Introduction to Perturbative QCD
—Foundation and Simple Applications

CTEQ Summer School, 2006

Wu-Ki Tung

Outline

I : Basic Ideas

- What is Quantum Chromodynamics (QCD)?
- Why do we believe in Quarks and Gluons?
  - Long-distance physics: the constituent QM;
  - Short-distance physics: the Parton Picture.

  QCD provides the foundation for both.
- How can a Strong-interaction theory, QCD, give rise to the simple Parton Picture?
  Separation of distance scales (Factorization):
  - Ultra-violet Renormalization and Asymptotic Freedom; (very short distance)
  - Coliner and Soft Singularities, Infra-red Safety & Factorization Thms. ("short"/"long" distances).

Outline

II: PQCD at Work: e^+e^- Annihilation

- Order α_s^0 (LO) process and the Parton Model;
- Order α_s^1 perturbative calculation (NLO QCD correction):
  - Colinear and Soft Singularities;
  - Infra-red Safe (IRS) Physical Observables;
  - Factorization of (non-IRS) Physical Observables into IRS (short-distance, calculable) and Universal (long-distance) pieces.
- General Statement of the Factorization Theorem and its physical interpretation.

Outline

III: PQCD at Work: Deep Inelastic Scattering

- Cross Sections and Structure Functions;
- Order α_s^0 (LO) processes and the Parton Model;
  - Parton Distributions; Sum Rules.
- Order α_s^1 (NLO) QCD corrections:
  A Simple Illustrative Example:
  - Physical origin of the Colinear Singularity—relation to *mass singularities*;
  - The Structure Function as a convolution of a (IRS) Wilson Coefficient and a (universal) Parton Distribution Function.
- Factorization at NLO.

IV: General Formalism of PQCD

- Factorization Theorem to all orders of the perturbative expansion (with non-zero quark masses);
- The importance of Scales—Factorization and Renormalization;
- General definition of Parton Distribution Functions (PDF);
- The Three Faces of the Magical Factorization Master Formula;
- Scale dependence of PDFs and QCD Evolution;
- Scale dependence (and independence) of Physical Predictions.

Outline

V: PQCD and Collider Physics

- Physical xSections and Partonic Interactions:
  - Probing the Standard Model and New Physics;
- Q-$\bar{Q}$ Annihilation and the Drell-Yan process:
  - Lepton-pair prod., $W/Z/\gamma$ prod., … etc.
- Gluon dominated process:
  - Direct Photon Production
  - Jet Cross Section: IRS and Jet Definition
- Heavy Quark Production
- PQCD and Top and Higgs Physics
- PQCD and Beyond the SM Physics.

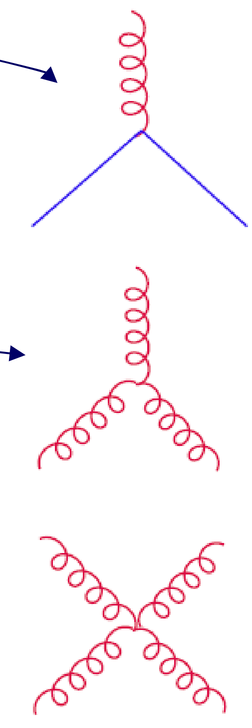
Factorization and
Global QCD Analysis of Parton Distribution Functions

Basic Elements of Quantum Chromodynamics (QCD)
– a Non-abelian Gauge Field Theory with SU(3) color Gauge Symmetry

Fields: Quarks ψ^{color}_{flavor} and Gluon $G^{color}(A{\cdot}T, g)$

$$G_{\mu\nu}\cdot t = (\partial_\mu A_\nu - \partial_\nu A_\mu)\cdot t - i\,g\,[A_\mu\cdot t,\, A_\nu\cdot t]$$

Interaction vertices



Basic Lagrangian:

$$\mathcal{L}_{class} = \bar{\psi}(i\,\not{\partial} - g\,\not{A}\cdot t - m)\psi - \frac{1}{4}Tr\,G_{\mu\nu}G^{\mu\nu}$$

- g: gauge Coupling Strength
- m_i: quark masses
- t & T: color SU(3) matrices in the fundamental and adjoint representations.

Group factors:

$$C_F\ \left(=\tfrac{4}{3}\right),\ C_A\ (= N_c = 3),\ \text{and}\ T_R\ \left(=\tfrac{1}{2}\right)$$

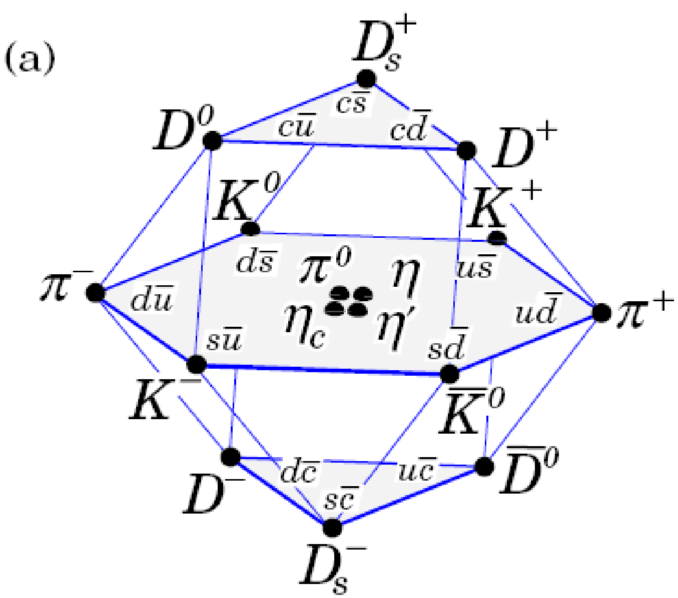
Experimental Foundation of QCD I
Long Distance Physics: Hadron Spectroscopy and the Constituent Quark Model (1960's)

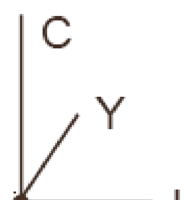
Quantum #'s of Mesons given by: L=0,1,2

$SU(3)_{flv}$ Octets (nonets) of q-qbar bound states.

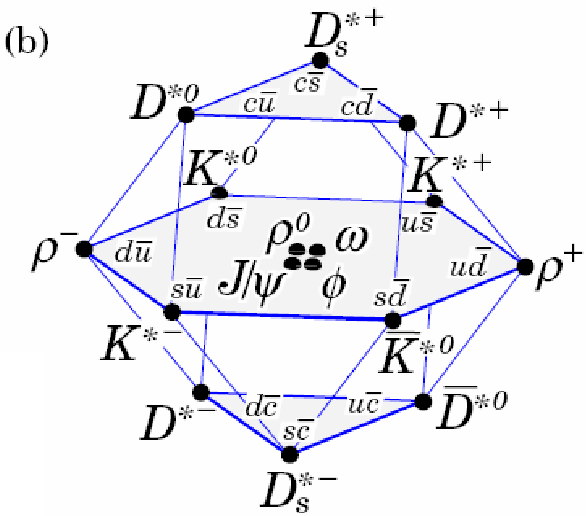
Addition of Charm Q.N. $\Rightarrow$ $SU(4)$

Pseudo-scalar mesons





Vector mesons



cf. PDG

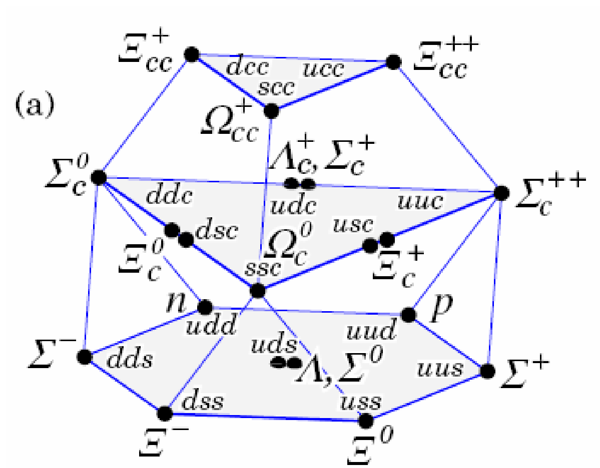
Experimental Foundation of QCD I
Long Distance Physics: Hadron Spectroscopy and the Constituent Quark Model (1960's)

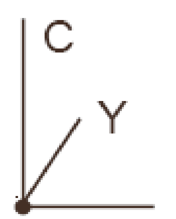
Combining of $SU(3)_{flavor}$ & $SU(2)_{spin}$ $\Rightarrow SU(6)$

Quantum #'s of Baryons given by: L=0,1,2

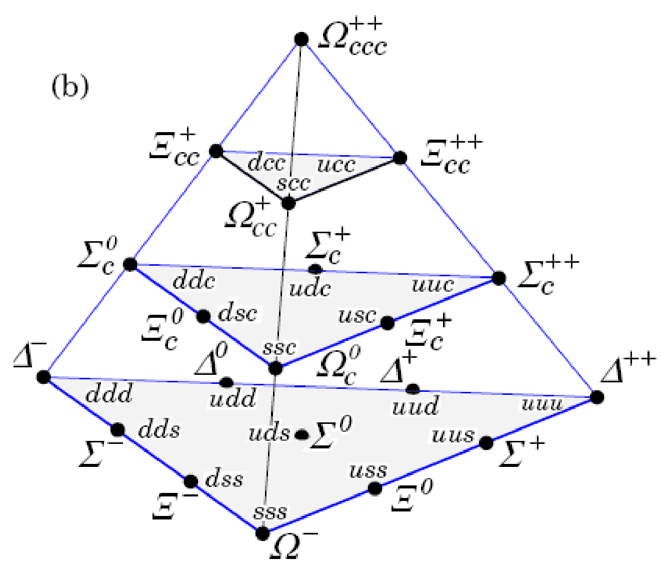
$SU(3)_{flv}$ Octets & decuplets of q-q-q bound states.

