Jets in e^+e^- Annihilation

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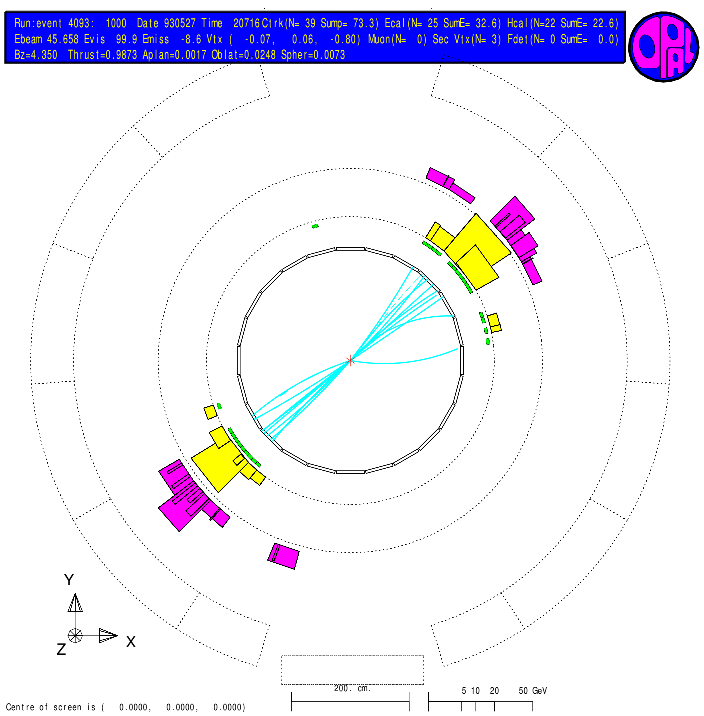
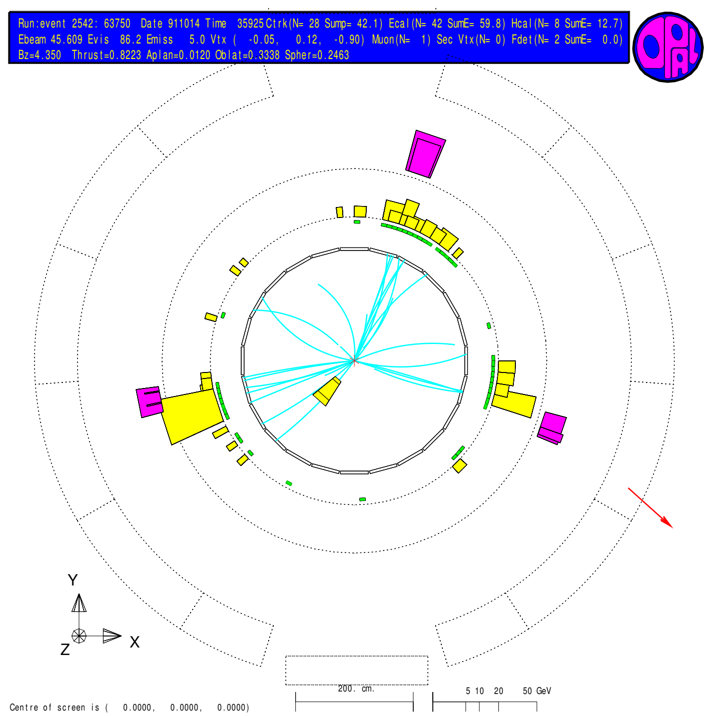
Structure of hadronic events

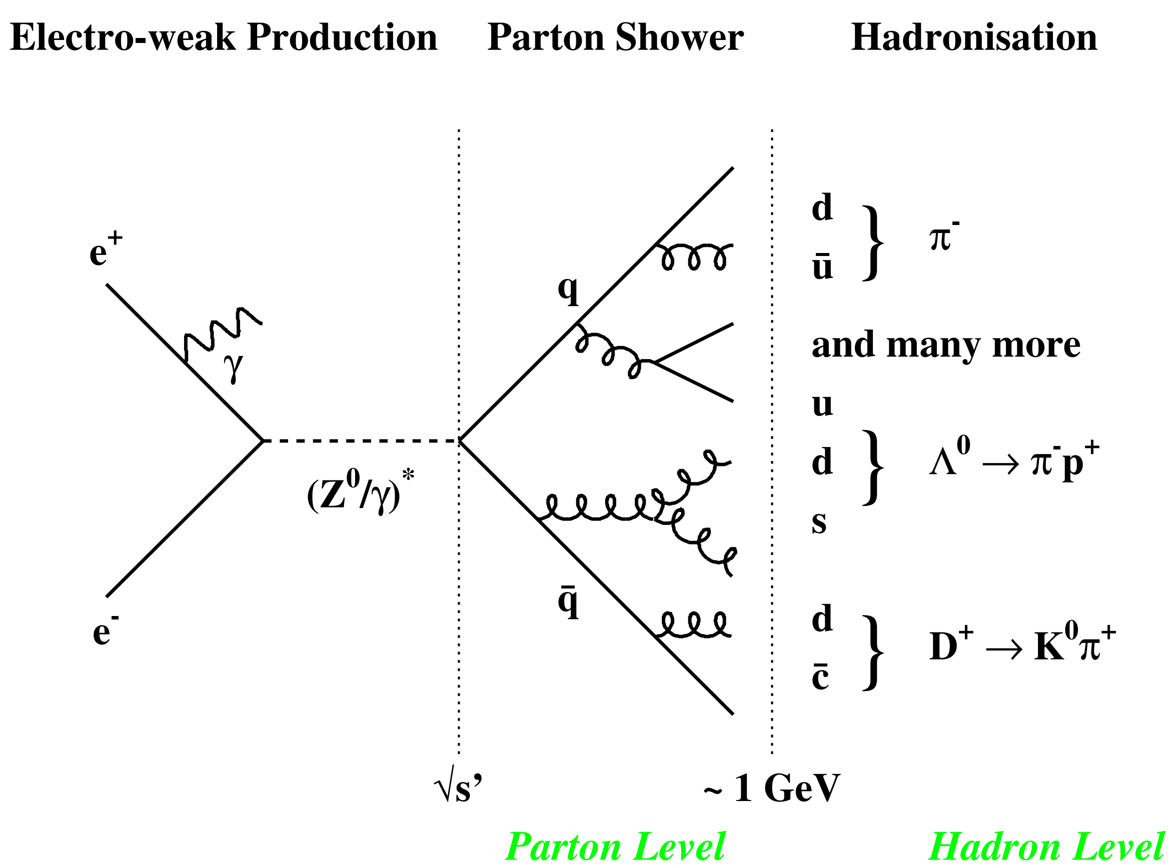
A two jet event



A three jet event



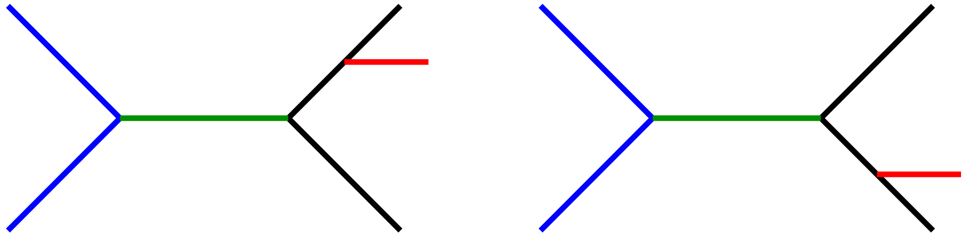
Structure of hadronic events



Here we will mostly discuss the hard scattering parton level phase

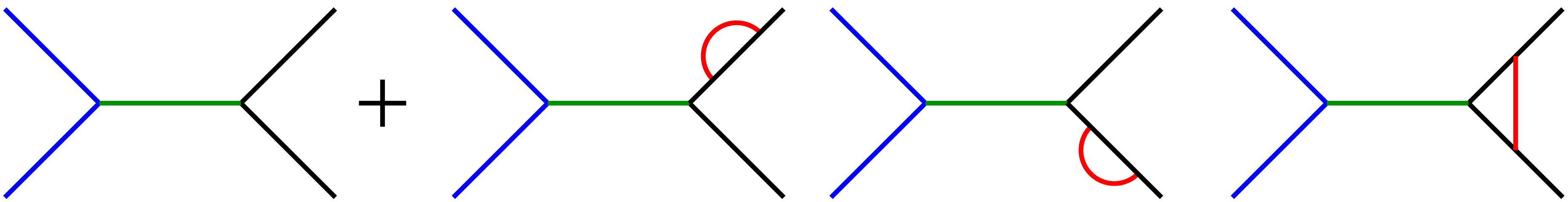
$\mathcal{O}(\alpha_s)$ corrections to $e^+e^- \rightarrow$ hadrons

