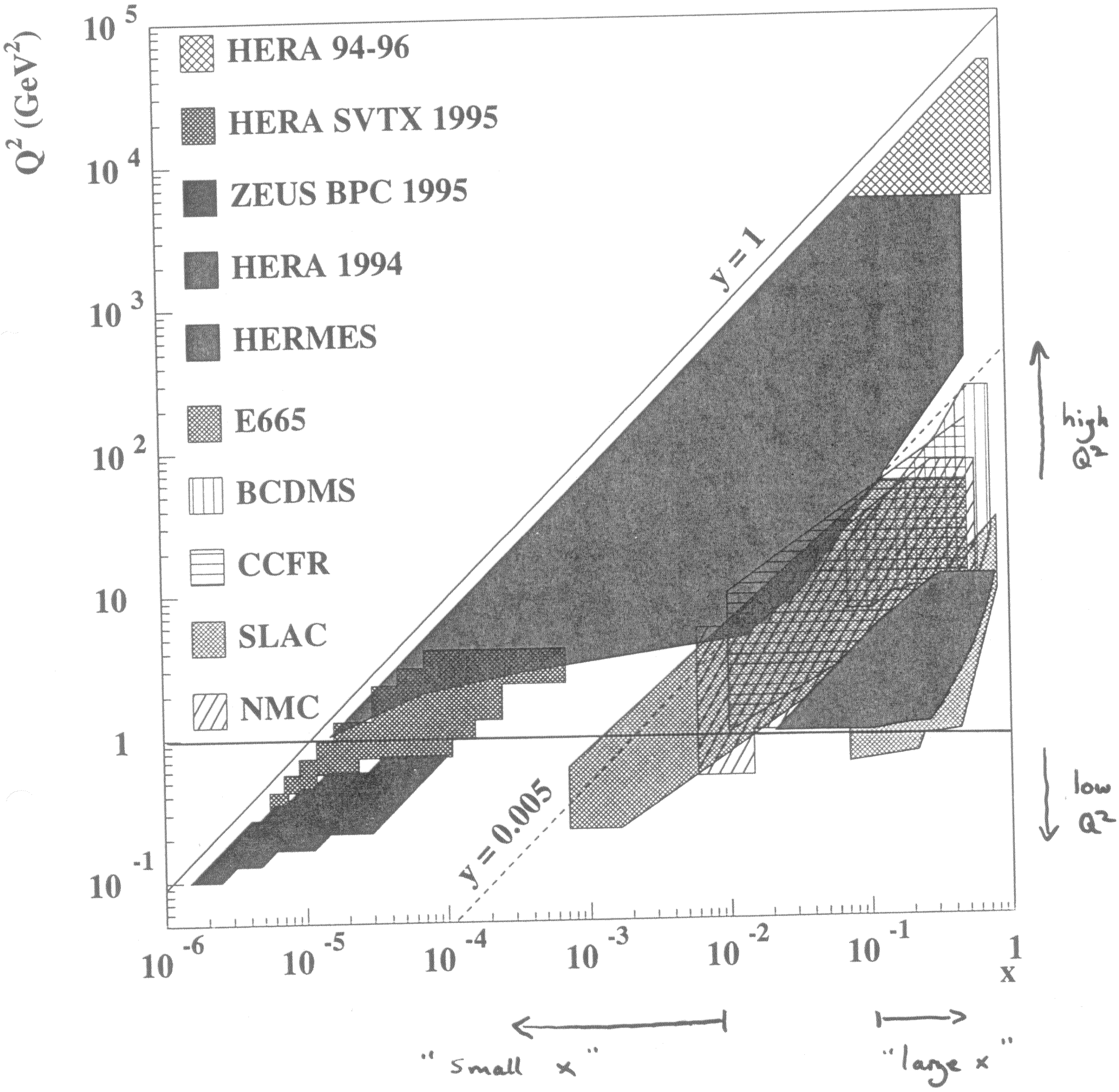
RDB
St. Andrews
June 2001.

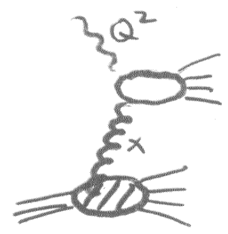
Small x Resummation

- $\log x$ vs $\log Q^2$
- AP $\leftrightarrow$ BFKL : duality
- HERA $\rightarrow$ Tevatron $\rightarrow$ LHC

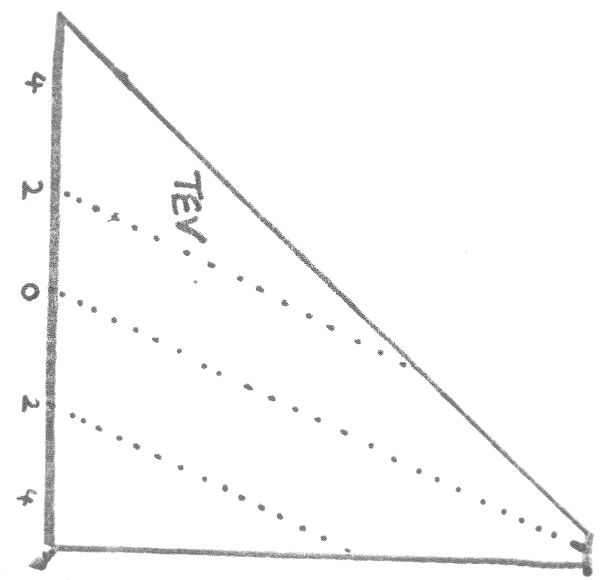
Fadin, Lipatov
Catani, Ciafaloni, Salam
Altarelli, Ball, Forte
Collins, Ellis
et al.

DIS kinematics





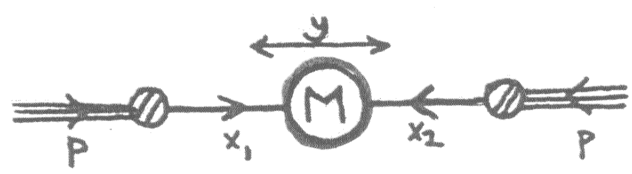
$Q^2 \simeq sxy$

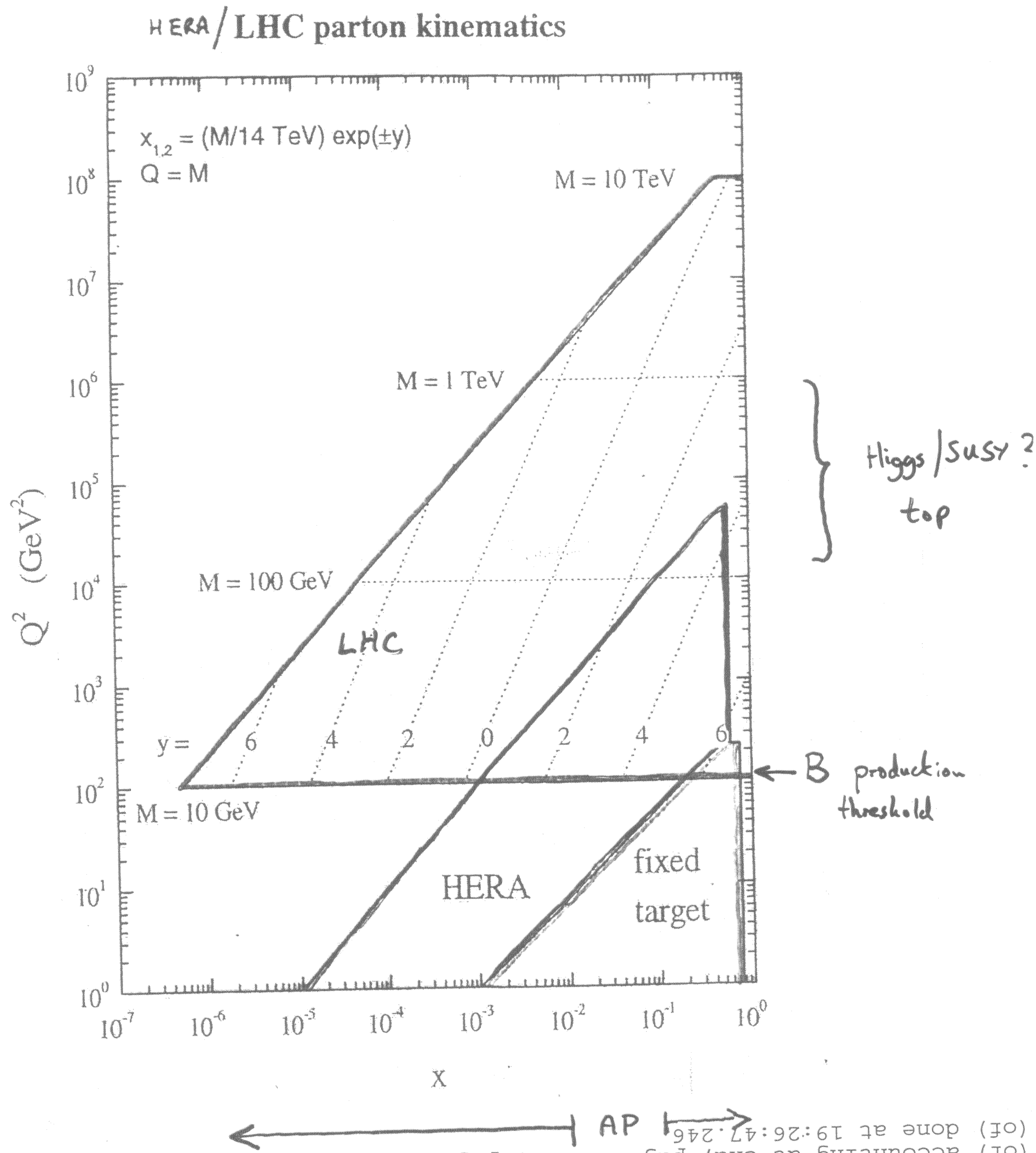


4
2
0
2
4
TEV

DIS :

Hadronic :





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(of) accounting at end, pagecount 0, pages 0 at 19:26:47.246
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Resummation

$$\sigma = \sum_0^\infty (\alpha_s L)^n + \alpha_s \sum_0^\infty (\alpha_s L)^n + \cdots$$

$\uparrow$ partonic xsec — LL — NLL — NNLL

- Large Q^2 : usual AP

$$L = \log Q^2/\Lambda^2$$

Known at NLLQ (1981) : NNLLQ progressing .....

Also (closely related) heavy quark resummation: $L = \log Q^2/M^2, \log M^2/\Lambda^2$

- Large x : threshold resummation

$$L = \log^2(1-x)$$

Known for many processes at NLL ($\approx$ 1990–95)

Important for Heavy quark | DY | Higgs | SUSY .... production.

[NB: NNLLQ not sufficient : need to resum.....]

- Small x : 'BFKL' resummation

$$L = \ln 1/x$$

Known at NLLx (Fadin – Lipatov : D1998)

But
- NLLx inconsistent with HERA data
- NLLx >> LLx $\approx$ LLQ ?

Important to resolve this if we want reliable predictions for LHC xsec.

[NB: NNLLQ not sufficient : need to resum.....]

F_2 rises at small x .....

