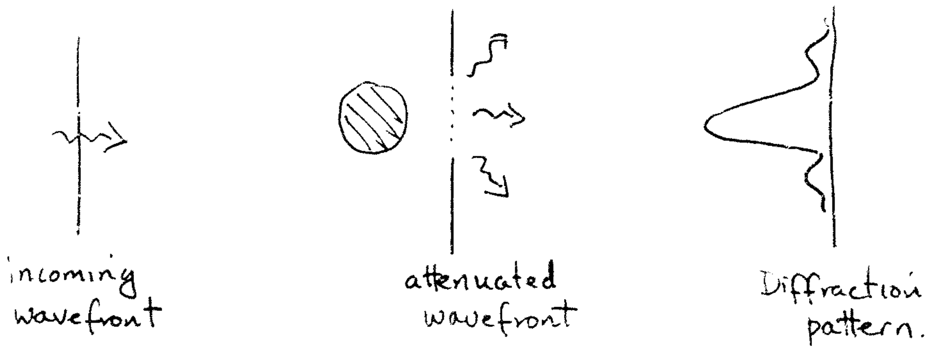
Diffraction

- General introduction: what is diffraction?
- Regge Theory
- Regge Phenomenology
- QCD and the BFKL equation
- BFKL Phenomenology

Diffraction & Small-x Physics

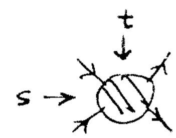
INTRODUCTION.



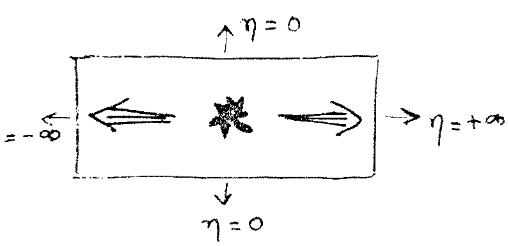
Diffraction is characterised by $\Delta\lambda \approx 0$

ie, $E_{beam} \gg \Lambda$

In $2 \to 2$ Scattering

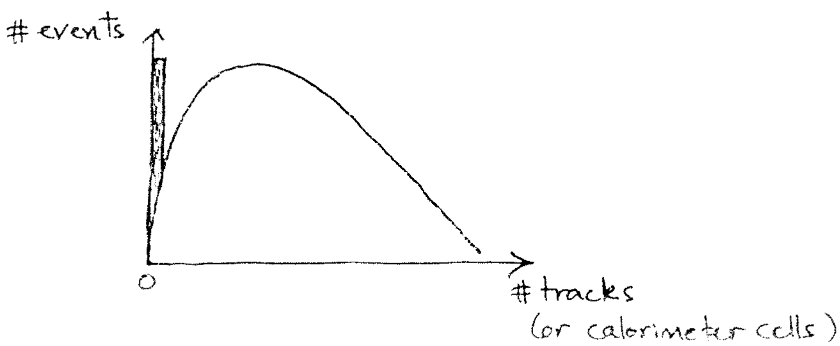


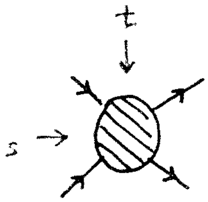
$s \gg -t, m^2$

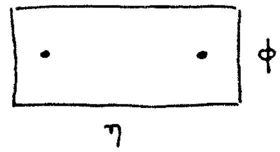


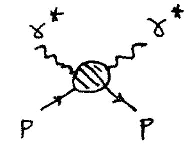
∴ RAPIDITY GAPS are synonymous with diffraction.

Diffraction is DISTINCT and not rare ......

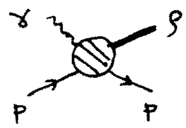




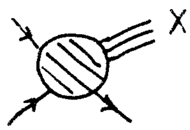


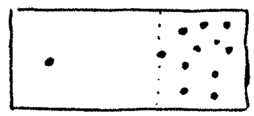
eg 

$\Rightarrow F_2(x, Q^2)$



"Elastic" Scattering

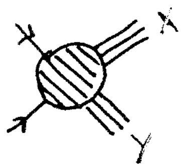


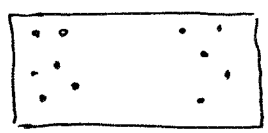


Single Diffraction

eg X contains jets, or $W^{\pm}$ etc (Tevatron, HERA)

we speak of "hard diffraction".



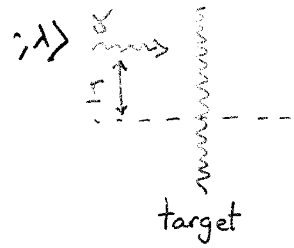


Double Diffraction

HERA ep collisions at $\sqrt{s} \simeq 300$ GeV

TEVATRON $p\bar{p}$ collisions at $\sqrt{s} \simeq 1.8$ TeV

Optical diffraction



Consider the incoming state

$$|in\rangle = \sum_{\lambda} \int d^2\underline{r}\; \psi_{in}^{(\lambda)}(\underline{r})\, |\underline{r},\lambda\rangle$$

Target attenuates incoming beam:

$$\int d^2\underline{r} \sum_{\lambda,\lambda'} \underline{\underline{\hat{T}_{\lambda\lambda'}}}(\underline{r})\; \psi_{in}^{(\lambda')}(\underline{r})\, |\underline{r},\lambda'\rangle$$

nb $\hat{T}$ keeps E and $\underline{r}$ fixed.

eg a sheet of polaroid takes $|in\rangle = \int d^2\underline{r}\; C\, |\underline{r},\lambda_0\rangle$

and turns it into $\hat{T}|in\rangle = \int d^2\underline{r}\; \frac{C}{\sqrt{2}} \left(|\underline{r},\lambda_0\rangle + |\underline{r},\lambda_1\rangle\right)$

if polaroid axis is 45° to initial pol$^{\underline{n}}$ (λ_0).

nb "new" state

Amplitude for scattering into $|\underline{b},\lambda_f\rangle$ is

$$\langle \underline{b},\lambda_f | \sum_{\lambda,\lambda'} \int d^2\underline{r}\; \hat{T}_{\lambda\lambda'}(\underline{r})\, \psi_{in}^{(\lambda')}(\underline{r})\, |\underline{r},\lambda'\rangle = \tilde{A}(\underline{b},\lambda_f)$$

Fourier transform to get angular distribution:

$$A(\underline{q},\lambda_f) = \int d^2\underline{b}\; e^{i\underline{q}\cdot\underline{b}}\; \tilde{A}(\underline{b},\lambda_f)$$

$|\underline{q}|$ = momentum transfer $= E \sin\theta$

Key characteristics :

1. Target "slices out/attenuates" the beam
   ie, impact parameter and energy are unaltered by the interaction.
   (energy: ie. λ.)

2. New states (i.e. of polarization) can be generated. This requires an energy degeneracy.

~~eg~~ Single slit $\quad |in\rangle = \sum_\lambda \int dr\, A_0\, |r,\lambda\rangle$

$$\hat{T}|in\rangle = \sum_\lambda \int_{-\frac{a}{2}}^{a/2} dr\, A_0\, |r,\lambda\rangle\, \Theta(a/2 - |r|)$$

$$\tilde{A}(b,\lambda_f) = A_0\, a\, \Theta(a/2 - |r|)$$

$$A(q,\lambda_f) = \int_{-\infty}^{\infty} db\, e^{iqb} A_0\, a\, \Theta(\tfrac{a}{2} - |r|)$$

$$\sim \int_{-a/2}^{a/2} db\, e^{iqb} \sim \frac{\sin(\frac{1}{2}qa)}{\frac{1}{2}qa}$$

Diffraction in strong interaction physics is completely analogous ....

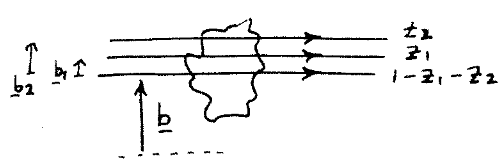
Consider a beam of N "partons" described by a wavefunction $\psi_Q(\{z_i\},\{\underline{b}_i\})$

Discrete quantum #'s. energy fraction (of beam) carried by parton i impact parameter of parton i relative to "centre" of beam.

$$|i\rangle = \sum_Q \prod_{j=1}^{N-1}\left(\int dz_j \int d^2\underline{b}_j\right) \psi_Q(\{z_j\},\{\underline{b}_j\}) \, |\{z_j\},\{\underline{b}_j\}\rangle$$

ie, We decompose the beam into a superposition of diffraction eigenstates.

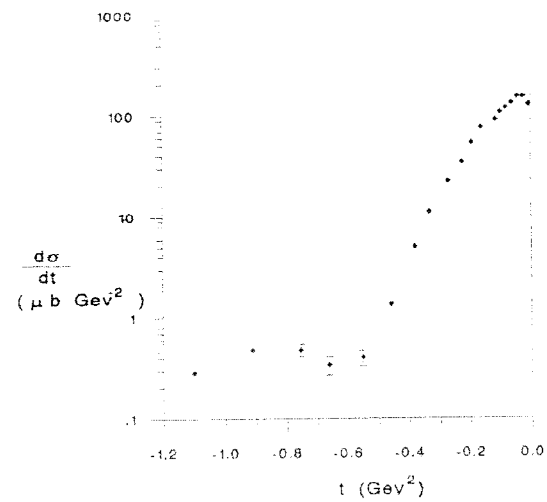
$$\hat{T}\,|\{z_j\},\{\underline{b}_j\}\rangle = T(\{z_j\},\{\underline{b}_j\};\underline{b},E)\,|\{z_j\},\{\underline{b}_j\}\rangle$$



q the momentum conjugate to $\underline{b}$ is the transverse momentum imparted to the target on scattering; "the momentum transfer".

As in the optical case

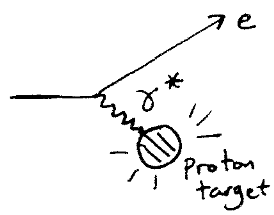
$$A(E,q) = \int d^2\underline{b}\; e^{iq\cdot\underline{b}} \langle f|\hat{T}|i\rangle$$

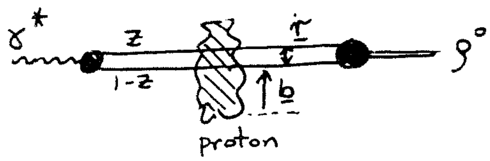


$\pi^- p \rightarrow \pi^0 n$

$E_{beam} = 20.8$ GeV.

A topical example : $\gamma^* p$ scattering





Since $E_{CM}^2 \gg m_\rho^2, Q^2, q^2$.

ρ° can be diffracted into existence by the strong interaction.

$$|\gamma_\lambda\rangle = |\gamma_\lambda\rangle_{bare} + \int dz\, d^2\underline{r}\; \psi_\lambda(z,\underline{r})\, |z,\underline{r}\rangle \; + \left(\begin{array}{c}\text{Higher Fock}\\ \text{states}\end{array}\right)$$

no strong interactions with the proton target.

Amplitude of scattering the ρ° to angle θ $\left(|q| \simeq E\sin\theta\right)$ is thus

$$A(E,q) = \int d^2\underline{b}\; e^{iq\cdot\underline{b}}\, \langle f|\hat{T}|i\rangle$$

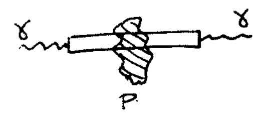
$$= \int d^2\underline{b}\; e^{iq\cdot\underline{b}} \int dz\, d^2\underline{r}\, dz'\, d^2\underline{r}'\; \psi_\rho^*(z',\underline{r}')\, \psi_\lambda(z,\underline{r})$$

$$* \langle z',r'|z,r\rangle\, T(z,\underline{r};\underline{b},E)$$

$$= \int d^2\underline{b}\; e^{iq\cdot\underline{b}} \int dz\, d^2\underline{r}\;\; \psi_\rho^*(z,\underline{r})\, \psi_\lambda(z,\underline{r})\; T(z,\underline{r};\underline{b},E)$$

Quantum mechanics + Definition of diffraction + a few simple properties of QCD = much progress !

Elastic Scattering $\gamma p \to \gamma p$



$$A^{el}(s,t) = \int d^2\underline{b}\, e^{i\underline{q}\cdot\underline{b}} \langle \gamma | \hat{T} | \gamma \rangle$$

$$= \int d^2\underline{b}\, e^{i\underline{q}\cdot\underline{b}} \int dz\, d^2\underline{r}\, |\psi(z,\underline{r})|^2 * T(z,\underline{r};s,\underline{b}) \quad \text{— (1)}$$

$s = 4E_{cm}^2$, $t = -q^2$ } Mandelstam variables.

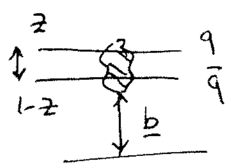
Optical theorem allows us to relate this to the TOTAL cross section for $\gamma p \to X$

$$\frac{\mathrm{Im}\, A^{el}(s,t=0)}{s} = \sigma_{tot}(\gamma p \to X) \quad \text{— (2)}$$

Put (1) in (2) $\Rightarrow$ $\sigma_{tot}(\gamma p \to X) = \int dz\, d^2\underline{r}\, |\psi(z,\underline{r})|^2\, \sigma(z,\underline{r};s)$

$$\equiv \int d^2\underline{b}\, \frac{\mathrm{Im}\, T(z,\underline{r};s,\underline{b})}{s}$$

= total cross-section for scattering the $q\bar{q}$ pair of size $\underline{r}$ off a proton at cm energy $\sim s$. The q & $\bar{q}$ sharing the beam γ energy in ratio $z:(1-z)$.

$\sigma(z,\underline{r};s) =$ 

(same object appears in other processes)

STRATEGY

(1) Compute ψ

$$\psi_L \sim e_q \sqrt{Q^2}\, z(1-z) K_0(\varepsilon r)$$

$$\psi_T \sim e_q \{z^2+(1-z)^2\}^{1/2}\, \varepsilon K_1(\varepsilon r)$$

$$\varepsilon^2 = Q^2 z(1-z) + m^2$$

Bessel functions

(2) Use total cross-section data to <u>EXTRACT</u> the dipole cross-section $\sigma(z, \underline{r}; s)$

$\longrightarrow$ Compare to predictions eg, pQCD; Regge theory; ...

(3) Knowing $\sigma(z, \underline{r}; s)$ can <u>EXTRACT</u> the meson wavefunction $\psi_V(z, \underline{r})$

$\longrightarrow$ Compare to predictions eg, QCD sum rules; Potential models; ..

(4) Make <u>PREDICTIONS</u> for the diffraction dissociation rate ....

