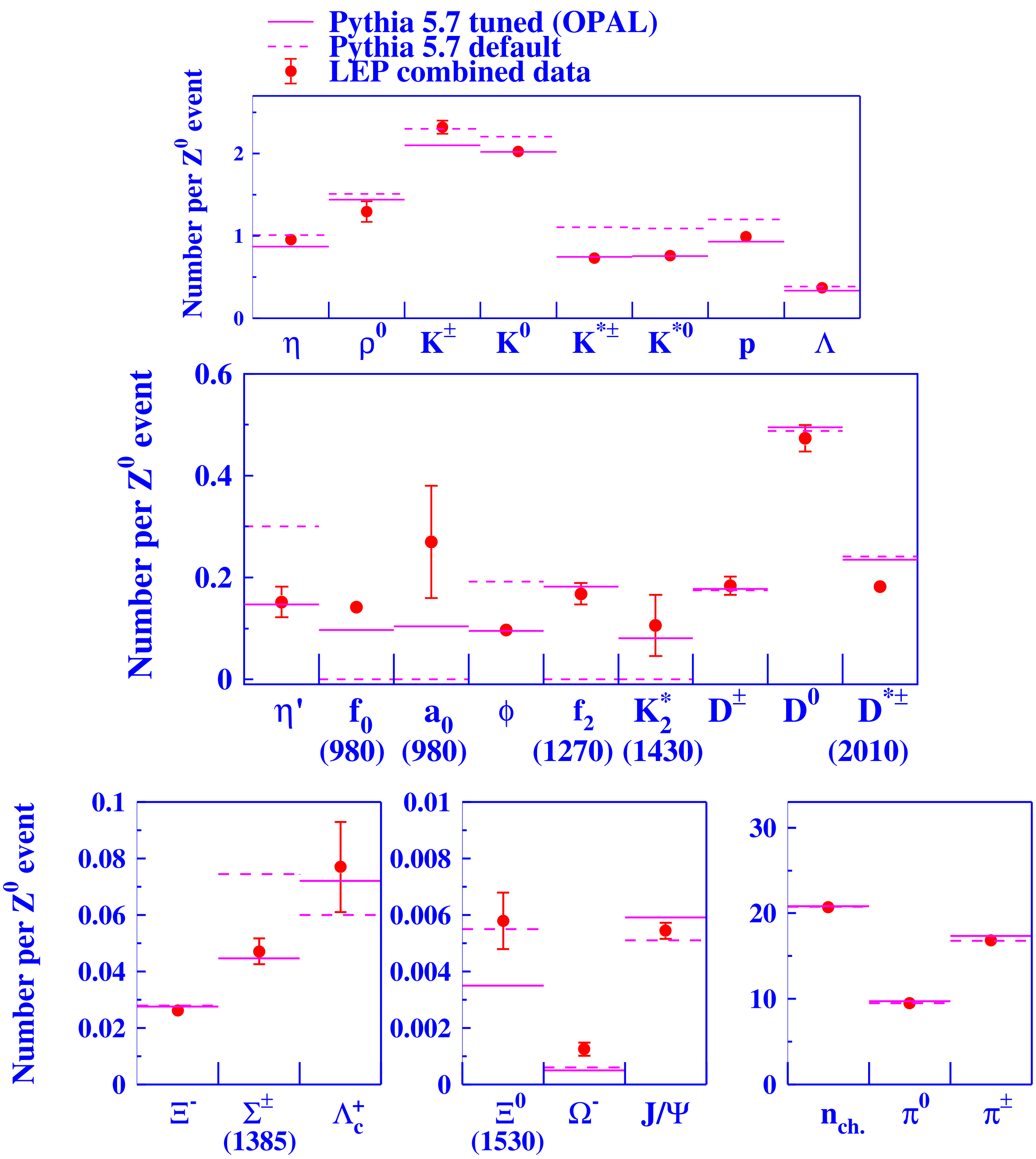
Parameters varied : Pythia 5.7 (ALEPH, DELPHI, OPAL)

