Global Fits and Parton Distributions

Robert Thorne

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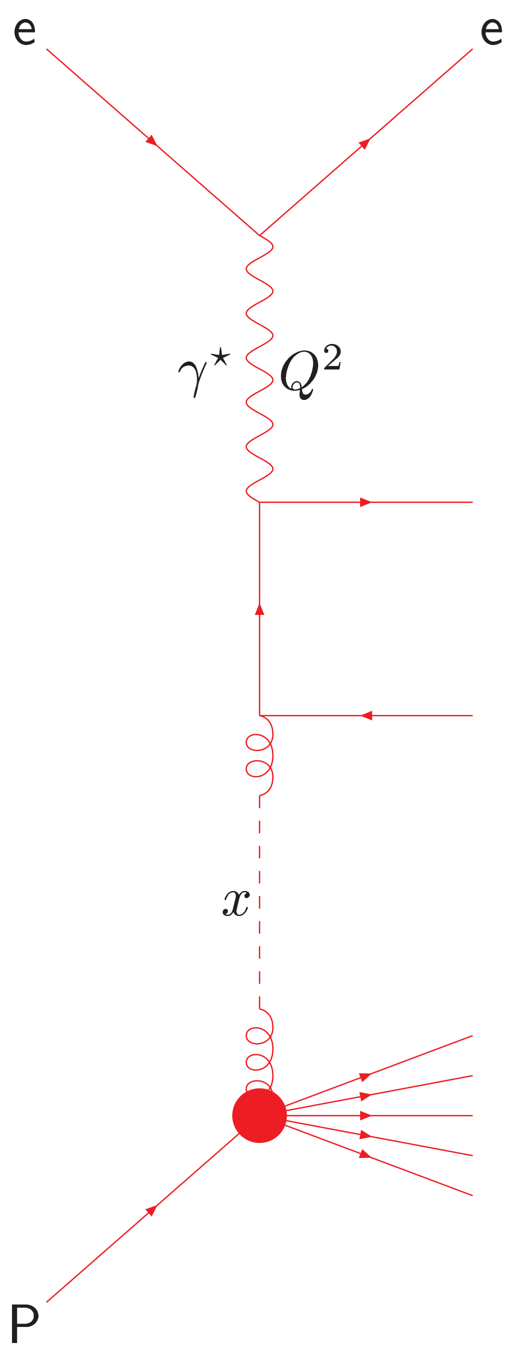
University of Cambridge

Royal Society Research Fellow

The fundamental quantities one requires in the calculation of scattering processes involving hadronic particles are the parton distributions. These can be derived from, and then used within, the quantum field theory QCD. Since this is a theory with strong interactions it is difficult to perform analytic calculations from first principles. However, using the **Factorization Theorem** it is possible to separate processes into nonperturbative parts which can be determined from experiment, and perturbative parts which can be calculated as a power-series in the strong coupling constant α_S.

Hadron scattering with an electron factorizes.



$C_i^P(x, \alpha_s(Q^2))$

$f_i(x, Q^2, \alpha_s(Q^2))$

$\alpha_s(Q^2)$ – Strong Coupling

$x = \frac{2m\nu}{Q^2}$ – Momentum fraction of Parton (ν=energy transfer)

The cross-section for this process can be written in the factorized form

$$\sigma(ep \to eX) = \sum_i C_i^P(x, \alpha_s(Q^2)) \otimes f_i(x, Q^2, \alpha_s(Q^2))$$

where $f_i(x, Q^2, \alpha_s(Q^2))$ are the parton distributions, i.e the probability to find a parton of type i carrying a fraction x of the momentum of the hadron.

Corrections to above formula of size $\Lambda^2_{\rm QCD}/Q^2$ - Higher Twist. These can be associated with multi-parton correlation functions.

Because they depend on the nonperturbative way in which partons are bound into the hadron, these parton distributions are not calculable from first principles. However, they do evolve with Q^2 in a perturbative manner

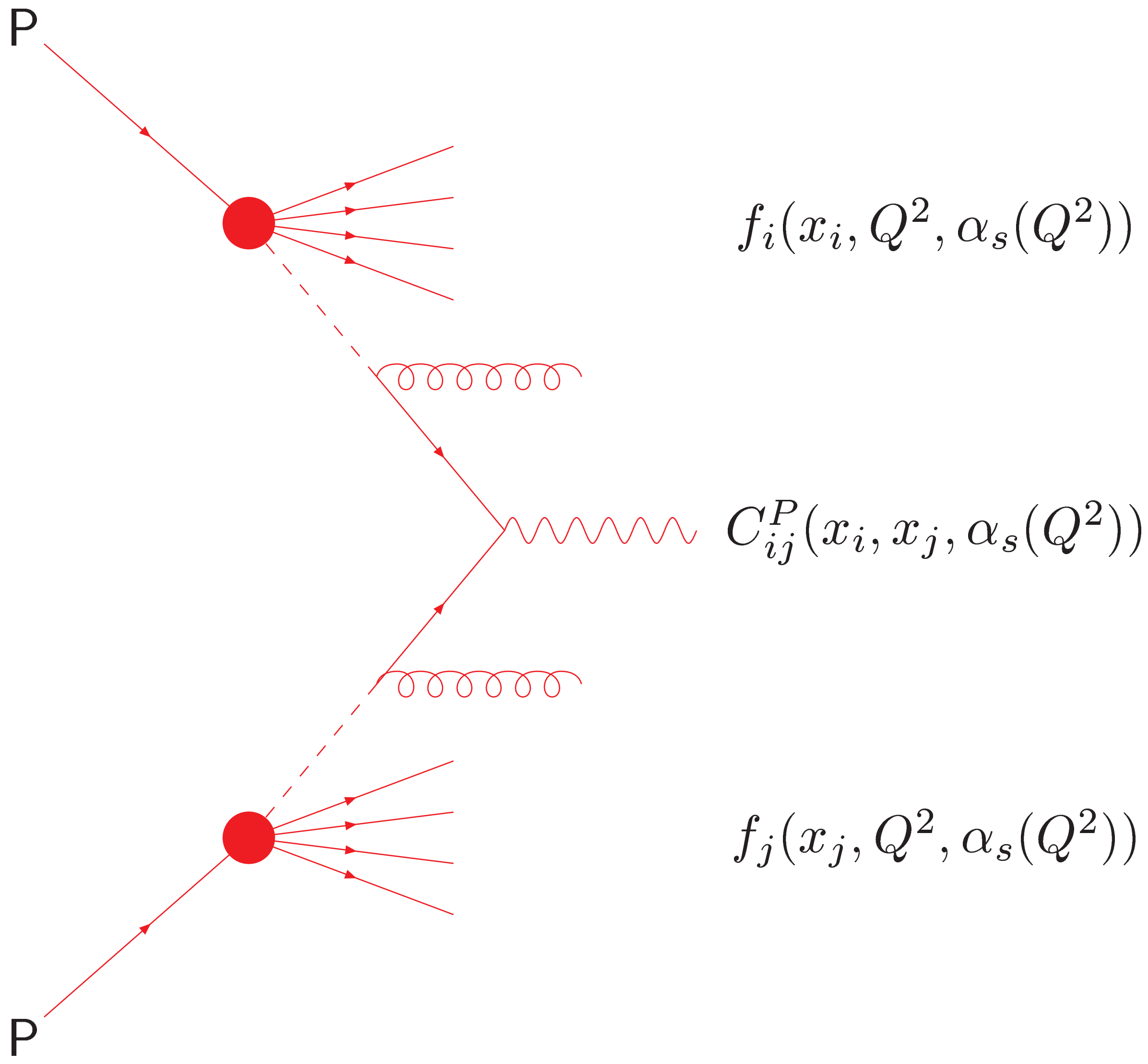
$$\frac{df_i(x, Q^2, \alpha_s(Q^2))}{d\ln Q^2} = \sum_i P_{ij}(x, \alpha_s(Q^2)) \otimes f_j(x, Q^2, \alpha_s(Q^2))$$

where the splitting functions $P_{ij}(x, Q^2, \alpha_s(Q^2))$ are calculable order by order in perturbation theory.

The coefficient functions $C_i^P(x, \alpha_s(Q^2))$ describing the hard scattering process are process dependent but are calculable as a power-series in the strong coupling constant $\alpha_s(Q^2)$.

$$C_i^P(x, \alpha_s(Q^2)) = \sum_k C_i^{P,k}(x)\alpha_s^k(Q^2).$$

Since the parton distributions $f_i(x, Q^2, \alpha_s(Q^2))$ are process-independent, i.e. **universal**, once they have been measured at one experiment, one can predict many other scattering processes.



Global fits to determine parton distributions use all available data - largely $ep \to eX$ (Structure Functions), and the most up-to-date QCD calculations to best determine **parton distributions** and their consequences. Currently use NLO–in–$\alpha_s(Q^2)$, i.e.

$$
\begin{aligned}
C_i^P(x, \alpha_s(Q^2)) &= C_i^{P,0}(x) + \alpha_S(Q^2) C_i^{P,0}(x). \\
P_{ij}(x, \alpha_s(Q^2)) &= \alpha_S(Q^2) P_{ij}^0(x) + \alpha_S^2(Q^2) P_{ij}(x).
\end{aligned}
$$

NNLO coefficient functions are known for some processes, e.g. structure functions, and NNLO splitting functions have considerable information, and may be known within a year or so. NNLO fits in near future.

Perturbation theory valid if $\alpha_s(Q^2) < 0.3$. Since running coupling constant $\alpha_s(Q^2)$

$$\alpha_s(Q^2) \approx \frac{4\pi}{(11 - 2/3N_f)\ln(Q^2/\Lambda^2_{QCD})}$$

where Λ_{QCD} is the scale of hadronic physics, i.e $\sim 150\mathrm{MeV}$, can use perturbation theory if $Q^2 > 2\mathrm{GeV}^2$. This cut should also remove influence of higher twists.

There are quite a number of groups performing fits in order to obtain parton distributions.

• MRST and CTEQ - both collaborations which fit all available data from DIS and hadron-hadron scattering in order to best determine all parton distributions in all regions of parameter space. Slight differences in philosophy, cuts on data etc. Until now add statistical and systematic errors in quadrature and minimize χ^2 with some consideration of weighting of experiments. Both improving this procedure.

• GRV - collaboration which makes theoretical assumption of valence-like partons at very low input scale ($\sim 0.3\mathrm{GeV}^2$). Combined with low $\alpha_S(M_Z^2)$. Never really a fit, but required global consistency in quantitative sense. Philosophy difficult to reconcile with most recent data.

• Giele, Keller and Kosower - dedicated attempt to produce uncertainties on parton distributions. So far fit only relatively small subset of data, all being DIS.

• H1 and ZEUS (Botje). Each experiment has groups which make fits centered on their own data. Treat correlated errors in a more sophisticated manner, but only fit subsets of data, again always DIS. Work on uncertainties using correlated errors.

Note Using only limited data sets can lead to results completely incompatible with other data!

MRST, CTEQ and GRV parton distributions available at http://durpdg.dur.ac.uk/hepdata/pdf.html

General procedure.

Start parton evolution at low scale $Q_0^2 \sim 1\mathrm{GeV}^2$, and fit data for scales above $2\mathrm{GeV}^2$. In principle 11 different parton distributions to consider

$$u, \bar{u}, \quad d, \bar{d}, \quad s, \bar{s}, \quad c, \bar{c}, \quad b, \bar{b}, \quad g$$

$m_c, m_b \gg \Lambda_{\mathrm{QCD}}$ so heavy parton distributions determined perturbatively. Assume $s = \bar{s}$. Leaves 6 independent combinations. Relate s to $1/2(\bar{u} + \bar{d})$ and use

$$u_V = u - \bar{u}, \quad d_V = d - \bar{d}, \quad \mathrm{sea} = 2*(\bar{u} + \bar{d} + \bar{s}), \quad \bar{d} - \bar{u}, \quad g,$$

Input partons parameterized as, e.g.

$$xf(x, Q_0^2) = (1-x)^{\eta}(1 + \epsilon x^{0.5} + \gamma x)x^{\delta}.$$

For non-singlet combinations, valence quarks, $\bar{d} - \bar{u}$, δ expected to be ~ 0.5. For singlet combinations, sea and gluon, δ expected to be ~ 0.

Also define the singlet quark distribution

$$\Sigma = u_V + d_V + \mathrm{sea} + (c + \bar{c}) + (b + \bar{b})$$

Assuming isospin symmetry $p \to n$ leads to

$$d(x) \to u(x) \qquad u(x) \to d(x).$$

Various sum rules constraining parton inputs and conserved order by order in α_S for evolution.

$$\int_0^1 u_V(x)\,dx = 2 \qquad \int_0^1 d_V(x)\,dx = 1$$

i.e. conservation of number of valence quarks.

Also conservation of momentum carried by partons

$$\int_0^1 x\Sigma(x) + xg(x)\,dx = 1.$$

Important constraint on form of gluon which is only probed indirectly.

In determining partons need to consider that not only are there 6 different combinations of partons, but also wide distribution of x from 0.75 to 0.00003. Need many different types of experiment for full determination.

Large x

Quark distributions are determined mainly by structure functions. Dominated by non-singlet valence distributions.

Simple evolution of non-singlet distributions and conversion to structure function

$$\begin{aligned}\frac{df^{NS}(x,Q^2)}{d\ln Q^2} &= P^{NS}(x,\alpha_s(Q^2))\otimes f^{NS}(x,Q^2)\\ F_2^{NS}(x,Q^2) &= C^{NS}(x,\alpha_s(Q^2))\otimes f^{NS}(x,Q^2,\alpha_s(Q^2))\end{aligned}$$

So evolution of high x structure functions good test of theory and of $\alpha_S(Q^2)$.

However - perturbation theory involves contributions to coefficient function $\sim \alpha_S^n(Q^2)\ln^{2n-1}(1-x)$ and higher twist known to be enhanced as $x\to 1$. Hence cut $W^2 = Q^2(1/x-1)+m_p^2$ at somewhere in region $10-15\mathrm{GeV}^2$ to avoid contamination of NLO theory.

Consider charged lepton proton scattering

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha_2}{Q^4}[1+(1-y)^2]F_2(x,Q^2) - y^2 F_L(x,Q^2)]$$

ignoring heavy vector meson exchange (small correction for HERA data), and where $y = Q^2/xs$. Both $F_L(x,Q^2)$ and y usually small so cross-section effectively measure of $F_2(x,Q^2)$.

$$\begin{aligned} F_2^p(x) &\approx x[4/9(u+\bar{u}+c+\bar{c})+1/9(d+\bar{d}+s+\bar{s})] \\ F_2^d(x) &\approx x[4/9(d+\bar{d}+c+\bar{c})+1/9(u+\bar{u}+s+\bar{s})] \end{aligned}$$

So SLAC, BCDMS NMC data on $F_2^p(x,Q^2)$ and NMC data on $F_2^d(x,Q^2)$ and $F_2^n(x,Q^2)/F_2^p(x,Q^2)$ help determine high x parton distributions dominated by valence quarks.