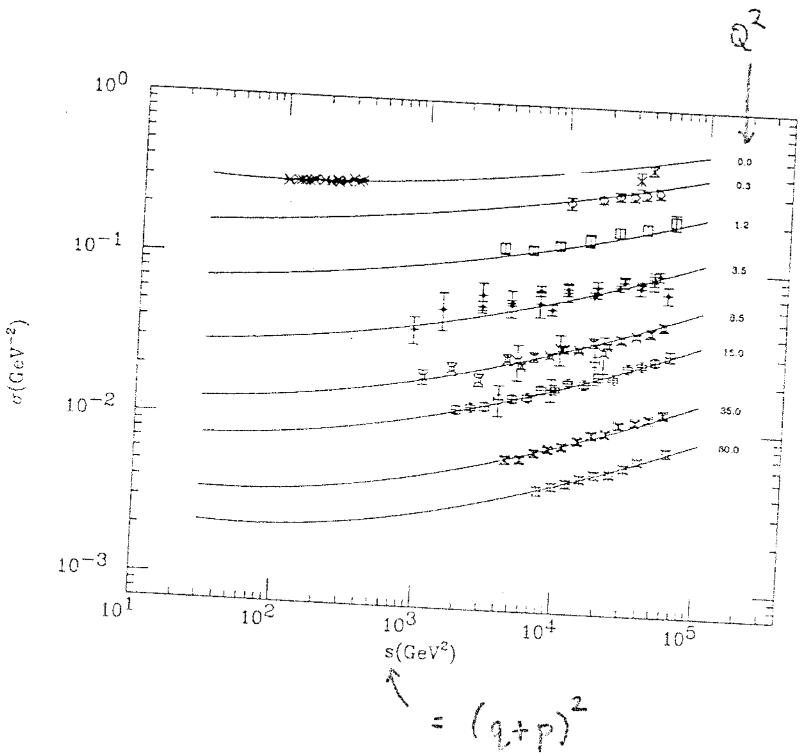


Figure 3: Representative sample of fitted data points for the total cross-section $\sigma_{\gamma p}^{tot}$ compared with curves calculated from the parameterised dipole cross-section for different Q^2 values (fit I).

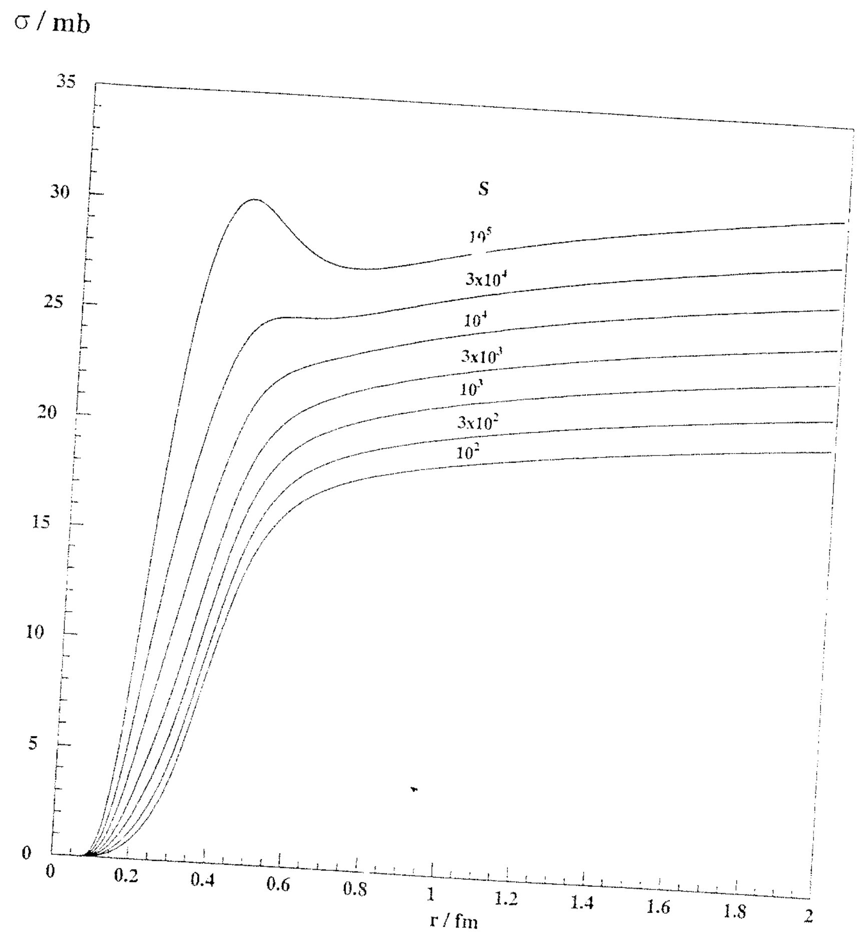
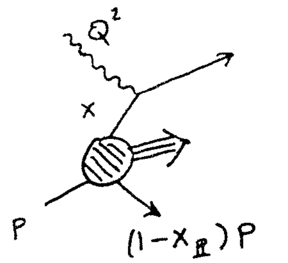
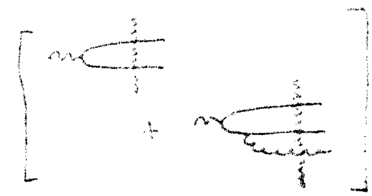
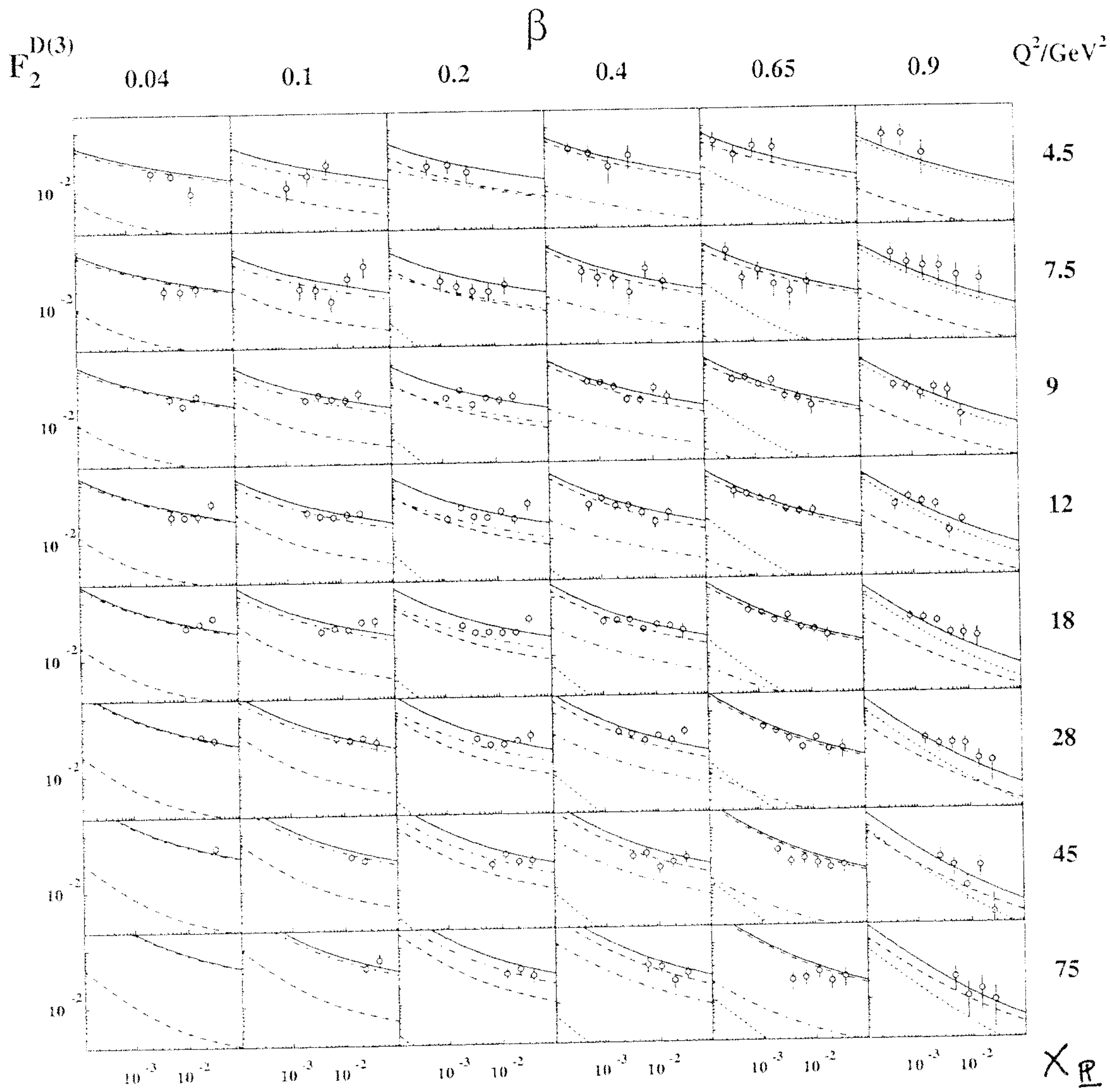


Figure 5: The dipole cross-section at different energies (fit I).





$$x = \frac{Q^2}{2p\cdot q} \equiv \beta\, x_{\mathbb{P}}$$

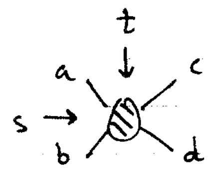


S-Matrix and REGGE THEORY

GOAL to extract physics from general properties of S-matrix

$$S_{ab} = {}_{out}\langle b | a \rangle_{in}$$

1. S-matrix is Lorentz invariant

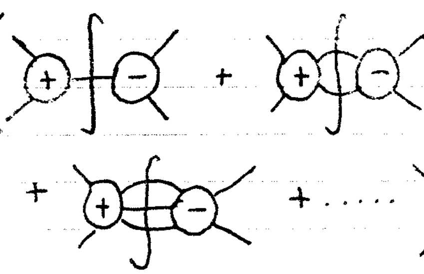


eg $2 \to 2$ scattering : $A(s,t)$ ← Mandelstam variables

2. S-matrix is unitary

$$S_{ab} = \delta_{ab} + i(2\pi)^4 \delta^{(4)}(P_F - P_i)\, A_{ab}$$

$$S S^\dagger = \mathbb{1} \quad \Rightarrow \quad 2\,\mathrm{Im}\, A_{ab} = (2\pi)^4 \delta^{(4)}(P_F - P_i) \sum_X A_{ax} A^\dagger_{xb}$$

$2\,\mathrm{Im}$ () $= (2\pi)^4 \delta^{(4)}(P_f - P_i)$ ()

" Cutkosky rules. "

Special case :

$$2\,\mathrm{Im}\, A_{aa} = (2\pi)^4 \delta^{(4)}(P_f - P_i) \sum_X |A_{a\to x}|^2$$

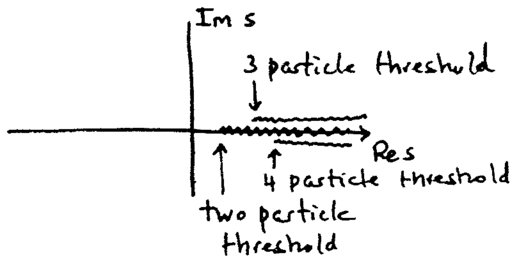
$$\simeq 2s\, \sigma_{total}(a \to X)$$

"optical theorem"

3. <u>S-matrix is an analytic function of Lorentz invariants with only those singularities required by unitarity</u>

(arises from causality)

- Physical thresholds $\Rightarrow$ cuts in complex s-plane



To see this: $\quad \mathrm{Im}\, A(s,t) = \dfrac{A(s,t) - A(s,t)^*}{2i}$

from Cutkosky $\mathrm{Im}\, A = 0$ below threshold ie A is real on a part of real s axis

$$\Rightarrow \quad A(s,t)^* \underset{\uparrow}{=} A(s^*,t) \qquad \text{"Schwarz reflection".}$$

true throughout domain of analyticity.

Hence $\mathrm{Im}\, A(s,t) = \lim\limits_{\varepsilon\to 0} A(s+i\varepsilon,t) = \dfrac{1}{2i} \lim\limits_{\varepsilon\to 0} \left[A(s+i\varepsilon,t) - A(s-i\varepsilon,t)\right]$

Discontinuity across Re s axis i.e. a cut.

- Crossing Symmetry

$$A_{ab\to cd}\,(s,t,u) \quad = \quad A_{a\bar{c}\to\bar{b}d}\,(t,s,u)$$

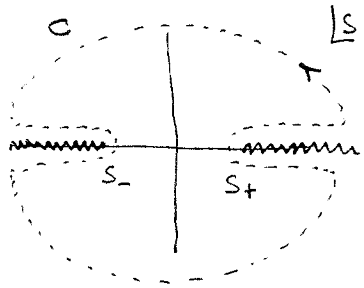
$s>0$, $t,u<0$ — physical region

$t>0$, $s,u<0$ — physical region

$\rightarrow$ Implies cuts along $-$ve Re s axis, due to cross channel thresholds.

- Dispersion relations

Given $\operatorname{Im} A(s,t)$ we can compute $\operatorname{Re} A(s,t)$



$$A(s,t) = \int_C \frac{ds'}{2\pi i} \frac{A(s',t)}{(s'-s)} \quad \text{(Cauchy)}$$

$$= \int_{s_+}^{\infty} \frac{\operatorname{Im} A(s',t)}{s'-s} \frac{ds'}{\pi} + \int_{-\infty}^{s_-} \frac{\operatorname{Im} A(s',t)}{s'-s} \frac{ds'}{\pi}$$

(assuming no single particle poles).

Valid if semi-circle at ∞ vanishes

(need "subtracted" dispersion rel$^{\underline{n}}$s otherwise)

$$\left\{ \text{eg } A(s,t) = A(s_0,t) + \frac{(s-s_0)}{\pi} \int_{s_+}^{\infty} \frac{\operatorname{Im} A(s',t)}{(s'-s_0)(s'-s)} ds' + \frac{(s-s_0)}{\pi} \int_{-\infty}^{s_-} \frac{\operatorname{Im} A(s',t)}{(s'-s_0)(s'-s)} ds' \right\}$$

eg Suppose $\operatorname{Im} A(s,t) \sim (\ln s)^n$

Then $\operatorname{Re} A(s,t) \sim -\frac{(\ln s)^{n+1}}{n+1}$

Focus now on $s \to \infty$ limit of amplitudes

i.e. "<u>Diffractive</u> / <u>Regge</u> <u>Limit</u>"

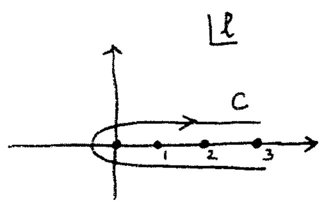
Consider PWE of cross channel process $a\bar{c} \to \bar{b}d$

$$A_{a\bar{c}\to\bar{b}d}(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, a_\ell(s)\, P_\ell(\cos\theta)$$

$$\cos\theta = 1 + 2t/s$$

By crossing symmetry

$$A_{ab\to cd}(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, a_\ell(t)\, P_\ell\left(1+2s/t\right)$$

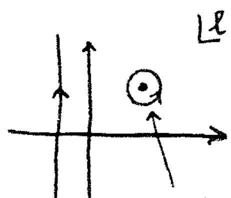


$$= \frac{1}{2i}\int_c d\ell\,(2\ell+1)\,\frac{a(\ell,t)}{\sin\pi\ell}\,P\left(\ell,1+2s/t\right)$$

not unique

$$= \frac{1}{2i}\int_c \sum_\eta d\ell\,(2\ell+1)\,a^{(\eta)}(\ell,t)\left(\frac{\eta+e^{-i\pi\ell}}{2}\right)P\left(\ell,1+\frac{2s}{t}\right)$$

$\eta = \pm 1$

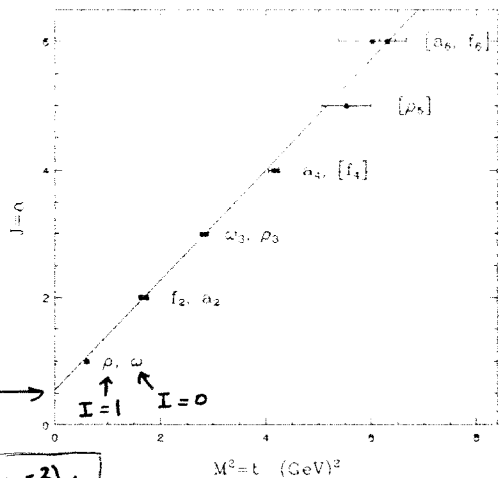


$$\underset{s\to\infty}{\sim} \boxed{\left(\frac{\eta+e^{-i\pi\alpha(t)}}{2}\right)\beta(t)\, s^{\alpha(t)}}$$

simple pole in $a^{(\eta)}(\ell,t)$
at $\ell = \alpha(t) \longleftarrow$ Regge trajectory

In general there can be many cuts and poles but the one with largest real part will dominate as $s \to \infty$.

