III: PQCD at Work: Deep Inelastic Scattering

- Cross Sections and Structure Functions;
- Order α_s^0 (LO) processes and the Parton Model;
  - Parton Distributions; Sum Rules.
- Order α_s^1 (NLO) QCD corrections:
  - Colinear (*mass)* Singularity from a different perspective;
  - Separation of long- and short-distance physics in the PQCD calculation of the Structure Functions: Physical origin of the universal Parton Distributions;
- Factorization in the NLO calculation.

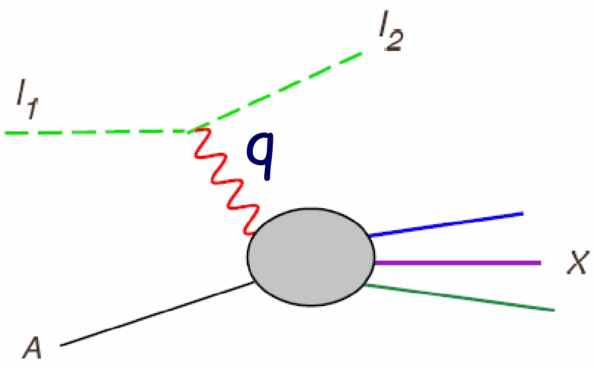
Deep Inelastic Scattering

$$\ell_1(\ell_1) + N(P) \rightarrow \ell_2(\ell_2) + X(P_X)$$

$$\nu = \frac{P \cdot q}{\sqrt{P \cdot P}} = E_1 - E_2$$

$$x = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}$$

$$y = \frac{P \cdot q}{P \cdot \ell_1} = \frac{\nu}{E_1}$$

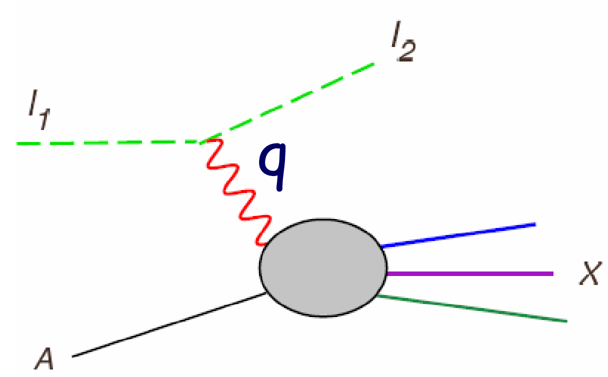


where $q = \ell_1 - \ell_2$, and E_1 and E_2 are the laboratory energies of the incoming and outgoing leptons

$$d\sigma = \frac{1}{2\Delta(s, m_{\ell_1}^2, M^2)} \overline{\sum_{spin}} \left|M^2\right| \, d\Gamma$$

$\Delta(a,b,c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)}$ = flux factor

To leading order in EW coupling (one vector boson exchange):



$$\mathcal{M} = J_\mu^*(P,q)\frac{g_B^2 G^\mu{}_\nu}{Q^2+M_B^2} j^\nu(q,\ell)$$

$$q=\ell_1-\ell_2,\ \ell=\ell_1+\ell_2,\ Q^2=-q^2>0,$$

and $G^\mu{}_\nu = g^\mu{}_\nu - q^\mu q_\nu/M_B^2$.

g_B : EW gauge coupling

B	γ	$W^\pm$	Z
g_B	$-e$	$\frac{g}{2\sqrt{2}}$	$\frac{g}{2\cos\theta_W}$

The lepton current is calculable:

$$j^\mu(q,\ell) = \langle \ell_2 | j^\mu | \ell_1 \rangle =$$

$$\overline{u}(\ell_2)\gamma^\mu[g_R(1+\gamma^5)+g_L(1-\gamma^5)]u(\ell_1)$$

lepton chiral couplings:

	γ	Z	$W^\pm$
g_V	Q_i	$T^i_{3L}-2Q_i\sin^2\theta_W$	$1\cdot V_{ij}$
g_A	0	T^i_{3L}	$1\cdot V_{ij}$
g_R	$\frac{Q_i}{2}$	$-Q_i\sin^2\theta_W$	0
g_L	$\frac{Q_i}{2}$	$T^i_{3L}-Q_i\sin^2\theta_W$	$1\cdot V_{ij}$

Cross Section and Structure Functions

$$\frac{d\sigma}{dx\,dy} = \frac{yQ^2}{8\pi} G_1\,G_2\,L\cdot W$$

where

$$L^{\mu}{}_{\nu} = \frac{1}{Q^2}\overline{\sum_{\rm spin}}\langle \ell_1 | j^{\dagger}_{\nu} | \ell_2\rangle\langle \ell_2 | j^{\mu} | \ell_1\rangle$$

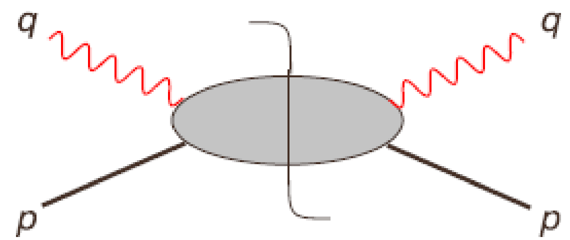
$$L^{\mu\nu} = \frac{8}{Q^2}\frac{1}{n_\ell}\left\{ g^2_{+\ell}\left[\ell_1^\mu \ell_2^\nu + \ell_2^\mu \ell_1^\nu - g^{\mu\nu}\frac{Q^2}{2}\right] - g^2_{-\ell}\left[i\epsilon^{\mu\nu\rho\sigma}\ell_{1\rho}\ell_{2\sigma}\right]\right\}$$

$$W^{\mu}{}_{\nu} = \frac{1}{4\pi}\overline{\sum_{\rm spin}}(2\pi)^4\delta^4(P+q-P_X)\langle P|J^\mu|P_X\rangle\langle P_X|J^\dagger_\nu|P\rangle$$

$$= -g^{\mu}{}_{\nu}W_1 + \frac{P^\mu P_\nu}{M^2}W_2 - i\frac{\epsilon^{Pq\mu}{}_{\nu}}{2M^2}W_3 +$$