Also use CCFR charged-current (neutrino) DIS data

$$\frac{d^2\sigma^{\nu,\bar\nu}}{dx dQ^2} \propto F_2^{\nu,\bar\nu}(x, \quad Q^2 \quad)\left[(1-y) + \frac{y^2}{2(1+R(x,Q^2))}\right]$$
$$\pm \quad xF_3^{\nu,\bar\nu}(x,Q^2)y(1-y/2),$$

where $R = F_L/(F_2 - F_L)$. For the proton

$$\begin{aligned}
F_2^{\nu} &= 2x[d+s+\bar u+\bar c] \\
F_2^{\bar\nu} &= 2x[u+c+\bar d+\bar s] \\
F_3^{\nu} &= 2x[d+s-\bar u-\bar c] \\
F_3^{\bar\nu} &= 2x[u+c-\bar d-\bar s].
\end{aligned}$$

Therefore

$$\begin{aligned}
F_2^{\nu} + F_2^{\bar\nu} &= 2x\sum_i (q+\bar q) = \Sigma \\
F_3^{\nu} + F_3^{\bar\nu} &= u_V + d_V.
\end{aligned}$$

In fact for an iso-scalar target, e.g. iron which is used by CCFR,

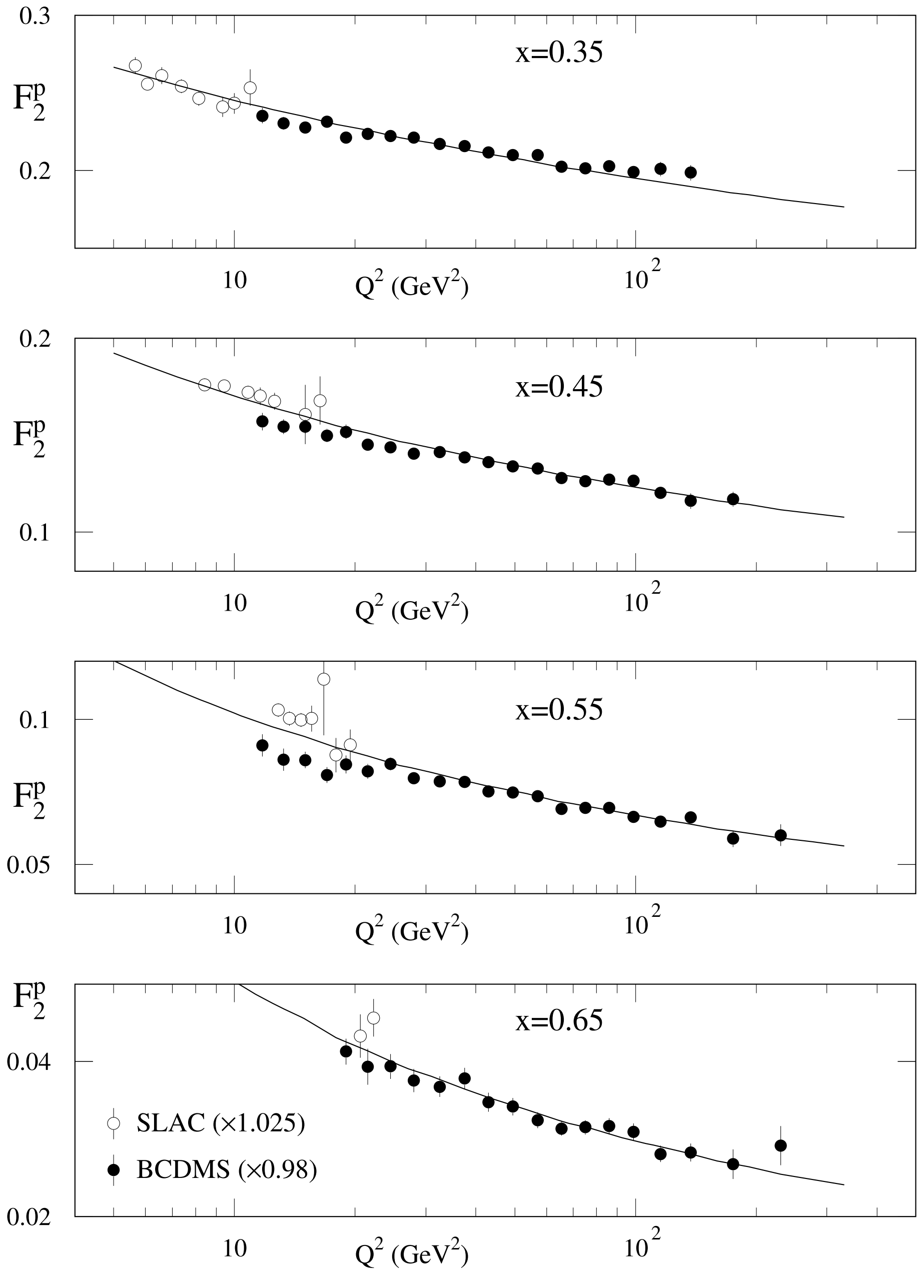
$$
\begin{aligned}
F_2^{\nu} &= F_2^{\bar{\nu}} = x\Sigma \\
F_3^{\nu} &= x(u_V + d_V) + 2x[s - \bar{c}] \\
F_3^{\bar{\nu}} &= x(u_V + d_V) - 2x[s - \bar{c}].
\end{aligned}
$$

And we must also correct for nuclear shadowing effects. (Parton distributions in nucleons in nuclei not the same as for free nucleons. Also applied for deuterium data.)

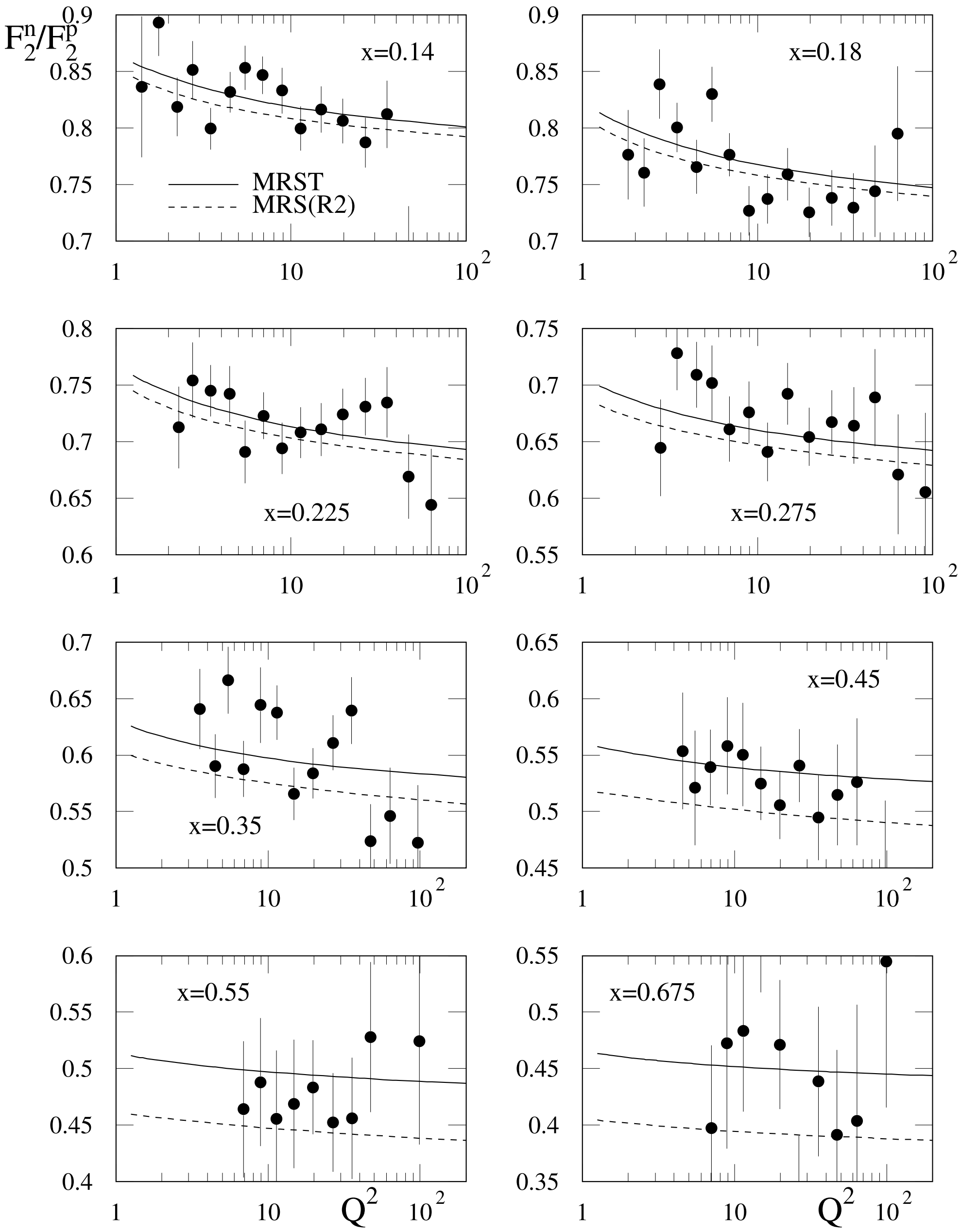
Comparison between theory and data good, and leads to determination of valence quarks at high x.

Also some HERA charged current-data at high Q^2 gives potential information on flavour decomposition. Currently low statistics. Improving.

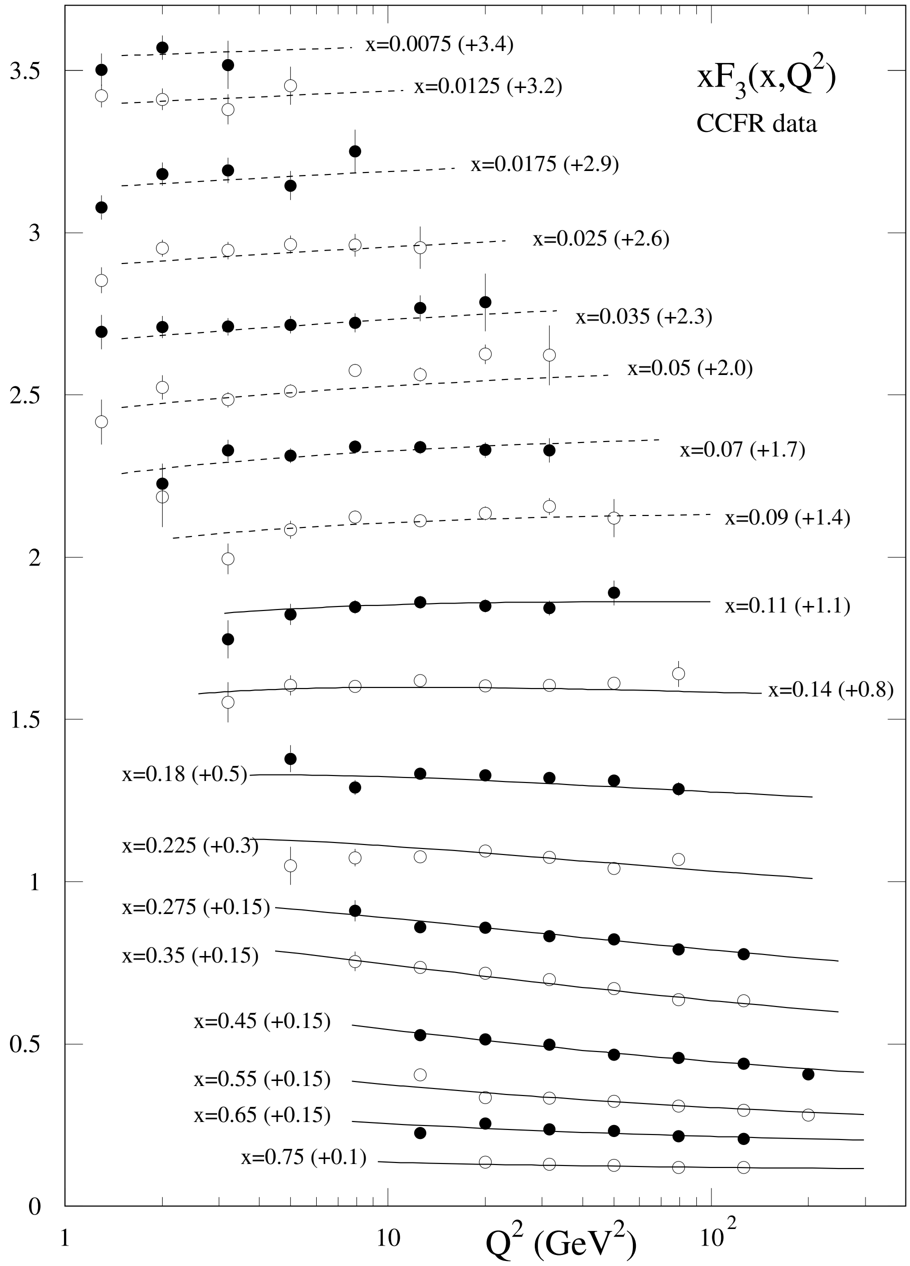
Find that at large x $u_V(x) > d_V(x)$, and that $xq_V(x) \sim (1-x)^3$, but no constraint in limit $x \to 1$. Some theory suggests $d_V(x)/u_V(x) \to$ constant as $x \to 1$, but no real evidence for or against.



Description of large x BCDMS and SLAC measurements of F_2^p.

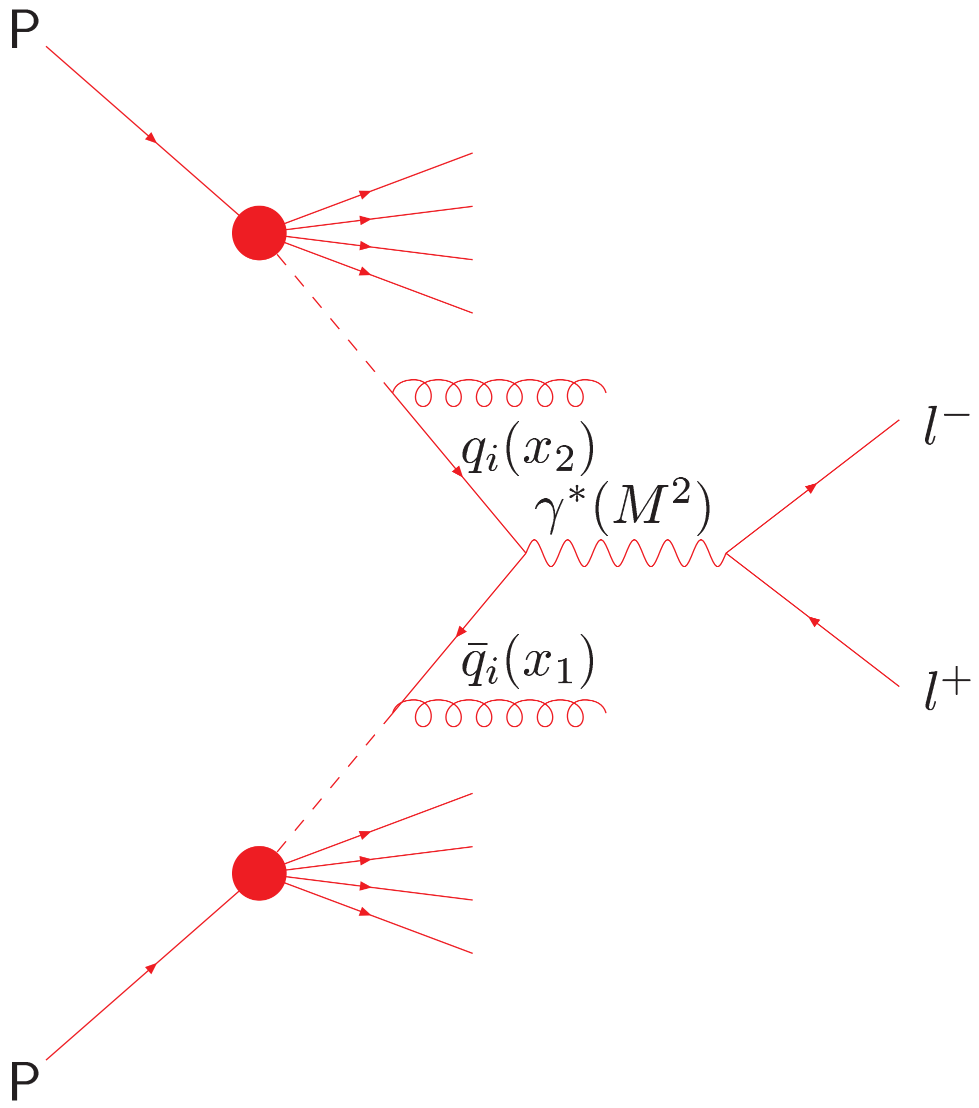


Description of large x NMC measurements of F_2^n/F_2^p.