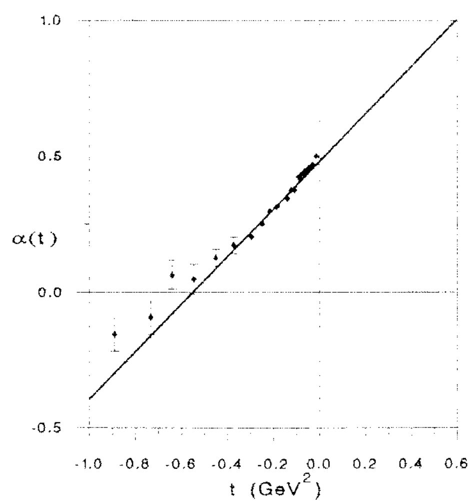
$\alpha(0) \simeq 0.5 \longrightarrow$



Natural parity meson trajectories (degenerate).

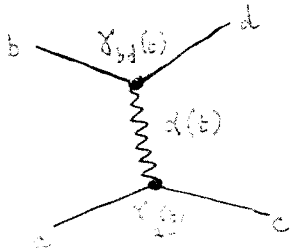
$$\boxed{\alpha(t) \simeq 0.5 + (0.9\ \mathrm{GeV}^{-2})\,t}$$

"REGGE trajectories are linear in t"



from $\pi^- p \to \pi^0 n$ ($I = 1$, even P $\Rightarrow$ ρ trajectory)

(20.8 GeV $< E <$ 199.3 GeV)



If $c = a$, $b = d$ $\Rightarrow$ elastic scattering

In this case poles with $\alpha(0) \geq 1$ are called

POMERON poles.

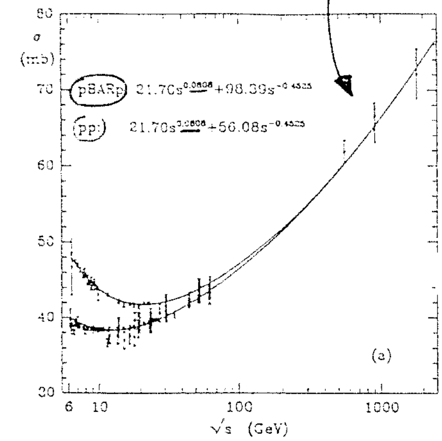
ie, Pomeron is a regge exchange with $\alpha(0) \geq 1$ and vacuum exchange $\left\{ \begin{array}{l} I = 0, \text{ even parity}, \\ \text{even } C \end{array} \right\}$

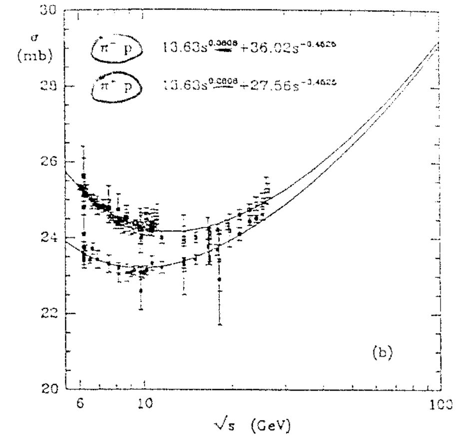
Such an exchange is required by data since total cross-sections are observed to <u>rise</u> as $s \to \infty$.

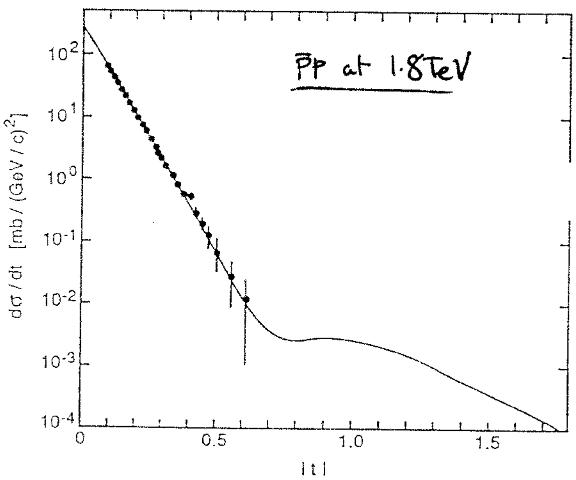
Simplest ansatz : there is a single $\mathbb{P}$ pole ....

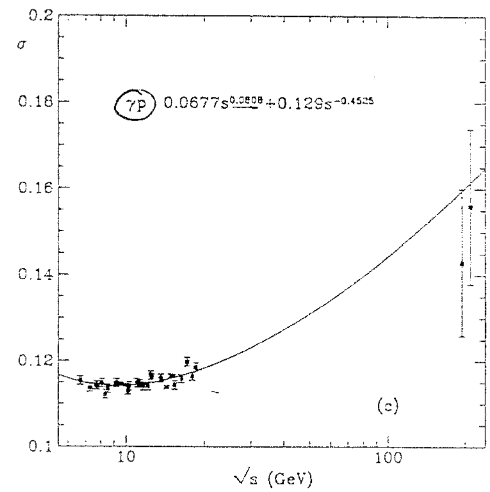
Pomeranchuk Theorem says vacuum exchange if $\alpha(0) \geq 1$.

The ℙ Rise (universal)









$(t) \simeq 1.08$
$\cdot\,(0.25\,\mathrm{GeV}^{-2})t$

⇑

trajectory

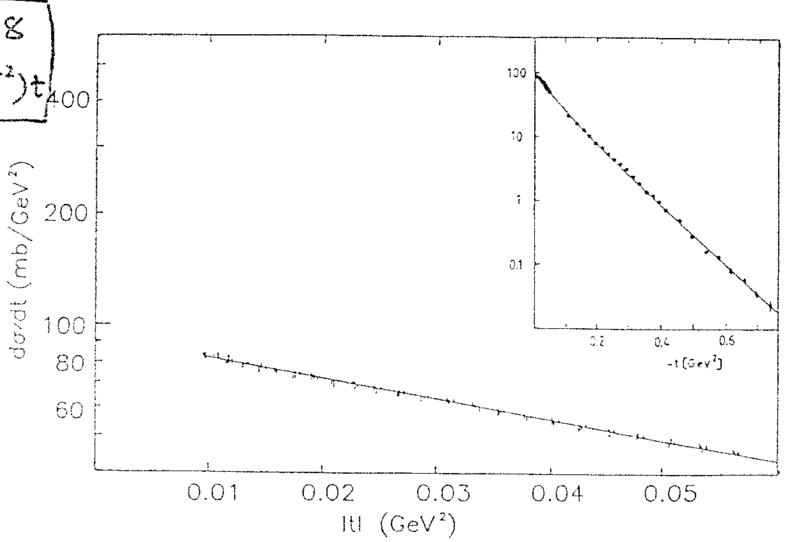


Fig. 2. The pp differential cross section at small $|t|$ at (a) $\sqrt{s} = 52.8$ GeV

CERN-TH.4062/84

JET STRUCTURE IN HIGH MASS DIFFRACTIVE SCATTERING

G. Ingelman
CERN, CH-1211 Geneva 23, Switzerland

P.E. Schlein
University of California *), Los Angeles, CA 90024, USA

ABSTRACT

We suggest that high-p_t jets may emerge from diffractively produced high mass states. Experimental measurements of such high-p_t structure would give new and valuable insight about the nature of the exchanged pomeron, or pomeron-like object. With the assumption of an effective gluon distribution for the pomeron structure, we estimate the cross-section for the process $\bar{p} + p \to \bar{p} + X$, where X contains two high-p_t jets. Observable rates are found at SPS and Fermilab collider energies.

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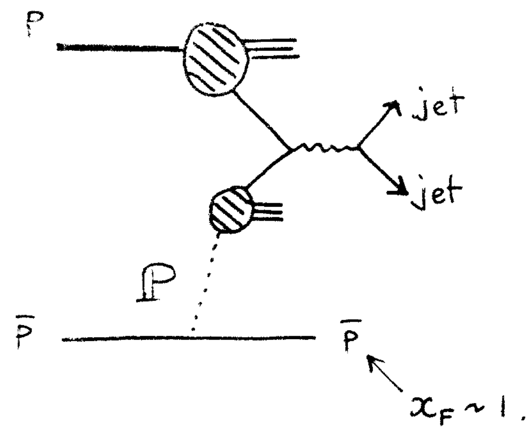
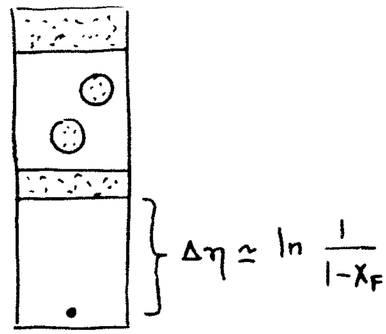
*) Supported by U.S. National Science Foundation Grant PHY79/24766.

CERN-TH.4062/84
November 1984

Ingelman - Schlein Conjecture ......



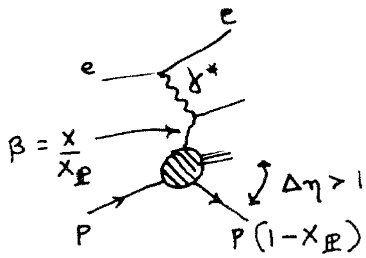


[UA8 (1992)
Evidence for a "super-hard" pomeron.]

$$M_X^2 \frac{d\sigma}{dt\, dM_X^2} = \frac{1}{16\pi} \left|\beta_{\bar{p}\bar{p}}(t)\right|^2 \left(\frac{s}{M_X^2}\right)^{2\alpha_{\mathbb{P}}(t)-2} \sigma_{p\mathbb{P}}(M_X^2, t)$$

"Regge Factorization"

a new $\mathbb{P}$?

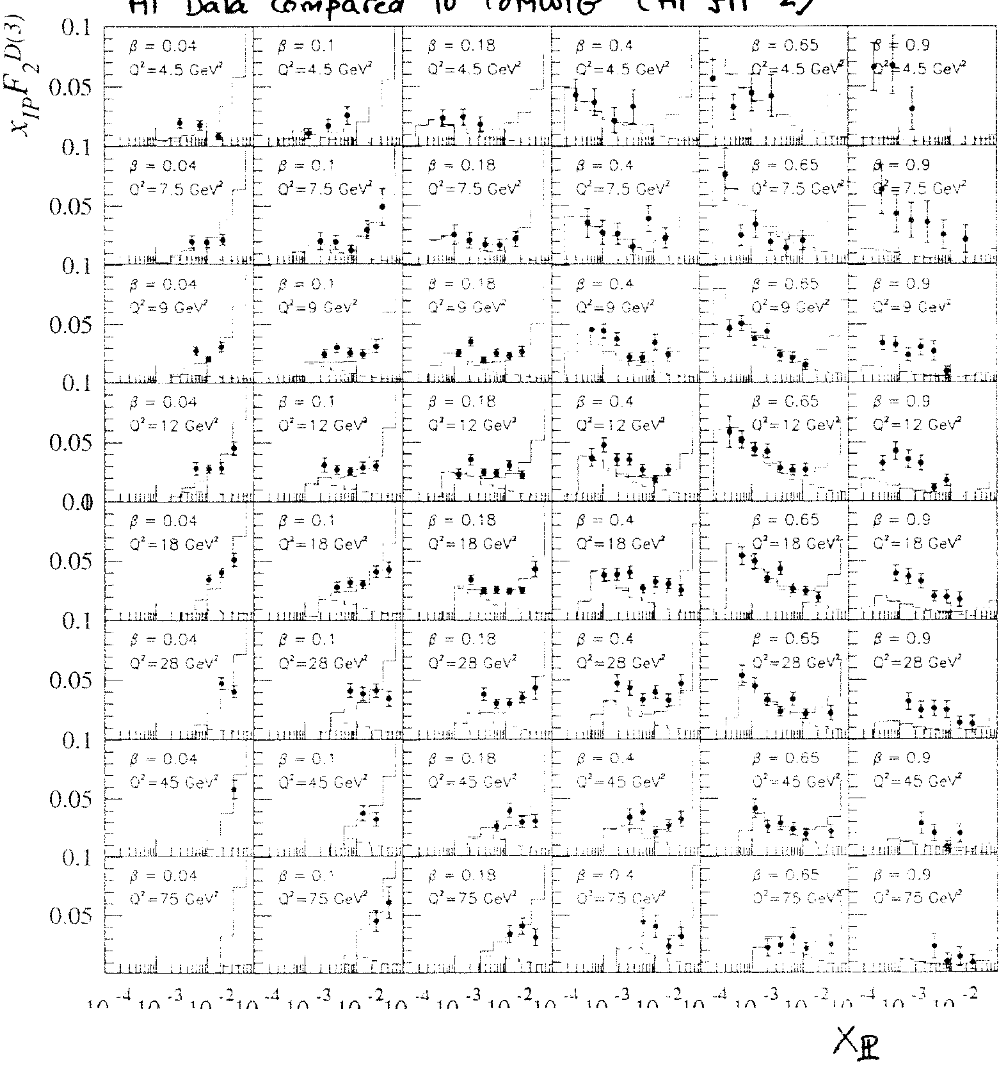


$$\boxed{\begin{array}{ll} \alpha_{\mathbb{P}}(0) \simeq 1.20 \pm 0.03 & \text{H1} \\ \alpha_{\mathbb{P}}(0) \simeq 1.16 \pm 0.04 & \text{ZEUS} \\ \alpha_{\mathbb{R}}(0) \simeq 0.50 \pm 0.10 & \text{H1} \end{array}}$$

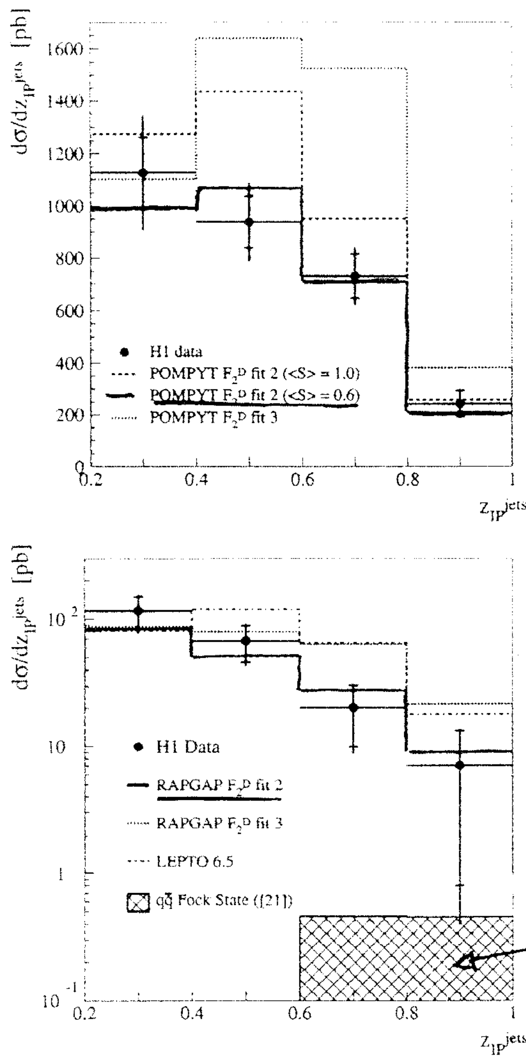
$$\sigma_{\gamma^*\mathbb{P}} \sim \frac{F_2^{D(3)}}{Q^2}$$ ← extract parton densities for $\mathbb{P}$.

