Drell-Yan process and heavy boson production at hadron colliders

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June 2, 2007

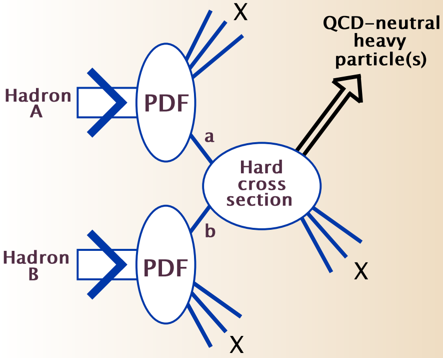
Outline

- Saturday
  - low-Q lepton pair production
  - heavy electroweak bosons
  - kinematics
  - leading-order (LO) cross sections in the helicity formalism
  - some NLO/NNLO results
- Monday
  - modern applications
  - QCD factorization and resummation of large logarithms
  - new physics searches

Special role of $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 ...)X$

Notations

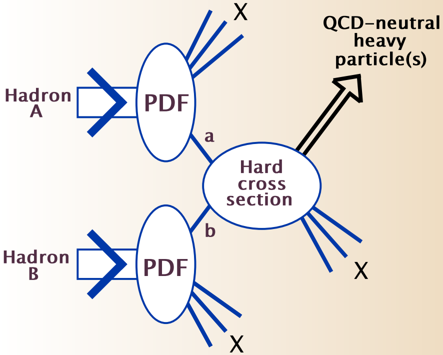
- A, B – initial-state hadrons (p, $\bar{p}$, n, nuclei, π, ...)
- V – a final-state QCD-neutral system (e.g., a virtual boson or boson pair with mass $Q \gg \Lambda_{QCD}$)
- v_1, v_2 – observed particles from decay of V (e.g., leptons)



Special role of $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 \ldots)X$

Examples

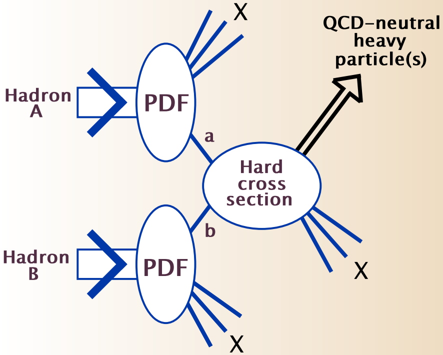
- $AB \to (\gamma^*, Z \to \ell^+\ell^-)X$ (with $\ell = e, \mu$)
- $AB \to (W \to \ell\nu_\ell)X$
- $AB \to$ Higgs $+ X$
- $AB \to VVX$ (with $V = \gamma, W, Z, \ldots$)
- $AB \to V_{BSM}X$ (with $V_{BSM} = Z'$, Randall-Sundrum graviton, etc.)



Special role of $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 \ldots)X$

"Simple" QCD factorization

- V does not interact with final-state hadrons, which are integrated out
$\Rightarrow$ no dependence on final-state nonperturbative functions
- An inclusive cross section factorizes into a hard-scattering cross section $\widehat{\sigma}$ and parton distribution functions $f_{a/A}(x,\mu)$



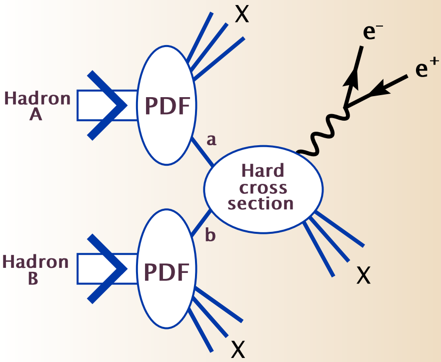
For example, the total cross section is given by
$\frac{d\sigma}{dQ^2} = \sum_{a,b} \int_\tau^1 \frac{d\xi}{\xi} f_{a/A}(\xi,Q) f_{b/B}(\frac{\tau}{\xi},Q) \frac{d\widehat{\sigma}_{ab}}{dQ^2}$, where $\tau \equiv Q^2/s$

