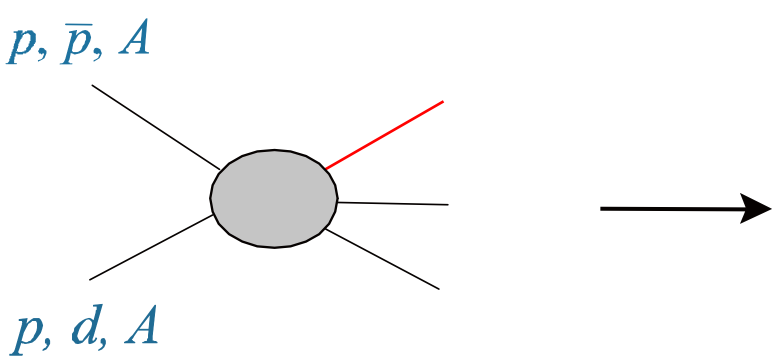
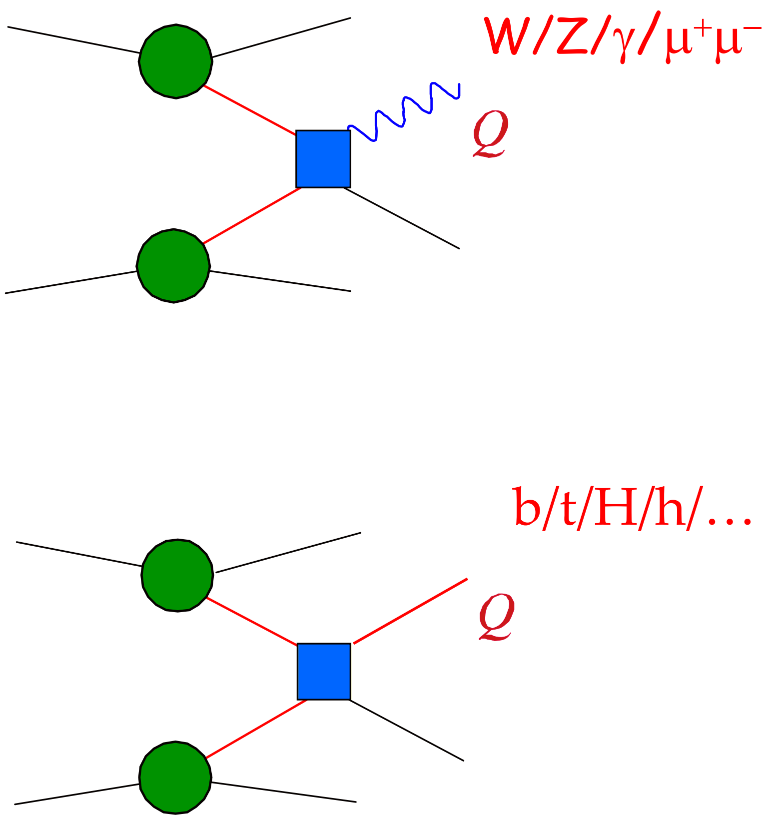
Part V: Applications of PQCD to High Energy Hadron-hadron Collisions

FNAL,
CERN,
Tevatron,
RHIC,
LHC





Kinematics: Rapidity

To apply the PQCD formalism, need a large scale in the process: invariant mass of $lepton - pair$, W, Z; transverse momentum p_T of $\gamma\ jet$.

Choose c.m. frame with z-axis in the beam. For any particle with mass m and momentum q: $q^\mu = (q^+, q^-, \mathbf{q})$.

$$\text{define rapidity} \quad : \qquad y = \frac{1}{2} \log\left(\frac{q^+}{q^-}\right) .$$

$$q^\mu = (e^y \sqrt{(\mathbf{q}^2 + m^2)/2},\ e^{-y} \sqrt{(\mathbf{q}^2 + m^2)/2},\ \mathbf{q}).$$

Transformation under a boost along z-axis:

$$q^+ \to e^\omega q^+, \quad q^- \to e^{-\omega} q^-, \quad \mathbf{q} \to \mathbf{q} \qquad \boxed{y \to y + \omega}$$

