Lattice QCD

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CTEQ/IPPP JUNE 2001

Lecture 1 - Methodology, problems, what to expect in next three years.
Lecture 2 - Current results, concentrating on heavy quark physics.

A short history of lattice QCD

- Invented 1974
  slow progress ....

- Renaissance in 1990s

  – Much improved understanding of systematic errors

  – 10-20% errors possible on spectrum, form factors etc

- Second lattice revolution - early 2000s

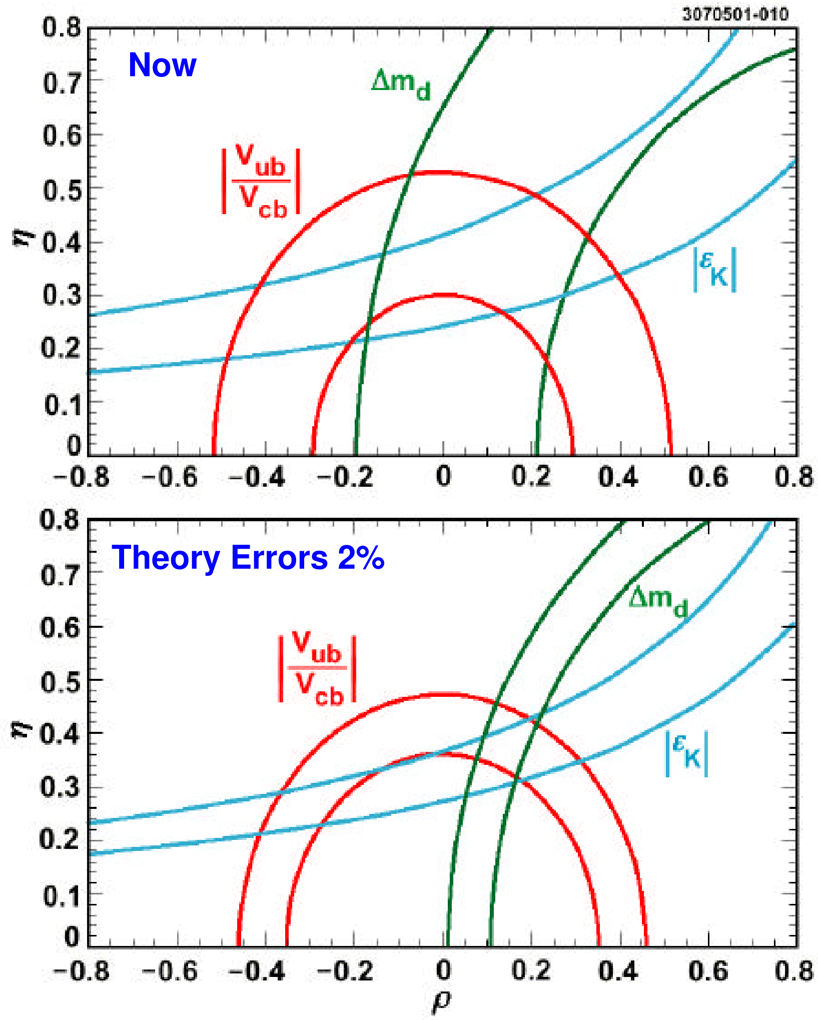
  – Teraflop computing power will enable simulations with ‘real’ dynamical quarks

  – carry improvement of systematic errors further

  – 2-3% errors will be possible

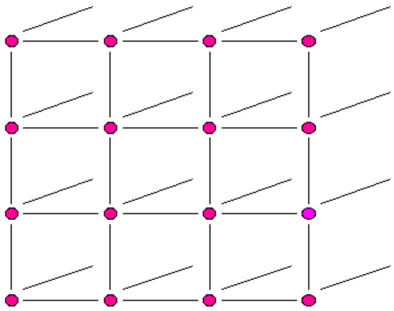
Why do you care?

Improved theoretical precision will add huge value to experiment, e.g. from B factories.



