The Standard Model of Electroweak Physics

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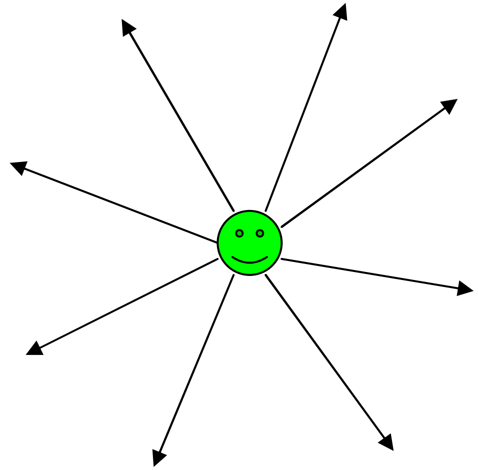
Lecture I: Incarnations of Symmetry

Noether's Theorem is as important to us now as the Pythagorean Theorem

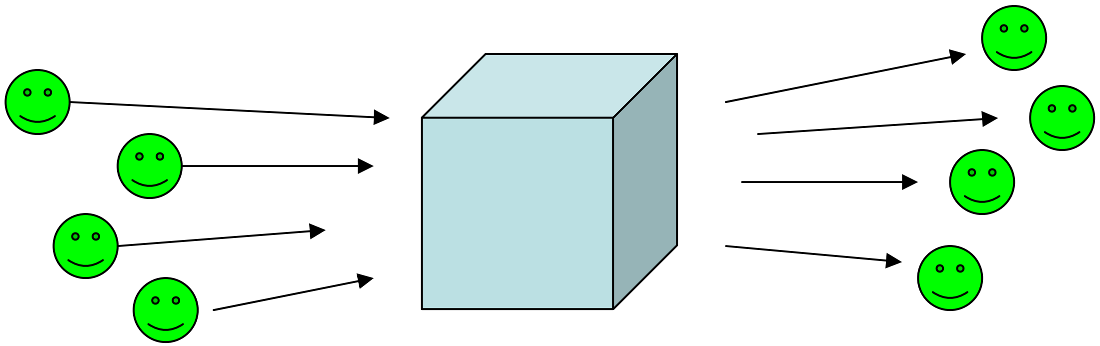
Emmy Noether 1882-1935

Electricity and Magnetism

Electric charge:



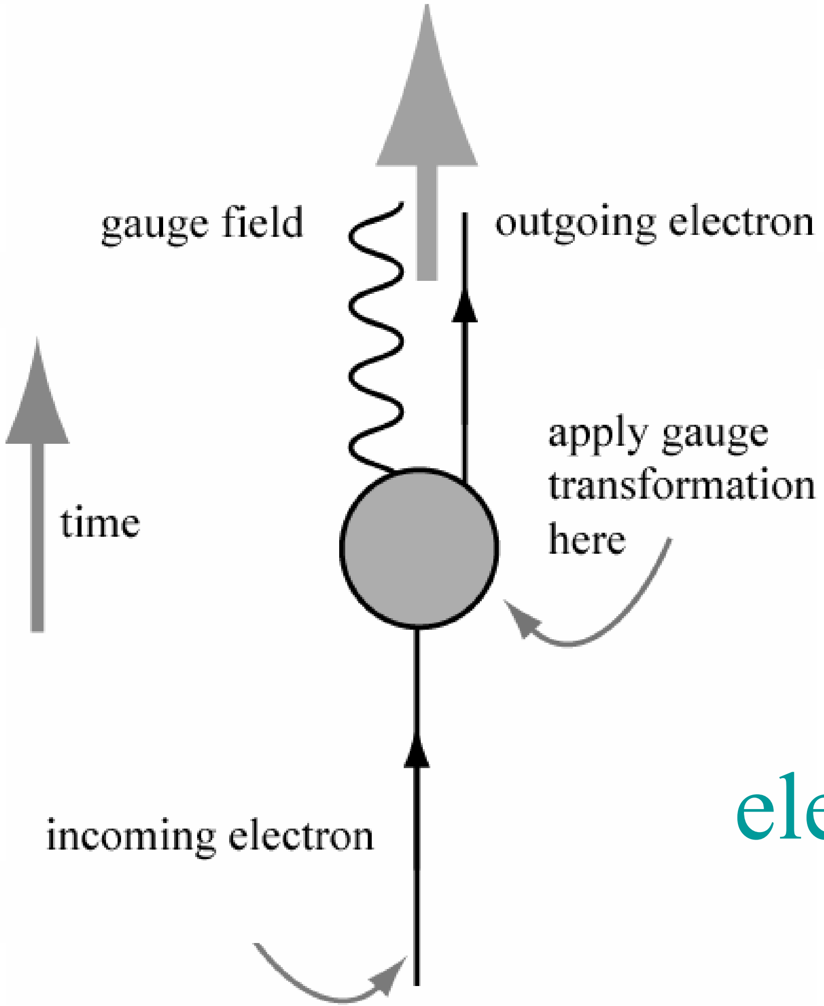
Electric charge is conserved:



What is the Symmetry that leads, by Noether's Theorem to electric charge conservation?