	PARJ	ADO	AD	AO	DO	A	D	O
General:								
Λ	81	X						
Q_0	82	X						
σ_q	21	X						
"A"	41						X	
"B"	42			X				
ϵ_c	54	X						
ϵ_b	55	X						
Identified hadrons:								
$\mathcal{P}(qq/q)$	1	X						
$\mathcal{P}(s/u)$	2	X						
$\mathcal{P}[(us/ud)/(s/u)]$	3	X						
$\mathcal{P}$(diquarks,S=1)	4				X			
$\mathcal{P}$(u& d mesons,S=1)	11			X				
$\mathcal{P}$(s mesons,S=1)	12			X				
$\mathcal{P}$(c,b mesons,S=1)	13			X				
$\mathcal{P}$(L=1,S=0 mesons)	14							X
$\mathcal{P}$(L=1,S=1,J=0 mesons)	15							X
$\mathcal{P}$(L=1,S=1,J=1 mesons)	16							X
$\mathcal{P}$(L=1,S=1,J=2 mesons)	17			X				
Leading baryon suppr.	19		X					
η suppr.	25						X	
η' suppr.	26		X					

DELPHI: Replace PARJ(11)-(17) with $\mathcal{P}$(S,L,f); S=0,1; L=0,1; f=ud,s,c,b
(24 parameters), related by 2J+1

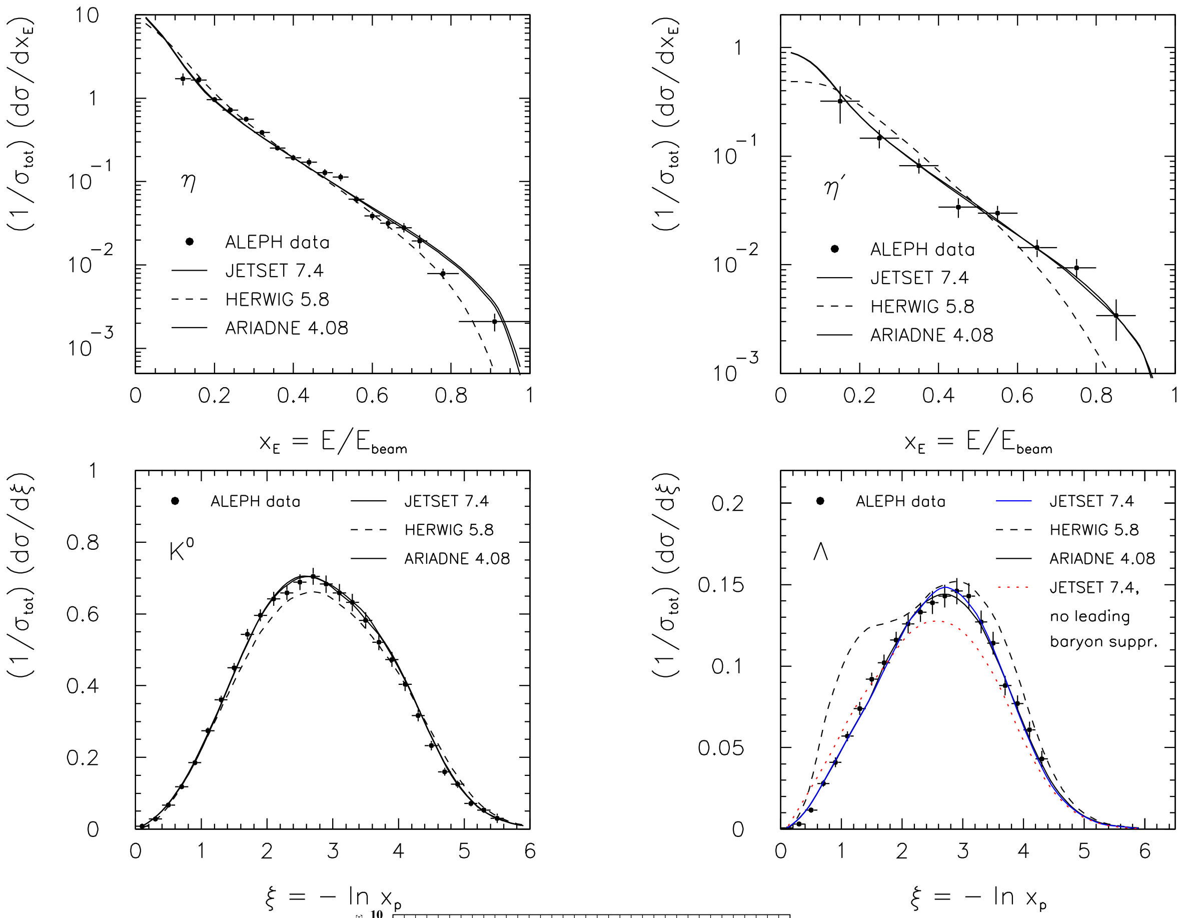
ALEPH: Assume 0^{+}:1^{+}:1^{-}:2^{+} mesons produced in ratios 1 : 3 : 3 : 5.