Pseudorapidity:

$$\eta = -\log\left(\tan(\Theta/2)\right) \ \approx\ y \text{ if } q \gg m$$

Vector Boson production in hadron-hadron collisions

Consider the "Drell-Yan" process:

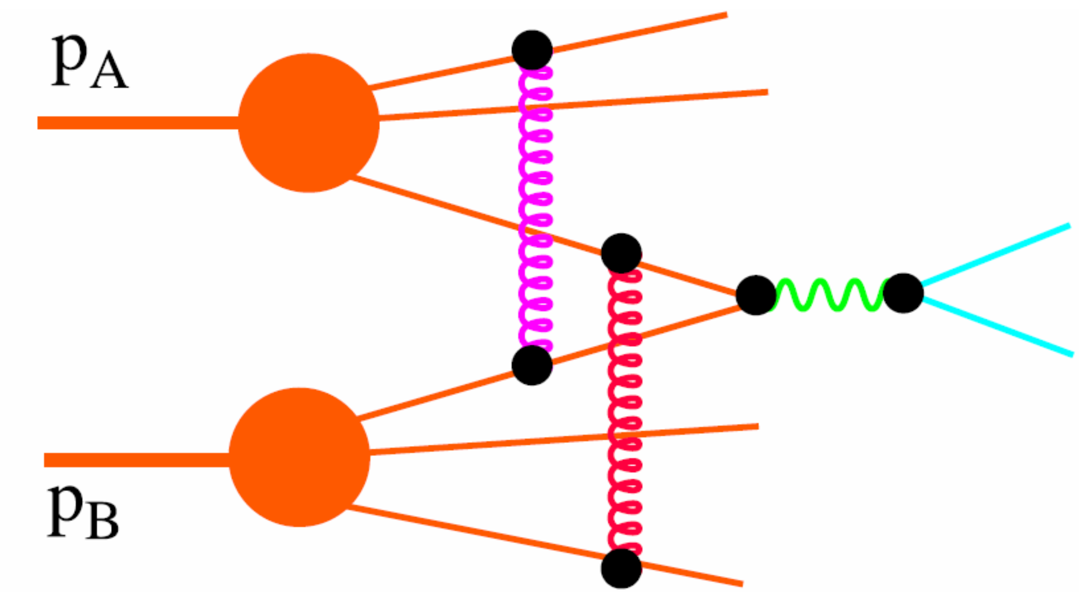
$$A + B \to (\gamma^*/W/Z) + X$$

Let $\quad x_A = e^{y}\sqrt{M^2/s}, \qquad x_B = e^{-y}\sqrt{M^2/s}.$

$$\frac{d\sigma}{dy} = \sum_{a,b}\int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B\, f_A^a(\xi_A,\mu)\, f_B^b(\xi_B,\mu)\, \frac{d\hat{\sigma}_{ab}(\mu)}{dy} + \mathcal{O}((\frac{m}{M})^p)$$

When $d\hat{\sigma}_{ab}/dy$ is calculated to order α_s^N then there are corrections of order α_s^{N+1}. We integrate over $\mathbf{q}$; γ^*, W, Z's are mostly at $\mathbf{q} \ll M$.

The proof of factorization is not obvious due to initial-state strong interactions—does not hold graph by graph



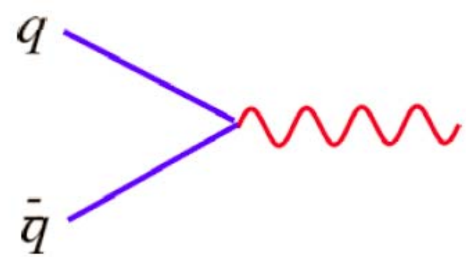
It is saved by the interplay between graphs, due to unitarity, causality, and gauge invariance.

Vector Boson Production Plays a Key Role in PQCD

Historically important on many accounts:

- Dominant production mechanism:

LO process:
$q\bar{q} \to \gamma^*/W/Z$
(original Drell-Yan)



This "clinched" the case for quark-parton model with successful account of $A + B \to \gamma^* + X \to \ell^+\ell^- + X$ (for pion and proton beams on nucleon targets), using $f_A^a(x, Q)$ derived from DIS.