Lattice QCD =

Euclidean space-time lattice



+ QCD Lagrangian (discretised)

$$\mathcal{L}_{QCD} = \mathcal{L}_g + \mathcal{L}_q$$
$$= \frac{1}{2g^2} Tr F_{\mu\nu}^2 + \overline{\psi}(\gamma \cdot D + m)\psi$$

Lattice spacing (a) is implicit u.v. cutoff
Parameters are :
Bare gauge coupling $\beta = 6/g^2$
Quark masses, $m_i a$

Running a lattice calculation :

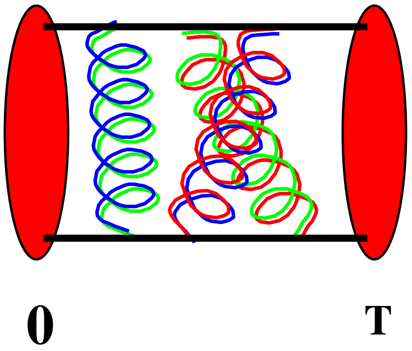
- Choose a volume and a rough lattice spacing. $(3\text{fm})^3 \times 6\text{fm}$ at $a = 0.1$ fm $\rightarrow 30^3 \times 60$. a chosen (roughly) sets β in $\mathcal{L}_{QCD}$.

- Choose formulation, number of flavours of dynamical quarks in the vacuum and their approx. masses $m_i a$ for $\mathcal{L}_{QCD}$.

- Generate configs of gluon fields with prob. $e^{-S_{QCD}}$, $S = \int d^4x\mathcal{L}$. Dynamical quarks appear implicitly through the quark determinant.

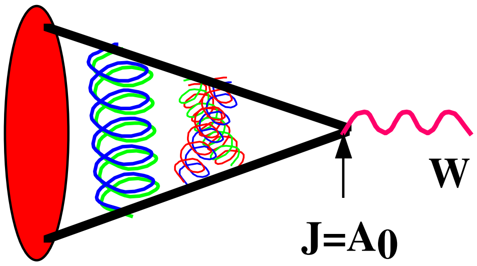
- Solve for valence quark props on each gluon field config.

- Put valence q props together, average, and fit to multi-exponential form:



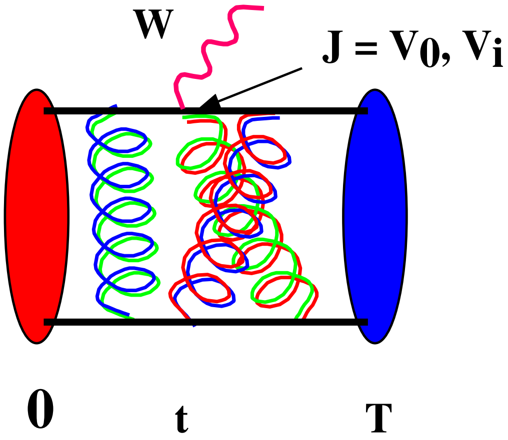
2pt function for spectrum

$$\langle 0|H^\dagger(T)H(0)|0\rangle \to Ae^{-MT}+\ldots$$



2pt function for decay constant

$$\langle 0|H^\dagger(T)A_0(0)|0\rangle \to \sqrt{A}\, f\sqrt{M}e^{-MT}$$

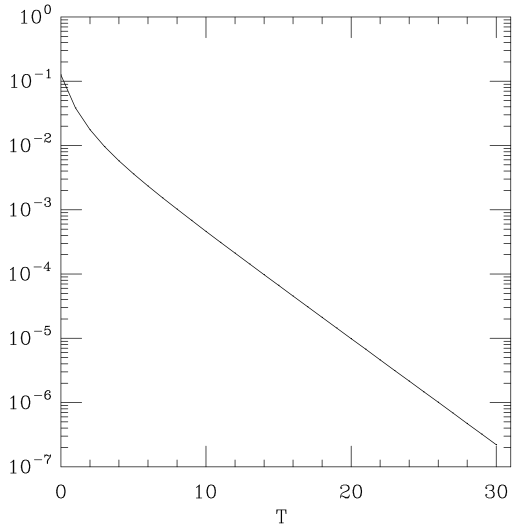


3pt function for SL decay

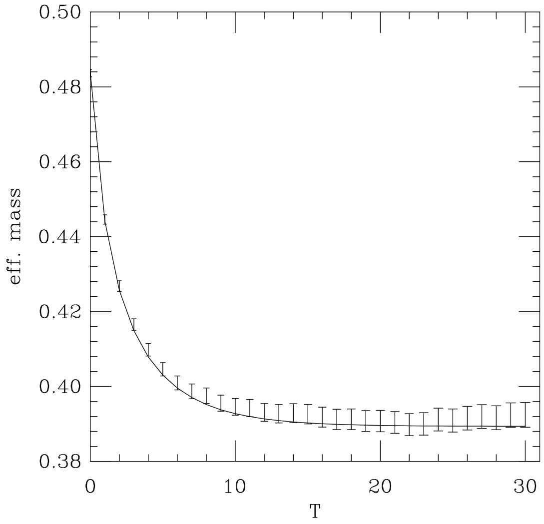
$$\langle 0|H_1^\dagger(T)J(t)H_2(0)|0\rangle \to$$

$$\sqrt{A_1}\sqrt{A_2}\langle H_1|J|H_2\rangle e^{-M_1(T-t)}e^{-M_2 t}$$

Raw lattice data for an Υ 2pt function.



Results usually presented as the log of the correlation function = 'effective mass'.



- Determine a by setting one hadron mass = its experimental value.
  e.g. $M_{lattice} = M_{expt} a \Rightarrow a$.
  All other masses can now be given in GeV.

- Rerun with different $m_i a$ until one other hadron containing quark i has mass = its experimental value, or interpolate/extrapolate results.

- Repeat at different a and compare. Physical quantities should be same or extrapolate to a=0.

Lack of computer power $\rightarrow$ compromise on lattice volume/number of lattice points/stats. Miss out dynamical quarks in S_{QCD} = Quenched Approximation(QA). Cost of light (u, d) dynamical quarks very high.

Control of lattice systematic errors.

- Lattice spacing errors.

  Find results in GeV show unphysical dependence on a (as a^n).

  Remove by extrapolation and/or reduce by *improvement* (increases n).

  Dependence arises from errors in discretising S_{QCD}. e.g. on the lattice

  $$p^2 \rightarrow (\frac{sin(pa)}{a})^2 = p^2(1 - \frac{(pa)^2}{3} + \ldots)$$

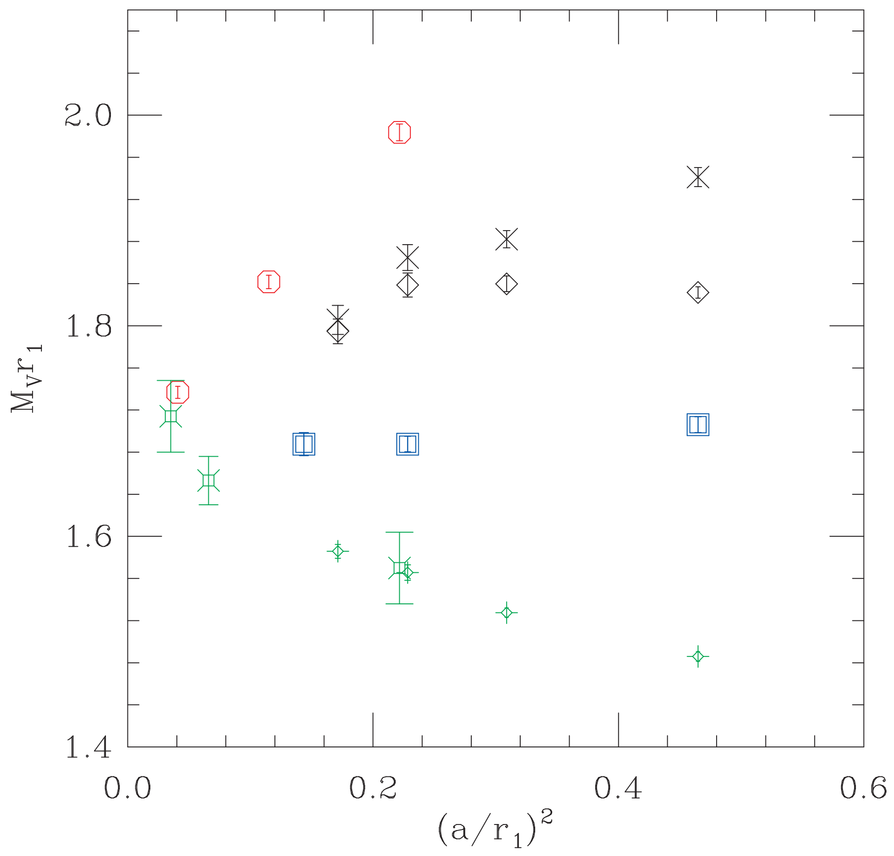
  Remove $a^2 \ldots$ errors by adding correction terms to S_{QCD} = improvement.