"Octet" Baryons





"Decuplet" baryons



Experimental Foundation of QCD II
Short Distance Physics: Deep Inelastic Scattering, e^+e^- Annihilation, and the Parton Model (~1969-72)

Evidences for the existence of Partons:

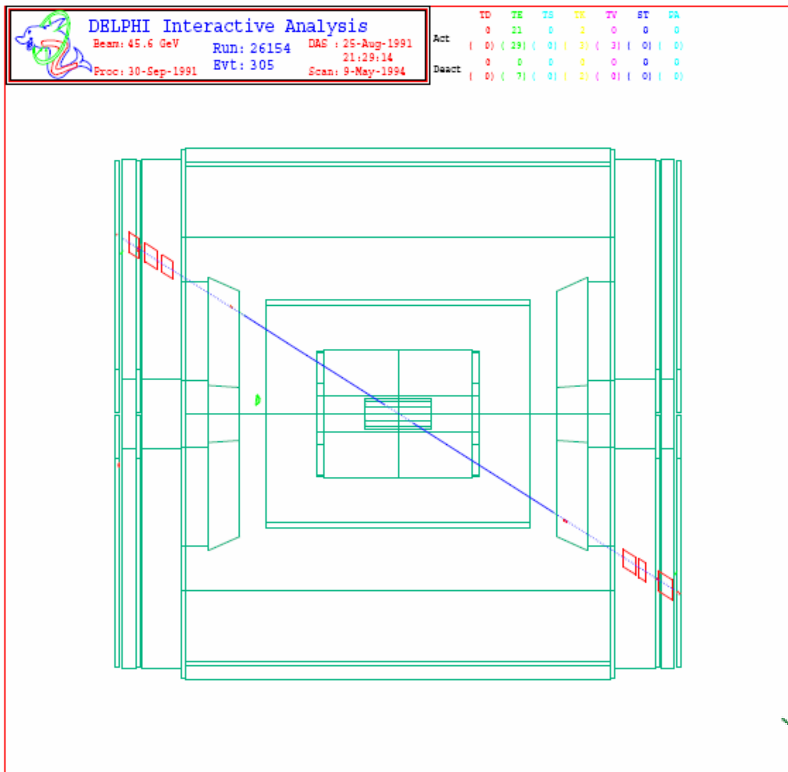
"direct": Most Hard Sc. events contain visible "jets" ⇒ fragments of underlying partons?

Are they point-like? "Rutherford expts"

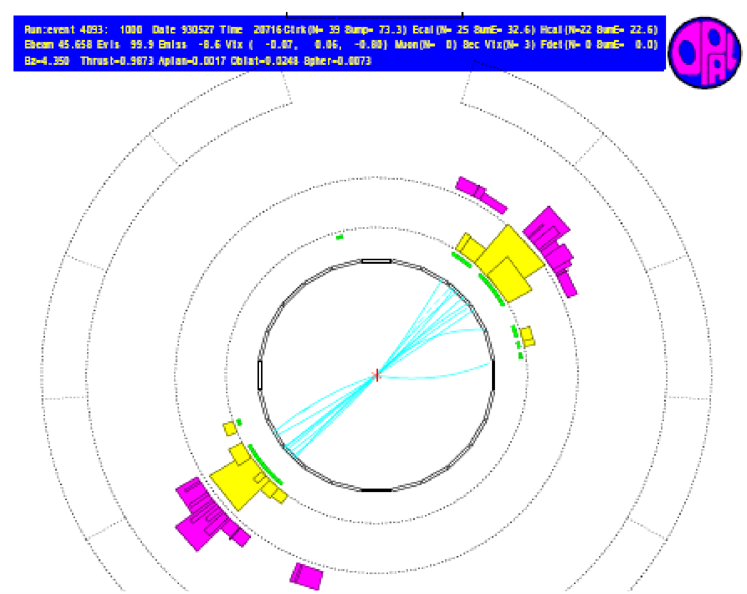
* (Bjorken) Scaling in DIS;
* annihilation into hadrons;
* Hadron-hadron scattering, ...

Two-jet Events in e^+e^- Annihilation
— Evidence for Quark - Anti-quark Production

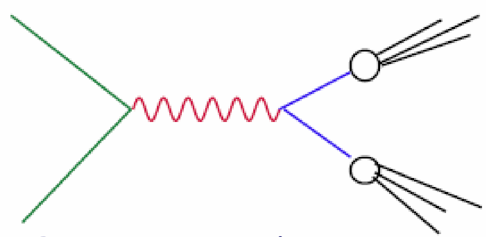
An elementary particle event
$$- e^+ e^- \longrightarrow \mu^+ \mu^-$$



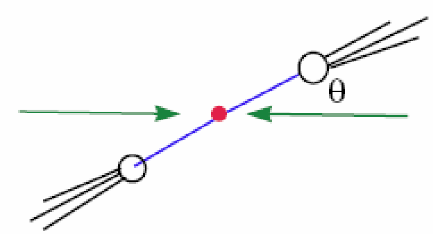
A typical event in
$e+e- \rightarrow$ hadron final state