Description of large x CCFR measurements of $F_3^{\nu N}$.

As x decreases the sea quarks become more important. These are determined at $x \sim 0.2$ by Drell-Yan scattering at Fermilab (E605, E772, E866). Process is production of lepton pairs from quark-antiquark annihilation in proton-proton scattering.

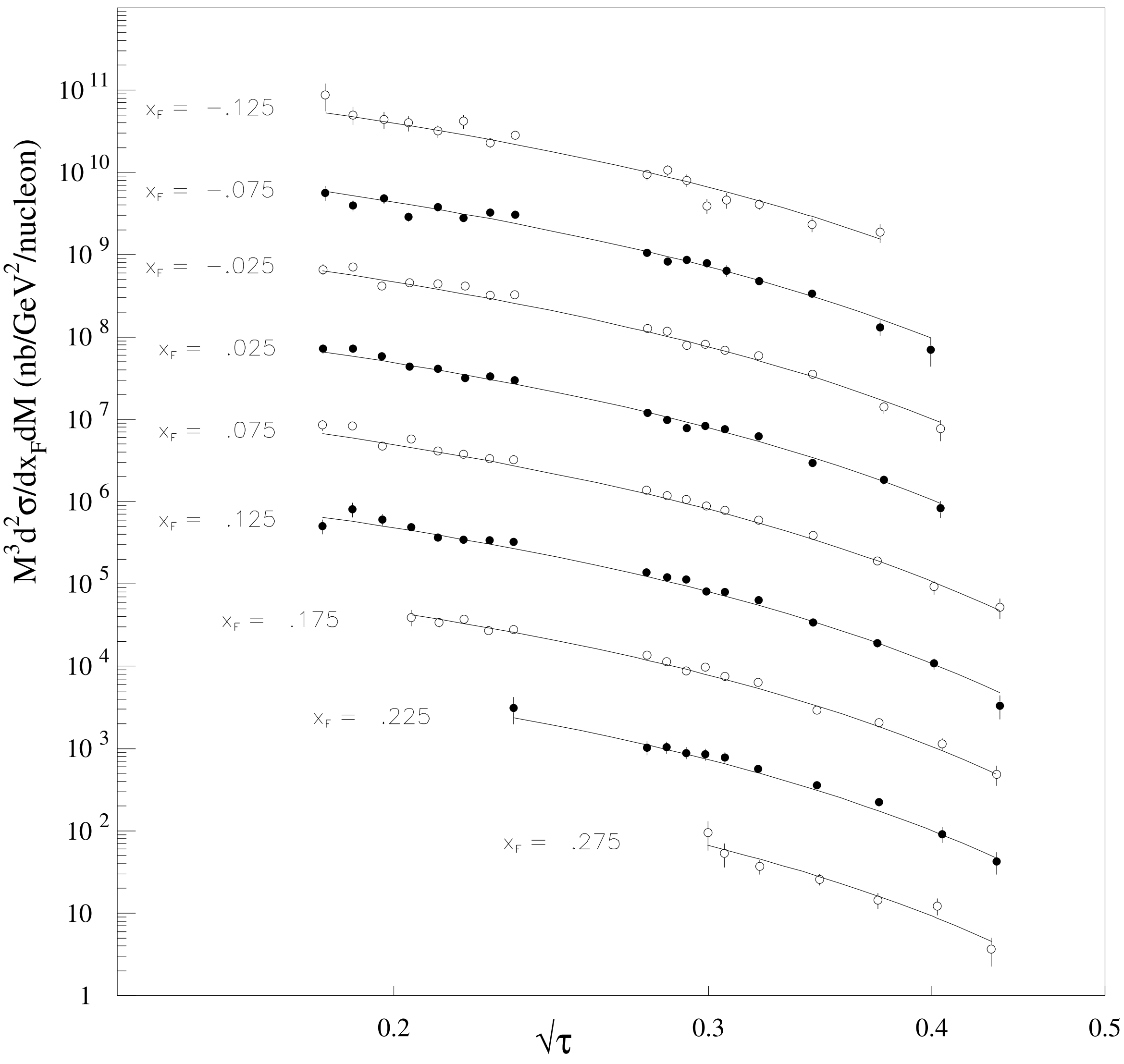


$x_F = x_1 - x_2$ and $\tau = x_1 x_2 = M^2/s$.

$$\frac{d\sigma}{dM^2} \propto \sum e_q^2 (q(x_1)\bar{q}(x_2) + q(x_2)\bar{q}(x_1)).$$

Probe of $\bar{u}$ and $\bar{d}$ in the proton at moderate x. Find $\bar{q} \sim (1-x)^7$.

E605 (p Cu $\rightarrow \mu^+\mu^-$ X) p_{LAB} = 800 GeV

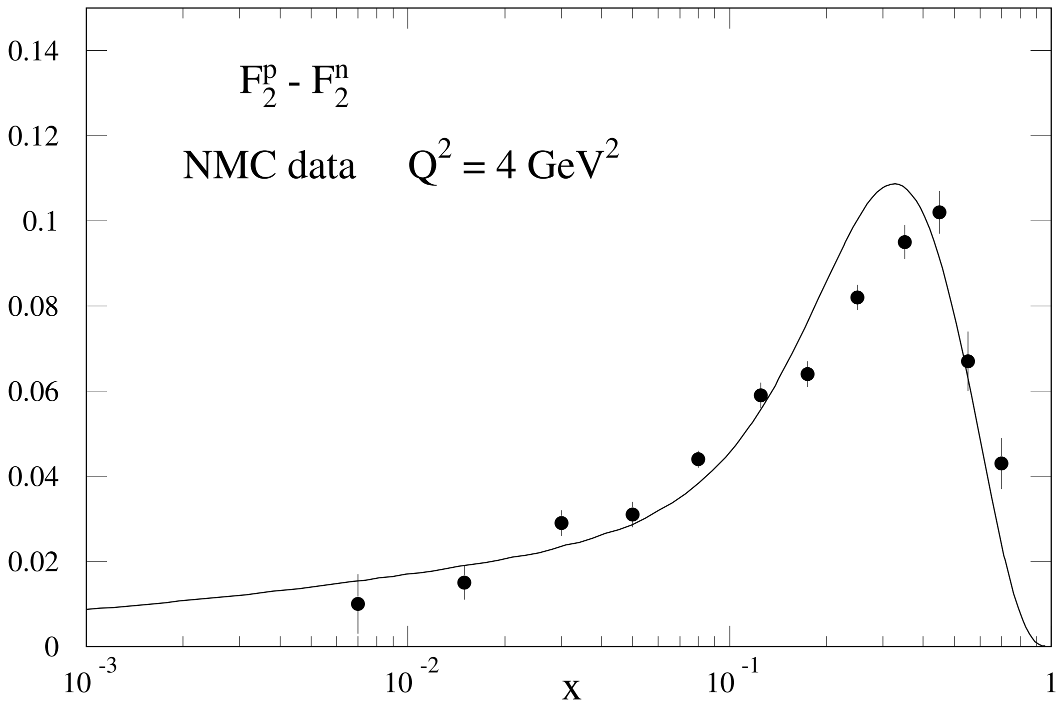


Description of Drell-Yan scattering.

Also need to consider the precise difference between $\bar{u}$ and $\bar{d}$. Related to Gottfried sum rule.

$$I_{GS} = \int_0^1 \frac{dx}{x}(F^{\mu p} - F^{\mu n}) = \frac{1}{3}\int_0^1 dx(u_V - d_V + \bar{u} - \bar{d})$$
$$= \frac{1}{3} + \frac{1}{3}\int_0^1 dx(\bar{u} - \bar{d})$$

This was measured by NMC to be 0.235 ± 0.026 which implies $\int dx(\bar{d} - \bar{u}) \approx 0.1$.



Information more directly available from Drell-Yan asymmetry

$$A_{DY} = \frac{\sigma_{pp} - \sigma_{pn}}{\sigma_{pp} + \sigma_{pn}} = \frac{1 - r}{1 + r},$$

where

$$r \approx \frac{4u_1\bar{d}_2 + d_1\bar{u}_2 + 4\bar{u}_1 d_2 + \bar{d}_1 u_2}{4u_1\bar{u}_2 + d_1\bar{d}_2 + 4\bar{u}_1 u_2 + \bar{d}_1 d_2},$$

and 1 labels the proton and 2 the neutron.

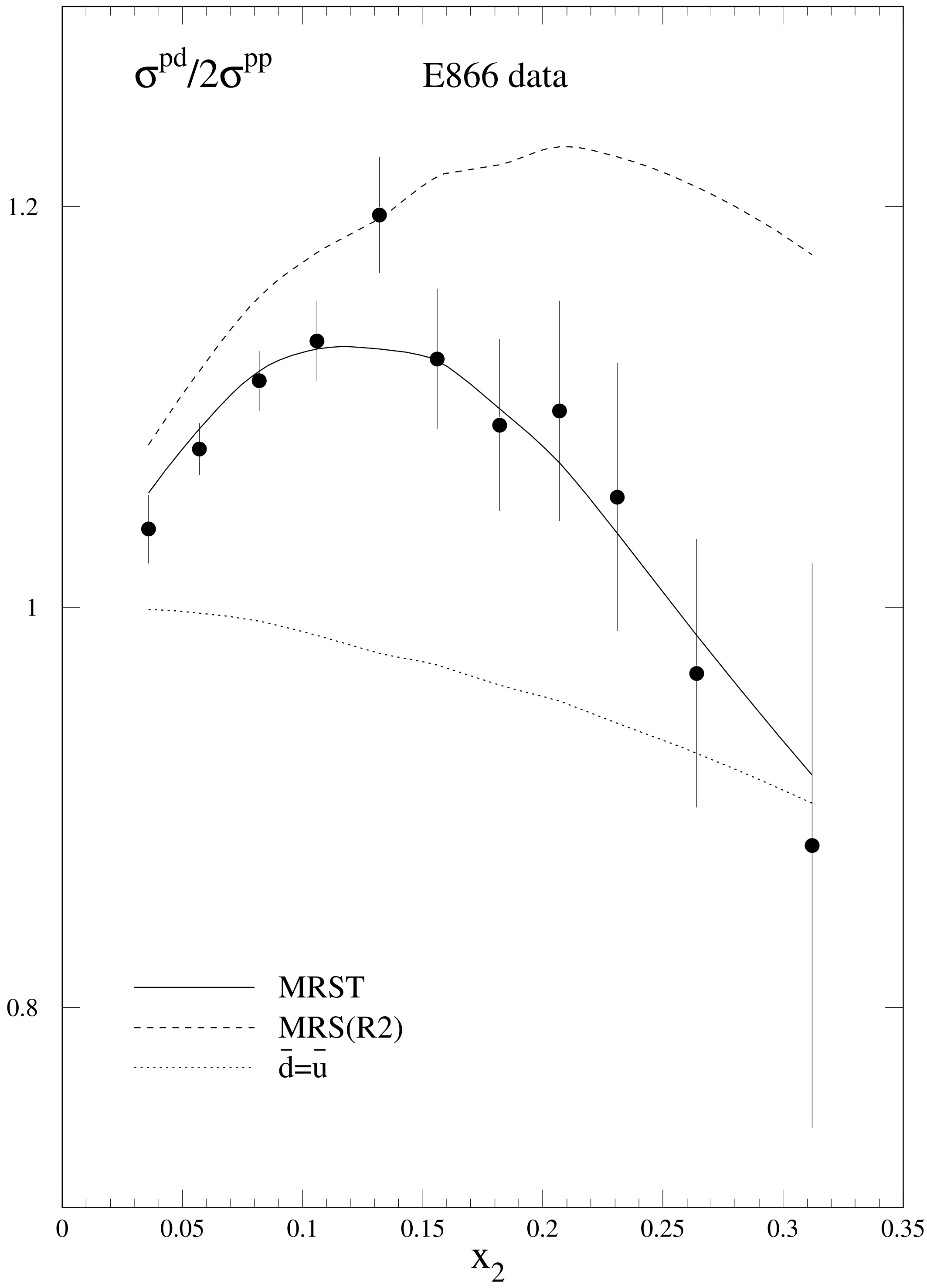
In fact measure the quantity

$$Rdp = \frac{\sigma_{pd}}{2\sigma_{pp}} = \frac{1}{2}(1 + r),$$

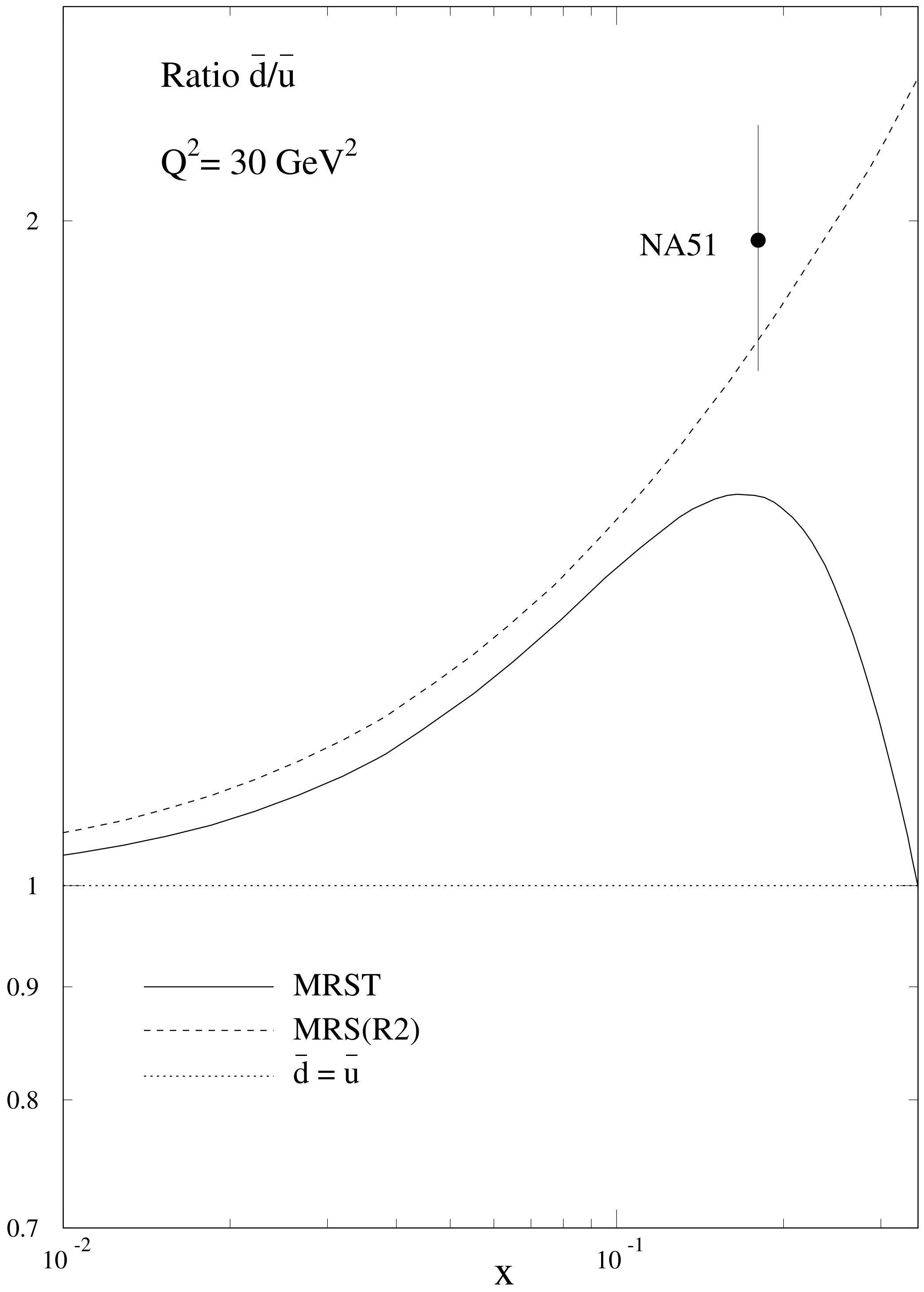
which contains the same information.