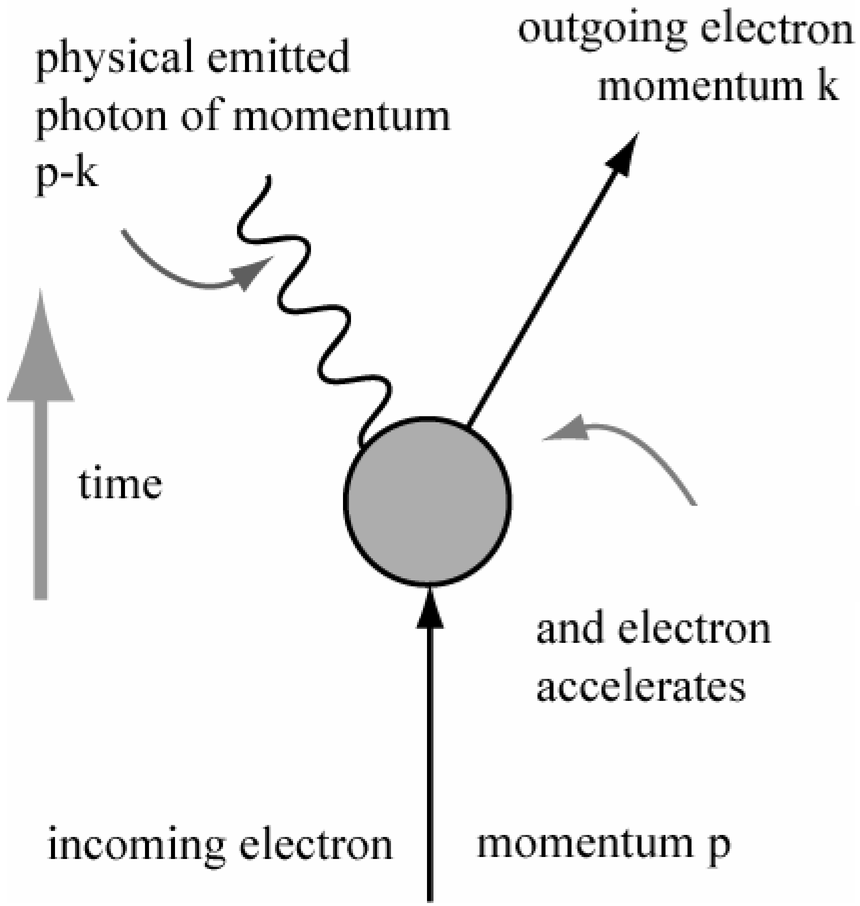
In Five Easy Pieces

I. Local Gauge Symmetry of Electrodynamics

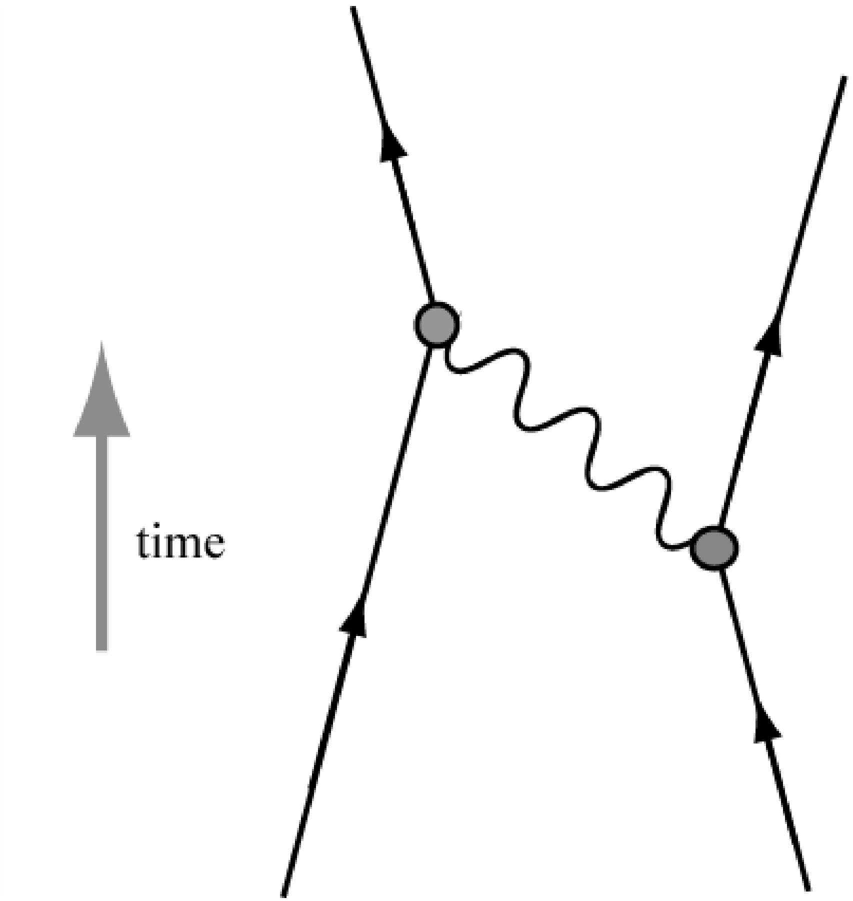


electron =
electron + gauge field

The "emission" of a photon

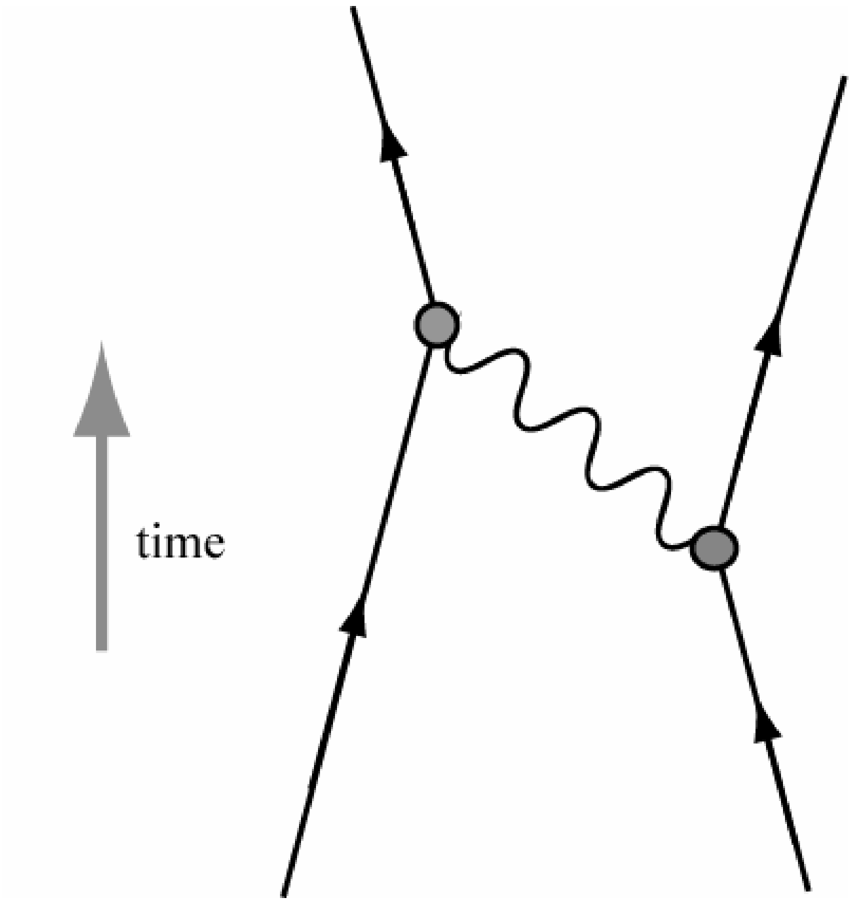


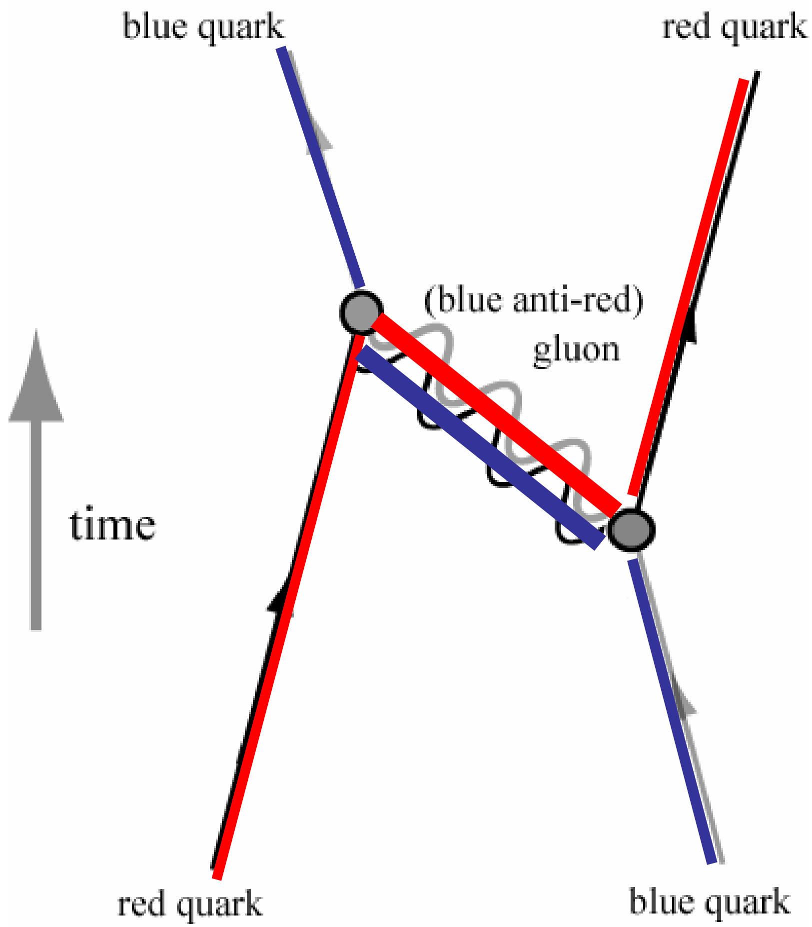
Electromagnetic force



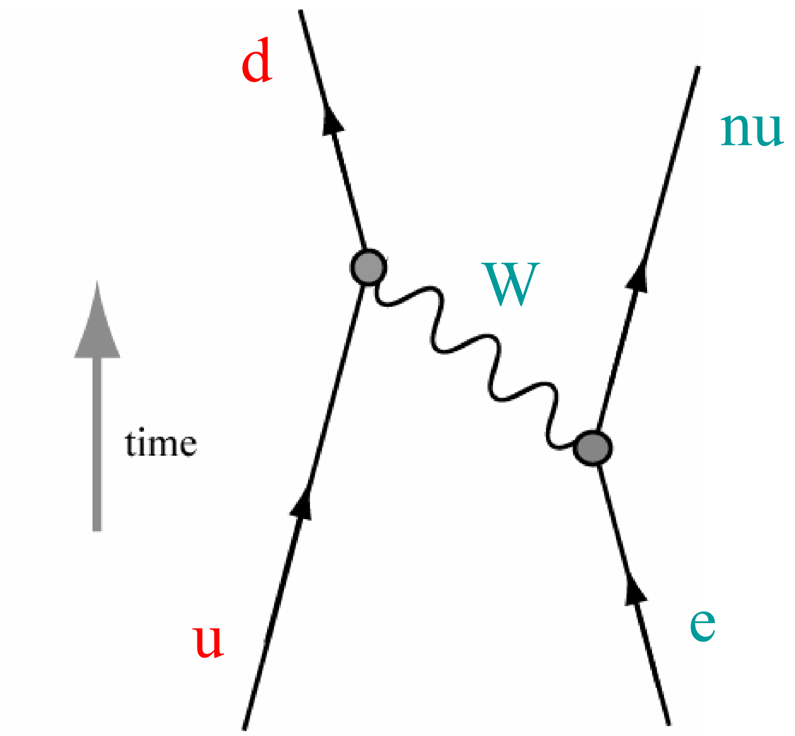
Electromagnetic force

Quark color force





Weak Force: SU(2) x U(1)



What gives rise to masses of W and Z boson?

SU(2)xU(1) is “Spontaneously broken Symmetry”

Higgs Field?

U(1) Local Gauge Invariance on a Wallet Card

$$\Psi(x,t) \longrightarrow e^{i\theta(x,t)}\Psi'(x,t)$$ Local

$$D_\mu\psi(x,t) \longrightarrow e^{i\theta(x,t)}D'_\mu\psi'(x,t)$$

$$D_\mu = \partial_\mu - ieA_\mu(x,t)$$ covariant derivative

$$A_\mu(x,t) \longrightarrow A_\mu(x,t)' + \frac{1}{e}\partial_\mu\theta$$

U(1) Local Gauge Invariance on a Wallet Card

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$D_\mu\psi(x,t) \longrightarrow e^{i\theta(x,t)}D'_\mu\psi'(x,t)$

$D_\mu = \partial_\mu - ieA_\mu(x,t)$

$A_\mu(x,t) \longrightarrow A_\mu(x,t)' + \frac{1}{e}\partial_\mu\theta$

field strength $F_{\mu\nu} = \frac{i}{e}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$

invariants $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Psi)^\dagger D^\mu\Psi - M^2\Psi^\dagger\Psi$

$\mathcal{L}(\Psi, A_\mu) = \mathcal{L}(\Psi', A_\mu')$ Gauge Invariance

Yang-Mills Local Gauge Invariance on a Wallet Card

$$\psi \rightarrow e^{i\phi^a Q^a} \psi$$

$$D_\mu = \partial_\mu - igA_\mu^a Q^a$$

$$[Q^a, Q^b] = if^{abc} Q^c$$

$$A_\mu^a \rightarrow A_\mu^a + \frac{1}{g}\partial_\mu \phi^a - f^{abc} A_\mu^b \phi^c$$

$$\frac{i}{g}[D_\mu, D_\nu] = Q^a \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - f^{abc} A_\mu^b A_\nu^c\right) \equiv Q^a G_{\mu\nu}^a$$

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}$$

Homework:

Verify all of the transformations laws stated on the previous pages for U(1) and Yang-Mills gauge theories.

II. Can a gauge field have a mass?

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$$

$$A_\mu(x,t) \longrightarrow A_\mu(x,t)' + \frac{1}{e}\partial_\mu\theta$$

$$\mathcal{L} \rightarrow -\frac{1}{4}F'_{\mu\nu}F^{\mu\nu\prime} + \frac{1}{2}m^2(A'_\mu + \frac{1}{e}\partial_\mu\theta)^2$$ NO!!!

Can a gauge field have a mass? Yes!

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$$

$$A_\mu(x,t) \longrightarrow A_\mu(x,t)' + \frac{1}{e}\partial_\mu\theta$$

$$\mathcal{L} \to -\frac{1}{4}F'_{\mu\nu}F^{\mu\nu\prime} + \frac{1}{2}m^2(A'_\mu + \frac{1}{e}\partial_\mu\theta)^2$$ NO!!!

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu + \frac{1}{2}(\partial_\mu\phi)^2 - mA_\mu\partial^\mu\phi$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2(A_\mu - \frac{1}{m}\partial_\mu\phi)^2$$

$$\mathcal{L} \to -\frac{1}{4}F'_{\mu\nu}F^{\mu\nu\prime} + \frac{1}{2}m^2(A'_\mu - \frac{1}{m}\partial_\mu\phi')^2$$

$$\phi \longrightarrow \phi' + \frac{m}{e}\theta$$ OK!!!

London's Theory of a superconductor:

$$L = -\frac{\hbar^2}{2m}|(\vec{\nabla} - i(e/c)\vec{A})\psi|^2 - i\psi^\dagger\partial_t\psi$$

Wave-function becomes "rigid:"

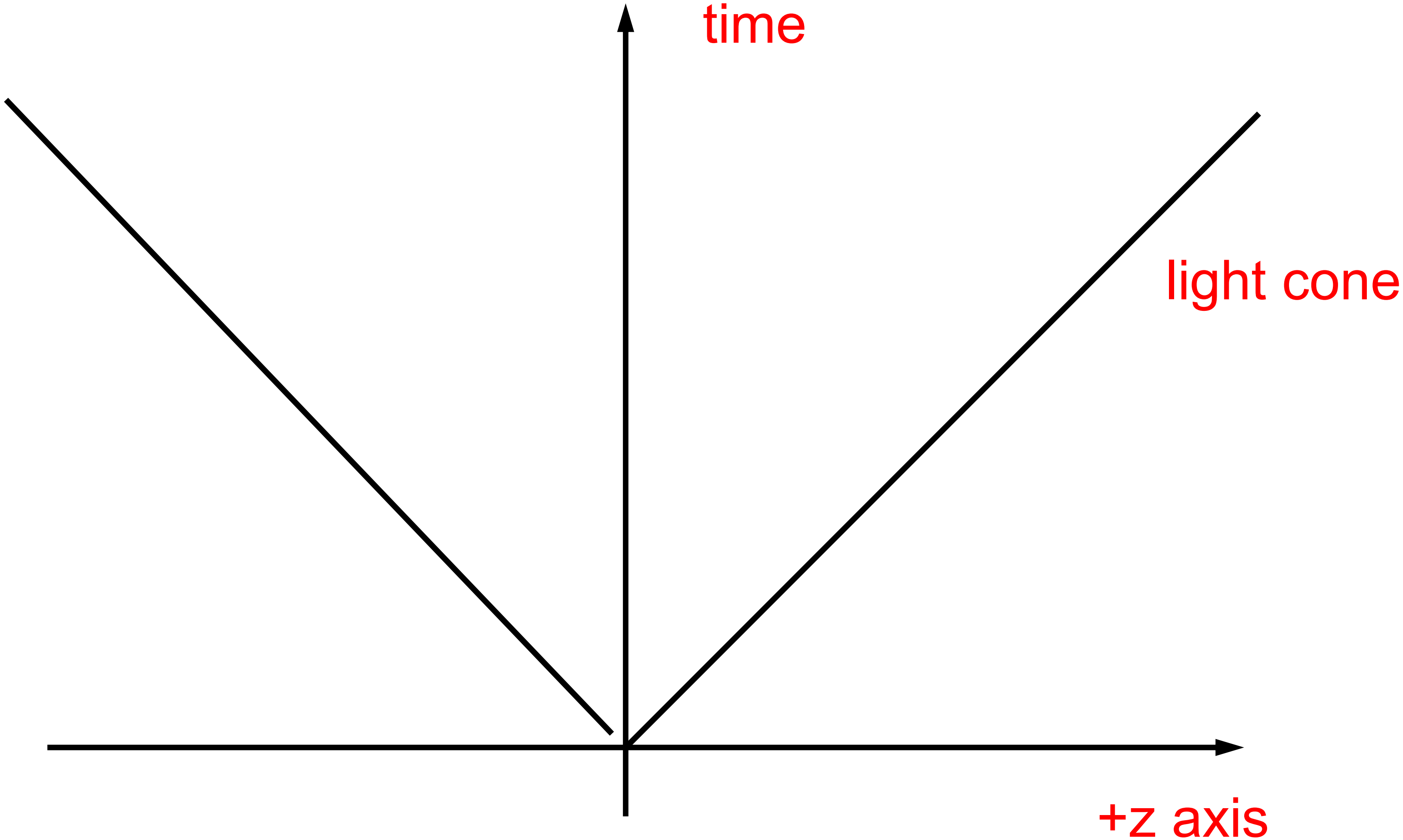
$$\psi \to \sqrt{n_e}e^{ie\phi/c}$$

$$L = -\frac{\hbar^2 e^2 n_e}{2mc^2}|(\vec{\nabla}\phi - \vec{A})\psi|^2 - i\psi^\dagger\partial_t\psi$$

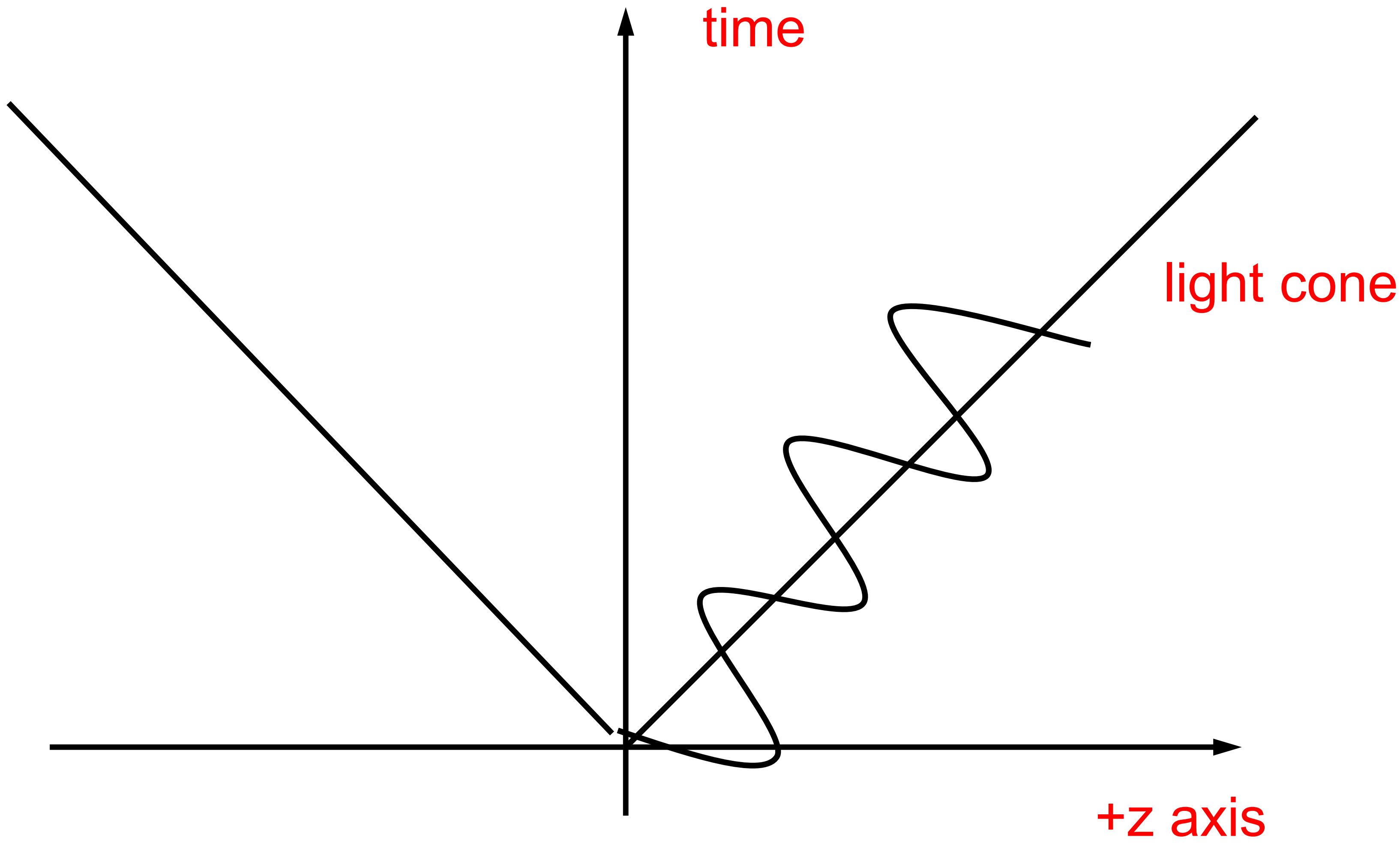
$$m_\gamma = \frac{\hbar e\sqrt{n_e}}{\sqrt{2m}c}$$