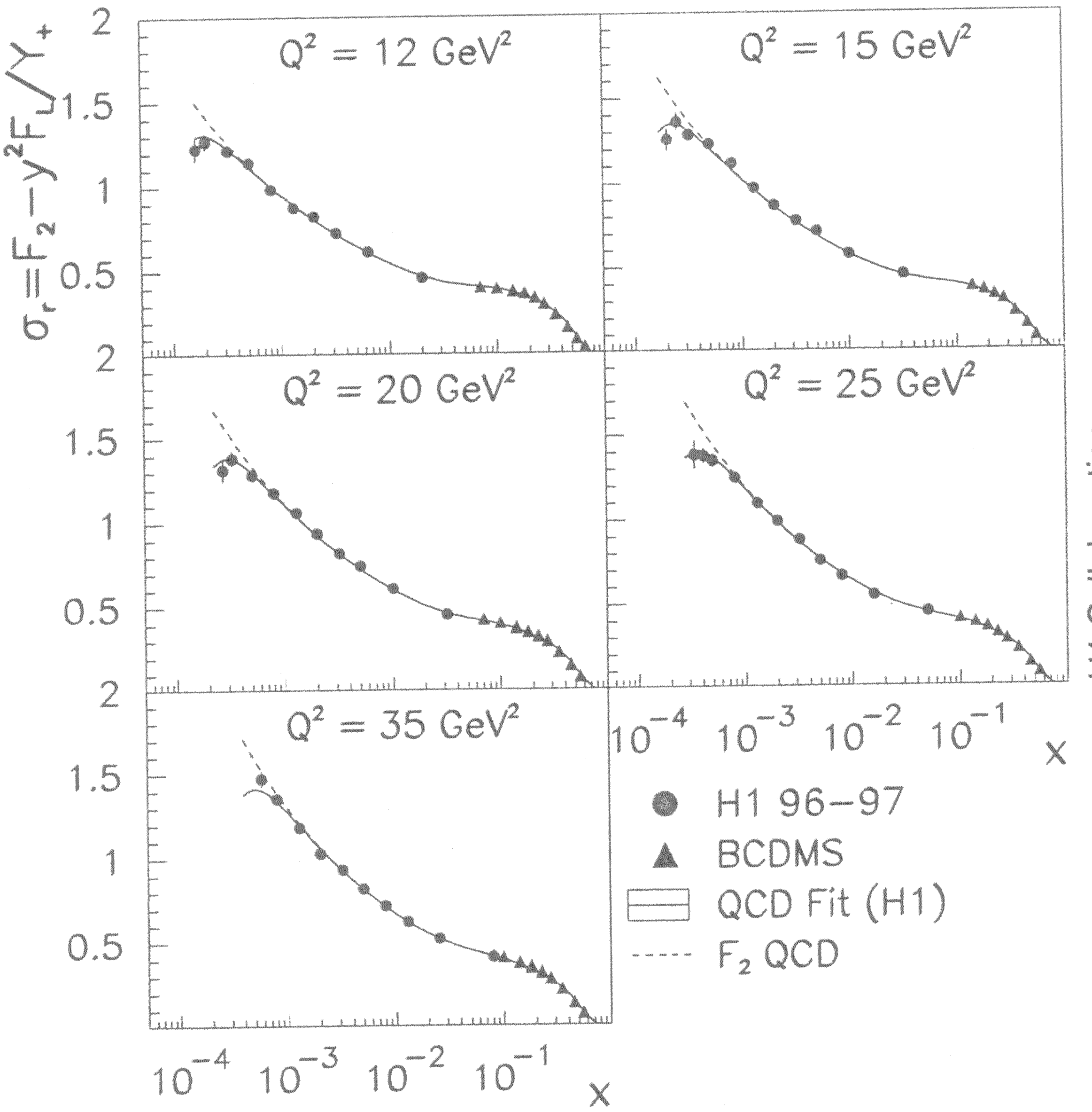
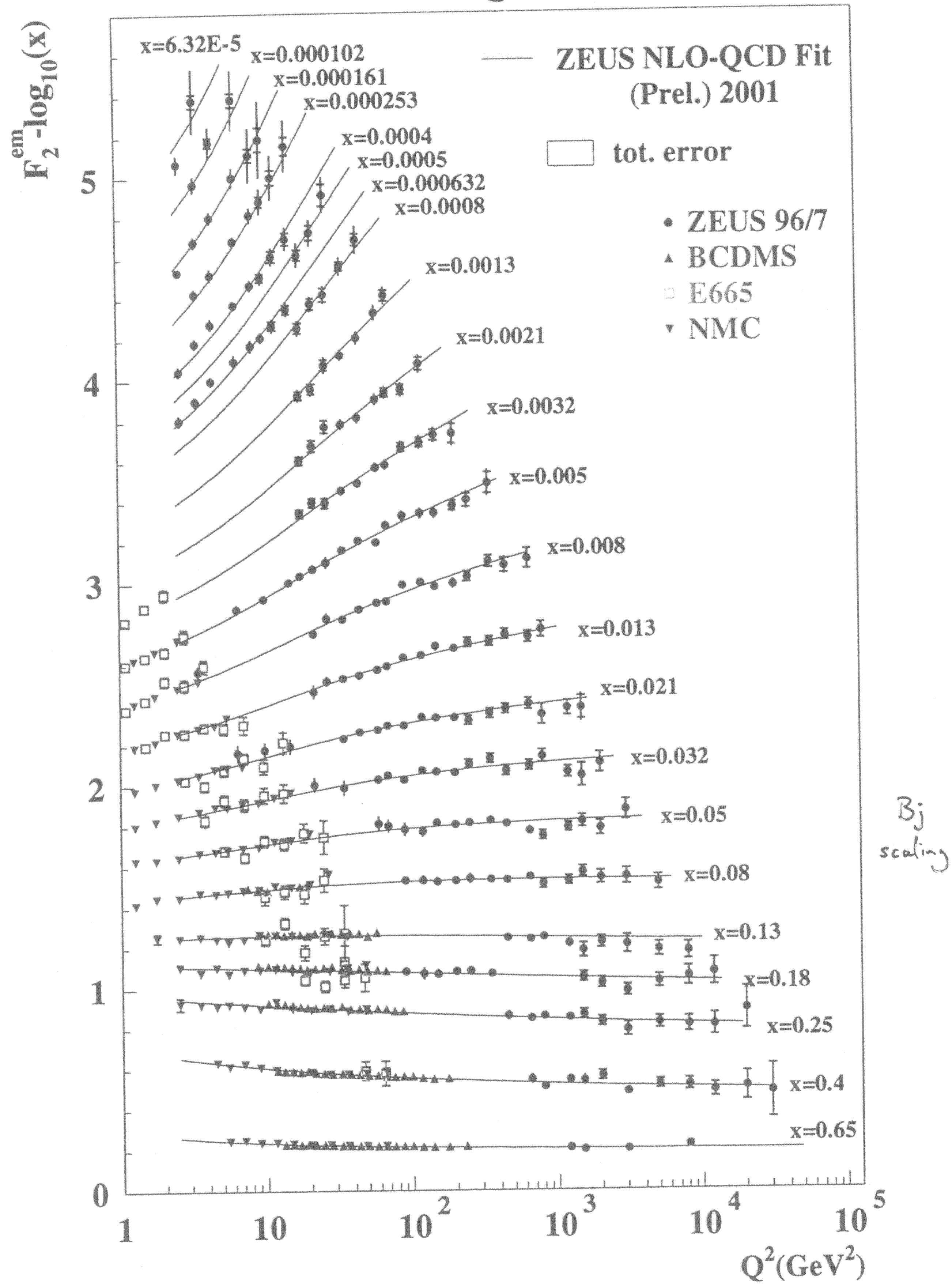


Figure 8: *Measurement of the reduced DIS scattering cross section (closed points). Triangles represent data from the BCDMS muon-proton scattering experiment. The curves represent a NLO QCD fit to the H1 data alone, using data with $y < 0.35$ and $Q^2 \geq 3.5$ GeV^2. The dashed curves show the F_2 structure function as determined with this fit. The error bands represent the experimental and model uncertainty of the QCD fit.*

.... and at large Q^2



- NLO-QCD fit describes data well both for ZEUS and fixed-target experiments.

Double Log Approx. : AP

Fixed coupling

Singlet AP: $G(x,Q) \equiv x g(x,Q)$ (large eval)

$$\frac{\partial}{\partial \ln Q} G(x,Q) = \int_x^1 \frac{dy}{y} P\left(\alpha, \frac{x}{y}\right) G(y,Q) \qquad \text{Resums LQ}$$

Mellin w.r.t. x

$$\frac{\partial}{\partial \ln Q} G(N,Q) = \gamma(\alpha, N)\, G(N,Q) \qquad G(N) = \int_0^1 dx\, x^{N-1} G(x)$$

Mellin w.r.t. Q

$$M\, G(N,M) - \underbrace{G_0(N)}_{\text{b.c.}} = \gamma(\alpha,N)\, G(N,M) \qquad G(M) = \int_0^\infty dQ^2\, Q^{2\,-M-1}\, G(Q)$$

Soln:

$$\boxed{G(N,M) = \frac{G_0(N)}{M - \gamma(\alpha,N)}} \leftarrow \text{simple pole.}$$

At small x, $P(\alpha,x) \sim \frac{\bar{\alpha}}{x}$: $\gamma(\alpha,N) \sim \frac{\bar{\alpha}}{N}$ $\qquad \bar{\alpha} \equiv \frac{C_A \alpha}{\pi}$

LLx $\qquad \Rightarrow \gamma$ fig $\times$ fig.

$$G(x,Q) \sim \int \frac{dN}{2\pi i} \int \frac{dM}{2\pi i}\, e^{N \ln 1/x + M \ln Q^2}\, \frac{1}{M - \bar{\alpha}/N}\, G_0(N)$$

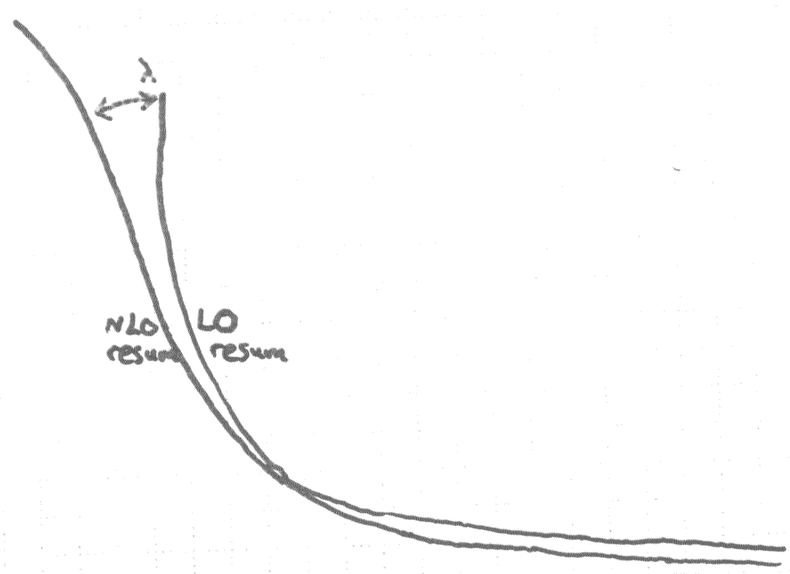
simple pole : $M = \bar{\alpha}/N$

$$\sim \int \frac{dN}{2\pi i}\, \underbrace{e^{N \ln 1/x + \frac{\bar{\alpha}}{N} \ln Q^2}}_{\text{saddle}:\ N_S = \sqrt{\frac{\bar{\alpha} \ln Q^2}{\ln 1/x}}}\, G_0(N)$$

$$\boxed{G(x,Q) \sim \exp 2\sqrt{\bar{\alpha} \ln Q^2 \ln 1/x}}$$

as $\ln Q^2 \to \infty$ and $x \to 0$ $\qquad \Rightarrow$ DIS fig $\qquad \Rightarrow$ NLO fits

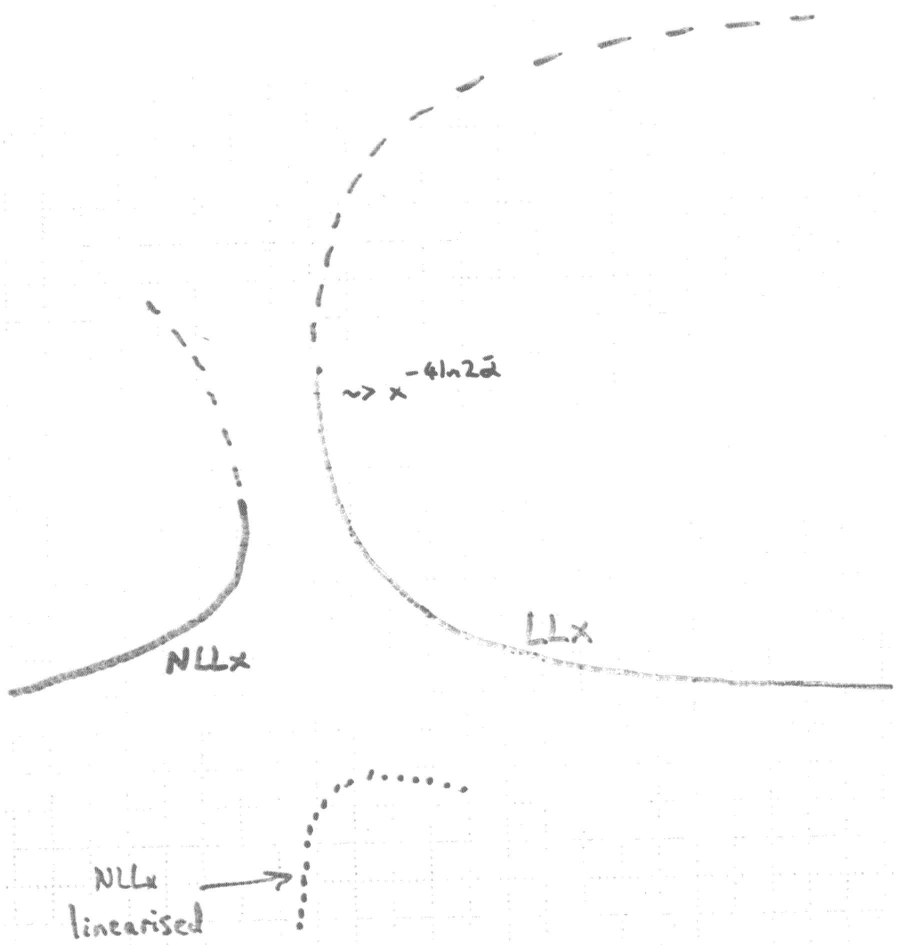
Rises at small x and large Q^2!



Resummed LO + NLO (from resummed χ using duality

Sensible

Remaining uncertainty near $\gamma \sim \frac{1}{2}$ parametrized by λ



$\gamma(\alpha, N)$ at LLx, NLLx, ... using BFKL + duality gives nonsense!

Approximating $\gamma(\alpha, N)$

Calculate $\gamma(\alpha,N)$ perturbatively in powers of α: $\Rightarrow$ fig (

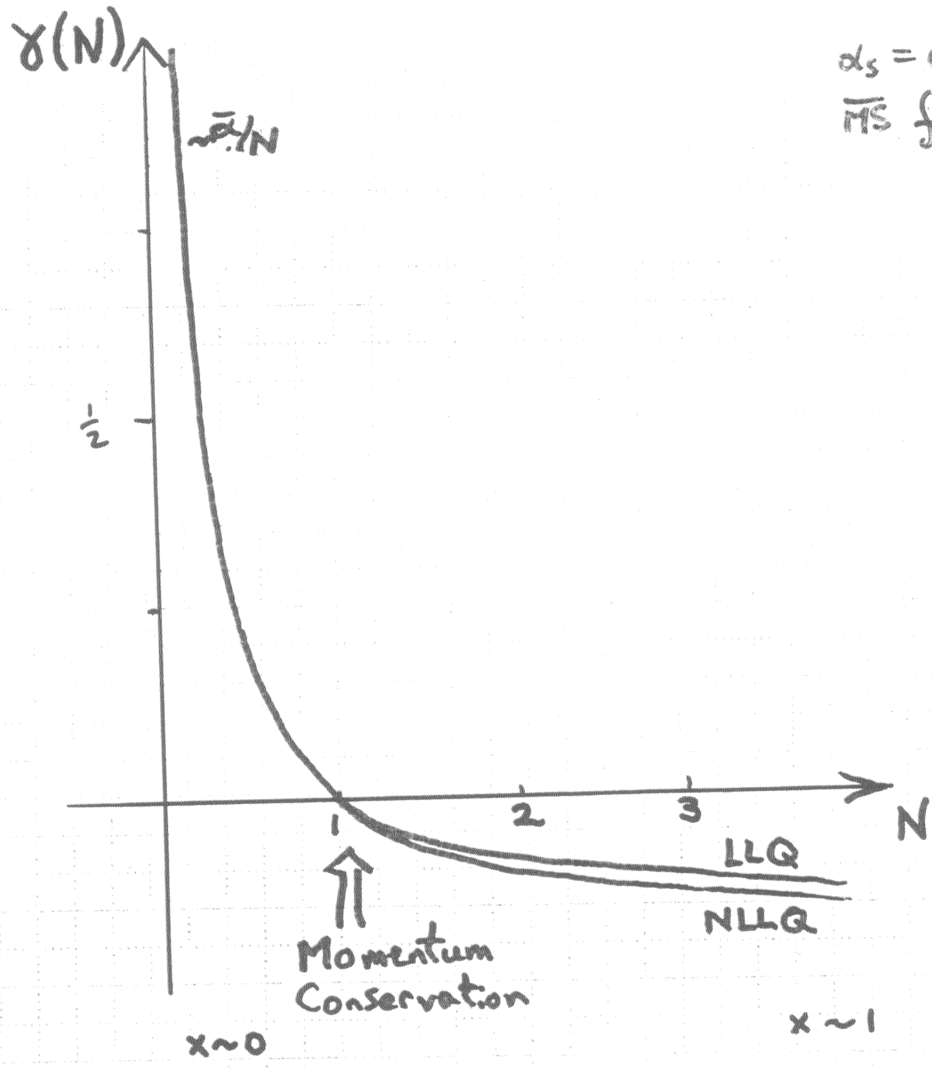
$$\gamma(\alpha,N) = \underbrace{\alpha\,\gamma_0(N)}_{\substack{\text{LLQ} \\ \text{GLAP}}} + \underbrace{\alpha^2\gamma_1(N)}_{\substack{\text{NLLQ} \\ \text{CFP}}} + \ldots\ldots$$