By the optical theorem (unitarity) the hadronic component is the Forward Compton Amplitude:

$$\gamma^*(q,\lambda) + A(P) \longrightarrow \gamma^*(q,\lambda') + A(P)$$



Cross Section and Structure Functions

$$\frac{d\sigma}{dx\,dy} = 2ME_1x\frac{d\sigma}{dx\,dQ^2} = 2ME_1^2y\frac{d\sigma}{dQ^2\,d\nu} = \frac{ME_1y}{E_2}\frac{d\sigma}{dE_2\,d\cos\theta}$$

3 equivalent sets of Structure Functions:

Historical (cf. Rosenbluth formula for elastic scattering):

$$\frac{d\sigma}{dE_2\,d\cos\theta} = \frac{2E_2^2}{\pi M}\frac{G_1G_2}{n_\ell}\left\{g_{+\ell}^2\left[2W_1\sin^2\frac{\theta}{2} + W_2\cos^2\frac{\theta}{2}\right] \pm g_{-\ell}^2\left[\frac{E_1+E_2}{M}W_3\sin^2\frac{\theta}{2}\right]\right\}$$

Modern (scaling structure functions):

$$\frac{d\sigma}{dxdy} = \frac{2ME_1}{\pi}\frac{G_1G_2}{n_\ell}\left\{g_{+\ell}^2\left[xF_1y^2 + F_2\left[(1-y) - \left(\frac{Mxy}{2E_1}\right)\right]\right] \pm g_{-\ell}^2\left[xF_3y(1-y/2)\right]\right\}$$

Helicity (scaling helicity structure functions):

$$\frac{d\sigma}{dxdy} = N\left\{g_{+\ell}^2\left[F_T(1+\cosh^2\psi) + F_L\sinh^2\psi\right] \mp g_{-\ell}^2\left[F_{PV}\cosh\psi\right]\right\}$$

analogues of $1 + \cos^2\theta$, $1 - \cos^2\theta$, $\cos\theta$

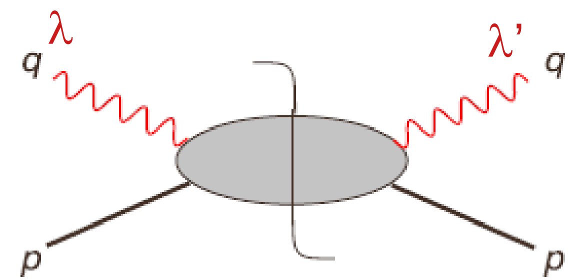
for space-like (vs. time-like) vector bosons

$$\cosh\psi = \frac{E_1+E_2}{\sqrt{Q^2+\nu^2}} \xrightarrow{m_A \to 0} \frac{(2-y)}{y}$$

Structure Functions

Scaling S.F.'s

$$
\begin{aligned}
F_1(x,Q) &= W_1 \\
F_2(x,Q) &= \frac{\nu}{M} W_2 \\
F_3(x,Q) &= \frac{\nu}{M} W_3
\end{aligned}
$$



Helicity S.F.'s

$$F_\lambda = \epsilon_\mu^{\lambda *}(P,q)\, W^\mu{}_\nu(P,q)\, \epsilon_\lambda^\nu(P,q)$$

Relations between invariant and helicity S.F.s

$$
\begin{aligned}
F_{right} = \quad F_+(x,Q) &= \quad F_1 - \tfrac{1}{2}\kappa\ F_3 \\
F_{left} = \quad F_-(x,Q) &= \quad F_1 + \tfrac{1}{2}\kappa\ F_3 \\
F_{long} = \quad F_0(x,Q) &= -F_1 + \tfrac{1}{2x}\kappa^2\ F_2
\end{aligned}
$$

At high energies

$$\kappa = \sqrt{1 + \frac{Q^2}{\nu^2}} \approx 1$$

Conversely,

$$
\begin{aligned}
F_1 &= \tfrac{1}{2}(F_+ + F_2) = \tfrac{1}{2}(F_{right} + F_{left}) = F_T \\
F_2 &= 2x(F_T + F_L)\ \frac{1}{\kappa^2} \\
F_3 &= (F_{right} - F_{left})\ \frac{1}{\kappa}
\end{aligned}
$$

(parity-violating)

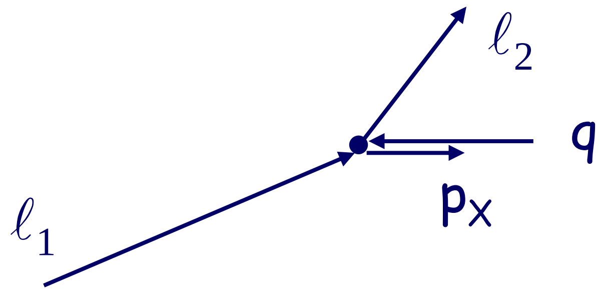
Space-time structure of DIS

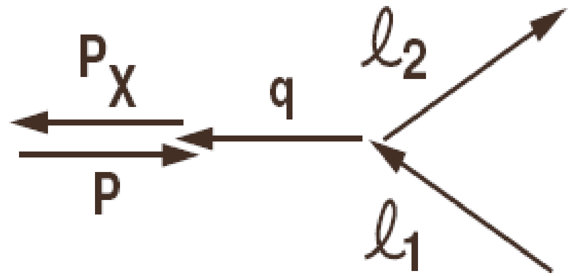
Use light-cone components $(k^+, k^-, \mathbf{k}_T)$.