Superconductivity → Massive Gauge Field

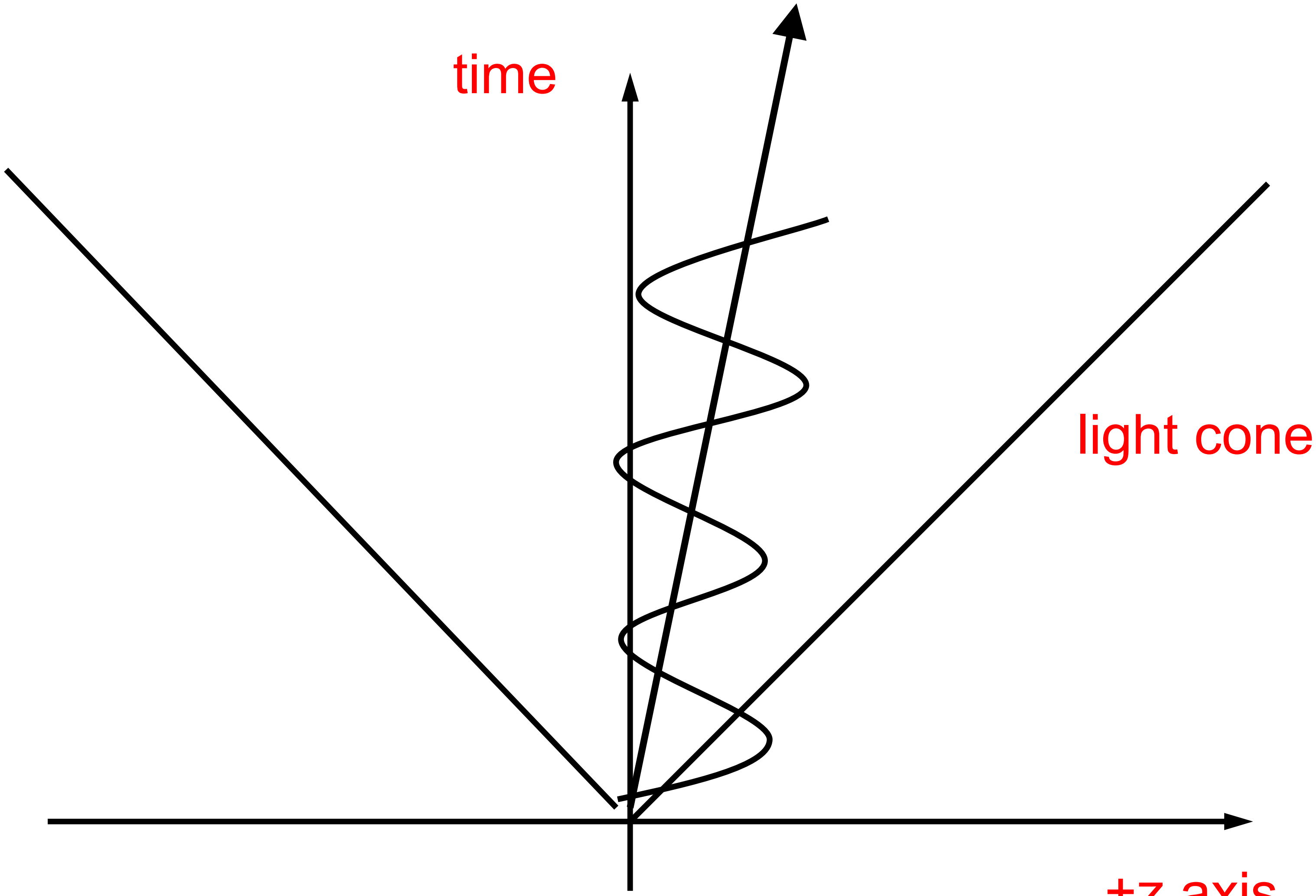
Cartoon Feature: Gauge Boson Mass



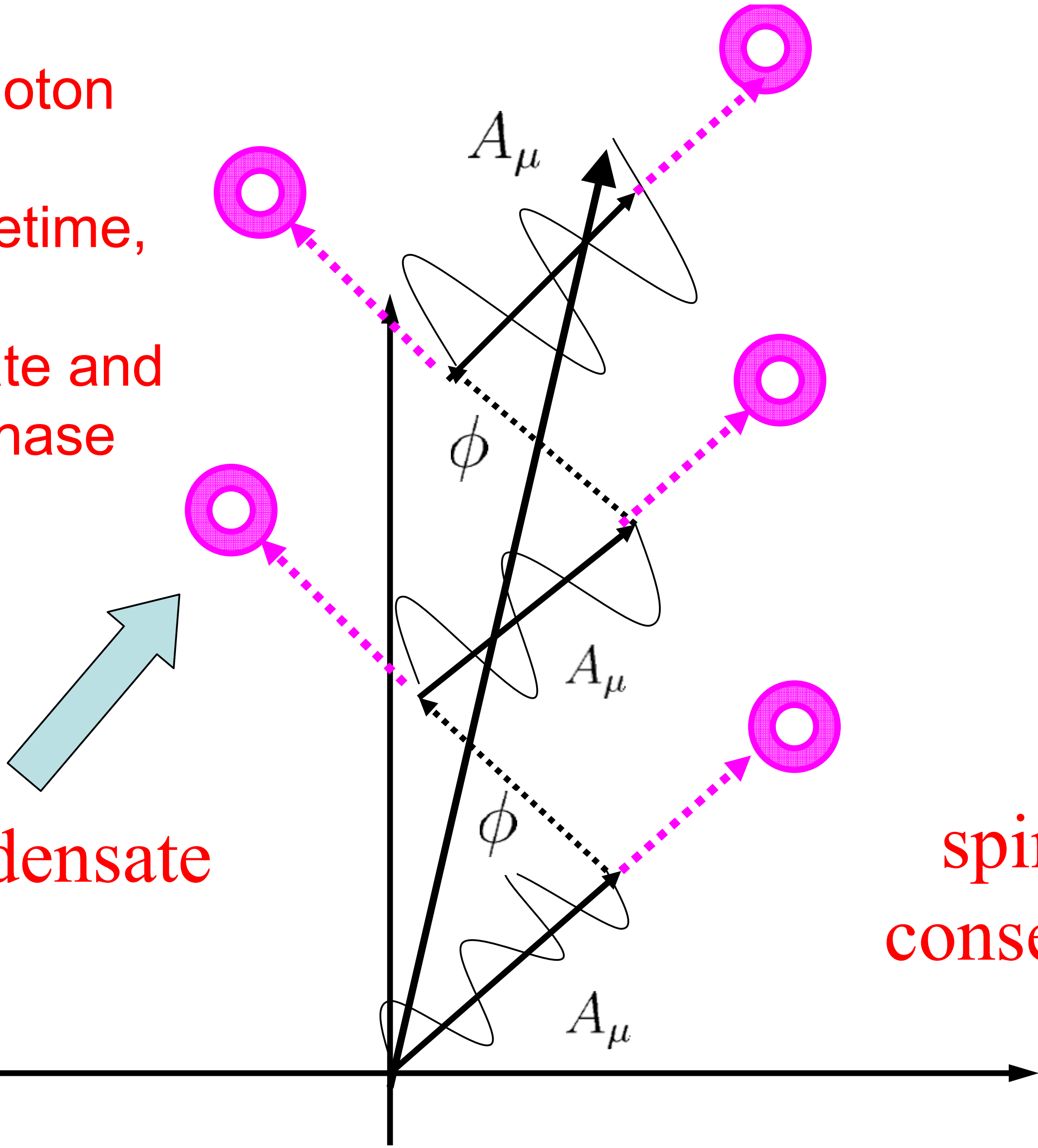
Massless Photon



A massive photon



A massive photon oscillates through spacetime, colliding with the condensate and mixing with phase degree of freedom:



Charge condensate

spin is conserved

Superconductivity =

Massive photon Field

Superconductivity →
Massive photon Field

Our vacuum is an electroweak
superconductor →
Massive W and Z bosons

Homework:

Consider a charged scalar field coupled to the photon and described the Lagrangian:

$$\mathcal{L}'_\Phi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(i\partial_\mu - eA_\mu)\Phi|^2 - V(\Phi)$$

$$V(\Phi) = -M^2|\Phi|^2 + \tfrac{1}{2}\lambda|\Phi|^4$$

Show that this is gauge invariant under:

$$\Phi \rightarrow e^{-ie\chi}\Phi \qquad A_\mu \rightarrow A_\mu + \partial_\mu\chi$$

Show that if : $\Phi = ve^{i\theta(x,t)}/\sqrt{2}$

then the photon acquires a gauge invariant mass: $M = ev$

III. Massless Fermions and Chiral Symmetry

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi \; ; \qquad \psi_R = \frac{1}{2}(1 + \gamma_5)\psi$$

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \, \psi = \bar{\psi}_L i \partial\!\!\!/ \, \psi_L + \bar{\psi}_R i \partial\!\!\!/ \, \psi_R$$

Massless Fermions have Chiral Symmetry

$$\psi_L = \tfrac{1}{2}(1-\gamma_5)\psi\ ; \qquad \psi_R = \tfrac{1}{2}(1+\gamma_5)\psi$$

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \, \psi = \bar{\psi}_L i \partial\!\!\!/ \, \psi_L + \bar{\psi}_R i \partial\!\!\!/ \, \psi_R$$

$$\text{“}\textit{chiral symmetry}\text{”}\ U(1)_L \times U(1)_R$$

$$\psi_L \to \exp(-i\theta)\psi_L \qquad \psi_R \to \exp(-i\omega)\psi_R$$

$$j_{\mu L} \equiv \frac{\delta \mathcal{L}}{\delta \partial_\mu \theta(x)} = \tfrac{1}{2}\bar{\psi}\gamma_\mu(1-\gamma_5)\psi \qquad j_{\mu R} \equiv \frac{\delta \mathcal{L}}{\delta \partial_\mu \omega(x)} = \tfrac{1}{2}\bar{\psi}\gamma_\mu(1+\gamma_5)\psi$$

vector current, $j_\mu = j_{\mu R} + j_{\mu L} = \bar{\psi}\gamma_\mu\psi$

axial vector current, $j^5_\mu = j_{\mu R} - j_{\mu L} = \bar{\psi}\gamma_\mu\gamma_5\bar{\psi}.$

Both currents are conserved when fermions are massless and chiral symmetry is exact (modulo anomalies)

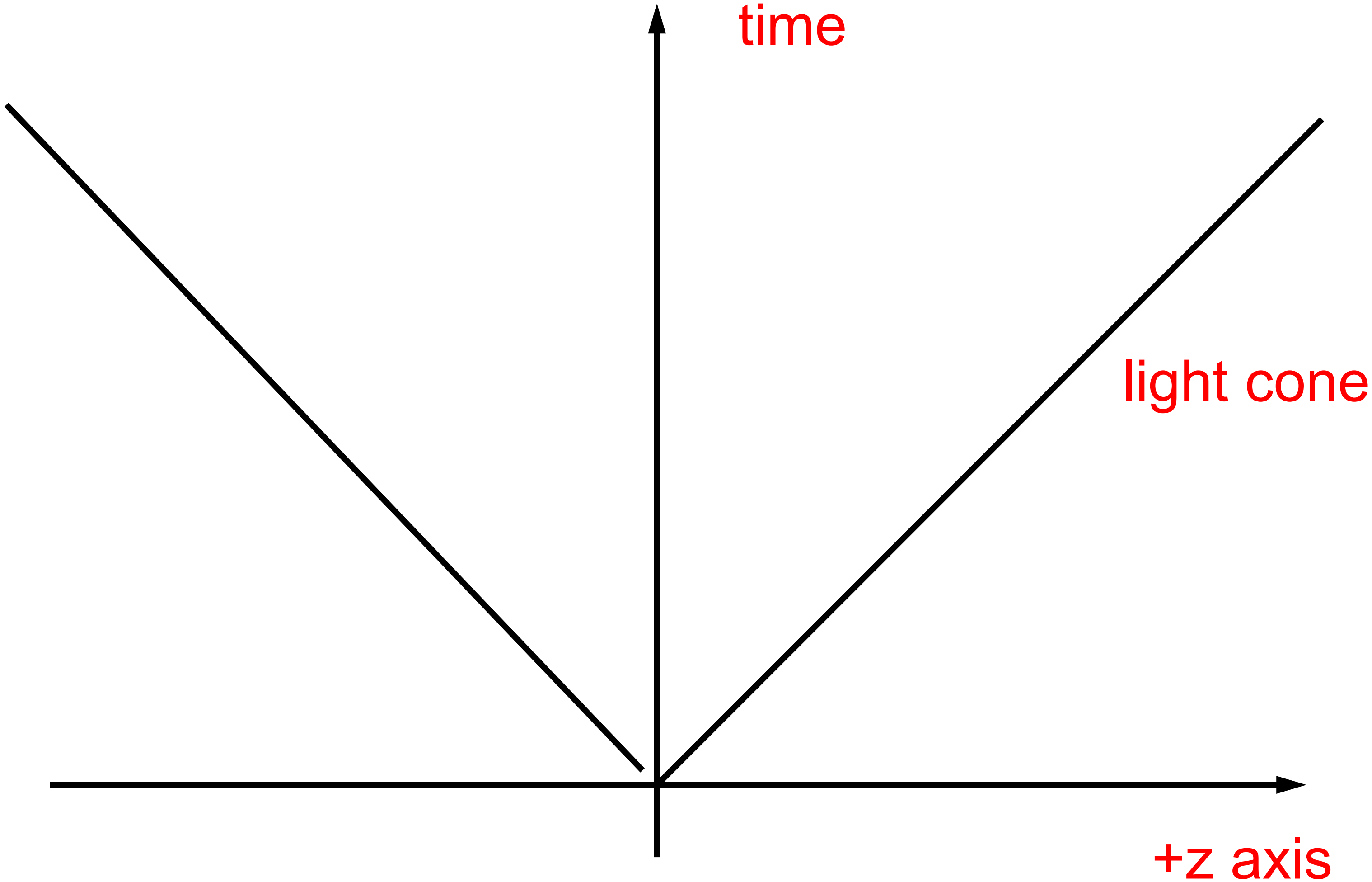
Each chiral current corresponds to a different "chiral charge"