Special role of $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 \ldots)X$

Drell-Yan process $AB \xrightarrow{\gamma^*} \ell^+\ell^- X$

In 1970, S. Drell and T. M. Yan, applied the parton model to predict scaling of $\mu^+\mu^-$ cross sections with respect to τ:

$$s\frac{d\sigma}{dQ^2} \sim K\mathcal{L}_{ab}(\tau),$$

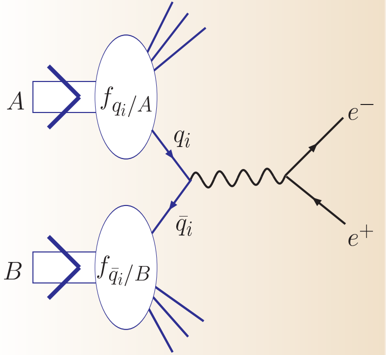


where $\mathcal{L}_{ab}(\tau)$ is the "parton luminosity", originally derived from DIS structure functions; K depends on the process and interaction model (parton spin, QED and QCD charges, ...), unknown in 1970!

Special role of $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 \ldots)X$

Drell-Yan process $AB \xrightarrow{\gamma^*} \ell^+\ell^- X$

In modern QCD, **approximate** scaling reflects the behavior of the Born $q\bar{q} \to \ell^+\ell^- X$ cross section if the logarithmic Q dependence of $f_{q/A}(x, Q)$ is neglected:



$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha_{EM}^2}{3N_c Q^2 s}$$

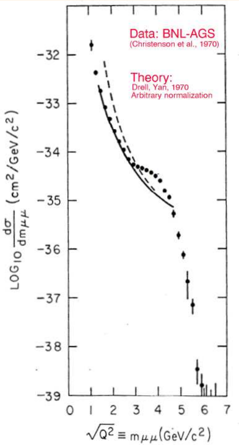
$$\times \sum_{i=u,d,s,\ldots} e_i^2 \underbrace{\int_\tau^1 \frac{d\xi}{\xi} \left[f_{q_i/A}(\xi, Q) f_{\bar{q}_i/B}(\frac{\tau}{\xi}, Q) + f_{\bar{q}_i/A}(\xi, Q) f_{q_i/B}(\frac{\tau}{\xi}, Q) \right]}_{\sim \mathcal{L}_{q_i\bar{q}_i}(\tau)},$$

with $N_c = 3$, $\alpha_{EM} \equiv e^2/(4\pi)$, ee_i is the fractional quark charge

Scaling in experimental data at Q of a few GeV

$$p + U \rightarrow \mu^+ \mu^- X$$

$\sqrt{s} = 7.4$ **GeV,** $Q = 1 - 6$ **GeV**



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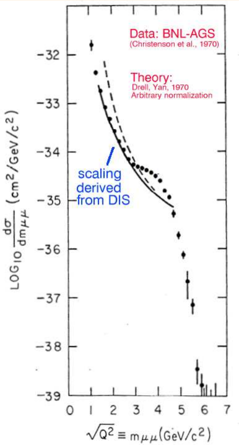

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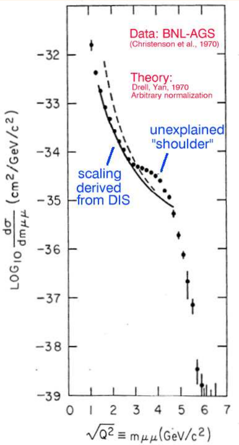

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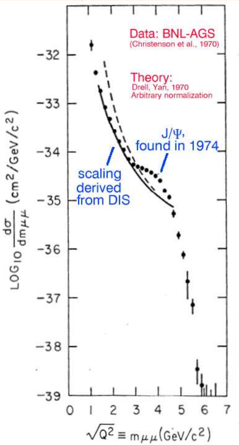

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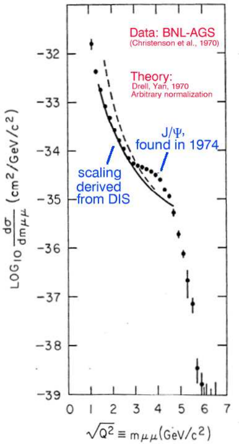
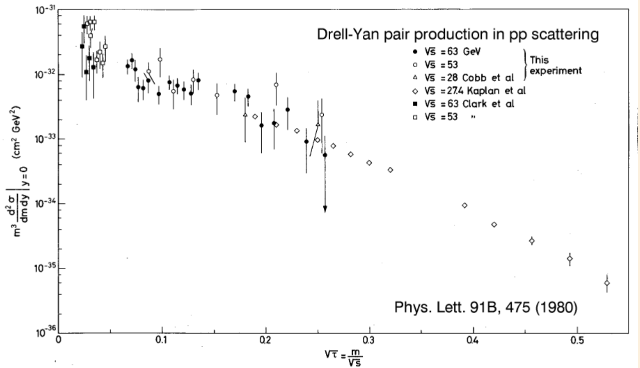