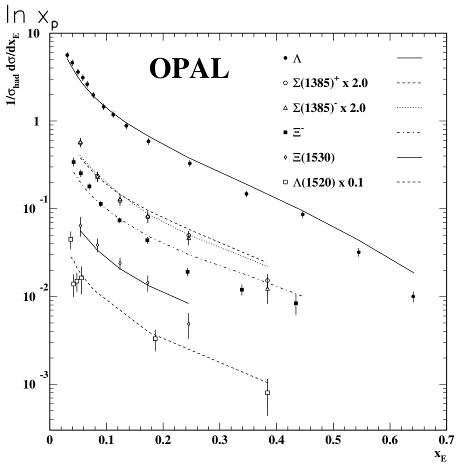
Production rates of identified particles



⟶ The overall description of the hadron production rates by the tuned Monte Carlo is quite good

Identified particle differential cross sections · · ·





Description by tuned version of Pythia (Jetset) is generally good

Prediction for the energy spectrum of baryons generally too hard

⟶ Suppress leading baryons, PARJ(19)

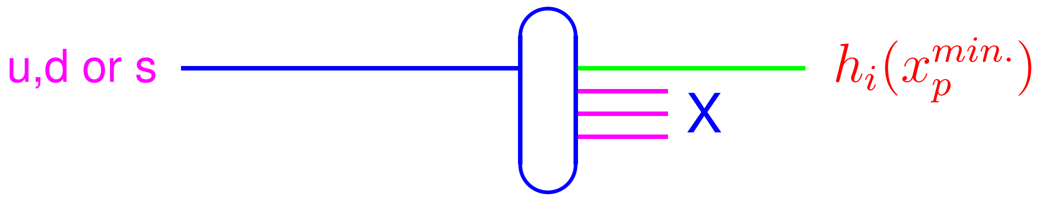
OPAL (1999) $\longrightarrow$

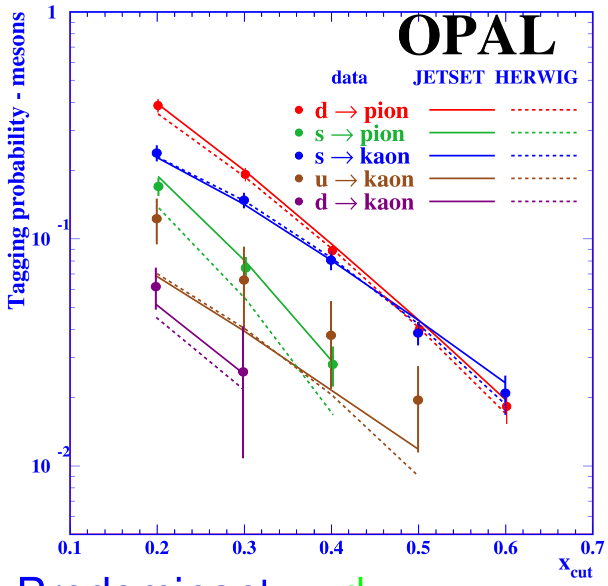
- Measure rates H_i for a hemisphere to contain an identified π^+, K^+, K^0_S, p, Λ or c.c. as its highest momentum particle
- Invert the equations

$$H_i = 2 \sum \eta_{q \to i} \, R_q$$

$$\text{with} \quad R_q = \Gamma(\mathrm{Z} \to \mathrm{q\overline{q}}) / \Gamma(\mathrm{Z} \to \text{ hadrons})$$

to find the tagging probabilities $\eta_{q \to i}$ for a quark of flavor q to appear in a leading hadron of type i