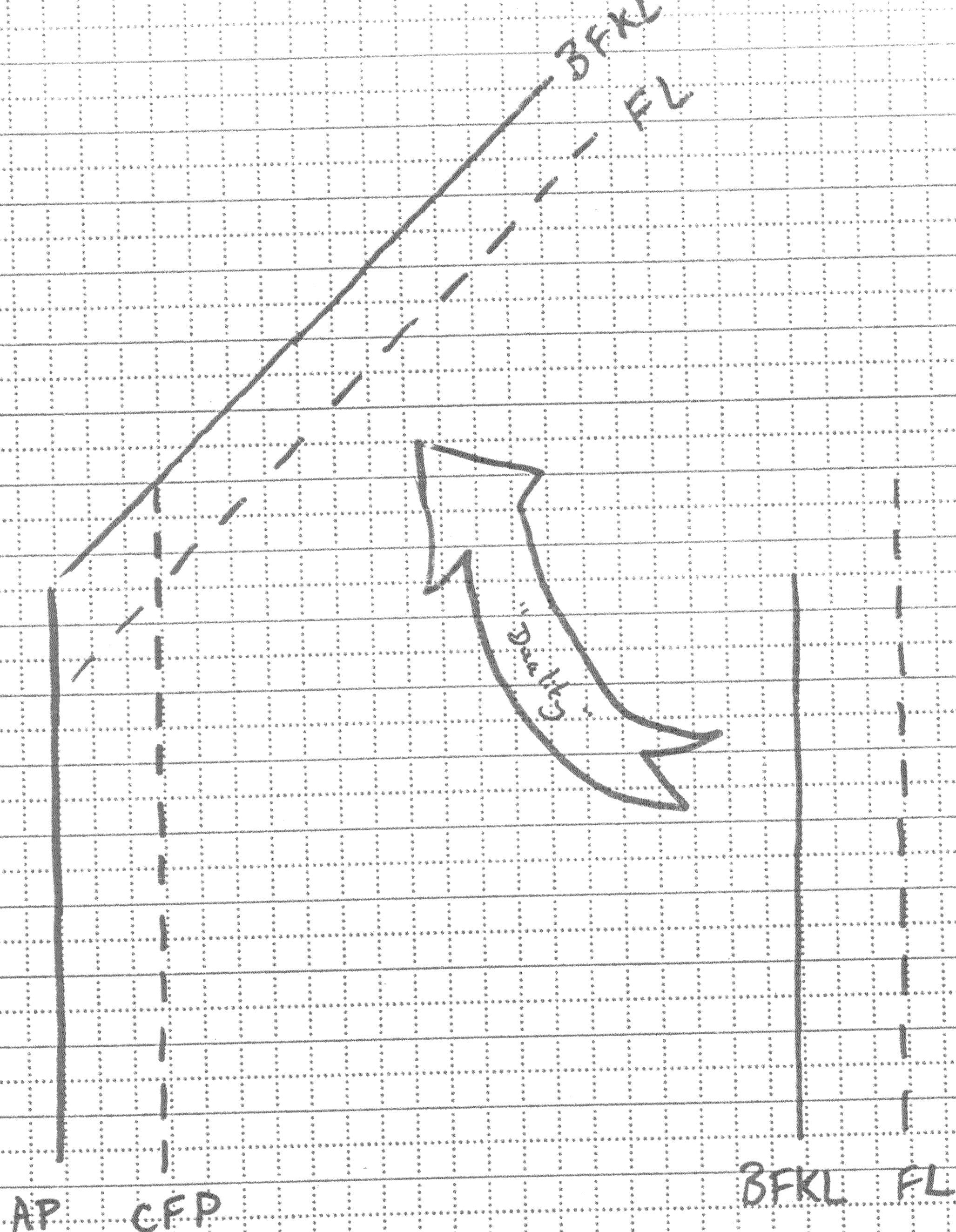
Sensible

except when $N \sim 0$: get $\left(\frac{\alpha}{N}\right)^n$

$\alpha_s = 0.2$

$\overline{MS}$ fact $\rightarrow$ ren.





At NLO

"CANCEL!"

$$\gamma_{DL}(\alpha, N) = \underset{\text{big}}{\gamma_{AP}(N) + \alpha\gamma_{CFP}(N)} + \underbrace{\left(\gamma_{BFKL}\left(\frac{\alpha}{N}\right) + \alpha\gamma_{FL}\left(\frac{\alpha}{N}\right) - d.c.\right)}_{\text{small}}$$

Sums up $\left(\frac{\alpha}{N}\right)^n$ in γ.

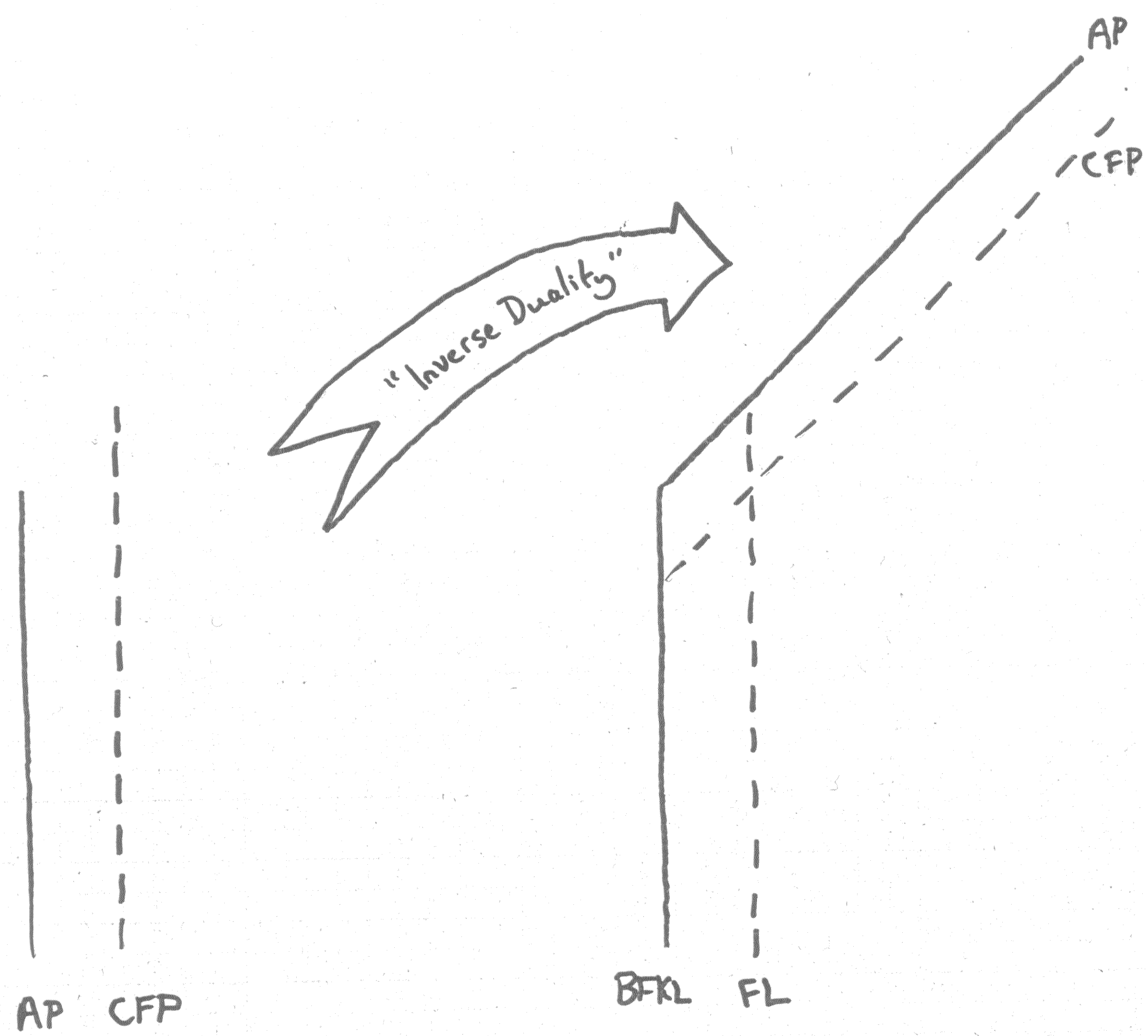
Crosses

$\gamma(\alpha, N)$

				$\frac{\alpha^5}{N^5}\times$	
			$\frac{\alpha^4}{N^4}\times$	$+$	$\cdot$
		$\frac{\alpha^3}{N^3}\times$	$\frac{\alpha^4}{N^3}+$	$\cdot$	
	$\frac{\alpha^2}{N^2}\times$	$\frac{\alpha^3}{N^2}+$	$\cdot$	$\cdot$	
$\frac{\alpha}{N}\otimes$	$\frac{\alpha^2}{N}+$	$\frac{\alpha^3}{N}\cdot$	$\cdot$	$\cdot$	
$\alpha\ \times$	α^2+	$\cdot$	$\cdot$	$\cdot$	
$\alpha N\ \times$	$\alpha^2 N+$	$\cdot$	$\cdot$	$\cdot$	
$\alpha N^2\times$	$\alpha^2 N^2+$	$\cdot$	$\cdot$	$\cdot$	
$\alpha N^3\ \times$	$+$	$\cdot$			

$\chi(\alpha, M)$

				$\frac{\alpha^5}{M^5}\times$
			$\frac{\alpha^4}{M^4}\times$	$+$
		$\frac{\alpha^3}{M^3}\times$	$\frac{\alpha^4}{M^3}+$	$\cdot$
	$\frac{\alpha^2}{M^2}\times$	$\frac{\alpha^3}{M^2}+$	$\cdot$	$\cdot$
$\frac{\alpha}{M}\otimes$	$\frac{\alpha^2}{M}+$	$\frac{\alpha^3}{M}\cdot$	$\cdot$	$\cdot$
$\alpha\ \times$	α^2+	$\cdot$	$\cdot$	
$\alpha M\times$	$\alpha^2 M+$	$\cdot$	$\cdot$	
$\alpha M^2\times$	$\alpha^2 M^2+$	$\cdot$	$\cdot$	
$\alpha M^3\times$	$\alpha^2 M^3+$	$\cdot$		



it NLO

~CANCEL!

$$\chi_{DL}(\alpha, M) = \chi_{BFKL}(M) + \alpha\chi_{FL}(M) + \Big(\underbrace{\chi_{AP}\left(\tfrac{\alpha}{M}\right) + \alpha\chi_{CFP}\left(\tfrac{\alpha}{M}\right)}_{\text{big}} - \text{d.c.}\Big)$$

Sums up $\left(\frac{\alpha}{M}\right)^n$ in χ

Double Asymptotic Scaling

Improve on DLLA by

- making coupling run: $\alpha_s \ln Q^2 \rightarrow \ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \sim \ln\ln Q^2$
- include subleading effects
- more careful treatment of b.c.

H1 2000

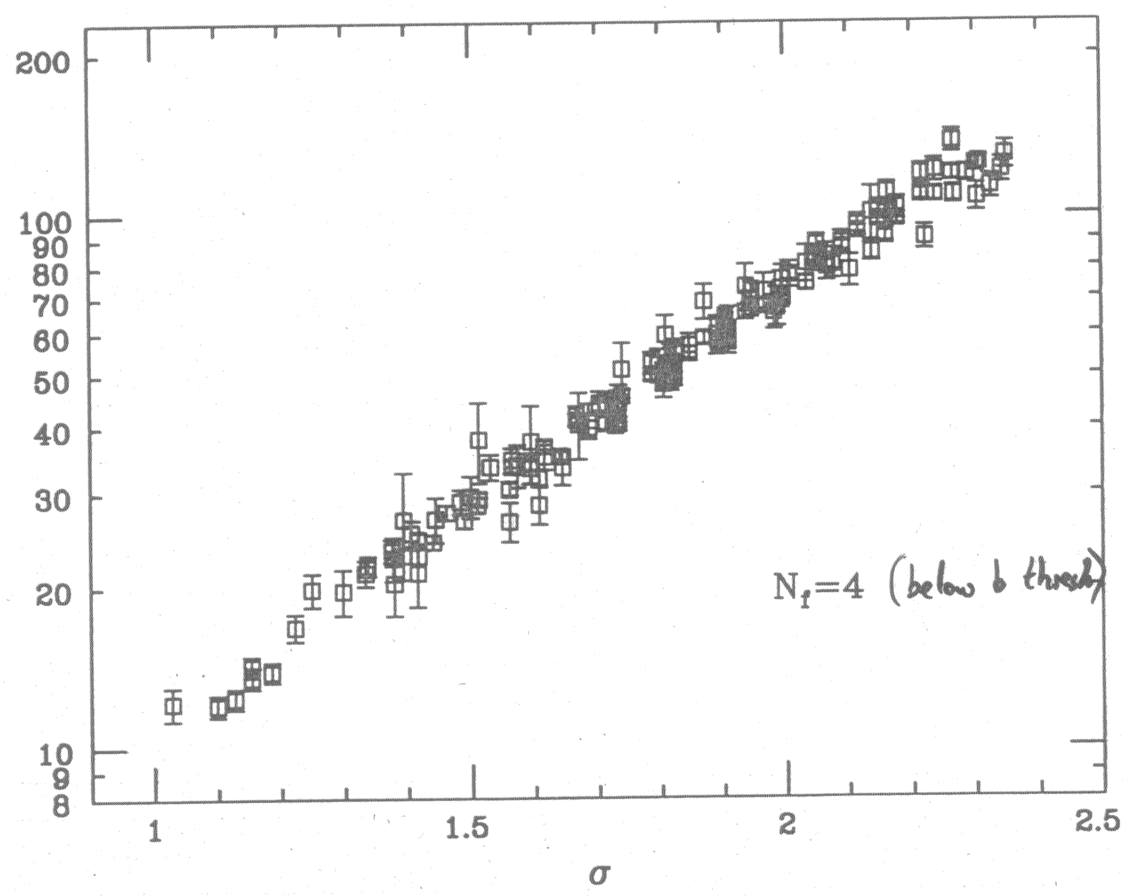
$R_F F_2$

$\uparrow$

$\rho\sqrt{\sigma}\, e^{\delta\sigma/\rho}$

$\rho \equiv \sqrt{\ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \Big/ \ln \frac{x_0}{x}}$