• NLO subprocesses:

· A relatively large "kfactor" $\equiv \frac{NLO}{LO} \sim 2$ caused concern about the validity of the perturbative series; but the origin for this was understood.

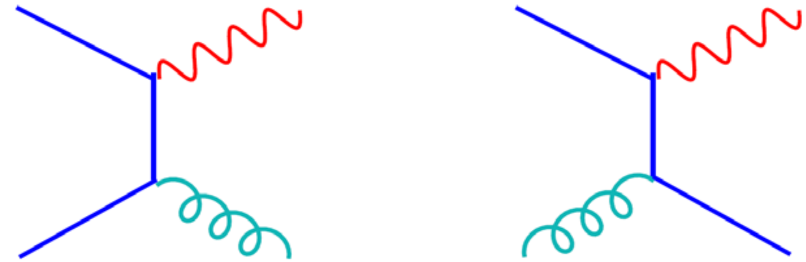
· With the NLO result, a factor of $\frac{1}{3}$ multiplying the naive formula was needed to get agreement in normalization. This corroborated the need for the *color* degree of freedom for quarks.

NLO calculation leads to non-trivial p_T distribution – vector boson can recoil against the final state parton-jet. But NLO result diverges at $p_T \to 0$, even if the integrated cross-section is finite. This is the first example of a "two-scale problem" which needs generalization of the PQCD formalism – resummation.

$W, \ Z$ production at hadron colliders is important both for quantitative understanding of the Standard Model (precision measurement of EW parameters and stringent tests of QCD), and for the study of signals and backgrounds for New Physics searches.

Direct photon production

$$A + B \to \gamma + X$$



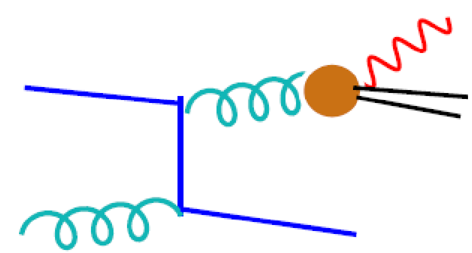
Leading subprocesses:
—same as the "NLO" graphs for DY.

Historically, this has been regarded as a primary process to probe the gluon parton distribution of the nucleon.

Complication #1: real photons consist of: a) a "point-like" component; and a "hadron-like" component, the latter in terms of photon fragmentation functions of quarks and gluons.

Hadronic contributions to direct photon production:

$$\begin{array}{rcl} a+b & \to & c+d \\ d & \to & \gamma + X \end{array}$$

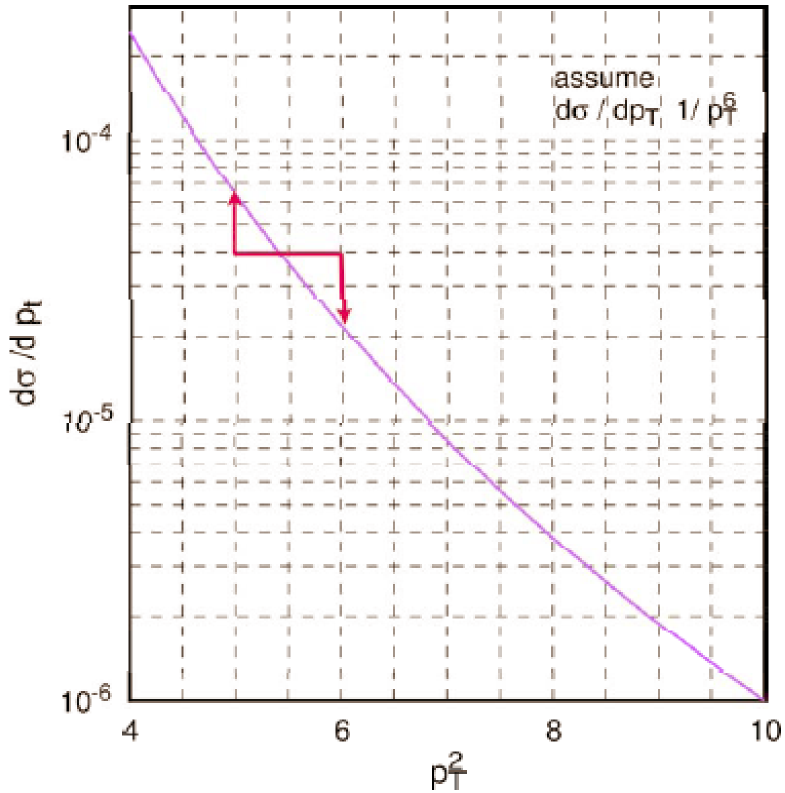


a, b, c, d : quarks & gluon

Complication #2: Both the k-factor and the scale-dependence of the NLO calculation are large over the range of current experimental measurements (up to a factor of 2).

Complication #3: The steeply falling p_T spectrum of γ is extremely sensitive to small broandening of initial-state parton "k_T".

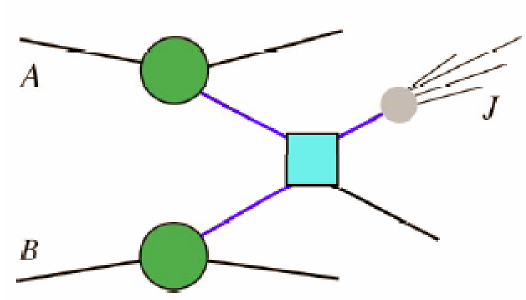
Source of this broadening: multi-gluon radiation at high energies; non-perturbative effects at low energies.



These complications make the direct photon production process a continuing challenge for theorists to gain control over.

Inclusive Jet Production

The basic idea is that final state partons manifest themselves as jets:



$$\frac{d\sigma}{dE_T d\eta} \approx \sum_{a,b} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \; f_A^a(\xi_A,\mu)\; f_B^b(\xi_B,\mu)\; \frac{d\hat{\sigma}^{ab}(\mu)}{dE_T d\eta}$$

But what constitutes a jet?

Minimum requirements on a good Jet definition ("algorithm"):

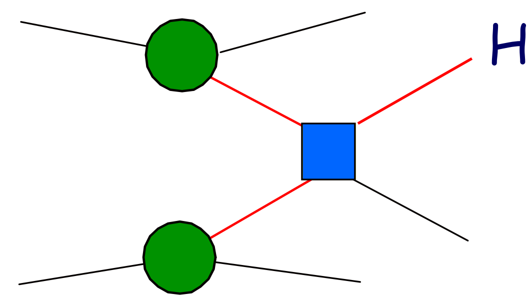
- Theoretically, it must be IRS (so that xSec's can be reliably calculated in PQCD;
- Experimentally, it must be conveniently, and unambiguously, implementable;
- Most importantly, everyone must agree on the same algorithm to use, so that meaningful comparisons can be made.

Guess which one of these is the hardest (impossible) to achieve?

Heavy Quark (H) Production

What are the scales involved in this process?

m/L_{QCD}, $Q(s)$, M_H,



This is a multi-scale problem; it is not straightforward!

Historically, there were two seemingly incompatible approaches—two opposite ways to approximate this process as a one-scale problem:

- $Q \gg M_H$: Zero-mass formalism, Variable-Flavor-Number-Scheme (VFNS);
- $M_H \gg Q$: Heavy Quark formalism, Fixed-Flavor-Number-Scheme (FFNS)

Collins' mass-indep. proof of Factorization provides a general formalism that is valid for all values of Q/M_H, that reduces to the two cases in their respective limits.

