Recent lattice results

Light quark results

Glueball results

Heavy quark results

See Proceedings of yearly Lattice conferences. LAT00 = Nucl. Phys. B (Proc. Suppl. 94) (2001), articles on hep-lat.

Light quark results

An important issue is QCD chiral symmetry on the lattice. Naive disc. of the Dirac action gives 16 quark 'flavours' (doubling problem). Several solutions to this:

Wilson : Dirac + Wilson term. Completely broken χal symmetry restored as $a \to 0$. 1 quark flavour. Improvement = clover.

Kogut-Susskind (staggered) quarks : 'Thinned' Dirac action. Keep some χal symmetry but 4 quark 'flavours' - become degenerate as $a \to 0$.

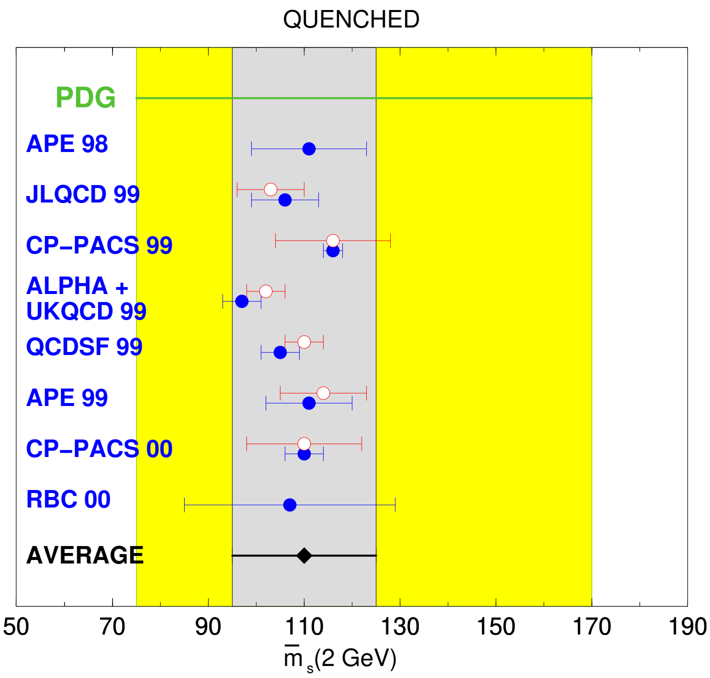
NEW! Ginsparg-Wilson (domain-wall) : Keep χal symmetry at finite a at large cost e.g. going to S_q defined in 5 dimensions.

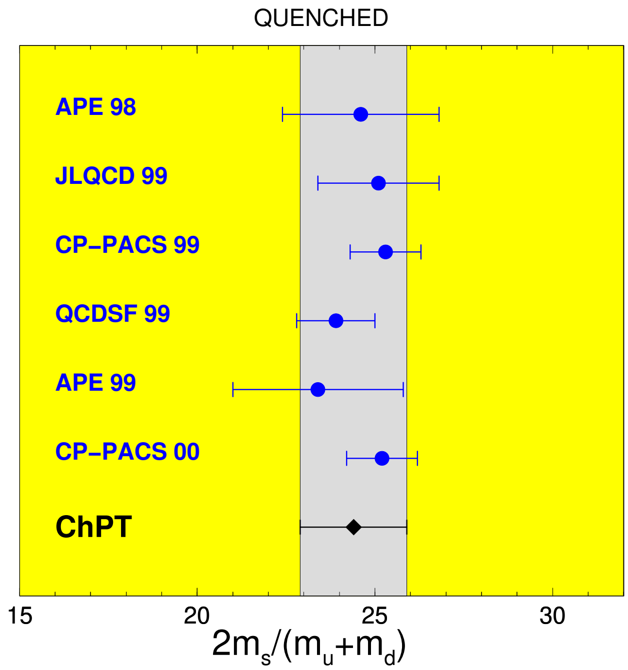
Which is best depends on quantity.

Light quark masses

Finding quark mass in S_{QCD} that gives right hadron mass $\Rightarrow$ a determination of the quark mass. Different formulations + different methods ($m_i a$ vs result from using PCAC relation). Convert to continuum ($\overline{MS}$) mass using pert. or nonpert. renorm.

Quenched results converging (Lubicz, LAT00)





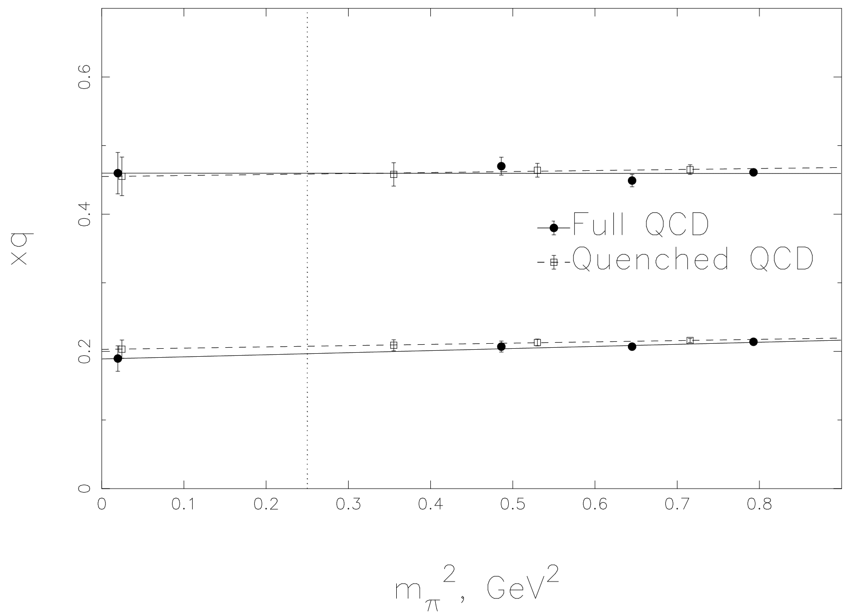
Unquenching increases the quark mass, but quote for now as error on quenched value.