$\delta \equiv \frac{11 + 2n_f/27}{11 - 2n_f/3}$



$= \sqrt{\ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \ln \frac{x_0}{x}}$

QCD predicts

$$R_F F_2 \sim e^{2\gamma\sigma} \quad \text{as} \quad \begin{array}{l} Q^2 \rightarrow \infty \\ x \rightarrow 0 \end{array}$$

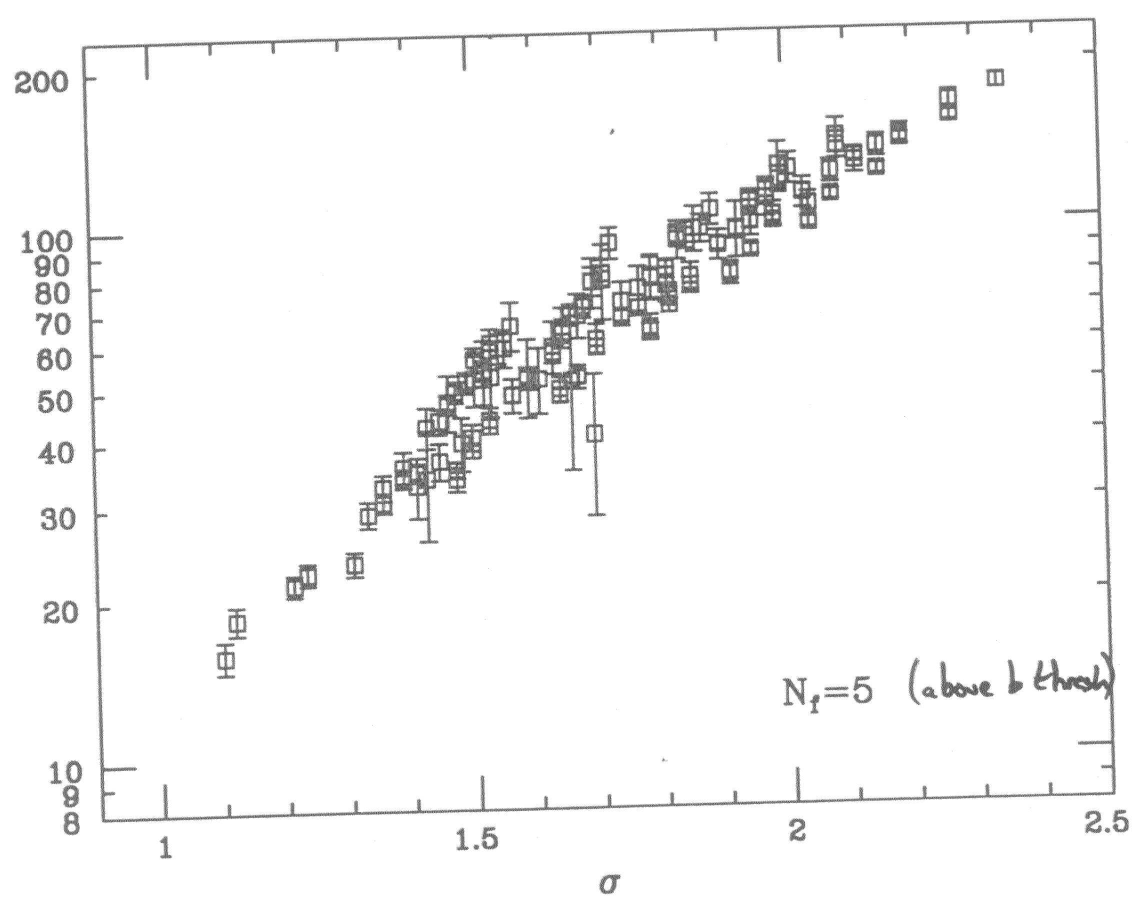
DGPTWZ 1974

ie data lie on straight line, with slope

$$2\gamma = 4\sqrt{\frac{C_A}{\beta_0}} = 2.4 \qquad n_f = 4.$$

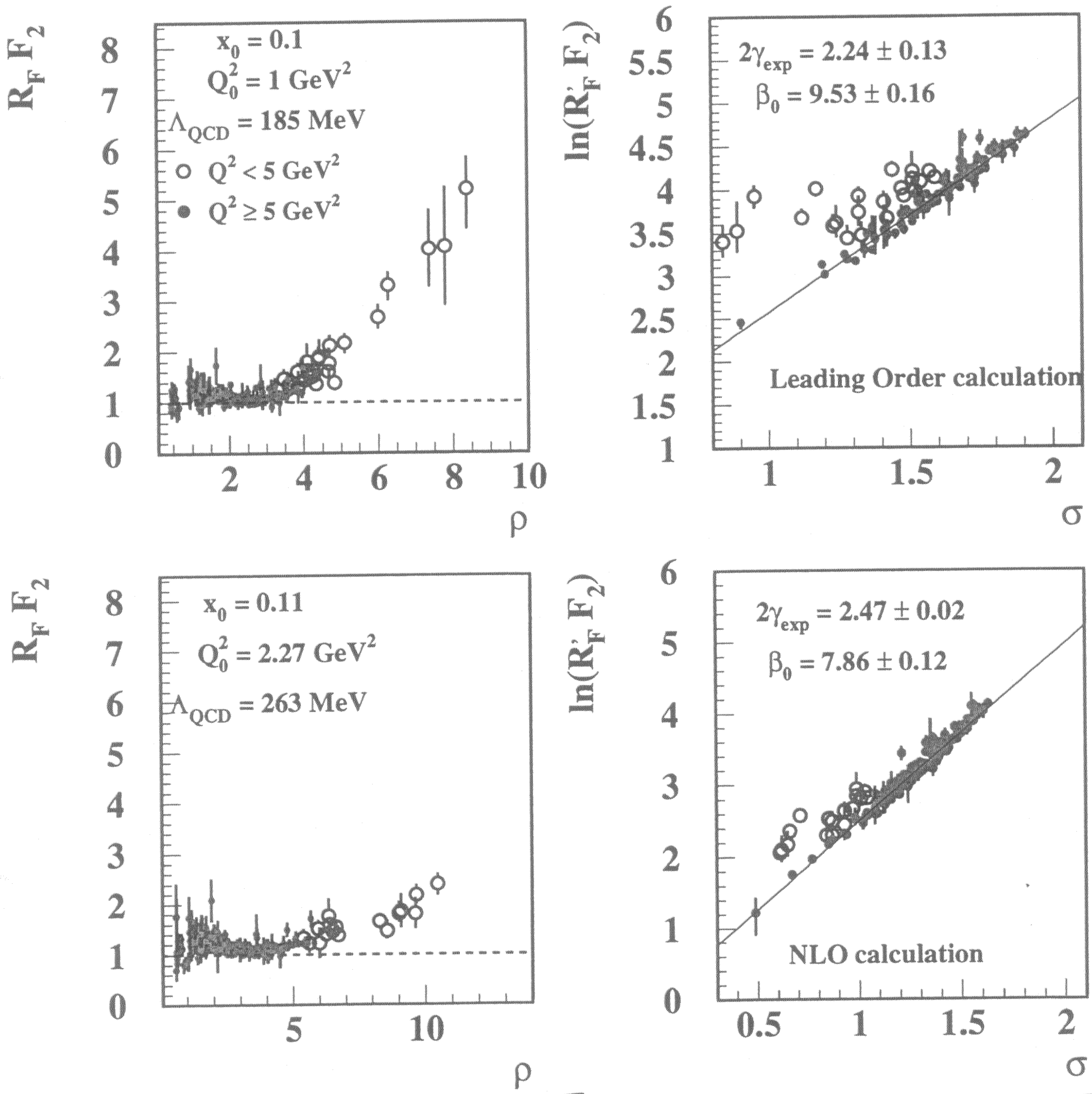
C_A: 3

β_0: $11 - \frac{2}{3}n_f$



200
100
90
80
70
60
50
40
30
20
10
9
8
1
1.5
2
2.5
σ
$N_f=5$ (above b thresh)

ZEUS analysis (1997)



$$\rho = \sqrt{\frac{\ln \alpha_s(Q_0^2)/\alpha_s(Q^2)}{\ln x_0/x}}$$

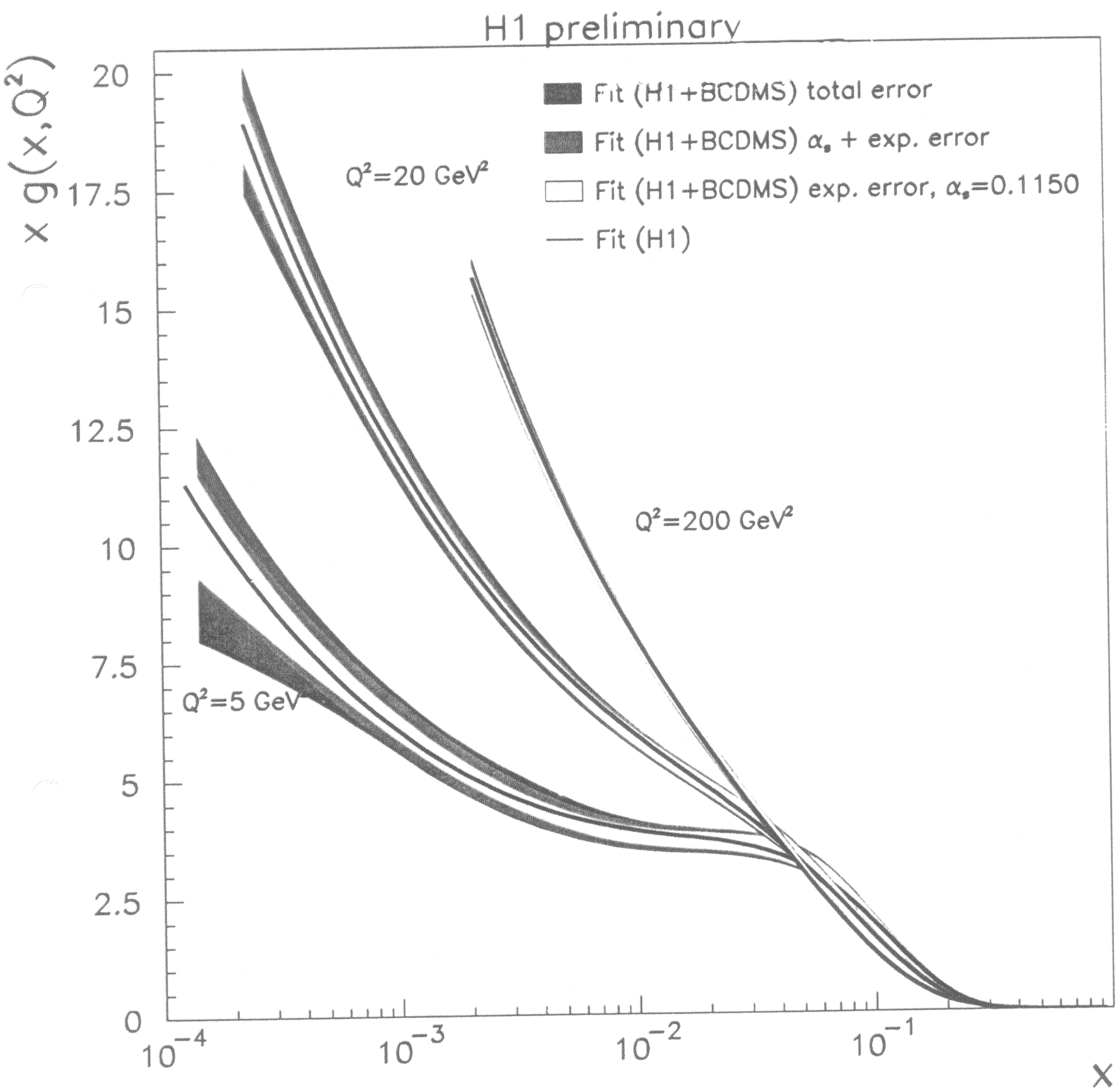
$$\sigma = \sqrt{\ln\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \ln\frac{x_0}{x}}$$

Should be flat

Should be straight line with slope $4\sqrt{3}/\sqrt{\beta_0}$ at LO.

$$\beta_0 = \begin{cases} 8.3 & n_f = 4 \\ 7.7 & n_f = 5 \end{cases}$$

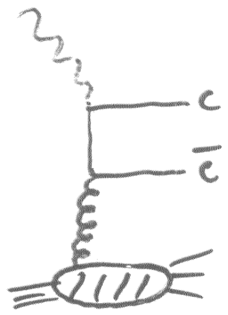
Extract gluon from scaling violations in F_2



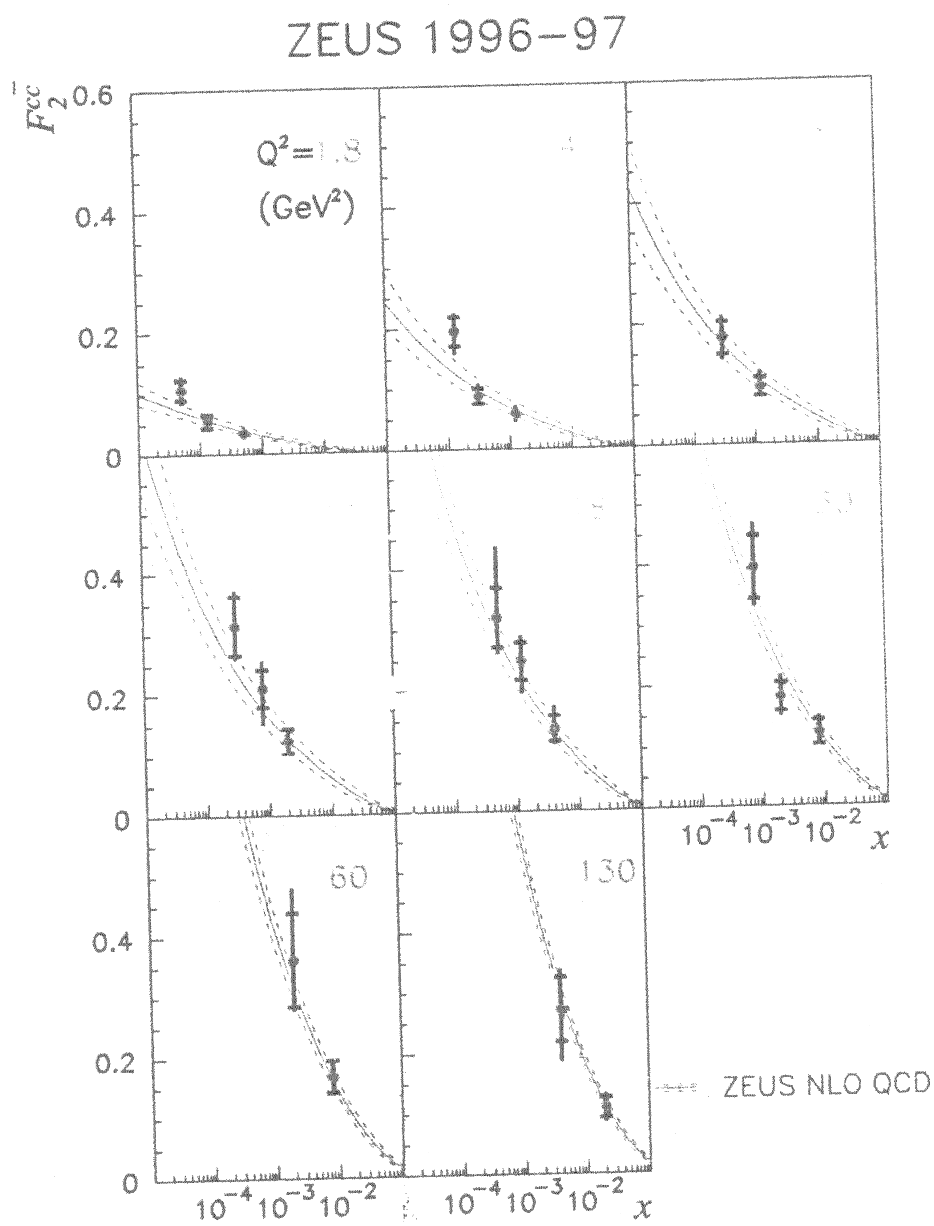
Rise of gluon at small x driven by $1/x$ sing. in P_{gg}

$\sim \frac{\alpha}{x}$

Charm production



Calculate $F_2^{c\bar{c}}$ using gluon extracted from F_2.

