NLO tools and MCFM Lecture II

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Slides available from http://theory.fnal.gov/people/ellis/Talks/CTEQ07/

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Hard scattering cross sections

- Why NLO?
- W production
- MCFM
- One loop integrals
  - ⋆ Reduction to basis set
  - ⋆ Passarino Veltman Decomposition of tensor integrals
- Combining NLO corrections and parton showers
  - ⋆ MC@NLO

Why NLO?

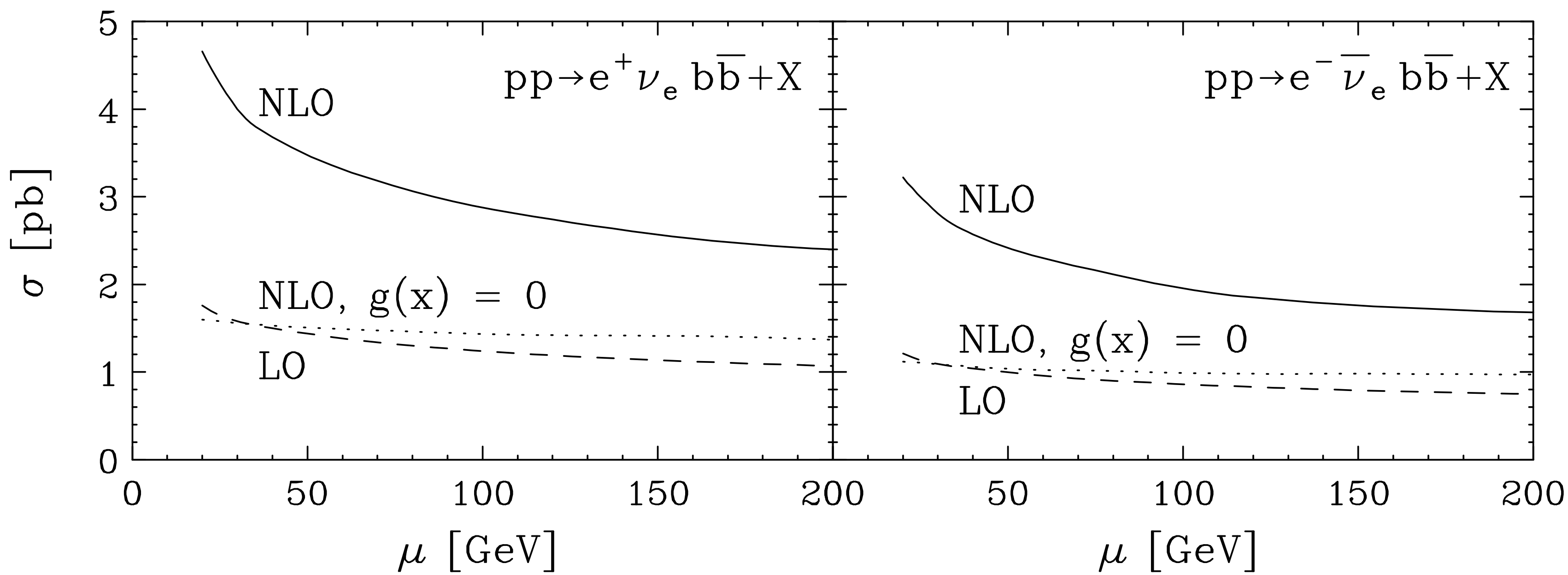
The benefits of higher order calculations are:-

- Less sensitivity to unphysical input scales (eg. renormalization and factorization scales)
- First prediction of normalization of observables at NLO
  - ⋆ Hence more accurate estimates of backgrounds for new physics searches.
  - ⋆ Confidence that cross-sections are under control for precision measurements.
- It is a necessary prerequisite for other techniques matching with resummed calculations, (eg. MC@NLO, see later).
- More physics
  - ⋆ Parton merging to give structure in jets.
  - ⋆ Initial state radiation.
  - ⋆ More species of incoming partons enter at NLO.

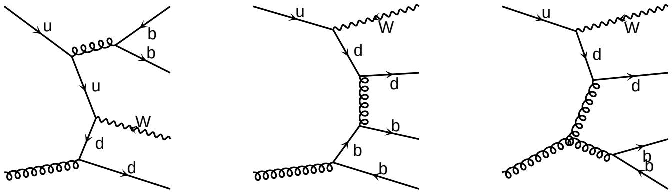
Influence of new processes at NLO

Campbell, Ellis, Rainwater, hep-ph/0308195

- Consider the process $pp \to W b\bar{b}$ at LHC



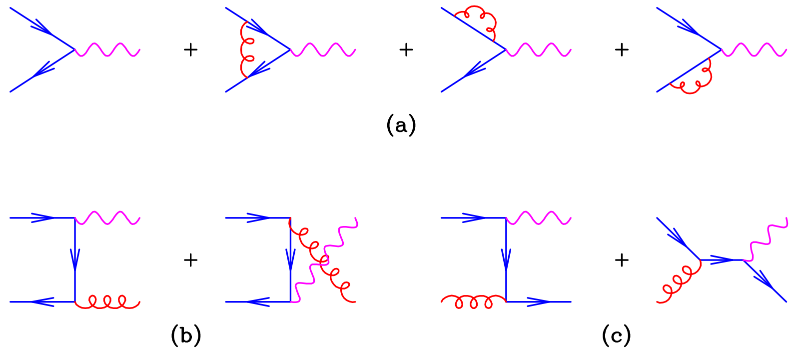
Diagrams by MadGraph



(a) (b) (c)

- new classes of gluon induced diagrams at NLO lead to large effects.

Next-to-leading order: Initial state



- The contribution of the real diagrams (in four dimensions) is

$$|M|^2 \sim g^2 C_F \left[\frac{u}{t} + \frac{t}{u} + \frac{2Q^2 s}{ut} \right] = g^2 C_F \left[\Big(\frac{1+z^2}{1-z} \Big) \Big(\frac{-s}{t} + \frac{-s}{u} \Big) - 2 \right]$$

  where $z = Q^2/s$, $s + t + u = Q^2$.

- Note that the real diagrams contain collinear singularities, $u \to 0, t \to 0$ and soft singularities, $z \to 1$.
- The coefficient of the divergence is the unregulated branching probability $\hat{P}_{qq}(z)$.
- Ignore for simplicity the diagrams with incoming gluons.