Two useful frames, related by $k_B^{\pm} = k_A^{\pm} (\frac{Q}{m x_{bj}})^{\pm 1}$:

4 − vector	rest frame of A $\vec{p} = 0$; $p^0 = m$	Breit frame $q^0 = q_T = 0$; $q_z = Q$
$(p^+, p^-, \mathbf{p})$:	$\frac{1}{\sqrt{2}} (m_A, m_A, \mathbf{0})$	$\frac{1}{\sqrt{2}} (\frac{Q}{x_{bj}}, \frac{x_{bj} m_A^2}{Q}, \mathbf{0})$
$(q^+, q^-, \mathbf{q})$:	$\frac{1}{\sqrt{2}} (-m_A x_{bj}, \frac{Q^2}{m_A x_{bj}}, \mathbf{0})$	$\frac{1}{\sqrt{2}} (-Q, Q, \mathbf{0})$

In its *rest frame*, constituents of hadron A interact at space-time distance $\sim$ the Compton wavelength: $\Delta x^+ \sim \Delta x^- \sim \frac{1}{m}$.

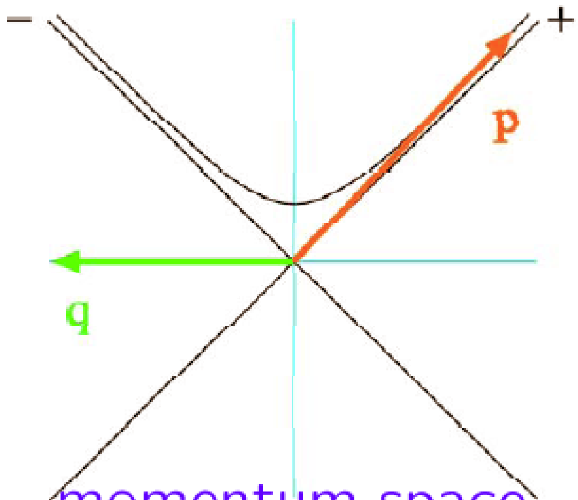




In the *Breit frame:* Lorentz transformation spreads out interactions: fast moving hadron has

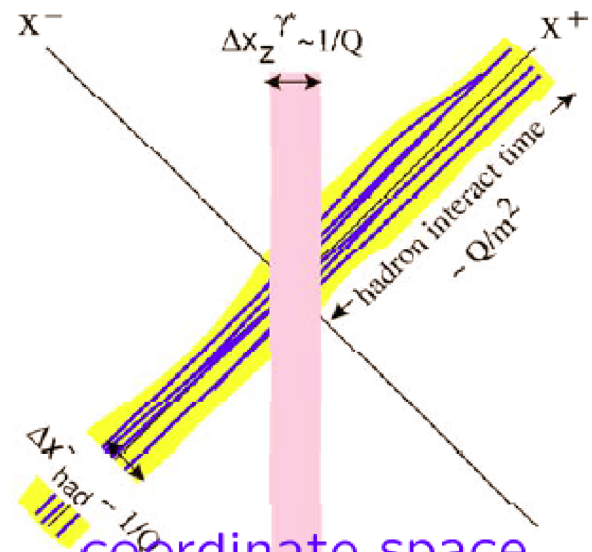
$$\Delta x^+ \sim \frac{1}{m}\times\frac{Q}{m x_{bj}} = \frac{Q}{m^2 x_{bj}}, \qquad \Delta x^- \sim \frac{1}{m}\times\frac{m x_{bj}}{Q} = \frac{x_{bj}}{Q}.$$

As $Q \to \infty$: $\Delta x^+ \sim \frac{Q}{m^2}$; $\Delta x^- \sim \frac{1}{Q}$, cf. plot:



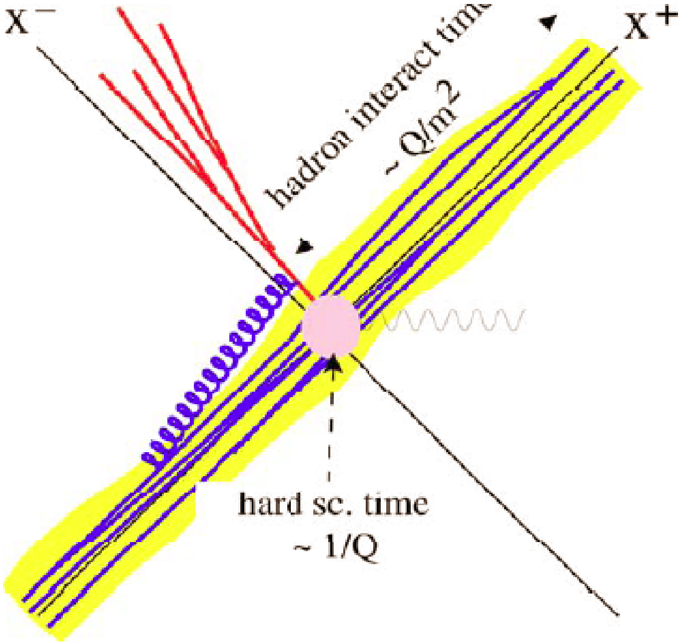
momentum space

Breit frame



coordinate space

The virtual photon ($q_z = Q$) probes the Breit frame hadron wavefunction with $\Delta x_z \sim 1/Q \Rightarrow$ Hard scattering takes place only within the short dist. $\Delta x^{\pm} \sim \frac{1}{Q}$ of the interaction point.



Thus, quark- and gluon- "**partons**" are effectively free in a DIS scattering event.

At a given x^+, one finds partons with an amp.

$$\psi(p_1^+, \vec{\mathbf{p}}_1; p_2^+, \vec{\mathbf{p}}_2; \cdots); \; 0 < p_i^+ \sim Q; \; \vec{\mathbf{p}}_i \ll p_i^+.$$

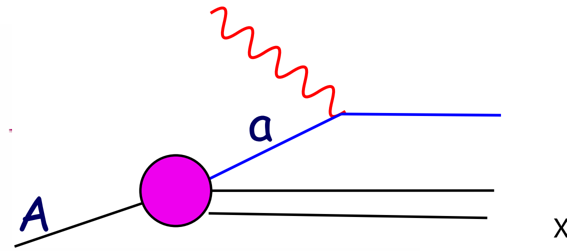
For p_i^+, use momentum fractions $\xi_i = p_i^+/p^+$ instead, where $0 < \xi_i < 1$.