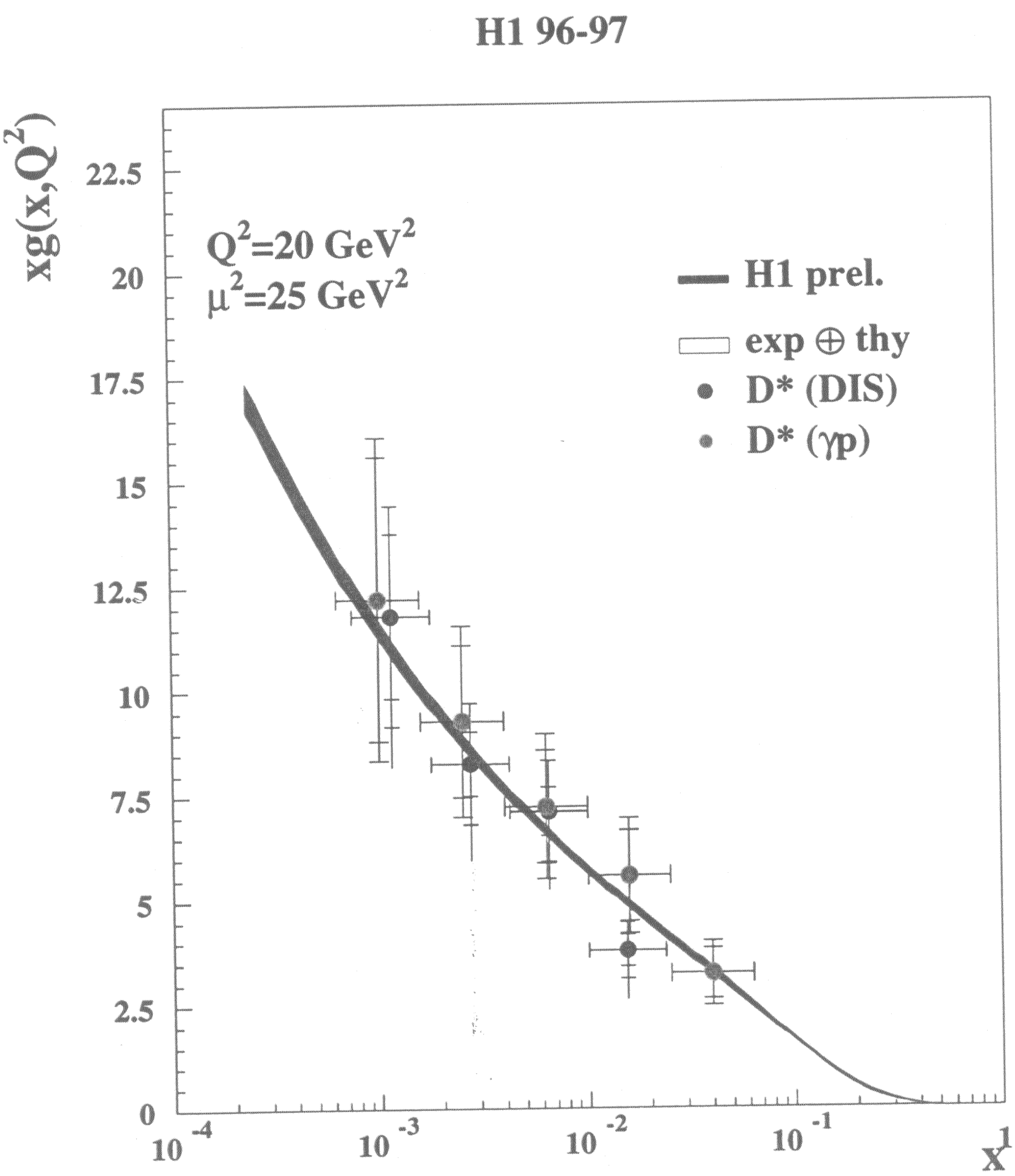


2

$F_2^{c\bar{c}}$ rises with x and Q^2 because the gluon does.

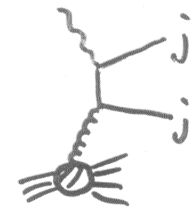
Direct Measurement of Gluon Density

- Reconstruct gluon momentum from D^* and scattered e
- Correct for higher order contributions and fragmentation by unfolding with NLO QCD programs



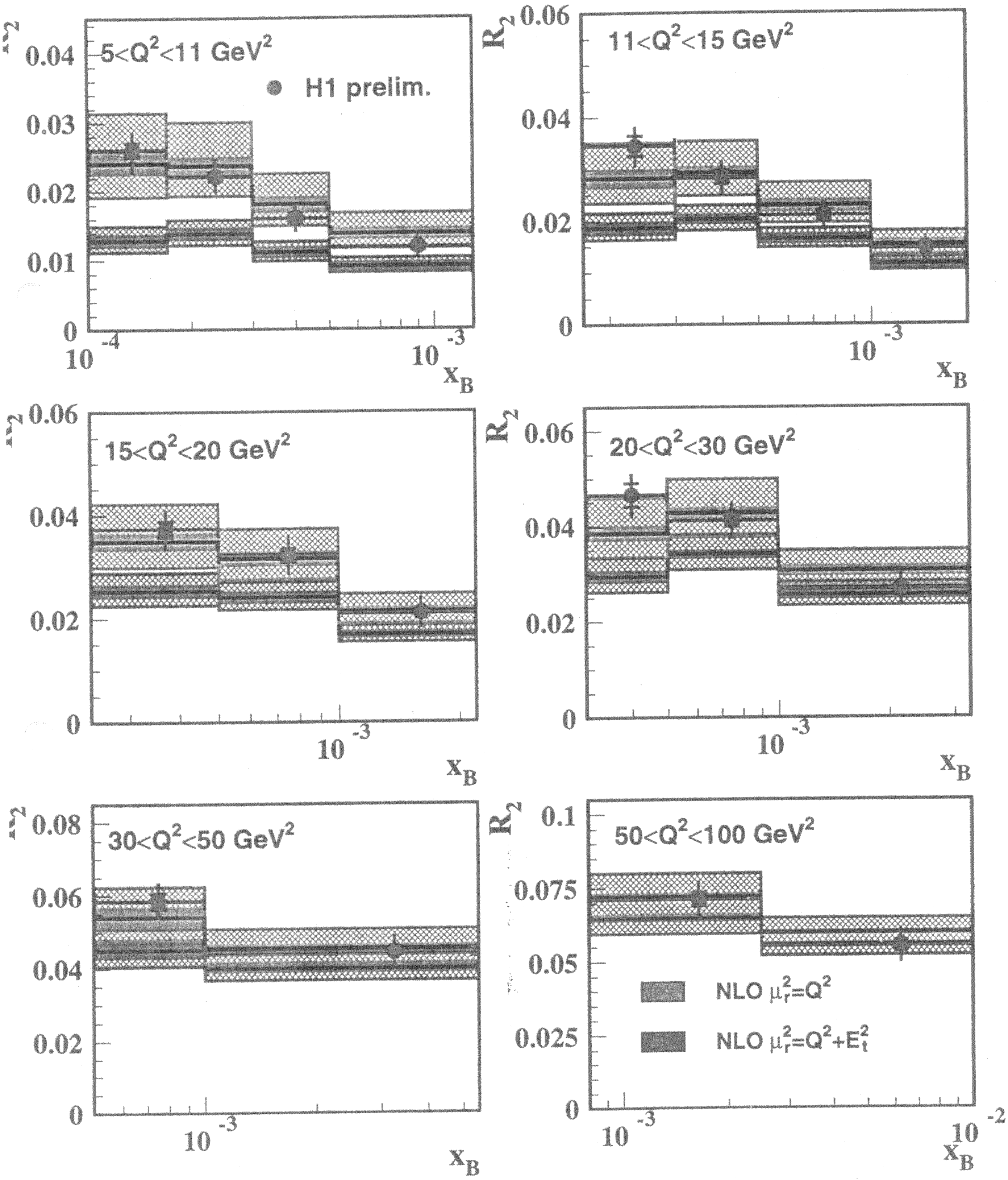
- Gluon distribution universal

Dijet rate at H1



E_T cut : $E_T > 2$ GeV

Dijet rate R_2 $\Delta=2$



Double Log Approx : BFKL

Fixed Coupling

$$\frac{\partial}{\partial \ln 1/x} G(x,Q) = \int_0^Q \frac{dk}{k} K\left(\alpha, \frac{Q}{k}\right) G(x,k)$$

Resums Lx

Mellin wrt Q

$$\frac{\partial}{\partial \ln 1/x} G(x,M) = \chi(\alpha, M)\, G(x,M)$$

$G(M) = \int_0 dQ\, Q^{-M-1} G(Q)$

Mellin wrt x

$$N\, G(N,M) - \bar{G}_0(M) = \chi(\alpha,M)\, G(N,M)$$

$G(N) = \int_0^1 dx\, x^{N-1} G(x)$

Soln:

$$\boxed{G(N,M) = \frac{\bar{G}_0(M)}{N - \chi(\alpha,M)}}$$

← simple pole

At large $Q \gg k$, $K(\alpha, Q/k) \sim$ const; $\chi(\alpha, M) \sim \frac{\bar{\alpha}}{M}$ $\longrightarrow \chi$ fig

LLQ

$$G(x,Q) \sim \int \frac{dM}{2\pi i} \int \frac{dN}{2\pi i}\, e^{M \ln Q^2 + N \ln 1/x} \frac{1}{N - \bar{\alpha}/M}\, \bar{G}_0(M)$$

simple pole: $N = \bar{\alpha}/M$

$$\sim \int \frac{dM}{2\pi i}\, \underbrace{e^{M \ln Q^2 + \frac{\bar{\alpha}}{M} \ln 1/x}}\, \bar{G}_0(M)$$

saddle: $M_S = \sqrt{\frac{\bar{\alpha} \ln 1/x}{\ln Q^2}}$

$$\boxed{G(x,Q) \sim \exp 2\sqrt{\bar{\alpha} \ln Q^2 \ln 1/x}}$$

as $\ln Q^2 \to \infty$ and $x \to 0$

DAS again!

Approximating $\chi(\alpha, M)$

Calculate $\chi(\alpha, M)$ perturbatively in powers of α: $\xrightarrow{\text{fix } (\alpha)}$

$$\chi(\alpha, M) = \alpha\,\chi_0(M) + \alpha^2\,\chi_1(M) + \ldots\ldots$$

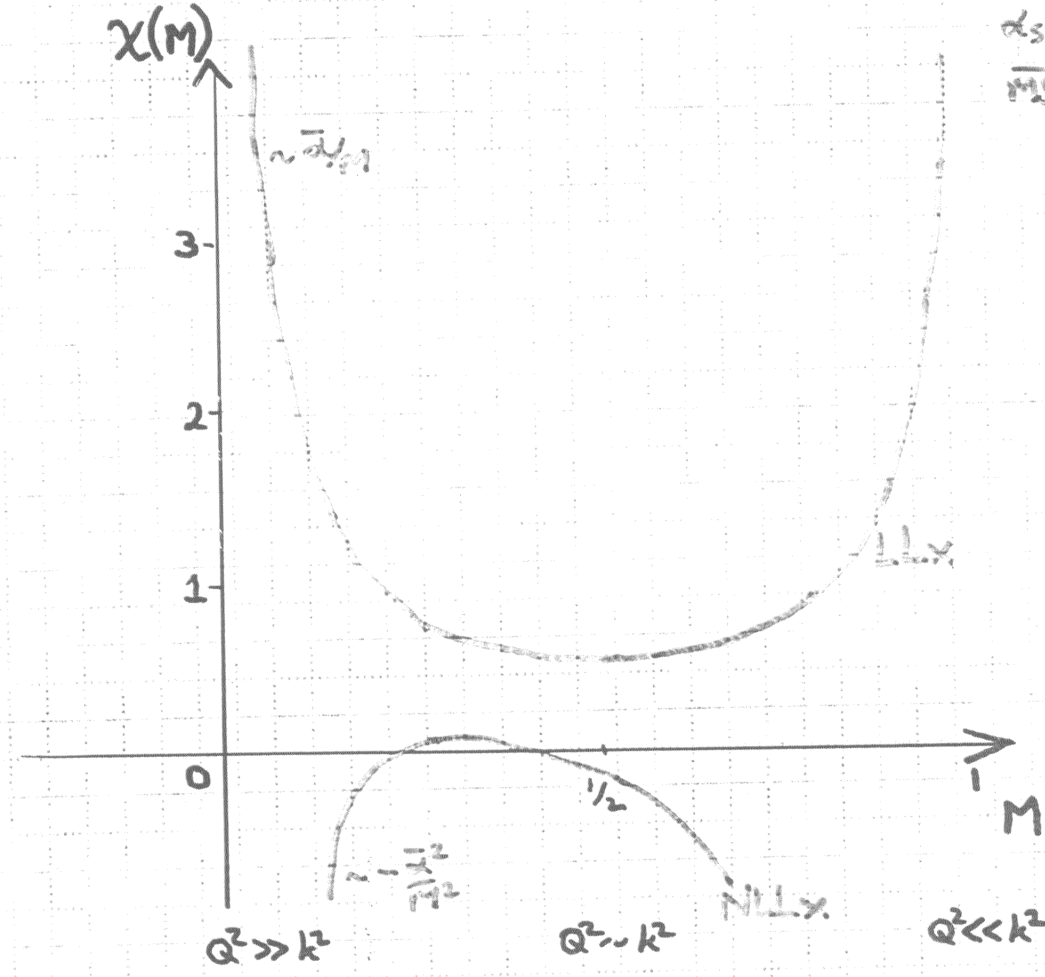
LLx (BFKL) — NLLx (FL)