Parton process underlying 2-jet events



Feynman diagram

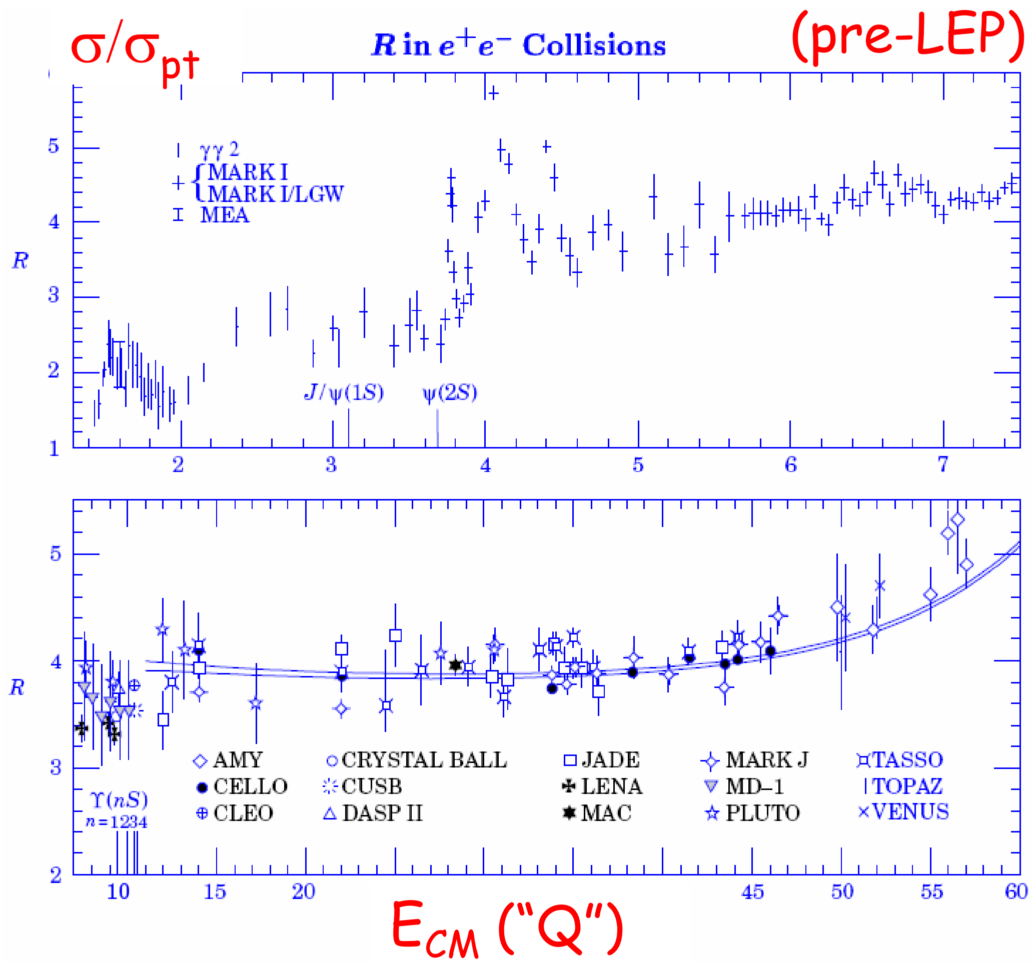


CM configuration

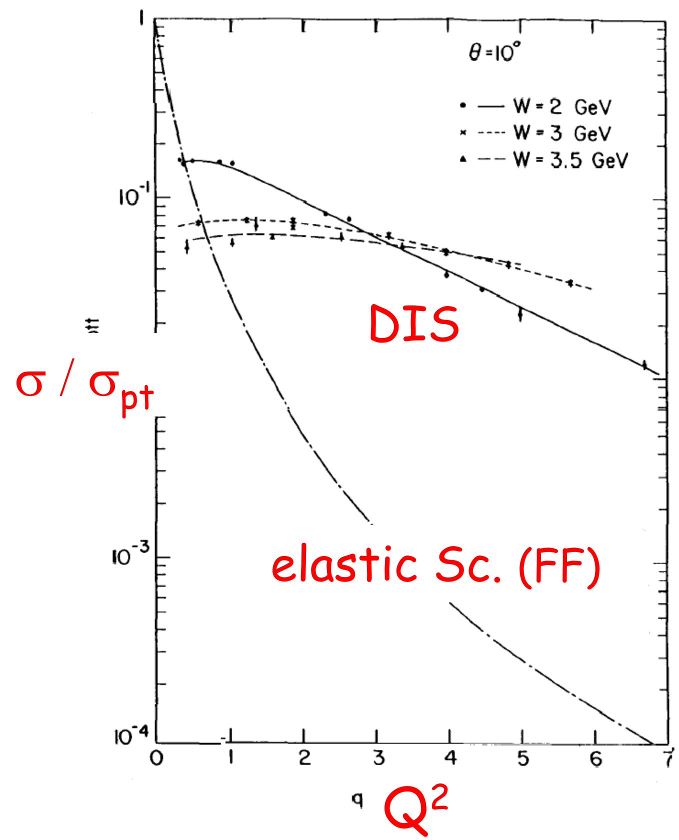
Partons are Point-like

Modern day "Rutherford Scattering":

In high energy inclusive e^+e^- annihilation and DIS, cross sections are *hard* (no form-factor like drop off for large Q)



First SLAC results on DIS (~ 1969)



Experimental Foundation of QCD II
Short Distance Physics: Deep Inelastic Scattering, e^+e^- Annihilation, and the Parton Model (~1969-72)

Properties of Partons:

2-Jet angular distributions in e^+e^- , DIS, DY proc. are the same as for scattering into leptons ⇒ underlying partons are fermionic

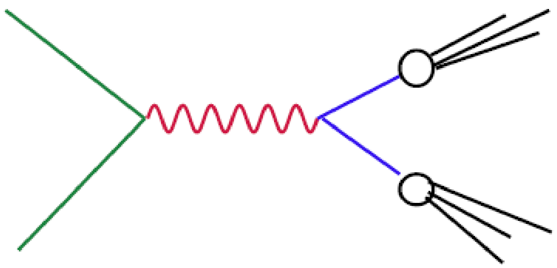
Expts. : EM & Weak Isospin couplings of partons = those of leptons ⇒ "Current Quarks"

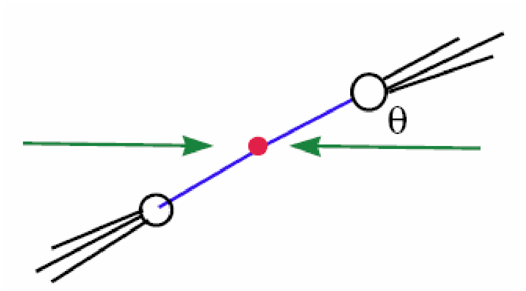
3-jet events and their detailed properties prove the existence, and spin of gluons

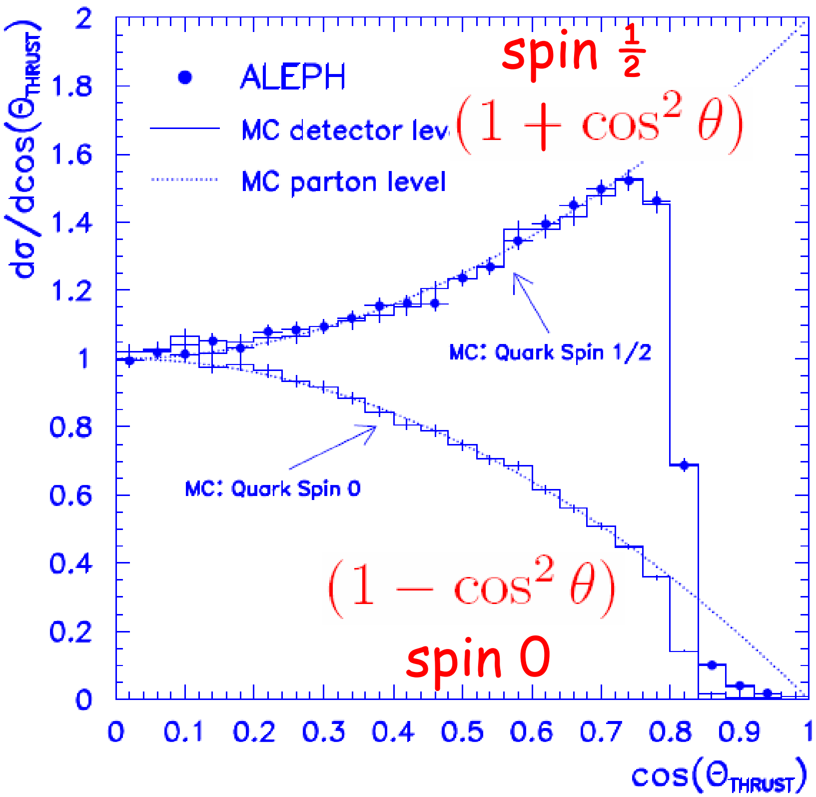
⇒ QCD-parton Model complete.

Measuring the Spin of the Quarks

- Use the angular distribution of 2-jet events





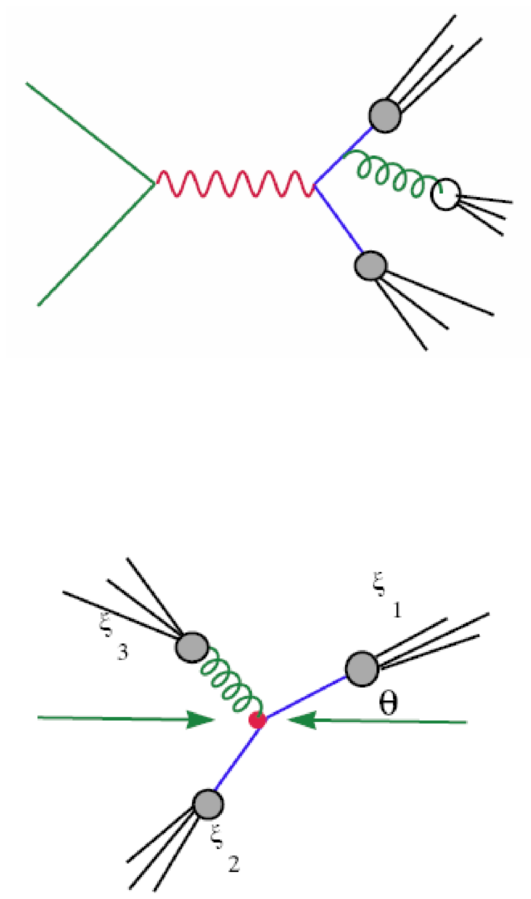


- In DIS: $F_L \sim 0$ (Callan-Gross) $\Leftrightarrow$ $\sigma \sim (1 + \cosh^2\psi)$ (cf. later)