$pN \to \ell^+\ell^- X$, **various energies**



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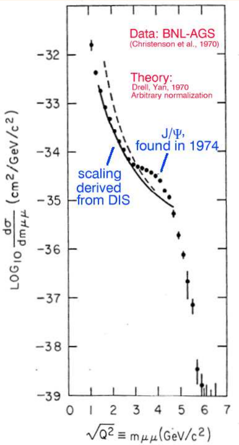

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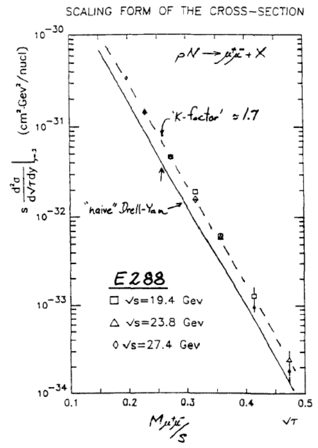

- Data exceeds the LO prediction by a substantial factor $K = 1.5 - 2$ due to a large NLO correction
- A useful quick estimate for $q\bar{q} \to VX$: $K_{NLO} \equiv \sigma_{NLO}/\sigma_{LO} \approx 1 + 3\alpha_s(Q)$, as will be demonstrated today

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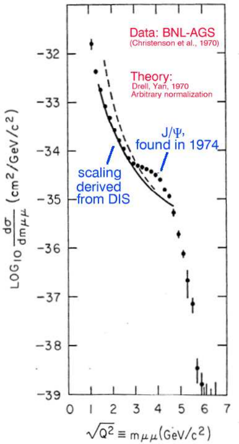

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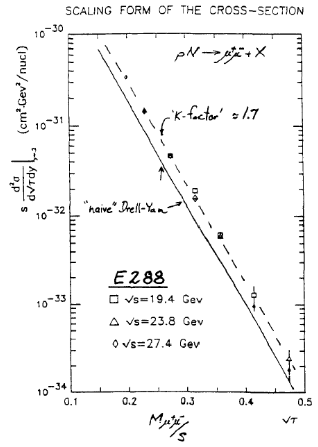

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Exercise: show that $K \approx 1.65$ **(**1.35**) at** $Q = 5$ (90) **GeV**

Heavy electroweak bosons W & Z

Symmetries of the minimal Standard Model

Forces between particles emerge from the local $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ symmetry of the quantum Lagrangian, broken as $SU(2)_L \otimes U(1)_Y \to U(1)_{EM}$ by interaction with Higgs scalar field doublet(s)

Spin-1 fields (force carriers)

- photons A^μ (electromagnetism)
- massive bosons $W^{\pm\mu}$, Z^μ (weak force)
- gluons $G^{a,\mu}$ (strong force)

Spin-1/2 fields ψ_f (matter fields)

	Charge		
	QCD	QED	Weak
quarks u, d, s, c, b, t	yes	yes	yes
charged leptons e, μ, τ	no	yes	yes
neutrinos ν_e, ν_μ, ν_τ	no	no	yes

The interaction Lagrangian

$$\mathcal{L}_{int}^{SM} = i \sum_{j,k} \sum_{V} \overline{\psi}_j \left[(1+\gamma_5) g_{R,ijV} + g_{L,ijV} (1-\gamma_5) \right] \gamma^\mu V_\mu \psi_k,$$

where

- ψ_j are fermion mass eigenstates
  - $(\gamma^\mu p_\mu - m_j)\psi_j = 0$; j, k run over all quark and lepton flavors
  - the weak and mass eigenstates for down-type quarks and neutrinos are related as

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V^{CKM} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \; V^{CKM} \left(V^{CKM}\right)^\dagger = 1$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V^{MNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \; V^{MNS} \left(V^{MNS}\right)^\dagger = 1$$

V^{CKM}, V^{MNS}: mass mixing (Cabibbo-Kobayshi-Maskawa and Maki-Nakagawa-Sakata) matrices