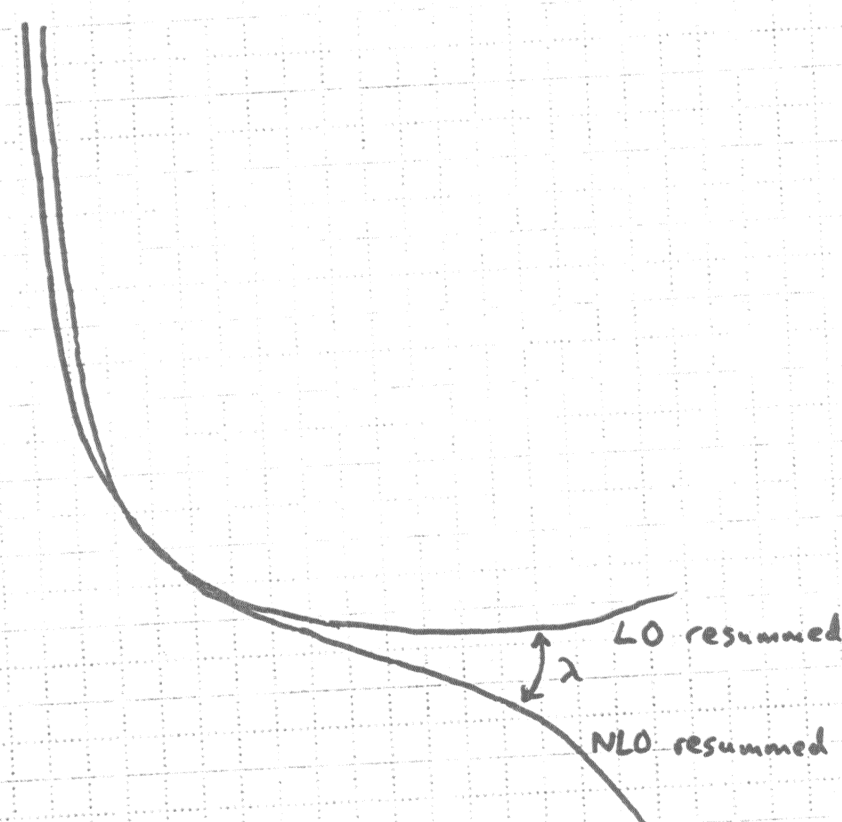
Nonsense when $M \sim 0$ (or $M \sim 1$) (inevitable)

Poor even when $M \sim 1/2$ (unlucky)

$\alpha_s = 0.2$

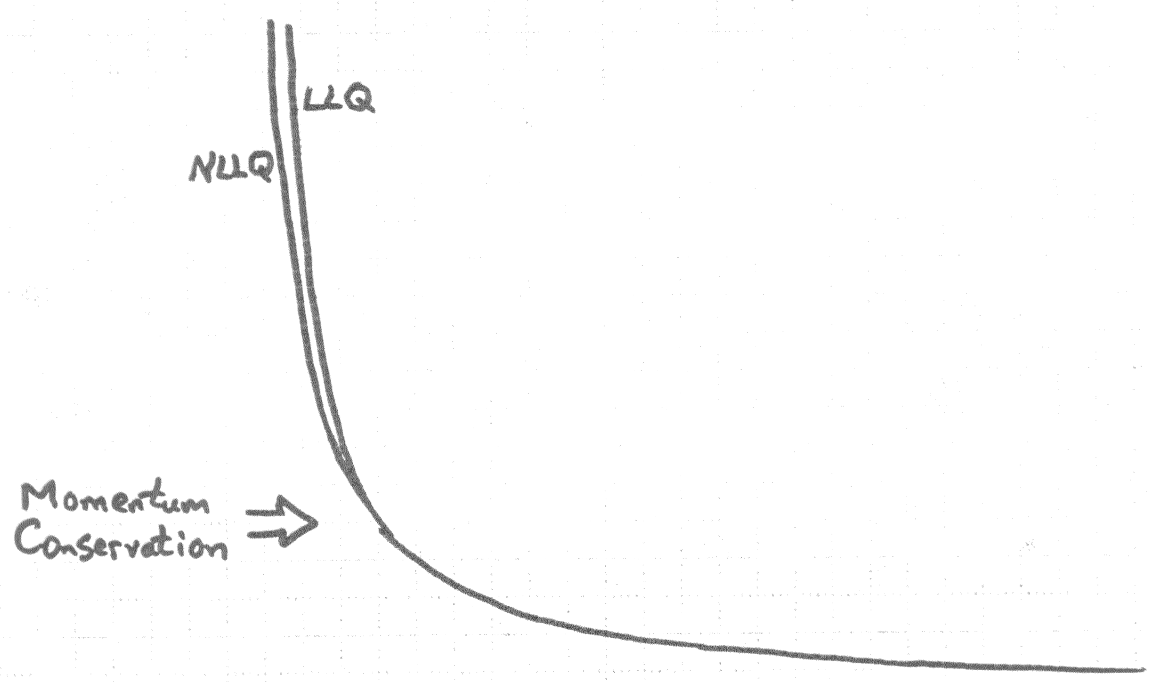
$\overline{MS}$ factorization





$\Rightarrow$ Resummed LO + NLO (add remaining LLx + NLLx: these now small)
Sensible for $M \lesssim \frac{1}{2}$

Parametrize remaining uncertainty around $M = \frac{1}{2}$ by λ.



$\chi(\alpha, M)$ at LLQ, NLLQ, ... using AP + duality

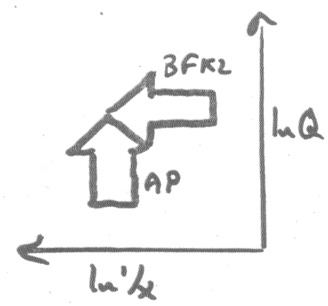
Sensible (resums the large logs of Q^2)

Duality

Fixed coupling

Assume that at small x and large Q^2

AP <u>and</u> BFKL true simultaneously



$$G(N,M) = \frac{G_0(N)}{M - \gamma(\alpha,N)} = \frac{\bar{G}_0(M)}{N - \chi(\alpha,M)}$$

AP soln. = BFKL soln

Only possible if

$$M = \gamma(\alpha,N) \iff N = \chi(\alpha,M)$$

so

$$\boxed{\begin{aligned} M &= \gamma(\alpha, \chi(\alpha,M)) \\ N &= \chi(\alpha, \gamma(\alpha,N)) \end{aligned}}$$

"Duality"
$M \leftrightarrow N$
$\gamma \leftrightarrow \chi$

i.e. $\gamma = \chi^{-1}$ and $\chi = \gamma^{-1}$. $\to$ γ, χ figs

eg $\gamma(\alpha,N) = \frac{\bar{\alpha}}{N} \iff \chi(\alpha,M) = \frac{\bar{\alpha}}{M}$ DLL

Also need to match b.c. :

$$G_0(N) = \frac{\bar{G}_0(\gamma(\alpha,N))}{-\chi'(\alpha,\gamma(\alpha,N))}, \quad \bar{G}_0(M) = \frac{G_0(\chi(\alpha,M))}{-\gamma'(\alpha,\chi(\alpha,M))}$$

N.B. $1 = \gamma'\chi'$

Momentum Conservation

$$\Rightarrow \quad \boxed{\begin{aligned} \gamma(\alpha, 1) &= 0 \\ \chi(\alpha, 0) &= 1 \end{aligned}} \qquad \begin{aligned} &\text{to all orders } \left[\partial^\mu T_{\mu\nu} = 0\right] \\ &\text{by duality} \end{aligned}$$

Only exact all order result for γ, χ.

e.g. $$\gamma(\alpha, N) \sim \frac{\bar{\alpha}}{N} - \bar{\alpha}$$

$\xrightarrow{\ \ }$ γ fig

$$M = \frac{\bar{\alpha}}{N} - \bar{\alpha} \quad \Rightarrow \quad N = \frac{\bar{\alpha}}{M + \bar{\alpha}}$$

so $$\chi(\alpha, M) \sim \frac{\bar{\alpha}}{M + \bar{\alpha}} = \frac{\bar{\alpha}}{M} - \left(\frac{\bar{\alpha}}{M}\right)^2 + \left(\frac{\bar{\alpha}}{M}\right)^3 - \cdots\cdots$$

$$\chi_0 \sim \frac{1}{M} \qquad \chi_1 \sim -\frac{1}{M^2} \qquad \chi_2 \sim \frac{1}{M^3}$$

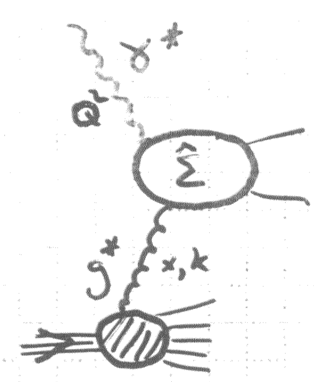
$\xrightarrow{\ \ }$ χ fig.

Collinear poles (ie LQ)

- Fixed order calcns of χ very poor near $M = 0$ $(Q^2 \gg k^2)$
- Momentum conservation resums the collinear poles in $\chi(\alpha, M)$.

$\xrightarrow{\ \ }$ $\gamma + \chi$ fig

Gluons $\longrightarrow$ Quarks



Fixed order (collinear fact)

$$F(x,Q) = \int_x^1 \frac{dy}{y} \underbrace{\sigma(\alpha, x/y)}_{\text{hard xsec}} \underbrace{G(y,Q)}_{\text{gluon pdf.}}$$

$\uparrow$ structure fn.

$$= \int \frac{dN}{2\pi i} x^{-N} C(\alpha, N) G(N,Q)$$

$\uparrow$ coeff fn.

"k_T-factorization" (for small x):

$$F(x,Q) = \int_x^1 \frac{dy}{y} \int_0^Q \frac{d^2k}{\pi} \underbrace{\Sigma(\alpha, Q^2/k^2, x/y)}_{\text{off shell hard xsec}} \underbrace{\mathcal{F}(y,k^2)}_{\text{"unintegrated" pdf}}$$

$$= \int \frac{dN}{2\pi i} \int \frac{dM}{2\pi i} x^{-N} (Q^2)^M \underbrace{J(\alpha, M, N)}_{\text{"impact factor"}} \mathcal{F}(M,N)$$

$\uparrow$ $= \dfrac{\mathcal{F}_0(M)}{N - \chi(\alpha, M)}$ (BFKL)

Pole at $M = \gamma(\alpha, N)$ (duality)

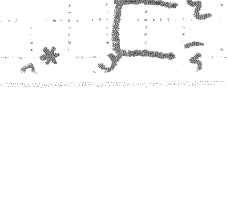
small x $\simeq \int \frac{dN}{2\pi i} x^{-N} J(\alpha, \gamma(\alpha, N), N)\, G(N,Q)$

$\uparrow$ $= -\dfrac{(Q^2)^\gamma}{\gamma} \dfrac{\chi(\gamma)}{\chi'(\gamma)} \mathcal{F}^0(\gamma)$

"integrated pdf".

Expand in α, $\frac{\alpha}{N}$: gives all the (N)LLx corrections to the coeff fn.

eg $C(\alpha, N)\big|_{\dots} = \alpha J_0(\gamma_0(\frac{\alpha}{N}), 0)$



Running Coupling

$t \equiv \ln Q^2/Q_0^2$

$$\alpha \longrightarrow \alpha(t) = \underbrace{\frac{\alpha}{1+\beta_0\alpha t}}_{\text{LLQ}} \underbrace{+ \cdots}_{\text{NLLQ}} = \alpha(1 \underbrace{- \alpha\beta_0 t}_{\text{LLQ, LLx}} \underbrace{+ (\alpha\beta_0 t)^2}_{\text{LLQ, NLLx}} \underbrace{- \cdots}_{\text{LLQ, NNLLx}})$$

$$\longrightarrow \hat{\alpha} = \frac{\alpha}{1 - \alpha\beta_0 \frac{d}{dM}} + \cdots \qquad \text{after } M\text{-Mellin.}$$

Factorization - duality remain valid (but proof much more difficult...)

$$N = \chi(\alpha(t), \gamma(\alpha(t), N))$$

At NLLx:

- Scheme change $\quad G(N,M) \longrightarrow u(M)\, G(N,M) \qquad$ LLx

$$\chi \longrightarrow \chi - \alpha\beta_0 \chi_0(M) \frac{d}{dM} \ln u(M)$$

$$\gamma \longrightarrow \gamma + \frac{d}{dt} \ln u\left(\gamma_0\left(\frac{\alpha(t)}{N}\right)\right)$$

NB LLx scheme change $\Rightarrow$ anom dim changes at NLLx

BFKL + FL + CH sufficient at NLLx.

- $M = 1/2$ singularity: when coupling runs

$$\chi \longrightarrow \chi + \frac{1}{2}\alpha\beta_0 \frac{\chi_0 \chi_0''}{\chi_0'} \longleftarrow \text{vanishes at } M = 1/2.$$

In $\overline{MS}$ this $M = 1/2$ sing in χ cancels: instead get

$\frac{1}{\sqrt{M - 1/2}}$ sing. in gluon normalization

(thus in $C(M)$, γ_{qg} etc.)

Can resum $M = 1/2$ sing to all orders (ie LLQ, N^nLLx)

using Airy fns $\Rightarrow$ further <u>reduction</u> in small x effects.