$\overline{m}_s(2\ \mathrm{GeV}) = 110 \pm 15 \pm 20\ \mathrm{MeV}$

$\overline{m}_{u,d}(2\ \mathrm{GeV}) = 4.5 \pm 0.6 \pm 0.8\ \mathrm{MeV}$

Light hadron structure functions.

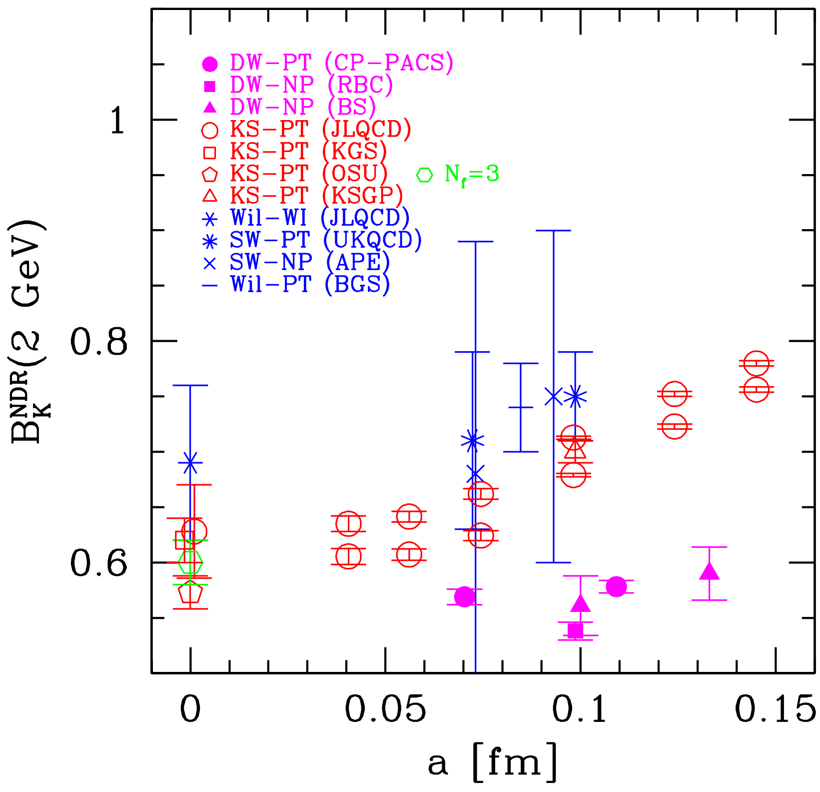
Calculate moments as matrix elements of appropriate operators in e.g. a proton using 3pt techniques.
Initial unquenched results same as quenched (Negele et al using SESAM unq configs.).



$< x >_u - < x >_d$ (4 GeV)($\overline{MS}$) = 0.27 vs. 0.18 from phenomenology.

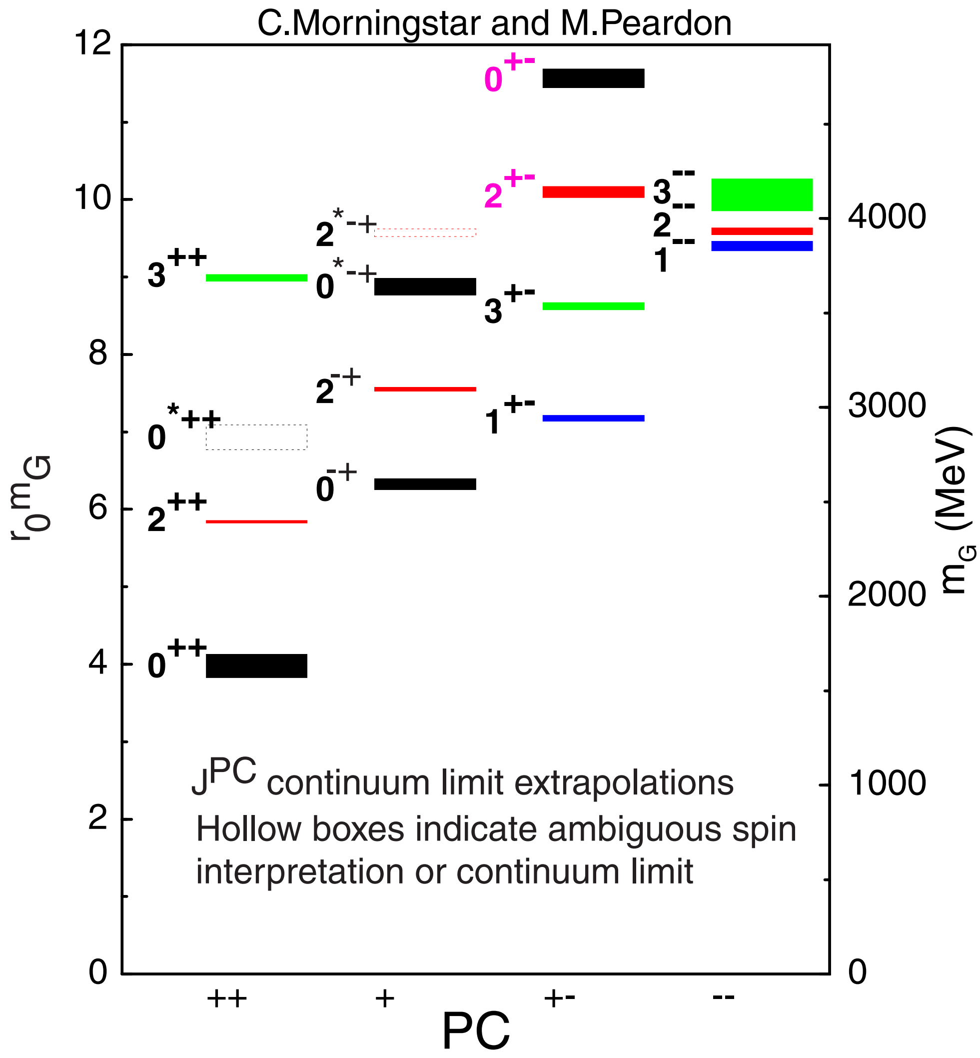
Kaon weak matrix elements.