- Control the divergences by continuing the dimensionality of space-time, $d = 4 - 2\epsilon$, (technically this is dimensional reduction). Performing the phase space integration, the total contribution of the real diagrams is

$$\begin{aligned}\sigma_R &= \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^\epsilon c_\Gamma \Bigg[\Big(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{\pi^2}{3}\Big)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) \\ &- 2(1-z) + 4(1+z^2)\Big[\frac{\ln(1-z)}{1-z}\Big]_+ - 2\frac{1+z^2}{(1-z)}\ln z\Bigg]\end{aligned}$$

with $c_\Gamma = (4\pi)^\epsilon / \Gamma(1-\epsilon)$.

- The contribution of the virtual diagrams is

$$\sigma_V = \delta(1-z)\left[1 + \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^\epsilon c'_\Gamma \Big(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 6 + \pi^2\Big)\right]$$

$$c'_\Gamma = c_\Gamma + O(\epsilon^3)$$

- Adding it up we get in dim-reduction

$$\sigma_{R+V} = \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \left[\left(\frac{2\pi^2}{3} - 6\right)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) - 2(1-z) \right.$$
$$\left. + \quad 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z \right]$$

- The divergences, proportional to the branching probability , are universal.
- We will factorize them into the parton distributions. We perform the mass factorization by subtracting the counterterm

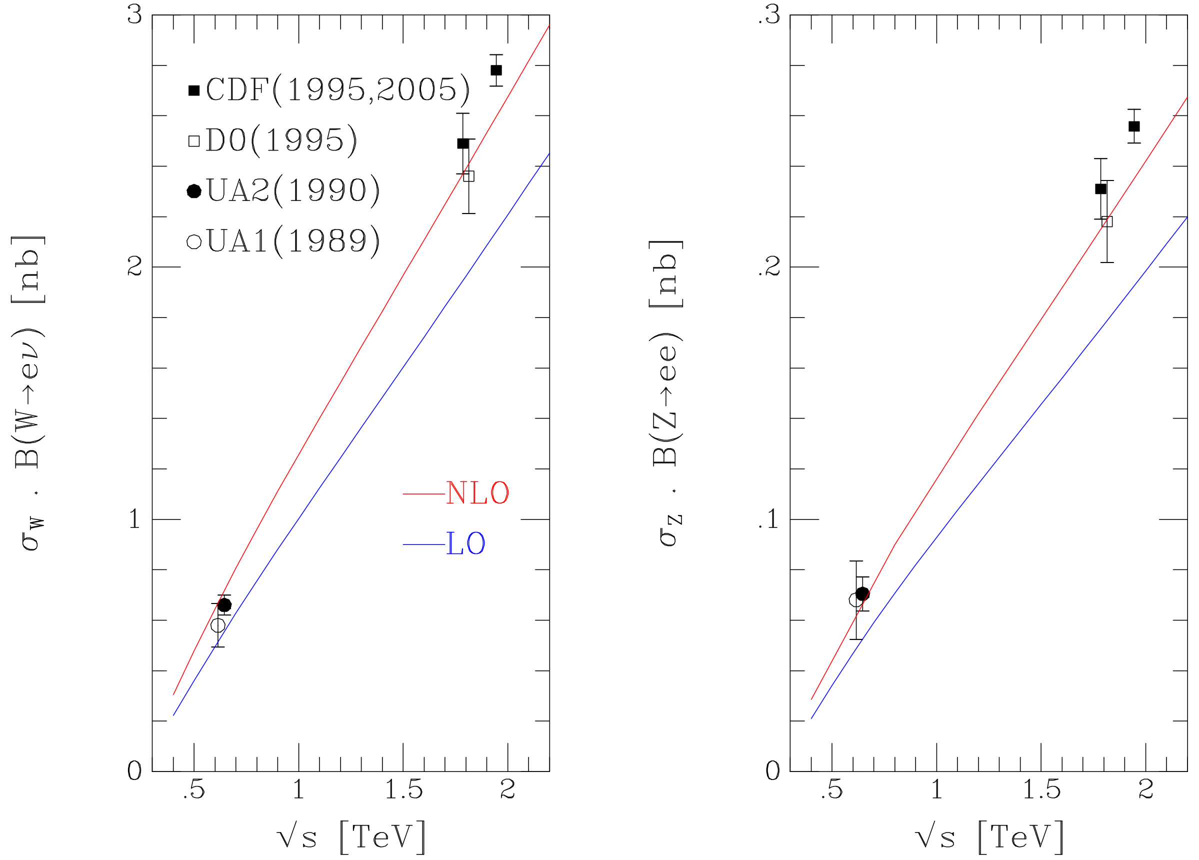
$$2\frac{\alpha_S}{2\pi} C_F \left[\frac{-c_\Gamma}{\epsilon} P_{qq}(z) - (1-z) + \delta(1-z)\right]$$

(The finite terms are necessary to get us to the $\overline{MS}$-scheme).

$$\hat{\sigma} = \frac{\alpha_S}{2\pi} C_F \left[\left(\frac{2\pi^2}{3} - 8\right)\delta(1-z) + 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2}\right]$$

- Similar correction for incoming gluons.

Application to W, Z production



- Agreement with NLO theory is good.
- LO curves lie about 25% too low.
- NNLO results are also known and lead to a further modest (4%) increase at the Tevatron.

General calculational method for NLO

- Direct integration is good for the total cross section, but for differential distributions, (to which we want to apply cuts), we need a Monte Carlo method.
- We use a general subtraction procedure at NLO.
- at NLO the cross section for two initial partons a and b and for m outgoing partons, is given by

$$\sigma_{ab} = \sigma_{ab}^{LO} + \sigma_{ab}^{NLO}$$

where

$$\begin{aligned} \sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^B \\ \sigma_{ab}^{NLO} &= \int_{m+1} d\sigma_{ab}^R + \int_m d\sigma_{ab}^V \end{aligned}$$

the singular parts of the QCD matrix elements for real emission, corresponding to soft and collinear emission can be isolated in a process independent manner

Calculational method (cont)

- One can use this the construct a set of counterterms

$$d\sigma^{ct} = \sum_{ct} \int_m d\sigma^B \otimes \int_1 dV_{ct}$$

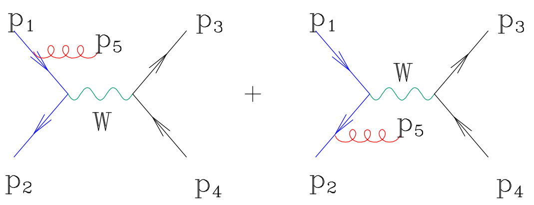
  where $d\sigma^B$ denotes the appropriate colour and spin projection of the Born-level cross section, and the counter-terms are independent of the details of the process under consideration.