Previously one point at $x_1 = x_2 = 0.18$ from NA51 implied $\bar{d} > \bar{u}$. Now E866 measured very accurately from $0.04 < x < 0.3$. Gives clear evidence of $\bar{u} - \bar{d}$ asymmetry, but not as much as suggested previously.

Seems to reach maximum at $x \approx 0.2$. Not clear what happens as $x \to 1$.



Drell-Yan asymmetry compared to E866 data.



Ratio of $\bar{d}$ to $\bar{u}$ in MRST partons.

It is also possible to relate the strange quark distribution to the $\bar{u}$ and $\bar{d}$ quarks. This is done using unlike sign dimuon production at CCFR.

$$\nu_\mu \to \mu^- + W^+$$

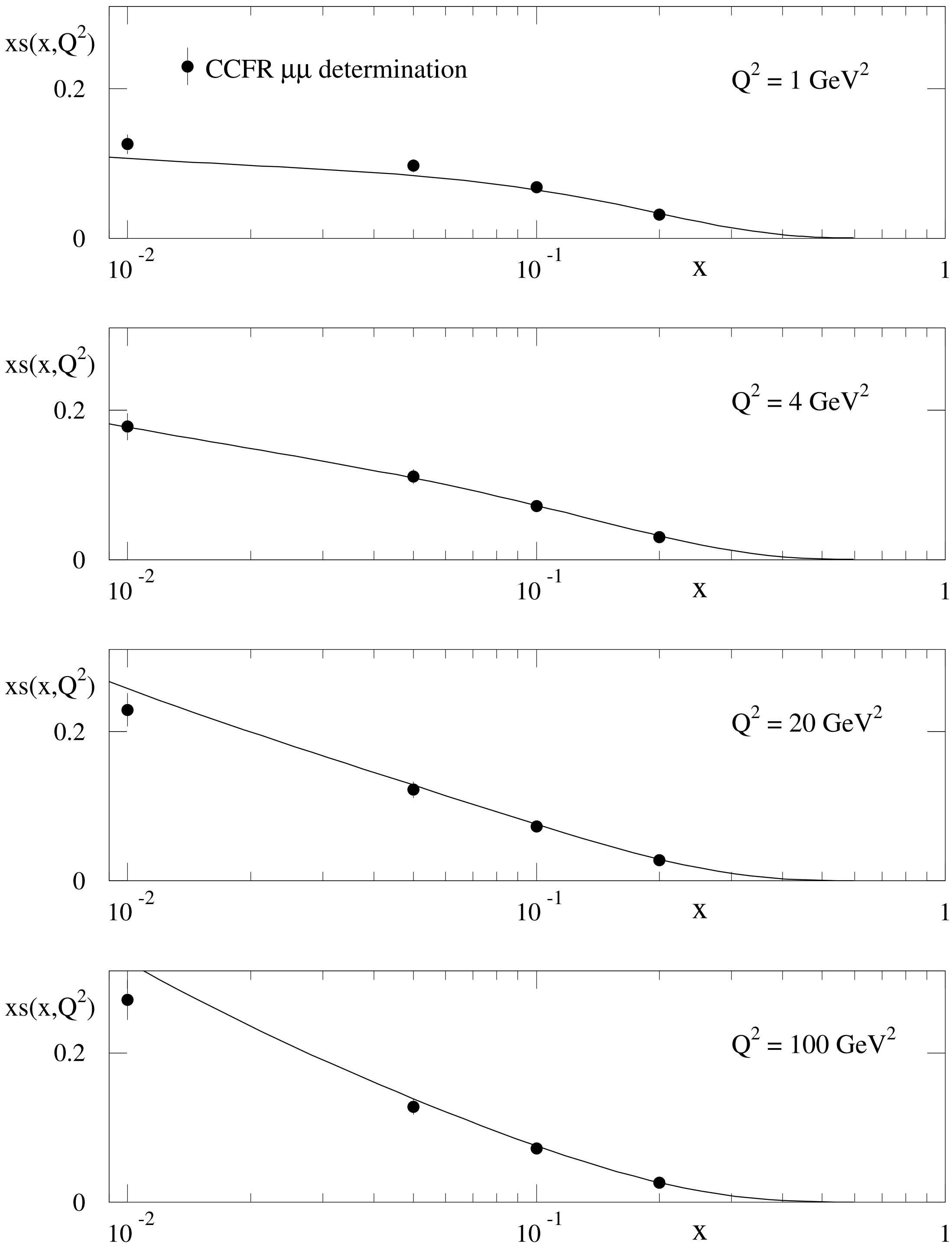
followed by

$$W^+ + s \to c \to D^+ \to \mu^+.$$

Using the production of dimuons, CCFR are able to construct the strange quark distribution in the proton in the region $0.01 < x < 0.2$ and for $1\mathrm{GeV}^2 < Q^2 < 100\mathrm{GeV}^2$.

In Global fits it is simply assumed that at Q_0^2 we have $s(x) = \kappa 0.5(\bar{u} + \bar{d})$. Using $Q_0^2 = 1\mathrm{GeV}^2$ and $\kappa = 0.5$ works very well, i.e. strange is 20% of the input sea. Since all quarks evolve equally this fraction increases as Q^2 increases.

It is assumed that the same simple parameterization is also true at smaller x, though there is no guarantee of this. (Naively the fraction might be larger at smaller x.)



Comparison of strange distribution to CCFR "data" for MRST partons.

The final piece of information on the light quarks at moderate to large x comes from W- or lepton asymmetry at the Tevatron.

$$\begin{aligned} A_W(y) &= \frac{d\sigma(W^+)/dy - d\sigma(W^-)/dy}{d\sigma(W^+)/dy + d\sigma(W^-)/dy} \\ &\approx \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}, \end{aligned}$$

where $x_{1,2} = x_0 \exp(\pm y), \quad x_0 = \frac{M_W}{\sqrt{s}}$.

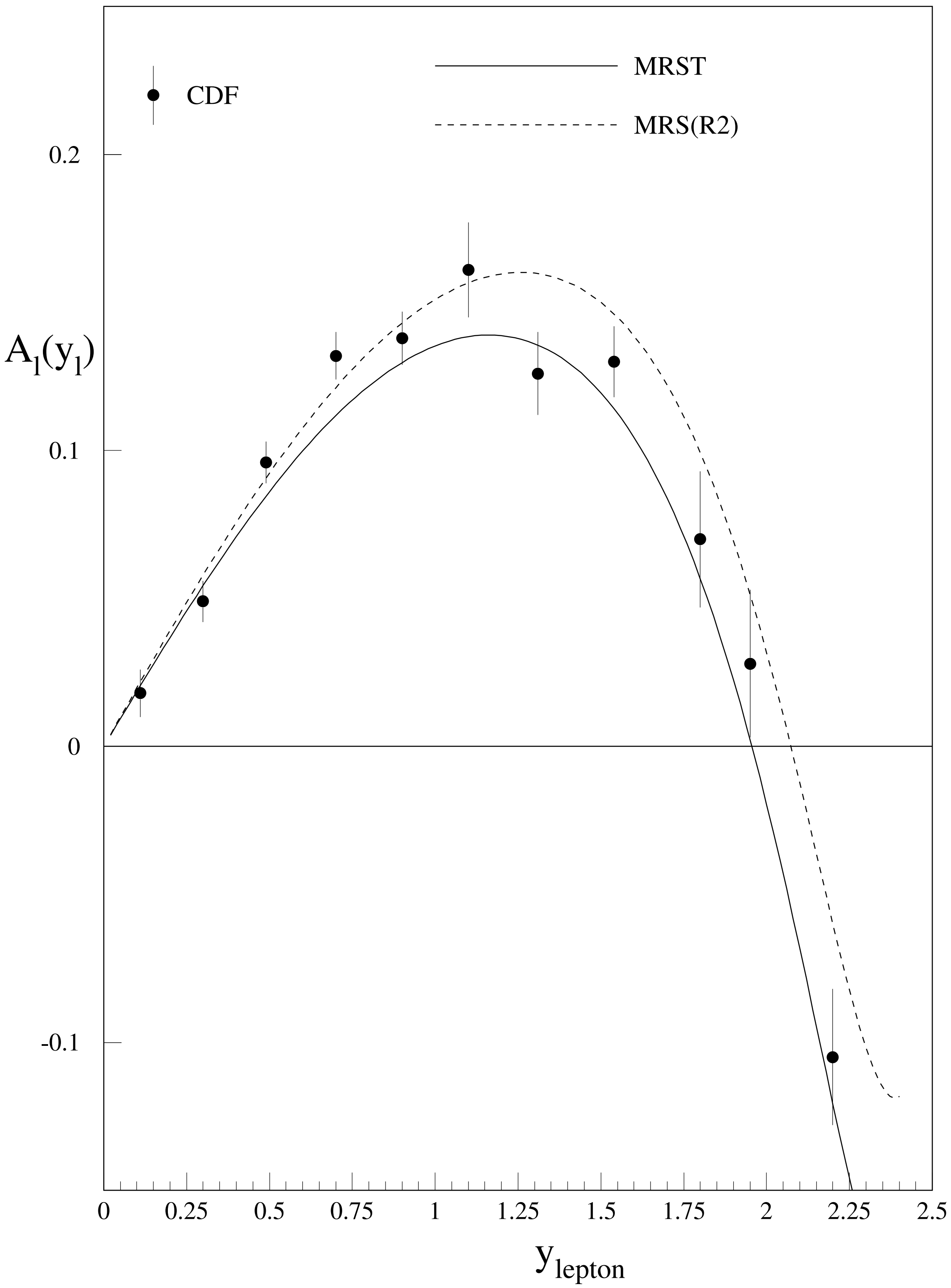
Since $u(x) > d(x)$ at large x, whereas they become roughly equal at smaller x, $A_W(y)$ is positive for $x_1 > x_0 = 0.05$ $(y > 1)$ where measurements are taken.

This helps pin down the u and d quarks in the region $x \sim 0.1$ as well as giving compatible information to NMC and CCFR at higher x and thus contributes to the determination of the two valence quark distributions.

In practice it is the final state leptons that are detected, so it is really the lepton asymmetry

$$A(y_l) = \frac{\sigma(l^+) - \sigma(l^-)}{\sigma(l^+) + \sigma(l^-)}$$

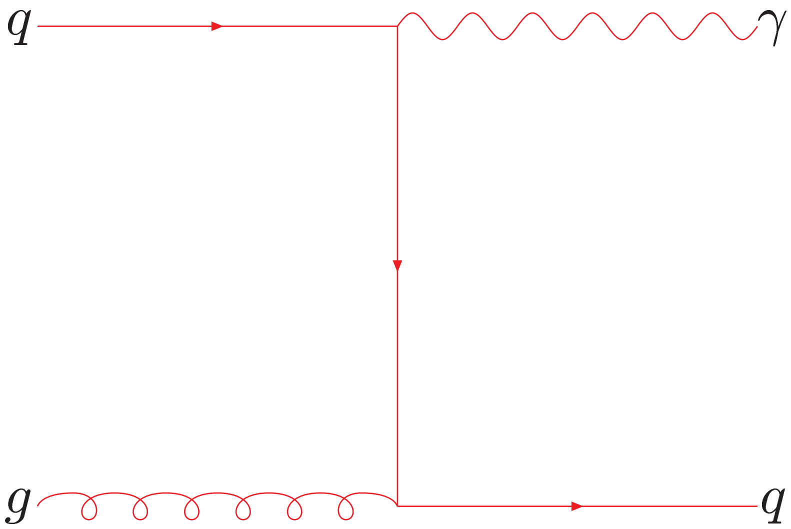
which is measured.



Comparison of lepton asymmetry to theory.

Gluon distribution

The above measurements constrain the high and moderate x quarks to a few percent or better. It is far more difficult to obtain precise information on the form of the high x gluon. Until recently many groups determined the gluon at high x via prompt photon production, e.g.



In principle this is a direct test of the large x gluon - $x_T = 2p_T/\sqrt{s}$.

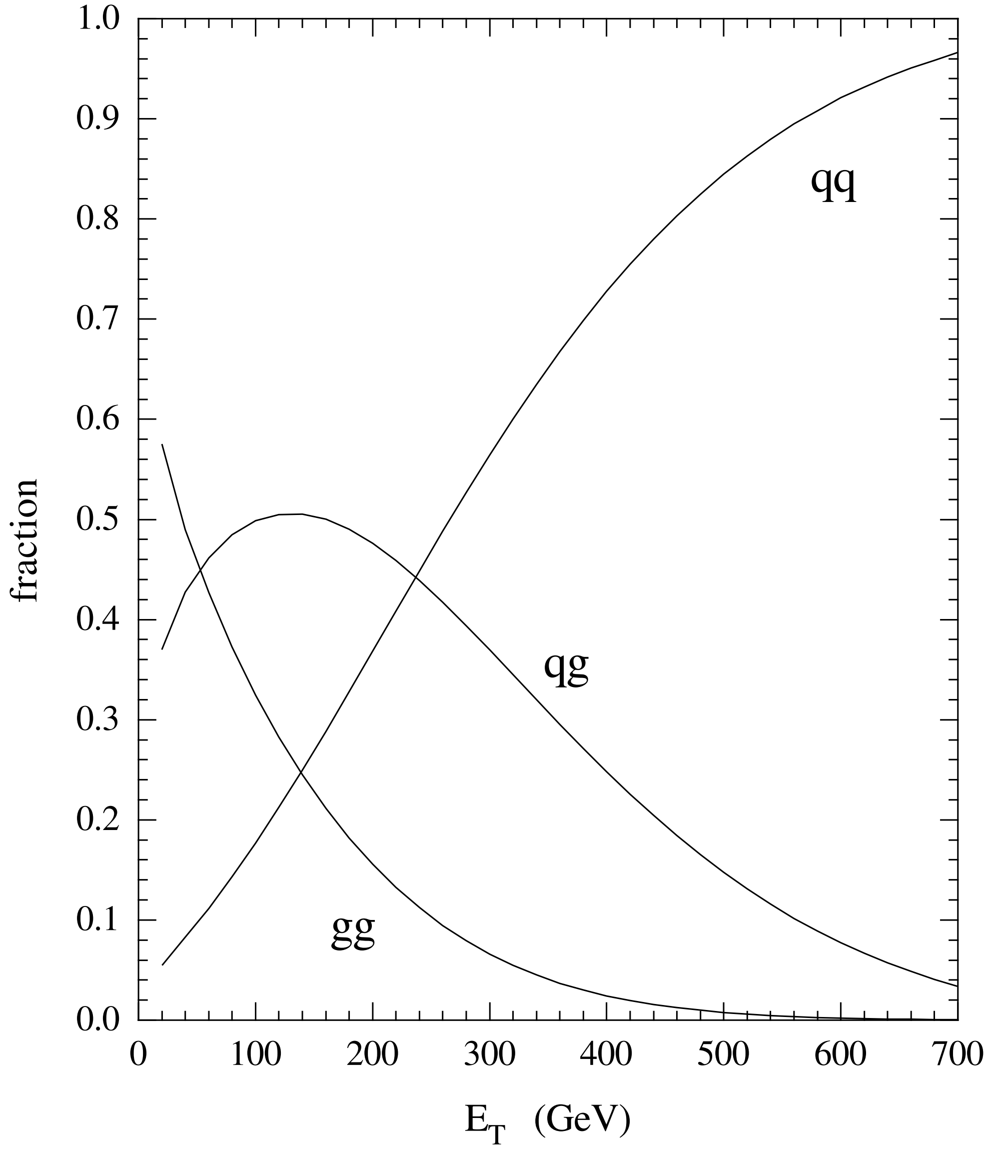
However, $d^2\sigma/dEdp_T$ is sensitive to nonperturbative information about the intrinsic k_T of the gluons in the proton, to resummation of threshold logarithms, i.e. $\ln(1-x_T)$, and to the interplay between the two. Also, some experiments probing similar regions of parameter space give results which are difficult to reconcile. Hence, this gives only a rough indication of the gluon distribution.