$\Rightarrow$ Hadron is like a collection of free, collinear, massless partons with fractional momenta $\{\xi_i\}$.

Note: The space-time picture suggests the possibility of separation of long- and short- distance physics; it *does not* provide a *proof* of factorization in QCD theory.

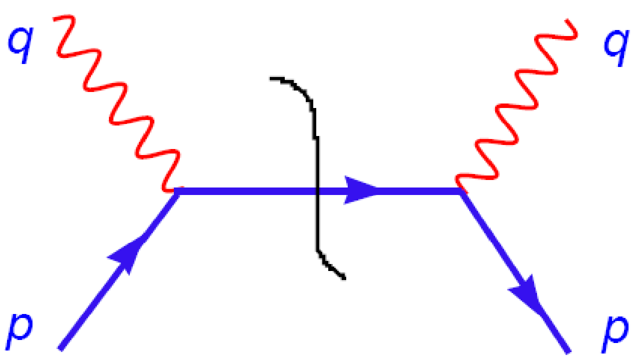
Parton Model results on Structure Functions

$$F_\lambda(x,Q^2) \sim \int_0^1 \frac{d\xi}{\xi} \sum_a f_A^a(\xi)\, \hat{F}_\lambda^a(x/\xi, Q^2)$$



where $\hat{F}_\lambda^a(z,Q^2)$ is the "partonic structure function" for DIS on the parton target a.

The Feynman diagram contributing to this elementary quantity and the result of a straight-forward calculation are (for electro-magnetic coupling case):



$$\begin{aligned}
\hat{F}_T^a(x/\xi, Q^2) &= Q_a^2\, \delta(x/\xi - 1) \\
\hat{F}_L^a(x/\xi, Q^2) &= 0 \\
\hat{F}_{PV}^a(x/\xi, Q^2) &= 0
\end{aligned}$$

$\Longrightarrow$ the simple scaling parton model results:

$$
\begin{aligned}
F_{\rm Trans}(x,Q^2) &= \sum_a Q_a^2\, f_A^a(x) && \text{(Bjorken. – Feynman)}\\
F_{\rm Long}(x,Q^2) &= 0 && \text{(Callan – Gross)}\\
F_{\rm P-V}(x,Q^2) &= 0 && \text{(EM Parity – cons.)}
\end{aligned}
$$

In terms of $F_{1,2}$

$$
\begin{aligned}
F_1(x,Q^2) &= \tfrac{1}{2}\sum_a Q_a^2\, f_A^a(x)\\
F_2(x,Q^2) &= x\sum_a Q_a^2\, f_A^a(x)\\
F_3(x,Q^2) &= 0
\end{aligned}
$$

The helicity version of these results is simpler and more physical, e.g. the commonly known C-G relation in terms of $F_{1,2}$ is: $F_2(x,Q^2) = 2xF_1(x,Q^2)$, which has no obvious physical meaning.

Simple Parton Model Formulas embody a lot of Physics of the SM

(Again, the helicity S.F.s provide the simplest and clearest results.)

Neutrino-proton Sc.

$$F^{\nu p}_{\mathrm{Left}} = x D(x)$$
$$F^{\nu p}_{\mathrm{Right}} = x \bar{U}(x)$$
$$F^{\nu p}_{\mathrm{Long}} = 0$$

Anti-neutrino-proton Sc.

$$F^{\bar{\nu} p}_{\mathrm{Left}} = x \bar{D}(x)$$
$$F^{\bar{\nu} p}_{\mathrm{Right}} = x U(x)$$
$$F^{\bar{\nu} p}_{\mathrm{Long}} = 0$$

where $D(x)$ represents a generic weak isospin $T_3 = -\frac{1}{2}$ down quark distribution, $U(x)$ a $T_3 = \frac{1}{2}$ up quark distribution, and $\bar{D}(x)$ & $\bar{U}(x)$ the corresponding anti-quark distributions.

Simple Parton Model Formulas (continued)

For a neutron target,

$$F_\lambda^{\nu n} = F_\lambda^{\nu p}(u \leftrightarrow d)$$
$$F_\lambda^{\bar{\nu} n} = F_\lambda^{\bar{\nu} p}(u \leftrightarrow d)$$

and for iso-scalar targets, denoted by $F_\lambda^{\nu N}$:

$$F_\lambda^{\nu N} = (F_\lambda^{\nu p} + F_\lambda^{\nu n})/2$$
$$F_\lambda^{\bar{\nu} N} = (F_\lambda^{\bar{\nu} p} + F_\lambda^{\bar{\nu} n})/2$$

One of the well-known, and useful, result is

$$F_{\mathrm{trans}}^{(\nu+\bar{\nu})N} \propto x \sum (U + D + \bar{U} + \bar{D}) = \text{tot. quark mom. frac.}$$

Parton Model Sum Rules

Quark Number Sum Rules:

$$\int (u(x) - \bar{u}(x))\, dx = 2$$
$$\int \left(d(x) - \bar{d}(x)\right) dx = 1$$
$$\int (s(x) - \bar{s}(x))\, dx = 0$$

Various linear combinations of the left-hand side can be formed to correspond to integrals of measurable structure functions.

Momentum Sum Rule: $\int x \left[g(x) + \sum (q(x) + \bar{q}(x))\right] dx = 1$

These sum rules remain valid even when QCD interaction is taken into account.