- these counterterm cancel all non-integrable singularities in $d\sigma^R$, so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} [d\sigma_{ab}^R - d\sigma_{ab}^{ct}] + \int_{m+1} d\sigma_{ab}^{ct} + \int_m d\sigma_{ab}^V$$

  The phase space integration in the first term can be performed numerically in four dimensions.

Matrix element counter-event for W production



In the soft limit $p_5 \rightarrow 0$ we have

$$|M_1(p_1,p_2,p_3,p_4,p_5)|^2 = g^2 C_F \frac{p_1\cdot p_2}{p_1\cdot p_5 \; p_2\cdot p_5} |M_0(p_1,p_2,p_3,p_4)|^2$$

- Eikonal factor can be associated with radiation from a given leg by partial fractioning

$$\frac{p_1\cdot p_2}{p_1\cdot p_5 \; p_2\cdot p_5} = [\frac{p_1\cdot p_2}{p_1\cdot p_5 + p_2\cdot p_5}][\frac{1}{p_1\cdot p_5} + \frac{1}{p_2\cdot p_5}]$$

- including the collinear contributions, singular as $p_1\cdot p_5 \rightarrow 0$, the matrix element for the counter event has the structure

$$|M_1(p_1,p_2,p_3,p_4,p_5)|^2 = \frac{g^2}{x_a p_1\cdot p_5} \hat{P}_{qq}(x_a) |M_0(x_a p_1,p_2,\tilde{p}_3,\tilde{p}_4)|^2$$

where $1 - x_a = (p_1\cdot p_5 + p_2\cdot p_5)/p_1\cdot p_2$ and $\hat{P}_{qq}(x_a) = C_F(1+x^2)/(1-x)$

Subtraction method for NLO

- For event $q(p_1) + \bar{q}(p_2) \to W^+(\nu(p_3) + e^+(p_4)) + g(p_5)$ with $p_1 + p_2 = \sum_{i=3}^5 p_i$
- generate a counter event $q(x_a p_1) + \bar{q}(p_2) \to W^+(\nu(\tilde{p}_3) + e^+(\tilde{p}_4))$ and $x_a p_1 + p_2 = \sum_{i=3}^4 \tilde{p}_i$ with $1 - x_a = (p_1 \cdot p_5 + p_2 \cdot p_5)/p_1 \cdot p_2$.
- A Lorentz transformation is performed on all j final state momenta $\tilde{p}_j = \Lambda^\mu_\nu p_j^\nu, j = 3, 4$ such that $\tilde{p}_j^\mu \to p_j^\mu$ for p_5 collinear or soft.
- The longitudinal momentum of p_5 is absorbed by rescaling with x.
- The other components of the momentum, p_5 are absorbed by the Lorentz transformation.
- In terms of these variables the phase space has a convolution structure,

$$d\phi^{(3)}(p_1, p_2; p_3, p_4, p_5) = \int_0^1 dx\ d\phi^{(2)}(p_2, xp_1; \tilde{p_3}, \tilde{p_4})[dp_5(p_1, p_2, x)]$$

where

$$[dp_5(p_1, p_2, x_a)] = \frac{d^d p_5}{(2\pi)^3} \delta^+(p_5^2) \Theta(x) \Theta(1-x) \delta(x - x_a)$$

- If k_i is the emitted parton, and p_1, p_2 are the incoming momenta, define the shifted momenta

$$\widetilde{k}_j^\mu = k_j^\mu - \frac{2k_j \cdot (K+\widetilde{K})}{(K+\widetilde{K})^2}\ (K+\widetilde{K})^\mu + \frac{2k_j \cdot K}{K^2}\ \widetilde{K}^\mu\ ,$$

where the momenta K^μ and $\widetilde{K}^\mu$ are,

$$K^\mu = p_1^\mu + p_2^\mu - p_i^\mu\ \ , \widetilde{K}^\mu = \widetilde{p}_{1i}^\mu + p_2^\mu\ \ .$$

- Since $2\sum_j k_j \cdot K = 2K^2$ and $2\sum_j k_j \cdot (K+\widetilde{K}) = 2K^2 + 2K\cdot\widetilde{K} = (K+\widetilde{K})^2$, $K^2 = \widetilde{K}^2$, the momentum conservation constraint in the $m+1$-parton matrix

$$p_1^\mu + p_2^\mu - \sum_j k_j^\mu - p_i^\mu = 0\ \ .$$

implies

$$\widetilde{p}_{1i}^\mu + p_2^\mu - \sum_j \widetilde{k}_j^\mu = 0\ \ .$$

- Note also that the shifted momenta can be rewritten in the following way:

$$\widetilde{k}_j^\mu = \Lambda^\mu{}_\nu(K,\widetilde{K})\, k_j^\nu \quad ,$$

$$\Lambda^\mu{}_\nu(K,\widetilde{K}) = g^\mu{}_\nu - \frac{2(K+\widetilde{K})^\mu (K+\widetilde{K})_\nu}{(K+\widetilde{K})^2} + \frac{2\widetilde{K}^\mu K_\nu}{K^2} \quad ,$$

- the matrix $\Lambda^\mu{}_\nu(K,\widetilde{K})$ generates a proper Lorentz transformation on the final-state momenta.
- If the emitted parton has zero transverse momenta, the Lorentz transformation reduces to the identity, because $K^\mu = \widetilde{K}^\mu$