Current best determination of high x gluon distribution given by inclusive jet measurements by D0 and CDF at Tevatron. Measure $d\sigma/dE_T d\eta$ for central rapidity CDF or in different bins of rapidity D0, where $\eta = -\ln\tan(\Theta/2)$ and Θ is the angle from the beam.

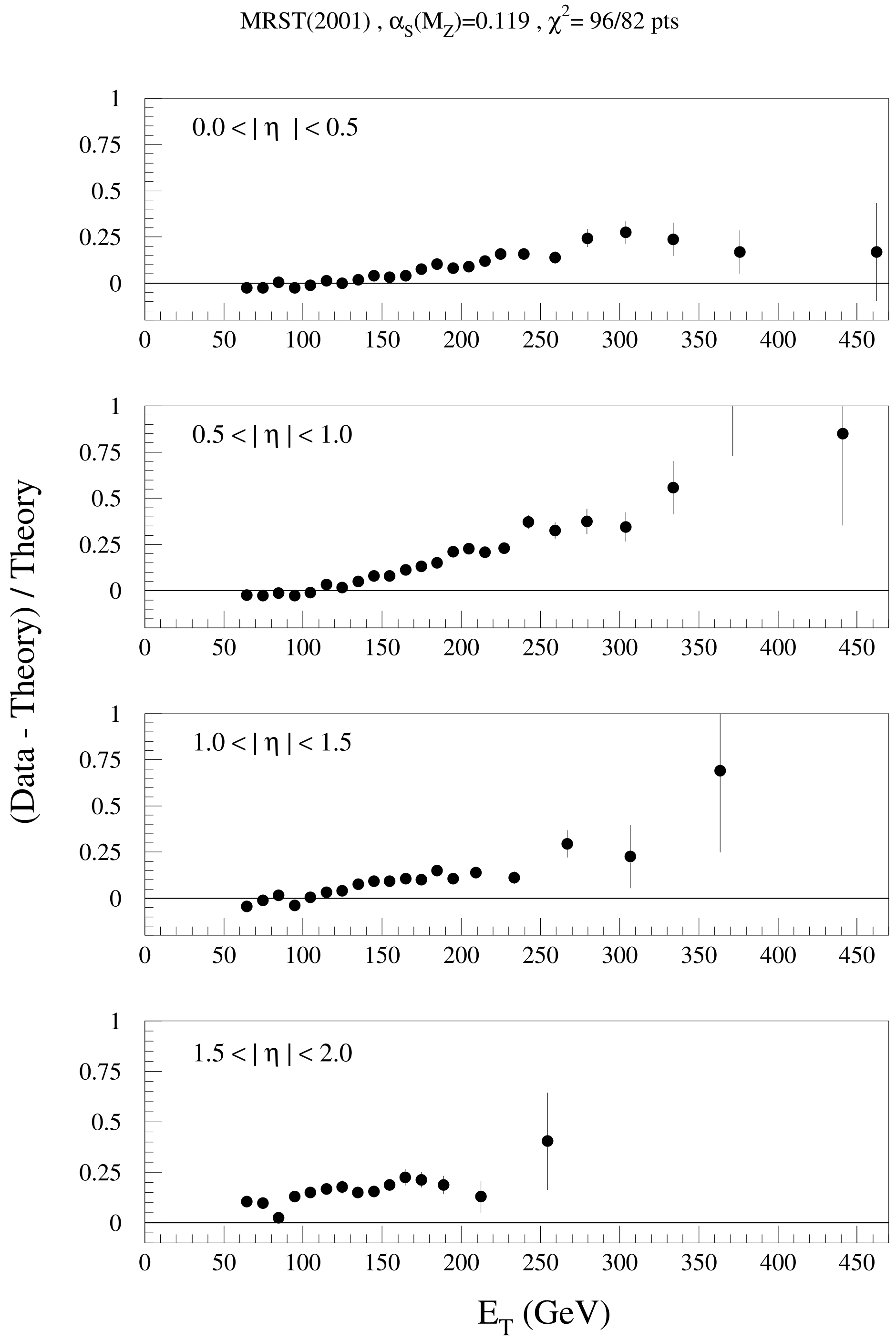
At central rapidity $x_T = 2E_T/\sqrt{s}$, and measurement extend up to $E_T \sim 400\mathrm{GeV}$, i.e. $x_T \sim 0.45$, and down to $E_T \sim 60\mathrm{GeV}$, i.e. $x_T \sim 0.06$.

At matrix element level gluon-gluon fusion dominates, but gluon distribution falls off more quickly as $x \to 1$ than quark distributions so there is a transition from gluon-gluon fusion at small x_T, to gluon-quark at intermediate x_T to quark-quark at high x_T. However, even at the highest x_T gluon-quark contributions are significant.

In fit to data difficult to achieve good apparent fit at highest E_T due to sum rule etc. However, normalization of data uncertain to $\sim 5\%$, and correlated error due to fragmentation models etc. more than 10% at highest E_T. Fit using full treatment of errors perfectly acceptable and determines gluon at $x \sim 0.4$ to about 20% - $xg(x) \sim (1-x)^5$. (Compatible with prompt photon indications.) The data are also sensitive to the value of $\alpha_S(M_Z^2)$ and prefer $\alpha_S(M_Z^2) \sim 0.115$.



Composition of the inclusive jet cross-section in terms of parton sub-processes.



Comparison of theory to data for the most recent D0 jet data. Systematic errors are not shown.

Small x.

All the above data constrain the partons for $x > 0.05$ (though NMC data extend from higher x down to $x \sim 0.01$). The extension to the region of very low x has been made in the past decade by HERA. In this region there is very great scaling violation of the partons from the evolution equations and also a great interplay between the quarks and gluons.

Evolution equations for singlet sector are coupled

$$\begin{aligned} \frac{d\Sigma}{d\ln Q^2} &= P_{qq} \otimes \Sigma + P_{qg} \otimes g \\ \frac{dg}{d\ln Q^2} &= P_{gq} \otimes \Sigma + P_{gg} \otimes g \end{aligned}$$

At very small x the splitting functions tend to

$$P_{gg}^0 \rightarrow \frac{3\alpha_S}{\pi}\frac{1}{x} \qquad P_{qg}^0 \rightarrow 2N_F \frac{\alpha_S}{6\pi},$$

and so the gluon grows very quickly with increasing Q^2 while the quark distribution also grows quickly driven by the gluon. Correlation between gluon and $dF_2(x,Q^2)/d\ln Q^2$.

At NLO the small x splitting functions become

$$P^1_{gg} \to -0.17\alpha_S^2 \frac{1}{x} \qquad P^1_{qg} \to 2N_F \frac{\alpha_S^2}{6\pi}\frac{2.1}{x}.$$

Hence, the gluon evolution is only slightly modified, whereas the quark evolution is greatly enhanced at NLO. $dF_2(x,Q^2)/d\ln Q^2$ not directly $\propto xg(x,Q^2)$

It is known that at each subsequent order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e.

$$P_{ij}(x,\alpha_s(Q^2)), \quad C_i^P(x,\alpha_s(Q^2)) \sim \alpha_s^m(Q^2)\ln^{m-1}(1/x).$$

and hence the convergence at small x is questionable.

The global fits usually assume that this turns out to be unimportant in practice, and proceed regardless.

Extra complication at small x - sometimes y and F_L are not small enough for longitudinal contribution to be neglected. HERA measure out to $y = 0.8$. Remember

$$\frac{d^2\sigma}{dxdQ^2} \propto F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2),$$

where $y = Q^2/xs$. So runs at different s allow extraction of $F_2(x,Q^2)$ and $F_L(x,Q^2)$. At HERA running only at a single value of s (to good approximation) so cannot use y-variation to extract $F_L(x,Q^2)$.

Can assume one understands $F_2(x,Q^2)$ to extrapolate into region of high y and hence "determine" $F_L(x,Q^2)$ (H1). Dangerous given previous arguments.

Alternatively - use NLO theory for both $F_2(x,Q^2)$ and $F_L(x,Q^2)$ and effectively fit to cross-section rather than structure function. Much more conservative - but more systematic.

Pay close attention to whether there is any particular problem with fit at high y. (Apparent in LO fit).

Fit quality good at small x with most recent data. Significant problem with $dF_2(x,Q^2)/d\ln Q^2$ at $x \sim 0.01$ where longest lever arm. However, $dF_2(x,Q^2)/d\ln Q^2$ depends strongly on gluon at small x. Momentum sum rule imposes global constraint on gluon - high x gluon affects possible form of small x gluon. Theoretically inconsistent to ignore high x constraint when performing small x fit. Often done nonetheless since high x constraint makes small x fit considerably more difficult (sign of $\ln(1/x)$ terms?).

Also, most recent data require negative input gluon at lowest x and $Q^2 \sim 1\mathrm{GeV}^2$, e.g. MRST extend gluon parameterization by adding contribution

$$-A_{-}(1-x)^{\eta_{-}}x^{-\delta_{-}}$$

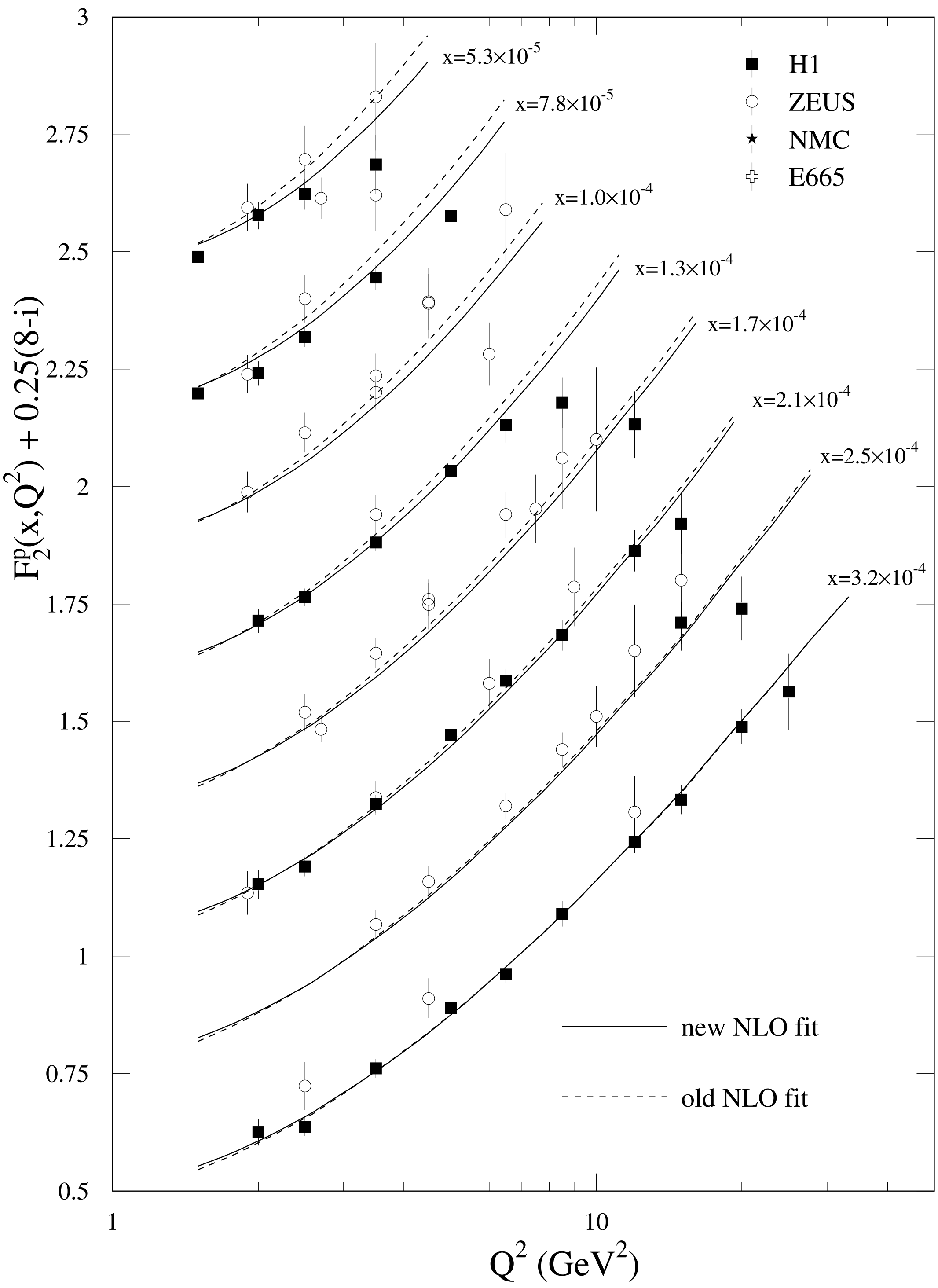
to $xg(x,Q_0^2)$. Not doing so $\rightarrow \Delta\chi^2 > 100$. H1 and ZEUS see same effect by evolving backwards.

Necessary both to get slope correct at low x and Q^2, but also at higher x to get enough gluon (cutting lowest x data makes effect worse). No problem in principle with negative gluon, but what about physical quantities?

$F_L(x,Q^2)$ closely related to gluon. Negative gluon leads to negative $F_L(x,Q^2)$ at lowest x and Q^2. Also instability between orders for prediction for $F_L(x,Q^2)$.