Predominant: d,u $\longrightarrow$ π
s $\longrightarrow$ K

Suppressed: d,u $\longrightarrow$ K
s $\longrightarrow$ π

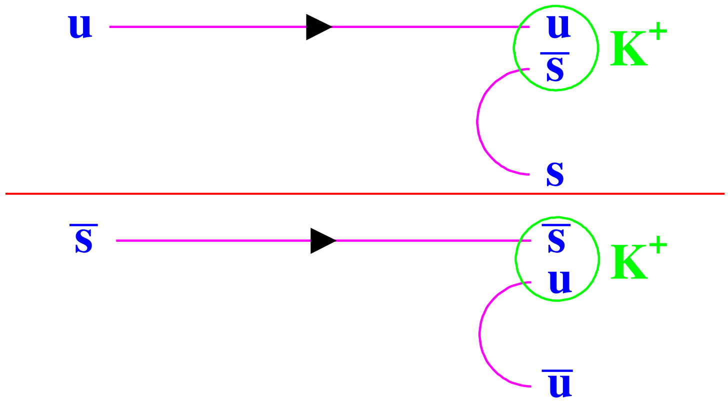
- The primary q, $\overline{\mathrm{q}}$ appear as valence quarks in the highest momentum hadrons (the leading particle effect)

Strangeness suppression factor:

$$\gamma_s \equiv \frac{\mathrm{Prob.(s)}}{\mathrm{Prob.(u,d)}}$$

$\mathrm{u\bar{u}}$ ($\mathrm{d\bar{d}}$) versus $\mathrm{s\bar{s}}$ pair production from the vacuum

$$\frac{\eta_{\mathrm{u}}^{\mathrm{K}^\pm}}{\eta_{\mathrm{s}}^{\mathrm{K}^\pm}} =$$



$$\approx \gamma_s$$

Similarly, $\frac{\eta_{\mathrm{u}}^{\mathrm{K^0_s}}}{\eta_{\mathrm{s}}^{\mathrm{K^0_s}}} \approx \gamma_s$

Contributions from decays of higher mass resonances, etc., predicted to be small (Jetset/Pythia)

$$\boxed{\gamma_s = 0.422 \pm 0.077}$$

$(x_p^{min.} > 0.20)$

Method does not compare yields for hadrons with different masses, e.g. K/π, or rely on tuning of MC parameters

$\longrightarrow$ $\gamma_s = 0.31$ in Jetset/Pythia (OPAL version)

$\longrightarrow$ Similar result, $\gamma_s = 0.26 \pm 0.12$, from SLD (PRL78 (1997) 3442)

(V) APPENDIX

(A1) Experimental studies of $r_{g/q}^{hadron}$

$\longrightarrow$ Two examples from history

(A2) Higher moments of multiplicity

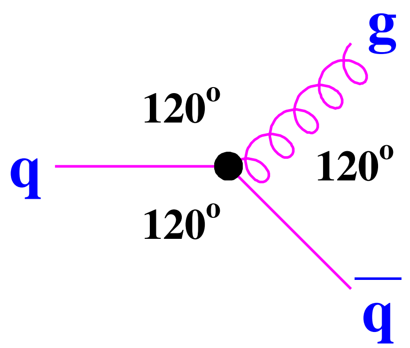
(A3) Dead cone effect

(A1) Experimental studies of $r_{g/q}^{hadron}$

$\longrightarrow$ Two examples from history

(1) HRS at PEP, e^+e^- $E_{c.m.}$=29 GeV (1985)

$\longrightarrow$ Select 3-fold symmetric 3-jet events (use jet finder)



NO TAGGING $\longrightarrow$ don't know which is the gluon jet (3-fold ambiguity)

$\longrightarrow$ Employ expression

$$\langle n_{ch.}\rangle_{event} = 2 \cdot \langle n_{ch.}\rangle_{quark} + \langle n_{ch.}\rangle_{gluon}$$

$\langle n_{ch.}\rangle_{gluon} \longrightarrow$ free parameter

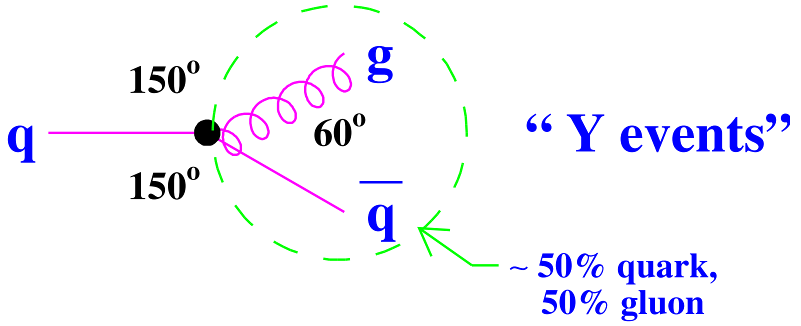
$\langle n_{ch.}\rangle_{quark} \longrightarrow$ use quark jets from 2-jet events with the same energy as the jets in the 3-fold symmetric events