Calculate matrix elements of effective $\mathcal{H}_{\mathcal{W}}$ between e.g. K and $\pi(\pi)$.
Mixing between operators occurs on lattice and matching to continuum is complicated. Correct χal behaviour is important (need GW?).
Most precise result for $K^0 - \overline{K}^0$ mixing param. B_K. (Lellouch, LAT00)



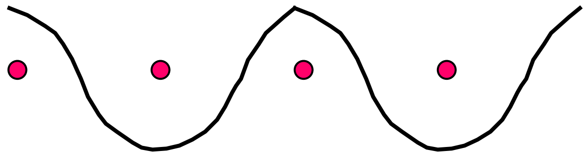
Glueball spectrum

Needs lots of configs for a good signal. Quenched analysis fairly complete. Unquenched calcs will take some time.



Heavy Quark Physics

b and c quarks have $m_i a \geq 1$ on current lattices.
$m_b a \sim 2 - 3$, $m_c a \sim 0.5 - 1$.
Large $ma, (ma)^2 \ldots$ errors appear if we naively use e.g. Wilson/clover quarks.
(e.g. $E(\vec{p} = 0) = ln(1 + ma) \Rightarrow 30\%$ error at $ma = 1$)
$\vec{p}a \approx m_Q a \approx 1$ very distorted on lattice.



BUT, b and c are non-relativistic in their bound states.
m_Q and $\vec{p} \approx m_Q$ are irrelevant dynamical scales.
Can treat b and c quarks accurately on the lattice with non-relativistic techniques.

Several ways to proceed:

1. Static Quarks : $m_Q = \infty$ limit. Spinless and flavourless. Quark prop = string of gluon fields in time dirn. Useful limit for understanding HQET.

2. NRQCD : Non-relativistic effective theory.

$$\mathcal{L}_Q = \overline{\psi}(D_t - \frac{\vec{D}^2}{2m_Q a} - c_4 \frac{\vec{\sigma}\cdot\vec{B}}{2m_Q a} + \ldots)\psi$$

ψ a 2-component spinor.
$m_Q a$ determined by getting heavy hadron mass correct.
c_i fixed by pert. or nonpert. matching to QCD.
$m_Q \to \infty$ is static.
Cannot take a to 0 but improve until a-dependence small enough.

3. Heavy Wilson quarks (FNAL method). :

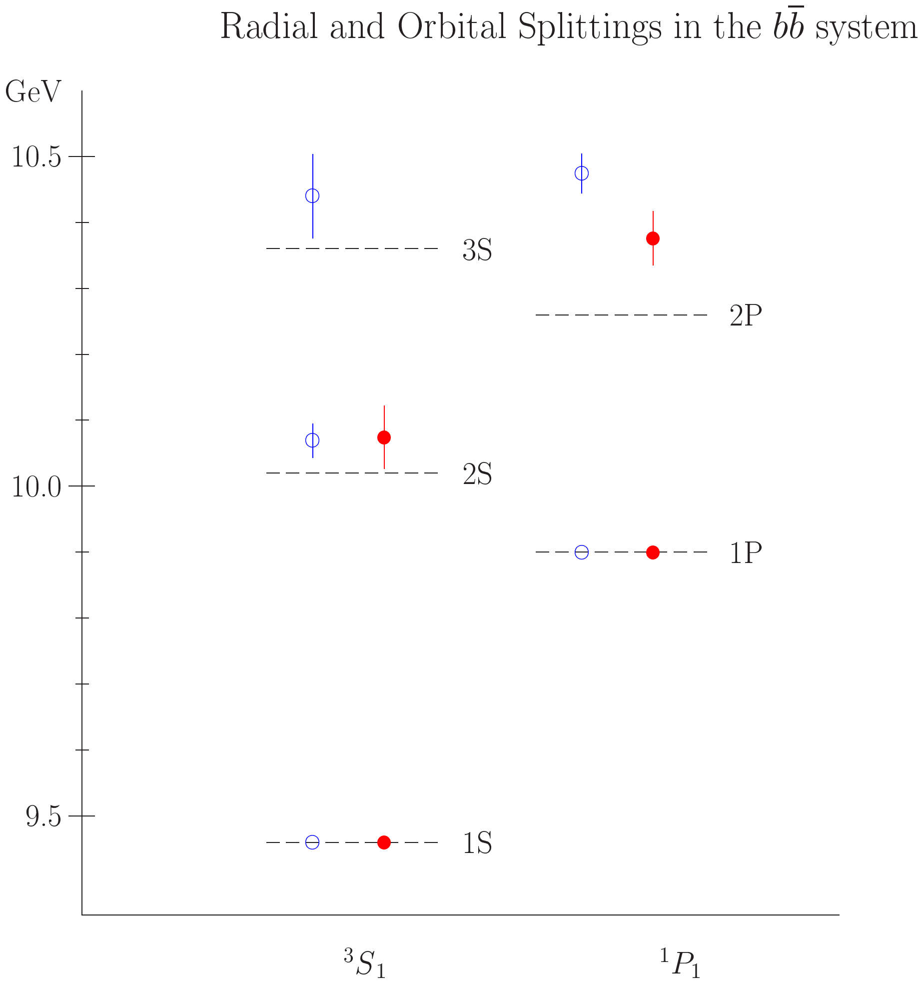
$$\mathcal{L}_Q = \overline{\psi}(\not{D} + m_Q a + \sigma_{\mu\nu}F^{\mu\nu})\psi$$

but interpret non-relativistically in fixing $m_Q a$. Match to QCD with $m_Q a$-dependent coefficients.
Small $m_Q a$ limit is light quarks.
Large $m_Q a$ limit is NRQCD.