Real Gluon emission



$$M_{q\bar{q}g} \propto g_s \Rightarrow |M_{q\bar{q}g}|^2 \propto \alpha_s$$

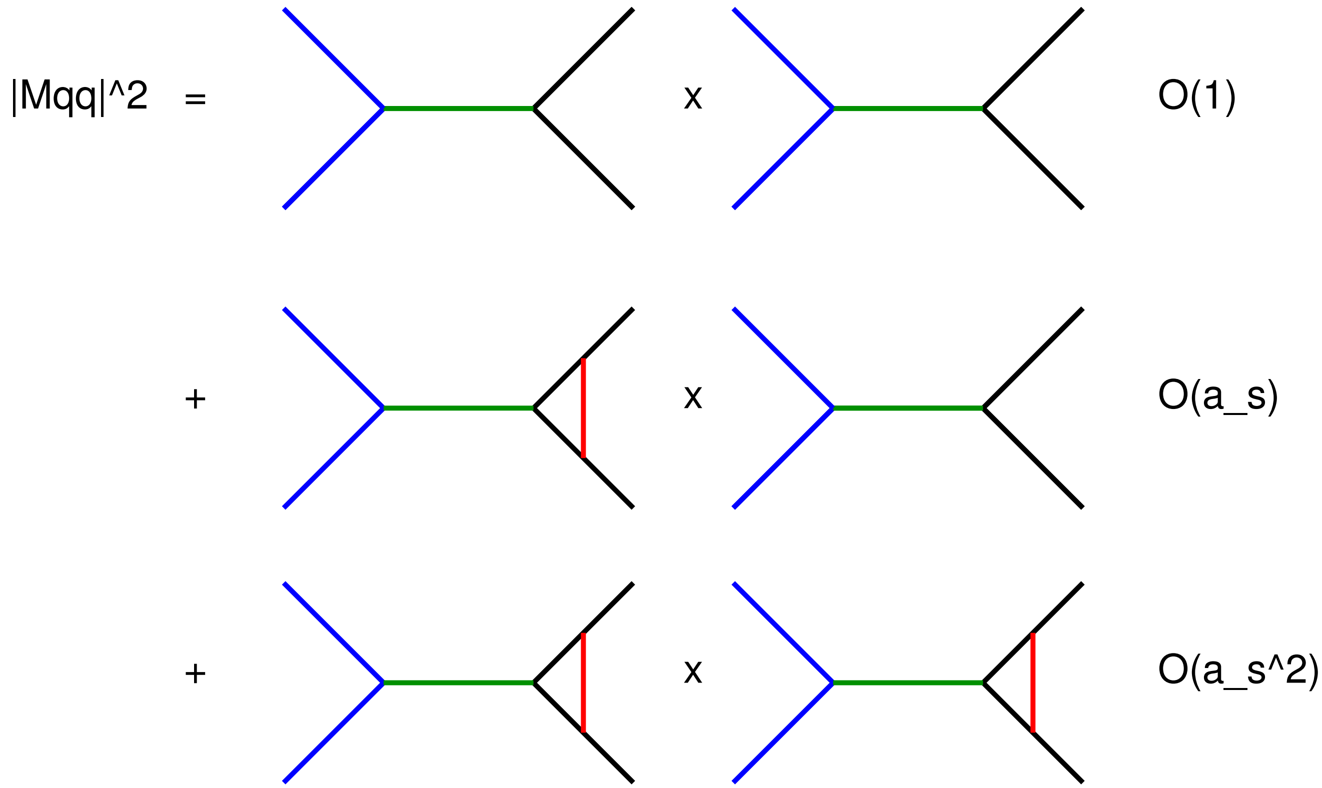
Virtual Gluon emission



$$M_{q\bar{q}} \sim 1 + \alpha_s$$

$\mathcal{O}(\alpha_s)$ corrections to $e^+e^- \to$ hadrons

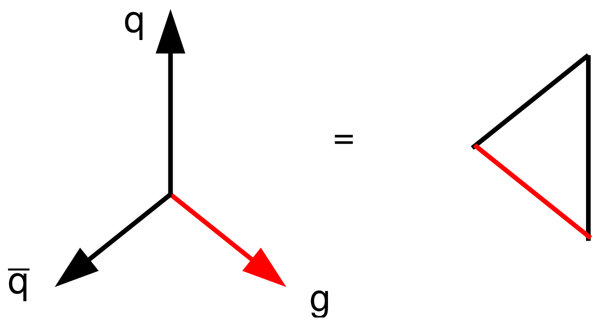
Note that



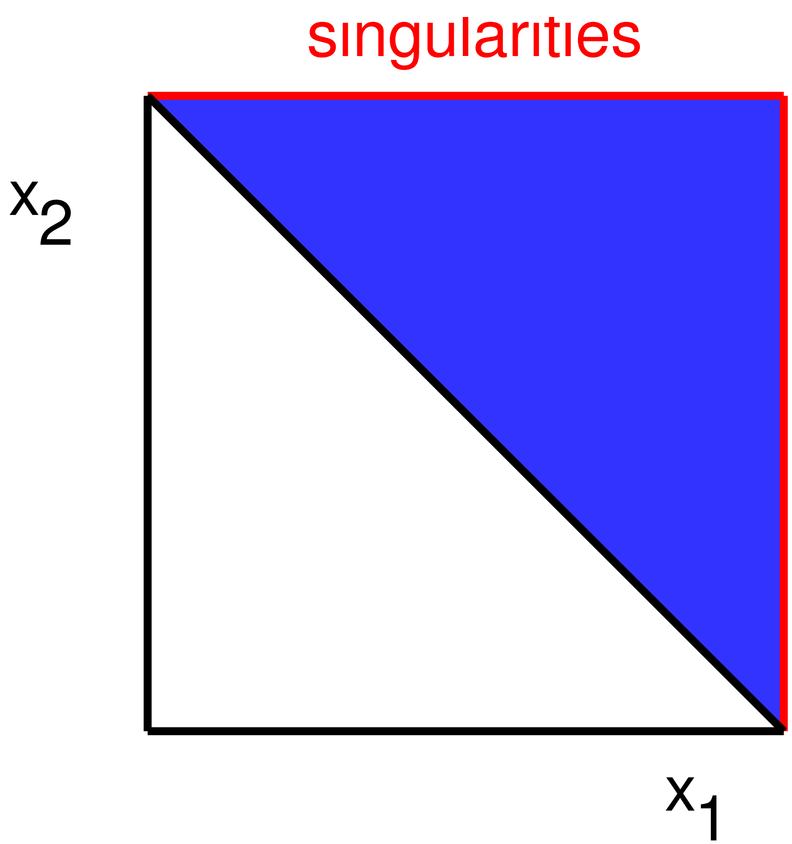
At NLO, we are only interested in the interference of the one-loop amplitude with the tree-graph

Phase space for real emission

Because of momentum conservation, q, $\bar{q}$ and g lie in a plane.



Useful variables are the energy fractions and invariant masses



$$x_i = \frac{2E_i}{\sqrt{s}}, \qquad x_1 + x_2 + x_3 = 2$$

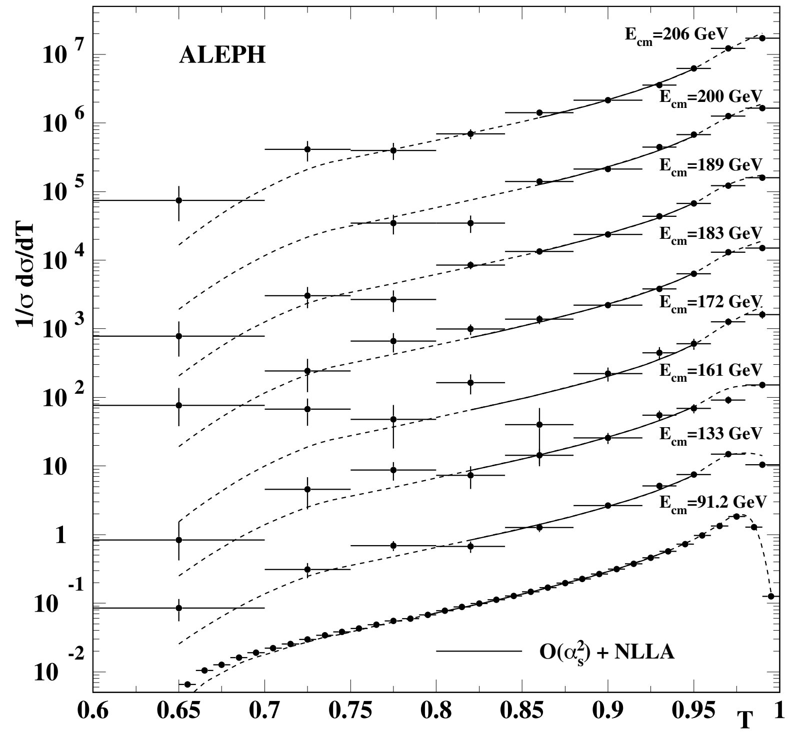
$$y_{ij} = \frac{2p_i.p_j}{\sqrt{s}} = 1 - x_k$$

Event shape variables

global observable characterising structure of hadronic event

e.g. Thrust in e^+e^-

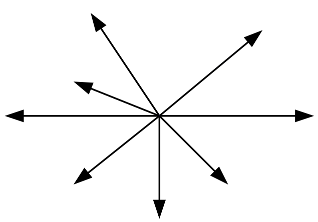
$$T = \max_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p_i}|}$$



limiting values:

- back-to-back (two-jet) limit: $T = 1$

- spherical limit: $T = 1/2$



$\mathcal{O}(\alpha_s)$ Thrust distribution

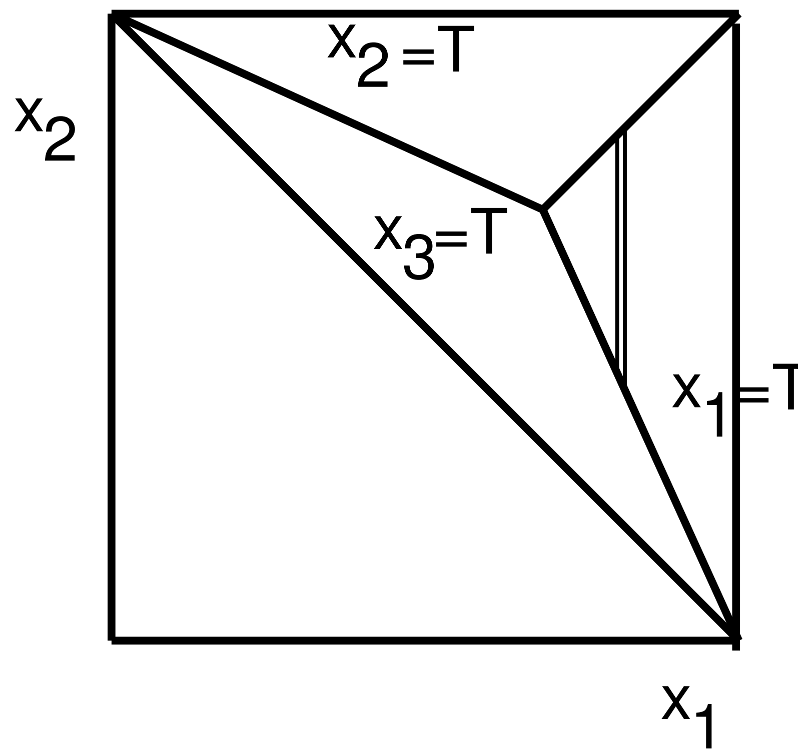
In terms of x_1 and x_2, the NLO cross section is given by

$$\frac{1}{\sigma^0_{q\bar{q}}}\frac{d^2\sigma}{dx_1 dx_2} = \frac{\alpha_s C_F}{2\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

For a three particle event,

$$T = \max x_i$$

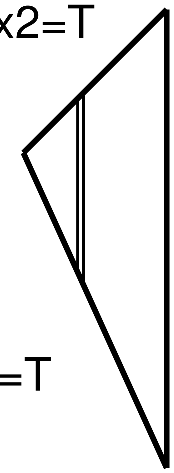
Therefore, we can divide the phase space into three regions corresponding to the Thrust value.



$\mathcal{O}(\alpha_s)$ Thrust distribution

For the region where $T = x_1$, the boundaries are

$$2(1-T) < x_2 < T$$



$$\begin{aligned}\frac{1}{\sigma^0_{q\bar{q}}}\frac{d\sigma}{dT}\Big|_1 &= \frac{\alpha_s C_F}{2\pi}\int_{2(1-T)}^{T}\frac{T^2+x_2^2}{(1-T)(1-x_2)} \\ &= \frac{\alpha_s C_F}{2\pi}\left[\frac{1+T^2}{(1-T)}\log\left(\frac{2T-1}{1-T}\right)+\frac{8-14T+3T^2}{2(1-T)}\right]\end{aligned}$$

Exercise: do the same for the other two regions.