$$\boxed{r_{g/q}^{hadron} = 1.29\,^{+0.29}_{-0.46}}$$

$\longrightarrow$ First published experimental result for $r_{g/q}$

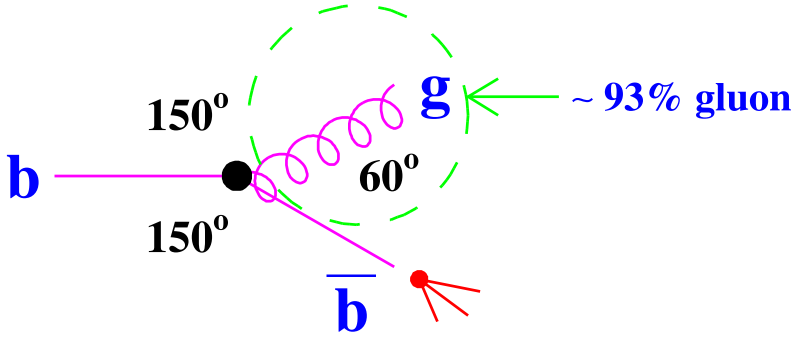
(2) OPAL at LEP-1, e^+e^- $E_{c.m.}$=91 GeV (1991, 1993)

$\longrightarrow$ Select 1-fold symmetric 3-jet events (use jet finder)



- Larger Statistics compared to 3-fold symmetric events
- Only a 2-fold ambiguity in gluon jet, rather than 3-fold
- Add results of 2 lower energy jets $\longrightarrow$ $\langle n_{ch.}\rangle_{gluon+quark}$

$\longrightarrow$ Anti-tag the gluon jet by identifying b quarks in $b\overline{b}g$ events



$\longrightarrow$ $\langle n_{ch.}\rangle_{gluon}$

$$\boxed{r_{g/q}^{hadron} = 1.25 \pm 0.04}$$ ($k_\perp$ jet finder)

$\longrightarrow$ First measurement of a multiplicity difference between gluon and quark jets

However these are not unbiased jets and this result for $r_{g/q}$ cannot be compared to the theoretical calculation !!

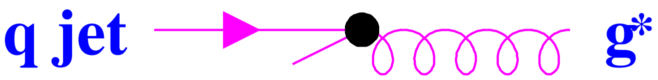
(A2) Higher moments of multiplicity (fluctuations from the mean)

Mean value $\langle n_{ch.} \rangle$, full phase space, $\boxed{\text{unbiased jets}}$

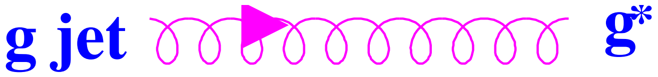
$\longrightarrow$ $r_{ch.} \approx 1.5$, i.e. $\boxed{\text{gluon result} > \text{quark result}}$

High multiplicity events mostly arise from a highly virtual gluon

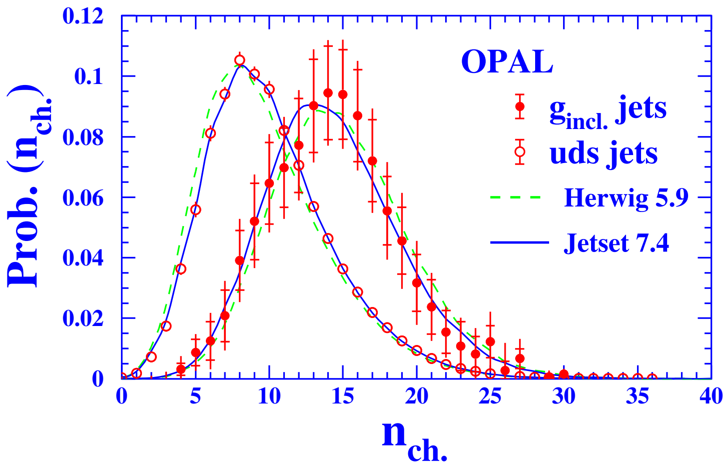
$\longrightarrow$ quark jets sometimes (not often) contain a highly virtual gluon