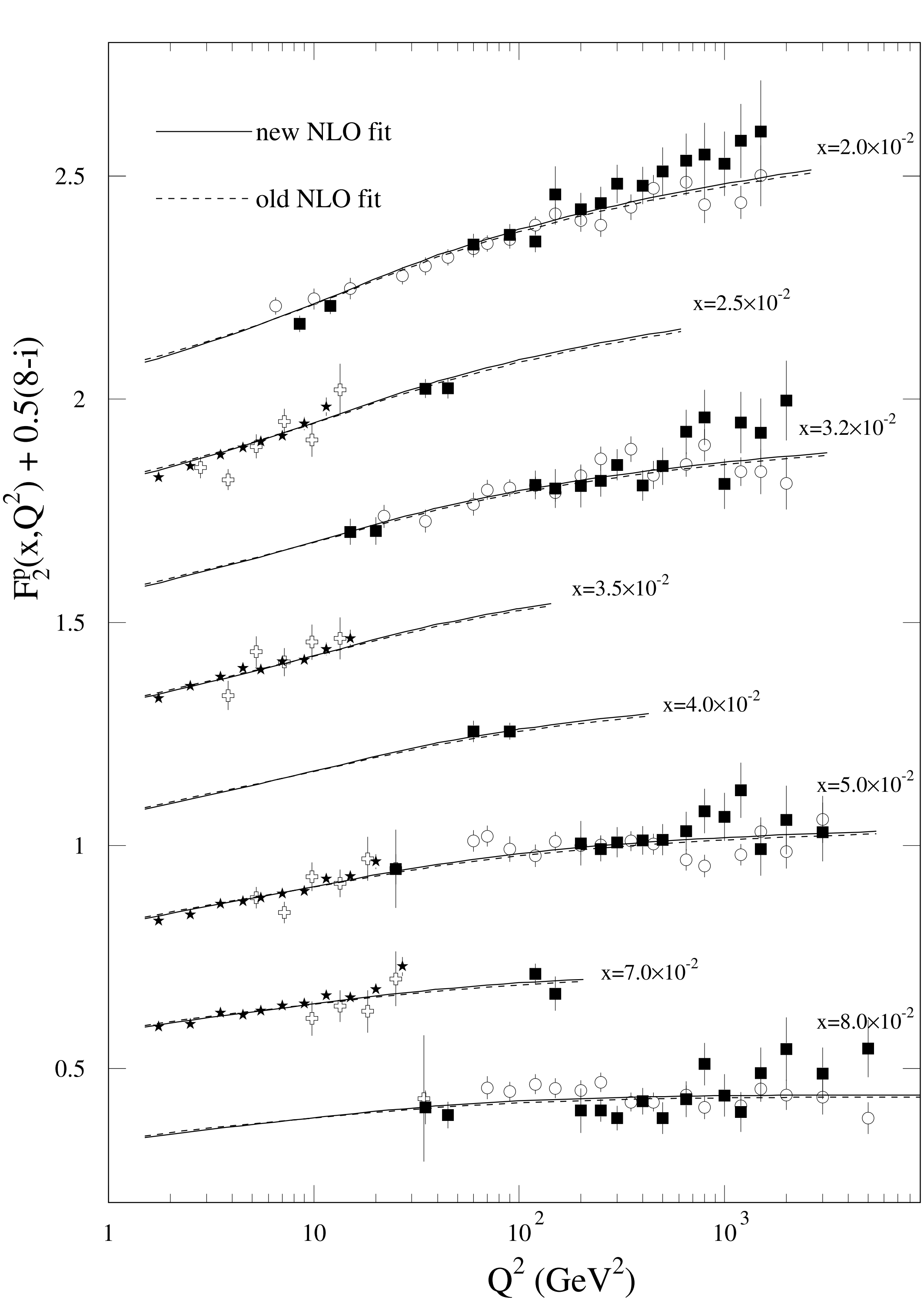
Therefore NLO small x fit reasonable, but there are significant caveats.

MRST(2001) NLO fit , x = 0.00005 - 0.00032

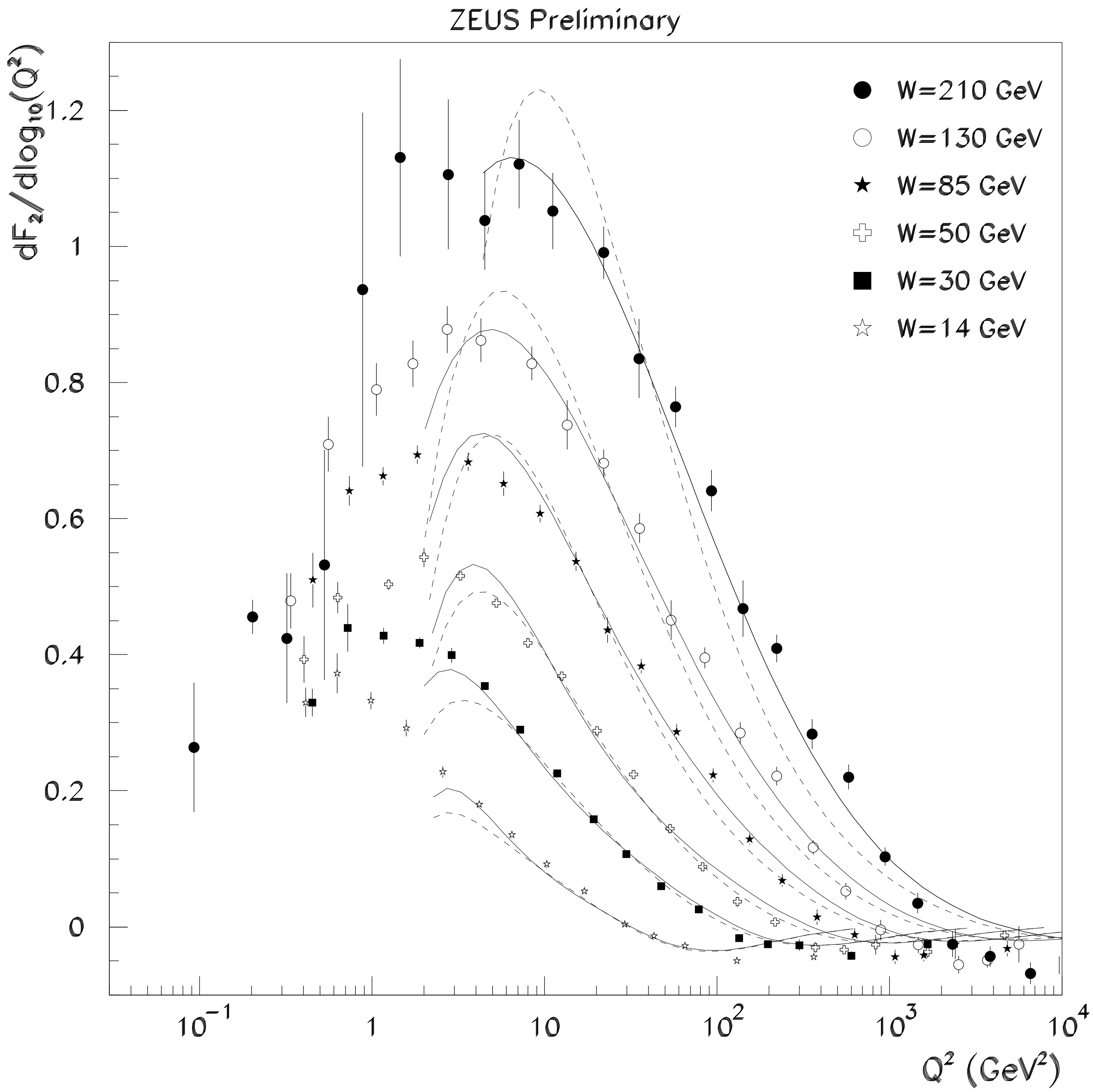


Comparison of MRST(2001) and MRST(1999)

MRST(2001) NLO fit , x=0.02 - 0.08



Comparison of MRST(2001) and MRST(1999)



Comparison of MRST values of $dF_2(x, Q^2)/d\ln Q^2$ (dashed lines) with the ZEUS data.

Heavy Quarks.

Mainly charm, small contribution from bottom. These are most important at small x. Not only necessary to have correct treatment because of direct data on charm from H1 and ZEUS (suffers from model dependence in extraction), but also because charm forms a large component of total structure function at small x.

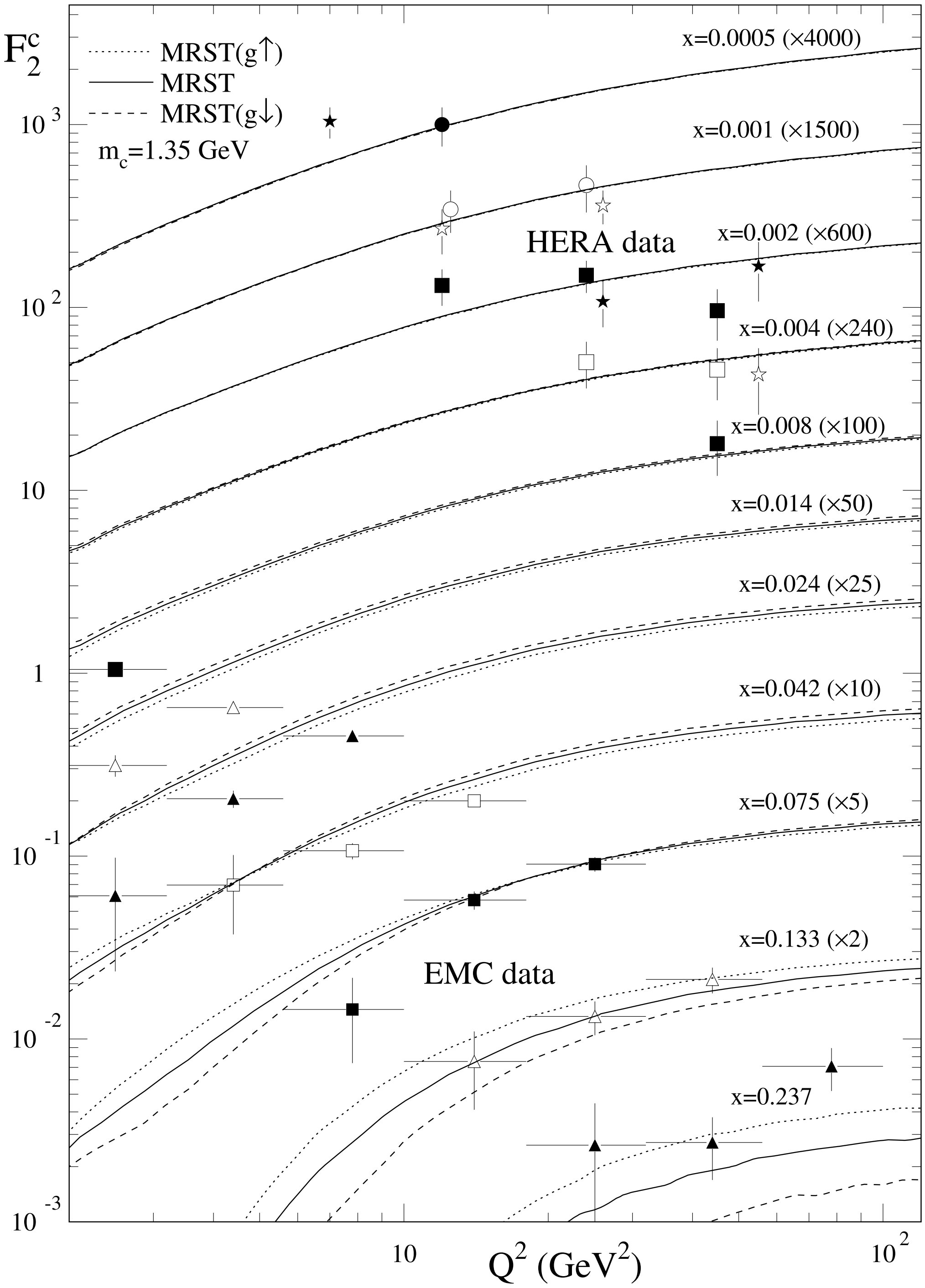
Two distinct regimes-

Near threshold $Q^2 \sim M_H^2$ massive quarks not partons. Created in final state.

High scales $Q^2 \gg M_H^2$ massless partons. Behave like up, down, strange.

Many analyses still use one or the other. However, more correct to use the theoretically correct description over whole range of Q^2. Known as variable flavour number scheme (**VFNS**) $\rightarrow$ precise definition of parton distributions and scattering at all scales.

Choose $M_c = 1.4\mathrm{GeV}$. Leads to good description of small x charm data (**not** good test of gluon universality) and higher x EMC data (again prefers similar gluon to jets), and to best overall global fit.



Comparison of charm structure function with MRST theoretical prediction.

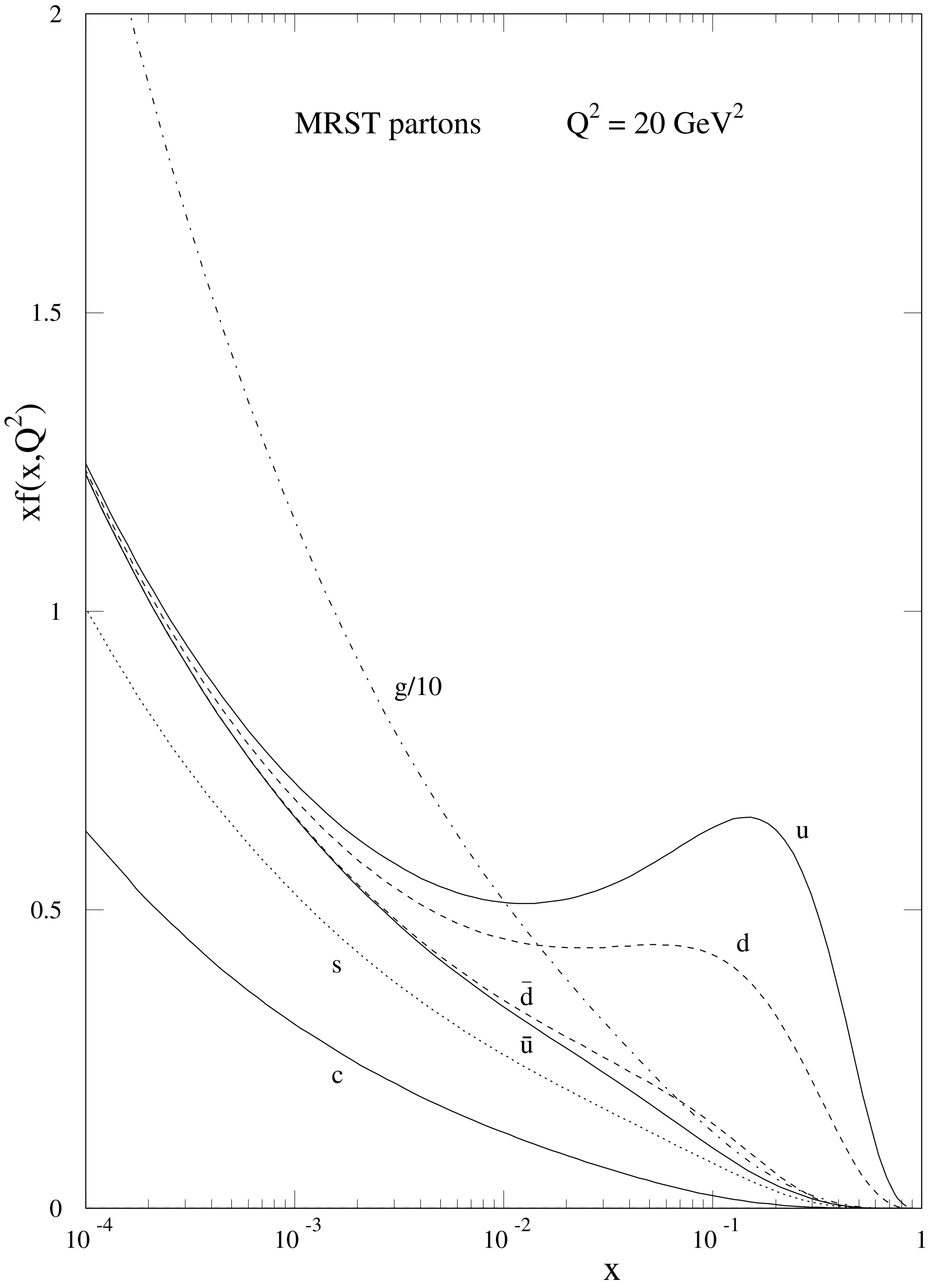
Results.