  Improvement terms get renormalised. Remove αa^2 errors by pert. or nonpert. matching to the continuum e.g. demand Lorentz invariance or correct Ward identities.

  Typical a^2 error at $a = 0.1$ fm is 1-2%.

Test improvement in QA by e.g. plotting m_ρ vs a.

MILC collab. results (□) with improved Kogut-Susskind quarks show best scaling ($\mathcal{O}(\alpha a^2)$). Clover (=Improved Wilson) (+) has $\mathcal{O}(\alpha a)$.



- Matching of lattice currents to the continuum for matrix elements.

  Lattice currents must be renormalized and improved to match continuum currents. Largest source of systematic error in QA.

$$J_{cont} = Z_0 J^{(0)}_{latt} + a Z_1 J^{(1)}_{latt} + \ldots$$

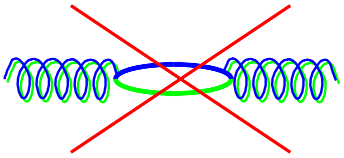
$$Z = 1 + c_1 \alpha_s(2/a) + c_2 \alpha_s^2(2/a) + \ldots$$

  At $a = 0.1$ fm $\alpha_s^2(2/a) \approx 7\%$ $\Rightarrow$ second order results are needed and/or non-pert. determination of Z for 2% errors.

- Unquenching

  QA has $m = \infty$ for dynamical (sea) quarks. No feedback from quark sector into gluon sector.

  $\Rightarrow \alpha_s$ runs incorrectly

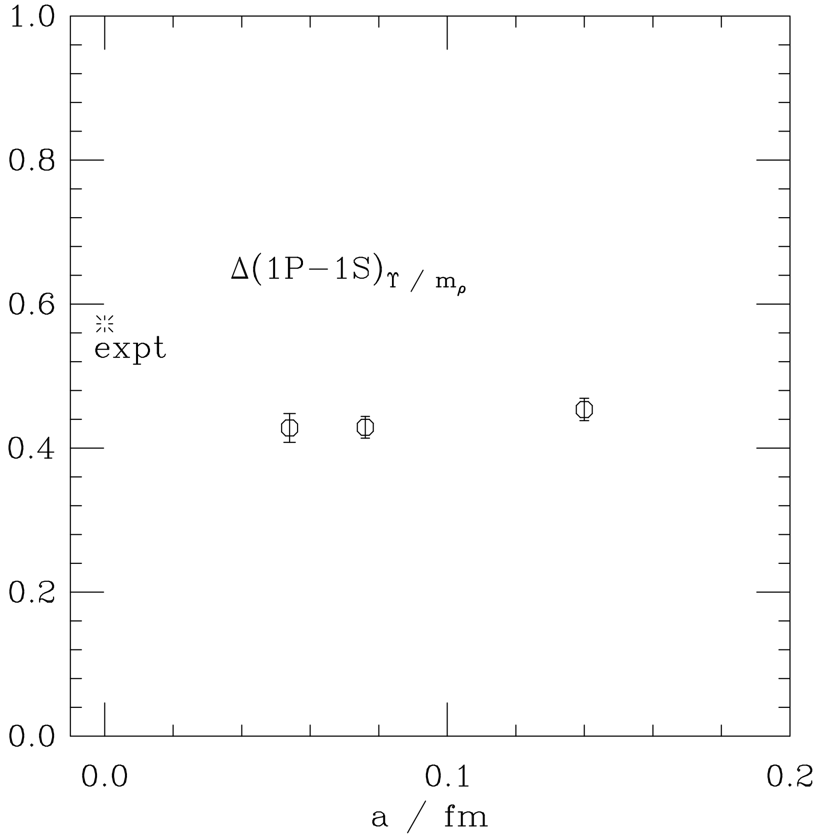
  

  + 'string doesn't break' i.e. coupling to decay channels is ignored.