4.Wilson/clover quarks : Same action as above but work only at small ma (OK for m_c ?). Extrapolate to large $m \Rightarrow$ large errors *and* expensive. Anisotropic lattices may be better. (fine $a_t \Rightarrow$ small ma_t even with larger a_s).

Results on the spectrum

1. $b\overline{b}$ (Υ) spectrum

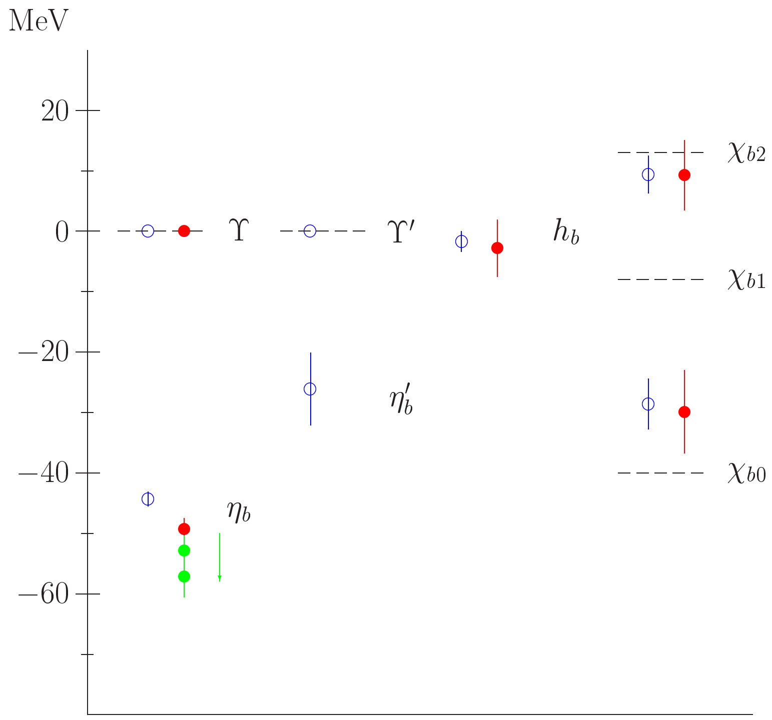


– – – : Experiment

○ : NRQCD for the b, glue: $n_f = 0, \beta = 6.0$.

● : NRQCD for the b, glue: $n_f = 2, clover, \kappa = 0.135, \beta = 5.2$, UKQCD ensemble.

Fine Structure in the $b\bar{b}$ system



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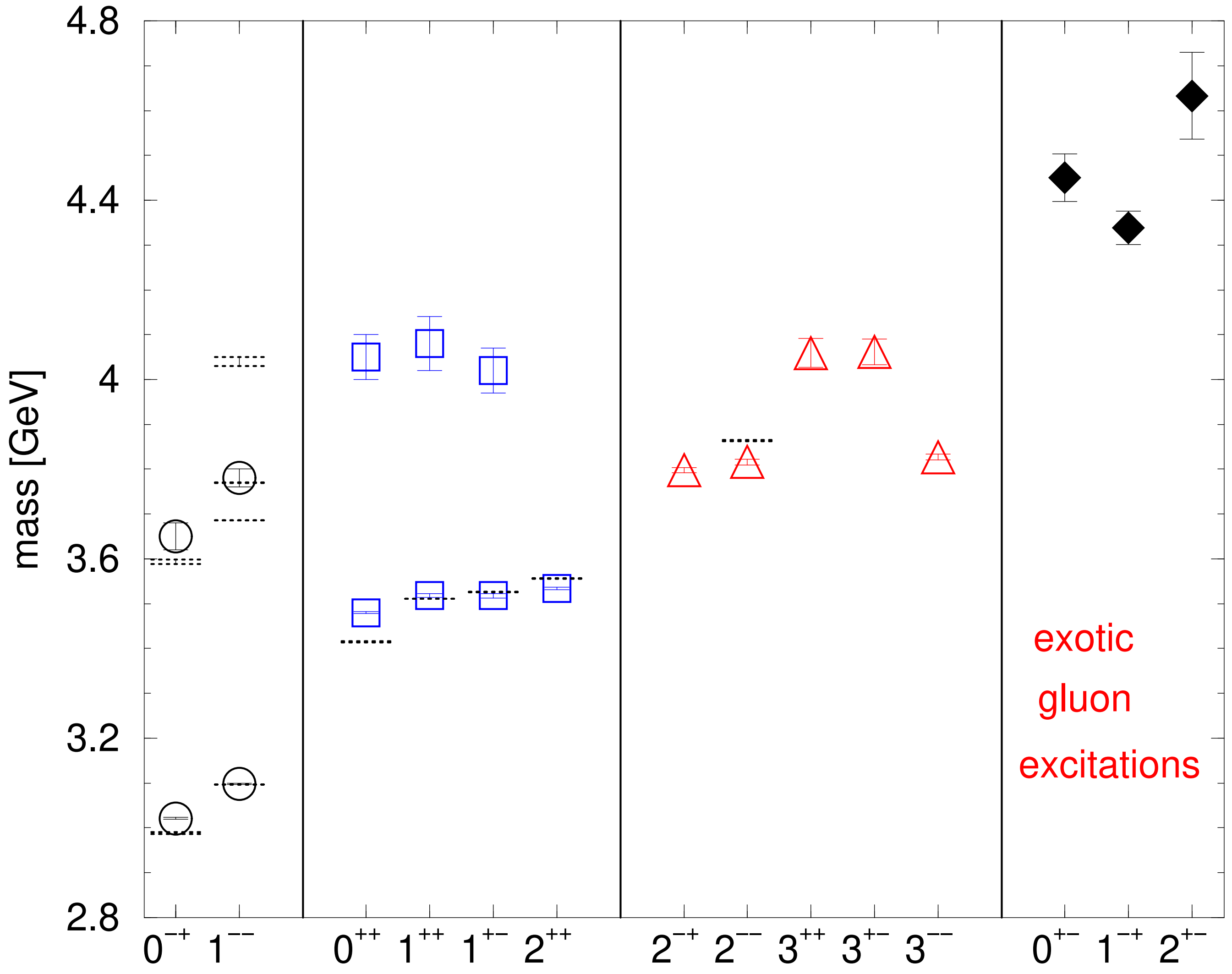
● ↓ : extrapolate to light dynamical mass and to $n_f = 3$.