MCFM overview

John Campbell and R.K. Ellis

- Parton level cross-sections predicted to NLO in α_S

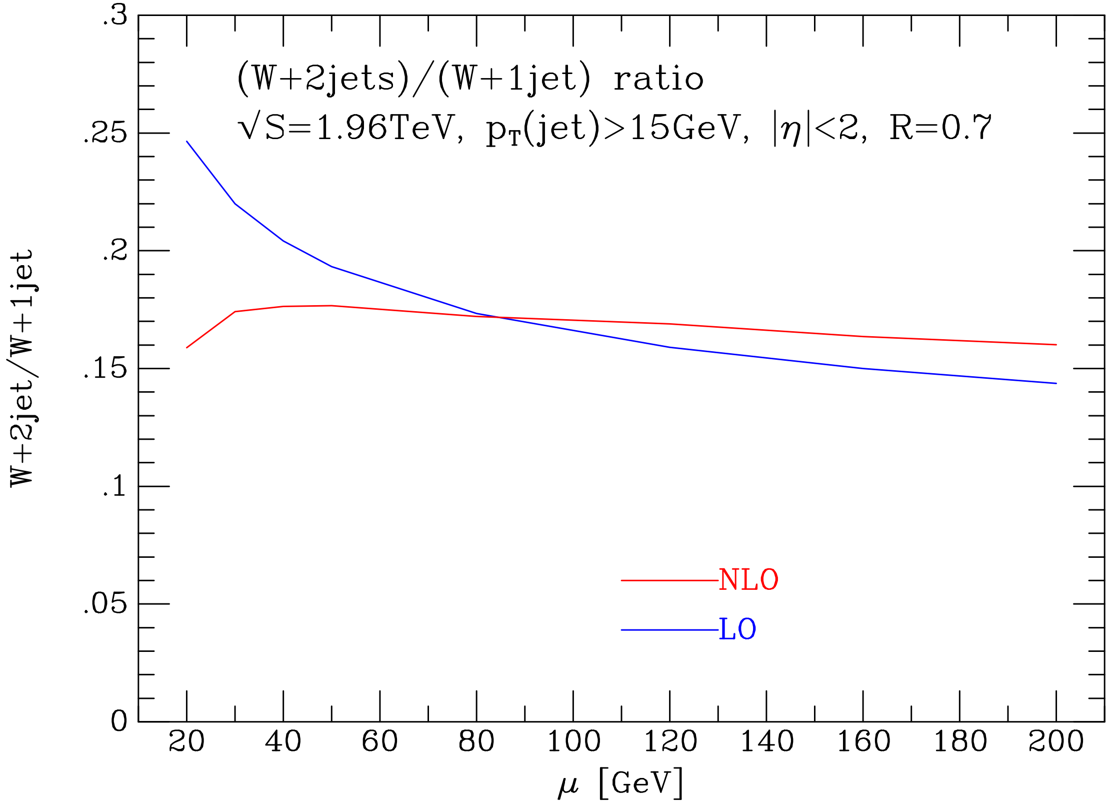
$p\bar{p} \to W^{\pm}/Z$	$p\bar{p} \to W^{+} + W^{-}$
$p\bar{p} \to W^{\pm} + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^{\pm} + \gamma$	$p\bar{p} \to W^{\pm}/Z + H$
$p\bar{p} \to W^{\pm} + g^{\star}\ (\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^{\pm}/Z +$ 1 jet	$p\bar{p} \to W^{\pm}/Z +$ 2 jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H +$ 1 jet
$p\bar{p}(VV) \to H +$ 2 jets	$p\bar{p} \to t + X$
$pp \to t + W$	

$\oplus$ less sensitivity to μ_R, μ_F, rates are better normalized, fully differential distributions.

$\ominus$ low particle multiplicity (no showering), no hadronization, hard to model detector effects

MCFM:examples

- (W+2 jet)/(W+1 jet)



An experimenter's wishlist

Run II Monte Carlo Workshop

Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		single top
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

A more realistic list

Les Houches workshop 2005

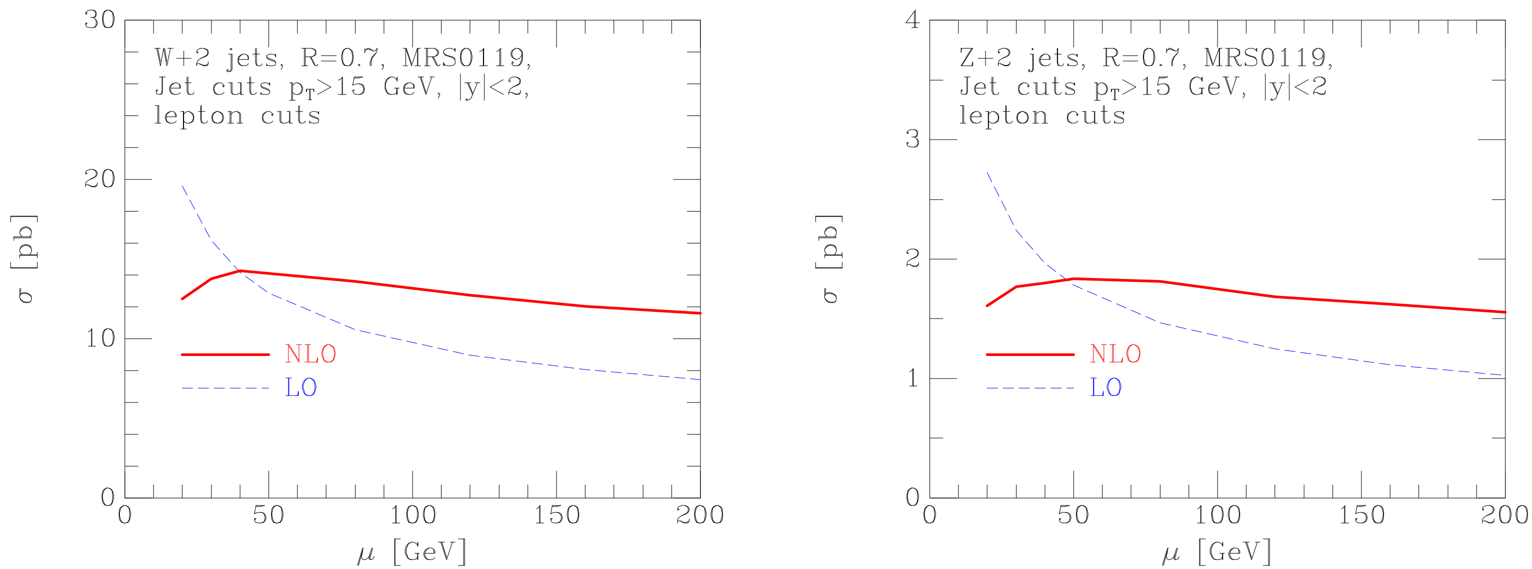
process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

- State of art
  - ⋆ Many calculations have been performed. for a list see, http://www.cedar.ac.uk/hepcode/
  - ⋆ MCFM is an attempt to collect together many results in a common framework

W+2 jet production

Campbell, Ellis,hep-ph/0202176

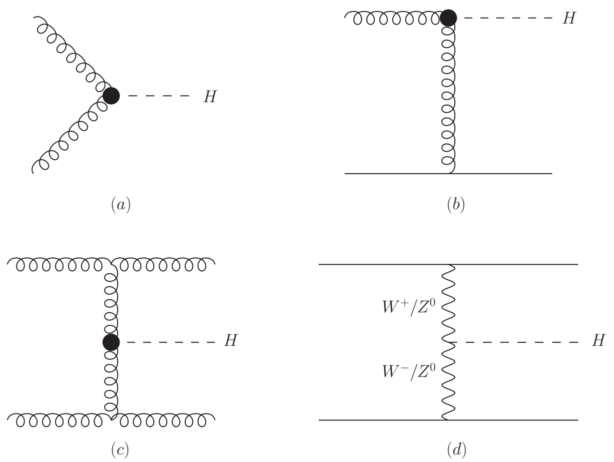
- Included in MCFM
- Calculated using analytic virtual matrix elements (Bern,Dixon,Kosower,hep-ph/9708239)



- Results for W + 2 jets at the Tevatron

Higgs + 2 jets

Campbell, Ellis, Zanderighi hep-ph/0608194



- sample diagrams representing the production of a Higgs boson at the LHC
- Process (d) is the expected to be the most significant discovery mode at the LHC for $115 < m_h < 160$.
- Process (c) represents a 'background', at least from the stand point of measuring the coupling of the Higgs boson to the W, Z bosons.
- We can calculate process (c) using an effective coupling of the Higgs boson to gluons, which represents a top quark loop in the limit of infinite top quark mass.