[meaningful in γ^* processes (Collins)]

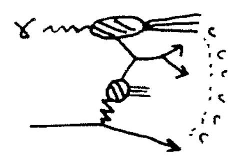
H1 Data Compared to POMWIG (H1 fit 2)



Dijets at HERA



≃ 60% "gap survival"



Need a "IP remnant"

Figure 6: Differential cross sections in $z_{I\!P}^{\rm jets}$ for the production of two jets in the component X of the process $ep \to eXY$. (a) Photoproduction and (b) DIS cross sections measured in the kinematic regions specified in table 1. The shaded bands show the overall normalisation uncertainties. The data are compared to the predictions of the POMPYT (photoproduction) and RAPGAP (DIS) Monte Carlo models with leading order parton densities for the pomeron at a scale set by $\hat{p}_T$ that are dominated by a 'flat' (labelled F_2^D fit 2) and a 'peaked' (labelled F_2^D fit 3) gluon distribution at $\hat{p}_T^2 = 3$ GeV^2 (see [16]). In (a), the prediction of POMPYT for the 'flat' gluon is also shown with a rapidity gap survival probability of 0.6 applied to events with $x_\gamma < 0.8$. Also shown in (b) are the RAPGAP implementation of a calculation [21] of the diffractive scattering of the $q\bar{q}$ fluctuation of the photon and the LEPTO 6.5 model with a probability of 0.5 for soft colour interactions to take place.

Also D^* production

in DIS   ZEUS: DIS ; $Q^2 \simeq 0$ $\langle S \rangle \simeq 50\%$ Conflict ?

H1 : DIS → $\frac{1}{2}$ × theory ?

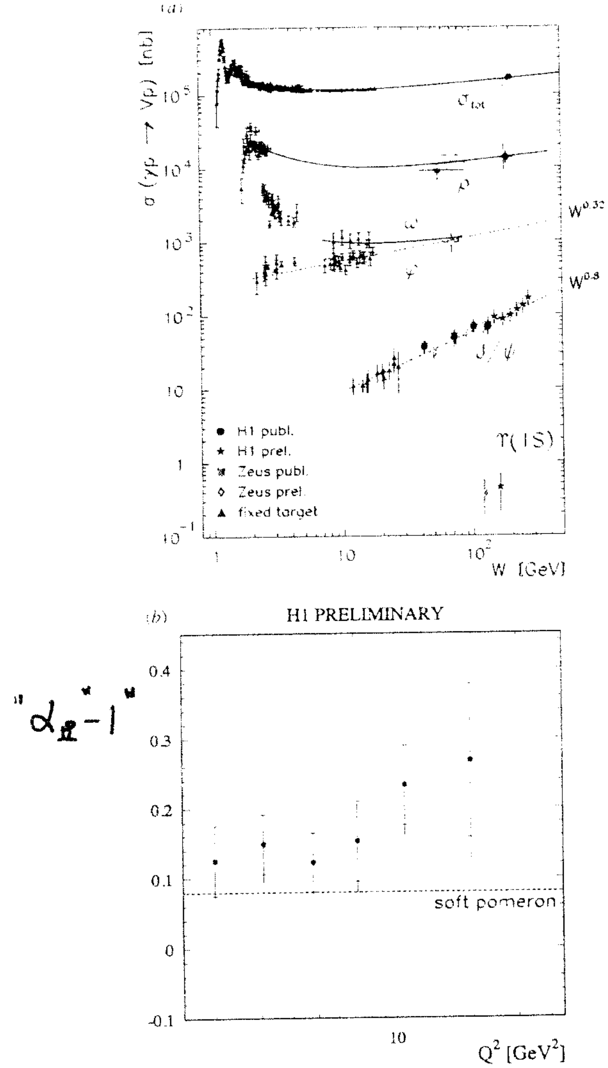


Figure 1. (*a*) The energy dependence of the total photoproduction cross sections for vector meson production at HERA and fixed target experiments, compared with the total γP cross section. (*b*) $\epsilon = \alpha(0) - 1$ in ρ production, as a function of the photon virtuality, Q^2.

Evidence for a steepening of the W distribution as Q^2 rises ....

	EXPERIMENT	CALCULATION	CALC./EXPT.
CDF W	1.15±0.51±0.20%	7%	6±3
CDF RG dijet	0.75±0.05±0.09%	16%	22±3
CDF pot dijet	0.109±0.003±0.016%	4%	34±5
D0 RG dijet	0.67±0.05%	12%	18±1
CDF HQ	0.18±0.03%	30%	167±28
CDF DPE	13.6±2.8±2.0 nb	3713 nb	273±69

* Theory (based on extrapolation of HERA parton densities) overestimates the Tevatron data by large, process dependent, factors.

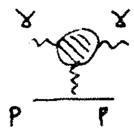
* Collins' proof of factorization only applies to processes where one beam particle is pointlike, e.g. Deep inelastic scattering at HERA.

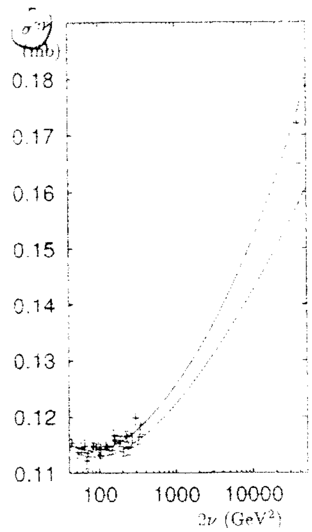
$\Rightarrow$ $\alpha_{\mathbb{P}} = 1.2$ fails spectacularly at Tevatron.
ie. pdfs from HERA.

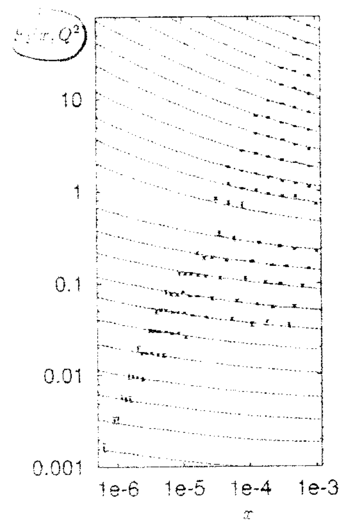
* Perhaps a ~~[illegible]~~ Two pomeron model will work better?

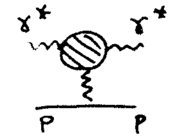
* Perhaps situation is more complicated than can be described by simple regge poles?
  QCD does not obviously lead to such a simple picture
  eg. scaling violations in F_2.
  BFKL ...

Donnachie & Landshoff : Two Pomerons (?)



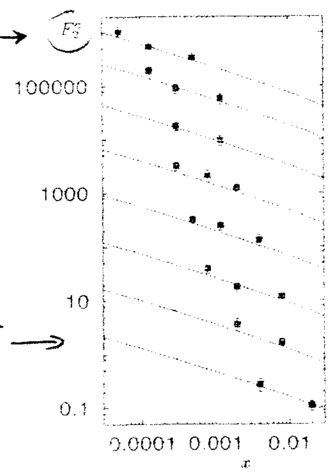




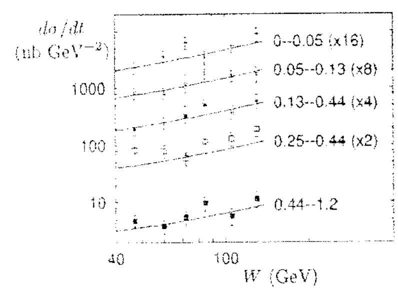


$$F_2 \sim \sum_j f_j(Q^2) \times x^{-(\alpha_j(0)-1)}$$

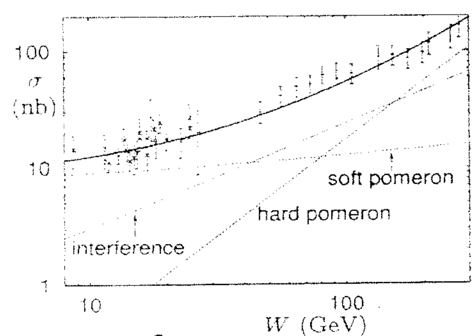
$Q^2 = 1.8$ GeV^2 $\longrightarrow$
$(\times 10^7)$



$Q^2 = 130$ GeV^2 $\longrightarrow$
$(\times 1)$



J/ψ

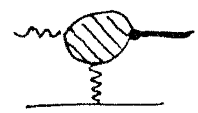


J/ψ

$$\boxed{\begin{aligned} \alpha_0(t) &= 1.08 + 0.25\,t \\ \alpha_1(t) &= 1.44 + 0.1\,t \end{aligned}}$$

Sum over 2 Pomerons and (f_2, a_2).

$$A \sim i \sum_j \beta_j(t)(\alpha_j' s)^{\alpha_j(t)-1} \times e^{-\frac{1}{2} i\pi(\alpha_j(t)-1)}$$

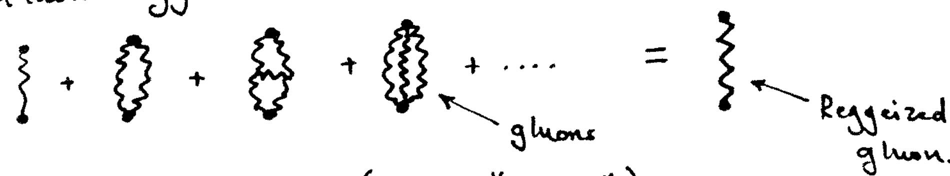


Deriving the BFKL Equation

1) Unitarity & analyticity

   ↓ Cutting rules ↓ Dispersion relations

2) Gluon "reggeization" – Octet exchange



- Soft gluon emission ("eikonal" approx^n)
- Multi-regge kinematics
- The gauge invariant vertex 

3) The BFKL Pomeron – Singlet exchange

Fadin, Kuraev, Lipatov (1976) Sov. Phys. JETP <u>44</u> 443

Balitsky & Lipatov (1978) Sov. J. Nucl. Phys. <u>28</u> 822

The Pomeranchuk singularity in nonabelian gauge theories

É. A. Kuraev, L. N. Lipatov, and V. S. Fadin

Institute for Nuclear Physics, Siberian Division, USSR Academy of Sciences, Novosibirsk
(Submitted June 7, 1976)
Zh. Eksp. Teor. Fiz. 72, 377–389 (February 1977)

An integral equation is derived for the t-channel partial wave amplitudes in the investigation of the multi-Regge form of the $2\to 2+n$ amplitude. For a t-channel state with isospin $T=1$ the solution of this equation is a Regge pole. The analytic properties of the isospin $T=0, 2$ partial wave amplitudes are investigated near the threshold for the production of two or three particles. It is shown that in the j-plane there are moving poles and cuts. For the $T=0$ vacuum channel it was found that the partial wave amplitude has a fixed square-root type branch point to the right of $j=1$.

PACS numbers: 12.40.Mm, 11.80.Et

1. INTRODUCTION

The most attractive models of strong interactions are at present models based on the gauge vector fields of the Yang-Mills[1] type. In distinction from quantum electrodynamics,[2] in these models the interaction vanishes at short distances, leading to an approximately scale-invariant behavior of the hadronic structure functions.[3] The infrared instability of the theory at large distances seems to be the mechanism which confines the quarks within the hadron.[4] The Yang-Mills theory is renormalizable. Moreover, this property is retained in the massive theory which arises from the massless one via the Higgs-Kibble mechanism.[5] For some of the models obtained in this manner factorization relations hold for the Born amplitudes, a necessary condition for the reggeization of vector bosons and spinor particles.[6]

In our preceding papers[7,8] the hypothesis that the Yang-Mills fields reggeize was confirmed to eighth order of perturbation theory (cf. also[9]). It was discovered that the inelastic amplitudes have a multiregge behavior.[8] This gave rise to the hope that in the nonabelian case, in distinction from quantum electrodynamics,[10] the total cross sections will not exceed the Froissart bound as the energy grows.[11] In our preceding note[12] we have shown that in the leading logarithmic approximation, in spite of the multiregge form of the inelastic amplitudes, the total cross sections increase with energy according to a power law. In the present paper we consider questions related to the Pomeranchuk singularity in nonabelian gauge theories in more detail.

In the following section we shall derive a multiregge equation for partial waves with different quantum numbers in the t-channel and show its self-consistency. In Sec. 3 we investigate the analytic properties in t of the partial-wave amplitudes and the moving singularities in the j-plane. In Sec. 4 we consider the leading singularity in the j-plane for the vacuum channel.

2. A MULTIREGGE EQUATION FOR THE t-CHANNEL PARTIAL WAVES

Below we shall consider the simplest model,[13] based on an isotriplet of Yang-Mills vector fields V_μ of mass m, the latter being the result of the appearance of a nonvanishing vacuum expectation value of an isodoublet complex field. In this model there is a scalar field φ necessary for the renormalizability of the theory. The calculation of the asymptotic behavior of the scattering amplitudes for large energies $s^{1/2}$ is carried out in the leading logarithmic approximation:

$$\boxed{g^2\ln\frac{s}{m^2}\sim 1,\quad g^2\ll 1,\quad s=(p_A+p_B)^2\gg m^2,\quad -t\sim m^2.}\tag{1}$$

In a preceding paper[8] we have shown that for an inelastic process in the multiregge kinematics (cf. Fig. 1)

$$s_i=(p_{D_{i-1}}+p_{D_i})^2\gg m^2,\quad -t_i=-q_i^2\sim m^2,\quad \prod_{i=1}^{n+1} s_i=s\prod_{i=1}^{n}(m_i^2+\mathbf{p}_{D_i}^2),\tag{2}$$

which yields the main contribution to the absorptive part of the s-channel elastic amplitude; the corresponding inelastic amplitude has the factorized form (cf. Eq. (55) in[8]):

$$A_{2\to 2+n}=s\Gamma_{AD_0}^{t_1}\frac{(s_1/m^2)^{\alpha(t_1)}}{t_1-m^2}\gamma_{t_1t_2}^{D_1}(q_1,q_2)\frac{(s_2/m^2)^{\alpha(t_2)}}{t_2-m^2}\cdots\gamma_{t_nt_{n+1}}^{D_n}(q_n,q_{n+1})\frac{(s_{n+1}/m^2)^{\alpha(t_{n+1})}}{t_{n+1}-m^2}\Gamma_{BD_{n+1}}^{t_{n+1}}\tag{3}$$

where

$$j=1+\alpha(t)=1+\frac{g^2}{(2\pi)^3}(t-m^2)\int\frac{d^2k}{[(k-q)^2-m^2][k^2-m^2]},\tag{4}$$
$$(t=q^2,\ q=q_\perp,\ k=k_\perp,\ k^2=-\mathbf{k}_\perp^2)$$

is the Regge pole trajectory. For $t=m^2$ this pole traverses the point $j=1$, corresponding to the spin of the original Yang-Mills boson, and this means the reggeization of the latter.