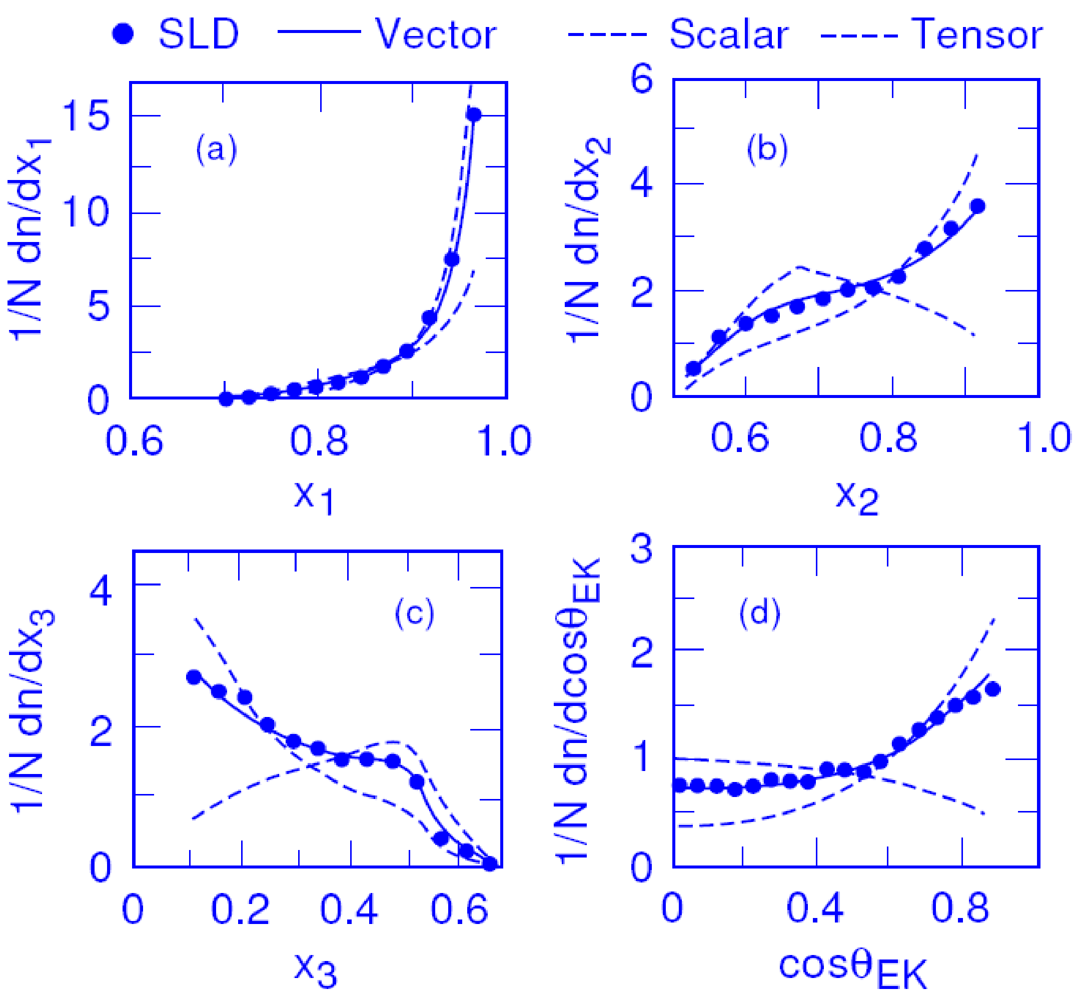
Quarks are spin ½ fermions

Measuring the Spin of the Gluon

Use the angular distributions of 3-jet events



Experimental result from SLD:



Very important question:
(central to PQCD, and to this course)

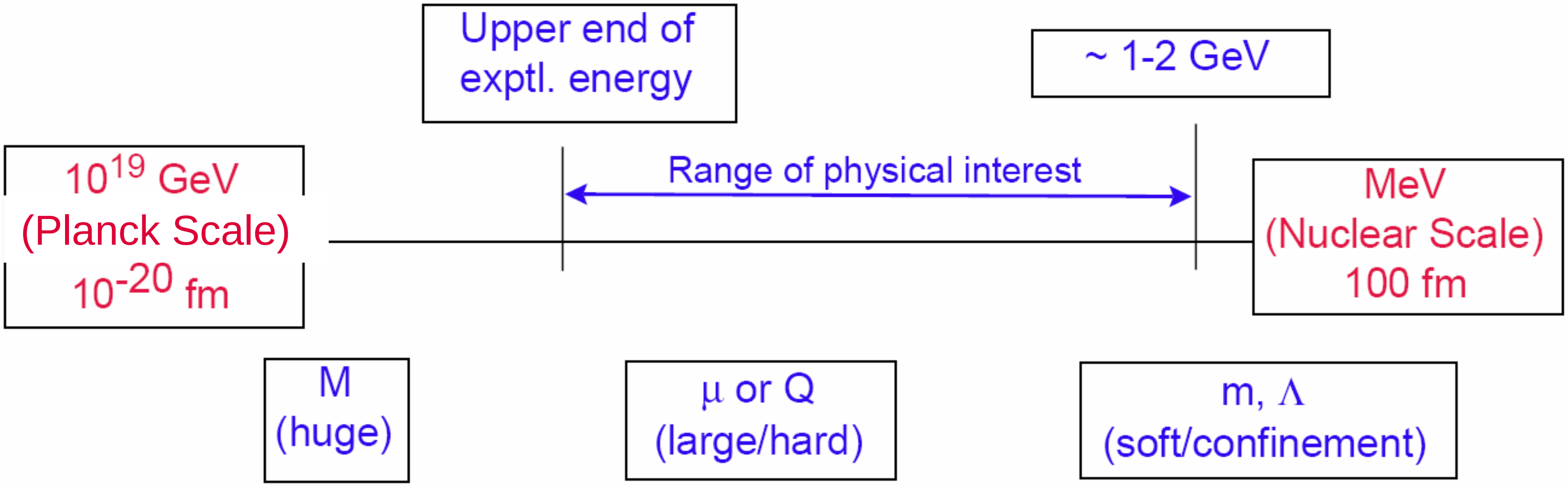
How could the simple parton picture (with almost non-interacting partons) possibly hold in QCD (—a strongly interacting quantum gauge field theory)?

Answer: 3 distinctive Features of QCD

- Asymptotic Freedom:
  A strongly interacting theory at long-distances (even confining) can become weakly interacting at short distances (due to scale dependence implied by the RGE).
- Infra-red Safety:
  There are classes of "infra-red safe" (IRS) quantities which are independent of long-distance physics, hence are calculable in PQCD.
- Factorization:
  There are an even wider class of physical quantities (inclusive cross sections) which can be *factorized* into long distance components (not calculable, but universal) & short-distance components (process-dependent, but infra-red safe, hence calculable).

The bulk of this course is devoted to exploring the ideas behind these features of QCD.

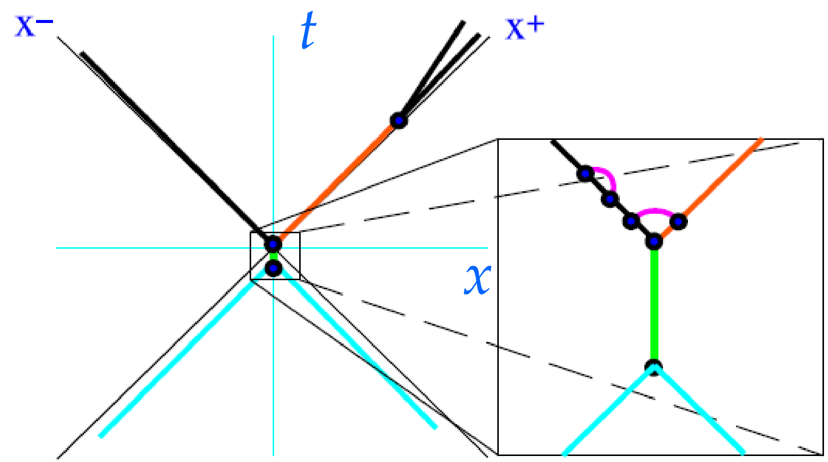
The importance of ***Scales*** -- Renormalization and Factorization



Ultra-violet Renormalization and Asymptotic Freedom
—the smallest time and shortest distances

What does renormalization do?

Say, MS renormalization introduces a *ren. scale* μ_R . In principle, μ_R is arbitrary; in practice, μ_R is chosen ~ a physical scale *Q, or* $\sqrt{s}$.



⋆ Physics of scales $|t| \ll 1/\mu$ removed from perturbative calculation; renormalization hides:

- the ugly: ultra-violet divergences; and
- the beautiful: short-distance physics $< \frac{1}{\sqrt{s}}$
  (*New Physics*: Q. Gravity, GUT, Super-xx, ...)

⋆ For QCD, $\alpha_s(\mu)$ decreases as μ increases — *Asymptotic freedom*.