$\longrightarrow$ gluon jets <u>always</u> contain a highly virtual gluon



$\longrightarrow$ quark jets exhibit more event-to-event fluctuations than gluon jets (a much longer tail on the quark jet distribution)



$\longrightarrow$ Expect $\boxed{\text{gluon result} < \text{quark result}}$ for fluctuations from the mean (higher moments)

Higher "central" moments

⟶ Dispersion D (RMS deviation from mean)

$$D^2 = \langle (n_{ch.} - \langle n_{ch.} \rangle)^2 \rangle$$

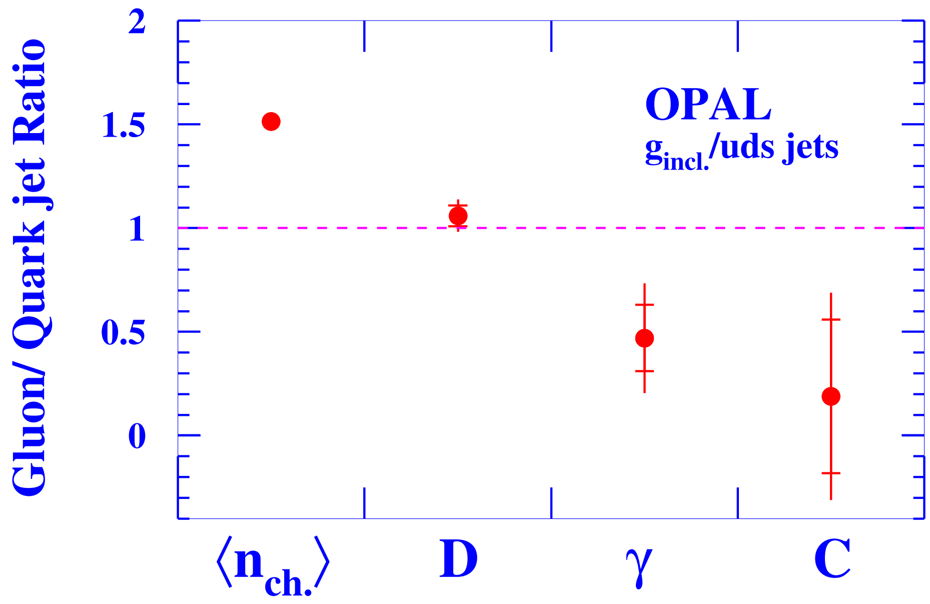
⟶ Skew γ: (asymmetry)

$$\gamma = \frac{\langle (n_{ch.} - \langle n_{ch.} \rangle)^3 \rangle}{D^3}$$

⟶ Curtosis C (deviation from a Gaussian)

$$C = \frac{\langle (n_{ch.} - \langle n_{ch.} \rangle)^4}{D^4} - 3$$

Unbiased gluon and quark jets ⟶



⟶ gluon result < quark result for higher moments, as expected

ANALYTIC CALCULATIONS

$\longrightarrow$ fluctuations characterized by binomial or "factorial moments"

$$\tilde{F}_r \equiv \langle n(n-1)\cdots(n-r-1)\rangle \quad r = \text{rank}$$

$$\text{i.e. } \; F_2 = \langle n\,(n-1)\rangle \;\;,\;\; F_3 = \langle n\,(n-1)\,(n-2)\rangle \;\;,\;\; \text{etc.}$$

$\longrightarrow$ $\tilde{F}_r$ appear naturally in common theoretical approaches to multiplicity (based on generating functions)

$\longrightarrow$ $\tilde{F}_r$ coincide with the ordinary moments $C_r \equiv \langle n^r \rangle$ asymptotically, $n \rightarrow \infty$

Normalized factorial moments

$$F_r \equiv \frac{\langle n(n-1)\cdots(n-r-1)\rangle}{\langle n \rangle^r}$$

$\longrightarrow$ Absolute predictions exist up to n.n.l.o., including energy conservation and proper phase space limits

$\longrightarrow$ Factorial cumulant moments K_r

$$K_r \equiv F_r - \sum_{m=1}^{r-1} \frac{(r-1)!}{m!\,(r-m-1)!}\, K_{r-m}\, F_r$$