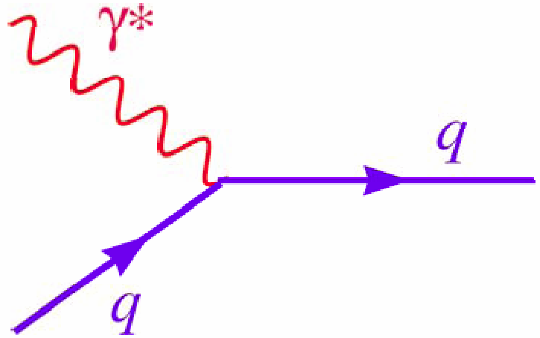
- The space-time picture is useful to guide our thinking. However, the order-by-order analytic calculation in momentum space is needed for quantitative applications of PQCD.
- PQCD is not equipped to calculate *Hadronic S.F.'s* $F_{\lambda}^{A}(x, Q)$, since hadronic wave functions are dominated by long-dist. physics.
- But, we can study the *Partonic S.F.'s in PQCD* $F_{\lambda}^{a}(x, Q)$, and establish the following important results:
  $\Rightarrow$ prove the factorization theorems;
  $\Rightarrow$ derive the scale dependence of the universal PDF's—the QCD evolution;
  $\Rightarrow$ derive the "hard cross-sections" …
  These results can then be applied to the physical world involving hadrons.

Order by order calculation of Partonic S.F.'s $F_\lambda^a(x,Q)$

Leading-order (LO) PQCD Calculation:

The lowest order in which lepton-hadron scattering can take place is α_s^0;

the scattering process is $\gamma^* q \to q$:

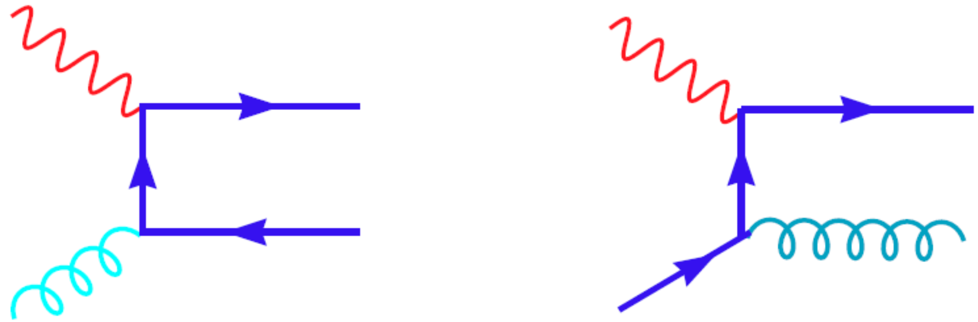


The calculation of ${}^0F_\lambda^a(x,Q)$ reduce to the same as that of the parton-model $\widehat{F}_\lambda^a(x)$.

Next-to-leading (NLO) Calculation:

To order α_s^1, two partonic processes contribute:

gluon-fusion: $\gamma^* g \to q\bar{q}$ *quark-sc.*: $\gamma^* q \to gq$



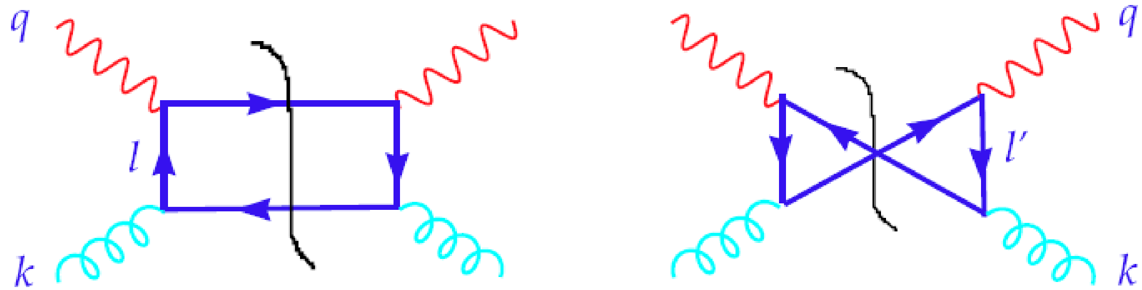
The two subprocesses do not interfere.

We shall study $\gamma^* g \to q\bar{q}$ in detail, since it is simpler. Much physics can be learnt from it without a lot of technical complications.

Will summarize the main results about $\gamma^* q \to gq$ afterwards.

$$\gamma^* g \rightarrow q\bar{q}$$

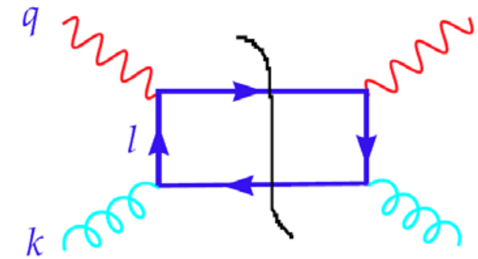
- The (partonic) S.F. calculation (squared amplitude) consists of two "cut-diagrams":



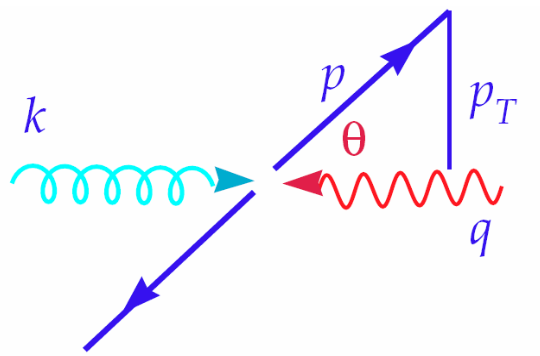
- We shall keep the ("physical") quark mass (e.g. m_R) in the calculation – for good reasons to be seen.
- The calculation is straightforward (entry-level QED text-book case—same as $\sigma_{\rm tot}$calculation for $\gamma\gamma \rightarrow e^+e^-$, or $\gamma e^- \rightarrow \gamma e^-$, or $e^+e^- \rightarrow \gamma\gamma$.)
- *The result is finite!* There is no ultra-violet or infra-red divergences to distract us.