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} A(1+\Delta) H G^a_{\mu\nu} G^{a\,\mu\nu}\,,$$

- We have evaluated the QCD virtual corrections, using a semi-numerical method for the one-loop diagrams.

Vector boson fusion cuts

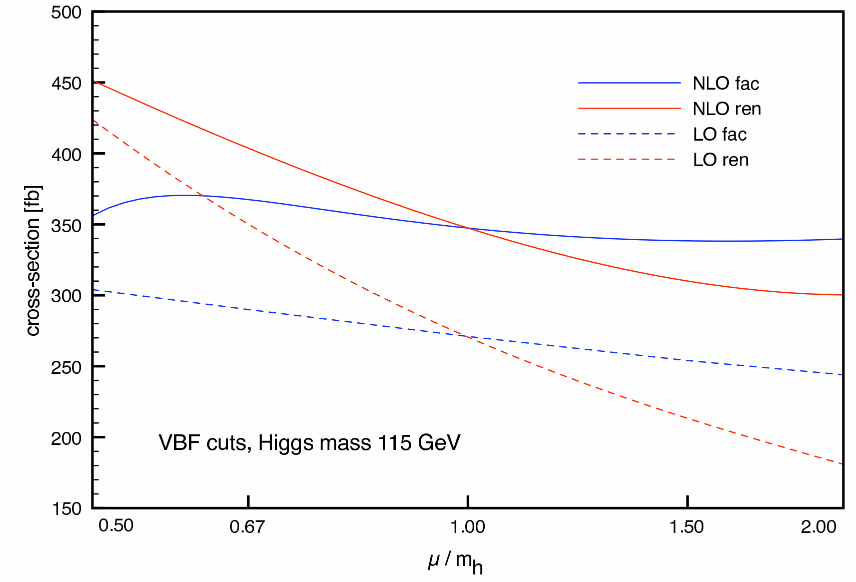
Higgs mass	115 GeV	160 GeV
$\sigma_{\rm LO}$ [pb]	3.50	2.19
$\sigma_{\rm NLO}$ [pb]	4.03	2.76
$\sigma_{\rm WBF}$ [pb]	1.77	1.32

- LO and NLO cross sections for the gluon-fusion process with the basic inclusive cuts, together with the weak boson fusion cross section at NLO.

Higgs mass	115 GeV	160 GeV
$\sigma_{\rm LO}$ [fb]	271	172
$\sigma_{\rm NLO}$ [fb]	346 ± 5	236 ± 3
$\sigma_{\rm WBF}$ [fb]	911	731

- LO and NLO cross sections with the weak boson fusion search cuts

$$|\eta_{j_1} - \eta_{j_2}| > 4.2, \qquad \eta_{j_1} \cdot \eta_{j_2} < 0 \; .$$



- Scale dependence with vector boson fusion cuts

One loop integrals

- Scalar box, triangles, bubbles and tadpoles give a complete basis. Once these integrals are known, all others can be derived,
- This is a great advantage since the scalar integrals are complicated objects, with analytic structure, branch cuts etc, as a function of invariants.
- Scalar pentagon integral can be reduced to a sum of five boxes, corresponding to the five pinchings of the propagators. Scalar hexagon integral can be reduced to a sum of six hexagons etc.
- Higher leg processes are important, eg. W=4 jets requires virtual corrections to a seven point process. This process is an important background for $t\bar{t}$ production.

Example: reduction of scalar hexagon

- Four dimensional scalar hexagon integral can be reduced to a sum of six pentagon integrals
- Consider a hexagon integral

$$\int d^4l \frac{1}{N_0 N_1 N_2 N_3 N_4 N_5} \text{ where } N_i = (l+q_i)^2 - m_i^2,\ N_0 = l^2 - m_0^2$$

- Take the Schouten identity

$$\epsilon^{q_1 q_2 q_3 q_4} l^\mu = \epsilon^{\mu q_2 q_3 q_4} l.q_1 + \epsilon^{q_1 \mu q_3 q_4} l.q_2 + \epsilon^{q_1 q_2 \mu q_4} l.q_3 + \epsilon^{q_1 q_2 q_3 \mu} l.q_4$$

$$v_1^\mu = \epsilon^{\mu q_2 q_3 q_4},\ v_2^\mu = \epsilon^{q_1 \mu q_3 q_4},\ v_3^\mu = \epsilon^{q_1 q_2 \mu q_4},\ v_4^\mu = \epsilon^{q_1 q_2 q_3 \mu},\ a = v_i.q_i = \epsilon^{q_1 q_2 q_3 q_4}.$$

$$l.q_i = \frac{1}{2}(N_i - N_0 - r_i),\ \ r_i = q_i^2 - m_i^2 + m_0^2$$

- Contracting Schouten identity with q_5

$$\begin{aligned} a\ l.q_5 &= \sum_i v_i.q_5\ l.q_i \\ 0 &= \int d^4l \frac{\sum_i r_i v_i.q_5 - a r_5 + a N_5 + N_0(\sum_i q_5.v_i - a) - \sum_i N_i v_i.q_5}{N_0 N_1 N_2 N_3 N_4 N_5} \end{aligned}$$

Tensor loop integrals

- Tensor loop can be reduced to sums scalar integrals using Passarino-Veltman decomposition.
- As an example consider the form factor decomposition of a simple rank 1 triangle diagram.