The vertices $\gamma_{t_it_j}^{D}$ are

$$\gamma_{ij}^{D}=mg\delta_{ij}\tag{5}$$

for the emission of scalar particles, and

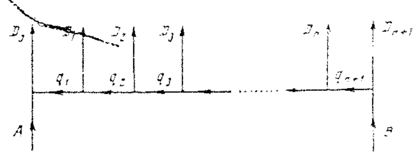
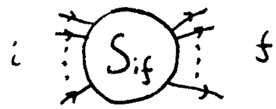


FIG. 1.

"The S-matrix is unitary."

$$S_{if} = \delta_{if} + i(2\pi)^4\,\delta^{(4)}(P_f - P_i)\,\underbrace{A_{if}}_{\text{scattering amplitude}}$$



Conservation of probability :-

$$\sum_x S_{ix} S^{\dagger}_{xi} = 1 \qquad \left(|S_{11}|^2 + |S_{12}|^2 + \dots |S_{1n}|^2 + \dots = 1 \right)$$

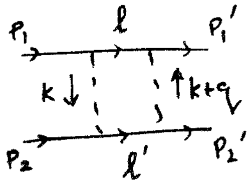
$$\therefore \quad SS^{\dagger} = \mathbb{1}$$

Hence

$$\boxed{2\,\mathrm{Im}\,A_{if} = \sum_n (2\pi)^4 \delta^4(P_n - P_i) \times d(PS^n)\, A_{in}\, A^{\dagger}_{nf}}$$

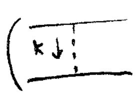
This is a useful way to get the imaginary part of an amplitude. ← avoids loop integrals.

eg/



$= A$

$$\mathrm{Im}\,A = \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^3}\,\delta(\ell^2)\,\frac{d^4\ell'}{(2\pi)^3}\,\delta(\ell'^2)\,.(2\pi)^4\delta(P_1 + P_2 - \ell - \ell')$$

$\times$ () ()†

Analyticity allows us to reconstruct the full amplitude. (given the imaginary part.)

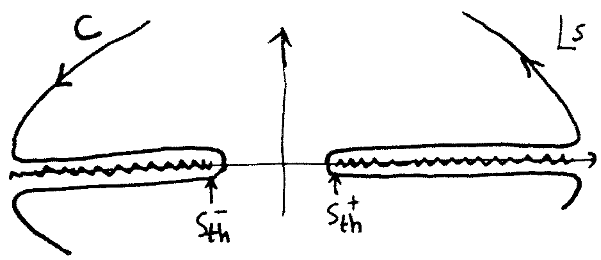
$$\mathrm{Im}\, A(s,t) = \frac{A(s,t) - A(s,t)^*}{2i}$$

$s > s_{th}^+$ $\quad \mathrm{Im}\, A \neq 0$ (unitarity)

$0 < s < s_{th}^+$ $\quad \mathrm{Im}\, A = 0 \quad \Rightarrow \quad A(s,t)^* = A(s^*,t)$

$\therefore$ Need a cut for $s > s_{th}^+$

Crossing implies a cut also for $s < s_{th}^-$



"We assume no more singularities"

So we can write a DISPERSION RELATION

Cauchy $\quad A(s,t) = \dfrac{1}{2\pi i} \oint_C \dfrac{A(s',t)}{s'-s}\, ds'$

Using $\quad \mathrm{Im}\, A(s,t) = \dfrac{1}{2i} \lim_{\varepsilon \to 0} \left[A(s+i\varepsilon,t) - A(s-i\varepsilon,t) \right]$

$$\Rightarrow A(s,t) = A(s_0,t) + \frac{(s-s_0)}{\pi} \int_{s_{th}^+}^{\infty} \frac{\mathrm{Im}\, A(s',t)}{(s'-s)(s'-s_0)}\, ds'$$

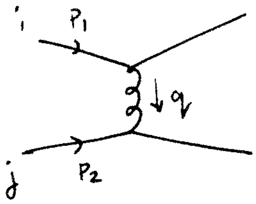
$$+ \frac{(s-s_0)}{\pi} \int_{-\infty}^{s_{th}^-} \frac{\mathrm{Im}\, A(s',t)}{(s'-s)(s'-s_0)}\, ds'$$

eg $\quad \mathrm{Im}\, A(s,t) = \ln^m s$

$\quad \Rightarrow A(s,t) \approx -\dfrac{1}{\pi(m+1)} \ln^{m+1} s \quad$ (for $\ln s \gg 1$)

Gluon Reggeization

Consider $q_i q_j \to q_i q_j$ scattering via octet exchange



& in the REGGE LIMIT $s \gg -t, m^2$

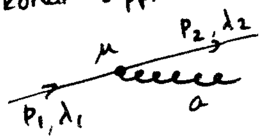
$= (p_1+p_2)^2$ $\quad = q^2$

lowest order contribution to amplitude.

$$A_0^{(8)}(s,t) = g^2 \, (2p_1^\mu) \, \frac{g_{\mu\nu}}{q^2} \, (2p_2^\nu) \, \tau^a \otimes \tau^a \, \delta_{\lambda_1\lambda_1'} \delta_{\lambda_2\lambda_2'}$$

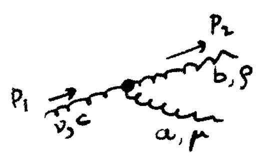
Eikonal factors

Eikonal approximation :



$$-ig\, \bar{u}(\lambda_2, p_2) \gamma_\mu u(\lambda_1, p_1) \tau^a_{ij}$$

$$= -2ig\, p_{1\mu} \delta_{\lambda_1\lambda_2} \tau^a_{ij}$$



$$ig\left(g^{\nu\rho}(2p_1 - q)^\mu + g^{\rho\mu}(2q - p_1)^\nu - g^{\mu\nu}(q + p_1)^\rho\right) T^a_{bc}$$

$(q = p_1 - p_2)$

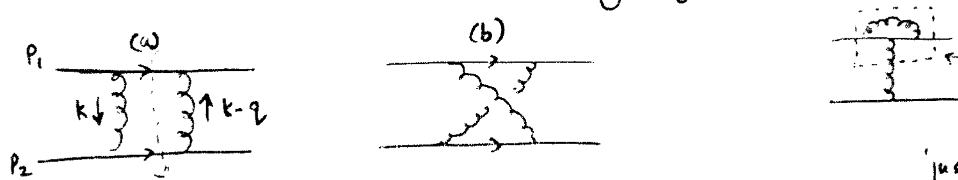
$$= 2ig\, p_1^\mu g^{\nu\rho} T^a_{bc}$$

Valid whenever emitted gluon is soft

(ie all its components are small relative to the largest one of the radiating particle.)

$\mathcal{O}(\alpha_s)$ Corrections

Since $s \gg$ all other invariants only graphs are:



just dresses the vertex i.e. no knowledge of $s = (p_1+p_2)^2$. $\therefore \sim g^4$ [in a covariant gauge]

These graphs will be $\sim g^4 \ln s \gg g^4$

Only need compute diagram (a), (b) is then fixed by crossing $s \leftrightarrow u$

Colour factors:

$$C.F.^{(a)} = (\tau^a \tau^b) \otimes (\tau^a \tau^b)$$

$$C.F.^{(b)} = (\tau^a \tau^b) \otimes (\tau^b \tau^a)$$

Computing the imaginary part:-

$$\mathrm{Im}\, A(s,t)_{(a)} = \frac{1}{2} g^4 \int d(P.S.^2) \left(\frac{2s}{\underline{k}^2}\right)\left(\frac{2s}{(\underline{k}-\underline{q})^2}\right) \delta_{\lambda_1\lambda_1'} \delta_{\lambda_2\lambda_2'} \, C.F.^{(a)}$$

$$\int d(P.S.^2) = \int \frac{d^4 k}{(2\pi)^2} \delta((p_1-k)^2)\, \delta((p_2+k)^2)$$

$p_1 = (E,0,0,E)$
$p_2 = (E,0,0,-E)$
$k_\perp = (0, \underline{k}, 0)$

Introduce SUDAKOV variables

$$k^\mu = \rho p_1^\mu + \lambda p_2^\mu + k_\perp^\mu \qquad (k_\perp^\mu k_{\perp\mu} = -\underline{k}^2)$$

So
$$\int d(P.S.^2) = \frac{s}{8\pi^2} \int d\rho\, d\lambda\, d^2\underline{k}\; \delta(-s(1-\rho)\lambda - \underline{k}^2)\, \delta(s(1+\lambda)\rho - \underline{k}^2)$$

Regge kinematics $\underline{k}^2 \ll s$ (typically)

$$\therefore \quad \rho \approx \underline{k}^2/s, \quad \lambda \doteq -\rho \qquad \lambda, \rho \ll 1$$

justifies use of eikonal approx$^{\underline{n}}$.

Putting $|\lambda|, \rho \ll 1$ in $d(P.S.^2)$ & $k_\mu k^\mu \approx -\underline{k}^2$

$$\Rightarrow \operatorname{Im} A(s,t)_{(a)} \simeq 8\pi\alpha_s \frac{s}{t} \delta_{\lambda_1\lambda_1'} \delta_{\lambda_2\lambda_2'} \, C.F.^{(a)} \qquad (t \simeq -q^2 < 0)$$
$$* \frac{\alpha_s}{2\pi} \int d^2\underline{k} \frac{-q^2}{\underline{k}^2 (\underline{k}-\underline{q})^2}$$

Using dispersion relation

$$\Rightarrow A(s,t)_{(a)} \approx \left(-\frac{\ln s}{\pi}\right) \operatorname{Im} A(s,t)_{(a)} \gg \operatorname{Im} A(s,t)_{(a)}$$

Adding the crossed graph $(s \rightarrow +u, \; u \approx -s, \; C.F.^{(a)} \rightarrow C.F.^{(b)})$

and using $C.F.^{(a)} - C.F.^{(b)} = -\frac{N_c}{2} \underbrace{\tau^a \otimes \tau^a}$ ← lowest order colour factor ie, octet projection.

$$\Rightarrow \boxed{A_1^{(8)}(s,t) = A_0^{(8)}(s,t)\, \varepsilon_g(t) \ln s}$$

$$\varepsilon_g(t) = \frac{N_c \alpha_s}{2\pi} \int \frac{d^2\underline{k}}{2\pi} \frac{t}{\underline{k}^2 (\underline{k}-\underline{q})^2} \quad \leftarrow \text{IR divergent.}$$

<u>NOTE</u> $\ln s \approx \ln s/\underline{k}^2 + c \qquad (c \ll \ln s/\underline{k}^2)$

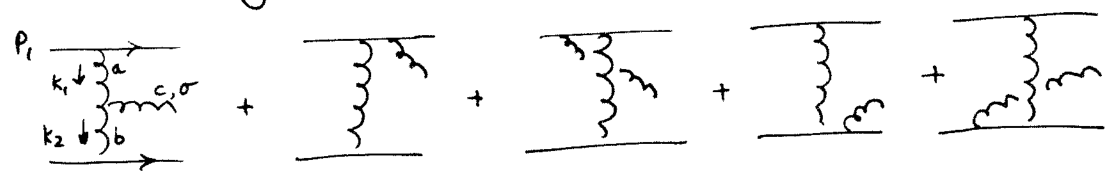
↖ scale is not fixed to leading logarithm accuracy.

[also note (b) has no imaginary part ; so for <u>singlet</u> exchange real parts <u>cancel</u> ($CF^{(a)} = CF^{(b)} = \operatorname{Tr}(\tau^a \tau^b)$) & we're left with a purely imaginary amplitude.]

$\mathcal{O}(\alpha_s^2)$ corrections

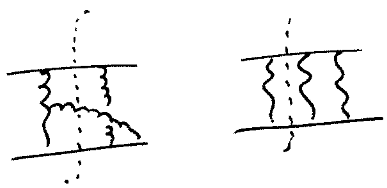
Again compute imaginary part to start.

There are only 5 relevant amplitudes radiating a gluon



plus the virtual graphs

(eg/



contribute to Im A.)

The only graphs we are throwing away involve radiative corrections which are insensitive to s & hence cannot generate logarithms. (as before)

Putting $k_i^\mu = \rho_i p_1^\mu + \lambda_i p_2^\mu + k_{i\perp}^\mu \qquad (i = 1, 2)$

To get the leading logarithmic behaviour

$$\boxed{\begin{array}{l} 1 \gg \rho_1 \gg \rho_2 \\ \text{and} \quad 1 \gg -\lambda_2 \gg -\lambda_1 \end{array}}$$

To see this,

$$\int d(\mathrm{P.S}^3) \simeq \frac{s^2}{128\pi^5}\int d\rho_1\, d\lambda_1\, d^2\underline{k}_1 \quad d\rho_2\, d\lambda_2\, d^2\underline{k}_2$$

$$\delta(-s\lambda_1-\underline{k}^2)\,\delta(s\rho_2-\underline{k}^2)\,\delta(-s\lambda_1\rho_2-\underline{k}^2)$$

$\underline{k}^2=(\underline{k}_1-\underline{k}_2)^2$ [this is derived ASSUMING strong ordering].

$$\simeq \frac{1}{128\pi^5}\int_{\rho_2}^{1}\frac{d\rho_1}{\rho_1}\, d\rho_2\, d^2\underline{k}_1\, d^2\underline{k}_2\;\delta(s\rho_2-\underline{k}^2)$$

$$\simeq \frac{1}{128\pi^5}\int\frac{d^2\underline{k}_1\, d^2\underline{k}_2}{s}\int_{\underline{k}^2/s}^{1}\frac{d\rho_1}{\rho_1}$$

Dominant (leading log) region
is $1 \gg \rho_1 \gg \rho_2 = \underline{k}^2/s$.

$$\text{cf}\int_{\underline{k}^2/s}^{1}\frac{d\rho_1}{\rho_1} = \left[\int_{\underline{k}^2/s}^{\underline{k}^2/s\epsilon_1} + \int_{\underline{k}^2/s\epsilon_1}^{1/\epsilon_2} + \int_{1/\epsilon_2}^{1}\right]\frac{d\rho_1}{\rho_1}$$

$$= \ln 1/\epsilon_1 + \left(\ln \epsilon_1/\epsilon_2 + \ln s/\underline{k}^2\right)$$

$$+ \ln \epsilon_2$$

So providing amplitudes don't induce anything, (eg logs from $\underline{k}_1$ integrals ?) then we have justified strong ordering assumption.