(no unstable oscillations)

Altarelli, RDB, Forte hep-ph/9911273

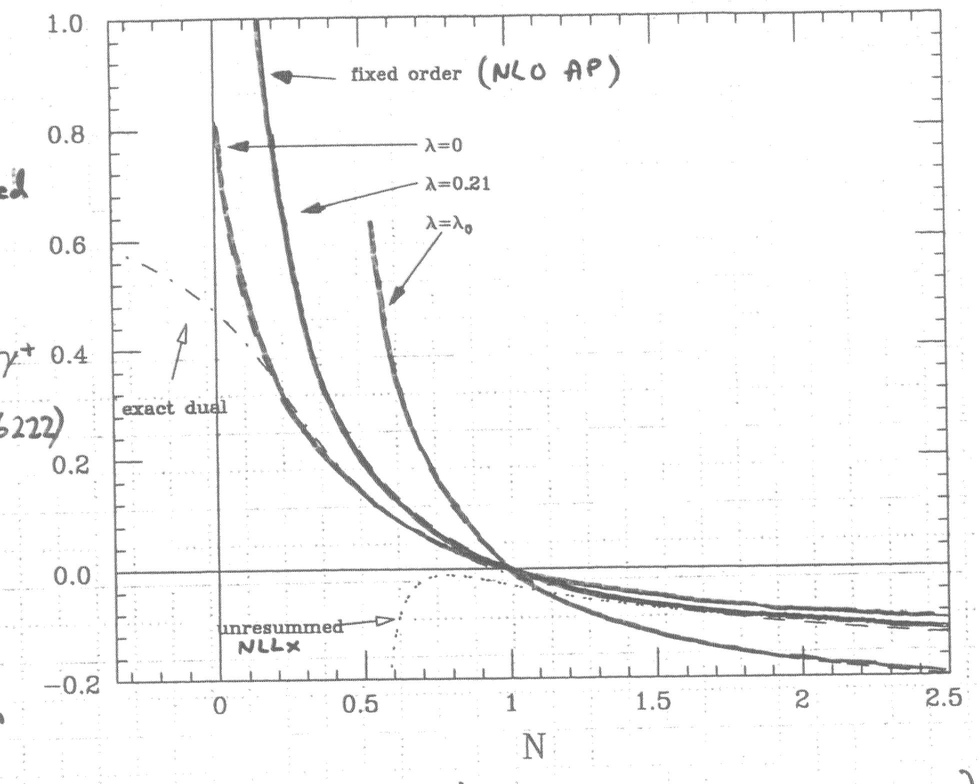
(Practical) Resummed Anom Dims at NLLx and NLLQ.

$\alpha_s = 0.2$

Treat resummed λ_{ev} exactly for perturbative stability....

RDB+SF hep-ph/9906222)

Technical....



Formally NLO: include in LO.

$\lambda_{ev} = 24 \ln 2 \frac{\alpha}{2\pi} + \Delta\lambda$

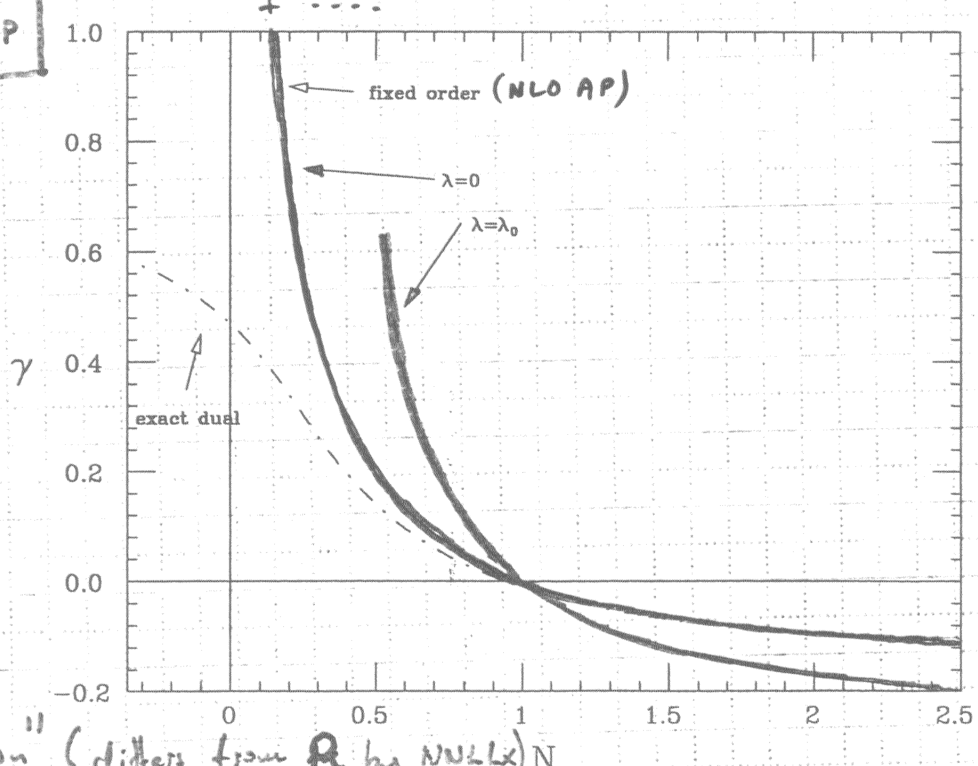
$= \chi(\alpha, M_{min})$

"R resummation"

$$\gamma^+(N,\alpha) = \left[\alpha\gamma_0(N) + \gamma_s\left(\frac{\alpha}{N-\Delta\lambda}\right) - \alpha\frac{n_c}{\pi N}\right] + \alpha\left[\alpha\gamma_1(N) + \tilde{\gamma}_{ss}\left(\frac{\alpha}{N-\Delta\lambda}\right) - \alpha\left(\frac{e_2}{N^2} + \frac{e_1}{N}\right) - e_0\right] + \ldots$$

$\boxed{\text{If } \lambda_{ev} \ll 0: \text{ recover NLO AP}}$

+ quark anom dim
+ coeff fn.
etc.....



"S resummation" (differs from R by NNLLx)

$$\gamma^+(N,\alpha) = \left[\alpha\gamma_0(N) + \gamma_s\left(\frac{\alpha}{N-\Delta\lambda}\right) - \alpha\frac{n_c}{\pi(N-\Delta\lambda)}\right] + \alpha\left[\alpha\gamma_1(N) + \tilde{\gamma}_{ss}\left(\frac{\alpha}{N-\Delta\lambda}\right) + \frac{n_c\Delta\lambda}{\pi(N-\Delta\lambda)^2} - \alpha\left(\frac{e_2}{(N-\Delta\lambda)^2} + \frac{e_1}{N-\Delta\lambda}\right) - e_0\right] + \ldots\ldots$$

+ .. etc

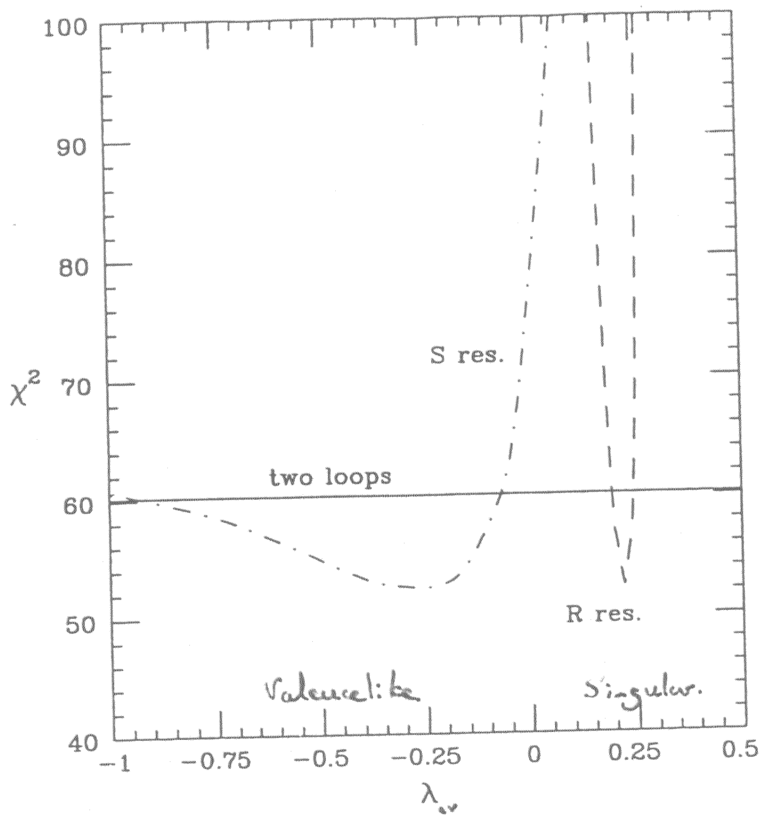
G. Altarelli + RDB + S. Forte hep-ph/0011?

Fits to data including NLO resummation.

$\log 1/x$ (at NLLx) and $\log Q^2$ (at NLLQ)

H1 1995-97 red. xsec data (i.e. F_2, F_L) 95 data pts.

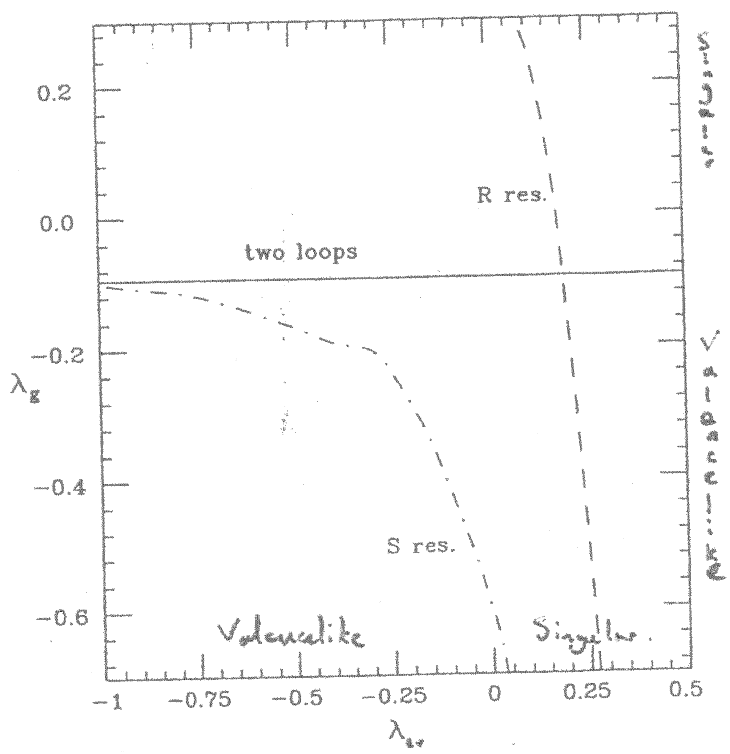
$\alpha_s(M_Z) = 0.119$ (fixed)



Resummed fits can be ~ as good as two loop fits

x. Splitting fn $\sim x^{-\lambda_{ev}}$

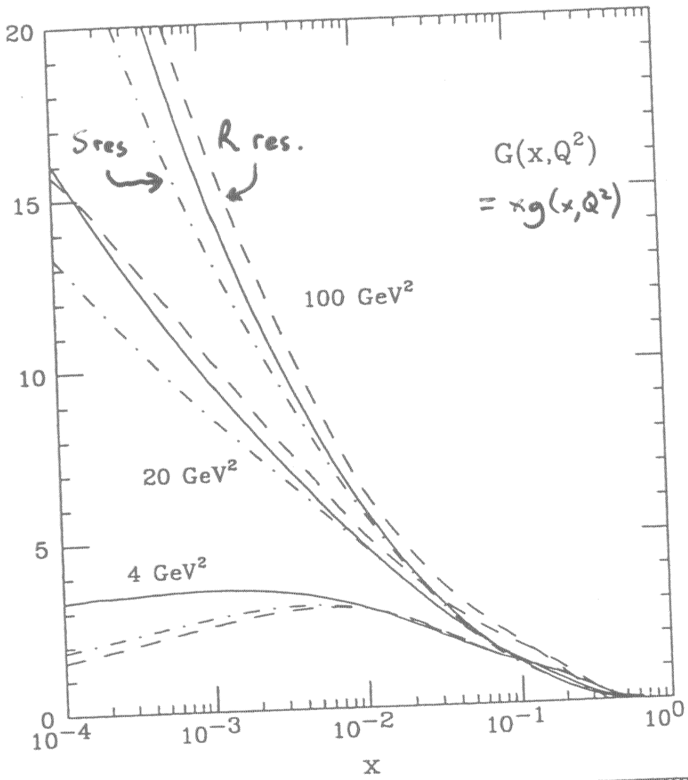
x. Initial gluon $\sim x^{-\lambda_g}$



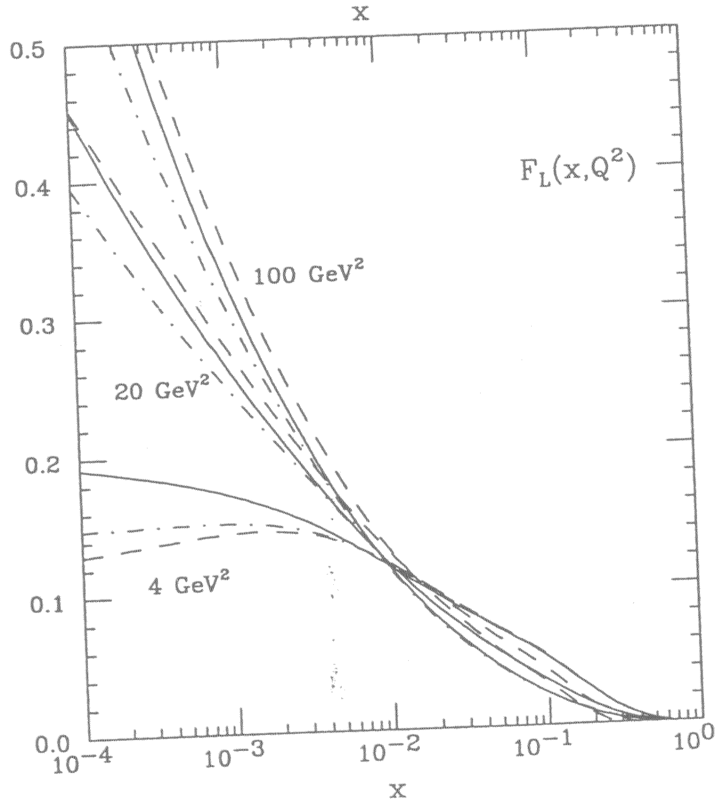
Compensation between evolution + initial gluon.