vector current, $j_\mu = j_{\mu R} + j_{\mu L} = \bar{\psi}\gamma_\mu\psi$

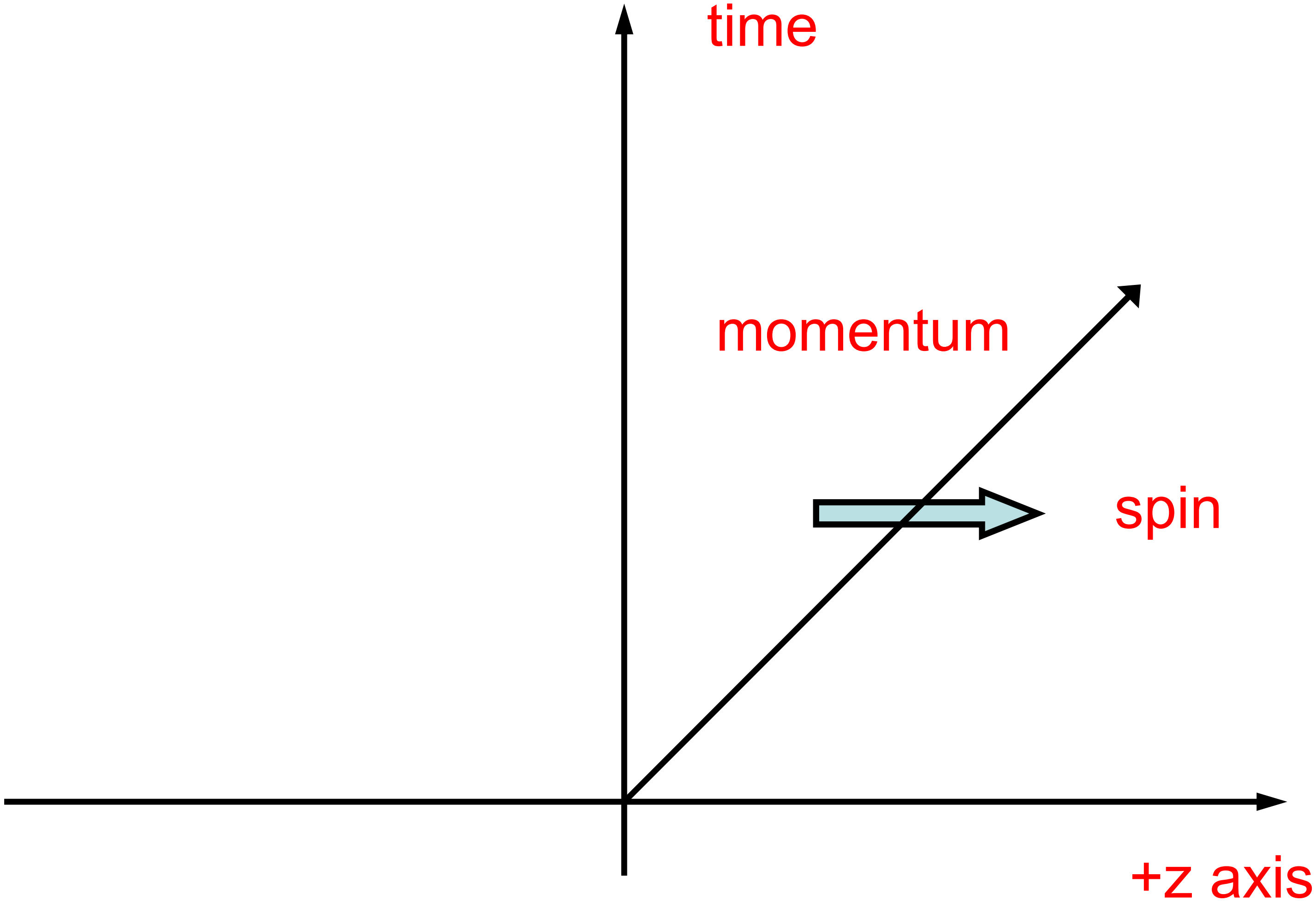
axial vector current, $j_\mu^5 = j_{\mu R} - j_{\mu L} = \bar{\psi}\gamma_\mu\gamma_5\bar{\psi}.$

e.g., L couples to W-bosons
while R does not
Standard Model is "flavor chiral"

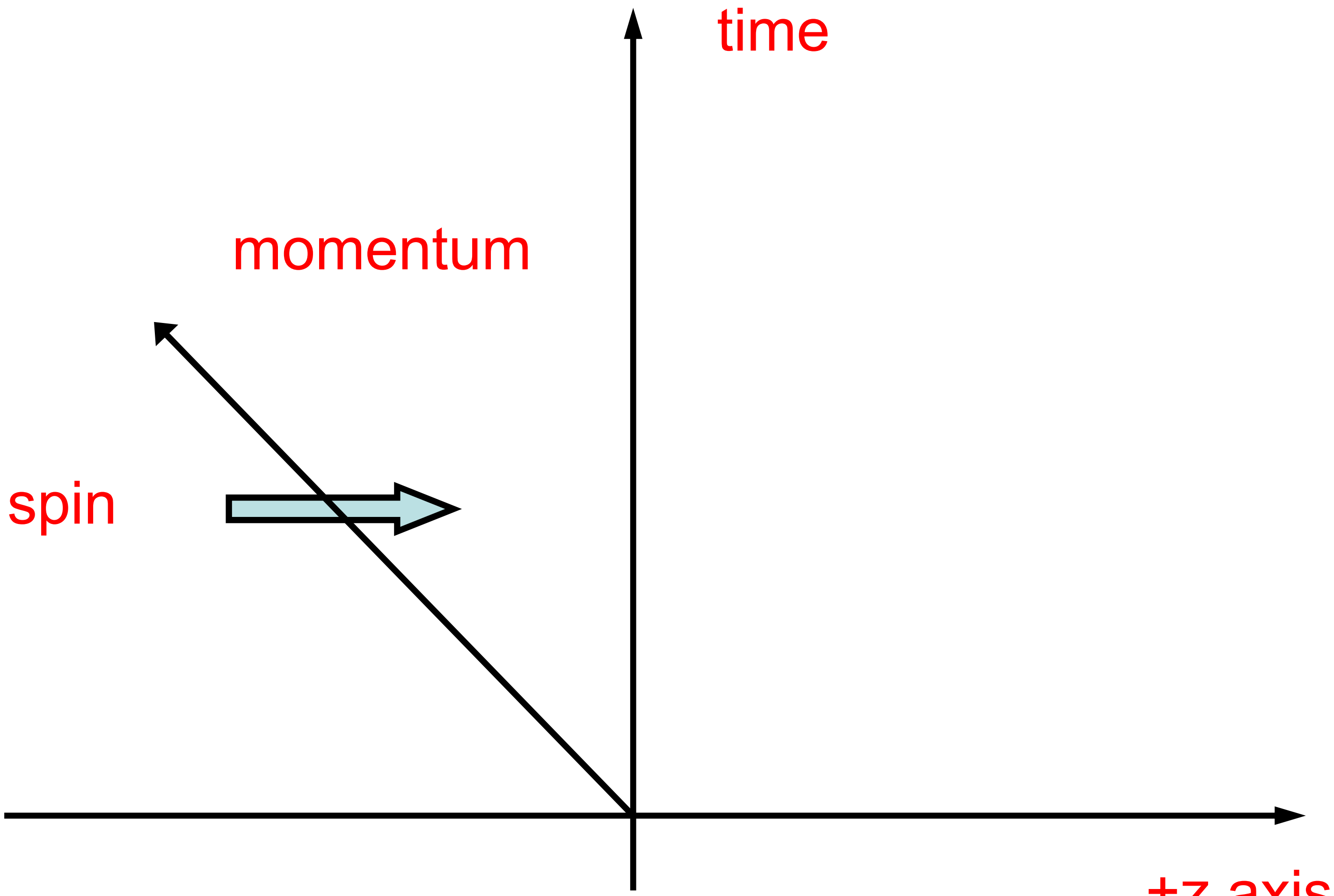
Cartoon Feature: Fermion Mass and Chirality



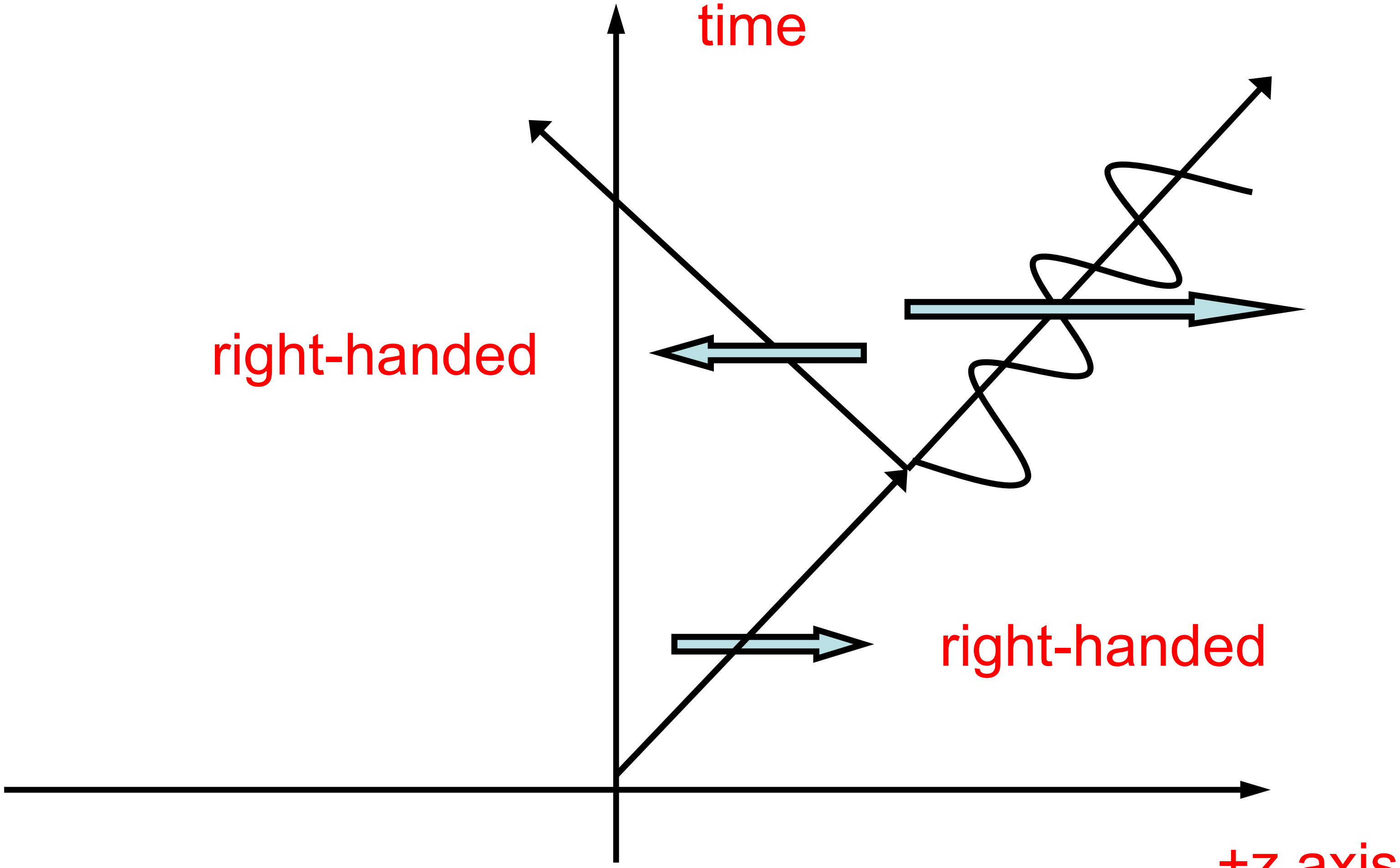
A massless right-handed fermion $s_z = +1/2$



A massless left-handed fermion $s_z = +1/2$

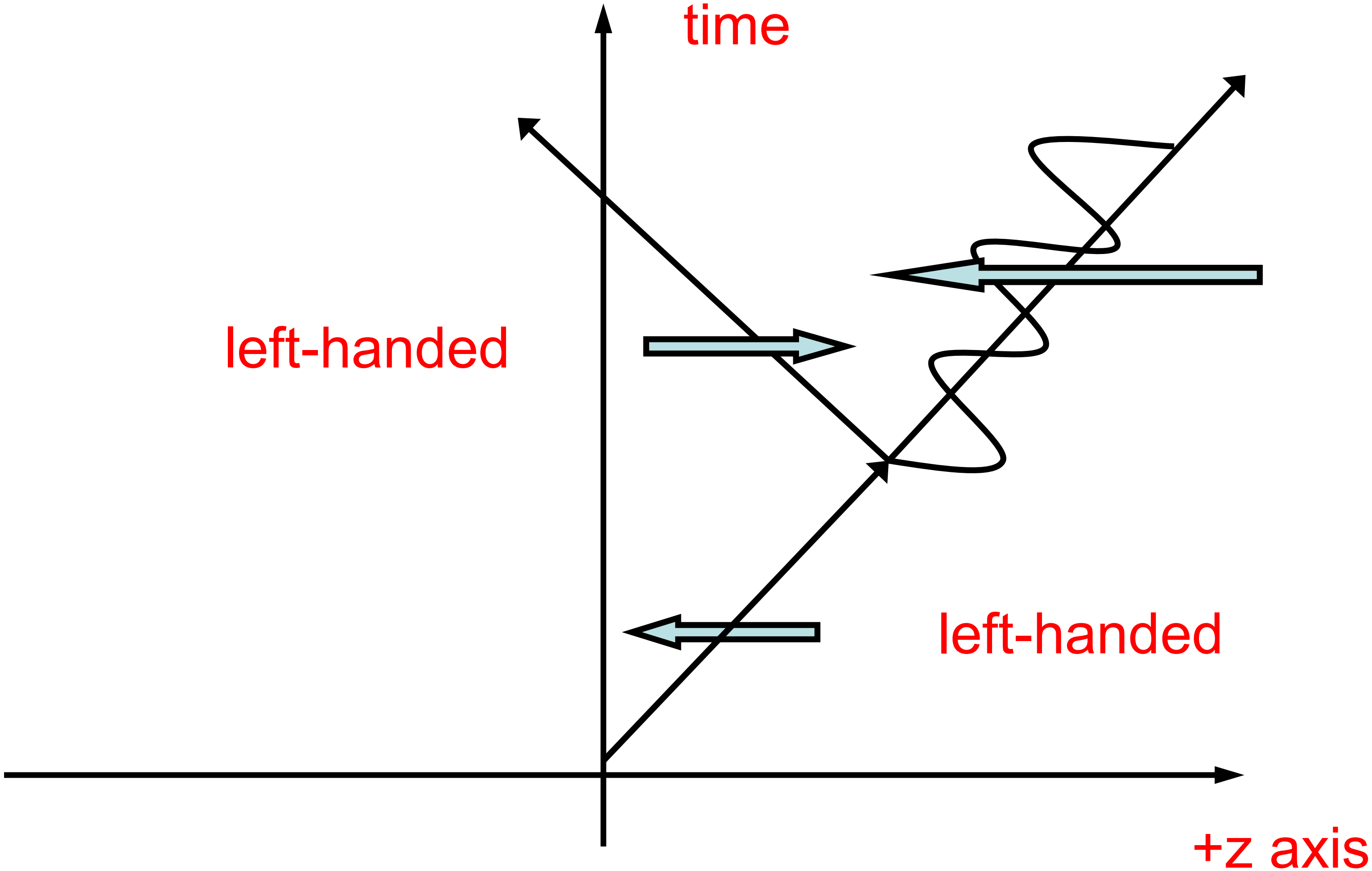


Can couple fermions to gauge bosons and preserve chiral symmetry:



Chirality is conserved

e.g., couple electron to the photon
in a L-R symmetric way



Chirality is conserved

The left-handed and right-handed electrons have the same electric charge

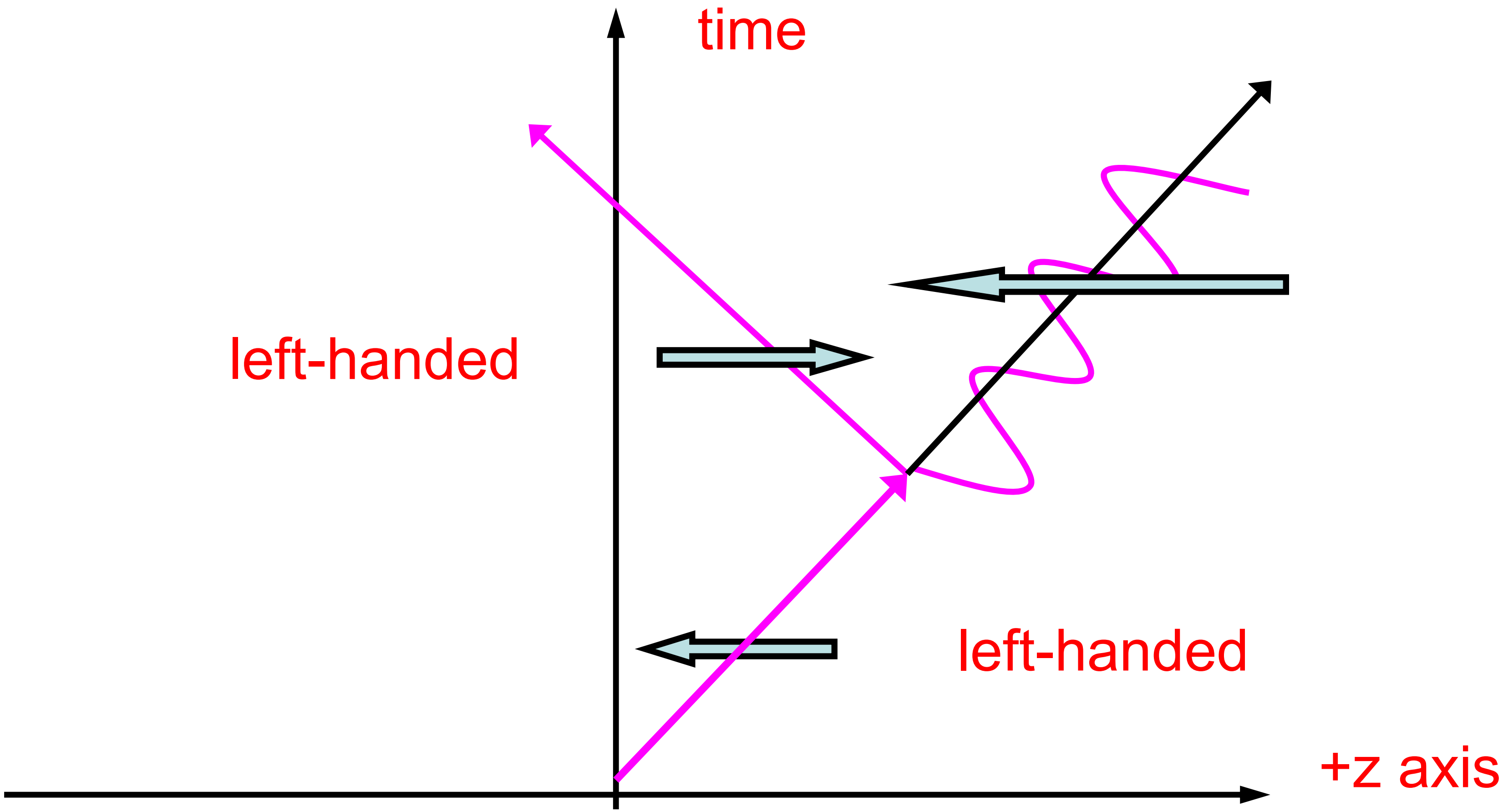
QED is “vectorlike”

Standard Model is not "vectorlike" !!!

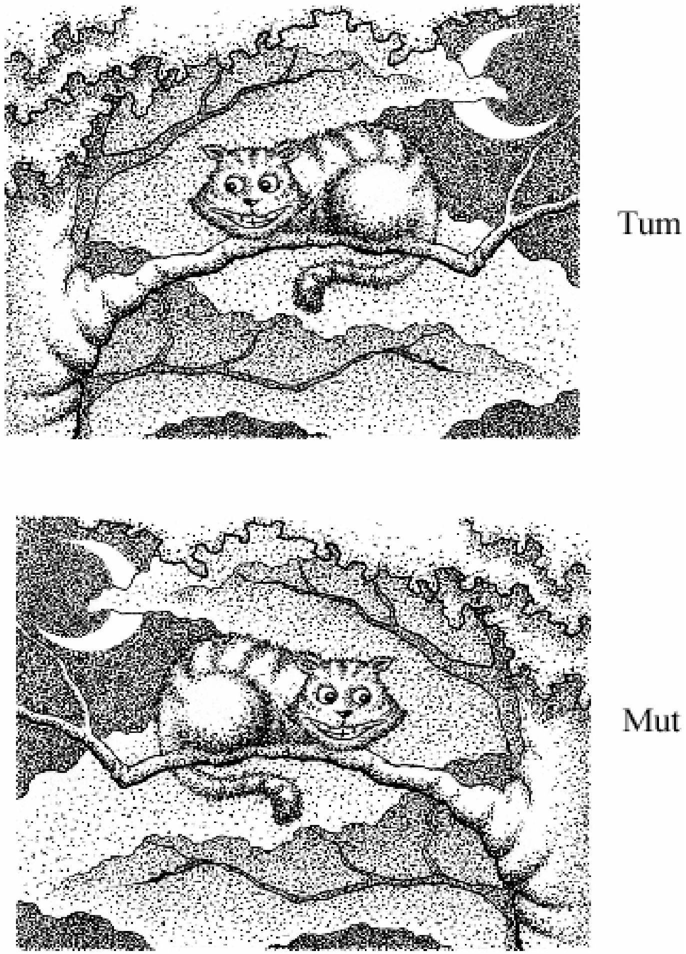
Only left-handed fermions have
electroweak charge and form doublets
under SU(2)

Right handed's are "sterile"
under SU(2)

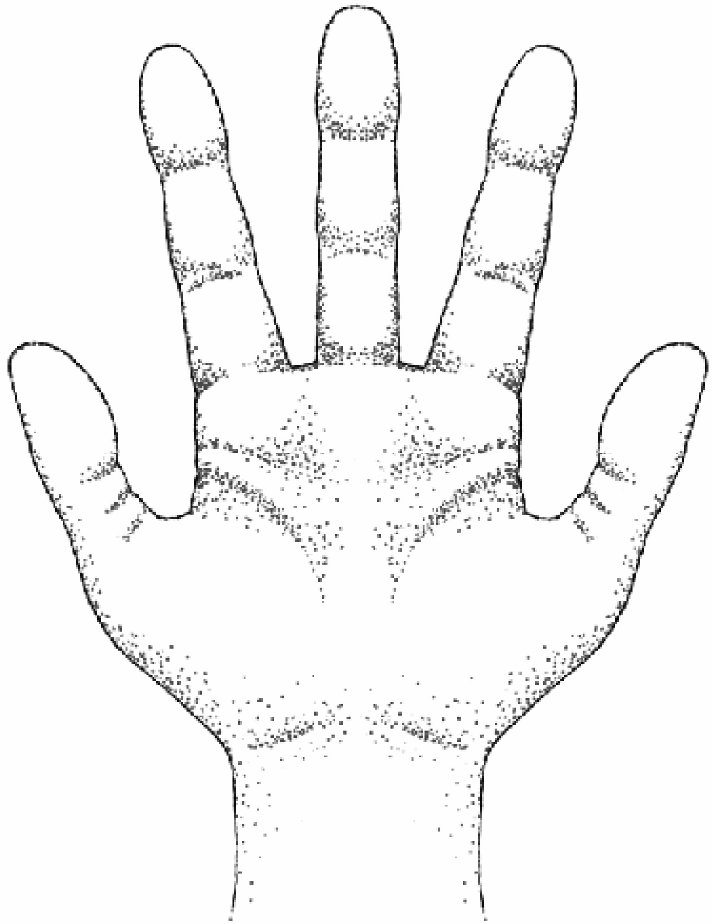
only L fermions couple to the W-boson



Reflection Symmetry (parity)

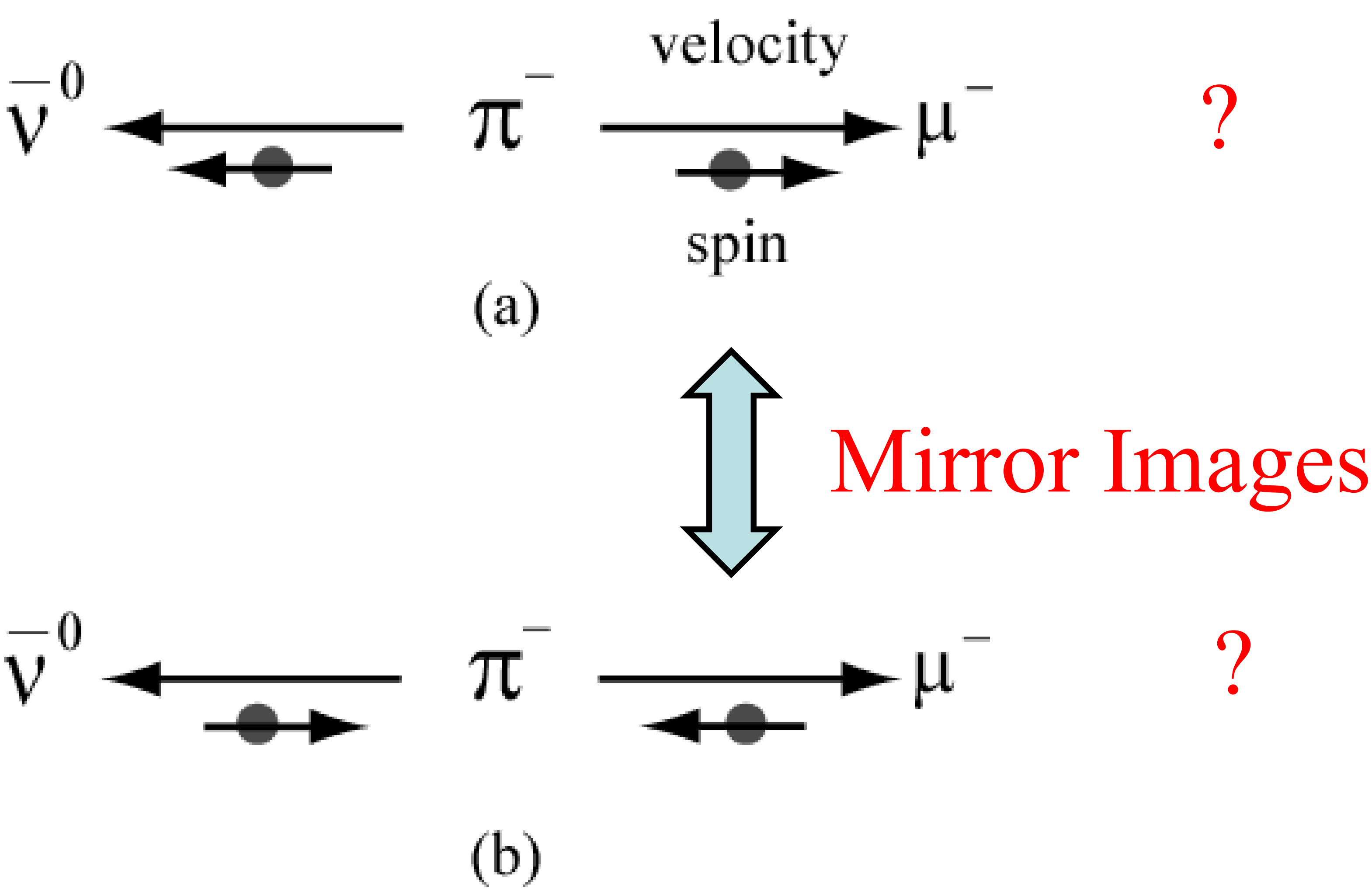


doublets

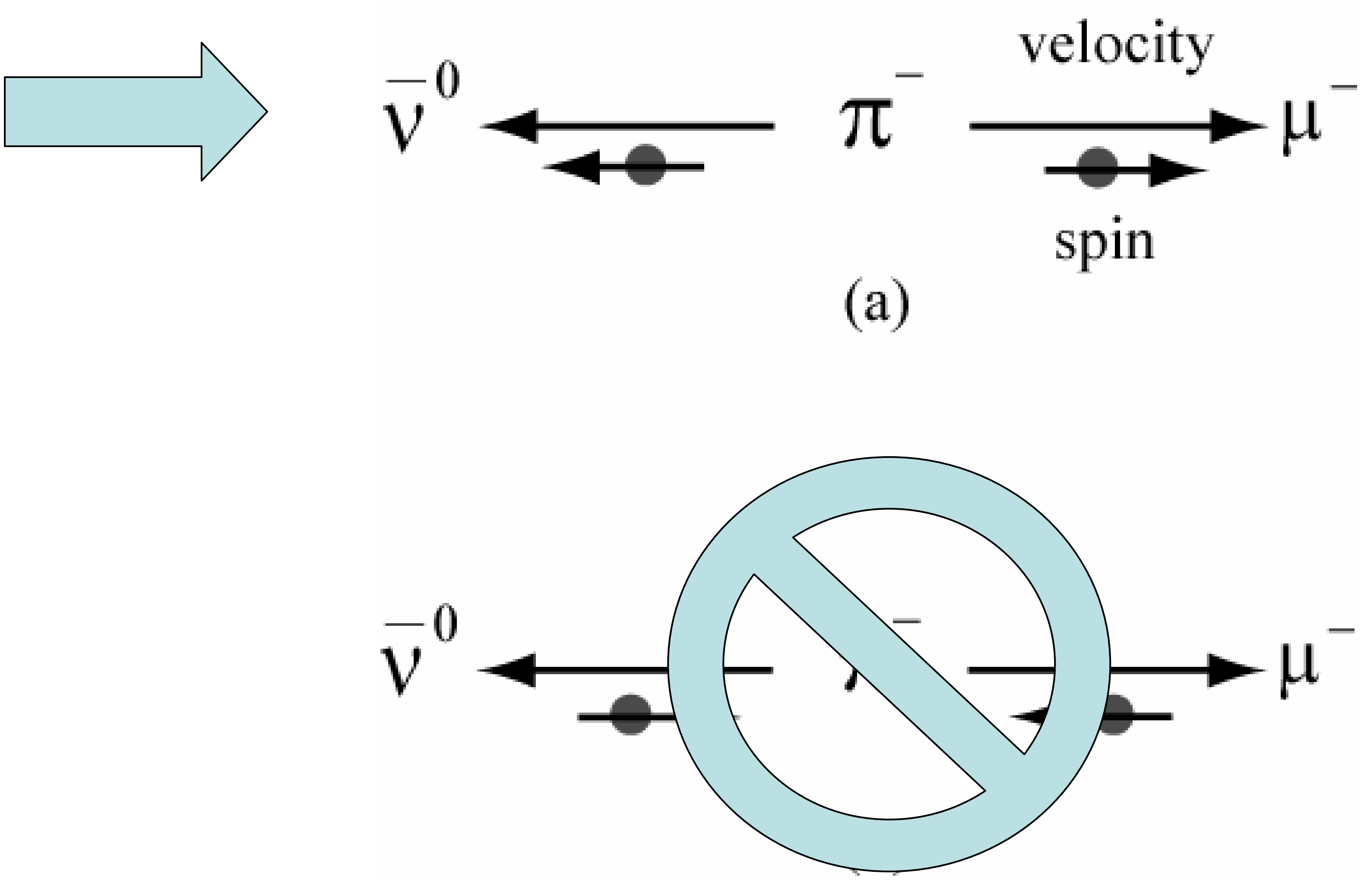


singlets

Helicity of decay products in pion decay:



Parity is violated in pion decay: (Lederman)



$$\textit{vector current,} \qquad j_\mu = j_{\mu R} + j_{\mu L} = \bar{\psi}\gamma_\mu\psi$$

$$\textit{axial vector current,} \qquad j_\mu^5 = j_{\mu R} - j_{\mu L} = \bar{\psi}\gamma_\mu\gamma_5\bar{\psi}.$$

Chiral symmetry is broken by mass

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi - m\bar{\psi}\psi = \bar{\psi}_L i \partial\!\!\!/ \psi_L + \bar{\psi}_R i \partial\!\!\!/ \psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

$U(1)_L \times U(1)_R$ chiral symmetry of the massless theory has now broken

residual $U(1)_{L+R}$, which is the vectorial symmetry of fermion number conservation.

$$\textit{vector current,} \qquad j_\mu = j_{\mu R} + j_{\mu L} = \bar{\psi}\gamma_\mu\psi$$

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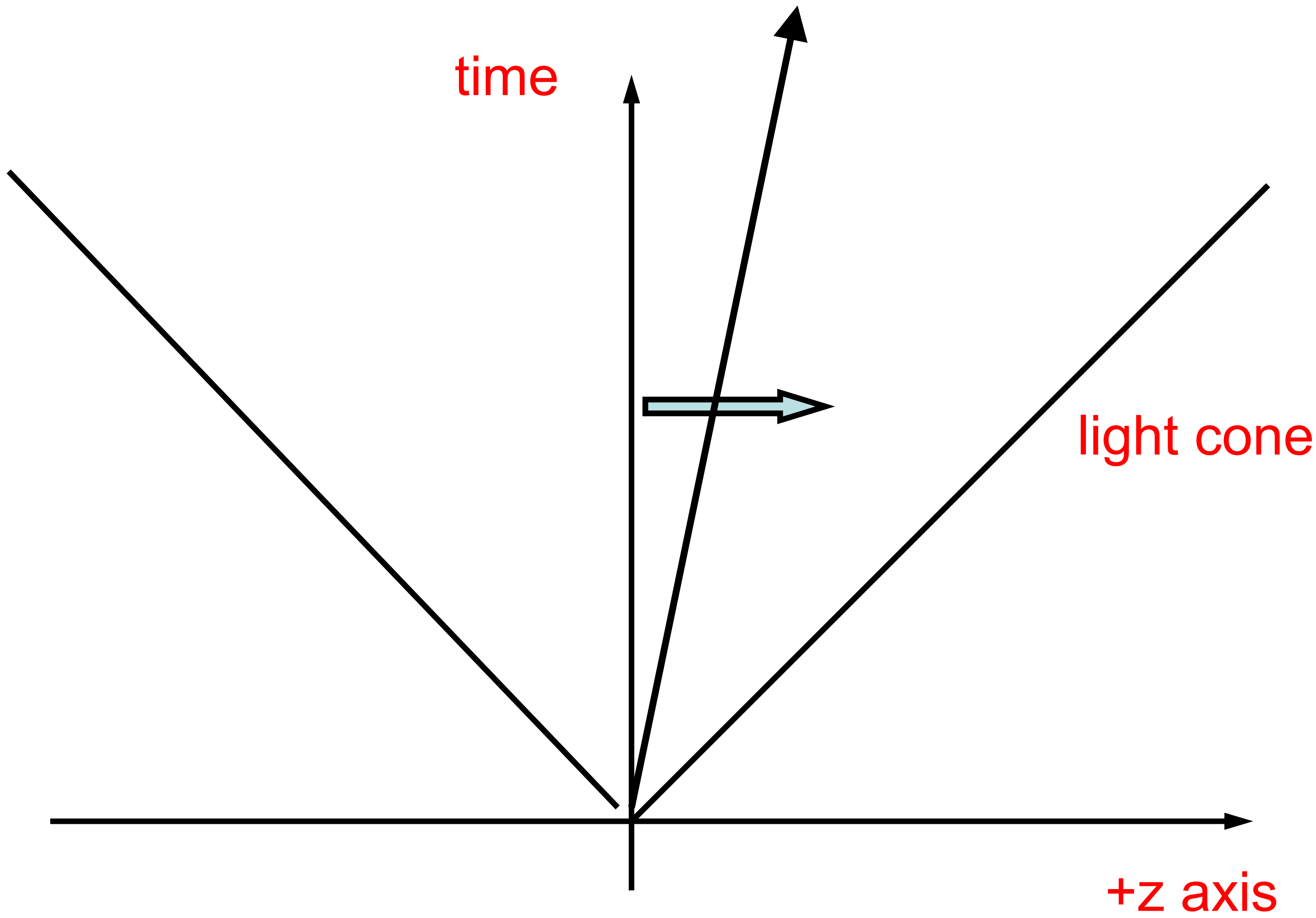
$U(1)_L \times U(1)_R$ chiral symmetry of the massless theory has now broken

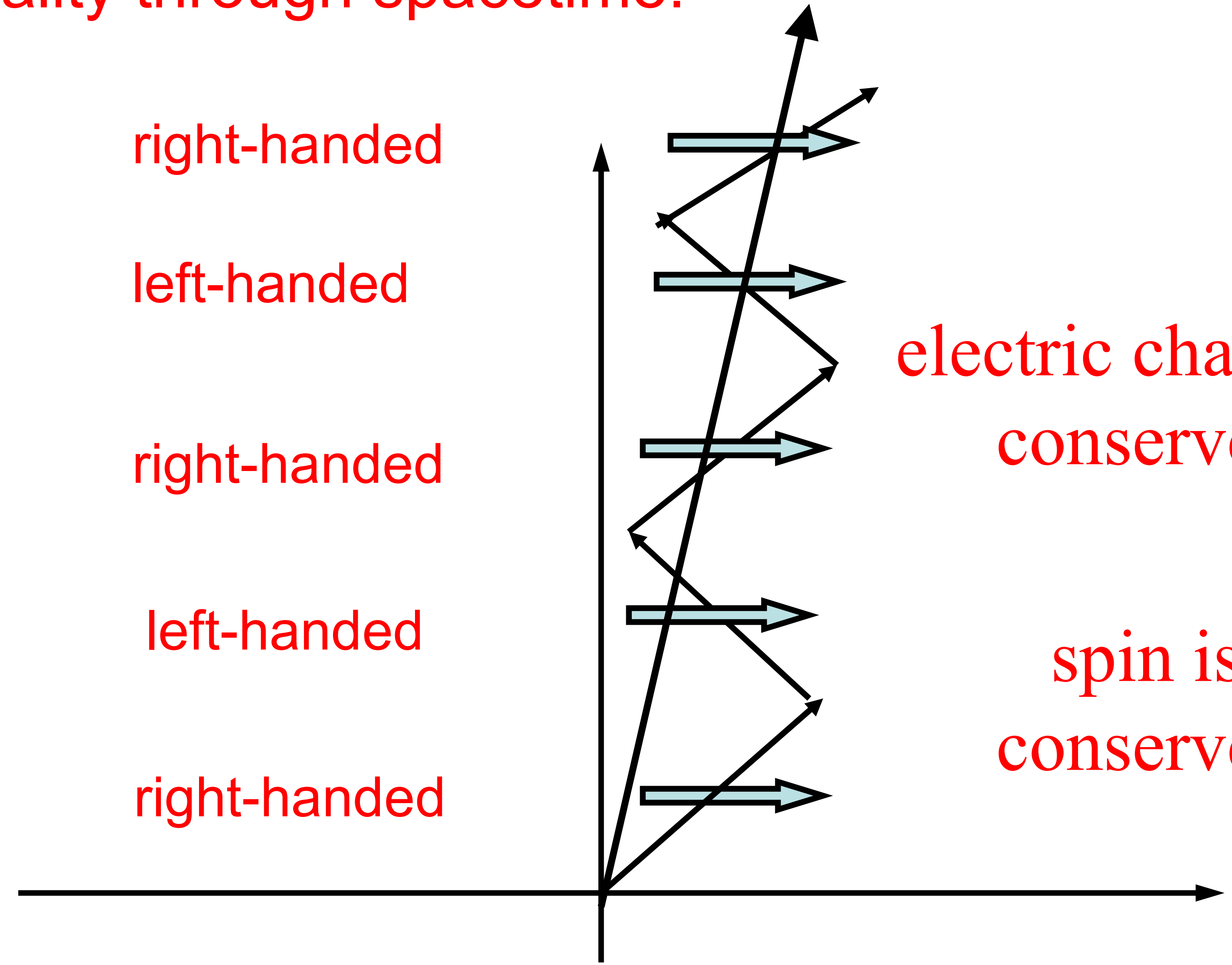
residual $U(1)_{L+R}$, which is the vectorial symmetry of fermion number conservation.

$$\begin{aligned} \partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi &= \bar{\psi} \overleftarrow{\partial\!\!\!/} \gamma_5\psi + \bar{\psi}\gamma_5 \overrightarrow{\partial\!\!\!/} \psi \\ &= -2im\bar{\psi}\gamma_5\psi \end{aligned}$$

Cartoon: a massive electron



A massive fermion oscillates in chirality through spacetime:



electric charge is conserved

spin is conserved

Chirality nonconservation by the mass term

$$vector\ \ current, \qquad j_\mu = j_{\mu R} + j_{\mu L} = \bar{\psi}\gamma_\mu\psi$$

$$axial\ \ vector\ \ current, \qquad j^5_\mu = j_{\mu R} - j_{\mu L} = \bar{\psi}\gamma_\mu\gamma_5\bar{\psi}.$$

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/\, \psi - m\bar{\psi}\psi = \bar{\psi}_L i \partial\!\!\!/\, \psi_L + \bar{\psi}_R i \partial\!\!\!/\, \psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

$$\begin{aligned} \partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi &= \bar{\psi} \overleftarrow{\partial\!\!\!/}\, \gamma_5\psi + \bar{\psi}\gamma_5 \overrightarrow{\partial\!\!\!/}\, \psi \\ &= -2im\bar{\psi}\gamma_5\psi \end{aligned}$$

Homework:

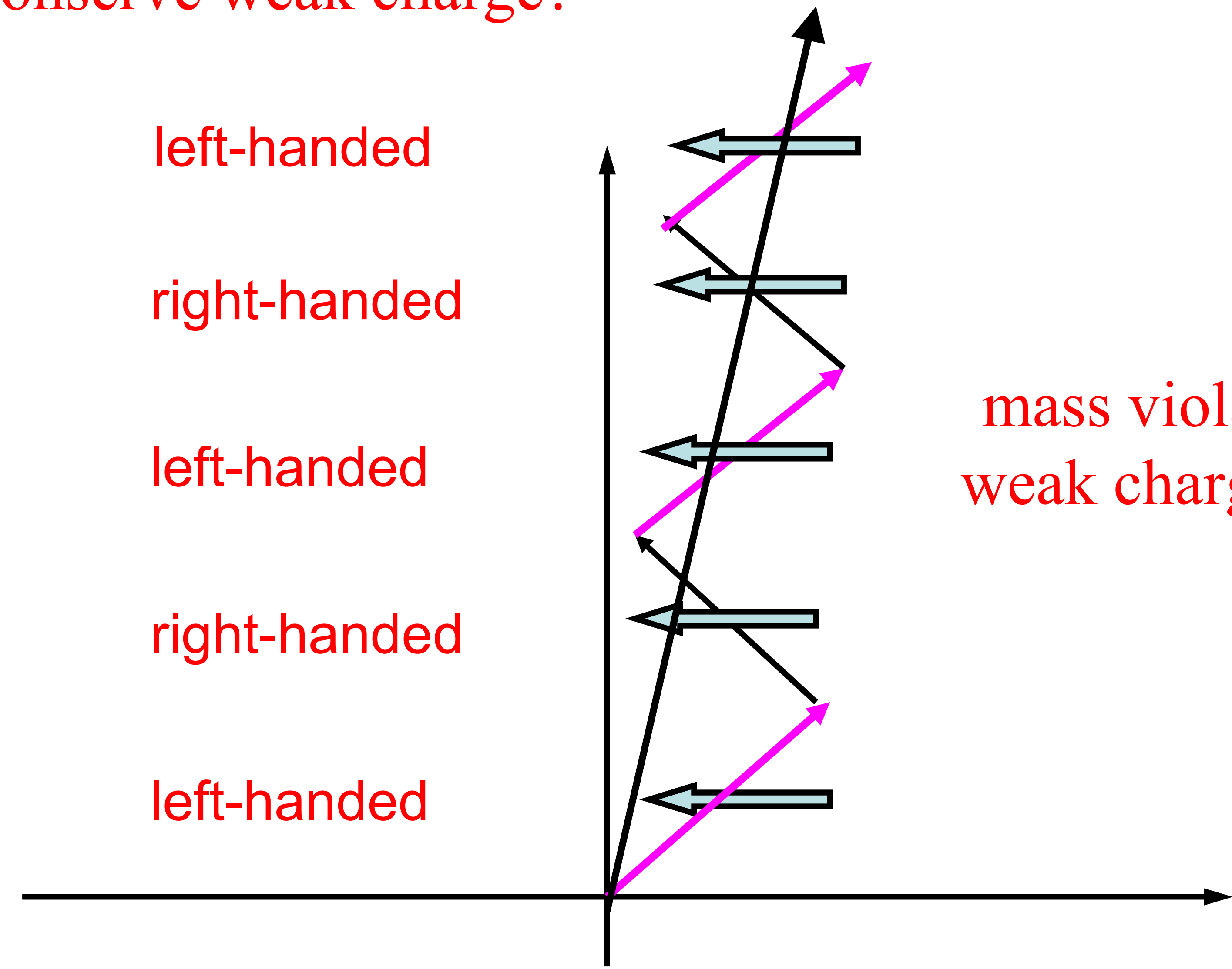
Verify that:

$$\partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi = \bar{\psi}\overleftarrow{\not{\partial}}\gamma_5\psi + \bar{\psi}\gamma_5\overrightarrow{\not{\partial}}\psi$$

$$= -2im\bar{\psi}\gamma_5\psi$$

Hint: use the Dirac equation

How do we make a massive fermion but conserve weak charge?