Incorrect physics shows up:

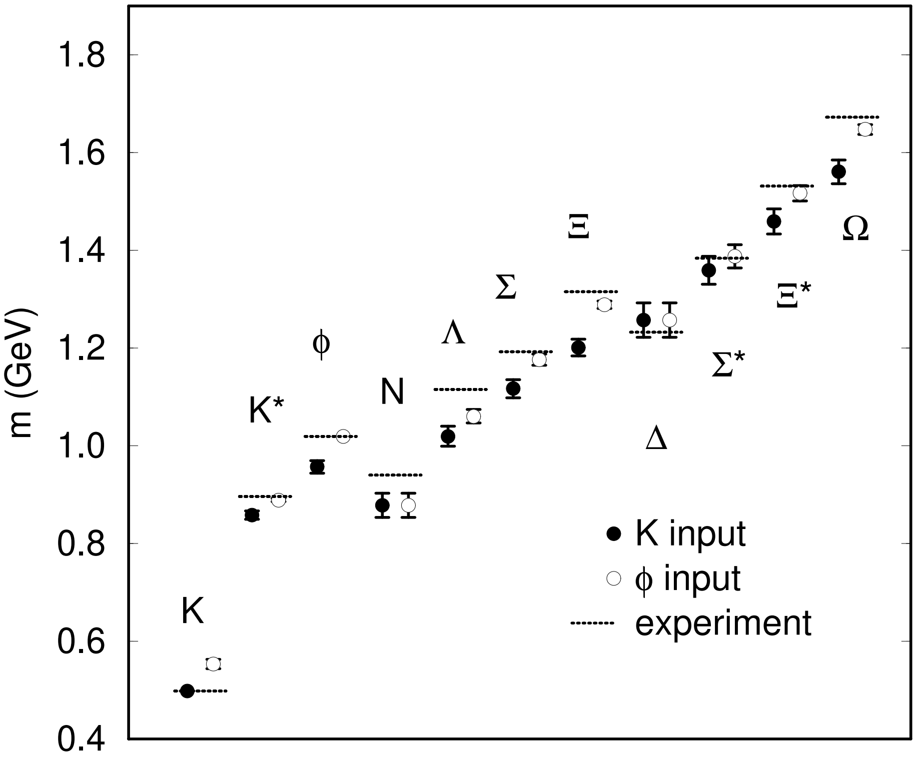
– a depends on quantity used to determine it at 10-20% level $\equiv$ dimensionless ratios of masses do not agree with experiment.

Results from UKQCD collaboration:



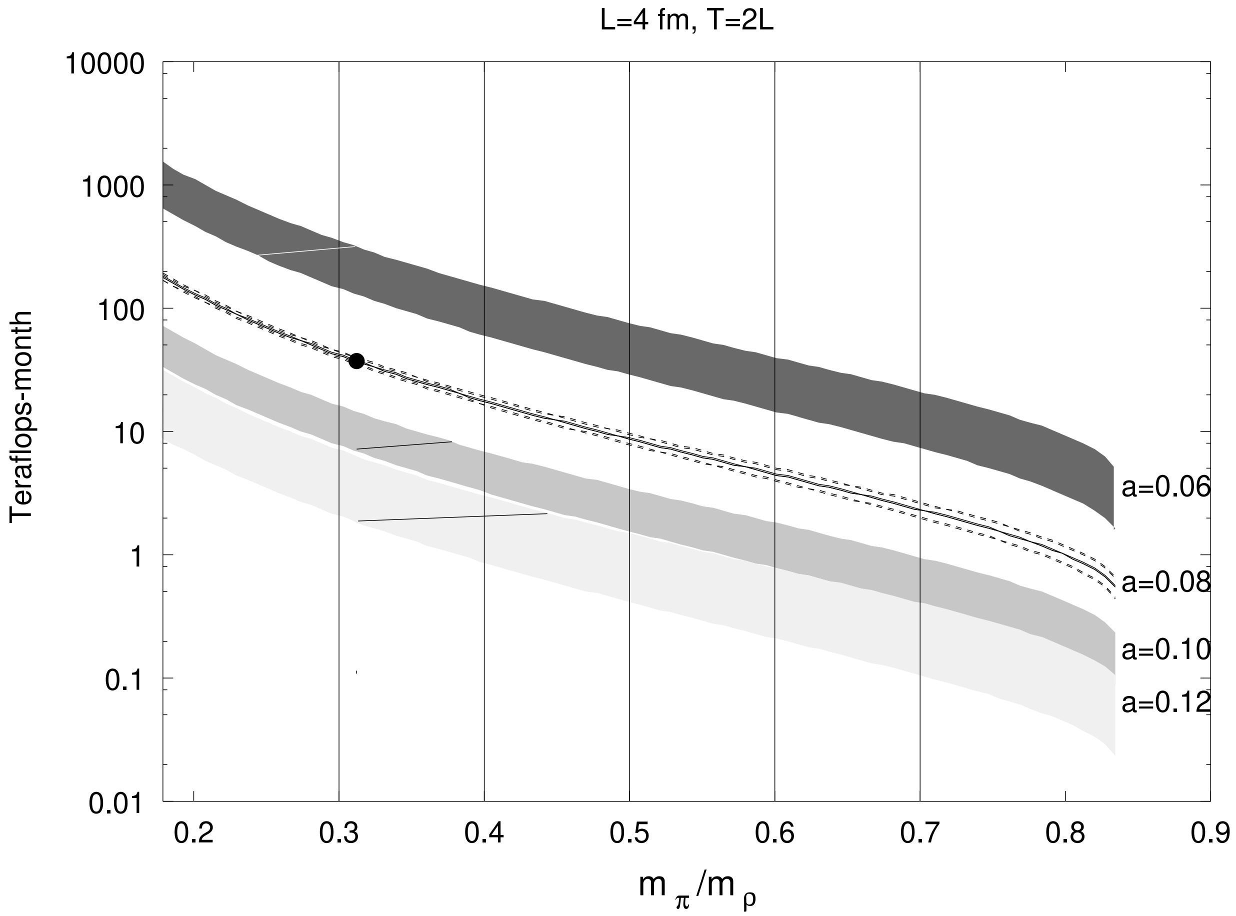
– The spectrum is wrong.

Results from CP-PACS collaboration using Wilson quarks in QA, extrapolated to $a = 0$.



Cost of unquenched calculations is high.

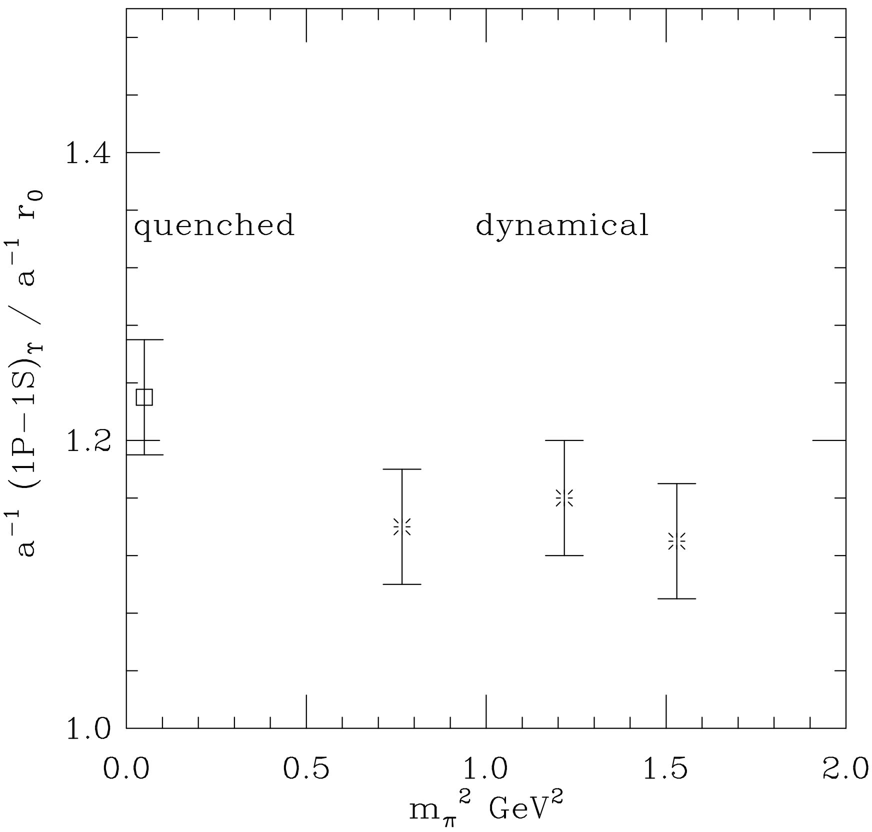
Estimates from SESAM collaboration for 100 configs with $N_f = 2$ Wilson dynamical quarks in terms of m_π/m_ρ reachable:



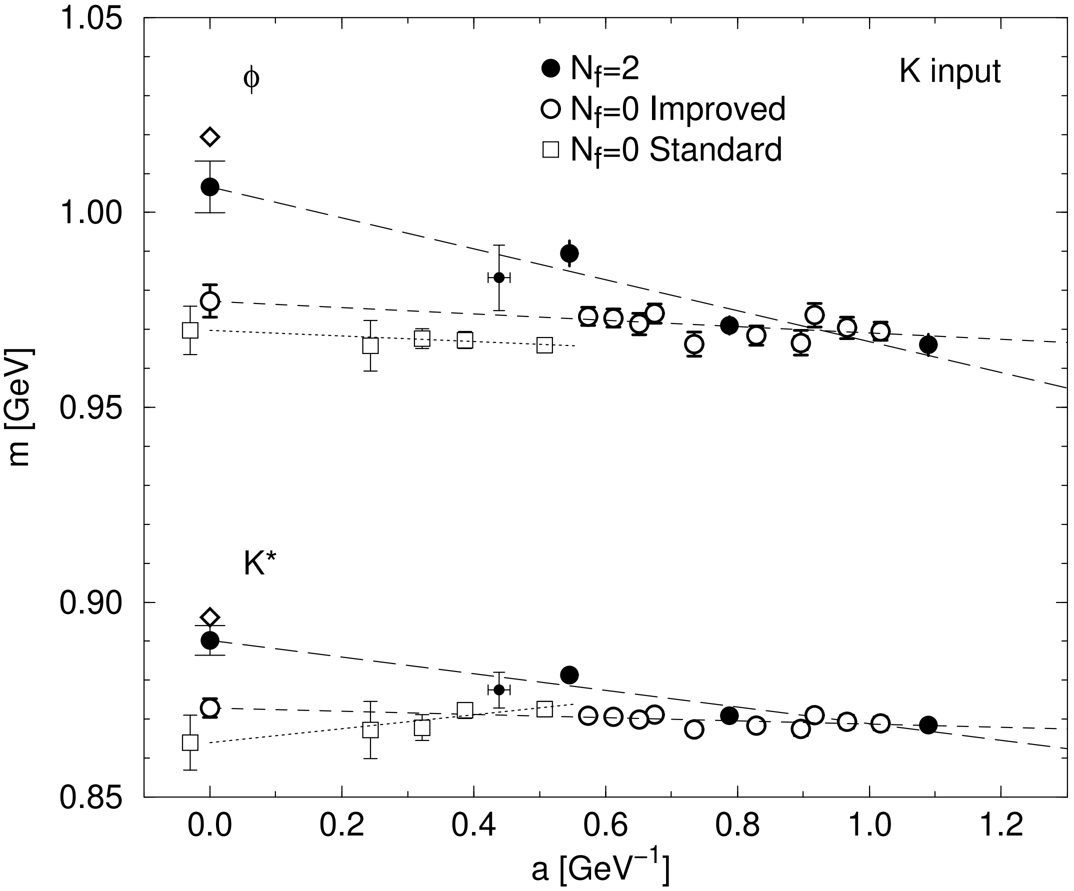
$m_\pi/m_\rho = 0.7 \Rightarrow m_{u,d} = m_s$
$m_\pi/m_\rho = 0.4 \Rightarrow m_{u,d} = m_s/4$

Current unquenched calculations show some progress.