$\longrightarrow$ The different K_r are much less correlated with each other for different ranks r than F_r

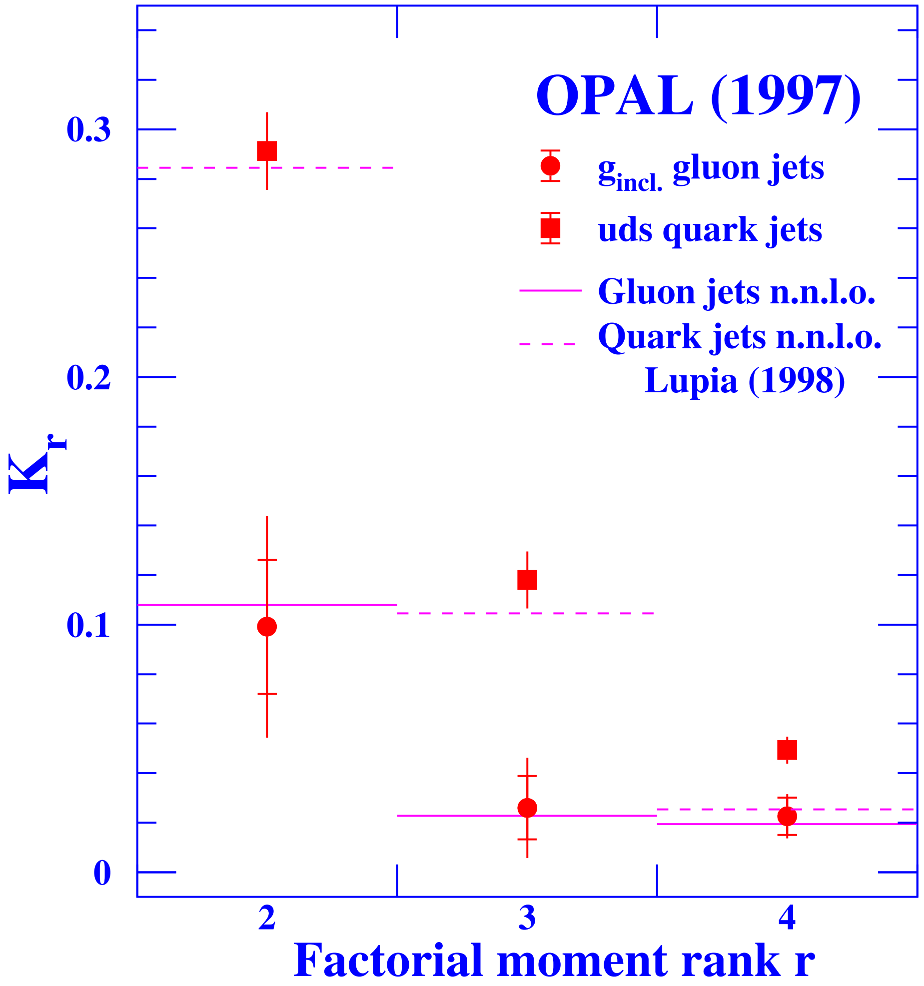
$K_2 \;\rightarrow\;$ a measure of dispersion D

$K_3 \;\rightarrow\;$ a measure of skew γ

$K_4 \;\rightarrow\;$ a measure of curtosis C

in a manner that is available from theory (Note: $K_1 \equiv 1$)

Factorial cumulants: calculation versus data



- ⟶ gluon result $<$ quark result for higher moments for both theory and data
- ⟶ Remarkable overall agreement between the QCD calculation and data not only for the mean $\langle n_{ch.}\rangle$ but for higher moments as well (at the Z^0 scale)
- ⟶ No hadronization corrections are applied