Hyperfine splitting sees dynamical quarks. Predict $m(\Upsilon) - m(\eta_b) = 60 \pm 15$ MeV. Aim for 1 – 2 % error for CLEO-c.

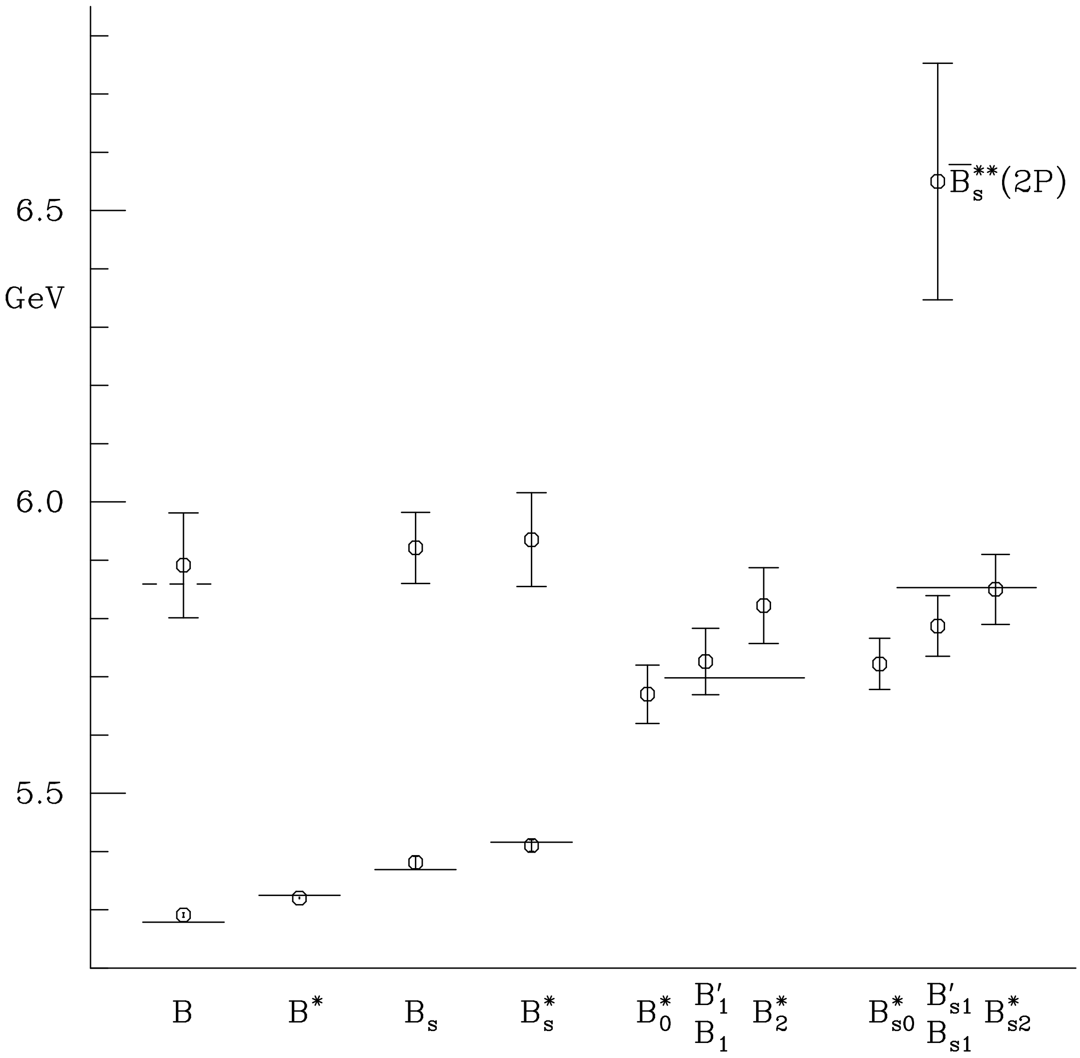
2. $c\bar{c}$ (ψ) spectrum

Columbia results on anisotropic lattices in the QA, including $c\bar{c}g$ hybrids.