Renormalization Group and the Running Coupling

- The μ dependence of $\alpha(\mu)$ is controlled by the *renormalization group equation* :

$$\frac{d\ \alpha_s(\mu)}{\pi\, d\ln(\mu^2)} = -\beta(\alpha_s(\mu)) = -\beta_0 \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 - \beta_1 \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 + \cdots .$$

- Solution of the RGE to 1-loop order sums leading quantum fluctuations to all orders of the fixed-coupling perturbative expansion.

$$\begin{aligned}\alpha_s(\mu) &\approx \alpha_s(M) - \ln(\frac{\mu^2}{M^2})\ \alpha_s^2(M) + (\frac{\beta_0}{\pi})^2 \ln(\frac{\mu^2}{M^2})\alpha_s^3(M) \\ &= \frac{\alpha_s(M)}{1 + \frac{\beta_0}{\pi}\alpha_s(M)\ln(\frac{\mu^2}{M^2})} = \frac{\pi}{\beta_0 \ln(\frac{\mu^2}{\Lambda^2})}.\end{aligned}$$

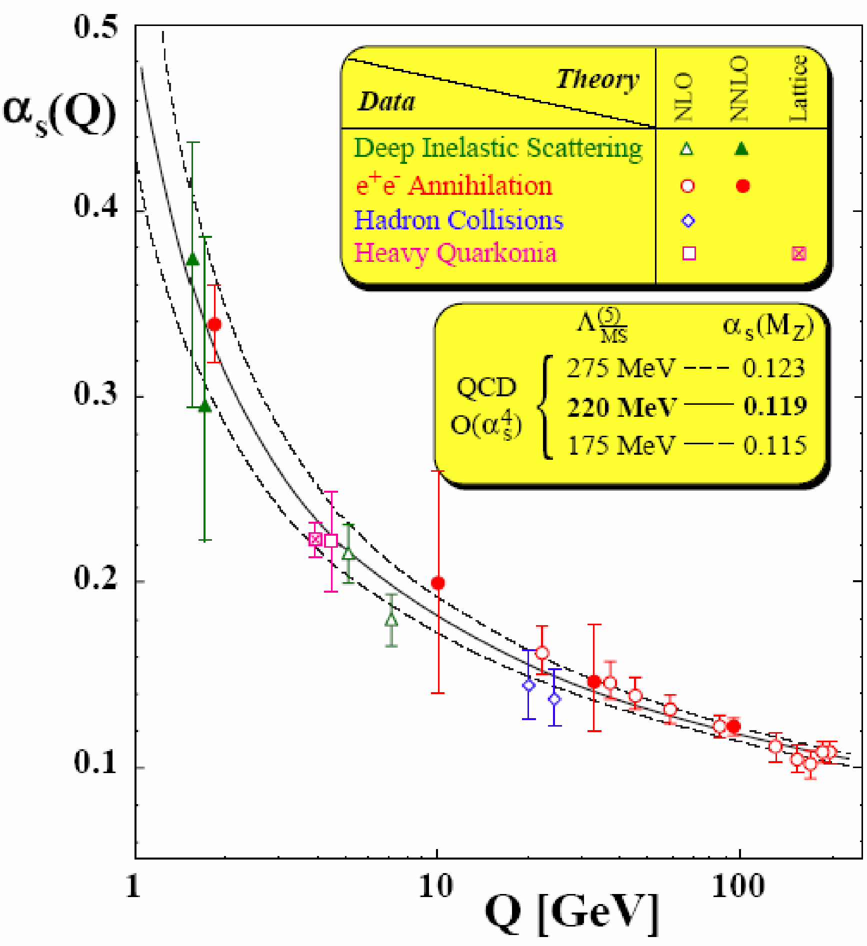
$\alpha_s(M)$, or Λ, is a parameter in the solution.

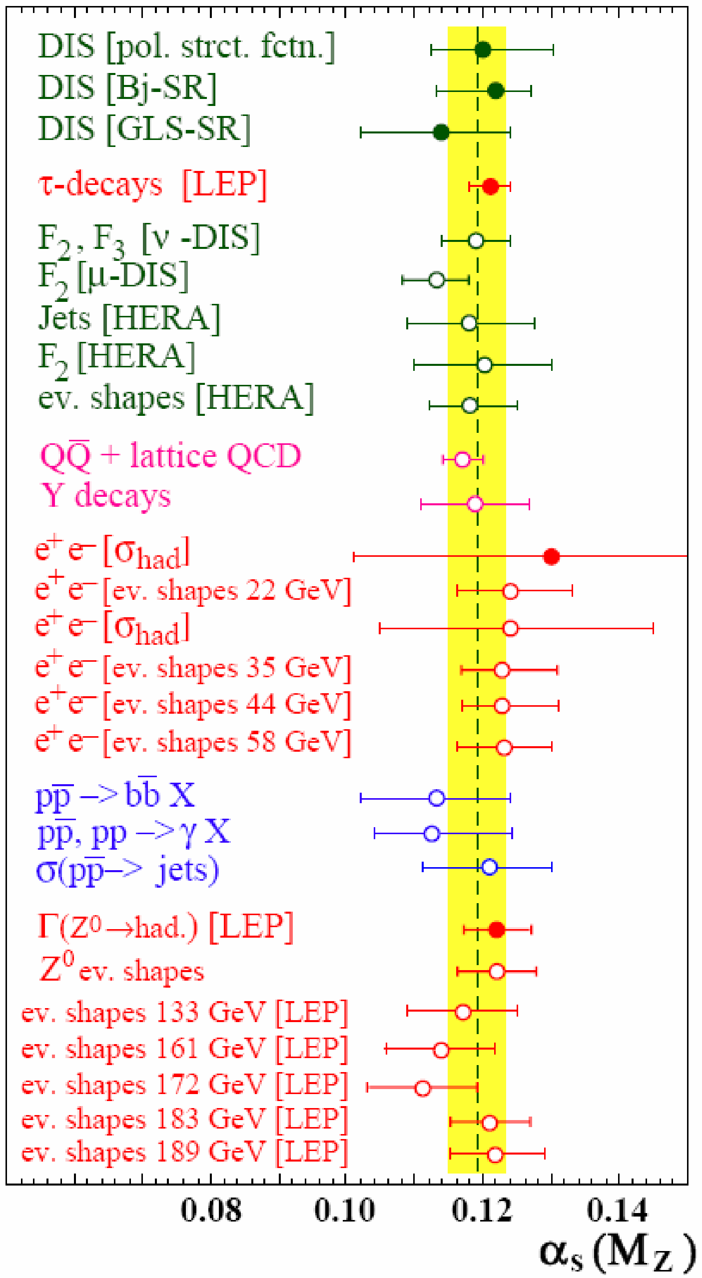
- $\beta > 0 \Rightarrow \alpha(\mu)$ decreases as μ increases—QCD is asymptotically free.

Asymptotic Freedom

Universal (running) coupling:

$$\alpha_s(Q) = \frac{g^2}{4\pi} = \frac{b}{\ln(Q/\Lambda)}(1+\ldots)$$





- How is $\alpha_s(\mu)$ measured in the variety of hadronic processes listed in the previous slide?
- In general, how can one relate PQCD calculations (on leptons, quarks and gluons) to physical observables measured in the lepton-hadron world?

Answer: (i) IRS; and (ii) Factorization …

II: PQCD at Work: e^+e^- Annihilation

- Order α_s^0 (LO) process and the Parton Model;
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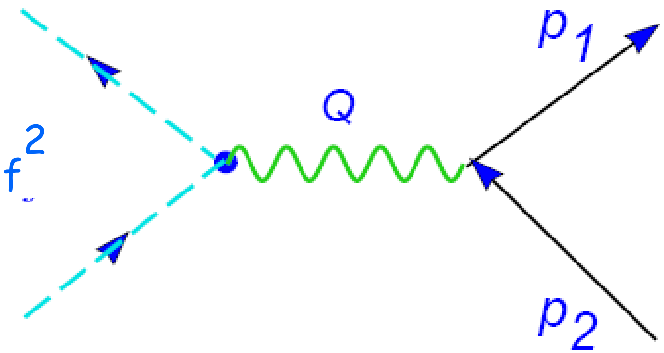
e^+e^- Annihilation into Hadrons: Leading Order in PQCD

Total cross section,
normalized to point-like cross section:

$$R = \frac{\sigma_T^{e^+e^- \to \mathrm{hadrons}}}{\sigma^T_{e^+e^- \to \mu^+\mu^-}}(E) = \sum_f q_f^2$$



Angular distribution:

$$\frac{d\sigma}{d\cos\theta} \propto (1 - \cos^2\theta)$$

e^+e^- Annihilation into Hadrons: Next to Leading Order (NLO) in PQCD

Kinematics:

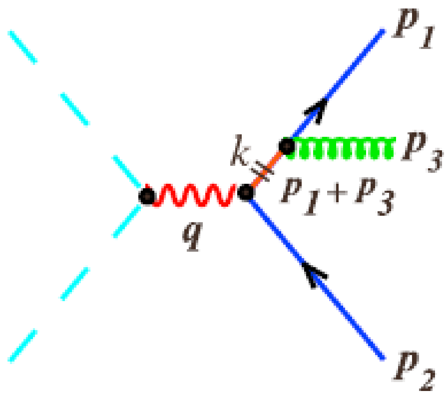
$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2q \cdot p_i}{s}$$

$$i = 1, 2, 3$$

$$\sum x_i = \frac{2q \cdot \sum p_i}{s} = 2$$

$$2(1 - x_1) = x_2 x_3 (1 - cos\theta_{23}), \quad cycl.$$

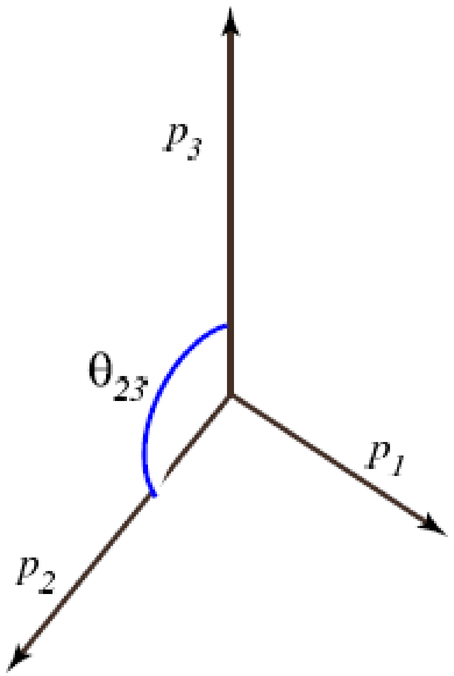
Tree diagram
(real gluon emission)

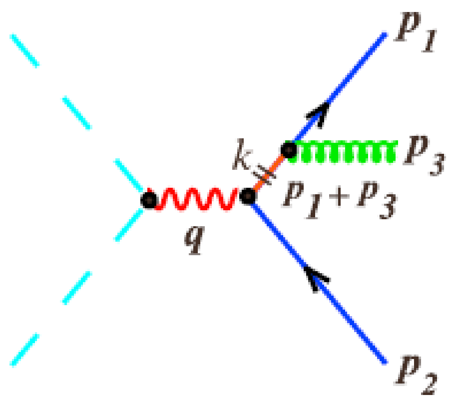


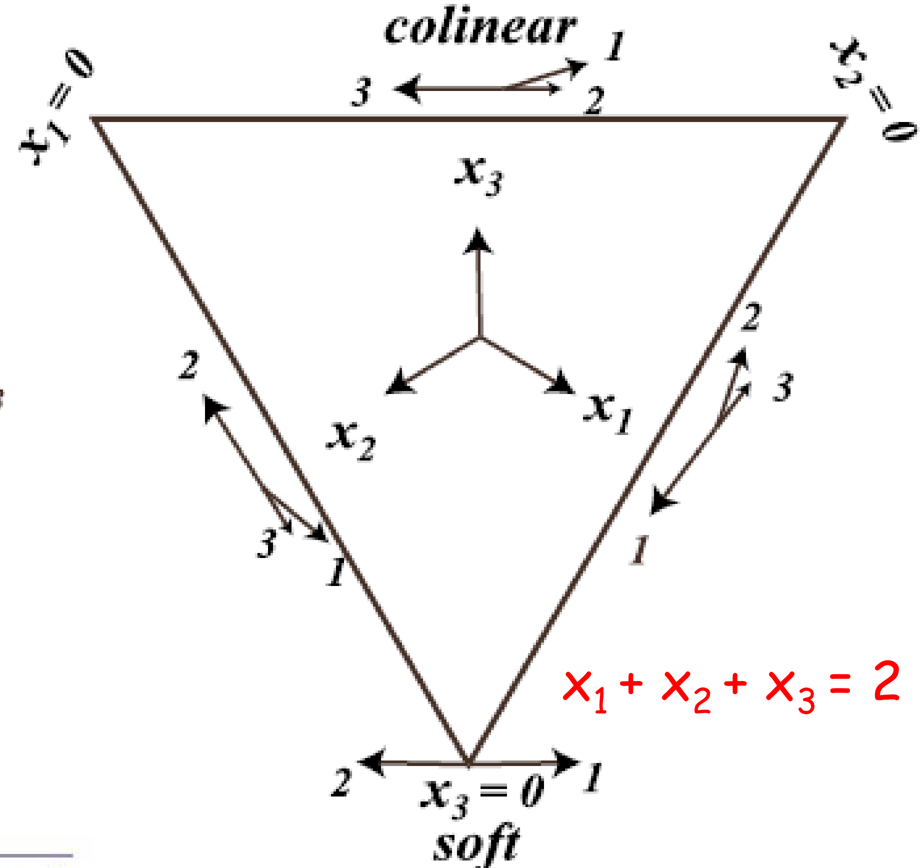
Differential cross section at the parton level:

$$\frac{d\sigma}{\sigma_0\, dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_1^2}{(1 - x_1)(1 - x_2)}$$

Colinear and Soft Singularities







$x_1 + x_2 + x_3 = 2$

$$\frac{d\sigma}{\sigma_0\, dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_1^2}{(1-x_1)(1-x_2)}$$

diverges when:

$x_i \rightarrow 1$ (colinear)

$x_i \rightarrow 0$ (soft)

In both configurations, the virtual propagator line goes on mass shell: $k^2 \rightarrow 0$

Moral: Singularities occur at boundaries of phase space (colinear/soft) where 2-->3 kinematics collapses to 2-->2 and *the 4-mom. k of the internal line goes on-mass shell.*

In general (and for theory students):

(These singularities correspond to solutions to the Landau equations for pinch surfaces of the Feynman diagrams.)

cf. TASI lecture notes of Sterman

Separation of Short- and Long-distance Interactions

Space-time Picture

Null Plane coordinates:

$$k^{\pm} = \frac{k^0 \pm k^3}{\sqrt{2}} \; ; \; k^2 = 2k^+k^- - \vec{k}_T^{\,2}$$

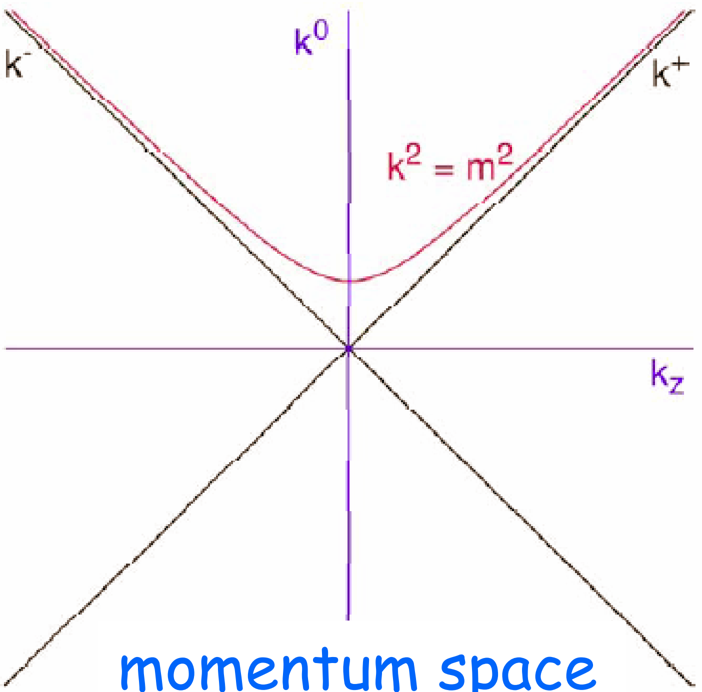
Space-time connection:

$$\int d^4x\, e^{i x\cdot k} \cdots$$

$$x \cdot k = x^- k^+ + x^+ k^- - \vec{x}_T \cdot \vec{k}_T$$

On mass shell:

$$k^- = \frac{\vec{k}_T^{\,2} + m^2}{2k^+}$$

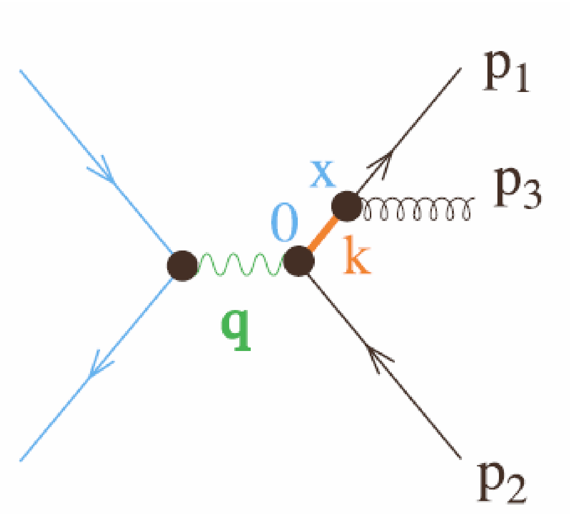


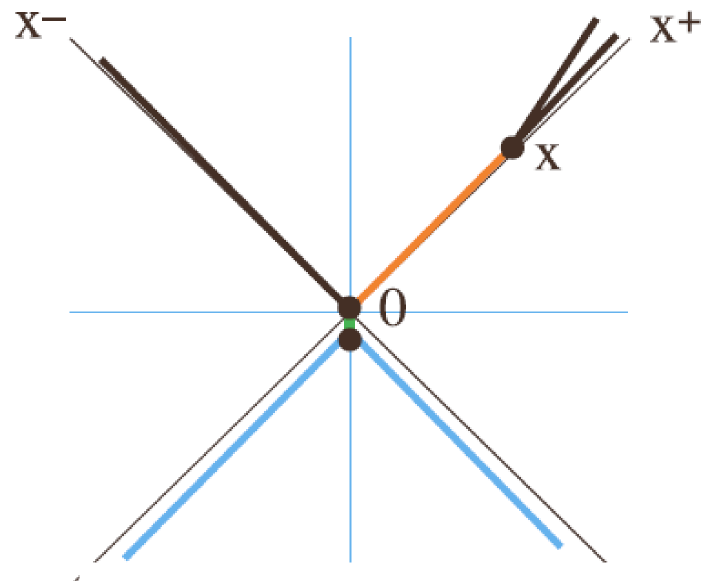
momentum space

High-energy interaction:

$$k^+ \to \infty \quad \Rightarrow \quad k^- \to 0 \quad \Rightarrow \quad x^+ \to \infty$$