The interaction Lagrangian

$$\mathcal{L}_{int}^{SM} = i \sum_{j,k} \sum_{V} \overline{\psi}_j \left[(1+\gamma_5) g_{R,ijV} + g_{L,ijV}(1-\gamma_5)\right] \gamma^\mu V_\mu \psi_k,$$

where

- $V = A^\mu, \mathcal{G}^\mu, W^{\pm\mu}, Z^\mu$
  - $\mathcal{G}_\mu \equiv G_\mu^a T^a$, T^a is the $SU(3)_C$ generator matrix ($\mathrm{Tr} T^a T^b = \delta^{ab}/2$)
- $g_{L,jkV}$, $g_{R,jkV}$: boson couplings to left- and right-handed fermions

The interaction Lagrangian

$$\mathcal{L}_{int}^{SM} = i \sum_{j,k} \sum_{V} \overline{\psi}_j \left[(1+\gamma_5) g_{R,ijV} + g_{L,ijV} (1-\gamma_5) \right] \gamma^\mu V_\mu \psi_k,$$

where

Fermions	Quarks	Leptons
Isospin $I_3 = 1/2$:	u, c, t	ν_1, ν_2, ν_3
$I_3 = -1/2$:	d, s, b	e^-, μ^-, τ^-
$g_{L,jkG} = g_{R,jkG}$	$g \frac{\delta_{jk}}{2}$	0
$g_{L,jkA} = g_{R,jkA}$	$e e_j \frac{\delta_{jk}}{2}$, $e_j \equiv I_3 + 1/6$	$e e_j \frac{\delta_{jk}}{2}$, $e_j \equiv I_3 - \frac{1}{2}$
$g_{L,jkW^+} = g^*_{L,kjW^-}$	$\frac{V_{jk}^{CKM} g_W}{2\sqrt{2}}$	$\frac{V_{jk}^{MNS} g_W}{2\sqrt{2}}$
$g_{R,jkW^+} = g^*_{R,kjW^-}$	0	0
$g_{L,jkZ}$	$\frac{g_W}{2c_W}(I_3 - e_j s_w^2)\delta_{jk}$	$\frac{g_W}{2c_W}(I_3 - e_j s_w^2)\delta_{jk}$
$g_{R,jkZ}$	$-\frac{g_W}{2c_W} e_j s_W^2$	$-\frac{g_W}{2c_W} e_j s_W^2$

$g = \sqrt{4\pi\alpha_S}$,

$e \equiv \sqrt{4\pi\alpha_{EM}}$,

$e = g_W \sin\theta_W$,

$c_W \equiv \cos\theta_W$,

$s_W \equiv \sin\theta_W$

Derive the LO cross section for a spin-1 boson

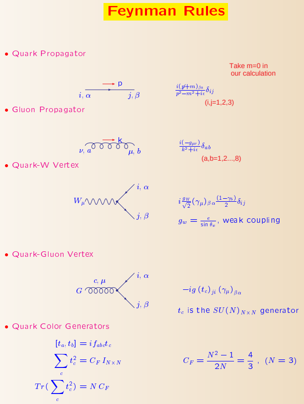
Traditional path
(see S. Dawson's lecture)

Lagrangian$\Rightarrow$Feynman rules $\Rightarrow$
$\sum_{spin} |\mathcal{M}|^2 \Rightarrow \mathrm{Tr}\,(\gamma^{\alpha_1}...\gamma^{\alpha_n}) \Rightarrow$cross section

Helicity amplitudes

Lagrangian$\Rightarrow$"Feynman rules" for helicity amplitudes$\Rightarrow \mathcal{M} \Rightarrow \sum_{spin} |\mathcal{M}|^2 \Rightarrow$ cross section

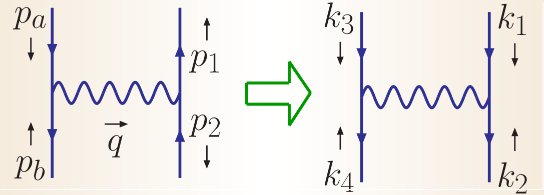
- Efficient computation of tree diagrams
- can be applied to 1-loop and 2-loop calculations (not discussed here)



- Many excellent reviews, e.g., *Mangano, Parke, Phys. Rep. 200, 301; Dixon, hep-ph/9601359*

Calculation of $\mathcal{M}\left(q(p_a)\bar{q}(p_b) \to \ell(p_1)\bar{\ell}(p_2)\right)$

1. Crossing



Compute $\mathcal{M}$ in the auxiliary process $\ell(k_1)\bar{\ell}(k_2)q(k_3)\bar{q}(k_4) \to 0$ to simplify the algebra; take the crossing to the physical channel $q(p_a)\bar{q}(p_b) \to \ell(p_1)\bar{\ell}(p_2)$ at the very end