$\mathcal{O}(\alpha_s)$ Thrust distribution

Adding up the three contributions together

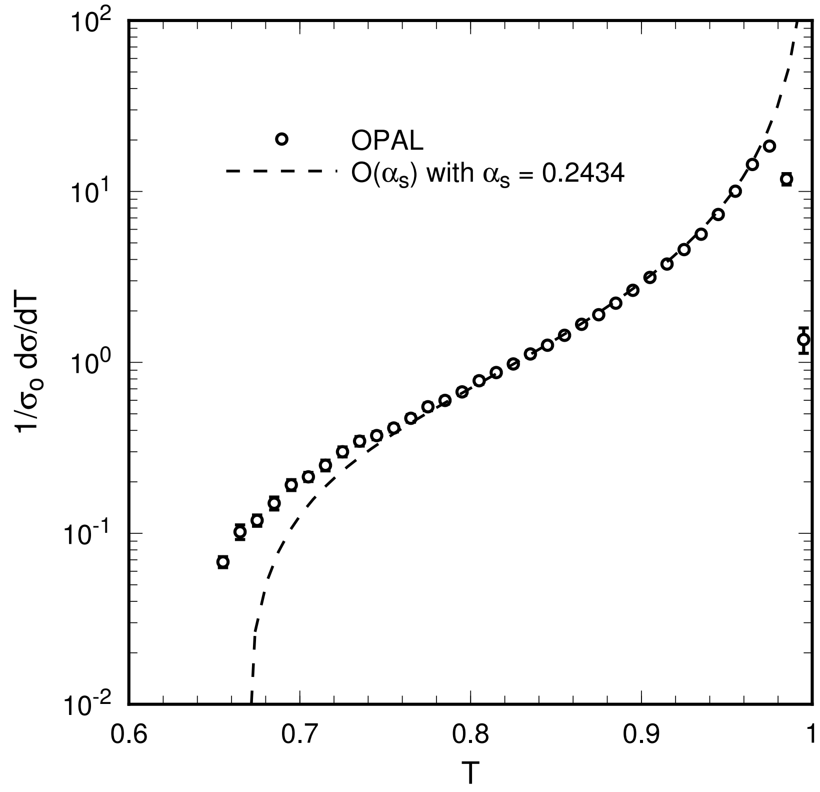
$$\frac{1}{\sigma^0_{q\bar{q}}}\frac{d\sigma}{dT} = \frac{\alpha_s C_F}{2\pi}\left[\frac{2(3T^2-3T+2)}{T(1-T)}\log\left(\frac{2T-1}{1-T}\right) - \frac{3(3T-2)(2-T)}{1-T}\right]$$

- $T > 2/3$ **when** $x_1 = x_2 = x_3 = T$
- **As** $T \to 1$**,**

$$\frac{1}{\sigma^0_{q\bar{q}}}\frac{d\sigma}{dT} \sim \frac{\alpha_s C_F}{2\pi}\left[\frac{4}{(1-T)}\log\left(\frac{1}{1-T}\right) - \frac{3}{1-T}\right]$$

- **Virtual contribution at** $T = 1$
- **Expect large hadronisation corrections as** $T \to 1$

$\mathcal{O}(\alpha_s)$ Thrust distribution



- **deficiency at small T due to kinematic bound**
- **shape good $0.75 < T < 0.95$**

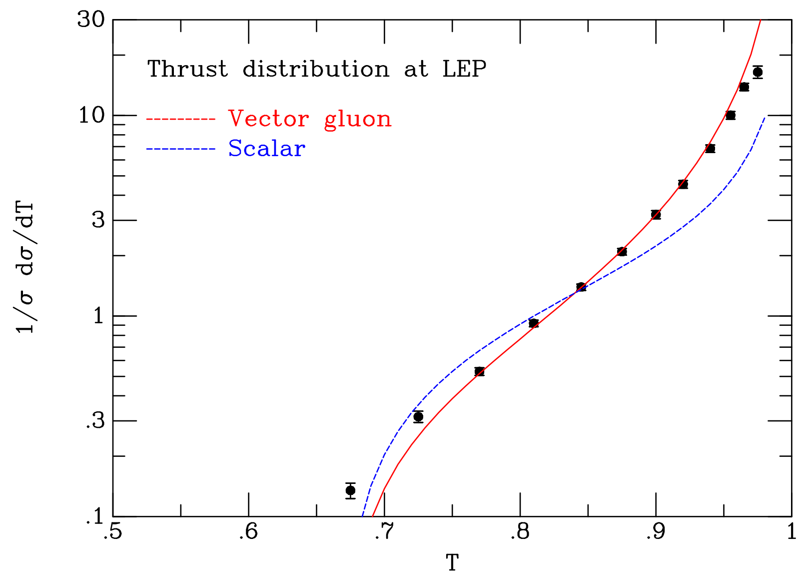
Spin of the gluon

If the gluon is a scalar, it would be evident in the event shape.

$$\mathcal{L}_{int} \sim \bar{g}_s \bar{\Psi}_i T^a_{ij} \Phi^a \Psi_j$$

leads to

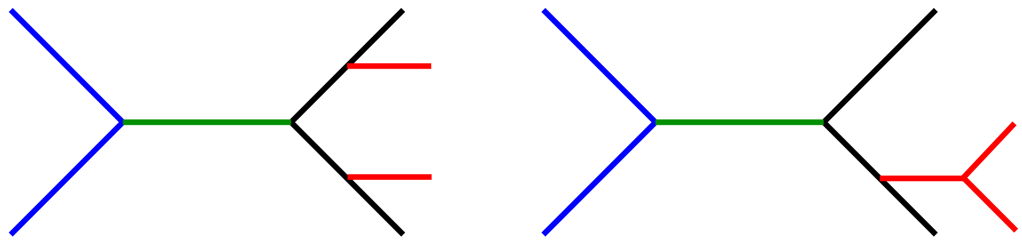
$$\frac{1}{\sigma^0_{q\bar{q}}} \frac{d^2\sigma}{dx_1 dx_2} \sim \frac{x_3^2}{2(1-x_1)(1-x_2)}$$



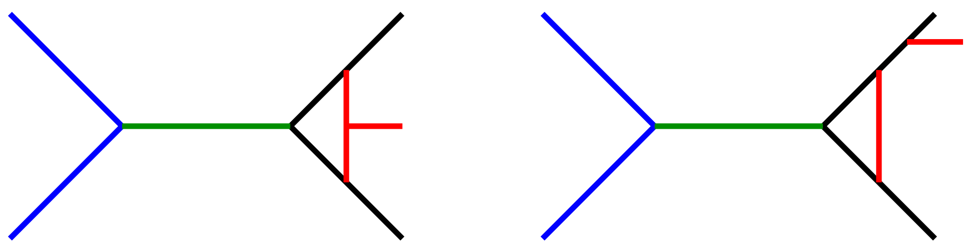
$$\frac{1}{\sigma^0_{q\bar{q}}} \frac{d\sigma}{dT} = \frac{\bar{\alpha}_s C_F}{2\pi} \frac{1}{2} \left[2 \log \left(\frac{2T-1}{1-T} \right) + \frac{(3T-2)(4-3T)}{1-T} \right]$$

NLO corrections to thrust distribution

At NLO, get contributions from double radiation



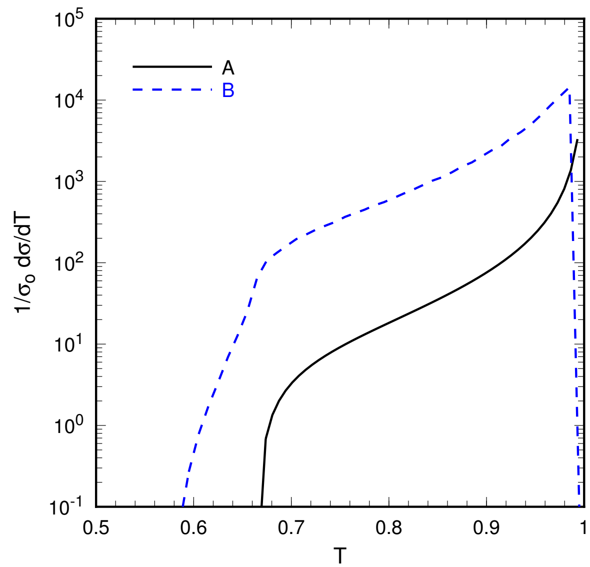
and virtual graphs

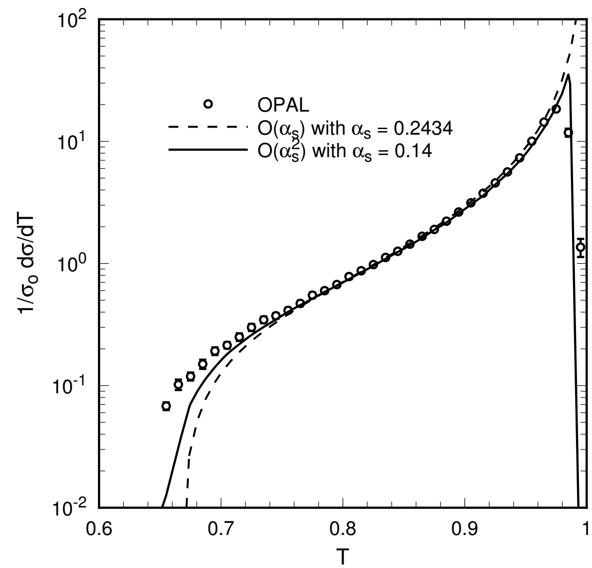


$$\frac{1}{\sigma_0}\frac{d\sigma}{dT} = \frac{\alpha_s(\mu)}{2\pi}A(T) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \left[b_0 A(T) \ln\left(\frac{\mu^2}{s}\right) + B(T) \right]$$

renormalisation term and **genuine NLO contribution**

NLO corrections to thrust distribution





- **As $T \to 1$, A diverges positive, B diverges negative**
- $T > 1/\sqrt{3}$ **at NLO**

- **Better agreement over wider range of T**
- **More sensible value of α_s**
- **Still problems as $T \to 1$**

Large logarithms in thrust distribution

As $T \to 1$,

$$\frac{1}{\sigma^0_{q\bar{q}}}\frac{d\sigma}{dT} \sim \frac{\alpha_s C_F}{2\pi}\left[\frac{4}{(1-T)}\log\left(\frac{1}{1-T}\right) - \frac{3}{1-T}\right]$$

define cross section for $T > \tau$ which is fraction of events with $T > \tau$,

$$\begin{aligned} R(\tau) &= \int_\tau^1 dT \frac{1}{\sigma^0_{q\bar{q}}}\frac{d\sigma}{dT} \\ &\sim 1 - \frac{\alpha_s C_F}{\pi}\ln^2(1-\tau) \end{aligned}$$

Singularity at $T = 1$ cancelled by one-loop two-parton contribution

When $\alpha_s \ln^2(1-\tau)$ large, i.e. $\tau \sim 0.95$ cannot trust perturbation theory. In fact,

$$R(\tau) \sim 1 - \frac{\alpha_s C_F}{\pi}\ln^2(1-\tau) + \frac{1}{2}\left(\frac{\alpha_s C_F}{\pi}\right)^2 \ln^4(1-\tau)$$

Large logarithms in thrust distribution

$$R(\tau) \quad \sim \quad \exp\left(-\frac{\alpha_s C_F}{\pi}\ln^2(1-\tau)\right)$$

so that as $\tau \to 1$, $R(\tau) \to 0$

This is the SUDAKOV form factor effect

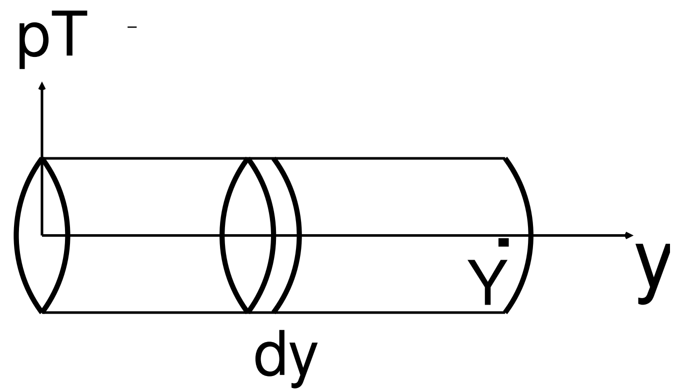
- for event to have very high thrust, must have radiated very few gluons

$\Rightarrow$ very improbable

c.f data, very improbable to have only 2 hadron event

- can also resum next-to-leading logs for many event shapes so that calculations believable when $\alpha_s \ln(1-\tau)$ small

Simple hadronisation model



Parton produces a tube in (y, p_T) space of light hadrons w.r.t. initial parton direction with transverse mass density μ

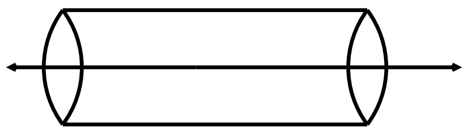
$$\begin{aligned} E_{jet} &= \mu \int_0^Y \cosh y \, dy = \mu \sinh Y \\ P_{jet} &= \mu \int_0^Y \sinh y \, dy = \mu(\cosh Y - 1) \end{aligned}$$