Correspondence between singularities in momentum space and the development of the system in space-time:





Space-time picture

Consider the fourier transform.

$$S_F(k) = \int dx^+ dx^- d\boldsymbol{x} \exp(i[k^+ x^- + k^- x^+ - \boldsymbol{k}\cdot\boldsymbol{x}])\ S_F(x).$$

Contributing values of x have small x^- large x^+.

Moral: Singularities associated with divergent perturbative X-sec <--> interactions a long time after the creation of the initial quark-antiquark pair.

Question:

What to do with the long-distance physics associated with these colinear/soft singularities?

Infra-red Safe Physical Observables

IRS observables are those that are insensitive to the colinear and soft singularities of the perturbative calculation, hence the long distance behavior of QCD.

Example: the total cross section $\sigma_T(e^+e^- \to \text{hadrons})$

$$\sigma_{tot}(s) = \sigma_0(s)[1 + \alpha_s(s)\, c_1 + \ldots]$$

$$\mathit{Block-Nordsieck\ Thm}\ (unitarity) \rightarrow c_{1,2,\ldots}\ \mathit{are\ finite,\ i.e.\ IRS}$$

Order α_s:

Cancellation of the colinear/soft singularities between real and virtual diagrams

Once the quark-antiquark pair is produced (at short distance), the probability for them to turn into some hadron state is unity, independent of the long-distance behavior of the theory.

Other Examples of IRS Observables

- Sterman-Weinberg Jet cross section and its modern variants
  - Jade, Cone, k_T, … cf. lecture on jets
- Shape Variables
  - Thrust, sphericity, aplanarity, oblateness, …
    cf. lecture on e+e-
- Energy-energy Correlations
- …

IRS: General Feature and Basic Physical Idea

Essential feature of a general IRS physical quantity:

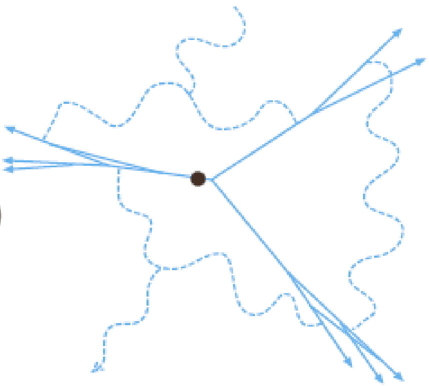
the observable must be such that it is insensitive to whether n or n+1 particles contributed -- if the n+1 particles has n-particle kinematics

e.g. a IRS "jet algorithm"

IRS: Analytic Definition

IRS observables are inclusive quantities

$$
\begin{aligned}
\mathcal{I} \;=\; & \frac{1}{2!}\int d\Omega_2 \, \frac{d\sigma[2]}{d\Omega_2}\, \mathcal{S}_2(p_1^\mu, p_2^\mu) \\
& + \frac{1}{3!}\int d\Omega_2 dE_3 d\Omega_3 \, \frac{d\sigma[3]}{d\Omega_2 dE_3 d\Omega_3}\, \mathcal{S}_3(p_1^\mu, p_2^\mu, p_3^\mu) \\
& + \frac{1}{4!}\int d\Omega_2 dE_3 d\Omega_3 dE_4 d\Omega_4 \\
& \quad \times \frac{d\sigma[4]}{d\Omega_2 dE_3 d\Omega_3 dE_4 d\Omega_4}\, \mathcal{S}_4(p_1^\mu, p_2^\mu, p_3^\mu, p_4^\mu) \\
& + \cdots .
\end{aligned}
$$



where Sn's satisfy, for $0 < \lambda < 1$ (colinear) or $\lambda = 0$ (soft):

$$
\mathcal{S}_{n+1}(p_1^\mu, \ldots, (1-\lambda) p_n^\mu, \lambda p_n^\mu) = \mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu).
$$

Example 1 : σ_{tot}

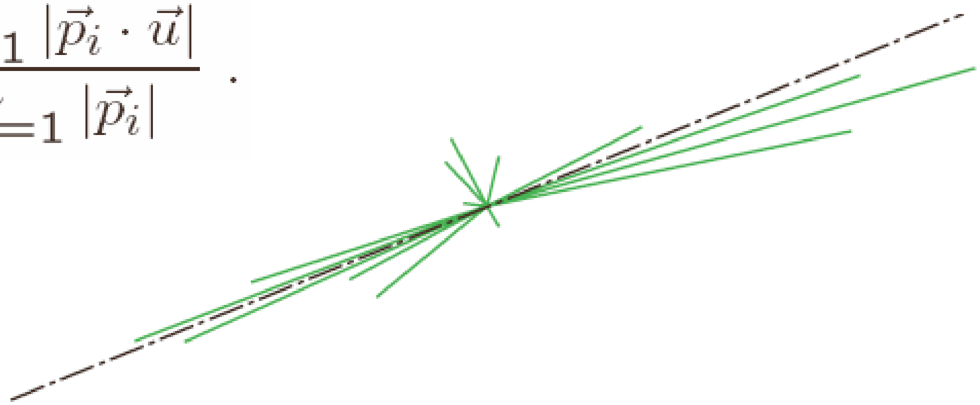
$$\mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu) = 1.$$

IRS condition obviously satisfied.

Example 2 : Thrust

$$\mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu) = \delta\left(T - \mathcal{T}_n(p_1^\mu, \ldots, p_n^\mu)\right) \; .$$

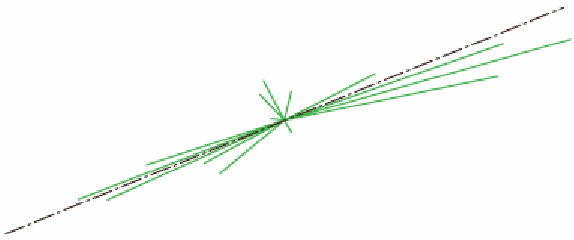
$$\mathcal{T}_n(p_1^\mu, \ldots, p_n^\mu) = \max_{\vec{u}} \frac{\sum_{i=1}^n |\vec{p}_i \cdot \vec{u}|}{\sum_{i=1}^n |\vec{p}_i|} \; .$$



cf. lectures on e+e-

Check the IRS criterion:

$$\mathcal{T}_n(p_1^\mu, \ldots, p_n^\mu) = \max_{\vec{u}} \frac{\sum_{i=1}^n |\vec{p}_i \cdot \vec{u}|}{\sum_{i=1}^n |\vec{p}_i|} \; .$$



- Contribution from a particle with $\vec{p} \to 0$ drops out.

- Replacing one particle by two collinear particles doesn't change the thrust:

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|,$$

and

$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|.$$

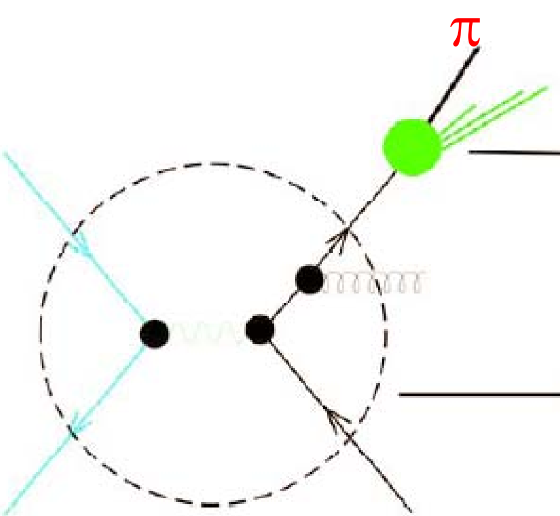
Are applications of PQCD confined to IRS physical observables?

(Most physical observables are not IRS!)

Fortunately not. In fact, the "QCD Parton Model" for lepton-lepton, lepton-hadron and hadron-hadron scattering cross sections at high energies provides a much more powerful framework for applying PQCD to study a vast range of SM and New Physics processes:
The basic idea behind this class of applications is the factorization of short-distance physics (of leptons, quarks, gluons, new particles) from long-distance physics (of hadrons).