- Denote $k^\mu_{1,2} = -p^\mu_{1,2}$, $k^\mu_{3,4} = p^\mu_{a,b}$;
- Assume $m_i^2 = 0$, $i = 1, .., 4$
- Particle spins are $s_i \equiv \lambda_i/2$, $\lambda_i = \pm 1$
- Convenient notation: $\{k_i, \lambda_i\} \equiv k_i^{\lambda_i}$

Calculation of $\mathcal{M}\left(q(p_a)\bar{q}(p_b) \rightarrow \ell(p_1)\bar{\ell}(p_2)\right)$

2. Color decomposition

- Decompose $\mathcal{M}$ into a sum of products of color $SU(N_c)$ factors $\left(T^{a_1}...T^{a_n}\right)_{c_1 c_{n+1}}$ and kinematical partial amplitudes $A_4(k_1^{\lambda_1}, k_2^{\lambda_2}, k_3^{\lambda_3}, k_4^{\lambda_4})$
  - trivial in our case:

$$\mathcal{M}\left(\ell(k_1^{\lambda_1}), \bar{\ell}(k_2^{\lambda_2}), q^{c_3}(k_3^{\lambda_3}), \bar{q}^{c_4}(k_4^{\lambda_4})\right) = \mathcal{I}_{c_3 c_4} A_4(k_1^{\lambda_1}, k_2^{\lambda_2}, k_3^{\lambda_3}, k_4^{\lambda_4})$$

$$\mathrm{Tr}\mathcal{I} = N_c$$

  - general formulas are given in the above references
- $A_n(1...n)$ satisfy several helpful symmetries, which often drastically reduce the number of independent amplitudes

$A_n(1, 2, ..., n)$ are gauge-invariant

$$A_n(1, ..., n) = (-1)^n A_n(n, n-1, ...1) \text{ (reflection identity)}$$

$$A_n(1^{\pm}, 2^{+}, ..., n^{+}) = 0 \text{ (effective supersymmetry)}$$

Massless spinor formalism in 4 dimensions

In the massless case, only 2 out of 4 components of the Dirac spinor field $\psi(k,\lambda)$ are independent

Introduce two 4-spinors $|k_i\pm\rangle \equiv |i\pm\rangle$:

$$|i\pm\rangle = u(k_i,\pm 1) = v(-k_i,\mp 1), \quad \langle i\pm| = \overline{u}(k_i,\mp 1) = \overline{v}(-k_i,\pm 1);$$

$$\frac{1}{2}(1\pm\gamma_5)|i\pm\rangle = |i\pm\rangle; \quad \langle i\pm|\frac{1}{2}(1\mp\gamma_5) = \langle i\pm|$$

On-shell conditions

$$\not{k}_i|i\pm\rangle = \langle i\pm|\not{k}_i = 0; \qquad \not{k}_i = |i+\rangle\langle i+| + |i-\rangle\langle i-|$$

Spinor products

$$\langle i-|j+\rangle \equiv \langle ij\rangle; \ \langle i+|j-\rangle \equiv [ij]$$

$$\langle ij\rangle^* = [ji]$$

$$\langle ij\rangle[ji] = 2k_i\cdot k_j \equiv s_{ij}$$

Tree amplitudes are rational functions of $\langle ij\rangle$ and $[ij]$

Some identities for spinor products

Gordon identity and projection operator:

$$\langle i^{\pm}|\gamma^{\mu}|i^{\pm}\rangle = 2k_i^{\mu}, \qquad |i^{\pm}\rangle\langle i^{\pm}| = \frac{1}{2}(1\pm\gamma_5)\,\not{k}_i \tag{19}$$

antisymmetry:

$$\langle j\,i\rangle = -\langle i\,j\rangle\,, \qquad [j\,i] = -[i\,j], \qquad \langle i\,i\rangle = [i\,i] = 0 \tag{20}$$

Fierz rearrangement:

$$\langle i^{+}|\gamma^{\mu}|j^{+}\rangle\langle k^{+}|\gamma_{\mu}|l^{+}\rangle = 2\,[i\,k]\,\langle l\,j\rangle \tag{21}$$

charge conjugation of current:

$$\langle i^{+}|\gamma^{\mu}|j^{+}\rangle = \langle j^{-}|\gamma^{\mu}|i^{-}\rangle \tag{22}$$