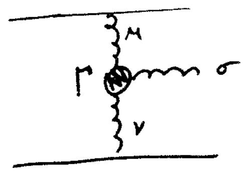
Using $k_i^2 \simeq k_{i\perp}^2 = -\underline{k}_i^2$

and on-shell condition for radiated gluon

ie, $\rho_1 \lambda_2 s = -(\underline{k}_1 - \underline{k}_2)^2$

$$A^{\sigma}_{2\to 3} = -\frac{4ig^3\, p_1^{\mu} p_2^{\nu}}{\underline{k}_1^2\, \underline{k}_2^2}\, \delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}\, f_{abc}\, \tau^a \otimes \tau^b$$

$$* \quad \underline{\Gamma^{\sigma}_{\mu\nu}(k_1,k_2)} \quad =$$



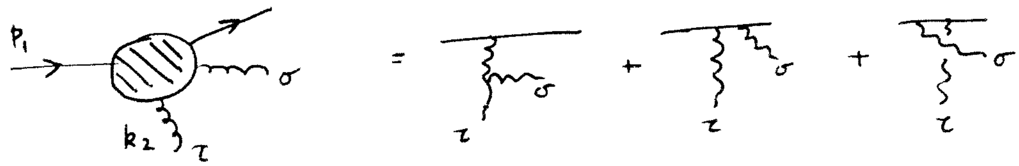
↑ gauge invariant effective vertex

$$\Gamma^{\sigma}_{\mu\nu}\, p_1^{\mu} p_2^{\nu} = \frac{1}{2} s \left[\left(\rho_1 + \frac{2\underline{k}_1^2}{\lambda_2 s}\right) p_1^{\sigma} + \left(\lambda_2 + \frac{2\underline{k}_2^2}{\rho_1 s}\right) p_2^{\sigma} - (k_1+k_2)_{\perp}^{\sigma} \right]$$

(from explicit calculation of the 5 diagrams – not too hard!).

Note $(k_1-k_2)_{\sigma}\, \Gamma^{\sigma}_{\mu\nu}(k_1,k_2) = 0$ (to leading log accuracy)

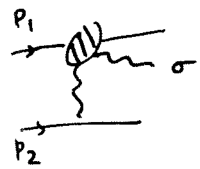
An alternative derivation of $\Gamma^{\sigma}_{\mu\nu}$ (GLR)

$M^{\sigma}_{\tau} =$ 

$$k_2^{\tau} M^{\sigma}_{\tau}(k_1,k_2) = 0$$

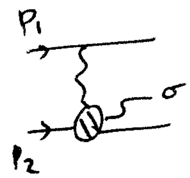
ie, $\lambda_2 p_2^{\tau} M^{\sigma}_{\tau} + k_{2\perp}^{\tau} M^{\sigma}_{\tau} = 0$ (ignore p_2 part of M)

Re-instating lower quark line:

$$A = 2p_2^{\tau} M^{\sigma}_{\tau} = -\frac{2k_{\perp 2}^{\tau}}{\lambda_2} M^{\sigma}_{\tau}(k_1,k_2) =$$ 

But  and  $\sim p_1^{\sigma} p_1^{\tau}$

& since $p_1^{\sigma} k_{\perp\sigma} = 0$ only  survives!

Similarly for $B = 2p_1^{\tau} N^{\sigma}_{\tau} = -\frac{2k_{1\perp}^{\tau}}{\rho_1} N^{\sigma}_{\tau} =$ 

So Ward identities tell us to replace p_i^{μ} (eikonal) vertex factor with $-k_{\perp 1}/\rho_1$ or $-k_{\perp 2}/\lambda_2$ as appropriate

& thus keep only the 1 graph 

$$A^{\sigma}_{2\to 3} = \frac{-4ig^3}{\underline{k}_1^2\,\underline{k}_2^2}\,\frac{k_{1\perp}^{\mu}\,k_{2\perp}^{\nu}}{\rho_1\lambda_2}\,\delta_{\lambda_1\lambda_1'}\,\delta_{\lambda_2\lambda_2'}\,f_{abc}\,\tau^a\otimes\tau^b$$

$$\left[-g_{\mu\nu}(k_1+k_2)^{\sigma} + g^{\sigma}_{\nu}(2k_2-k_1)_{\mu} + g^{\sigma}_{\mu}(2k_1-k_2)_{\nu}\right]$$

Using $\rho_2 \ll \rho_1$ & $|\lambda_1| \ll |\lambda_2|$ gives

$$\Gamma^{\sigma}_{\mu\nu}(k_1,k_2) = (\text{as before}) + \underbrace{\sim (k_1-k_2)_{\sigma}}$$

vanish on contracting with outgoing (on-shell) gluon pol$^{\underline{n}}$ vector.

So

$$\mathrm{Im}\,A_2^{(8)} = -\frac{g_{\sigma\tau}}{2}\int d(P.S.^3)\,A^{\sigma}_{2\to3}(k_1,k_2)\,A^{\tau\,\dagger}_{2\to3}(k_1-q,\,k_2-q)$$

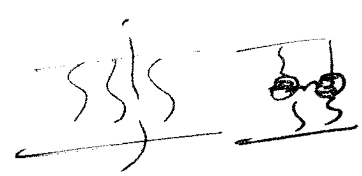
$$+\ (\text{virtual graphs})$$

$$\text{Colour factor} = -f_{abc}\,f_{dec}\,(\tau^a\tau^d)\otimes(\tau^b\tau^e)$$

Anticipating that we'll be adding u-channel contribution to real part, lets antisymmetrize in (b,e)

$$\to\ -\tfrac{1}{2}\left(f_{abc}f_{dec} - f_{aec}f_{cdb}\right)(\tau^a\tau^d)\otimes(\tau^b\tau^e)$$

$$= \frac{N_c^2}{8}\,\tau^a\otimes\tau^a$$



Turning the handle....

$$\mathrm{Im}\, A_2^{(8)} = -\frac{N_c^2 \alpha_s^2}{32\pi^3} A_0^{(8)} q^2 \int_{\underline{k}^2/s}^{1} \frac{d\rho_1}{\rho_1}\, d^2\underline{k}_1\, d^2\underline{k}_2$$

$$* \left[\frac{q^2}{\underline{k}_1^2 \underline{k}_2^2 (\underline{k}_1-q)^2 (\underline{k}_2-q)^2} - \cancel{\frac{1}{\underline{k}_1^2 (\underline{k}_1-\underline{k}_2)^2 (\underline{k}_2-q)^2}} - \cancel{\frac{1}{\underline{k}_2^2 (\underline{k}_1-q)^2 (\underline{k}_1-\underline{k}_2)^2}} \right.$$

$\cancel{(+\ \text{virtuals})}$

nb terms $\sim \dfrac{1}{\underline{k}_1^2 \underline{k}_2^2 (\underline{k}_i-q)^2}$ cancelled

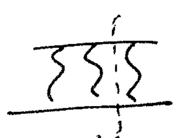
Such terms are UV divergent $\left(\int^\infty d^2\underline{k}_i\right)$ for $\underline{k}_i^2 \lesssim s$ they would have INDUCED additional logarithms.

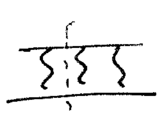
The virtual graphs are straightforward, they contribute

$$-\frac{4g^4}{2}\frac{N_c}{4}\int d(\mathrm{P.S.}^2)\, \frac{s}{\underline{k}_2^2}\, \epsilon_g(\underline{k}_2^2) \ln\frac{s}{\underline{k}^2}\, \frac{s}{(\underline{k}_2-q)^2}\, \delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}$$

(from  calculation)

$$= -\frac{N^2\alpha_s^2}{32\pi^3} A_0^{(8)} q^2 \int \frac{d^2\underline{k}_1\, d^2\underline{k}_2}{\underline{k}_1^2 (\underline{k}_1-\underline{k}_2)^2 (\underline{k}_2-q)^2} \ln\frac{s}{\underline{k}^2}$$

= 

 is obtained by replacing $\underline{k}_1 \leftrightarrow \underline{k}_2$.

$\Longrightarrow$ Cancellation marked above.

So, $A_2^{(8)}$ has an imaginary part given by uncrossed graphs & a dominant real part given$_s$ by

$$\boxed{\mathrm{Re}\, A_2^{(8)} = \frac{1}{2}\, \epsilon_g^2(t) \ln^2\left(\frac{s}{\underline{k}^2}\right) A_0^{(8)}}$$

after using dispersion rel^{n}.

Very suggestive that, to all orders, (ie, fully summing all leading logs)

$$A^{(8)}_{\substack{\text{leading}\\ \text{log}}} = A_0^{(8)}\left(1 + \epsilon_g(t) \ln s/\underline{k}^2 + \tfrac{1}{2}\epsilon_g(t)^2 \ln^2 \frac{s}{\underline{k}^2} + \ldots\right)$$

$$= A_0^{(8)} \left(\frac{s}{\underline{k}^2}\right)^{\epsilon_g(t)} \quad \text{———} \circledast$$

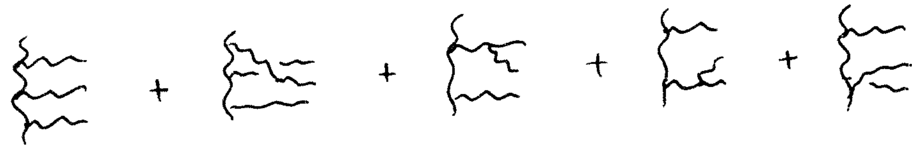
(Cheng & Lo (1976) did $O(\alpha_s^3)$ and showed trend continues)

$\circledast$ is what we mean by "the gluon is "reggeized""

n-gluon emission

Consider a portion of a graph :-

$\circledast$ =



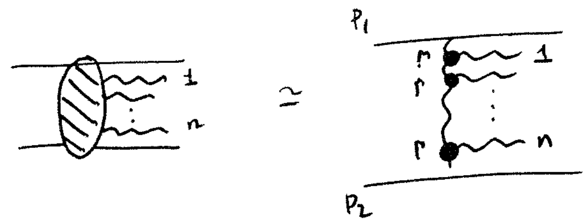
Strong ordering of Sudakov variables continues

$$\Rightarrow \quad d(PS^{n+2}) \sim \left(\prod_{i=1}^{n} \int_{\rho_{i+1}}^{1} d\rho_i/\rho_i\right)\left(\prod_{j=1}^{n+1} d^2\underline{k}_j\right) d\rho_{n+1}\, \delta(s\rho_{n+1}-\underline{k}^2)$$

∴ Eikonal approximation means

$\circledast \simeq \Gamma$

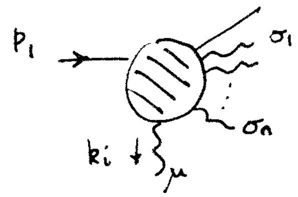
Suggests



$$\rho_1 \gg \rho_2 \ldots \gg \rho_n \;; \quad |\lambda_1| \ll |\lambda_2| \ldots \ll |\lambda_n|$$

$$\left(\underline{k}^2/s \ll \rho_i, |\lambda_i| \ll 1\right)$$

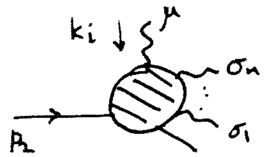
Using the Ward identity "trick" can prove this is so



$$k_i M_\mu^{\sigma_1..\sigma_n}(p_1, k_1, .., k_n, k_i) = 0$$

i.e. $$k_{i\perp}^\mu M_\mu^{\sigma_1..\sigma_n} = -\lambda_i p_2^\mu M_\mu^{\sigma_1..\sigma_n}$$

& Similarly



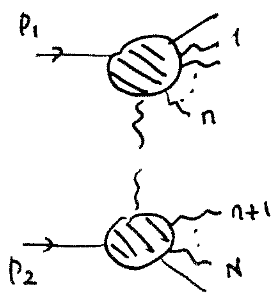
$$k_{i\perp}^\mu N_\mu^{\sigma_1..\sigma_n} = -\rho_i p_1^\mu N_\mu^{\sigma_1..\sigma_n}$$

Now $M_\mu^{\sigma_1..\sigma_n} = p_{1\mu} M^{\sigma_1..\sigma_n}$ (+ small)

$$\therefore M^{\sigma_1..\sigma_n} = \frac{2 M_\mu^{\sigma_1..\sigma_n} p_2^\mu}{s}$$

& $N_\mu^{\sigma_1..\sigma_n} = p_{2\mu} N^{\sigma_1..\sigma_n}$ (+ small)

$$\therefore N^{\sigma_1..\sigma_n} = \frac{2 N_\mu^{\sigma_1..\sigma_n} p_1^\mu}{s}$$



$$= M_\mu^{\sigma_1...\sigma_n} N^{\mu\, \sigma_{n+1}..\sigma_N} / k_i^2$$

$$= \tfrac{1}{2} s\, M^{\sigma_1..\sigma_n} N^{\sigma_{n+1}..\sigma_N} / k_i^2$$

$$= \frac{2}{s}\left(-\frac{k_{i\perp}^\alpha}{\lambda_i}\right)\left(-\frac{k_{i\perp}^\beta}{\rho_i}\right) \frac{M_\alpha^{\sigma_1..\sigma_n} N_\beta^{\sigma_{n+1}\sigma_N}}{k_i^2}$$

So imposing the Ward identity allows us to write down a gauge invariant subset of graphs for n-gluon emission :-

ie, $$A^{2\to 2+n}_{\sigma_1\ldots\sigma_n} = 2isg^2 \,\frac{i}{k_1^2}\, \delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}\, \tau_{a_1}\otimes\tau_{a_{n+1}}$$

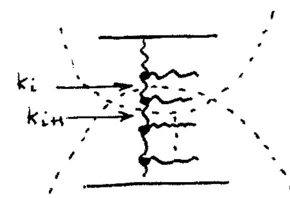
$$\prod_{i=1}^{n}\left[f_{a_i a_{i+1} b_i}\,\frac{ig}{k_{i+1}^2}\left[\frac{2k_{i\perp}^{\mu_i}\,k_{i+1\perp}^{\nu_i}}{\rho_i\,\lambda_{i+1}\,s} \;\Lambda_{\mu_i\nu_i\sigma_i}(k_i,k_{i+1})\right]\right]$$

$$= \frac{2p_2^{\mu_i}\,p_1^{\nu_i}}{s}\,\Gamma_{\mu_i\nu_i\sigma_i}$$

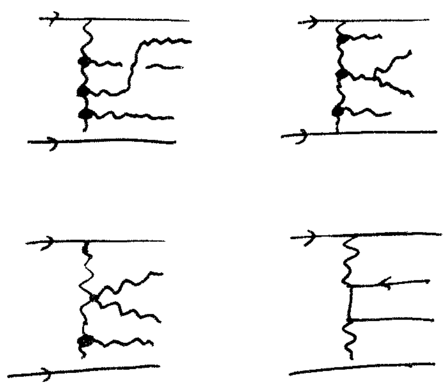
usual triple gluon vertex

We have seen that, for $n=1$, this gives the dominant contribution.

It also gives the dominant contribution for general n



ie,



} sub-leading

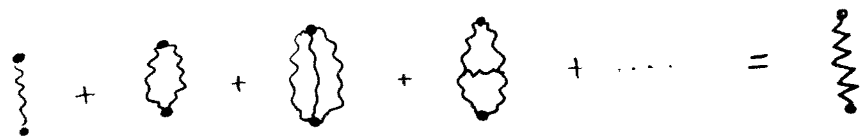
$\Rightarrow$ $\boxed{\text{Effective vertex gives leading logarithm contrib}^{\underline{n}}\text{ as sum of simple (uncrossed) ladders.}}$

Virtual corrections – "Ladders within ladders"

We suspect that leading logarithmic contribution to $qq \to qq$ via octet exchange is

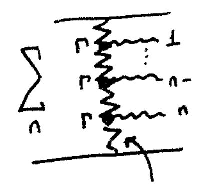
$$= A_0^{(8)} \left(\frac{s}{-t}\right)^{\epsilon_g(t)} \qquad \text{—— ①}$$

ie, virtual corrections dress (Reggeize) the bare t-channel gluon.