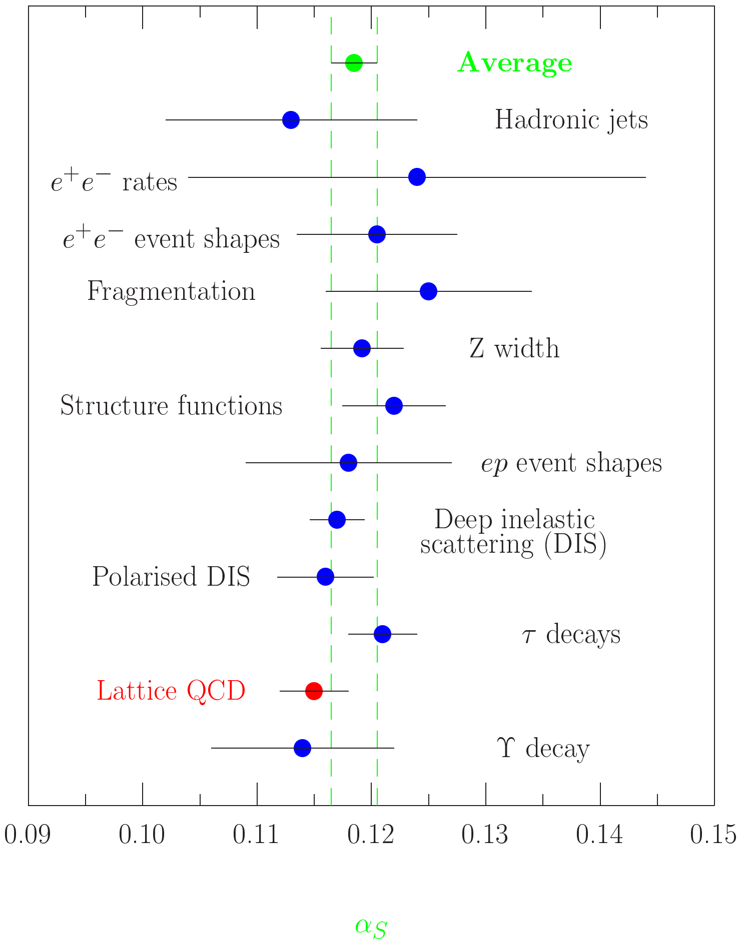
3. $b - light$ (B) spectrum

NRQCD results in the QA.



Determining parameters of S_{QCD}

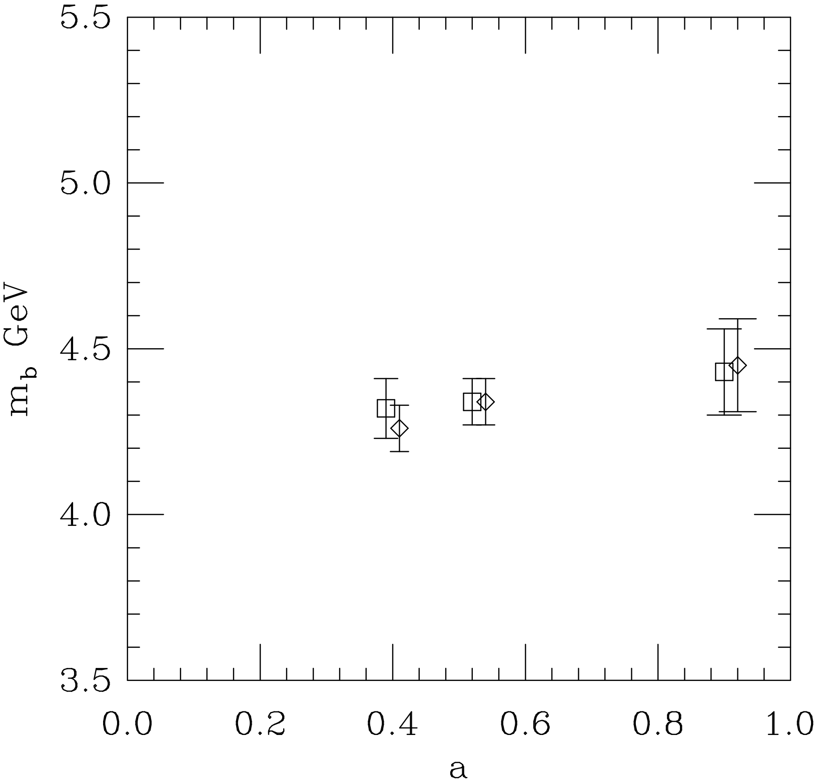
Splitting between 1P and 1S states in $b\bar{b}$ gives a good method to find a. This is $\equiv$ to finding Λ_{QCD} and $\Rightarrow$ α_s. Convert to $\overline{MS}$ and run to M_Z to compare with other determinations.



m_b determined from fixing s.t. B or Υ mass correct. best determination is from B in static limit in QA.

$$\overline{m}_b(\overline{m}_b) = Z_{cont}(m_B - E_B(\vec{p}=0) + E_0)$$

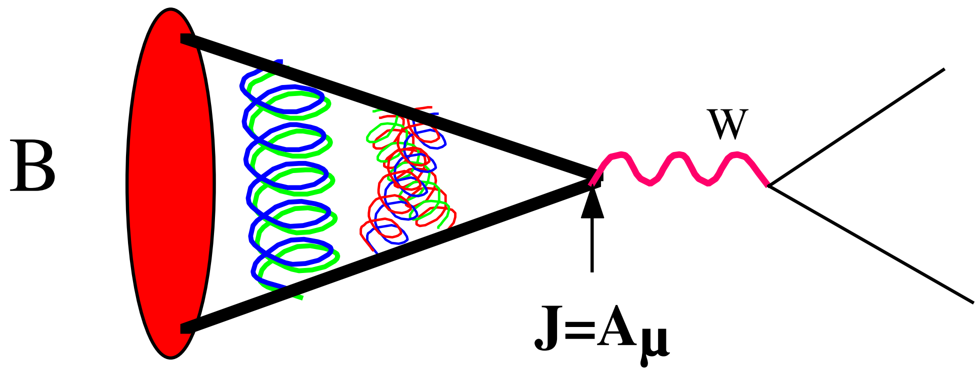
Z_{cont} and E_0 known in pert. th. through α_s^3.
Result is 4.34(7) GeV.
$\sim$ 50 MeV lighter unquenched ?