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{l^2 (l+p)^2 (l+q)^2} = \begin{pmatrix} p^\mu & q^\mu \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

- We can solve for C_1, C_2 by contracting with the external momenta, p, q.

$$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} [2l \cdot p] \\ [2l \cdot q] \end{pmatrix} = G \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \equiv \begin{pmatrix} 2p \cdot p & 2p \cdot q \\ 2p \cdot q & 2q \cdot q \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

where the notation is $[2l \cdot p] = \int \frac{d^n l}{(2\pi)^n} \frac{2l \cdot p}{l^2 (l+p)^2 (l+q)^2}$

- by expressing $2l \cdot p, (2l \cdot q)$ as a sum of denominators $2l \cdot p = (l+p)^2 - l^2 - p^2$ we can express R_1, R_2 as a sum of scalar integrals
- Solving we get

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = G^{-1} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

Tensor loop integrals II

- G is the Gram matrix

$$G = \begin{pmatrix} 2p\cdot p & 2p\cdot q \\ 2p\cdot q & 2q\cdot q \end{pmatrix}, \quad \Delta_2(p,q) = |G| = 4(p^2q^2 - (p\cdot q)^2)$$

$$G^{-1} = \frac{\begin{pmatrix} 2q\cdot q & -2p\cdot q \\ -2p\cdot q & 2p\cdot p \end{pmatrix}}{\Delta_2(p,q)}$$

- Thus the solution is $C = G^{-1}R$
- This solution appears to have a problem when $p \parallel q$ and the Gram determinant vanishes; the original tensor integral had no special problems when $p \parallel q$.
- G can be diagonalized by an orthogonal transformation $G = ODO^T$, $D =$ diag$\{\lambda_+, \lambda_-\}$
- Defining modified form factors C' and inhomogeneous terms, R' by the transformations $C' = O^T C, \;\; R' = O^T R$, we have the solution:-

$$\begin{pmatrix} C'_1 \\ C'_2 \end{pmatrix} = \begin{pmatrix} 1/\lambda_+ & 0 \\ 0 & 1/\lambda_- \end{pmatrix} \begin{pmatrix} R'_1 \\ R'_2 \end{pmatrix}$$

- In the singular region one of the eigenvalues, say λ_- will vanish

Singular region

- Now consider the approach to the singular region by setting $q_\mu = \kappa p_\mu + \delta_\mu$ and keeping only the leading terms in δ. The eigenvalues are

$$\lambda_+ = 2p^2(1+\kappa^2), \quad \lambda_- = \frac{2(\delta^2 p^2 - (\delta \cdot p)^2)}{p^2(1+\kappa^2)}, \quad |G| = 4(\delta^2 p^2 - (\delta \cdot p)^2)$$

- λ_- vanishes like $O(\delta^2)$
- The matrix of eigenvectors is

$$O \sim \frac{1}{\sqrt{1+\kappa^2}} \begin{pmatrix} 1 - \frac{\kappa\delta.p}{p^2(1+\kappa^2)} & \kappa + \frac{\kappa\delta.p}{p^2(1+\kappa^2)} \\ \kappa + \frac{\kappa\delta.p}{p^2(1+\kappa^2)} & -1 + \frac{\kappa\delta.p}{p^2(1+\kappa^2)} \end{pmatrix}$$

Singular region II

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{l^2(l+p)^2(l+q)^2} = \begin{pmatrix} p'_\mu & q'_\mu \end{pmatrix} \begin{pmatrix} C'_1 \\ C'_2 \end{pmatrix}$$

$$= \begin{pmatrix} p'_\mu & q'_\mu \end{pmatrix} \begin{pmatrix} 1/\lambda_+ & 0 \\ 0 & 1/\lambda_- \end{pmatrix} \begin{pmatrix} R'_1 \\ R'_2 \end{pmatrix}$$

- The momentum corresponding to the singular eigenvalue is

$$q'_\mu = -\delta_\mu + \frac{\delta \cdot p \kappa (1+\kappa)}{p^2(1+\kappa^2)} = O(\delta)$$

$$R'_2 \sim \kappa[2l \cdot p] - [2l \cdot q] \sim O(\delta)$$

- As expected the result for the tensor integral is finite in the limit $\delta \to 0$, but the vanishing of R'_2 is not manifest; it is realized as a property of a combination of scalar integrals.
- This can cause numerical instabilities, which have to be protected against.

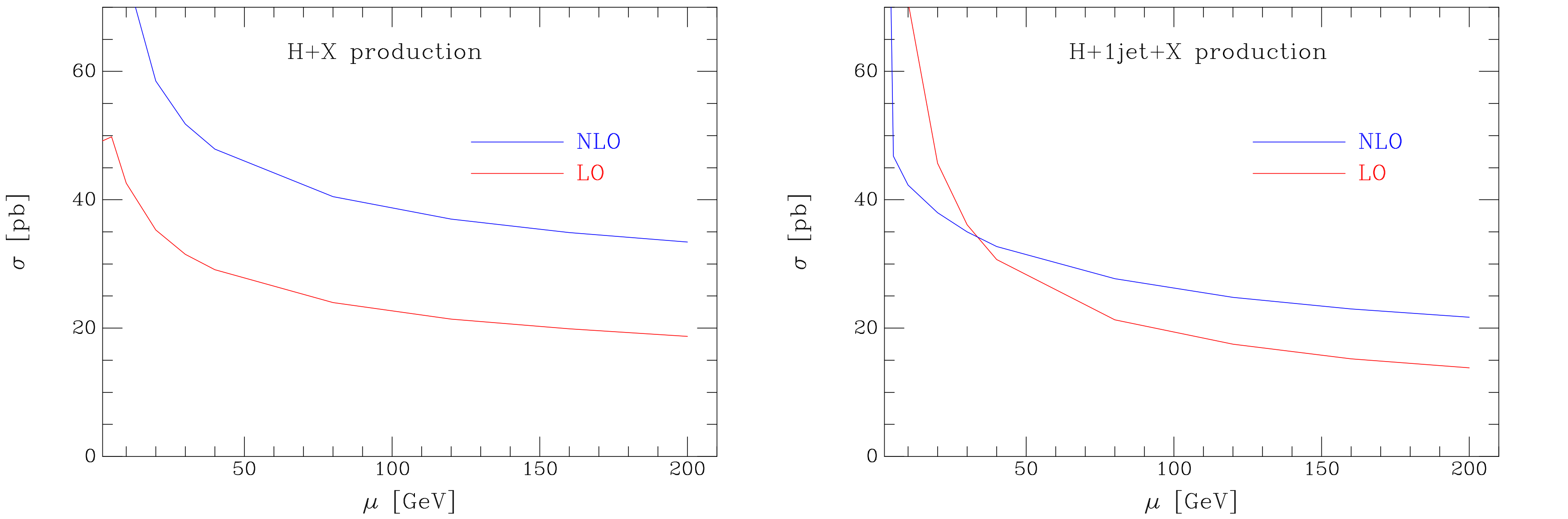
Limitations of parton level programs

- No resummation of large corrections, (soft, collinear, threshold) at phase space boundaries
- Only one additional parton
- Not a good description of more exclusive observables
- event weights can be negative
- Only parton level events

Some of these limitations are overcome by shower Monte Carlo event generators. We shall now illustrate a procedure to combine NLO calculations with parton showers.