UKQCD has clover m_{sea} down to m_s ($N_f = 2$) and $a \approx 0.1$ fm and see signs of convergence in a determinations.

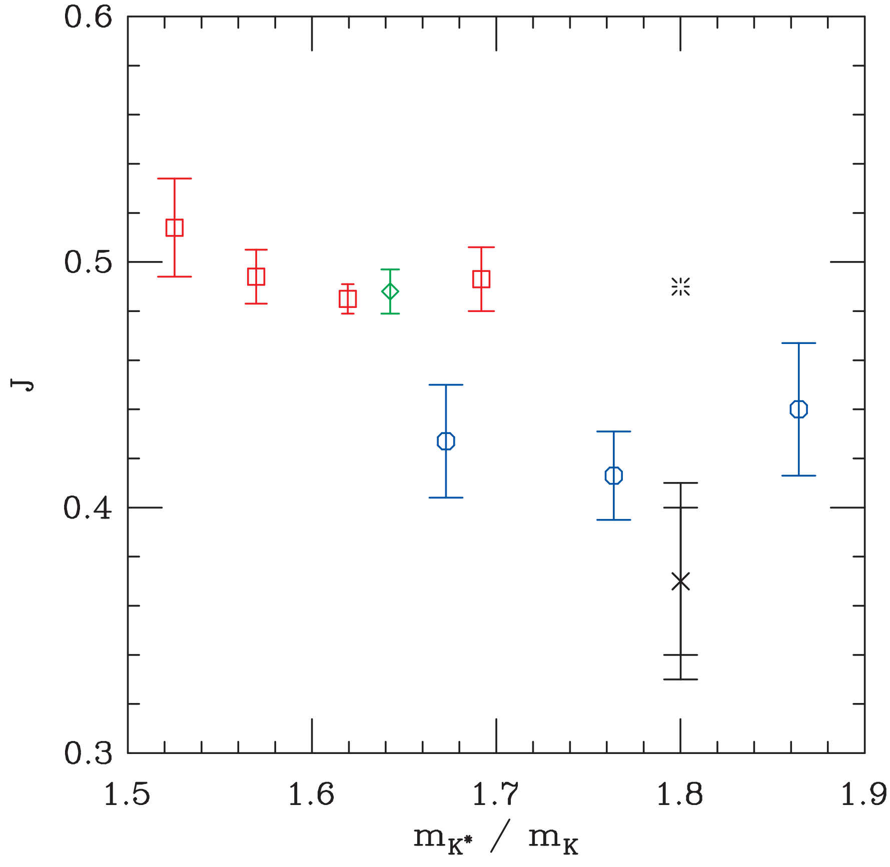


CP-PACS has clover m_{sea} down to m_s ($N_f = 2$) and a range of a values and see improvements in the spectrum.



MILC has improved KS $m_{sea,u,d}$ down to $m_s/4$ ($N_f = 2{+}1$) with $a = 0.13$ fm and see an improvement in 'J'.

$$J = m_{K^*} \frac{(m_\phi - m_\rho)}{2(m_K^2 - m_\pi^2)}$$



Lattice calculations over next three years:

With 5 Tflops machine (UKQCD 2003) can make 500 config ensembles with 2 ($m_s/4$) + 1 (m_s) sea quarks and $a \leq 0.1$fm using improved KS formulation.
Clover is more expensive. Ginsparg-Wilson is even more expensive.

Need to extrapolate to the chiral (real $m_{u,d}$) limit, using form from chiral pert. th.

Need various a to check lattice spacing errors.

Need improved pert. or nonpert. matching to continuum for matrix elements.

Should enable errors at 2-3% level for most of non-singlet quark spectrum and matrix elements