Determining B matrix elements

1. f_B

Simplest 2pt m.e. for B leptonic decay.



$$< 0|A_\mu|B >= p_\mu f_B$$

Match A_μ on lattice to continuum. Done to $\mathcal{O}(\alpha_s, 1/m_Q, a)$ for NRQCD and heavy Wilson.

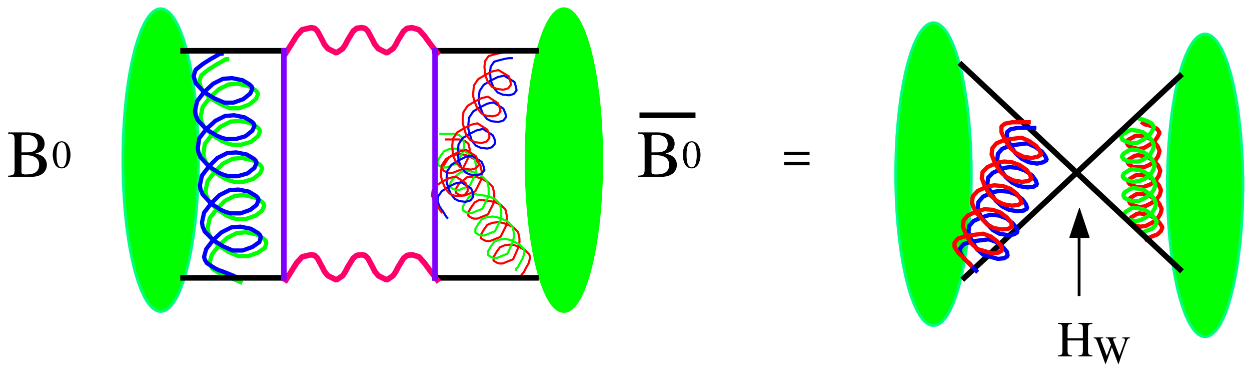
World averages (Bernard, LAT00)
$f_B^{(QA)} = 175 \pm 20$ MeV, 20% larger unq. ?
$f_{D_s}^{(QA)} = 220 \pm 20$ MeV
$f_{B_s}/f_B = 1.15(4)$; $f_{D_s}/f_D = 1.10(6)$
Need to reduce pert. errors by $\mathcal{O}(\alpha_s^2)$ calcs.

2. B_B

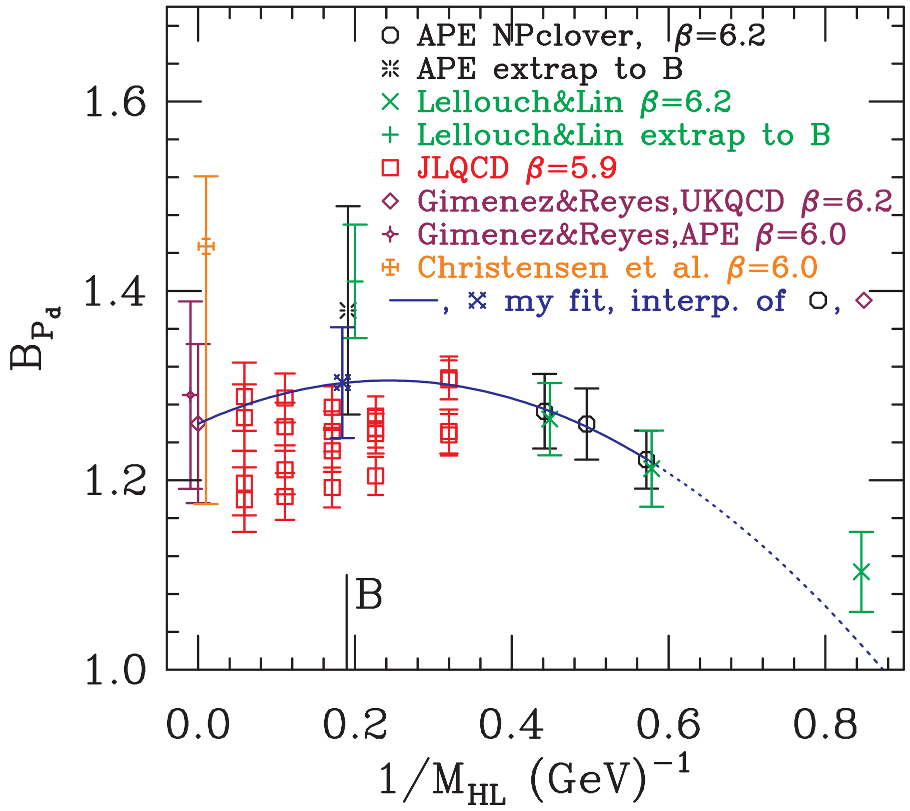
Matrix element for 'box' diagram for $B^0 - \overline{B}^0$ mixing – becomes m.e. of 4-q operator from H_W. Convenient to take ratio to f_B^2, call result B_B.

$\Delta m_B \propto |V_{tb}^* V_{tq}|^2 f_B^2 B_B$.

$\Delta\Gamma$ depends on B_S.