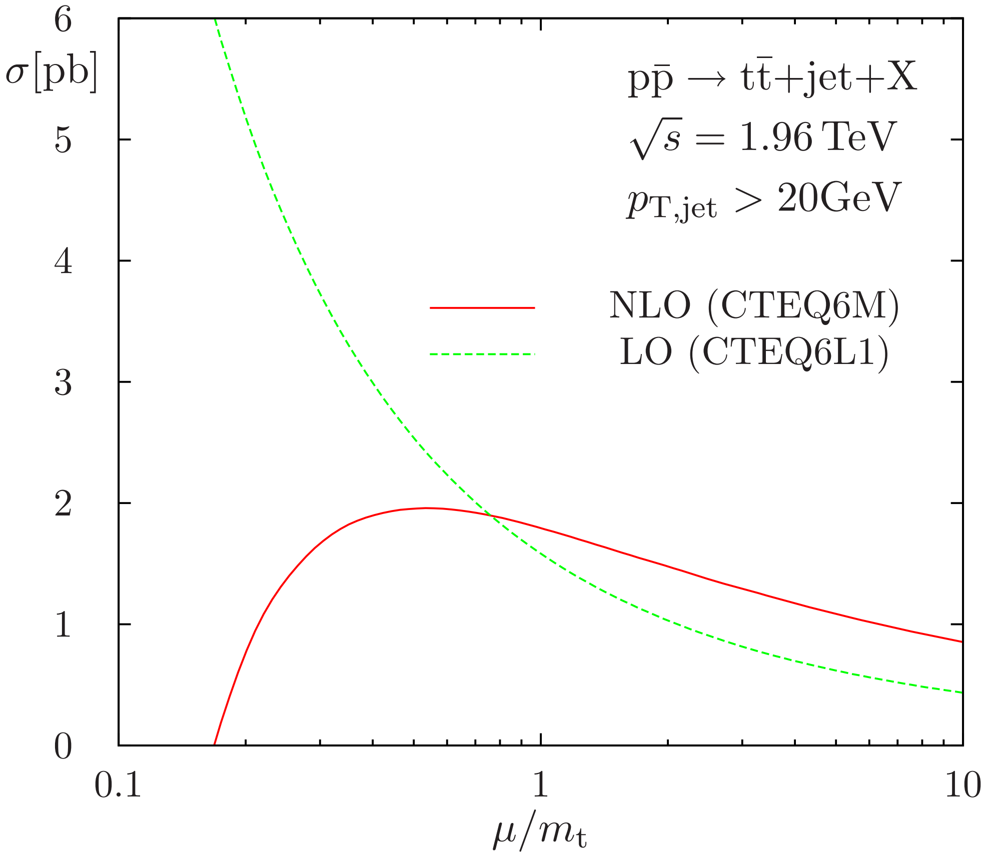
MCFM example

- Production of a $m_H = 120$ GeV Higgs, using effective Lagrangian $HG^{\mu\nu}G_{\mu\nu}$, obtained in heavy top limit.
- Cross sections for Higgs+anything or Higgs+1 jet+anything are the same.
- Radiation probability is one, and NLO is clearly inadequate.
- what is needed is a combination of NLO and shower Monte-Carlo, (MC@NLO)



t tbar + jet production

S. Dittmaier, P. Uwer, S. Weinzierl hep-ph/0703120

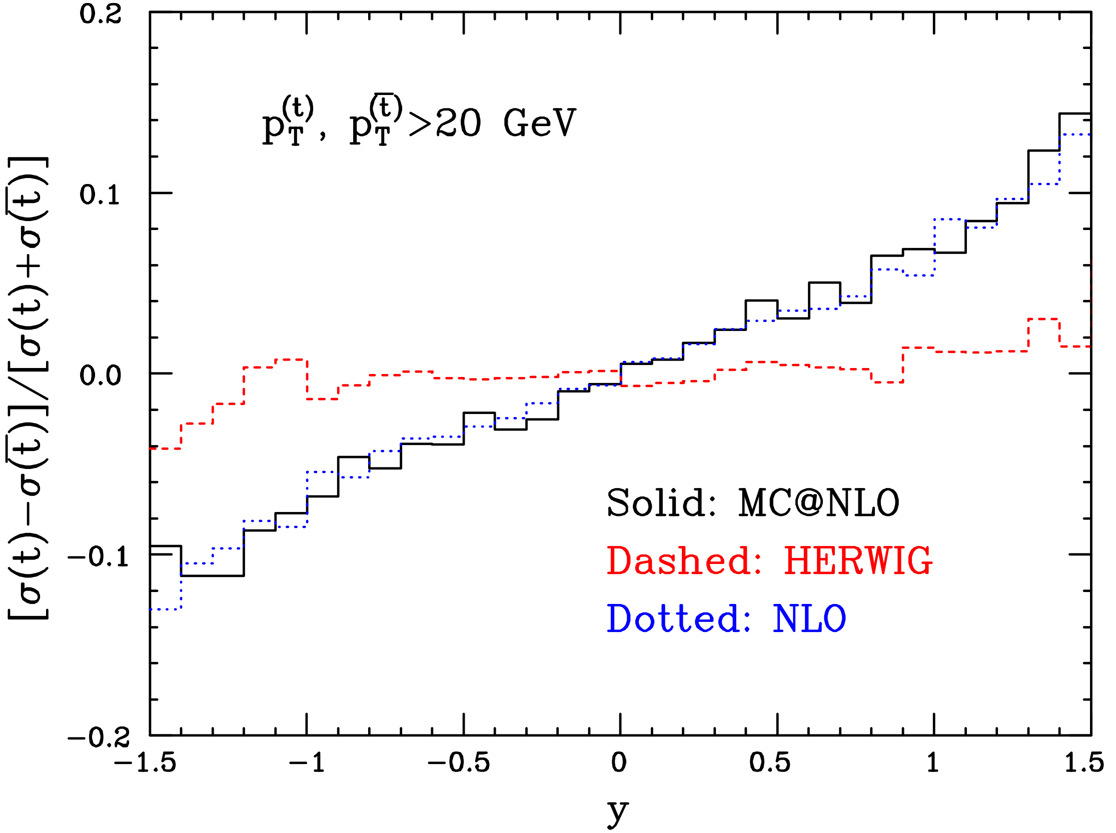


- Stability of jet + $t\bar{t}$ cross section
- Cross section of similar order to inclusive cross section, $\sigma_{t\bar{t}} \sim 6$ pb
- Virtual corrections calculated by improved Passarino-Veltman decomposition.