Outline of Calculation:

$$B(q) + g(k) \rightarrow \overline{q}_1(p_1) + q_2(p_2)$$



- Kinematics in the CM frame:



$$\begin{array}{lcllll}
k^\mu & : & (\;k, & 0, & 0, & k\;) \\
q^\mu & : & (E_q, & 0, & 0, & -k) \\
p_1^\mu & : & (E, & p_T & 0, & p_L) \\
p_2^\mu & : & (E, & -p_T & 0, & -p_L)
\end{array}$$

- Results:

$$F_g^T(Q^2, s, m^2) = L\,\frac{(Q^4 + s^2)}{(Q^2 + s)^2} - \frac{2(s - Q^2)^2 p}{(Q^2 + s)^2 \sqrt{s}}$$

$$F_g^L(Q^2, s, m^2) = \frac{8(Q^2 - m^2)p}{(Q^2 + s)^2 \sqrt{s}} + \mathcal{O}\left(\frac{m^2}{Q^2} L\right)$$

where $L = 2 \log \left[\frac{\sqrt{s} + \sqrt{s - 4m^2}}{2m} \right]$

Fun part: physics in the Bjorken limit ...

The Bjorken limit:

$$Q^2 \gg m^2, \;\; s \gg m^2, \;\; x = \frac{Q^2}{2k \cdot q} = \frac{Q^2}{s + Q^2} \approx O(1).$$

The "finite" partonic structure functions, e.g. $F_g^T(x, Q^2, m/Q)$, contains the large logarithm

$$L = 2 \log \left[\frac{\sqrt{s} + \sqrt{s - 4m^2}}{2m} \right] \longrightarrow \log \frac{s}{m^2} = \log \frac{Q^2}{m^2} (\frac{1}{x} - 1)$$

Since $\alpha_s L \sim 1$, this will render the perturbative expansion useless for sufficiently large Q/m!

Crucial Question:

Can this problem be isolated and controlled?

Answer: yes!

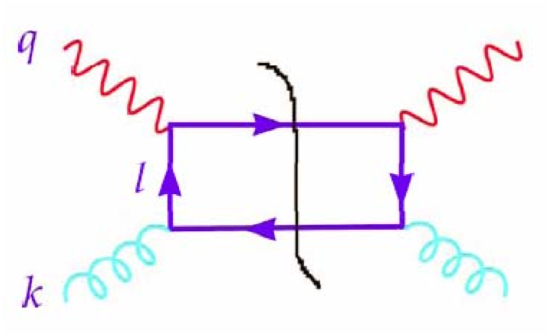
Key ideas:

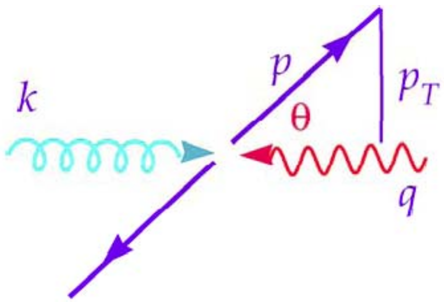
- The large logarithm is due to the collinear region of the momentum phase space $\Leftrightarrow$ long-distance physics in coordinate space;
- The long- and short- distance physics can be systematically separated (factorized);
- The short distance part will be kept. The long-distance part is "universal"; it can be resummed into parton distributions.

This is how it works ...