$\Rightarrow \; m_{jet}^2 = E_{jet}^2 - P_{jet}^2 = 2\mu P_{jet}$ **or** $E_{jet} \sim P_{jet} + \mu$

where $\mu \sim 0.5 - 1$ GeV from experiment

Hadronisation and Thrust

2 jet event:



$$T_{parton} = 1$$

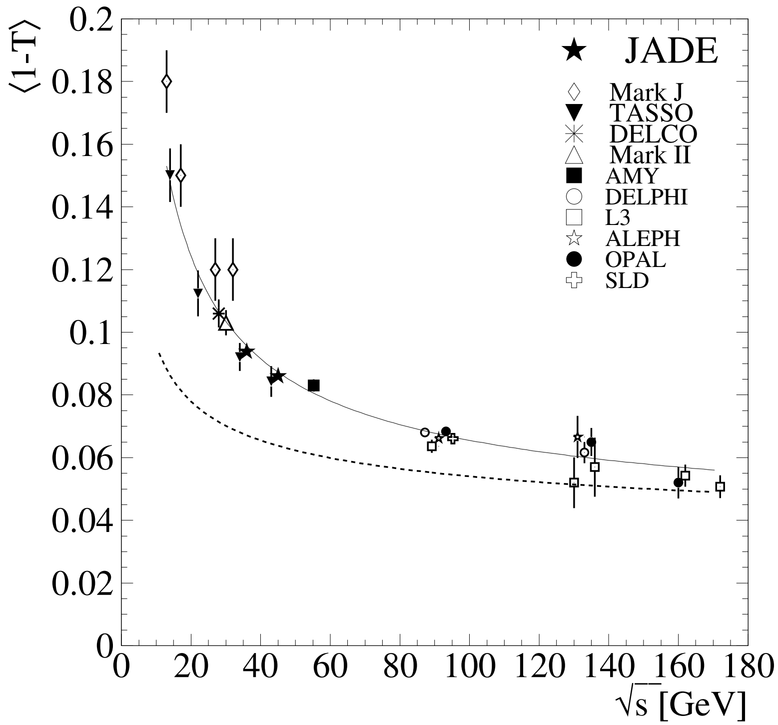
$$T_{hadron} = \frac{2P_{jet}}{Q} = \frac{2(E_{jet} - \mu)}{Q} = 1 - \frac{2\mu}{Q}$$

i.e.

$$\delta T = -\frac{2\mu}{Q}$$

Mean value of thrust

$$\langle 1 - T \rangle = 0.33\alpha_s + 1.0\alpha_s^2 + \frac{1\ GeV}{Q}$$



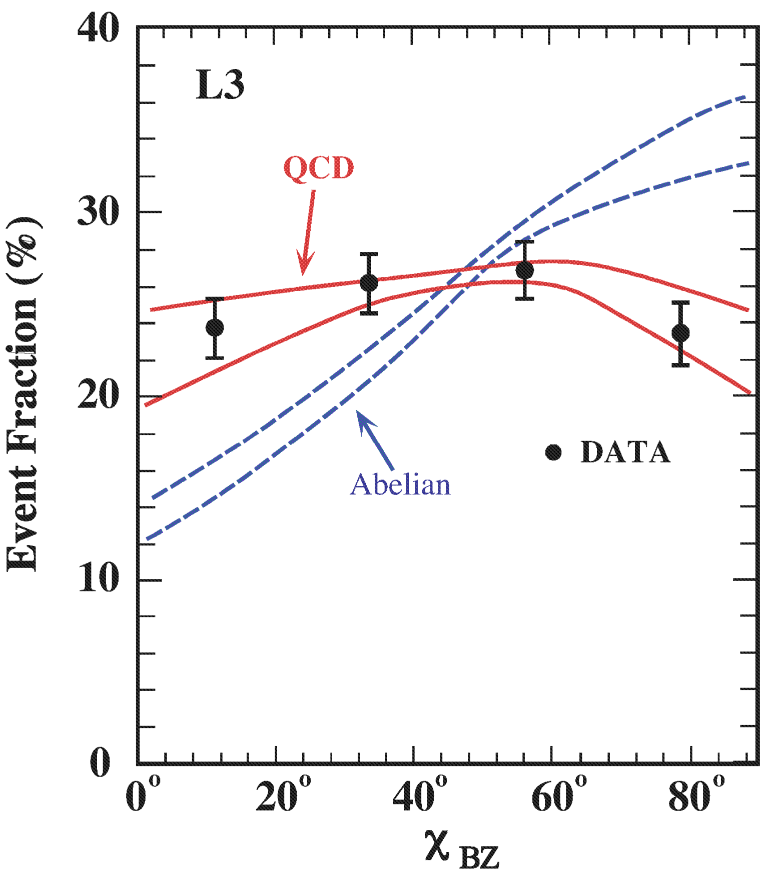
The triple gluon vertex

Four jet matrix elements are sensitive to the triple gluon vertex - (makes event more planar)

To quantify this effect study e.g.Bengtsson-Zerwas angle

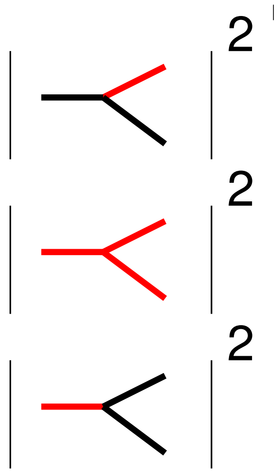
$$\cos\theta_{BZ} = \frac{(\vec{p_1} \times \vec{p_2}).(\vec{p_3} \times \vec{p_4})}{|\vec{p_1}||\vec{p_2}||\vec{p_3}||\vec{p_4}|}$$

for four jets $(E_i, \vec{p_i})$ **with** $E_1 > E_2 > E_3 > E_4$



Probing non-abelian structure of QCD

Probabilities for parton splitting



$$q \to qg \quad \propto \quad C_F \alpha_s$$

$$g \to gg \quad \propto \quad C_A \alpha_s$$

$$g \to q\bar{q} \quad \propto \quad T_R \alpha_s$$

In QCD,

$$C_F = \frac{N^2 - 1}{2N}, \qquad C_A = N, \qquad T_R = \frac{1}{2}$$

All splittings present in $\mathcal{O}(\alpha_s^2)$ event shapes, i.e.

$$B(T) = C_F \alpha_s^2 \left(C_F B_{C_F}(T) + C_A B_{C_A}(T) + T_R B_{T_R}(T) \right)$$

which can be fit to data

Probing non-abelian structure of QCD

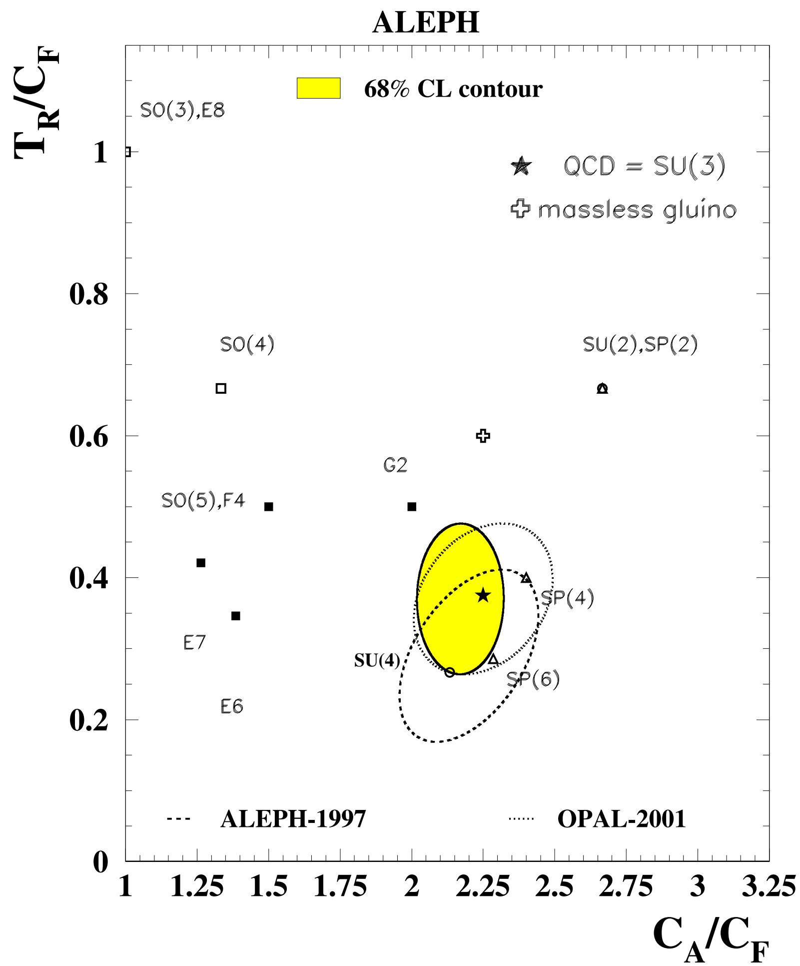
Fixing the quadratic casimirs of QCD in 4 jet events

OPAL

C_A	$3.02 \pm 0.25 \pm 0.49$
C_F	$1.34 \pm 0.13 \pm 0.22$
$\alpha_s(M_Z)$	$0.120 \pm 0.011 \pm 0.020$

ALEPH

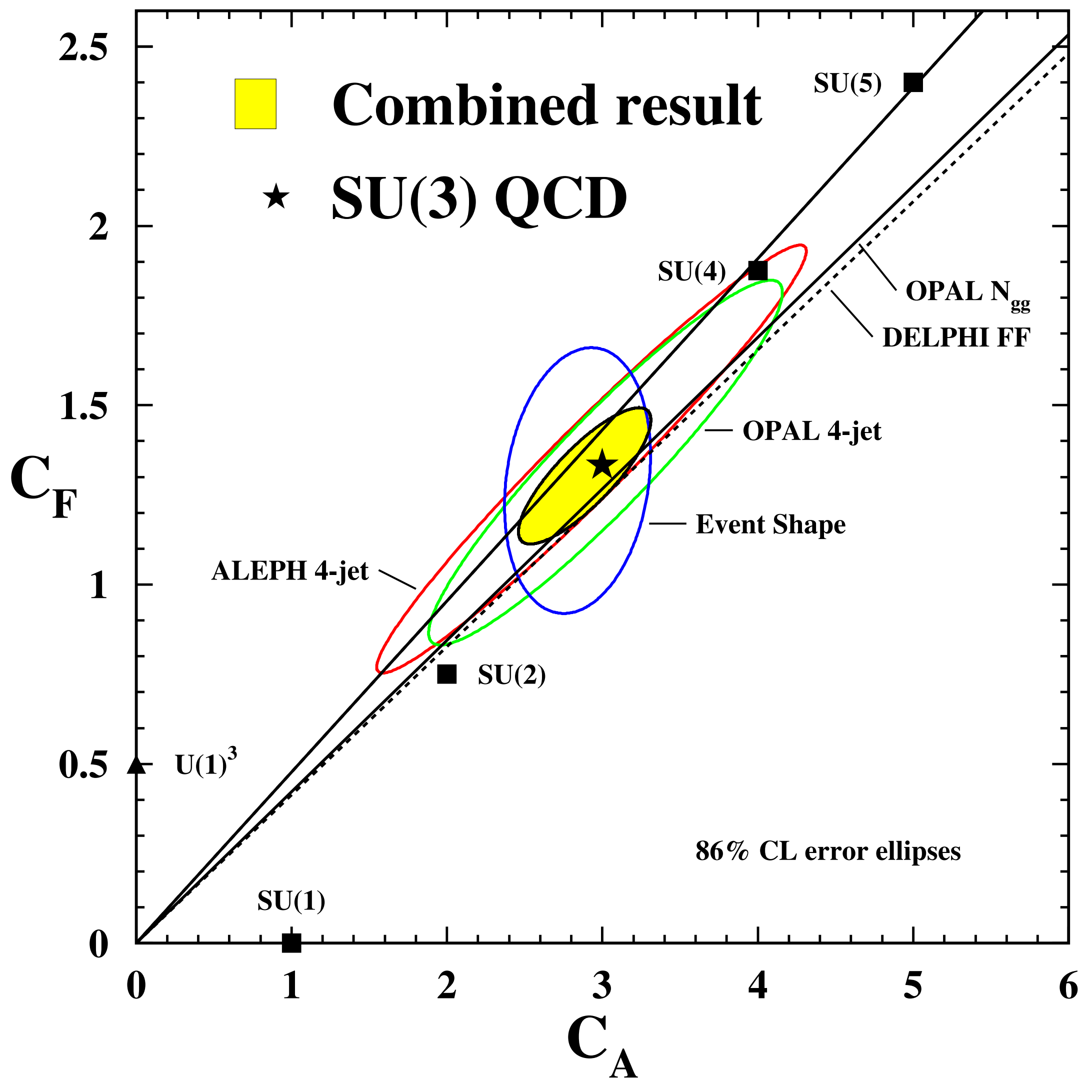
C_A	$2.93 \pm 0.14 \pm 0.49$
C_F	$1.35 \pm 0.07 \pm 0.22$
$\alpha_s(M_Z)$	$0.119 \pm 0.006 \pm 0.022$