Asymmetry in top production

Frixione,Nason,Webber

- MC@NLO, Basic processes are Drell-Yan +variants, Vector boson pairs, single top production, and $Q\bar{Q}$ production
- Example of $t\bar{t}$-production using MC@NLO
- NLO curve (in blue, dotted).



NLO: Schematic description

- A schematic description of a NLO calculation is as follows.

$$
\begin{aligned}
\left(\frac{d\sigma}{dx}\right)_B &= B\delta(x) \\
\left(\frac{d\sigma}{dx}\right)_V &= a\left(\frac{B}{2\epsilon}+V\right)\delta(x) \\
\left(\frac{d\sigma}{dx}\right)_R &= a\frac{R(x)}{x}
\end{aligned}
$$

- In terms of the above the calculation of any observable O can written as

$$
\langle O\rangle = \lim_{\epsilon\to 0}\int_0^1 dx x^{-2\epsilon} O(x)\left[\left(\frac{d\sigma}{dx}\right)_B + \left(\frac{d\sigma}{dx}\right)_V + \left(\frac{d\sigma}{dx}\right)_R\right]
$$

Subtraction method

We can isolate the divergent part of the real radiation contribution

$$\langle O \rangle_{\mathrm{R}} = aBO(0) \int_0^1 dx\, \frac{x^{-2\epsilon}}{x} + a \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x^{1+2\epsilon}} \; .$$

The second term does not contain singularities so we can set $\epsilon = 0$

$$\langle O \rangle_{\mathrm{R}} = -a\frac{B}{2\epsilon}O(0) + a \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x} \; .$$

The NLO prediction using the subtraction method is

$$\langle O \rangle_{\mathrm{sub}} = BO(0) + a \left[VO(0) + \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x} \right] .$$

Toy Monte Carlo

Frixione-Webber

- Rewrite the basic NLO formula in a different which allows simpler matching with the Monte Carlo:

$$\langle O \rangle_{\rm sub} = \int_0^1 dx \left[O(x) \frac{aR(x)}{x} + O(0) \left(B + aV - \frac{aB}{x} \right) \right].$$

- Introduce Sudakov form factor for the toy model

$$\Delta(x_1, x_2) = \exp \left[-a \int_{x_1}^{x_2} dz \frac{Q(z)}{z} \right],$$

where $Q(z)$ is a radiation function with the following general properties:

$$0 \leq Q(z) \leq 1, \quad \lim_{z \to 0} Q(z) = 1, \quad \lim_{z \to 1} Q(z) = 0.$$

If x_s is the energy of the system before the first branching occurs, then $\Delta(x, x_s)$ is the probability that no photon be emitted with energy z such that $x \leq z \leq x_s$.

Matching NLO and MC

$$\left(\frac{d\sigma}{dO}\right)_{\mathrm{MC@LO}} = B I_{\mathrm{MC}}(O,1).$$

$$\left(\frac{d\sigma}{dO}\right)_{\mathrm{naive}} = \int_0^1 dx \left[I_{\mathrm{MC}}(O, x_{\mathrm{M}}(x)) \frac{aR(x)}{x} + I_{\mathrm{MC}}(O,1) \left(B + aV - \frac{aB}{x} \right) \right].$$

This equation suggests the following procedure:

- Pick at random $0 \leq x \leq 1$.
- Generate an MC event with $x_{\mathrm{M}}(x)$ as maximum energy available to the photon in the first branching; attach to this event the weight $w_{\mathrm{EV}} = aR(x)/x$.
- Generate another MC event (a "counter-event") with $x_{\mathrm{M}} = 1$; attach to this event the weight $w_{\mathrm{CT}} = B + aV - aB/x$.
- Repeat the first three steps N times, and normalize with $1/N$.

This procedure fails, since the weights w_{EV} and w_{CT} diverge as $x \to 0$.

Modified subtraction method

$$\left(\frac{d\sigma}{dO}\right)_{\rm msub} = \int_0^1 dx \Bigg[I_{\rm MC}(O, x_{\rm M}(x)) \frac{a[R(x) - BQ(x)]}{x} + I_{\rm MC}(O,1)\left(B + aV + \frac{aB[Q(x)-1]}{x}\right)\Bigg].$$

- We subtract and add the quantities

$$I_{\rm MC}(O,1)\frac{aBQ(x)}{x}, \quad I_{\rm MC}(O,x_{\rm M})\frac{aBQ(x)}{x}$$

- The two terms involving $Q(x)$ are not identical, so this is not a subtraction in the usual sense of an NLO computation.
- The two terms do not contribute to the observable O at $\mathcal{O}(a)$, because they are compensated by terms in the parton shower $BI_{\rm MC}(O,1)$

Expansion to $O(\alpha_S)$

- Expansion of Monte Carlo piece is

$$I_{\rm MC} = (1 - a\int_{x_0}^1 dt \frac{Q(t)}{t}\ \delta(O - O(0)) + a\int_{x_0}^1 dt \frac{Q(t)}{t}\ \delta(O - O(t)) + O(a^2)$$

- Insertion of this piece in the modified Monte-Carlo formula gives

$$\begin{aligned}
\left(\frac{d\sigma}{dO}\right)_{\rm msub} \quad = \quad & \int_0^1 dx \Bigg[\delta(O - O(x)) \frac{a[R(x) - BQ(x)]}{x} \\
& + \delta(O - O(0)) \left(B + aV - \frac{aB}{x} \right) \\
& + aB\delta(O - O(0)) \left(\frac{Q(x)}{x} - \int_{x_0}^1 dt \frac{Q(t)}{t} \right) \\
& + aB \int_{x_0}^1 dt \delta(O - O(t)) \frac{Q(t)}{t} \Bigg] + \mathcal{O}(a^2).
\end{aligned}$$

Expansion (continued)

- Collecting terms we obtain the starting formula for a NLO correction, plus power suppressed terms which are anyway not controlled in the Monte Carlo

$$
\begin{aligned}
\left(\frac{d\sigma}{dO}\right)_{\rm msub} &= \int_0^1 dx \left[\delta(O-O(x))\frac{aR(x)}{x} + \delta(O-O(0))\left(B + aV - \frac{aB}{x}\right)\right] \\
&+ aB\int_0^{x_0} dx \frac{Q(x)}{x}\left[\delta(O-O(0)) - \delta(O-O(x))\right] + \mathcal{O}(a^2).
\end{aligned}
$$

- It can also be shown that the normal summation of branching logarithms is not compromised by this procedure.