Above procedure completely determines parton distributions at present. Adding statistical and systematic errors in quadrature (except for jets) fit reasonably good.

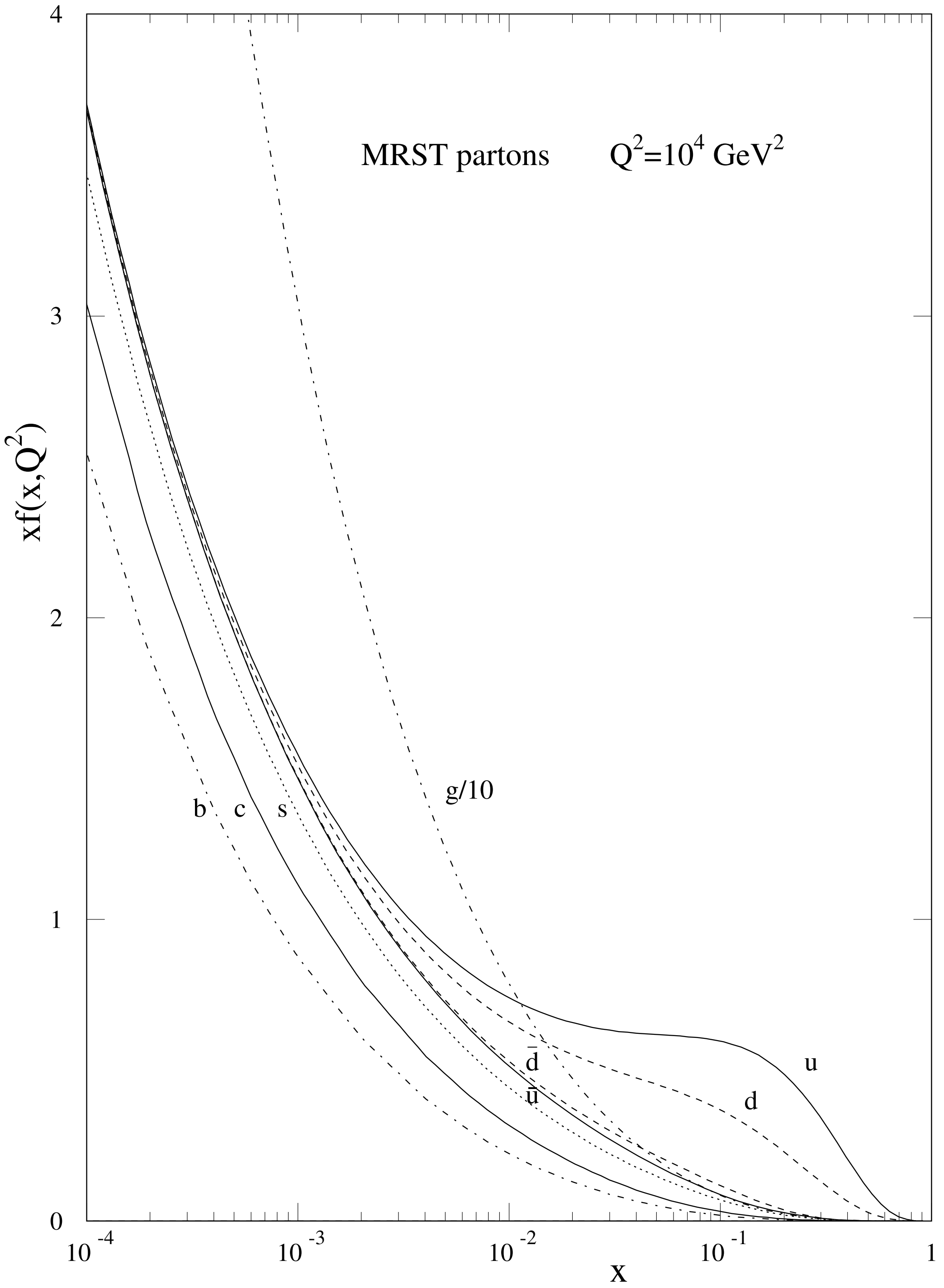
Data set	No. of data pts	MRST
H1 ep	400	377
ZEUS ep	272	253
BCDMS μp	167	205
NMC μp	126	136
NMC μd	126	100
SLAC ep	53	71
E665 μp	53	52
E665 μd	53	60
CCFR $F_2^{\nu N}$	63	57
CCFR $F_3^{\nu N}$	63	57
NMC n/p	156	144
Jets	113	152

And about 1 per point for smaller sets. $\alpha_S(M_Z^2) = 0.119$. Compromise between different data sets. Total $\chi^2 = 1754/1723$ - but errors overestimated. Some areas where theory perhaps needs to be improved.

Resulting partons over-leaf.



MRST98 partons at $Q^2 = 20\mathrm{GeV}^2$. CTEQ very similar.



MRST98 partons at $Q^2 = 10^4 \mathrm{GeV}^2$. CTEQ very similar.

Parton Uncertainties - currently an issue attracting a lot of work. Number of approaches.

Statistical Approach (Giele, Keller and Kosower) constructs an ensemble of distributions labelled by $\mathcal{F}$ each with probability $P(\{\mathcal{F}\})$. Mean μ_O and deviation σ_O of observable O then given by

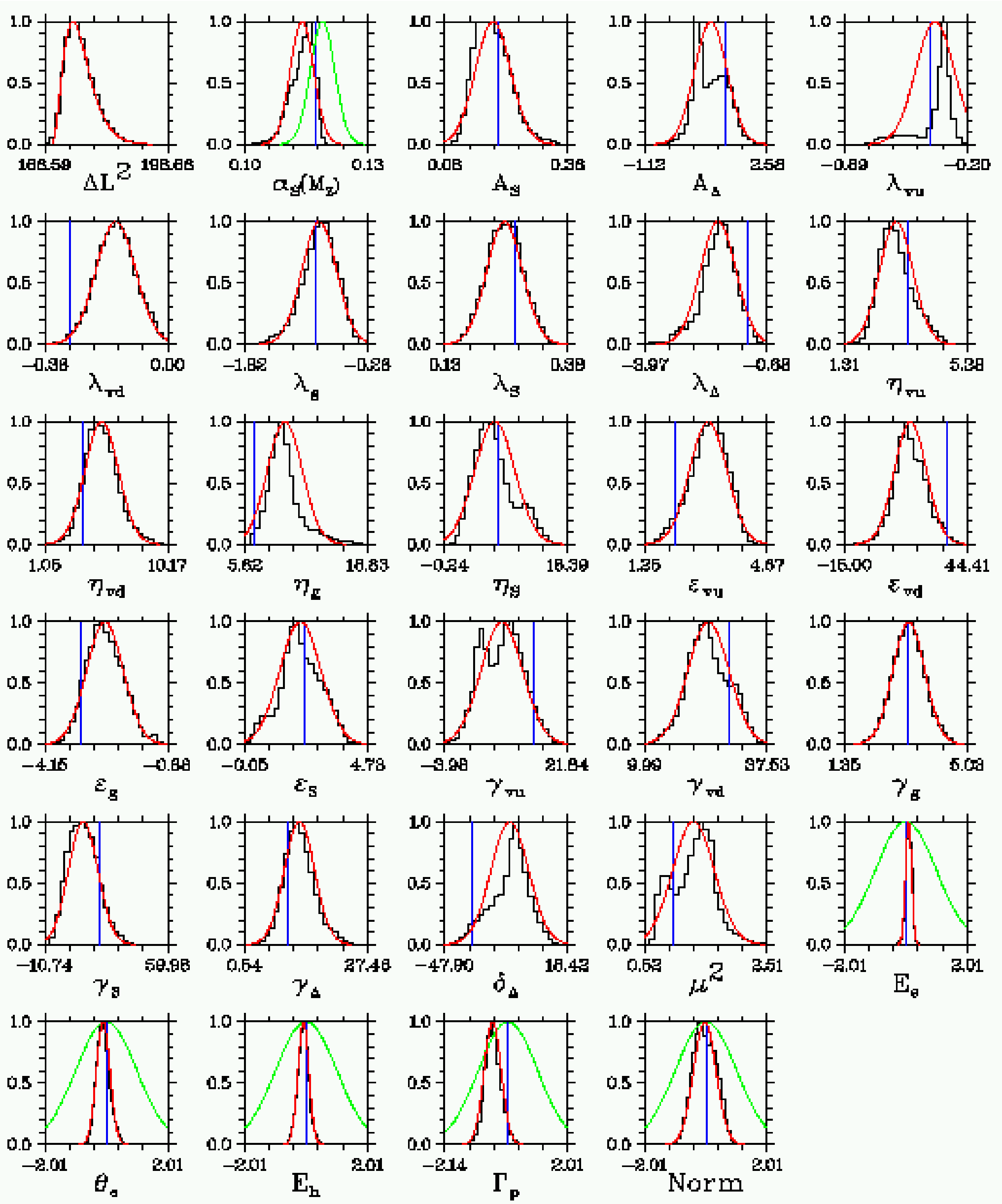
$$\mu_O = \sum_{\{\mathcal{F}\}} O(\{\mathcal{F}\})P(\{\mathcal{F}\}), \quad \sigma_O^2 = \sum_{\{\mathcal{F}\}} (O(\{\mathcal{F}\}) - \mu_O)^2 P(\{\mathcal{F}\}).$$

While this is statistically correct it is inefficient and in practice they generate N_{pdf} different distributions with unit weight but distributed according to $P(\{\mathcal{F}\})$ where N_{pdf} can be made as small as 100. Then

$$\mu_O = \frac{1}{N_{pdf}} \sum_1^{N_{pdf}} O(\{\mathcal{F}\}), \quad \sigma_O^2 = \frac{1}{N_{pdf}} \sum_1^{N_{pdf}} (O(\{\mathcal{F}\}) - \mu_O)^2.$$

Can incorporate full information about measurements and their error correlations in the calculation of $P(\{\mathcal{F}\})$.

Currently uses only proton DIS data sets in order to avoid complicated uncertainty issues such as shadowing effects. Difficult to obtain consistency between different DIS experiments. Lack of full data leads to “unusual” values for some parameters. Shows that this is a complicated game.



H1 set of parton parameters from GKK approach. Red curve Gaussian approx and blue line MRST value. Green curve for α_S is LEP result.

Hessian (Error Matrix) approach first used by H1 and ZEUS, recently extended by CTEQ.

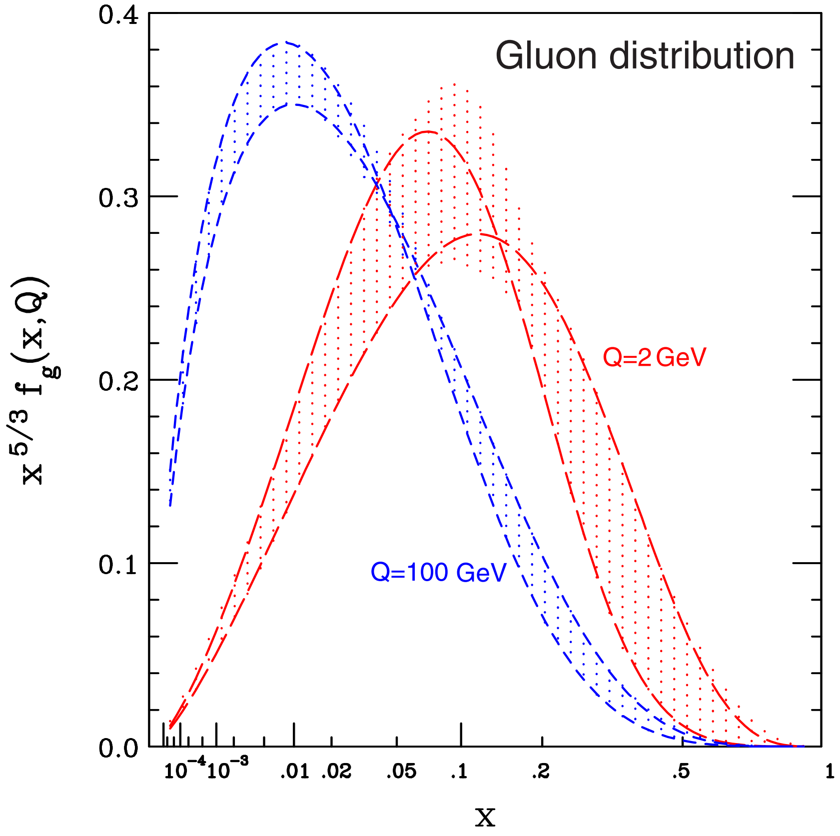
$$\begin{aligned}\Delta\chi^2 &= \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)}) \\ (\Delta F)^2 &= \Delta\chi^2 \sum_{i,j} \frac{\partial F}{\partial a_i}(H)^{-1}_{ij}\frac{\partial F}{\partial a_j},\end{aligned}$$

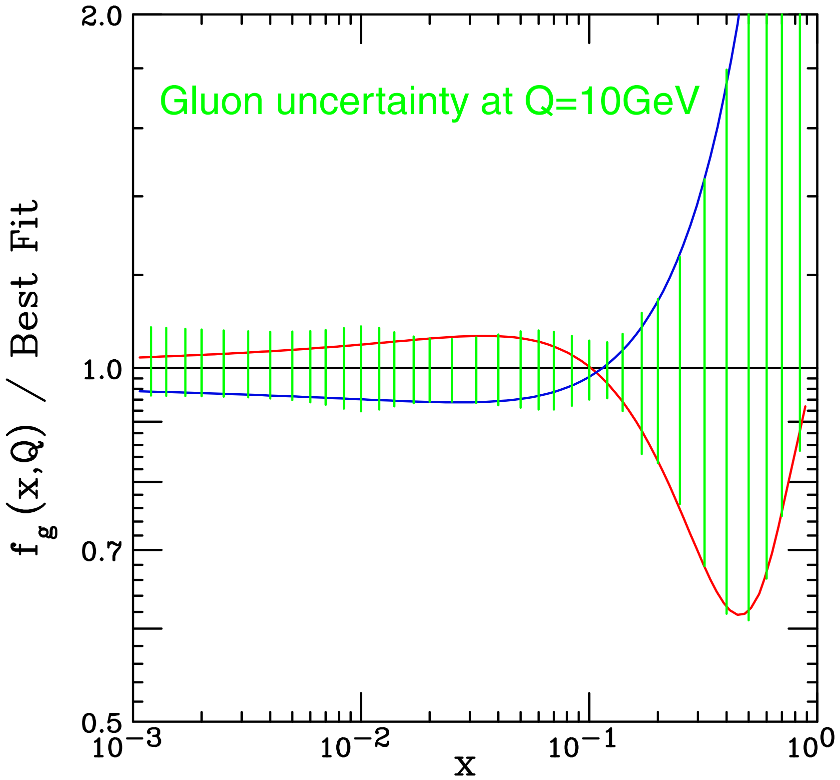
where a_i are the parton parameters, F is some physical quantity, and the Hessian matrix H is the inverse of the error matrix. Problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space.

Solved by finding and rescaling eigenvectors of H leading to diagonal form

$$\begin{aligned}\Delta\chi^2 &= \sum_i z_i^2 \\ (\Delta F)^2 &= \sum_i \left(F(S_i^{(+)}) - F(S_i^{(-)})\right)^2,\end{aligned}$$

where $S_i^{(+)}$ and $S_i^{(-)}$ are PDF sets displaced along eigenvector direction by given $\Delta\chi^2$. Art in choosing "correct" $\Delta\chi^2$ given complication of errors in full fit. CTEQ choose $\Delta\chi^2 \sim 100$.