Strategy : (a) Assume ① is true

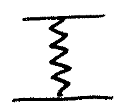
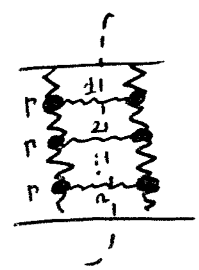
(b) Compute $A_{2\to 2+n}$ from $\sum_n$



t-channel gluons dressed up.

(c) Use $A_{2\to 2+n}$ to compute octet amplitude

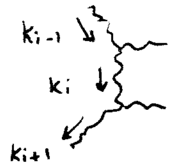
ie, Demonstrate that

Im  $= \sum_{n=0}^{\infty}$ 

This demonstrates the self-consistency of the ansatz

So $$A^{\sigma_1 \ldots \sigma_n}_{2\to 2+n} = i\, 2s\, g^{n+2}\, \delta_{\lambda_1\lambda_1'}\, \delta_{\lambda_2\lambda_2'}\, \tau^{a_1} \otimes \tau^{a_{n+1}} \frac{i}{\underline{k}_1^2}$$

$$\times \underline{\underline{\left(\frac{1}{\rho_1}\right)}}^{\epsilon_g(k_1^2)} \prod_{i=1}^{n} \frac{2p_1^{\mu_i} p_2^{\nu_{i+1}}}{s} \Gamma^{\sigma_i}_{\mu_i \nu_{i+1}} \frac{i}{\underline{k}_{i+1}^2} \underline{\underline{\left(\frac{\rho_i}{\rho_{i+1}}\right)}}^{\epsilon_g((k_{i+1})^2)} f_{a_i a_{i+1} b_i}$$



$$\hat{s}_i = (k_{i-1} - k_{i+1})^2$$

$$\approx -\rho_{i-1} \lambda_{i+1} s$$

$$\approx \frac{\rho_{i-1} (\underline{k}_i - \underline{k}_{i+1})^2}{\rho_i}$$

$$(k_i - k_{i+1})^2 = 0$$

$$\Rightarrow -\rho_i \lambda_{i+1} s = (\underline{k}_i - \underline{k}_{i+1})^2$$

$$\mathrm{Im}\, A^{(8)}(s,t)$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \int d(\mathrm{P.S.}^{n+2})$$

$$* A^{\sigma_1 \ldots \sigma_n}_{2\to 2+n} (k_1, \ldots, k_n)$$

$$* A^{+}_{2\to 2+n\, \sigma_1 \ldots \sigma_n} (k_1 - q, \ldots, k_n - q)$$

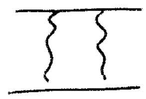
Colour factor $= \tau^a \otimes \tau^a \left(\frac{N_c}{2}\right)^n \frac{N_c}{4}$

Skipping the algebra

$$\Rightarrow \boxed{A^{(8)} = 8\pi \alpha_s \frac{\underline{k}^2}{t} \tau^a \otimes \tau^a\, \delta_{\lambda_1\lambda_1'}\, \delta_{\lambda_2\lambda_2'} \left(\frac{s}{\underline{k}^2}\right)^{1+\epsilon_g(t)} \times \frac{\left(1 - e^{i\pi(1+\epsilon_g(t))}\right)}{2}}$$

$\alpha_g(t) = 1 + \epsilon_g(t)$ is the gluon "Regge trajectory".

Singlet Exchange – The Pomeron

Lowest order  gives $\text{Im}\, A_1^{(1)} = 4i\alpha_s^2\, s\, \delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}$

$$* \left(\frac{\text{Tr}(\tau_a\tau_b)\,\text{Tr}(\tau_a\tau_b)}{N_c^2} \right) \leftarrow \frac{N_c^2-1}{4N_c^2}$$

[as in octet exchange but different colour factor.]

$$* \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{q})^2}$$

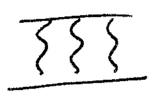
$\text{Re}\, A_1^{(1)} = 0$ $\because$ u-channel cancels (same colour factor)

1 gluon emission

As in octet but colour factor

$$\frac{1}{N_c}\text{Tr}(\tau_a\tau_b)\,\text{Tr}(\tau_c\tau_d)\, f_{ace}\, f_{bde}$$

$$= N_c \left(\frac{N_c^2-1}{4N_c^2}\right)$$



$$\frac{1}{N_c^2}\,\text{tr}(\tau_a\tau_b\tau_c)\,\text{tr}(\tau_a\tau_b\tau_c) = \frac{N_c}{2}\left(\frac{N_c^2-1}{4N_c^2}\right)$$

$\therefore$ Colour factors are different we no longer get the cancellation of terms that occurred for octet

ie, 2-loop is not $\propto$ 1-loop

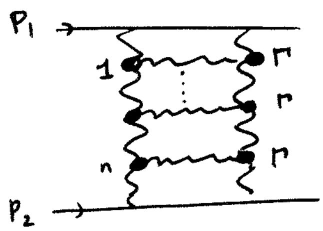
(no exponentiation)

$$\mathrm{Im}\, A_2^{(1)} = \frac{-2N_c\alpha_s^3}{\pi^2}\, s\, \delta_{\lambda_1\lambda_1'}\,\delta_{\lambda_2\lambda_2'} \left(\frac{N_c^2-1}{4N_c^2}\right) \ln\frac{s}{\underline{k}^2}$$

$$\int d^2\underline{k}_1\, d^2\underline{k}_2 \left[\frac{q^2}{\underline{k}_1^2\underline{k}_2^2\,(\underline{k}_1-\underline{q})^2(\underline{k}_2-\underline{q})^2} - \frac{1}{2}\,\frac{1}{\underline{k}_1^2(\underline{k}_1-\underline{k}_2)^2(\underline{k}_2-\underline{q})^2} - \frac{1}{2}\,\frac{1}{\underline{k}_2^2(\underline{k}_1-\underline{k}_2)^2(\underline{k}_1-\underline{q})^2}\right]$$

Don't cancel any more.

In general, as for octet, need



Basic "cell"  is iterated,
ie, easy enough to set up an integral equation to do the sum.

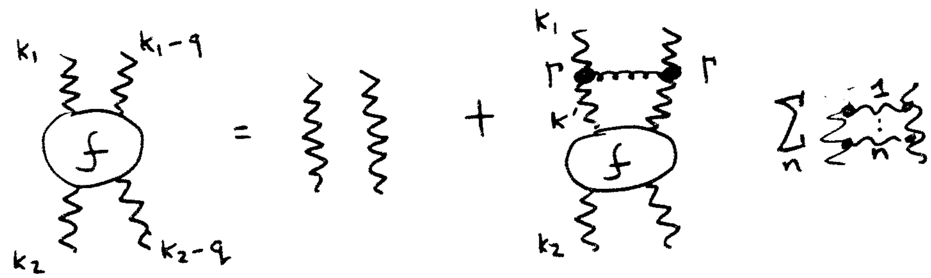
To undo all the ρ-integrals, take a Mellin transform
ie, Define

$$\int_1^\infty d\left(\frac{s}{\underline{k}^2}\right)\left(\frac{s}{\underline{k}^2}\right)^{-\omega-1}\frac{A^{(1)}(s,t)}{s}$$

$$\equiv 4i\,\alpha_s^2\,\delta_{\lambda_1\lambda_1'}\,\delta_{\lambda_2\lambda_2'}\,\frac{N_c^2-1}{4N_c^2}$$

$$* \int \frac{d^2\underline{k}_1\, d^2\underline{k}_2}{\underline{k}_2^2(\underline{k}_1-\underline{q})^2}\, \underline{\underline{f(\omega,\underline{k}_1,\underline{k}_2,q)}}$$

Integral equation for f



$$\boxed{\begin{aligned}&[\omega - \epsilon_g(\underline{k}_1^2) - \epsilon_g((\underline{k}_1-\underline{q})^2)]\, f(\omega, \underline{k}_1, \underline{k}_2, \underline{q}) \\ &= \delta^{(2)}(\underline{k}_1 - \underline{k}_2) - \frac{N_c \alpha_s}{2\pi^2} \int d^2\underline{k}' \left[\frac{\underline{q}^2}{(\underline{k}'-\underline{q})^2 \underline{k}_1^2} - \frac{1}{(\underline{k}'-\underline{k}_1)^2} \right. \\ &\left. \times \left(1 + \frac{(\underline{k}_1 - \underline{q})^2 \underline{k}'^2}{(\underline{k}'-\underline{q})^2 \underline{k}_1^2} \right) \right] f(\omega, \underline{k}', \underline{k}_2, \underline{q})\end{aligned}}$$

"BFKL Equation"

Just algebra to get this from what we've written,

nb if $h(s) = \underline{k}^2 \prod_{i=1}^{n} \int_{\rho_{i+1}}^{1} d\rho_i/\rho_i \; f_i\left(\frac{\rho_{i+1}}{\rho_i}\right) \delta(\rho_n s - \underline{k}^2)$

then $H(\omega) = \int_1^\infty d\left(\frac{s}{\underline{k}^2}\right) \left(\frac{s}{\underline{k}^2}\right)^{-\omega-1} h(s)$

$$= \prod_{i=1}^{n} \tilde{f}_i(\omega) \qquad \tilde{f}_i(\omega) = \int_0^1 d\tau_i \, \tau_i^{\omega-1} f_i(1/\tau_i)$$

$$\left[\text{eg. } f_i(1/\tau_i) \sim \tau_i^{-\epsilon_g(k_1^2) - \epsilon_g((k_1-q)^2)}\right]$$

$$\Rightarrow [\omega - \epsilon_g(k_1^2) - \epsilon_g((k_1-q)^2)] \text{ factor.}$$

Solution for $t = 0$

We can write $\omega f(\omega, \underline{k}_1, \underline{k}_2) = \delta^{(2)}(\underline{k}_1 - \underline{k}_2)$

$$+ K \otimes f(\omega, \underline{k}_1, \underline{k}_2)$$

(drop q label since $q^2 = 0$ is being considered)

Skipping details,

$$f(\omega, k_1, k_2) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} d\nu \left(\frac{k_1^2}{k_2^2}\right)^{i\nu} \frac{e^{in(\theta_1 - \theta_2)}}{2\pi^2 k_1 k_2} \frac{1}{\omega - \frac{N_c \alpha_s}{\pi} \chi_n(\nu)}$$

$$\underline{k} = (k, \theta)$$

$$\chi_n(\nu) = 2\left(-\gamma_E - \mathrm{Re}\left[\psi\left(\tfrac{n+1}{2} + i\nu\right)\right]\right)$$

↑ Digamma function.

$$F(s, k_1, k_2) \approx \frac{1}{\sqrt{k_1^2 k_2^2}} \left(\frac{s}{\underline{k}^2}\right)^{\omega_0} \frac{1}{\sqrt{\pi \ln s/\underline{k}^2}}$$

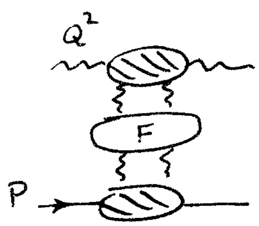
↑ (inverse transform of f)

$$* \frac{1}{2\pi a} \exp\left(\frac{-\ln^2(\underline{k}_1^2/\underline{k}_2^2)}{4a^2 \ln(s/\underline{k}^2)}\right)$$

$$a = 14\,\zeta(3)\,\frac{N_c \alpha_s}{\pi}$$

$$\omega_0 = 4 \ln 2\,\frac{N_c \alpha_s}{\pi}$$

eg, DIS



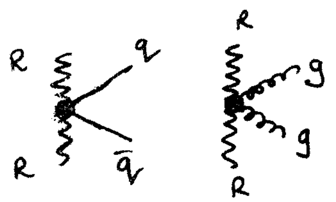
optical thm $\sigma_{tot}^{\lambda} = \dfrac{\mathrm{Im}\, A(\gamma p \to \gamma p)}{s}$

$$\sigma_{tot}^{T} + \sigma_{tot}^{L} \sim F_2(x, Q^2)/Q^2$$

Next-to-leading logs

BFKL equation keeps its form.

Additional building blocks needed :



and also loop corrections to RRg vertex and reggeized gluon propogator.

Put $\beta_0 = 0$ means Laplace transform works again :

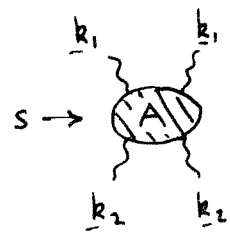
$$\omega_0 \longrightarrow \omega_0 \left(1 - 6.5\, \alpha_s\right)$$

huge NLL correction !

But this is not physical ......

$$A(s) \sim \int d\omega \, d\gamma \left(\frac{s}{k_1 k_2}\right)^{\omega} \frac{1}{\omega - \chi(\gamma)} \left(\frac{k_1^2}{k_2^2}\right)^{\gamma}$$

$$\chi(\gamma) = \chi_{LO}(\gamma) + \alpha_s \chi_{NLO}(\gamma)$$



Contains spurious $\log Q/\mu$ terms at NNLO and beyond

eg $\sim \alpha_s^3 \log^5 Q/\mu$ etc.

forbidden by DGLAP & beyond control of NLO BFKL.

(DGLAP limit $\underline{k}_1^2 \gg \underline{k}_2^2$)

$= Q^2$ $\quad = \mu^2$

Following Andersson, Gustafson & Samuelsson,

Salam showed how one can modify $\chi(\gamma)$

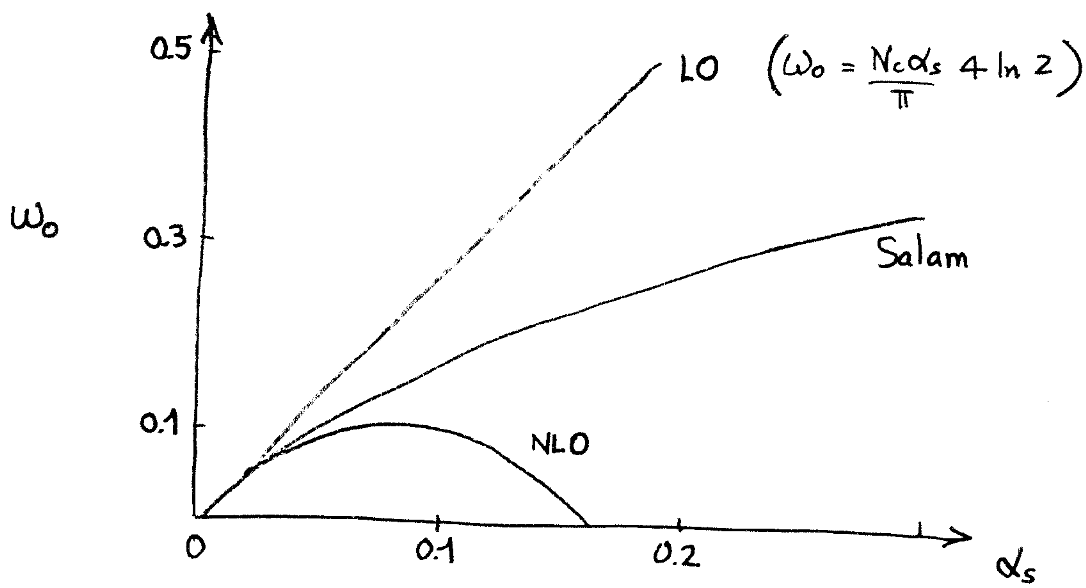
to exclude these terms

ansatz for beyond NLO.