Physical Picture of the collinear/mass logarithm

Step back to examine the Feynman integral



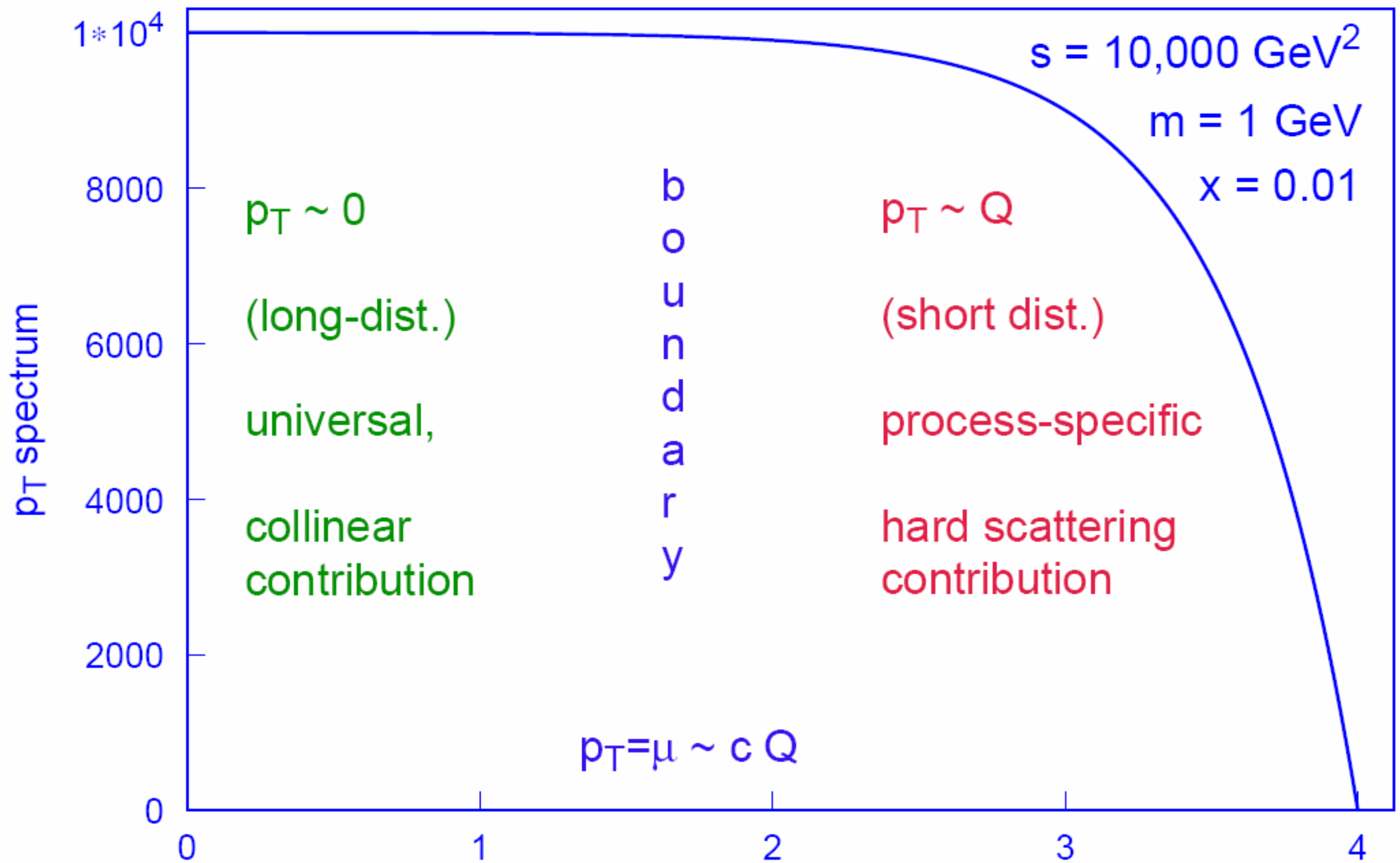


$$
\begin{aligned}
F_T(x,\tfrac{m}{Q}) &= \int d\Gamma_2 \frac{N}{(\ell_1^2 - m^2)(\ell_2^2 - m^2)} &&= \int dp_T^2 \frac{N}{(2k\cdot p_1)(2k\cdot p_2)} \\
&= \int dp_T^2 \frac{N'}{(E-p_L)(E+p_L)} &&= \int dp_T^2 \frac{1}{E^2 - p_L^2} N' \\
&= \int_0^p \frac{dp_T^2}{p_T^2 + m^2} N'(p_T, x\ldots) &&= \int_0^{\log(s/4m^2)} d\eta\, N'(\eta, x\ldots)
\end{aligned}
$$

where $\eta = \log(p_T^2 + m^2)/m^2$, and N' is well behaved in the limits $p_T, \eta \to 0$ and $m \to 0$.

Separation of long/short distance physics

– physical, intuitive ideas



$$\eta = \log(p_T^2 + m^2)/m^2 \quad \sim \log Q/m$$

• The boundary, $p_T = \mu_F$ (the factorization scale), is "arbitrary", provided it lies near the upper end of the flat region which separates long/short distance physics.

• A shift of the value of μ_F results in shifting a finite term between the long/short distance pieces; the sum (the "physical" structure function) is independent of the choice of μ_F.

• Important: the separated (long/short) pieces have quite distinct properties, as listed on the plot.

These results can be realized analytically ....

Analytic separation of long/short dist. pieces

- The separation of the long- and short-distance physics can be achieved by introducing an (arbitrary) intermediate scale $\mu_{(F)}$, cf. the η-plot.

$$F_T(x,\frac{m}{Q}) = P(x)\log\frac{\mu^2}{m^2} + P(x)\log\frac{s}{4\mu^2} + \tilde{F}(x,\frac{m}{Q})$$

$$= P(x)\log\frac{\mu^2}{m^2} + \hat{F}(x,\frac{Q}{\mu},\frac{m}{\mu})$$

The first term is manifestly isolated from the hard sc.– it consists of the long dist. log factor $\log\frac{\mu^2}{m^2}$, a universal function $P(x)$, the famous "splitting function" for $g \to q\bar{q}$, and the degenerate $\gamma q \to q$ partonic S.F..

The second term, $\hat{F}(x,\frac{Q}{\mu},\frac{m}{\mu})$, is "infra-red safe", i.e. it is finite in the limit $m \to 0$; and it contains all the hard scattering physics.

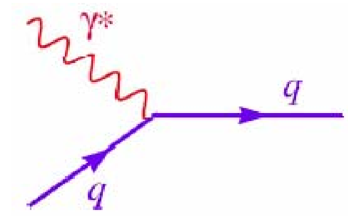
The Emergence of Factorization

Rewrite our results on Partonic Str. Fns. as:

LO:

$$\gamma q \to q \qquad : \quad {}^0F_q^\lambda\left(x, \frac{Q}{m}\right) \quad = {}^0f_q^{q'} \otimes {}^0\hat{F}_{q'}^\lambda$$

(No gluon term at LO.)

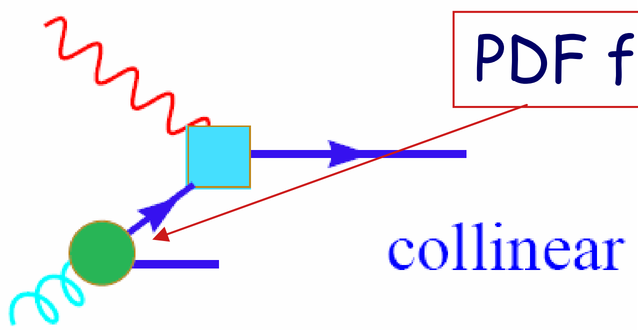


NLO (gluon fusion term)

$$\gamma g \to q\bar{q} \qquad : \quad {}^1F_g^\lambda\left(x, \frac{Q}{m}\right) \quad = {}^1f_g^q \otimes {}^0\hat{F}_q^\lambda + {}^0f_g^g(x) \otimes {}^1\hat{F}_g^\lambda$$