Resummed gluon distbn.

Gluon distbn ($\overline{MS}$)



F_L

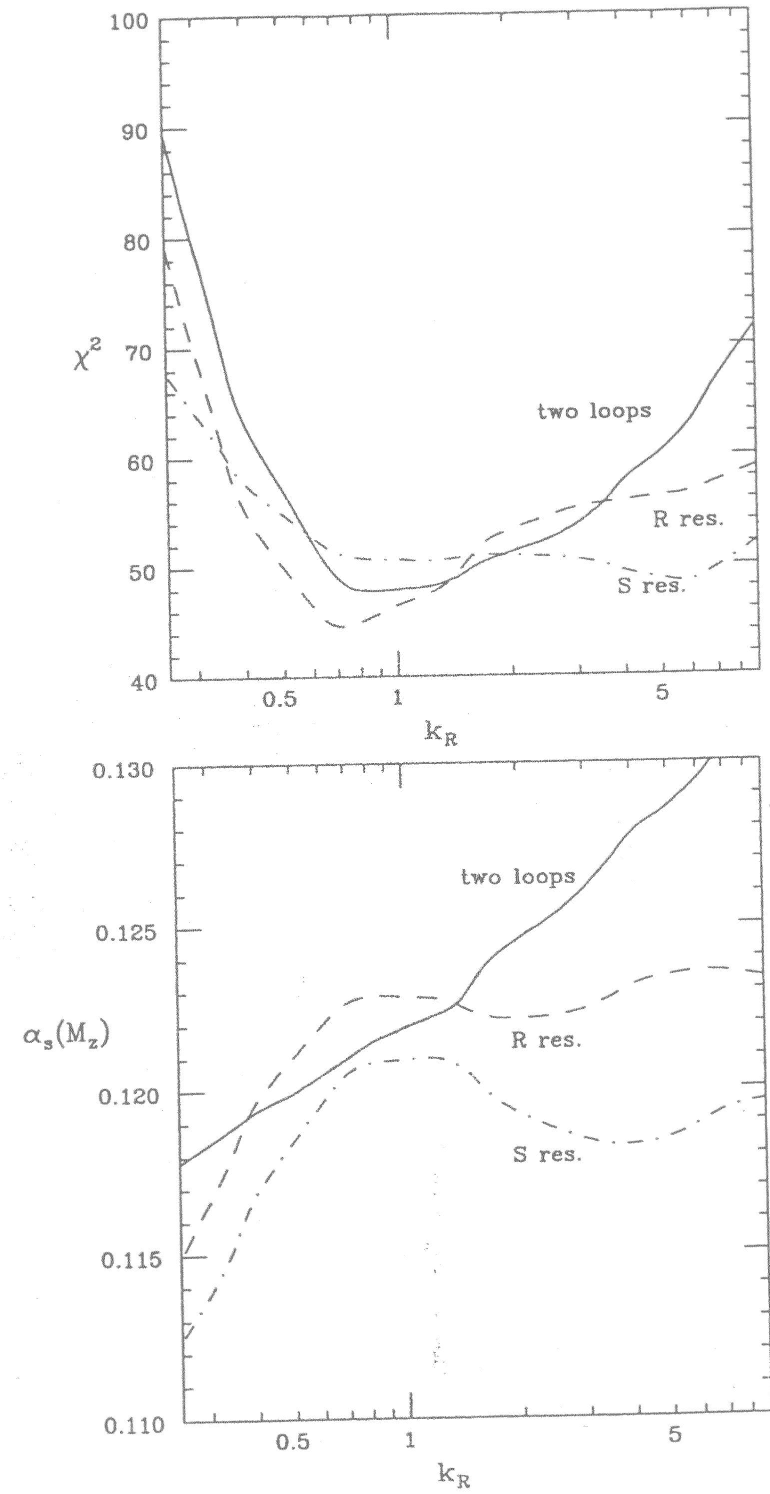


$\longleftarrow$ small x

Gluon $\to$ F_L reduced at low Q^2

α_s → ren. scale dependence

$$\mu_R^2 = k_R Q^2$$



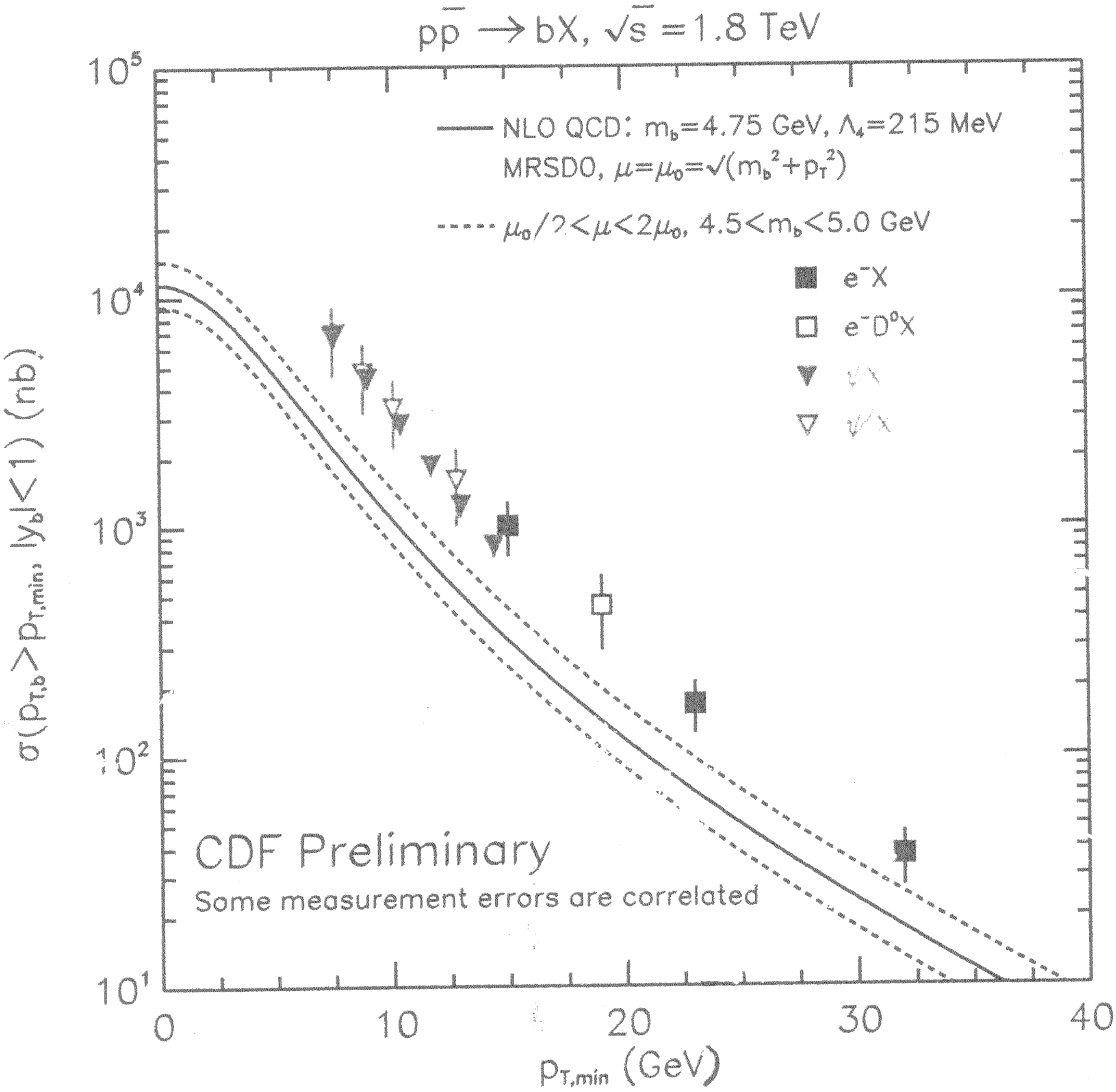
Theoretical uncertainty at two loops

Theoretical uncertainty after resummation

χ^2 & $\alpha_s(M_Z)$ both ~ flatter after resummation.

Need small x resummation for reliable α_s determination at small x.

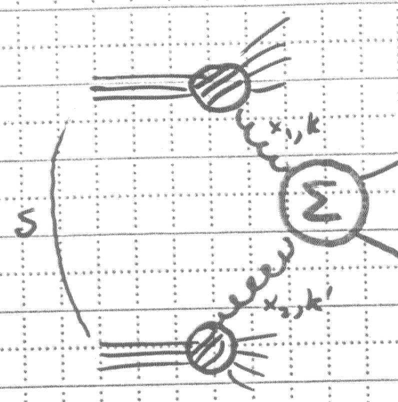
B production



Data $\simeq$ factor of 2 above theory.

Small x effect?

Resummation of Hadroprod. of Heavy Quarks



$$\rho = 4M^2/S$$

Fixed order (collin. fact.)

$$\Sigma(\rho) = \sum_{ij} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \underbrace{\Sigma^{FO}_{ij}\left(\frac{\rho}{x_1 x_2}\right)}_{\text{Hard xsec: NLO (Nason, Dawson, Ellis)}} F_i(x_1)\, F_j(x_2)$$

$$= \sum_{ij} \int \frac{dN}{2\pi i}\, \rho^{-N}\, \hat{\Sigma}^{FO}_{ij}(N)\, F_i(N)\, F_j(N)$$

Mellin wrt ρ

k_t factorisation:

$$\Sigma(\rho) = \sum_{ij} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \int \frac{d^2k}{\pi} \frac{d^2k'}{\pi} \underbrace{\Sigma_{ij}\left(\underline{k}, \underline{k}'; \frac{\rho}{x_1 x_2}\right)}_{\text{off-shell hard xsec}} \underbrace{\mathcal{F}_i(\underline{k}, x_1)\, \mathcal{F}_j(\underline{k}', x_2)}_{\text{unintegrated pdf}}$$

$$= \sum_{ij} \int \frac{dN}{2\pi i}\, \rho^{-N}\, h_{ij}(\underbrace{\gamma(N), \gamma(N)}, N)\, F_i(N)\, F_j(N)$$

Mellin wrt $\underline{k}, \underline{k}'$;

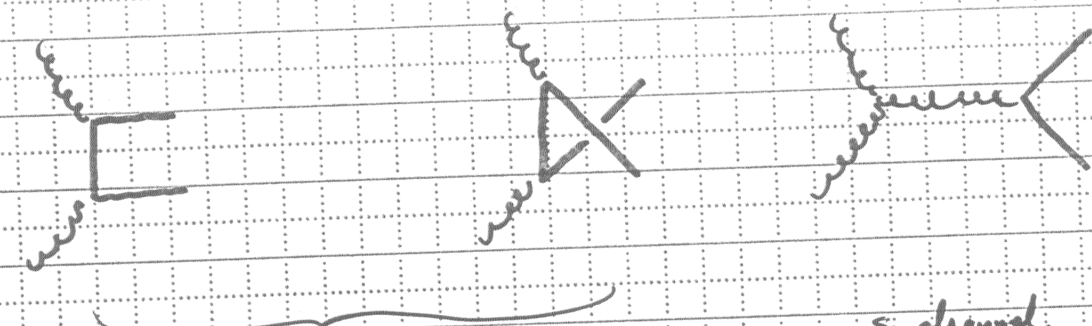
$M = \gamma(N)$ by duality

$$\mathcal{F}(M, N) = \frac{\mathcal{F}^0(M)}{1 - \frac{\bar{\alpha}}{N}\chi(M)} \qquad F_i(N) = -\frac{1}{\gamma} \frac{\chi(\gamma)}{\chi'(\gamma)} \mathcal{F}^0_i(\gamma)$$

simple pole in M   integrated pdf   unintegrated pdf

Calculation of "Impact Factor" $h(M_1, M_2, N)$

Calculate off-shell xsec $\Sigma_{ij}(k, k', \rho)$ → Mellin wrt $\underline{k}, \underline{k}', \rho$



t-channel

s-channel.

Ans:

$$h(M_1, M_2; N=0) = \frac{\pi\alpha^2}{V_c} M_1 M_2 \, B(M_1, 1-M_1)\, B(M_2, 1-M_2)$$

Catani, Ciafaloni hep-ph/9604?50, 9701303

Ball, RKE hep-ph/0101199

$$\times \left\{ 4N_c \left[\frac{B(3-M_1-M_2, 3-M_1-M_2)}{1-M_1-M_2} + \frac{B(3-M_1-M_2, 3-M_1-M_2)}{(1-M_1-M_2)^3 \, B^2(1-M_1, 1-M_2)} \right] - \frac{2}{N_c} \frac{\Gamma(2-M_1)\Gamma(2-M_2)\Gamma(2-M_1-M_2)}{\Gamma(4-2M_1)\Gamma(4-2M_2)} \, \frac{7-5(M_1+M_2)+3M_1M_2}{1-M_1-M_2} \right\}$$

- matches onto fixed order results (set $M_1 = M_2 = \gamma(\frac{\alpha}{N})$ → expand in α/N)

- near $M_1 = M_2 = 1/2$

$$h(M_1, M_2; 0) \sim \frac{\pi\alpha^2}{V_c} \frac{N_c}{6} \frac{1}{(1-M_1-M_2)^3}$$

Catani et al NPB366(1991)135

Triple pole ⇒ large enhancement at small ρ from s-channel gluon.

RDB + RKE
hep-ph/0101177

Asymptotic estimates:

as $\rho \to 0$

$$\Sigma(\rho) \sim \frac{\pi^2}{8 V_c} \frac{\ln M^2/\mu^2}{\bar{\alpha}(M^2)} \frac{\mu^2}{M^2} \underbrace{e^{-2\bar{\alpha}_s(M^2)}}_{\substack{\text{power growth} \\ \text{(from triple pole in xsec)}}} \left[G(\bar{x}, M^2)\right]^2 \qquad \bar{x} = \left(\frac{\mu^2}{M^2}\right)^{1/4\bar{\alpha}_s}$$

Estimate K-factor $\sim$ 1.4 Tevatron
2 LHC.

Large enhancement even when gluon pdf is not enhanced

but detailed numerical studies needed

- Diff xsec (p_T distbn) much more difficult...

- Can the triple pole at $M_1 = M_2 = 1/2$ be resummed?
or is it physical?

Summary

- Small x resummation requires also collinear resummation

  Duality : AP $\Leftrightarrow$ BFKL

- Resummation effects v. small in most inclusive processes but
  - changes extraction of gluon + α_s from HERA data
  - may give large effect in some processes

    eg hadroproduction of heavy quarks

- LHC : watch this space.