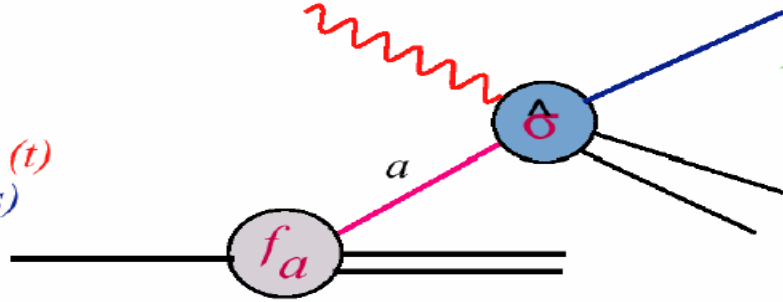
Conventional QCD Parton Model

Zero-mass Variable Flavor Number Scheme (ZVFNS)

Zero-mass Factorization Theorem

$\sum$

$a = g, u, d, s, c, b, (t)$
(all active flavors)



All parton masses
$m_a = 0$
MSbar Subtr.

active flavor : all quarks with $m_H < Q$: $n_{fl}(Q)$

Usual parton distributions are generated in this scheme:
EHLQ, ... , MRS, CTEQ

i.e. $F_2(x,q) = u + d + s + c + b + \ldots + \text{NLO}$

Useful in Global QCD Analysis – simple to implement;
NLO hard-Xsec calculations exist for all processes.

All popular applications are based on this scheme:
Pythia, Herwig, Isajet ; EKS, JetRad, ...

....

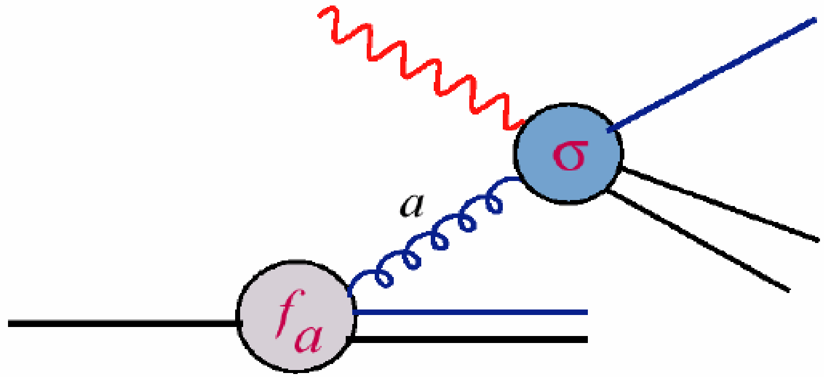
Advantage: Simple and intuitive (Almost folklore)

Limitations: mass effects neglected (except for thresholds)
Not appropriate for $Q \sim m_H$

Fixed-flavor-number (FFN) Scheme

1988 - 1992

$\sum_{a = g, u, d, s}$ *(light fl. only)*

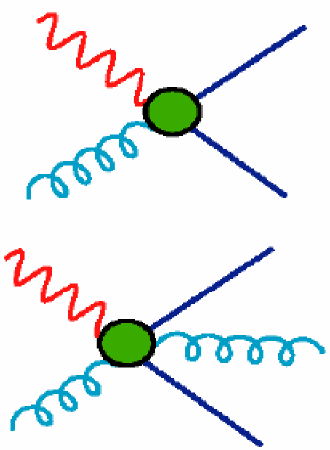


heavy quark:
m_H "large"
No col. Subtr.

number of quark flavor n_{fl} =3 (for c) fixed, indep. of Q

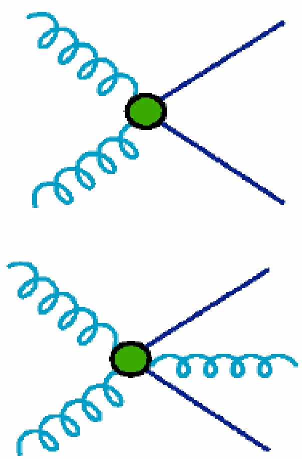
Lepto-production

hadro-production



"Heavy flavor creation"
(HC)

"Gluon-fusion process"



NLO calculation:

Laenen, Smith,
van Neerven ...et.al

Nason, Dawson, Ellis
Mangano, Ridolfi, Fixione .

Advantage: Standard one-scale calculation.