Schouten identity:

$$\langle i\,j\rangle\,\langle k\,l\rangle = \langle i\,k\rangle\,\langle j\,l\rangle + \langle i\,l\rangle\,\langle k\,j\rangle\,. \tag{23}$$

In an n-point amplitude, momentum conservation, $\sum_{i=1}^{n} k_i^{\mu} = 0$, provides one more identity,

$$\sum_{\substack{i=1\\ i\neq j,k}}^{n} [j\,i]\,\langle i\,k\rangle = 0. \tag{24}$$

Exercises

1. In Weyl representation,

$$\gamma^0 = \begin{pmatrix} 0 & \mathbf{I} \\ \mathbf{I} & 0 \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & \tilde{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix},$$

σ_i $(i = 1, 2, 3)$ are Pauli matrices. The massless spinors satisfy

$$|p+\rangle = \left(\begin{array}{c} \xi_+(p) \\ 0 \end{array} \right), |p-\rangle = \left(\begin{array}{c} 0 \\ \xi_-(p) \end{array} \right);$$

$$\langle p+| = \left(\begin{array}{cc} 0 & \xi_+^\dagger(p) \end{array} \right), \langle p-| = \left(\begin{array}{cc} \xi_-^\dagger(p) & 0 \end{array} \right),$$

where $\xi_\lambda(p)$ is a 2-component spinor for a massless fermion with momentum p and helicity λ, normalized by $\xi^\dagger_{\lambda_1}(p)\xi_{\lambda_2}(p) = 2p^0\delta_{\lambda_1\lambda_2}$. Show that some spinor products vanish:

$$\langle p\pm|q\pm\rangle = \langle p\pm|\gamma^\mu|q\mp\rangle = 0.$$

Exercises

2. One possible representation for $\xi_\pm(p)$ is

$$\xi_\pm(p) = \frac{1}{2^{1/4}} \begin{pmatrix} \pm\sqrt{p^+} e^{-\varphi_P/2} \\ \sqrt{p^-} e^{\varphi_p/2} \end{pmatrix},$$

where I introduced light-cone coordinates for p,

$$p^\pm \equiv \frac{p^0 \pm p^3}{\sqrt{2}}, \, \vec{p}_T = \sqrt{2p^+p^-}(\cos\varphi_p, \sin\varphi_p).$$

We have $p^2 = 2p^+p^- - p_T^2 = 0$, $p \cdot q = p^+q^- + q^+p^- - \vec{p}_T \cdot \vec{q}_T$, etc.

(a) Check that $\xi^\dagger_{\lambda_1}(p)\xi_{\lambda_2}(p) = 2p^0\delta_{\lambda_1\lambda_2}$.

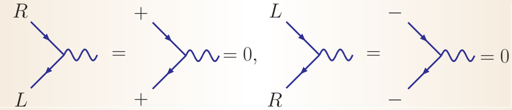
(b) Prove antisymmetry, Gordon identity, Fierz rearrangement on slide 12

Partial amplitudes

The rule

$$\langle p \pm | \gamma^{\mu} | q \mp \rangle = 0$$

reflects chirality conservation in the $\bar{\psi}\not{V}\psi$ vertex:

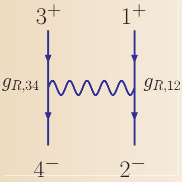


This condition and effective supersymmetry of massless QCD,

$$A_n(1^{\pm}, 2^{+}, ..., n^{+}) = 0,$$

imply that the only non-vanishing LO amplitudes are
$A_4(+-+-)$, $A_4(+--+)$, $A_4(-++-)$, $A_4(-+-+)$.

Partial amplitudes



Denote the couplings as $g_{P,12} \equiv f_P$ and $g_{P,34} \equiv f_P$ for $P = L, R$

$$A_4(+-+-) = -\frac{i}{q^2 - M_V^2} f_R g_R \langle 4+|\gamma^\mu|3+\rangle\langle 2+|\gamma_\mu|1+\rangle$$

$$= -\frac{i}{q^2 - M_V^2} g_{R,12} g_{R,34} [42]\langle 13\rangle$$

$$A_4(+--+) = -\frac{i}{q^2 - M_V^2} f_L g_R [41]\langle 23\rangle$$

$$A_4(-++-) = -\frac{i}{q^2 - M_V^2} f_R g_L [32]\langle 14\rangle$$

$$A_4(-+-+) = -\frac{i}{q^2 - M_V^2} f_L g_L [31]\langle 24\rangle$$

Spin sum

$$\sum_{spin} |A_4|^2 = \frac{1}{\left(q^2 - M_V^2\right)^2} \Bigg((f_R^2 g_R^2 + f_L^2 g_L^2) s_{42} s_{13} + (f_R^2 g_L^2 + f_L^2 g_R^2) s_{41} s_{23} \Bigg)$$

$$= \frac{1}{\left(q^2 - M_V^2\right)^2} \Bigg((f_R^2 g_R^2 + f_L^2 g_L^2) s_{13}^2 + (f_R^2 g_L^2 + f_L^2 g_R^2) s_{14}^2 \Bigg),$$