Factorization in e^+e^- interaction at high energies

Example: One particle inclusive cross-section



Fragmentation function: $D_a^h(z, \mu)$

Long-distance physics;

Universal.

Hard scattering: $\hat{\sigma}^a(s, z, \mu)$

Short distance physics;

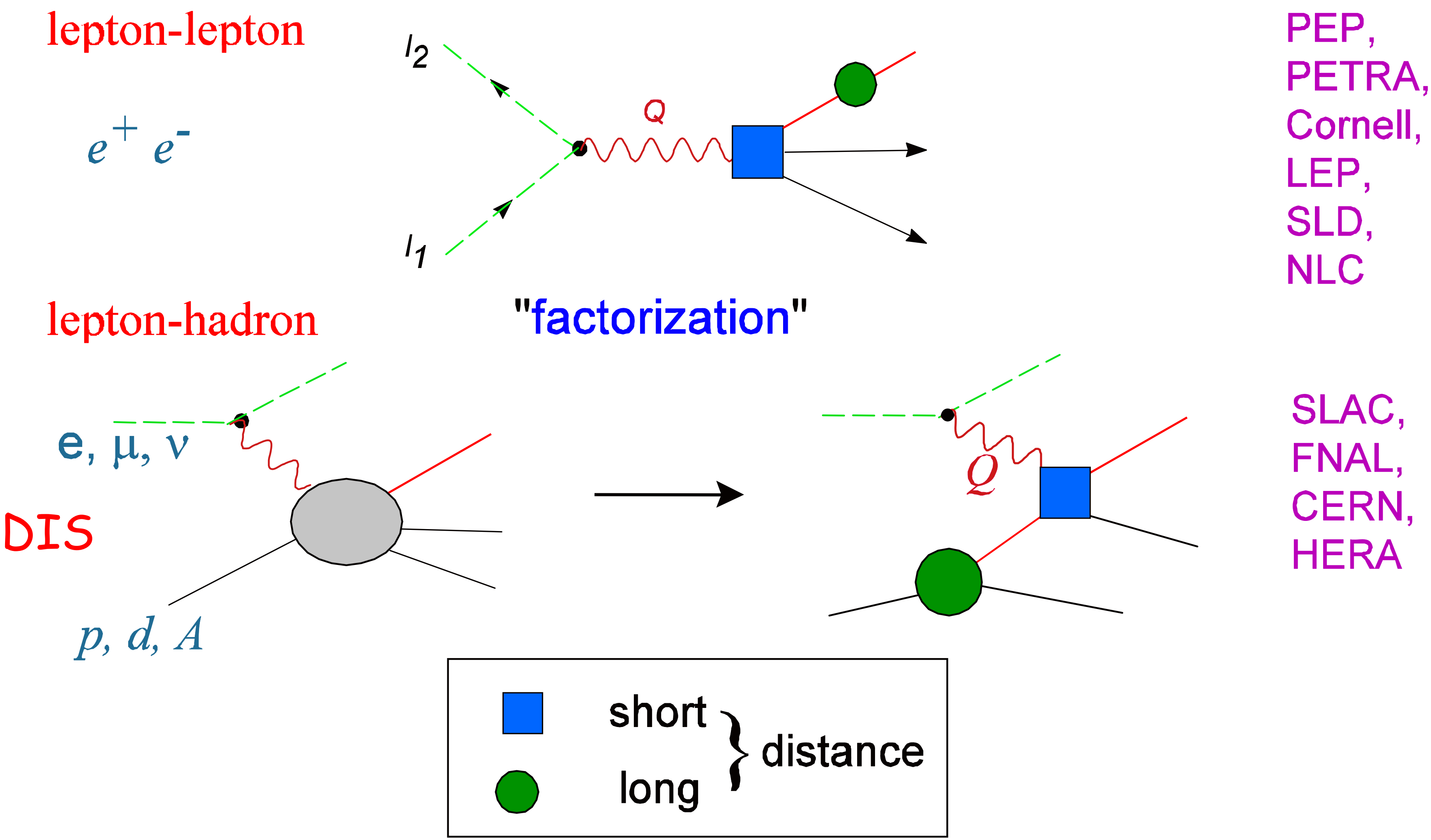
IRS, perturbatively cal.

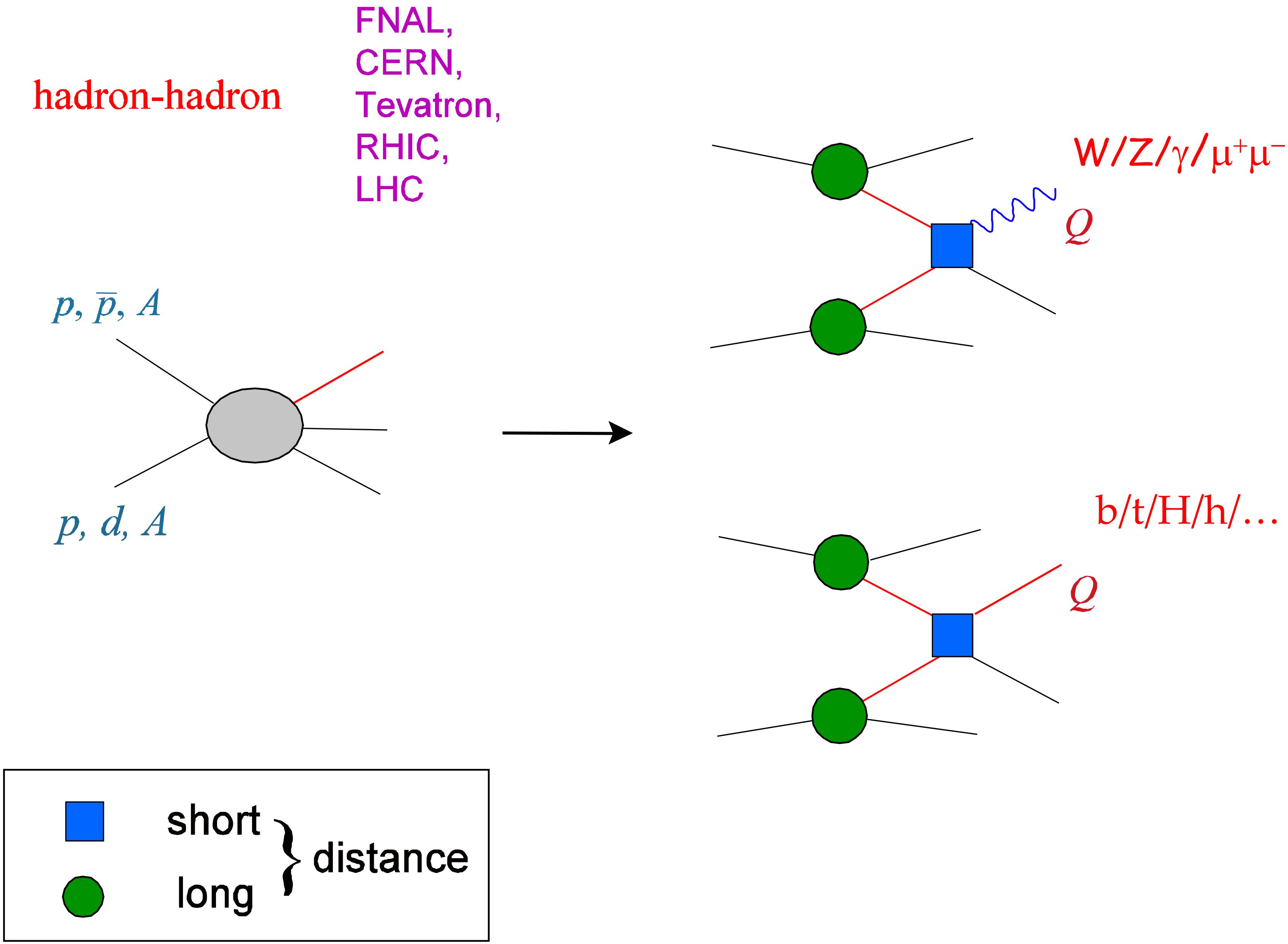
$$\sigma(s, z) = \int_z^1 \frac{d\varsigma}{\varsigma}\, \hat{\sigma}^a\left(\frac{s}{\mu}, \frac{z}{\varsigma}, \alpha_s(\mu)\right) \cdot D_a(\varsigma, \mu)$$

Details will be discussed in the context of DIS (to follow).

Factorization and the Parton Picture at High Energies

(Requires at least one large scale Q—hard scattering)




hadron-hadron
FNAL,
CERN,
Tevatron,
RHIC,
LHC
$p, \bar{p}, A$
p, d, A
$W/Z/\gamma/\mu^+\mu^-$
Q
b/t/H/h/…
Q
short
long
distance