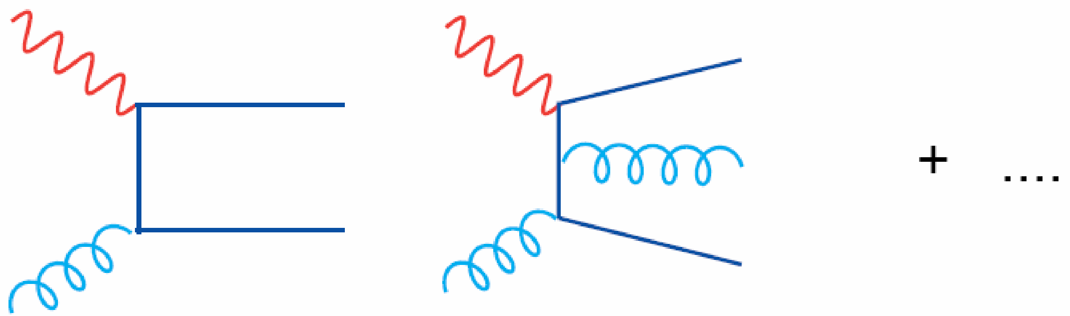
Limitations: no active H parton at any energy; contains powers of $log(Q^2/m_H^2)$ => Large theoretical uncertainty, hence suspect for $Q >> m_H$.

Example: Charm Production in DIS:

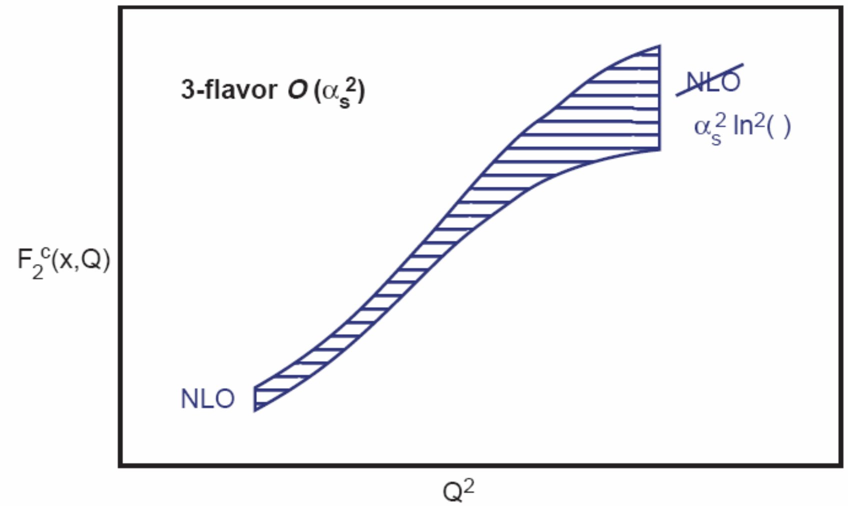
3-flavor order α_S^2 calculation

(no charm parton at any energy scale)

Perturbative series to calculate:



Expected accuracy:

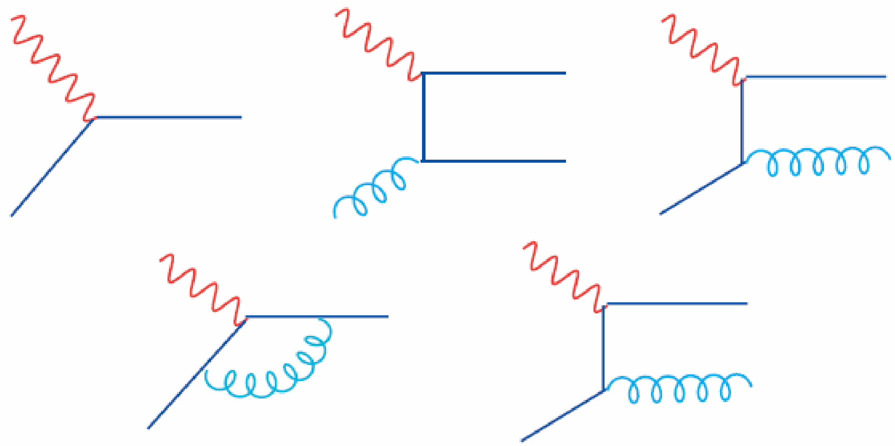


- Expansion is in powers of $\alpha_s ln(Q^2/m_c^2)$ which is *not small when* $Q^2 \gg m_c^2$ – the uncertainty band becomes wide in this region.