Expect

$C_A/C_F = 9/4 \quad T_R/C_F = 3/8$

Probing non-abelian structure of QCD



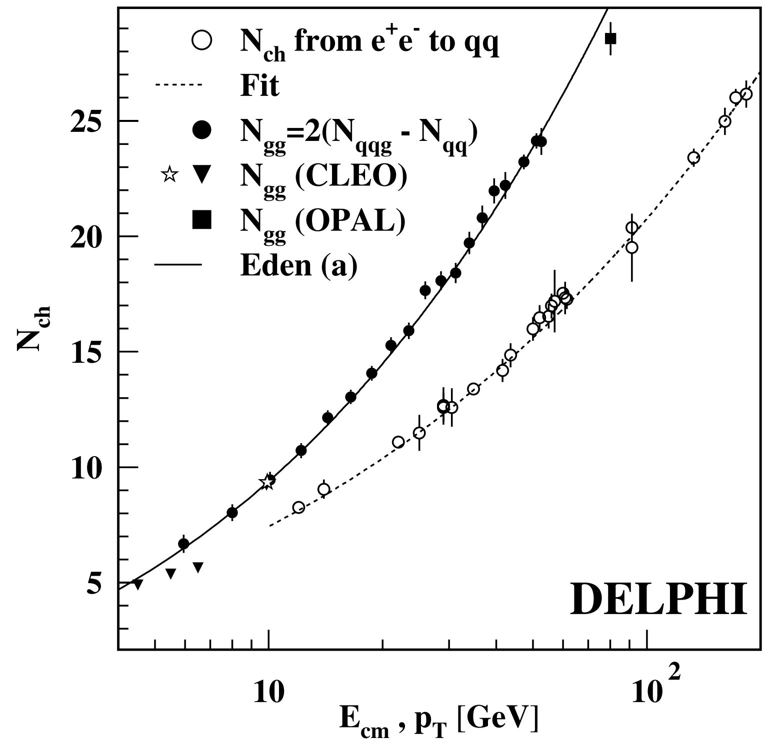
Differences between quark and gluon jets

QCD predicts that quarks and gluons fragment differently because of their different colour charges

Fundamental prediction:
the number of soft gluons emitted within a gluon jet should be **~twice** that for quark jet

$$r_{g/q} \equiv \frac{\langle n \rangle_{gluon}}{\langle n \rangle_{quark}} \sim \frac{C_A}{C_F} = 2.25$$

Valid for soft particles and unbiased jets.
For hard gluon emission, quarks and gluons are similar $r_{g/q} \sim 1$

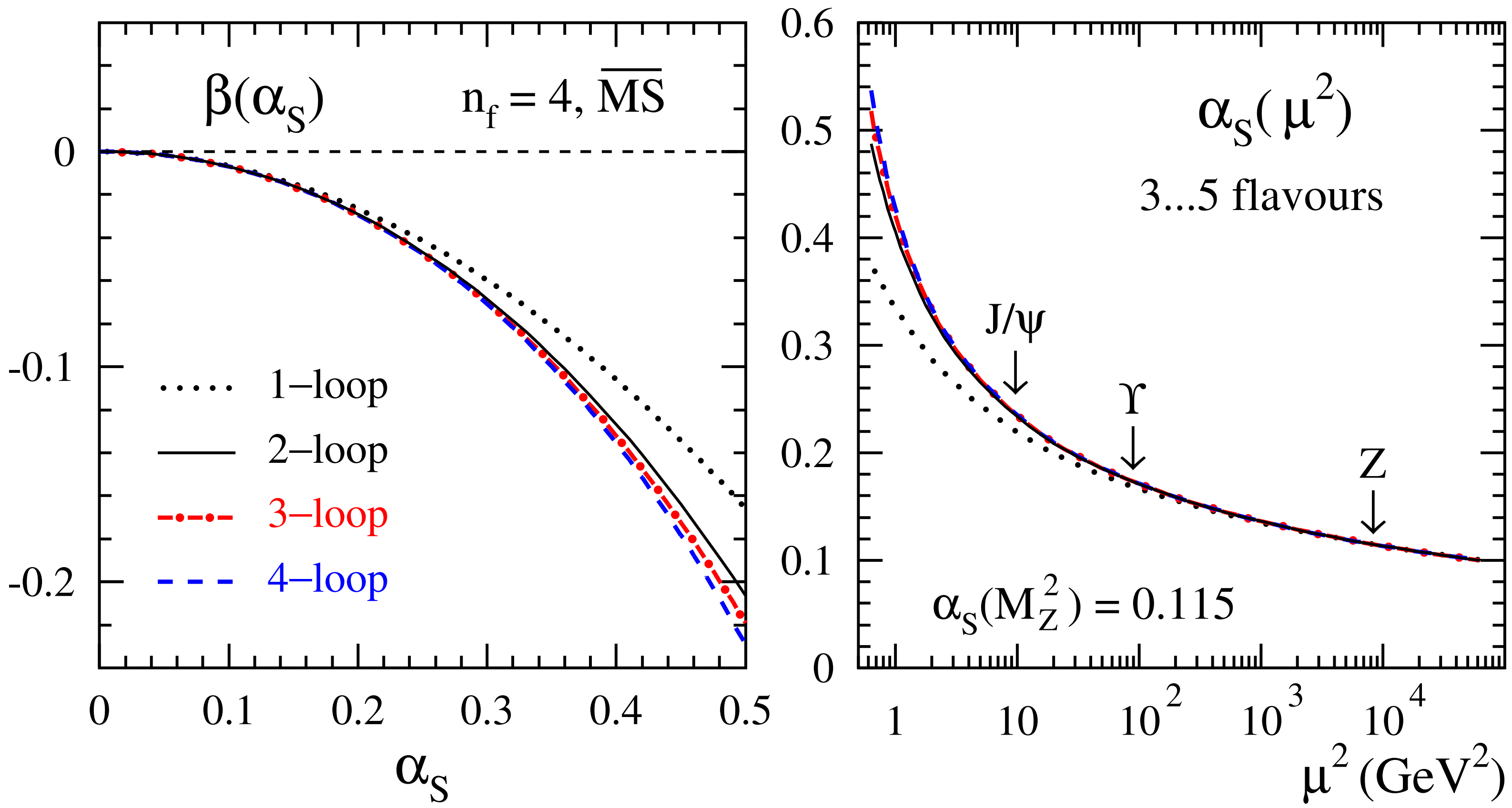


The running coupling in perturbative QCD

$$d\alpha_s/d\ln\mu^2 = -\beta_0\,\alpha_s^2 - \beta_1\,\alpha_s^3 - \beta_2\,\alpha_s^4 - \beta_3\,\alpha_s^5 - \ldots$$

Four-loop coeff.:

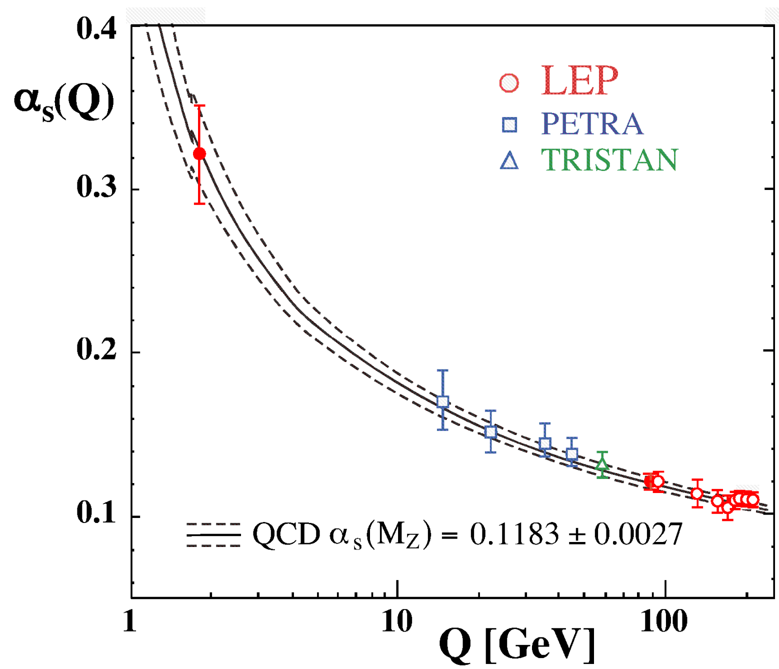
van Ritbergen, Vermaseren, Larin; Czakon



The running coupling from LEP

Combined results for $\alpha_s(M_Z)$

$$\tau \text{ decays}: \quad = \quad 0.1180 \pm 0.0030$$
$$R_Z: \quad = \quad 0.1226^{+0.0058}_{-0.0038}$$
$$\text{shapes}: \quad = \quad 0.1202 \pm 0.0050$$



Final combined result from LEP

$$\alpha_s(M_Z) = 0.1195 \pm 0.0034$$

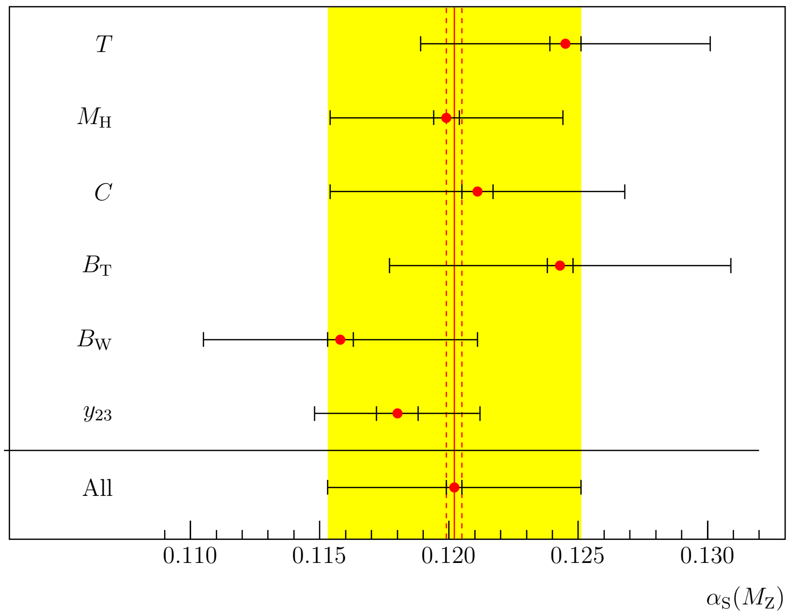
Errors dominated by theoretical uncertainties.