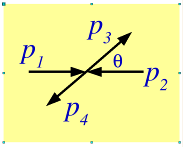
where I used

$$\langle ij \rangle [ji] = 2 p_i \cdot p_j = s_{ij},$$

$$s_{12} = s_{34},\ s_{13} = s_{24},\ s_{14} = s_{23}$$

A rest frame of the vector boson

Return to the physical channel and consider the rest frame of V:



$$p_a = \frac{Q}{2}(1,0,0,1);\; p_b = \frac{Q}{2}(1,0,0,-1);$$
$$p_1 = \frac{Q}{2}(1,0,0,\cos\theta_*);\; p_2 = \frac{Q}{2}(1,0,0,-\cos\theta_*);$$

For $q(p_a)\bar{q}(p_b): p_a = -k_3,\, p_b = -k_4$

For $\bar{q}(p_a)q(p_b): p_a = -k_4,\, p_b = -k_3$

$$|\mathcal{M}|^2 = \frac{1}{(q^2 - M_V^2)^2}\frac{Q^4}{4N_c}\Big[(f_R^2 + f_L^2)(g_L^2 + g_R^2)(1+\cos^2\theta_*) + \epsilon_{q\bar{q}}(f_R^2 - f_L^2)(g_L^2 - g_R^2)(2\cos\theta_*)\Big],$$

$\epsilon_{q\bar{q}} = 1\ (-1)$ for $q\bar{q}$ $(\bar{q}q)$

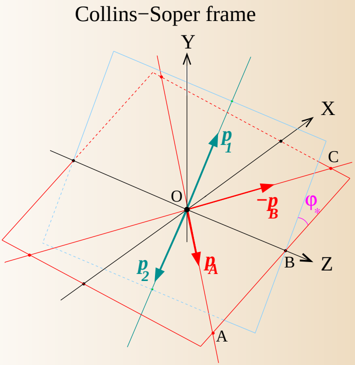
Inclusive kinematics of the lepton pair

The momenta p_1^μ, p_2^μ are fully specified by

- the mass Q, transverse momentum Q_T, rapidity $y = \frac{1}{2}\ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$ of the intermediate boson V in the lab frame
- angles θ_* and φ_* of lepton momenta in the special rest frame of V (Collins-Soper frame)

$$\frac{d^3\vec{p}_1}{2p_1^0}\frac{d^3\vec{p}_2}{2p_2^0} = \frac{1}{8}d^4q\,\underbrace{d\cos\theta_* d\varphi_*}_{d\Omega}$$

$$= \frac{\pi}{16}dQ^2dydQ_Td\Omega$$



$\measuredangle AOB = \measuredangle BOC$

$p_A^x, p_B^x \propto -Q_T \qquad p_A^y = p_B^y = 0$

$p_1 = (Q/2)(1, \sin\theta_*\cos\varphi_*, \sin\theta_*\sin\varphi_*, \cos\theta_*)$

$p_2 = (Q/2)(1, -\sin\theta_*\cos\varphi_*, -\sin\theta_*\sin\varphi_*, -\cos\theta_*)$

At Born level, $Q_T = 0$

Covariant definitions for Q_T and y

Exercise. Convince yourself that y and Q_T can be introduced in a covariant form as