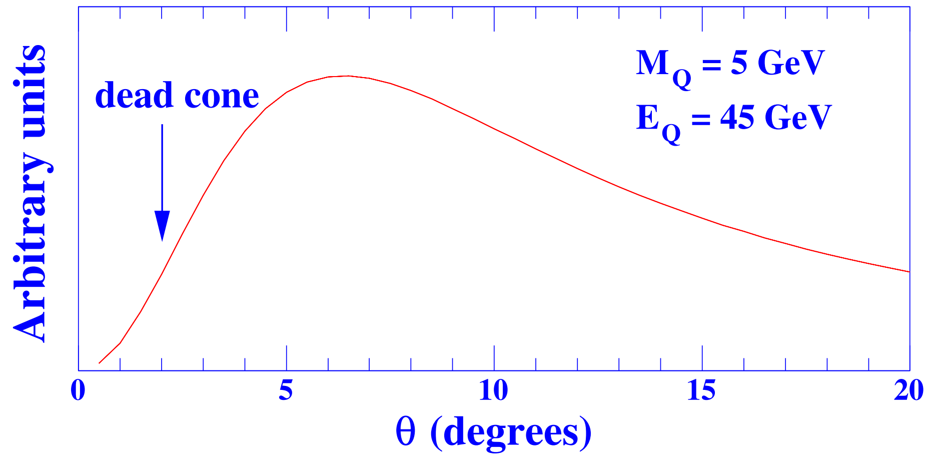
(A3) Dead cone effect

In QCD, gluons at small angles θ are radiated according to

$$\frac{\theta^2}{(\theta^2 + \theta_M^2)^2}$$

$$\theta_M = \frac{M_Q}{E_Q} \;\; ; \;\; M_Q,\, E_Q \; = \; \text{mass \& energy of the quark}$$

$\longrightarrow$ There is a "dead cone" in the forward region, i.e. a cone around the quark's direction of motion containing a suppressed level of radiation (it is not really "dead")



$\longrightarrow$ The larger M_Q, the larger the suppression of forward multiplicity, $\theta < \theta_M$

$\longrightarrow$ b quark events have a smaller multiplicity at the parton level than uds events, in contrast to the hadron level

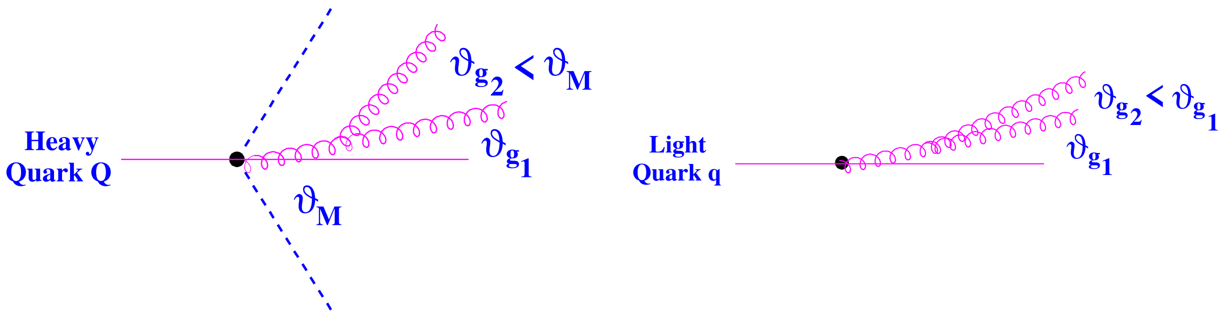
$\longrightarrow$ Analogy with QCD: muons radiate less than electrons

- Because of the dead cone, the soft gluon multiplicity in b and uds quark jets (the “accompanying multiplicity”) is NOT the same for $\theta < \theta_M$
- Rather *non-intuitively*, the multiplicity difference of the accompanying multiplicity,

$$\delta_{b\ell}^{parton} = \langle n \rangle_b^{parton} - \langle n \rangle_{uds}^{parton} ,$$

is predicted to be independent of the quark jet energy E_Q
(Dokshitzer, Khoze & Fadin (1982); Dokshitzer, Khoze & Troyan (1991))

$\longrightarrow$ For $\theta < \theta_M$, the evolution of a radiated gluon is not restricted by angular ordering because the heavy quark Q and radiated gluon g_1 separate quickly (due to their difference in velocity) and radiate incoherently



- Convolute the distribution of gluons g_1 emitted by the cm Q or q with the multiplicity generated by the subjet g_1

$\longrightarrow$ It is found that $\delta_{b\ell}^{parton}$ depends on M_Q but not E_Q

- Estimate the corresponding multiplicity difference for hadrons using data on $\langle n_{ch.}^{uds} \rangle$ at low energies and $\langle n_{ch.} \rangle$ of B hadron decays (Schumm et al. (1992); Petrov & Kisselev (1995))

$$\boxed{\delta_{b\ell}^{hadron} \approx 3.7}$$