$\Downarrow$

Stable ω_0

$$\left[\begin{aligned}\chi_{\text{Salam}}(\gamma) &= \chi_{NLO}(\gamma)\\ &\quad + \mathcal{O}(\alpha_s^3)\end{aligned}\right]$$



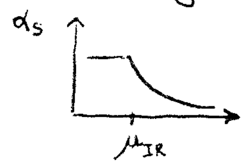
(But we should remember $\beta_0 \neq 0$ ....)

Running the coupling

Is important!

Integrals like $\int_{-\infty}^{\infty} d\log k^2 \; \alpha_s(k^2)$ mean that we have a problem.

Ansatz: regularise the coupling in some way (new scale μ_{IR}).



Solution in "collinear model" (Ciafaloni, Colferai, Salam)

$$F(s, \underbrace{k_1, k_2}_{k_1 \sim k_2 \sim Q}, 0) \sim \frac{s^{\omega_s(Q)}}{\sqrt{\ln s}} \left(1 + \mathcal{O}\left(\alpha_s^5 \ln^3 s\right)\right)$$

pQCD part; "fake" $\mathbb{P}$. cut means no pole at $j > 1$. Not a power as $s \to \infty$.

eg $\gamma^*\gamma^* \to X$ forward jets etc.

$$+ \sim s^{\omega_{\mathbb{P}}(\mu_{IR})} \left(\mu_{IR}^2 / Q^2\right)^{1+\omega_{\mathbb{P}}(\mu_{IR})}$$

Pomeron i.e. dominates asymptotically ($s \to \infty$)

Non-pert. part is power suppressed

ie There is an intermediate region in $\ln s$ where pQCD dominates before eventually giving way to non-perturbative pomeron.

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A collinear model for small-x physics*

Marcello Ciafaloni, Dimitri Colferai

Dipartimento di Fisica, Università di Firenze and INFN, Sezione di Firenze
Largo E. Fermi 2, 50125 Firenze, Italy
E-mail: `Ciafaloni@fi.infn.it` `Colferai@fi.infn.it`

Gavin P. Salam

INFN, Sezione di Milano
Via Celoria 16, 20133 Milano, Italy
E-mail: `Gavin.Salam@mi.infn.it`

ABSTRACT: We propose a simple model for studying small-x physics in which we take only the collinearly enhanced part of leading and subleading kernels, for all possible transverse momentum orderings. The small-x equation reduces to a second order differential equation in $t \equiv \log k^2/\Lambda^2$ space, whose perturbative and strong-coupling features are investigated both analytically and numerically. For two-scale processes, we clarify the transition mechanism between the perturbative, non-Regge regime and the strong-coupling Pomeron behaviour.

KEYWORDS: QCD, NLO Computations.

$$\omega f_\omega(t_1,t_2) = \delta(t_1-t_2) + \int_{-\infty}^{\infty} dt' \left[\bar\alpha(t)\, e^{\frac{t'-t}{2}}\,\theta(t-t') + \bar\alpha(t')\, e^{\frac{t-t'}{2}}\,\theta(t'-t)\right] \cdot f_\omega(t,t_2)$$

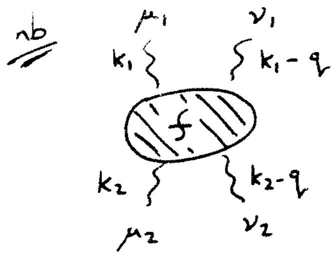
$$t = \ln \underline{k}^2/\Lambda^2$$

$$\bar\alpha(t) = \frac{1}{bt}\,\theta(t-\bar t) + \frac{1}{b\bar t}\,\theta(\bar t-t)$$

frozen QCD coupling.

*Work supported by E.U. QCDNET contract FMRX-CT98-0194.

nb



$$\frac{f(\omega, \underline{k}_1, \underline{k}_2, q)}{\underline{k}_2^2 (\underline{k}_1 - q)^2} = \frac{s}{(2\pi)^4} \int d\rho_1 \, d\lambda_2 \times \frac{4 p_1^{\mu_1} p_1^{\nu_1} \, p_2^{\mu_2} p_2^{\nu_2}}{s^2} \underline{G^{(1)}_{\mu_1 \nu_1 \mu_2 \nu_2}(\omega, k_1, k_2, q)}$$

4-point Green function.

Generalizing away from $qq \to qq$,

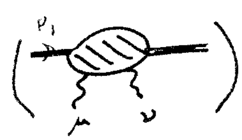
ie, $AB \longrightarrow AB$

$$A^{(1)}(\omega, t) = \frac{\mathcal{G}}{(2\pi)^4} \int d^2\underline{k}_1 \, d^2\underline{k}_2 \; \underline{\Phi_A(\underline{k}_1, q) \; \Phi_B(\underline{k}_2, q)} \times \frac{f(\omega, \underline{k}_1, \underline{k}_2, q)}{\underline{k}_2^2 (\underline{k}_1 - q)^2}$$

Colour factor.

"Impact factors"

$\Phi_A \sim$



$p_1^\mu p_1^\nu$ part

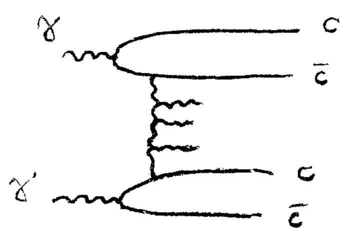
"High energy factorization"

Balitsky & Lipatov (August 1978)

$$\omega f(\underline{k}_1,\underline{k}_2,\underline{q}) = \delta^{(2)}(\underline{k}_1-\underline{k}_2) + \frac{3g^2}{(2\pi)^3}\int\frac{d^2\underline{k}'}{(\underline{k}_1-\underline{k}')^2} \times \Bigg[$$

$$\left\{\frac{(\underline{k}_1-\underline{k}')^2\,\underline{q}^2}{\underline{k}'^2(\underline{q}-\underline{k}')^2} + \frac{\underline{k}_1^2}{\underline{k}'^2} + \frac{(\underline{q}-\underline{k}_1)^2}{(\underline{q}-\underline{k}')^2}\right\} f(\underline{k}',\underline{k}_2,\underline{q}) \; -$$

$$\left\{\frac{\underline{k}_1^2}{\underline{k}'^2+(\underline{k}_1-\underline{k}')^2} + \frac{(\underline{q}-\underline{k}_1^2)}{(\underline{q}-\underline{k}')^2+(\underline{k}_1-\underline{k}')^2}\right\} f(\underline{k}_1,\underline{k}_2,\underline{q}) \Bigg]$$

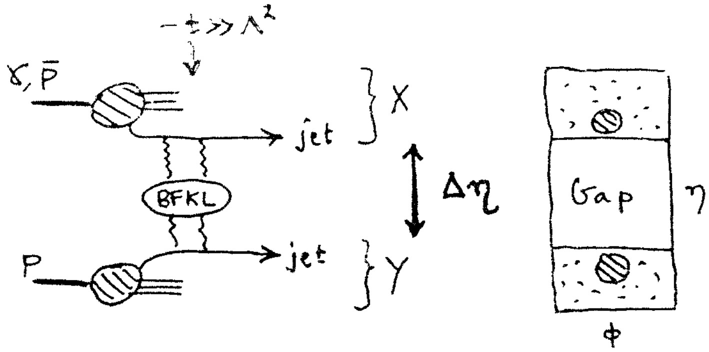


For $\sqrt{s}_{e^+e^-} \simeq 22$ GeV (!)

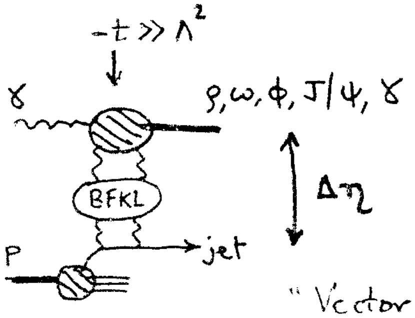
"For $t \neq 0$, with good accuracy we can replace .... by a diffusion equation with a boundary condition corresponding to a wall at $\underline{k}_\perp^2 = \underline{q}^2$ "

A new more accessible regime ?

High-t Phenomena



"gaps between jets"



$$\boxed{p_\perp^{jet} \simeq \sqrt{-t}}$$

IR cutoff

"Vector particle production"

* BFKL can be solved for $-t \neq 0$ (Lipatov) in LLA.

* Calculations complete for all above processes

  Mueller & Tang $\quad qq \to qq \quad (gg \to gg)$

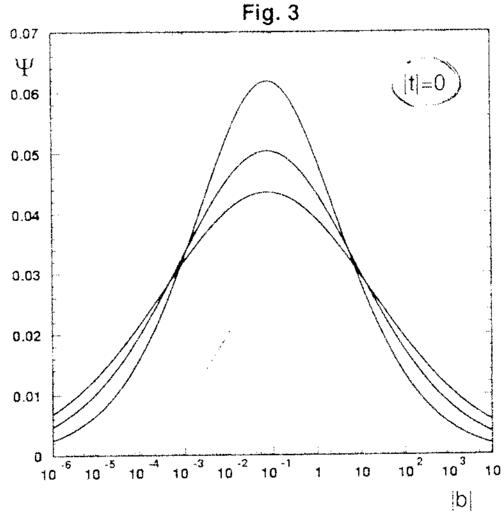
  Evanson, Forshaw; Ivanov, Wüsthoff $\quad \gamma p \to \gamma + X$, $\gamma\gamma \to \gamma\gamma$

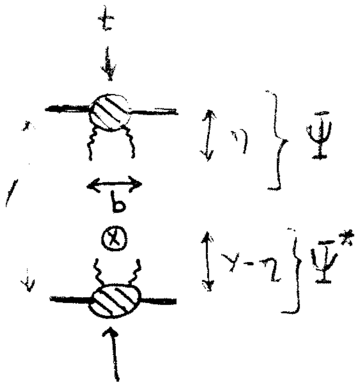
  Forshaw, Ryskin $\quad \gamma p \to J/\psi + X$

  Ivanov $\quad \gamma p \to \rho, \phi + X$

* Incorporated into HERWIG $\quad$ Cox, Forshaw; Hayes, Butterworth, Seymour.

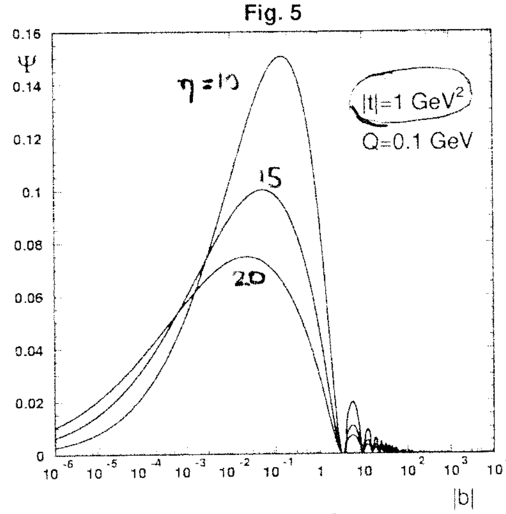
Fig. 3





external states of size $1/Q$

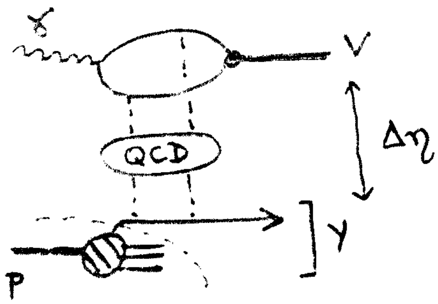
Fig. 5



exponential suppression of infra-red scale physics.

Forshaw & Sutton

LARGE t VECTOR MESON PRODUCTION



$$P_{\perp V}^2 \simeq -t$$

- Inclusive over system Y $\left(M_Y < M_Y^{max} \ll W \equiv s_{\gamma p}\right)$
  - ⇒ BIG EXTENSION IN RAPIDITY REACH
    ($\Delta\eta \lesssim 7$ @ HERA)

- No gap survival problems

- Can look at intermediate → low t.

BUT

- Need to know how meson is formed

- m_V is fixed (info on different mesons would be very useful)

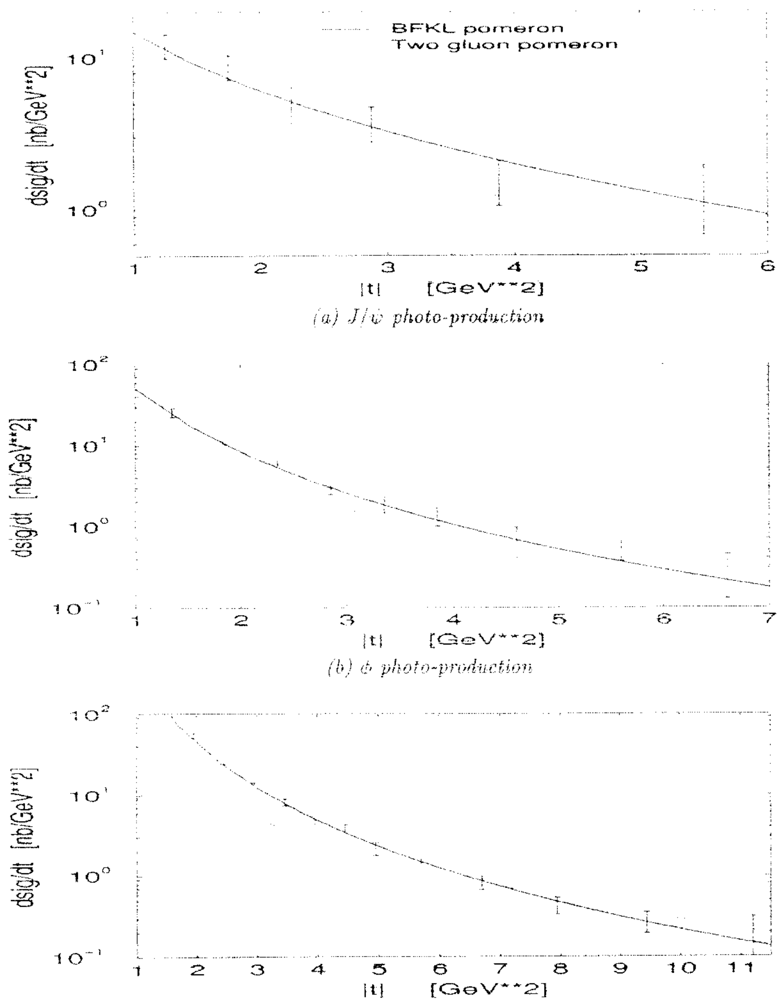
QCD calculation for J/ψ [no Fermi motion $\delta(z-\frac{1}{2})$]

in HITVM monte carlo.

$\tau \equiv \frac{-t}{[illegible]}$

[JRF + Ryskin
Bartels, JRF, Lotter, Wüsthoff.]

[Relativistic Corrections ⇒ Ivanov et al]



(a) J/ψ photo-production

(b) ϕ photo-production

(c) ρ photo-production

Fig.4: *$d\sigma/dt$ for the BFKL and two gluon Pomeron for J/ψ, ϕ and ρ vector mesons. The two gluon Pomeron fits are for (a) $\alpha_s = 0.36$, (b) $\alpha_s = 0.35$ and (c) $\alpha_s = 0.27$. The BFKL is a simultaneous fit for the three mesons with $\alpha_s = 0.25$ and $\beta = 15.0$.*