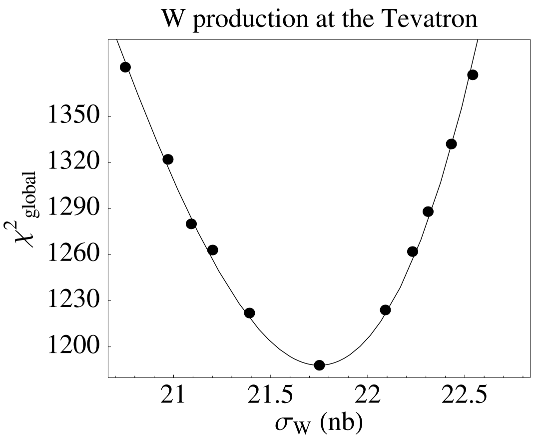


Results of CTEQ Hessian approach for gluon uncertainty.

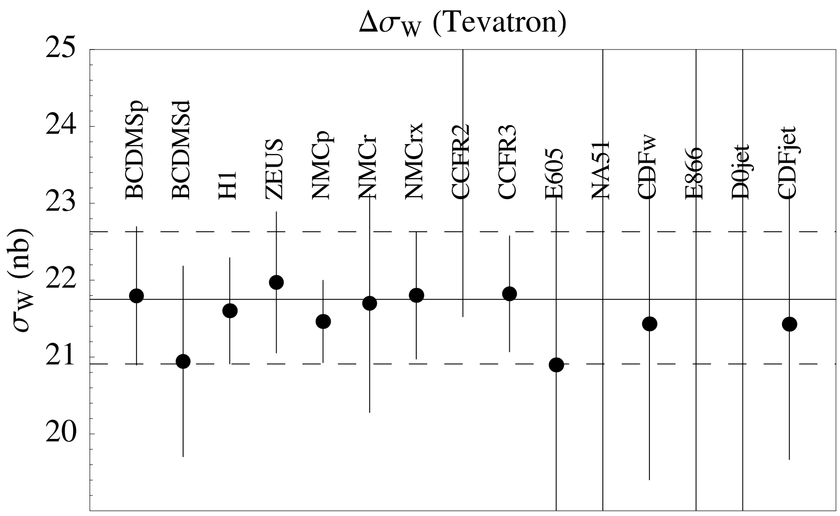
Can also look at uncertainty on a given physical quantity using Lagrange Multiplier method, first suggested by CTEQ i.e. perform fit while constraining value of some physical quantity. Minimize

$$\Psi(\lambda, a) = \chi^2_{global}(a) + \lambda F(a)$$

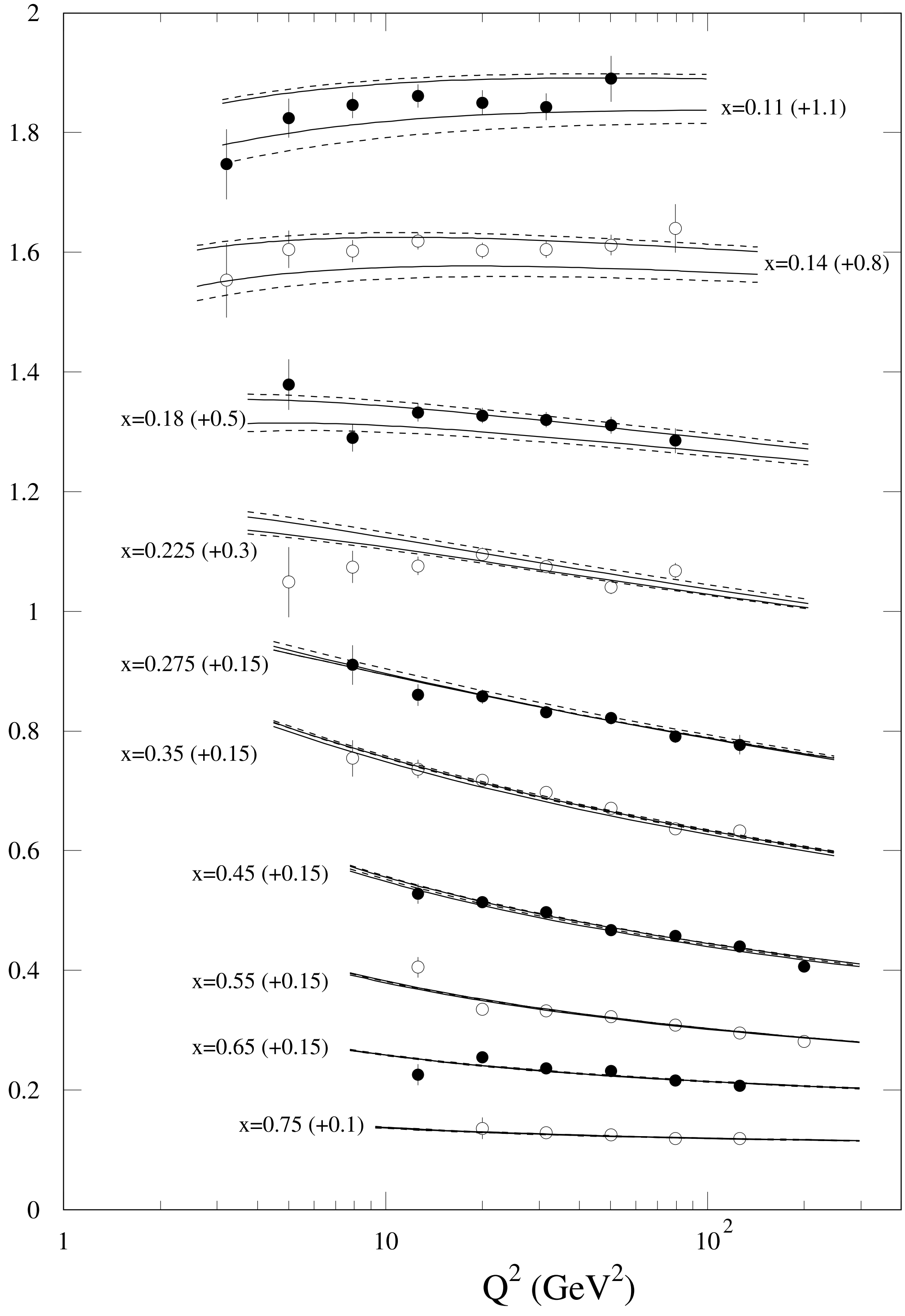
for various values of λ. Give set of best fits for particular values of parameter $F(a)$. Useful example W cross-section at Tevatron.



Uncertainty then determined by deciding allowed range of $\Delta\chi^2$. Can also look at range of allowed variation in $F(a)$ for each of the experiments in the global fit. Checks if total $\Delta\chi^2$ is coming specifically from one region.



CCFR XF_3 data , $W_{TEV}(1.021)$, $W_{TEV}(0.973)$, $W_{TEV}(1.034)$, $W_{TEV}(0.956)$ fit



Deterioration in fit for increase and decrease in $\sigma_W(\mathrm{Tev})$ obtained by MRST.

Once again, no unique prescription for deciding when a fit is actually acceptable or not. Use global χ^2 only or look at individual sets. What are criteria for limit for acceptable fit globally, or for one experiment?

CTEQ find $\Delta\sigma_{exp}(LHC) \approx \pm 8\%, \Delta\sigma_{exp}(\mathrm{Tev}) \approx \pm 5\%$

Procedure also used by MRST for most up-to-date data, and using stricter criteria. Find $\Delta\sigma_{exp}(LHC) \approx \pm 2\%, \Delta\sigma_{exp}(\mathrm{Tev}) \approx \pm 2\%$.

Can repeat for other processes.

- Higgs production sensitive to small x gluon rather than small x quarks,

- HERA charged current data sensitive to very high x quarks

- ....

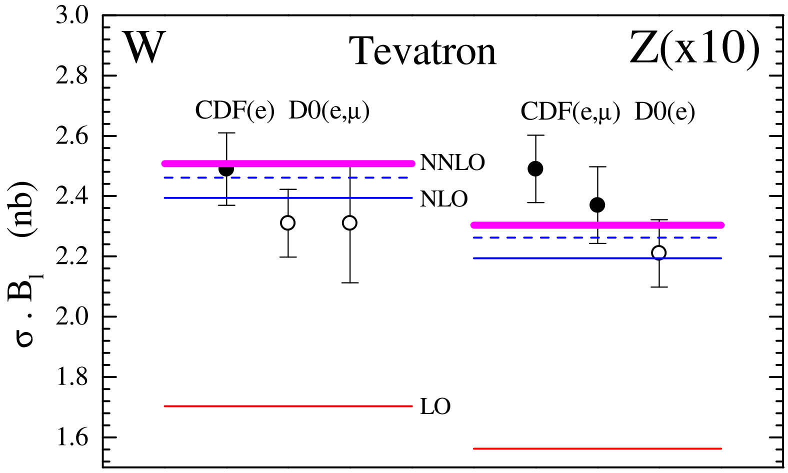
Uncertainty due to experimental errors rather small. Only more than few $\%$ for quantities determined by high x gluon.

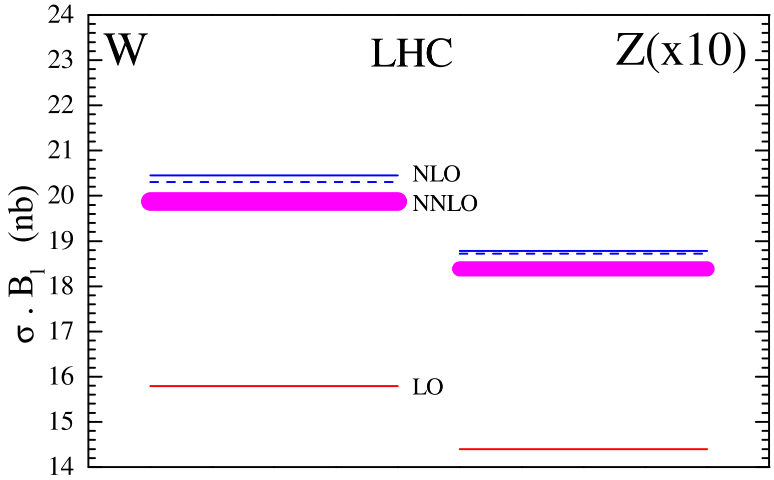
Also need to consider theoretical errors. Note renormalization/factorization scale variation is **not** reliable in general. In order to investigate true theoretical error must consider some way of performing correct large x and small x resummations (former easier), and/or use what we already know about NNLO.

Coefficients functions known at NNLO

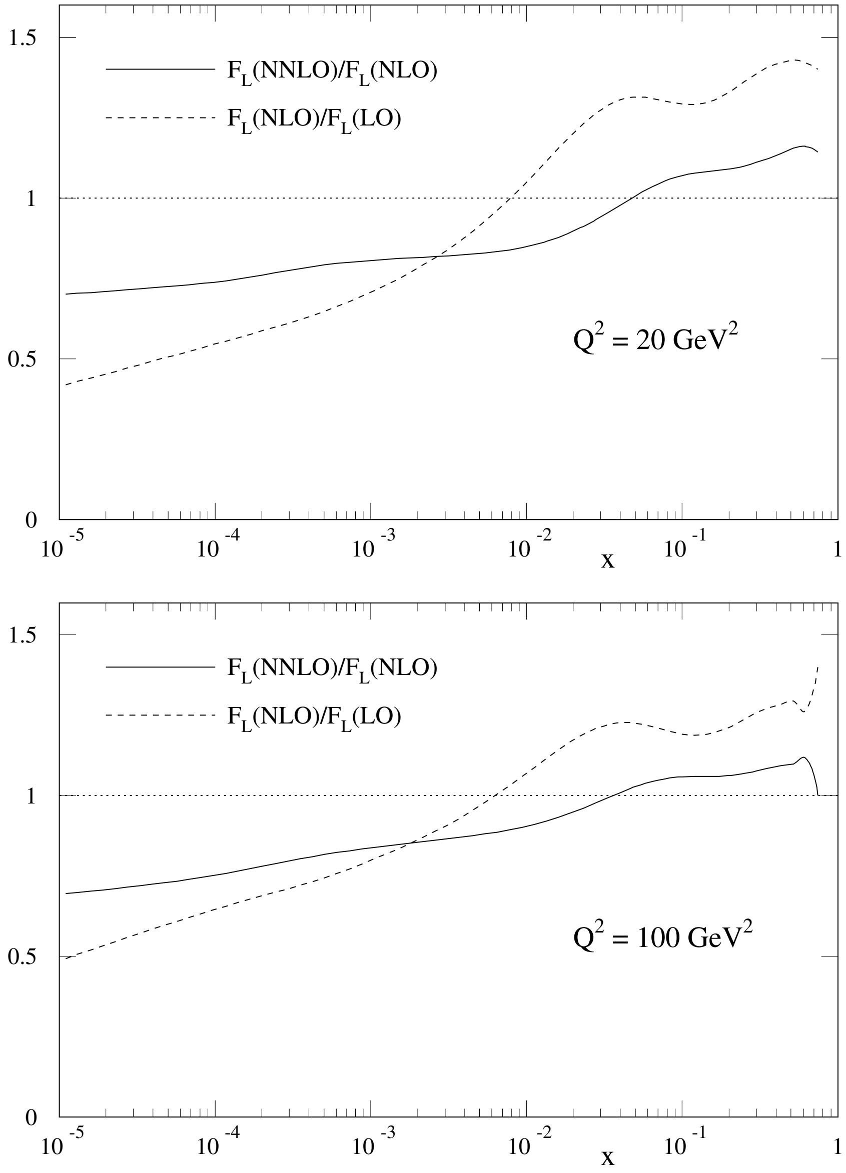
Singular limits $x \to 1$, $x \to 0$ known for NNLO splitting functions as well as limited moments (Larin, Nogueira, van Ritenberg Vermaseren, Retey). Approximate NNLO splitting functions devised by van Neerven and Vogt.

Used in global fit by MRST. Improve quality of fit. Implies reasonable stability for (quark-dominated) W and Z cross-sections. (Possible danger from unsatisfactory $dF_2/d\ln Q^2$)





More danger in gluon dominated quantities - gluon not directly constrained by data. Possible/probable poor convergence order by order for $F_L(x, Q^2)$ at small x.



Note K-factor for Higgs production at LHC ~ 1.6 Spira, Djouadi, Graudenz and Zerwas.

Conclusions

One can perform global fits to all up-to-date data over wide range of parameter space. Fit fairly good - some slight worries:

low W^2 - $\alpha_S^n \ln^{2n-1}(1-x)$, higher twist. Remove via cuts.

$dF_2/d\ln Q^2$ at small x - $\alpha_S^n \ln^{n-1}(1/x)$. Also remove via cuts? Rather Draconian.

High x gluon most uncertain. Constrained (to 20%) by new jet data. Consistent with other indications.

Various ways of looking at uncertainties due to errors on data alone. Useful to concentrate on W (and Z) cross-sections. Methods can be applied in same manner to other quantities.

Uncertainties from this source rather small. Even for quark dominated processes uncertainty from input assumptions e.g. $\alpha_S(M_Z^2)$, cuts on data, varying experimental normalization to limits ..., and from higher orders/resummation comparable and potentially larger.

For some processes theory probably the dominant source of uncertainty at present. Systematic study needed. Much harder than uncertainties due to errors. Just beginning.