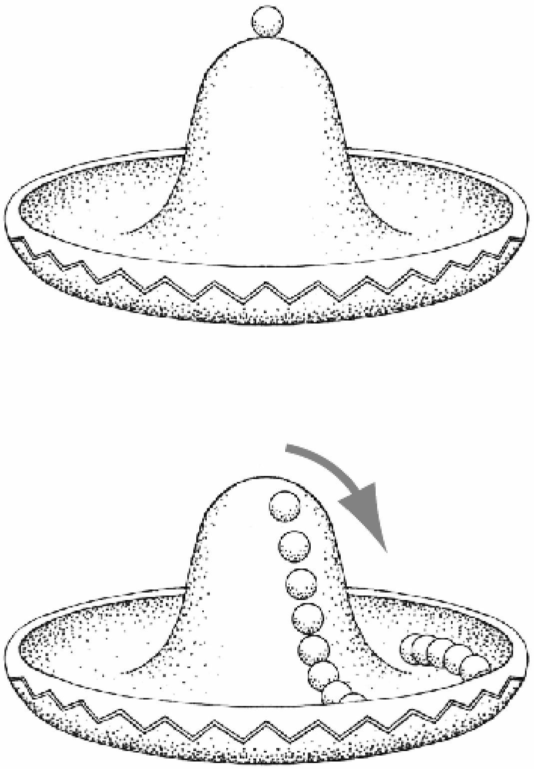


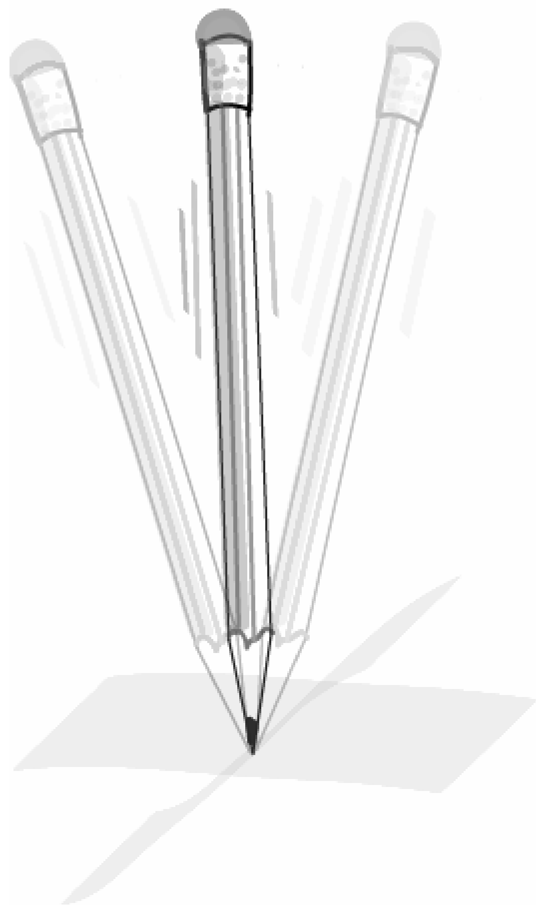
mass violates weak charge!!!

Mass Violates Electroweak Gauge Symmetry!!!

IV. Spontaneous Symmetry Breaking

Assume the potential for the field Φ is: $$V(\Phi) = -M^2|\Phi|^2 + \frac{1}{2}\lambda|\Phi|^4$$





$$V(\Phi) = -M^2|\Phi|^2 + \frac{1}{2}\lambda|\Phi|^4$$

The vacuum built around the field configuration $\langle\Phi\rangle = 0$ is unstable.

$$\langle\Phi\rangle = v/\sqrt{2} \qquad \frac{v}{\sqrt{2}} = \frac{M}{\sqrt{\lambda}}$$

We can parameterize the "small oscillations" around the vacuum state by writing:

$$\Phi = \frac{1}{\sqrt{2}}(v + h(x))\exp(i\phi(x)/f)$$

Substituting this anzatz into the scalar Lagrangian

$$\begin{aligned}\mathcal{L}_\Phi &= \tfrac{1}{2}(\partial h)^2 - M^2 h^2 - \sqrt{\frac{\lambda}{2}} M h^3 - \frac{1}{8}\lambda h^4 \\ &\quad + \frac{v^2}{2f^2}(\partial\phi)^2 + \frac{1}{2f^2} h^2 (\partial\phi)^2 + \frac{\sqrt{2}M}{\lambda f^2} h (\partial\phi)^2 + \Lambda\end{aligned}$$

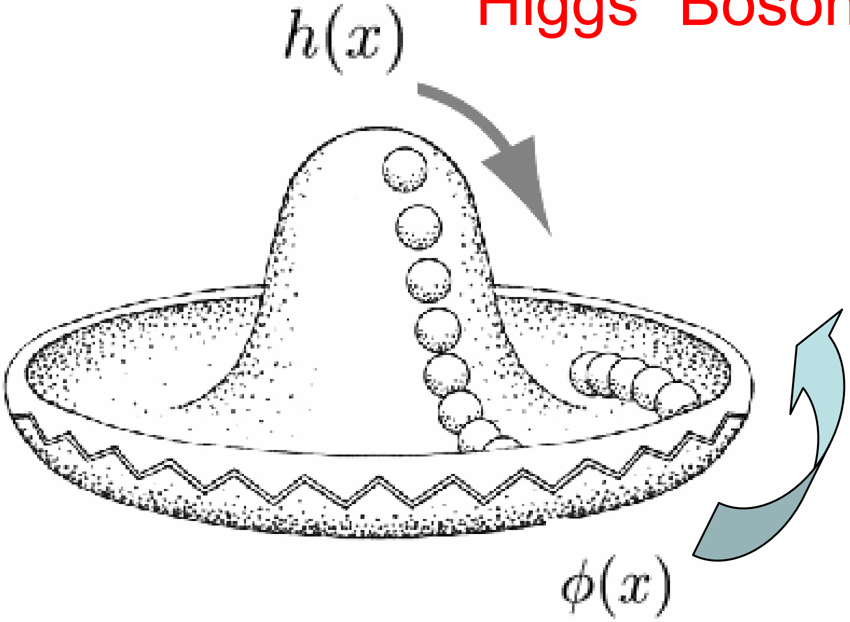
$$\Lambda = -M^4/2\lambda$$

We see that $\phi(x)$ is a massless field (a Nambu–Goldstone mode).

symmetry $\phi \to \phi + \xi$

$$f = v$$

“Higgs” Boson



Nambu-Goldstone Boson

limit $M \to \infty$, and $\lambda \to \infty$

hold $v^2 = f^2 = 2M^2/\lambda$ fixed.

} a *nonlinear* σ *model*

suppresses fluctuations in the h field

$$\Phi = (f/\sqrt{2})\exp(i\phi/f)$$

$$j^5_\mu = \overline{\psi}\gamma_\mu\gamma^5\psi - 2f\partial_\mu\phi$$

Engineer a Coupling to a Complex Scalar Boson that conserves chirality

we can preserve the full $U(1)_L \times U(1)_R$

and *still give the fermion a mass!*

$$\Phi \to \exp[-i(\theta - \omega)]\Phi$$

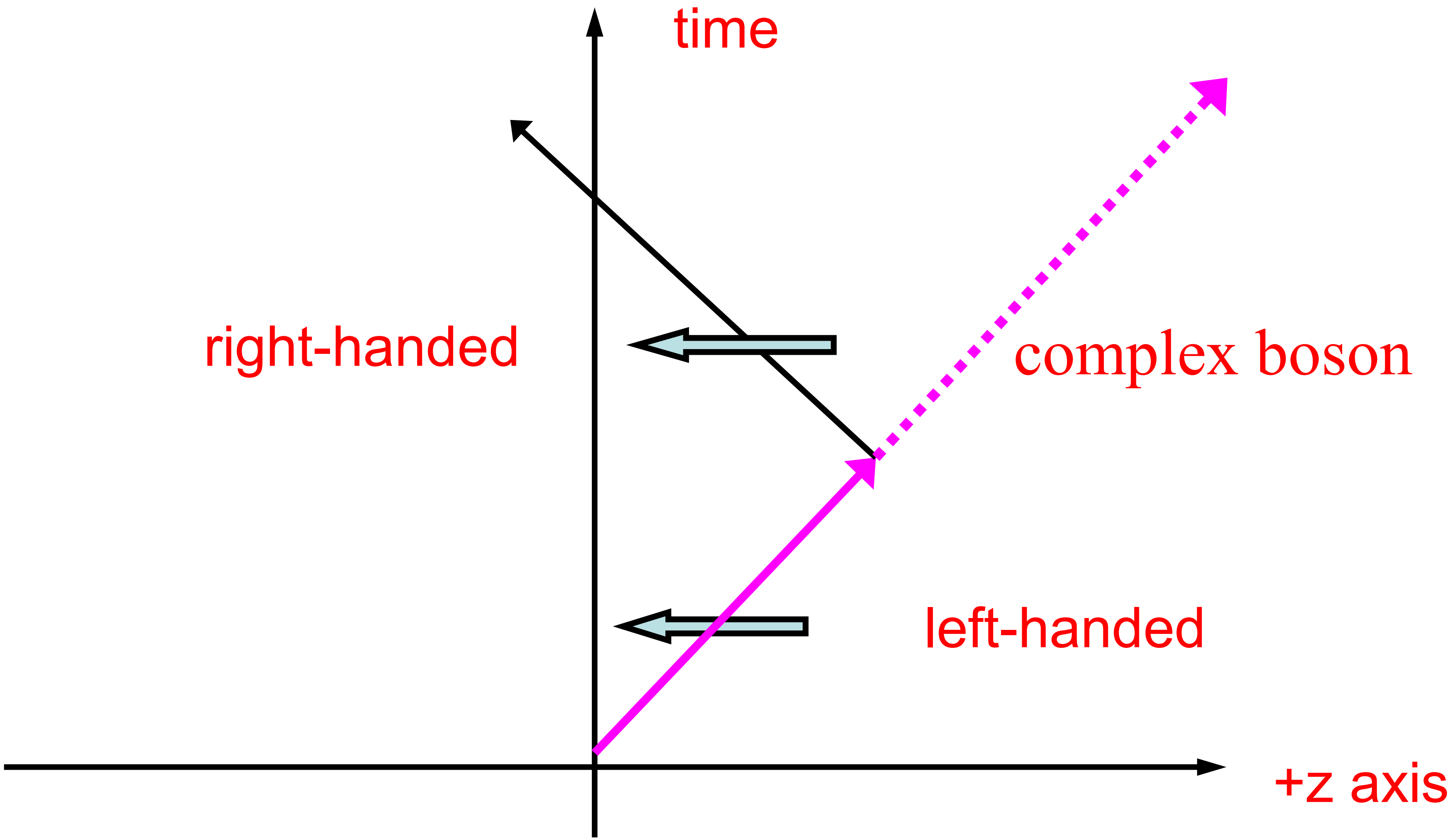
$$\mathcal{L} = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R - g(\bar{\psi}_L \psi_R \Phi + \bar{\psi}_R \psi_L \Phi^*) + \mathcal{L}_\Phi$$

$$\mathcal{L}_\Phi = |\partial\Phi|^2 - V(|\Phi|)$$

the axial current is now changed to:

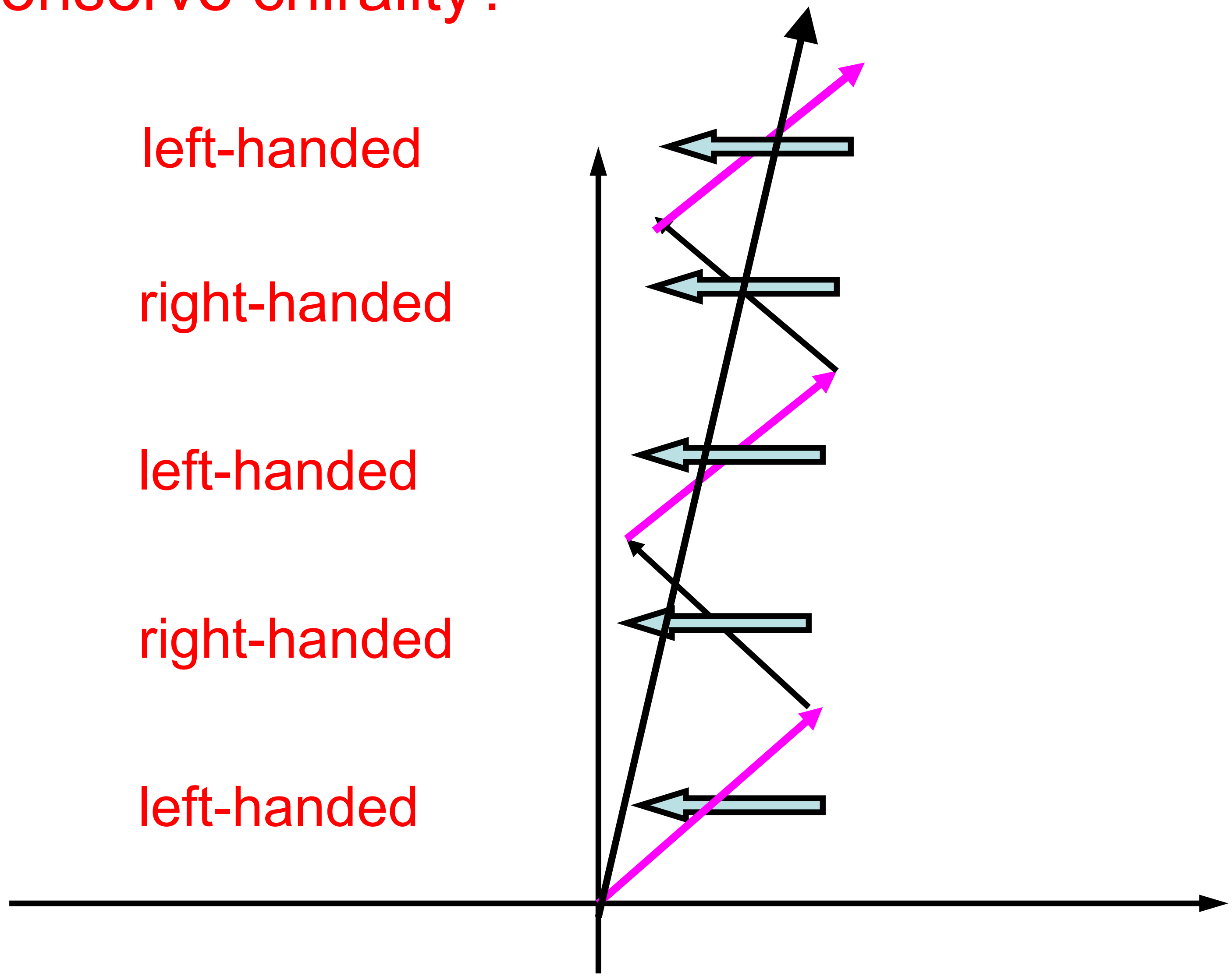
$$j^5_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi + 2i\Phi^*(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\Phi$$

Couple to a Complex Scalar

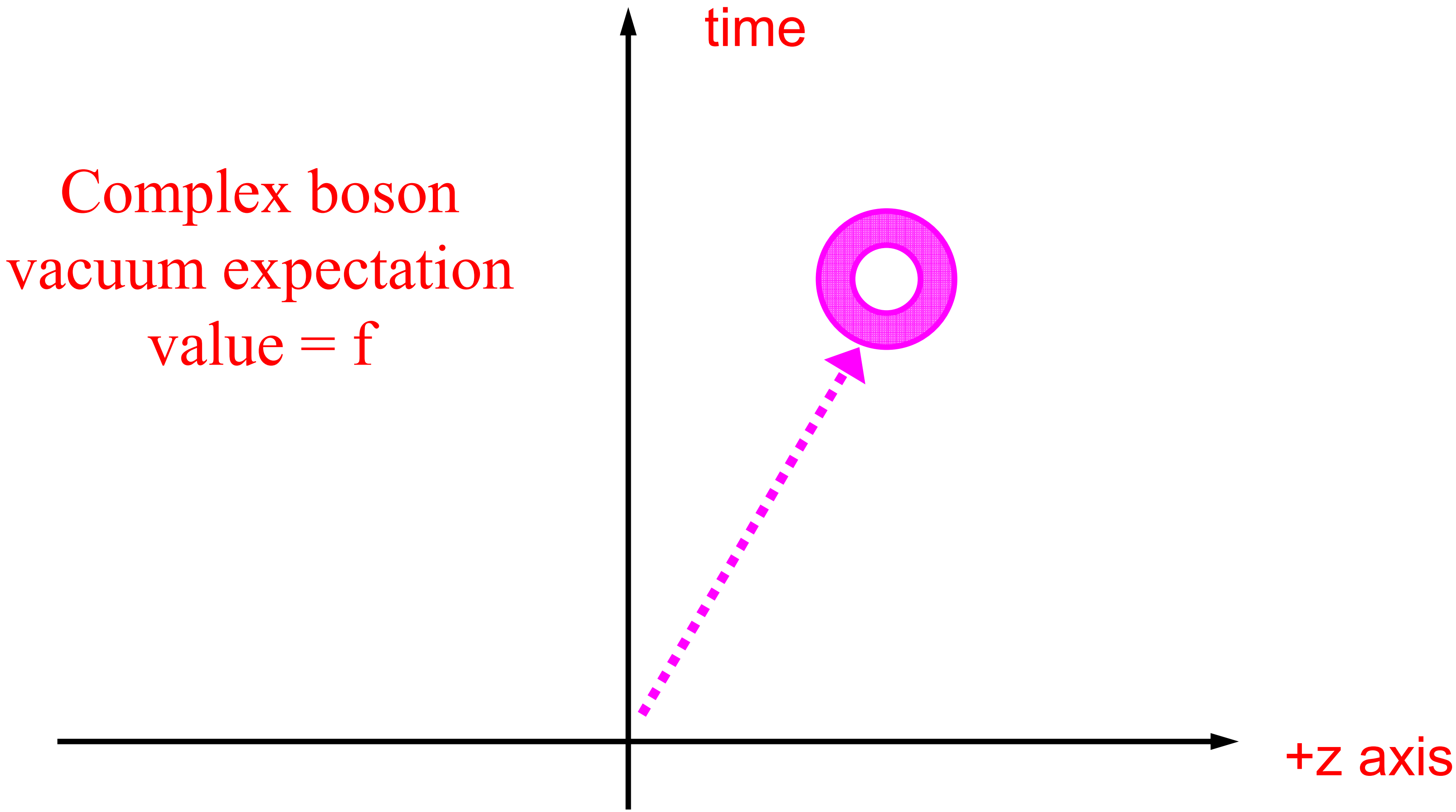


Chiral charge is conserved

Can we have a massive fermion but conserve chirality?

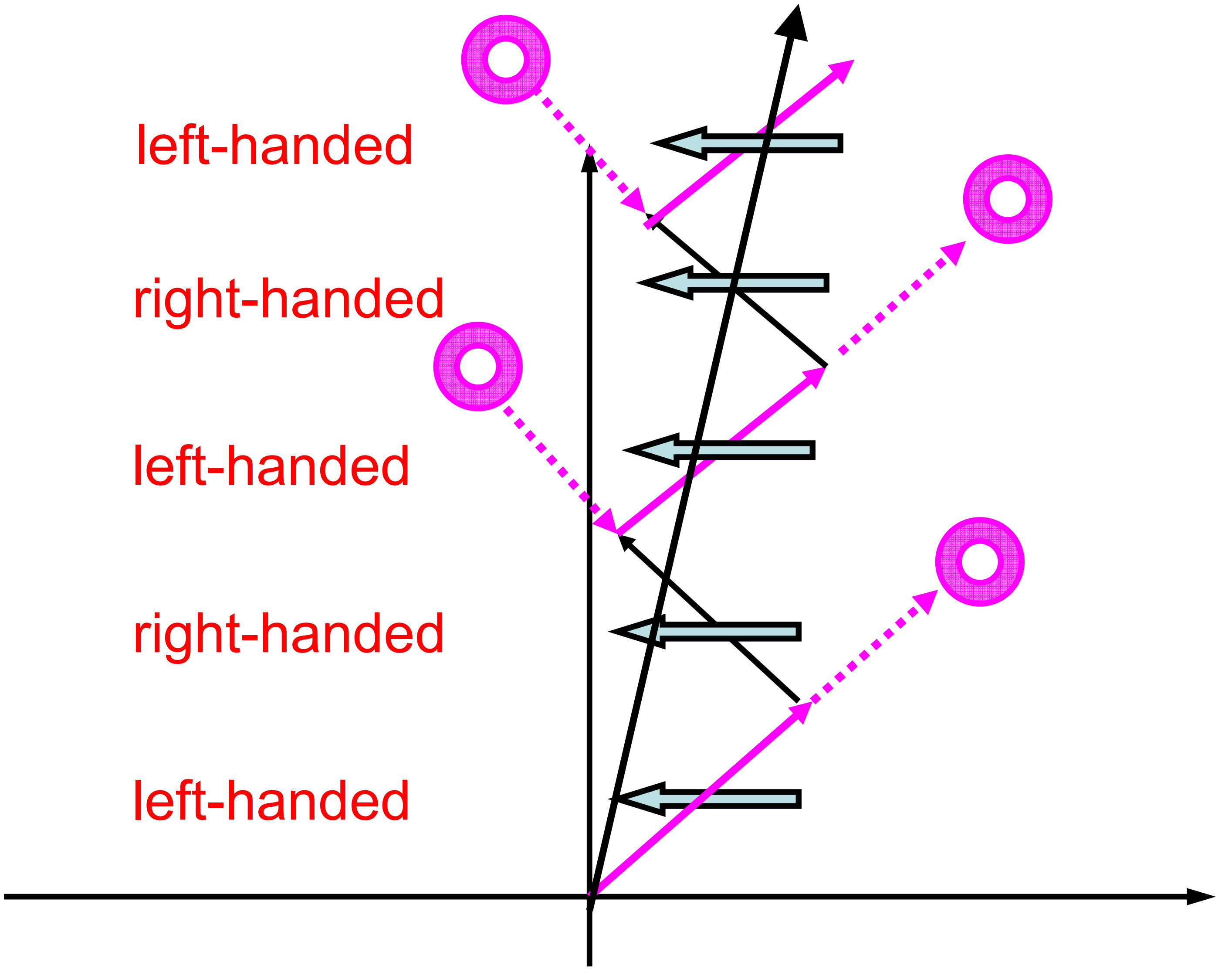


Boson Condenses in vacuum!



Chiral charge is hidden in vacuum !!

Fermion Masses generated spontaneously



Ungauged Spontaneously Broken Chiral Symmetry

$$\mathcal{L} = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R + \frac{1}{2}(\partial\phi)^2 - (gf/\sqrt{2})(\bar{\psi}_L\psi_R e^{i\phi/v} + \bar{\psi}_R\psi_L e^{-i\phi/f})$$

If we expand in powers of ϕ/f we obtain:

$$\mathcal{L} = \bar{\psi} i\partial\!\!\!/ \psi + \bar{\psi} i\partial\!\!\!/ \psi + \frac{1}{2}(\partial\phi)^2 - (gf/\sqrt{2})\bar{\psi}\psi - i(g/\sqrt{2})\phi\bar{\psi}\gamma^5\psi + \ldots$$

this Lagrangian describes a Dirac fermion of mass $m = gf/\sqrt{2}$

pseudoscalar Nambu-Goldstone boson ϕ, which is coupled to $i\bar{\psi}\gamma_5\psi$

$g = \sqrt{2}m/f$ Goldberger-Treiman relation

Gauged Spontaneously Broken Chiral Symmetry

what happens if Φ is a charged scalar field

$$\mathcal{L}'_\Phi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(i\partial_\mu - eA_\mu)\Phi|^2 - V(\Phi)$$

gauge invariant $\quad A_\mu \to A_\mu + \partial_\mu \chi \quad \Phi \to e^{-ie\chi}\Phi$

$$\Phi = (f/\sqrt{2})\exp(i\phi/f) \qquad B_\mu = A_\mu - \frac{1}{e}\partial_\mu \phi$$

$$\mathcal{L}'_\Phi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}e^2v^2B_\mu B^\mu + \tfrac{1}{2}(\partial_\mu h)^2$$
$$- M^2h^2 - \sqrt{\frac{\lambda}{2}}Mh^3 - \frac{1}{8}\lambda h^4 + \tfrac{1}{2}e^2\left(h^2 + \frac{\sqrt{2}M}{\lambda}h\right)B_\mu B^\mu$$

Higgs boson has a mass $\sqrt{2}M$

Landau-Ginzburg model of superconductivity.

"abelian Higgs model."

the Nambu-Goldstone boson has been "eaten"

V. Putting it all together: Gauged Spontaneously Broken Chiral Symmetry

$$\mathcal{L} = \mathcal{L}'_\Phi + \bar{\psi}_L(i\partial\!\!\!/ - eA\!\!\!/)\psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R - g(\bar{\psi}_L\psi_R\Phi + \bar{\psi}_R\psi_L\Phi^\dagger)$$

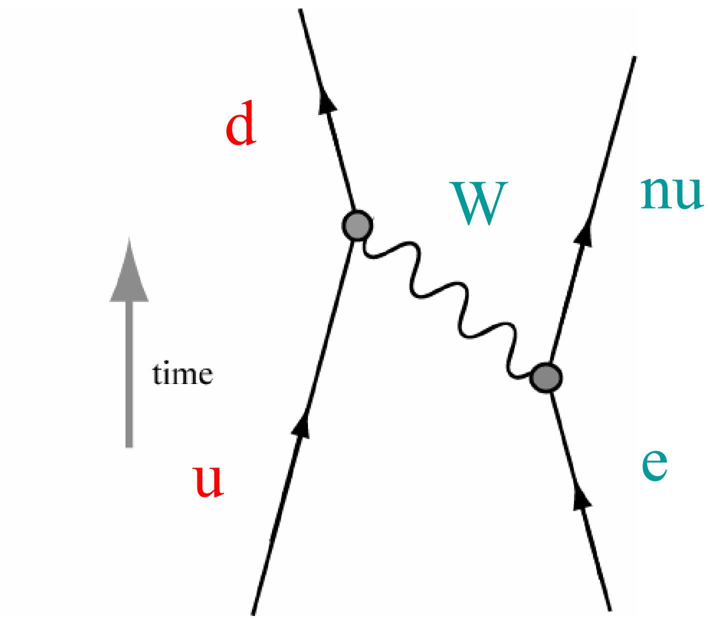
$$\Phi = (f/\sqrt{2})\exp(i\phi/f) \qquad B_\mu = A_\mu - \frac{1}{e}\partial_\mu\phi$$

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}e^2v^2B_\mu B^\mu + \tfrac{1}{2}(\partial_\mu h)^2 \\
& -M^2h^2 - Mh^3 - \frac{1}{8}\lambda h^4 + \tfrac{1}{2}e^2h^2B_\mu B^\mu \\
& +\bar{\psi}i\partial\!\!\!/ \psi - eB^\mu\bar{\psi}_L\gamma_\mu\psi_L - m\bar{\psi}\psi - \frac{1}{\sqrt{2}}gh\bar{\psi}\psi
\end{aligned}$$

We're ready for the Standard Model !!!

Weak Force:

SU(2) x U(1)



What gives rise to masses of W and Z boson?

SU(2)xU(1) is "Spontaneously broken Symmetry"

Higgs Field?