Results and matching in QA not as good as for f_B. (Bernard, LAT00)



World averages:

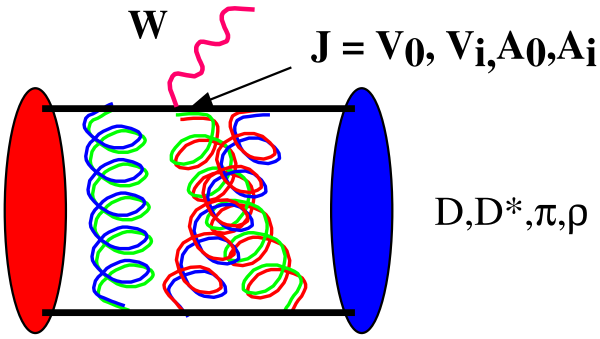
$\hat{B}_{B_d} = 1.30(12)(13)$; $f_{B_d}\sqrt{\hat{B}_{B_d}} = 230(40)$ MeV.

$\hat{B}_{B_s}/\hat{B}_{B_d} = 1.00(4)$;

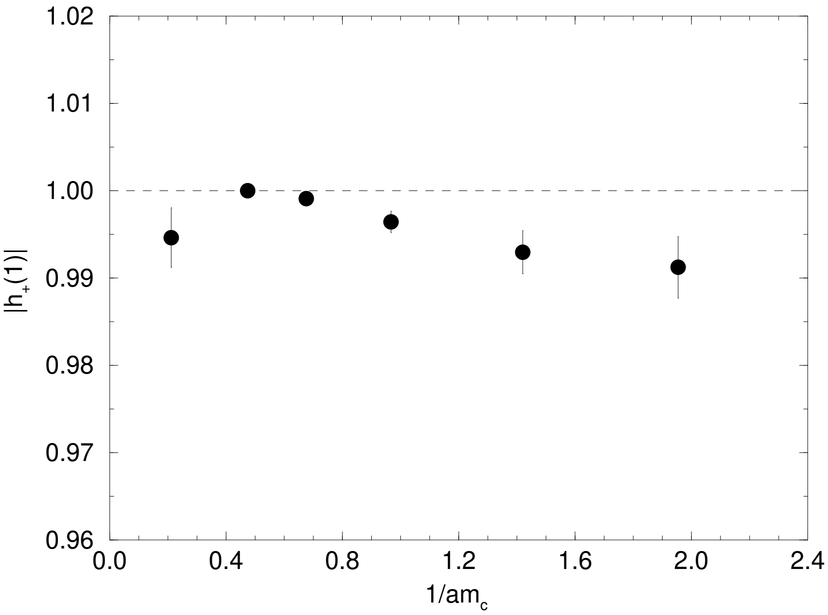
$$\xi \equiv \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.16(5)$$

$B_S^{\overline{MS}}(m_b) = 0.82(8)$

3. B SL decay.

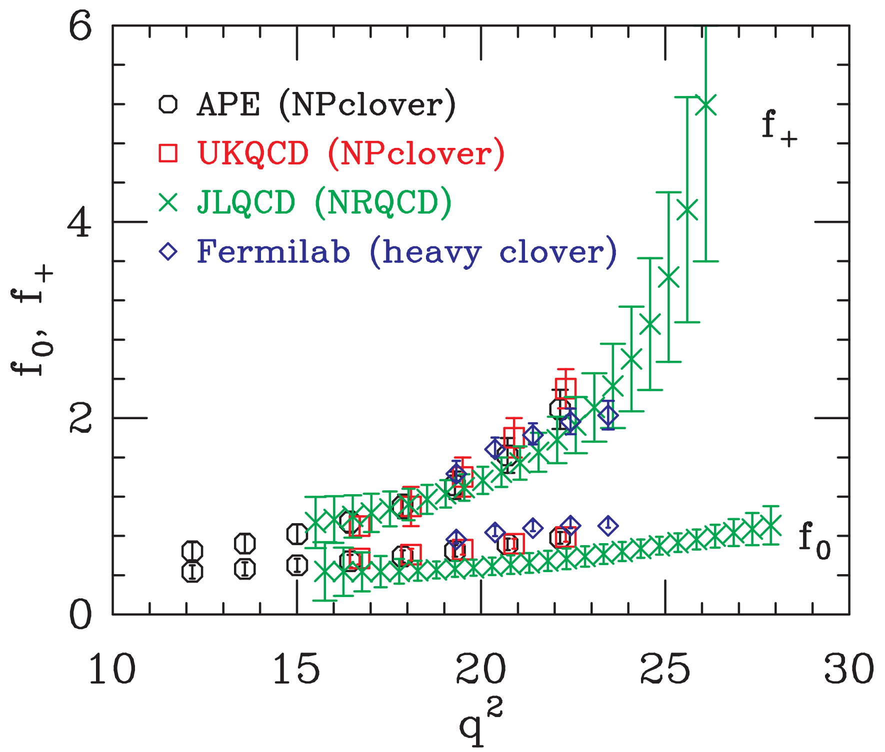


$B \to D^{(*)}$ decay.
exptl rate$(v_B \cdot v_D = 1) = |V_{cb}|^2 \mathcal{F}(1)$; $\mathcal{F}$ from lattice m.e.
Best results from FNAL (QA): $\mathcal{F}_{B\to D}(1) = 1.058(20)$.



$B \to \pi, \rho$ decay.

Lattice calculations (all QA) work far from physical region (small $\vec{p}_B, \vec{p}_\pi$). Require extrapolation,interpolation, etc. Smooths out v. rough raw data. More work is needed.



Soft pion theorem $f_0(q^2_{max}) = f_B/f_\pi$ doesn't work well.

Conclusions

Lattice spectrum calculations are in relatively good shape. Precise calculations possible with unq. ensembles with light dynamical quarks.

Matrix element calculations for B_B, B SL decay need more work including higher order matching to continuum.

This can be done over next 3 years.