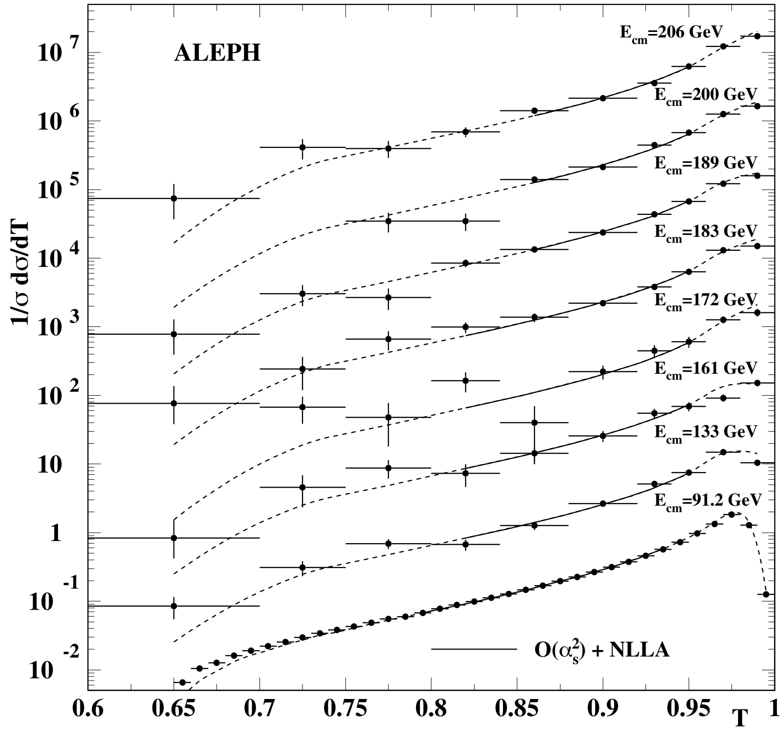
Bethke, hep-ex/0406058

e^+e^- event shapes

Current state of the art is

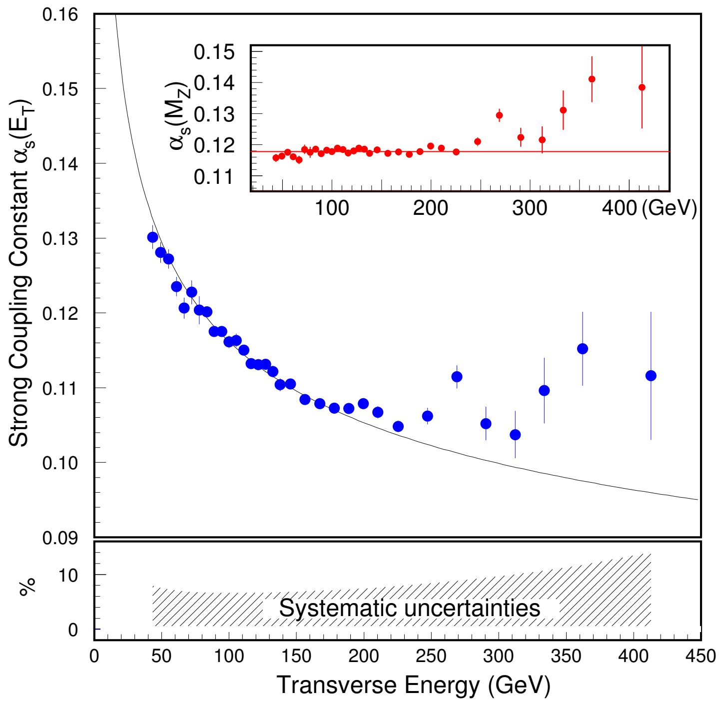
- ✓ NLO perturbation theory,
- ✓ more sophisticated NLL resummation
- ✓ better modelling of hadronisation corrections

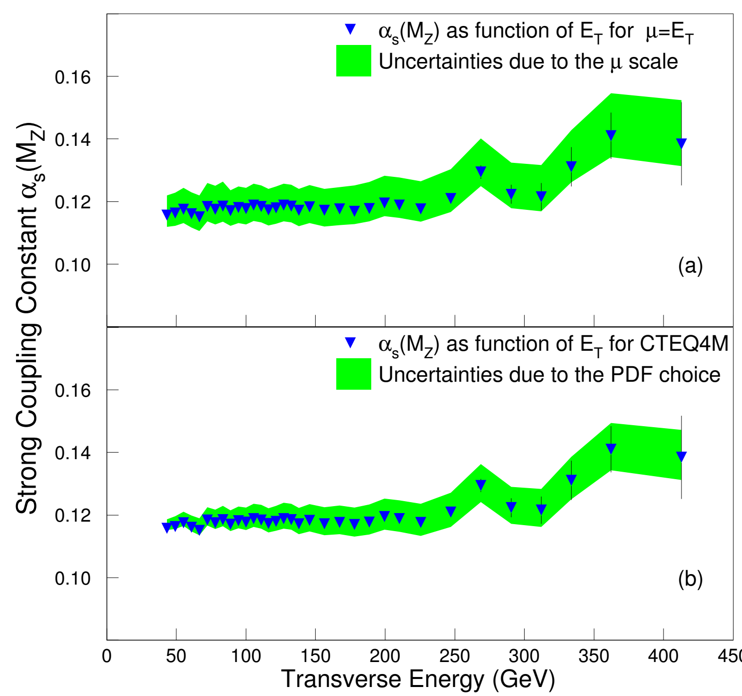




Why go beyond NLO?

In many cases, the uncertainty from the pdf's and from the choice of renormalisation scale give uncertainties that are as big or bigger than the experimental errors.
e.g. theoretical uncertainties in α_s extraction from $p\bar{p} \to$ jet are due to renormalisation scale and pdf's





$$\alpha_s(M_Z) = 0.1178 \ {}^{+6\%}_{-4\%}(scale) \ {}^{+5\%}_{-5\%}(pdf)$$

Why do we vary renormalisation scale?

- The theoretical prediction should be independent of μ_R
- The change due to varying the scale is formally higher order. If an observable $\mathcal{O}bs$ is known to order α_s^N then,

$$\frac{\partial}{\partial \ln(\mu_R^2)} \sum_0^N A_n(\mu_R)\alpha_s^n(\mu_R) = \mathcal{O}\left(\alpha_s^{N+1}\right).$$

- So the uncertainty due to varying the renormalisation scale is way of **guessing** the uncalculated higher order contribution.

Why do we vary renormalisation scale?

- . . . but the variation only produces copies of the lower order terms

$$\mathcal{O}bs = A_0\alpha_s(\mu_R) + \left(A_1 + b_0 A_0 \ln\left(\frac{\mu_R^2}{\mu_0^2}\right)\right)\alpha_s(\mu_R)^2$$

  A_1 will contain logarithms and constants that are not present in A_0 and therefore **cannot be predicted** by varying μ_R.
  For example, A_0 may contain infrared logarithms L up to L^2, while A_1 would contain these logarithms up to L^4.
- μ_R variation is **only an estimate** of higher order terms
- A large variation probably means that **predictable** higher order terms are large - but doesnt say anything about A_1.

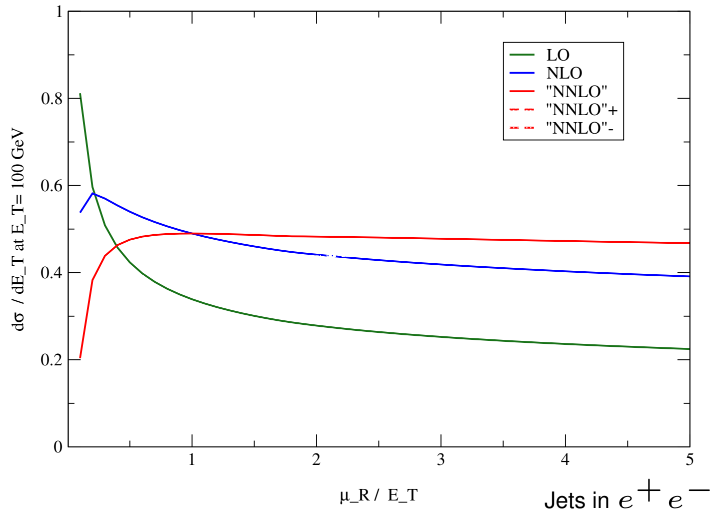
Renormalisation scale dependence

For example, $p\bar{p} \rightarrow$ jet, scale dependence

$$\begin{aligned} \frac{d\sigma}{dE_T} &= \alpha_s^2(\mu_R) A \\ &+ \alpha_s^3(\mu_R)\left(B + 2b_0 L A\right) \\ &+ \alpha_s^4(\mu_R)\left(C + 3b_0 L B + (3b_0^2 L^2 + 2b_1 L) A\right) \end{aligned}$$

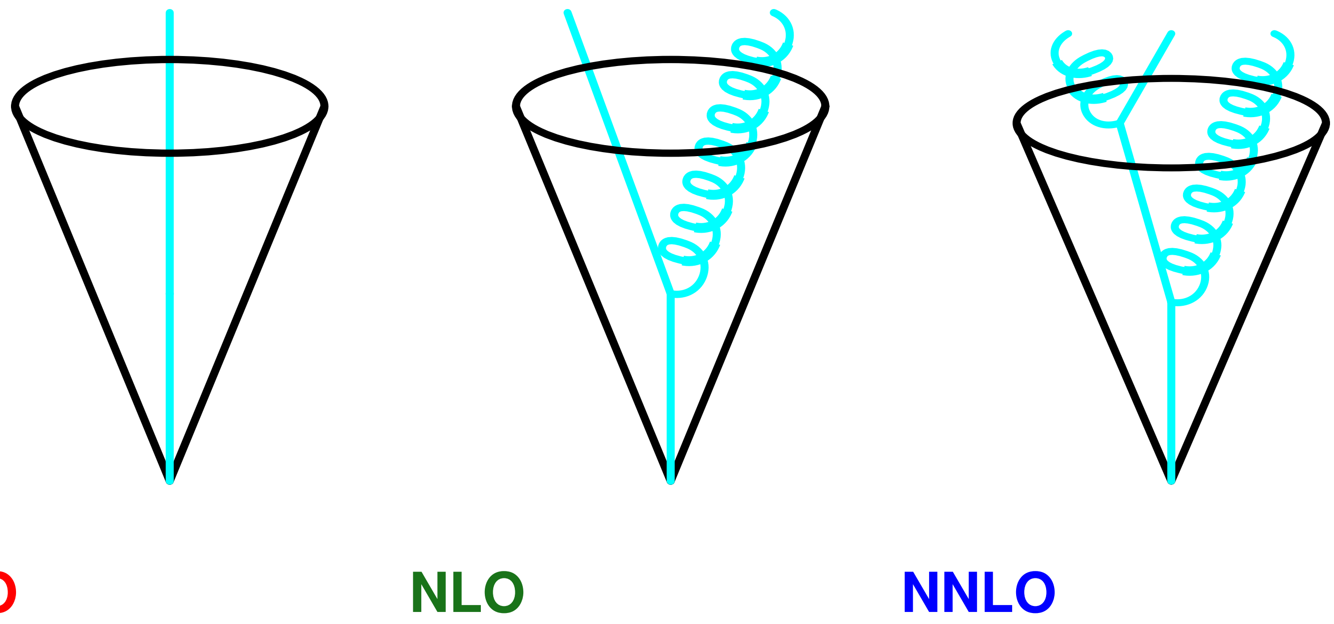
with $L = \log(\mu_R/E_T)$. The NNLO coefficient **C** is unknown.