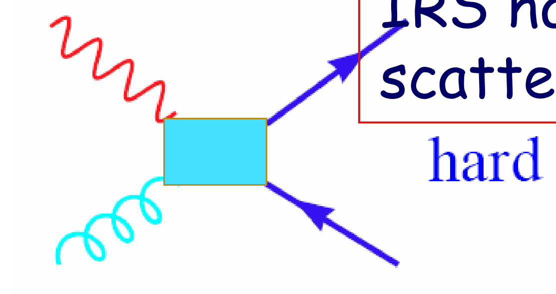
$$(Additive \quad factorization) \quad = {}^1f_g^q \otimes {}^0\hat{F}_q^\lambda + {}^1\hat{F}_g^\lambda$$

PDF for q/g



+

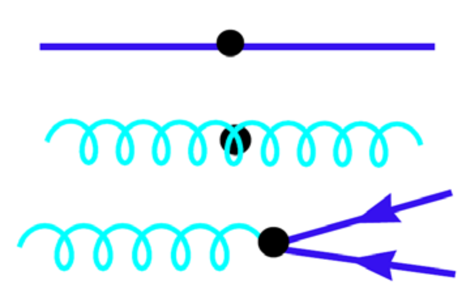
IRS hard scattering



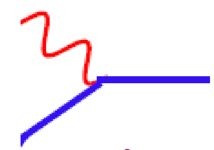
Perturbative PDFs and Hard Cross Sections

perturbative *Parton distribution functions*:

$$
\begin{aligned}
{}^{0}f_q^{q'}(x) &= \delta(x)\,\delta_q^{q'} \\
{}^{0}f_g^{g}(x) &= \delta(x) \\
{}^{1}f_g^{q}\left(x, \tfrac{m}{\mu}, \mu\right) &= \tilde{\alpha}_s(\mu)\, P_{gq}(x)\, \log\left(\tfrac{\mu}{m}\right)
\end{aligned}
$$

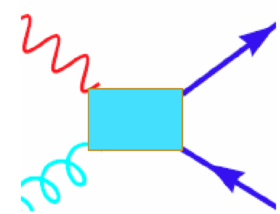


Hard cross-sections (Wilson coefficients):



$$
{}^{0}\hat{F}_q^{\lambda}\left(x, \tfrac{Q}{m}\right) = {}^{0}F_q^{\lambda}(x, Q) = e_q^2\, \delta(x-1) \qquad cf.\ parton\ sec.
$$

$$
{}^{1}\hat{F}_g^{\lambda}\left(x, \tfrac{Q}{\mu}, \tfrac{m}{Q}\right) = {}^{1}F_g^{\lambda}\left(x, \tfrac{Q}{m}\right) - {}^{1}f_g^{q} \otimes {}^{0}\hat{F}_q^{\lambda} \qquad cf.\ cal.\ this\ sec.
$$



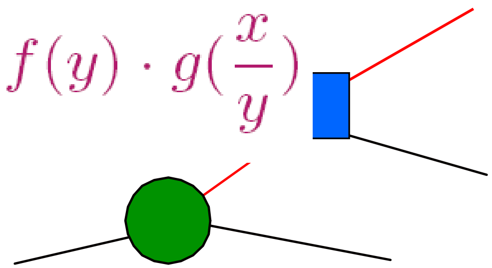
These results can be interpreted as the order $\alpha_s(\mu)$ expansion of a multiplicative factorization formula …

Statement of Factorization Theorem for DIS

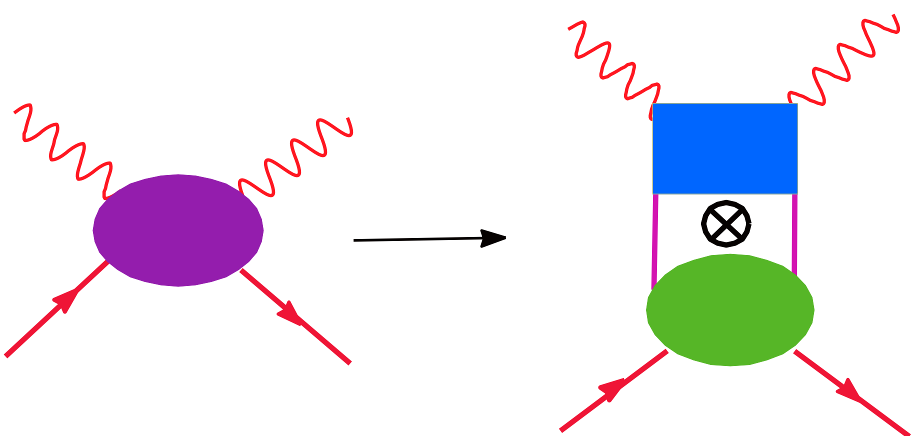
$$F_a^{\lambda}(x, \frac{Q}{m}) = \sum_{a'} f_a^{a'}(x, \frac{m}{\mu}, \alpha_s) \otimes \hat{F}_{a'}^{\lambda}(x, \frac{Q}{\mu}, \frac{m}{Q}, \alpha_s)$$

where the convolution $\otimes$ is defined as: version:

$$f(x) \otimes g(x) \equiv \int \frac{dy}{y} f(y) \cdot g(\frac{x}{y})$$



Pictorial representation, full (more accurate) version:



From the Illustrative Example to the Real World . . .
What issues need to be addressed?

- What happens when we consider the other NLO subprocess $\gamma q \rightarrow g q$?
- What happens when we go to higher and higher orders in $\alpha_s(\mu)$; and to other processes?
- What is the meaning and use of the perturbative "parton distribution functions" encountered in the calculations? (They are associated with long-distance physics and contain colinear and soft singularities, hence are infra-red unsafe.)
- What does factorization of *partonic* cross-sections have to do with *hadronic* X-sections?

- Operationally, how does one calculate the IRS hard cross-sections which are needed in the hadronic factorization formulas?
- How are the universal hadronic parton distribution functions defined and used?
- Where does the scale-dependence of PDF's come from? — Origin of the QCD evolution (DGLAP) equation.
- What about the scale- and scheme-dependence of PQCD predictions on physical X-sections?