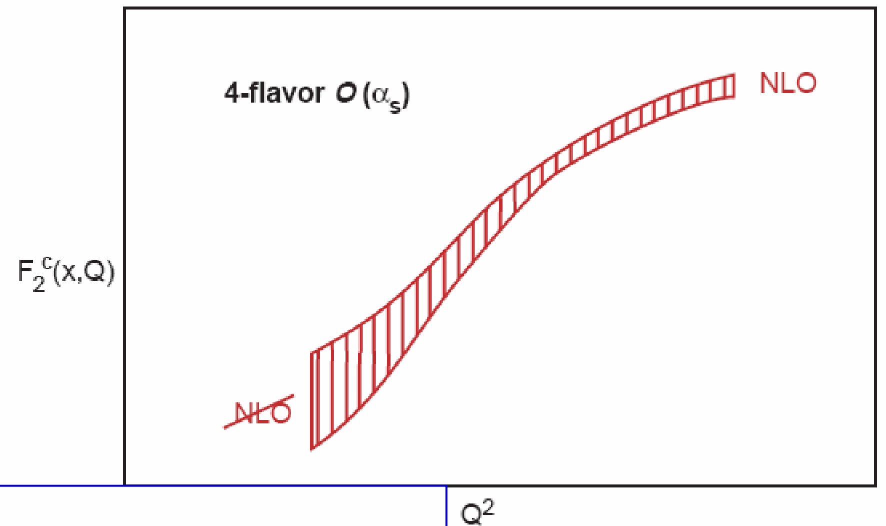
4-flavor order α_s calculation

(including the charm parton)

Perturbative series to calculate:



Expected accuracy:

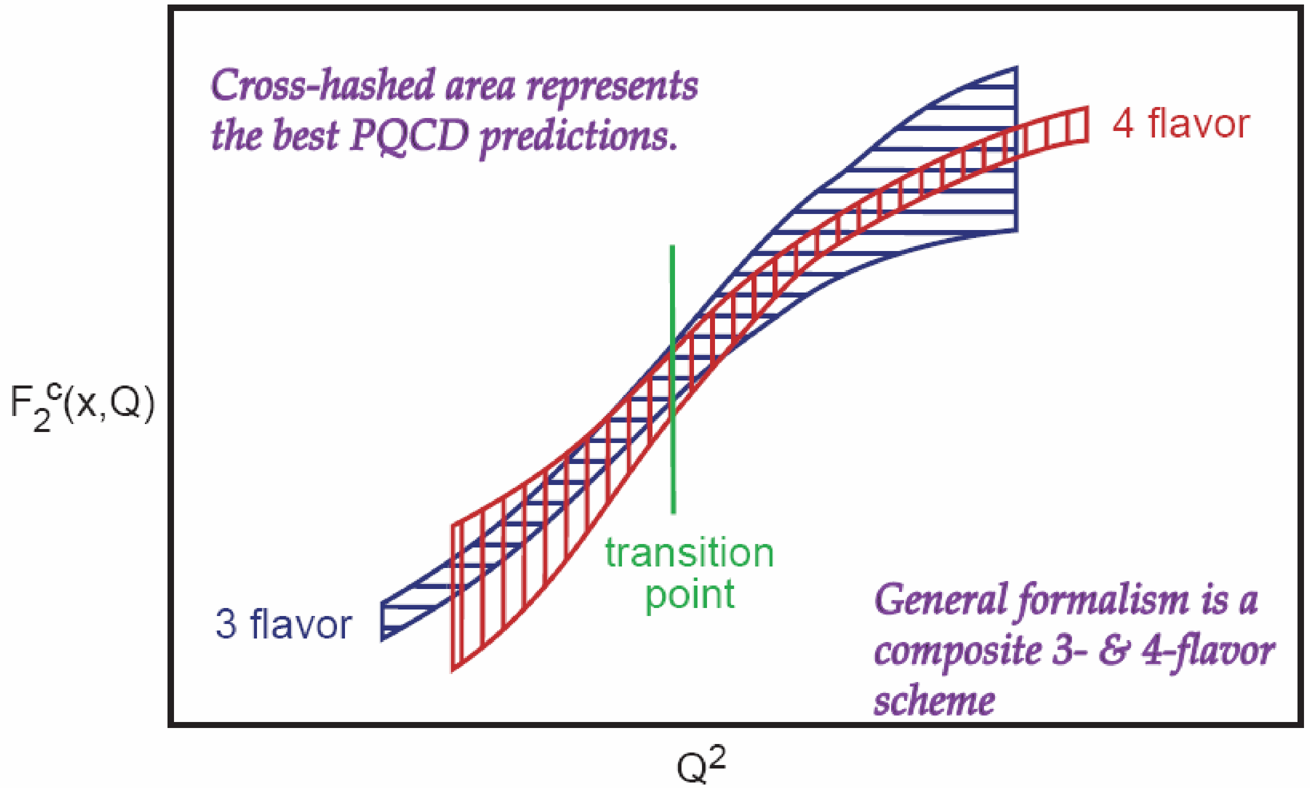


Implicitly assumes $m_c^2 \ll Q^2$;
In the conventional approach, set $m_c \to 0$ in hard scattering calculation. But there is no need to do so in general.
$\Rightarrow$ Not a good approximation when $m_c^2 \sim Q^2$.

The Modern Approach

– Generalized $\overline{MS}$ Formalism of Collins (ACOT)

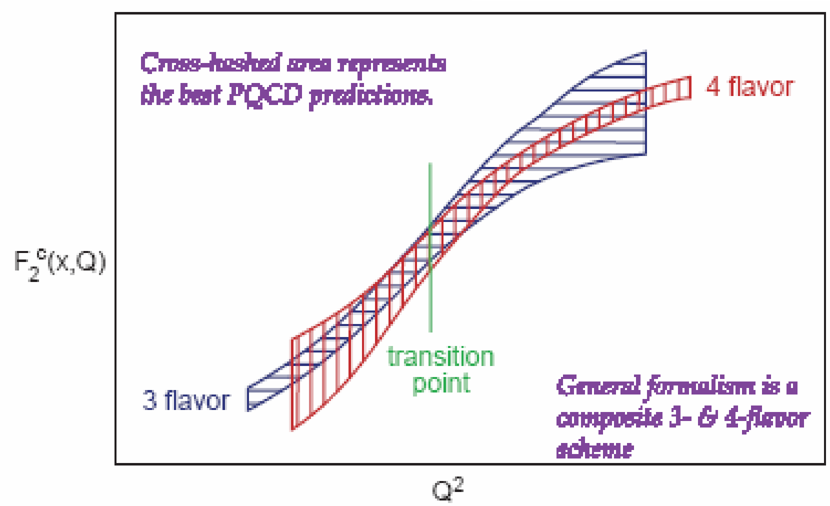
Intuitively *obvious*: a *composite scheme*



1978: CWZ : EW
'85, C-T : QCD
'88, O-T : Higgs
'94, ACOT : HQ
'97, Collins : FacThm

Technically *precise*: essential ingredients:

- *3-flavor scheme* near $Q \sim m_c$ and up;
- *4-flavor $m_c \neq 0$ scheme* at asymptotic $Q \gg m_c$ and down;
- a set of matching conditions which relate the two schemes at some *matching scale* μ_m;
- a suitably chosen *transition* scale μ_t at which one switches from one scheme to the other.



Other Multi-scale Processes and Resummations

- Small-x ($x \to 0$) and BFKL;
- Transverse momentum distribution and Sudakov Effects;
- Large-x ($x \to 1$) and Threshold resummation;
- Combined Resummations
  - BFKL and DGLAP;
  - p_T and threshold; …
- Power-law corrections
  - Renormalons
  - Higher-twist …

Have fun!