The curves show guesses $C = 0$ (solid) and $C = \pm B^2/A$ (dashed).
Scale dependence is significantly reduced.



Jet algorithms

Also there is a mismatch between the number of hadrons and the number of partons in the event. At NLO at most two partons make a jet - while at NNLO three partons can combine to form the jet



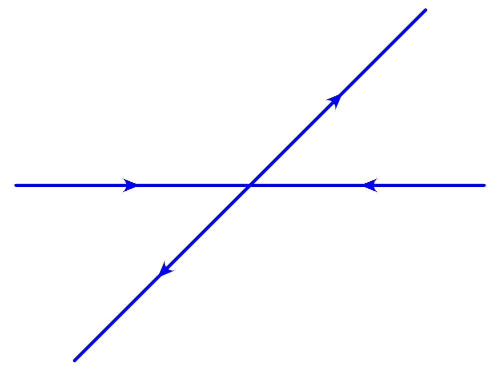
Perturbation theory starts to reconstruct the shower

$\Rightarrow$ better matching of jet algorithm between theory and experiment

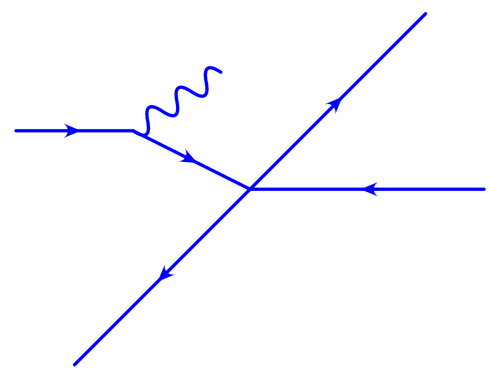
$\Rightarrow$ need for better jet algorithms

Description of the initial state

LO At lowest order final state has no transverse momentum

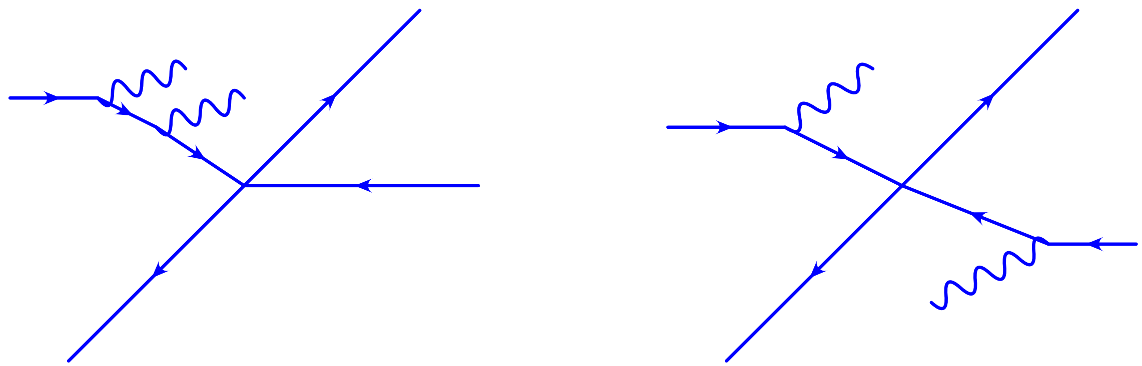


NLO Single hard radiation gives final state transverse momentum, even if no additional jet observed



Description of the initial state

NNLO Double radiation on one side or single radiation off each incoming particle gives more complicated transverse momentum to final state



Higher orders and power corrections

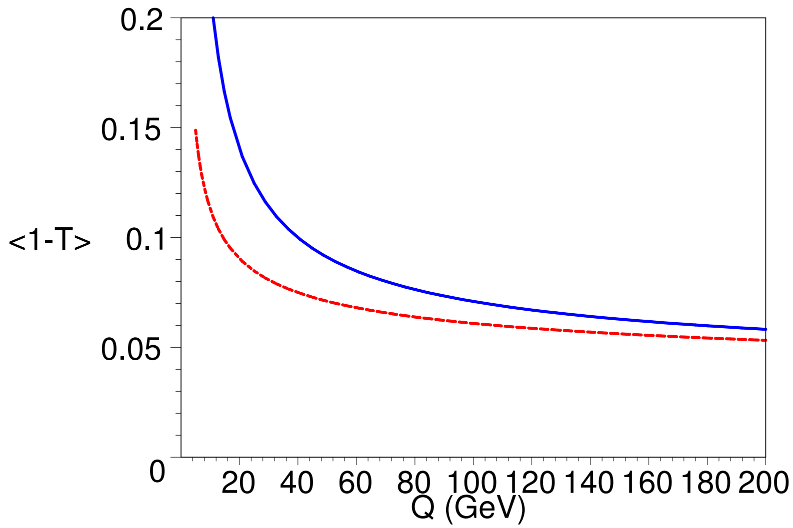
NLO Phenomenological power corrections match data with coefficient of $1/Q$ extracted from data.

$$\langle 1-T\rangle \sim 0.33\alpha_s + 1.0\alpha_s^2 + \frac{\lambda}{Q}$$

At NLO, $\lambda \sim 1$ GeV gives a good description of the data.

$\langle 1-T\rangle$ with **NLO** and no power correction and **NLO** with power correction $\lambda = 1$ GeV.

The power correction parameterises the unknown higher orders as well as the genuine non-perturbative correction



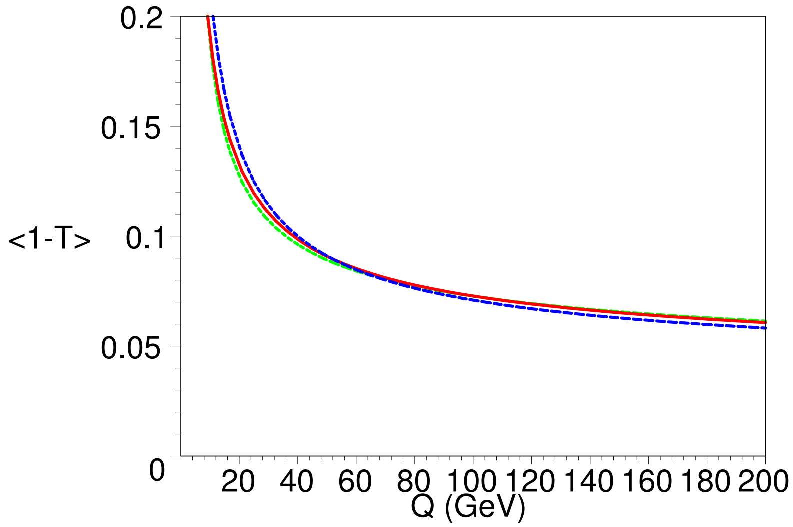
Higher orders and power corrections

NNLO Higher orders partially remove need for power correction

$$\langle 1-T\rangle \sim 0.33\alpha_s + 1.0\alpha_s^2 + A\alpha_s^3 + \frac{\lambda\ \mathrm{GeV}}{Q}$$

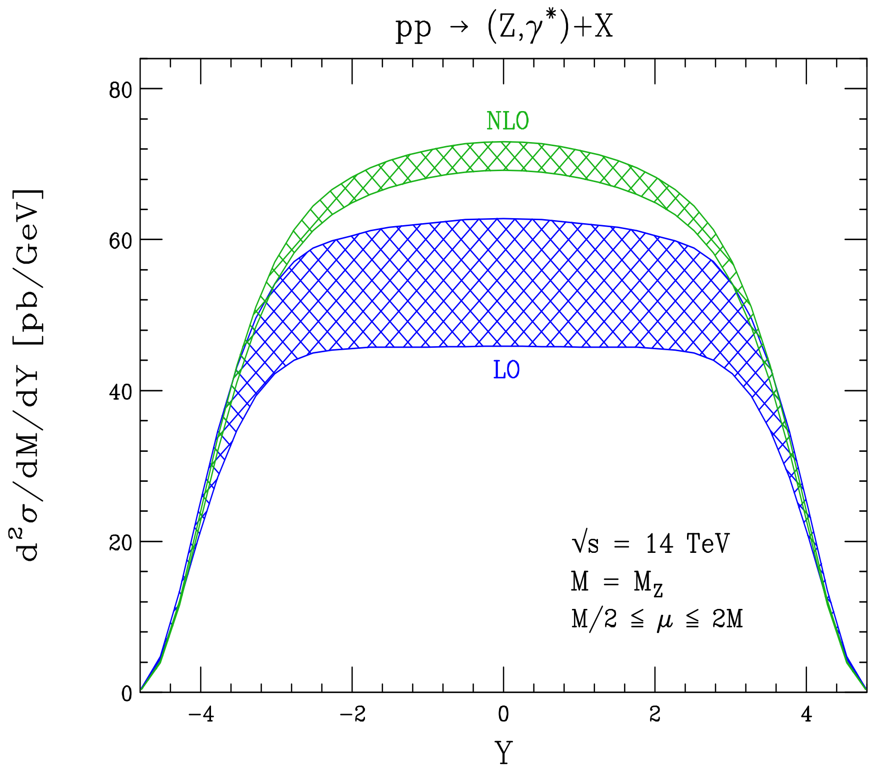
If we **guess** $A=3$, then $\lambda = 0.5$ GeV is good fit.

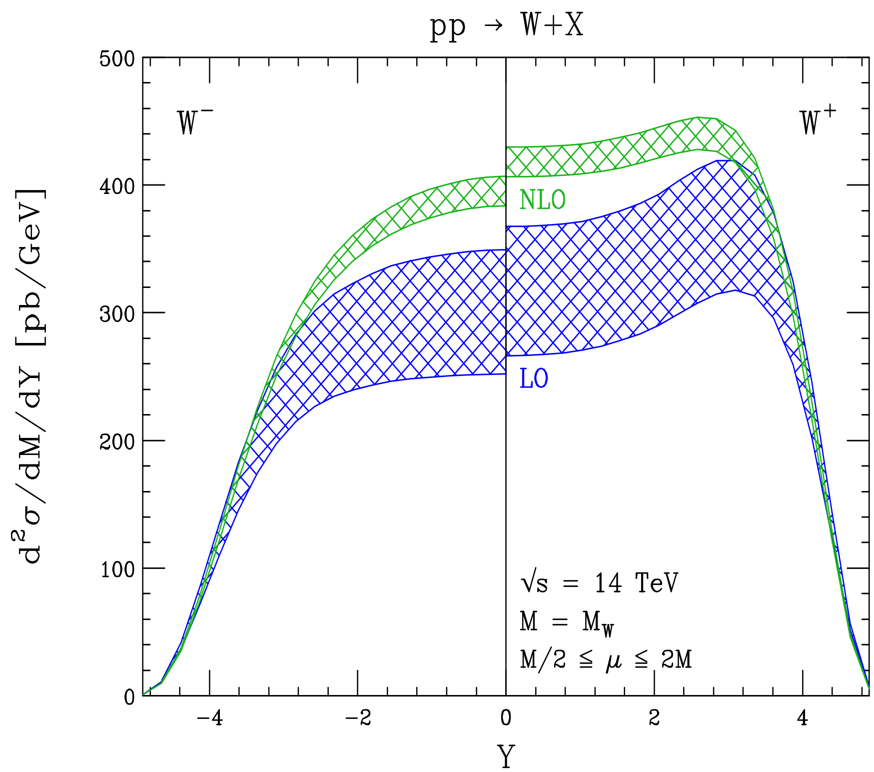
$\langle 1-T\rangle$ with **NLO** and $\lambda = 1$ GeV, **"NNLO"** with $\lambda = 0.5$ GeV and **"All orders"** with no power correction.



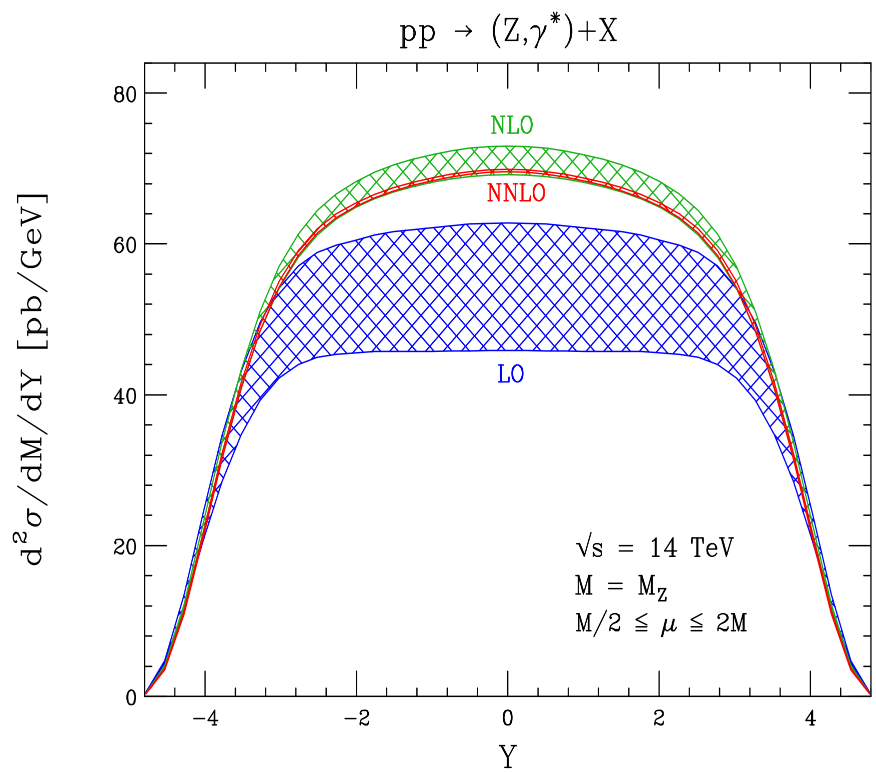
At present data not good enough to tell difference between $1/Q$ and $1/\log(Q/\Lambda)^3$.

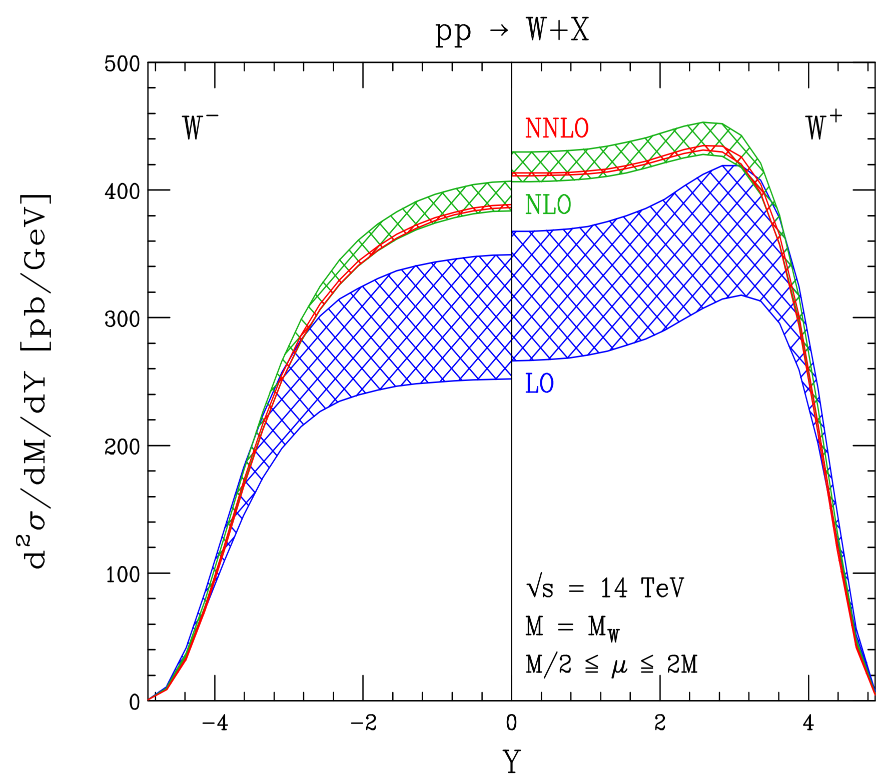
Gauge boson production at the LHC





Gauge boson production at the LHC





Gold-plated process

Anastasiou, Dixon, Melnikov, Petriello

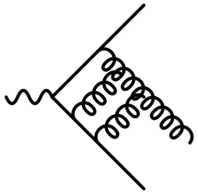
At LHC NNLO perturbative accuracy better than 1%

$\Rightarrow$ **use to determine parton-parton luminosities at the LHC**

Event shapes at NNLO

- Two-loop **matrix elements**

$|\mathcal{M}|^2_{\text{2-loop},3\text{ partons}}$

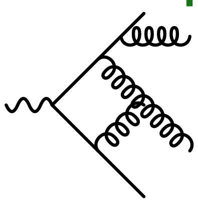


explicit infrared poles from loop integrals

Garland, Gehrmann, Glover, Koukoutsakis, Remiddi:
Moch, Uwer, Weinzierl

- One-loop **matrix elements**

$|\mathcal{M}|^2_{\text{1-loop},4\text{ partons}}$

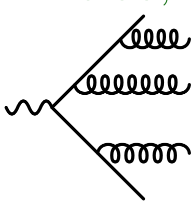


explicit infrared poles from loop integral and implicit infrared poles due to single unresolved radiation

Bern, Dixon, Kosower, Weinzierl;
Campbell, Miller, Glover

- Tree level **matrix elements**

$|\mathcal{M}|^2_{\text{tree},5\text{ partons}}$



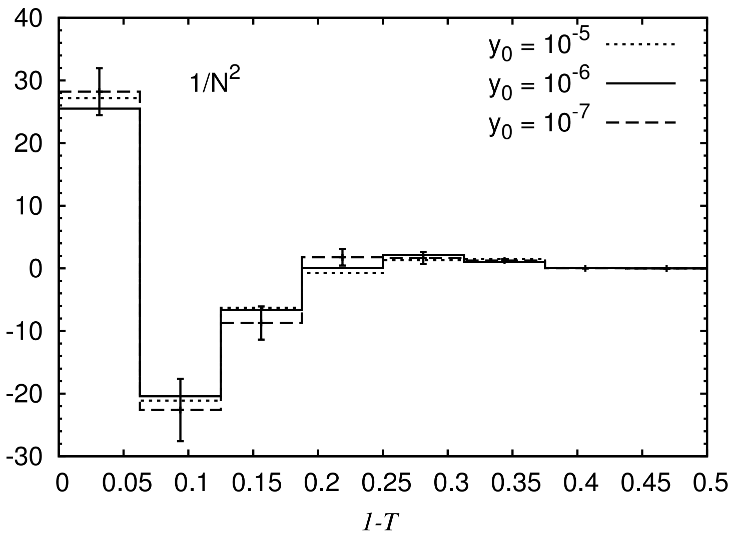
implicit infrared poles due to double unresolved radiation

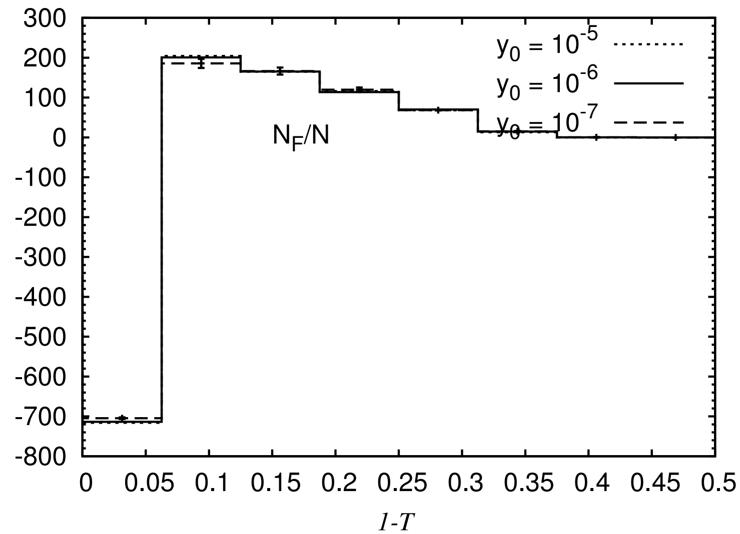
Hagiwara, Zeppenfeld;Berends, Giele, Kuijf

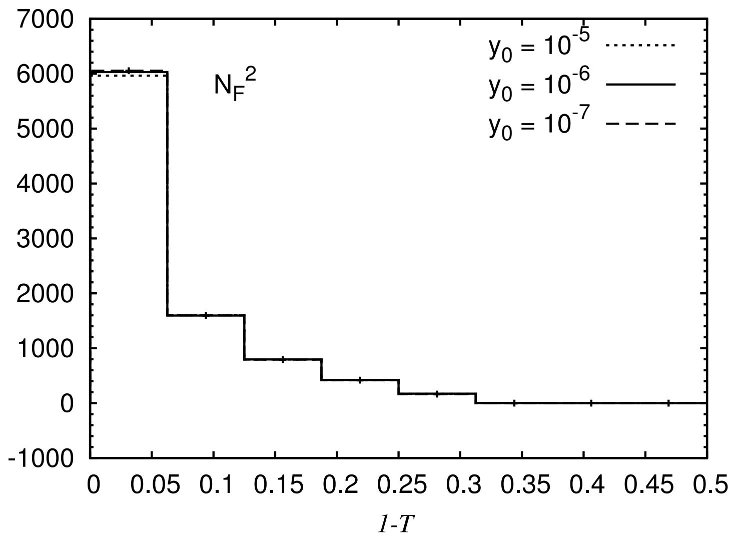
Infrared Poles cancel in the sum

QED-type contributions to $e^+e^- \to 3$ jets

Gehrmann-De Ridder, Gehrmann, Glover, Heinrich







- independent of phase space cut y_0
- CPU time about 1 day on 2.8 GHz Athlon

Summary

QCD studies at LEP have led to a significant increase of knowledge about hadron production and the dynamics of quarks and gluons at high energies.

These studies demonstrate QCD as a consistent theory which accurately describes the phenomenology of the Strong Interaction

Future developments in this field are within reach: NNLO QCD calculations and predictions for jet and event shape observables will soon be available; they will initiate further analyses of the LEP data which will provide even more accurate and more detailed determinations of α_s

Bethke, hep-ex/0406058