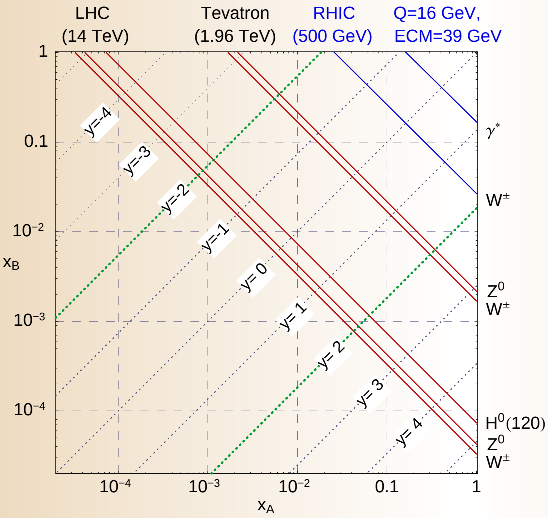
$$y = \frac{1}{2}\ln\left(\frac{p_B \cdot q}{p_A \cdot q}\right),$$

$$Q_T^2 = -q_{t\mu} q_t^\mu, \text{ with}$$

$$q_t^\mu \equiv q^\mu - \frac{(p_A \cdot q)}{(p_A \cdot p_B)} p_B^\mu - \frac{(p_B \cdot q)}{(p_A \cdot p_B)} p_A^\mu$$

As a result, they can be a part of the Lorentz-invariant phase space

$x_{A,B}$, typical parton momentum fractions



$$x_{A,B} \equiv \frac{Q}{\sqrt{s}} e^{\pm y}$$

Born level: $p_a^\mu = x_A p_A^\mu$, $p_b^\mu = x_B p_B^\mu$

Typical rapidities in the experiment: $|y| \lesssim 2$

- experiments at higher energies are sensitive to PDF's at smaller x

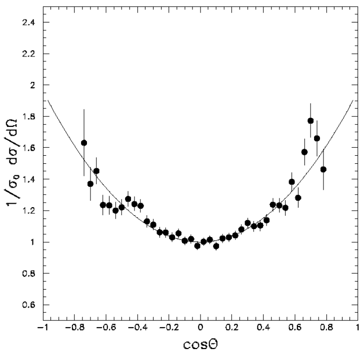
Complete Born cross section

$$\frac{d\sigma}{dQ^2 dy d\Omega} = \frac{1}{16\pi N_c^2 s} \frac{Q^2}{(Q^2 - M_V^2)^2 + \Gamma_V^2 Q^4/M_V^2}$$

$$\times \sum_{j,\bar{k}=u,\bar{u},d,\bar{d},\ldots} \Big\{ (f_R^2 + f_L^2)(g_{L,j\bar{k}}^2 + g_{R,j\bar{k}}^2)(1+\cos^2\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) + \bar{q}_{\bar{k}}(x_A) q_j(x_B)\right]$$

$$+ (f_R^2 - f_L^2)(g_{L,j\bar{k}}^2 - g_{R,j\bar{k}}^2)(2\cos\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) - \bar{q}_{\bar{k}}(x_A) q_j(x_B)\right] \Big\}$$

- The $2\cos\theta_*$ term vanishes in the parity-conserving case ($f_L = f_R$ or $g_L = g_R$)
- The $(1+\cos\theta_*^2)$ dependence in the experimental data confirms the vector (spin-1) nature of low-Q Drell-Yan process



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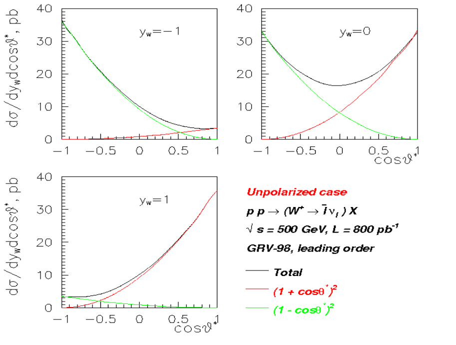
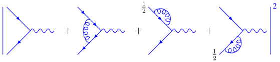
$$+ (f_R^2 - f_L^2)(g_{L,j\bar{k}}^2 - g_{R,j\bar{k}}^2)(2\cos\theta_*)\,[q_j(x_A)\bar{q}_{\bar{k}}(x_B) - \bar{q}_{\bar{k}}(x_A) q_j(x_B)] \Big\}$$

- W boson production: $f_R = g_R = 0$
- W cross section depends on two functions $(1 \pm \cos\theta_*)^2$ weighted by different parton luminosities
- non-trivial correlation between y and θ_* in the acceptance, etc.

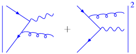


NLO cross section

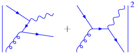
- NLO: $(\alpha_s^{(1)})$ virtual corrections $(q\overline{q}')_{virt}$



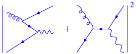
- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(q\overline{q}')_{real}$



- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(qG)_{real}$



- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(G\overline{q}')_{real}$



Virtual contributions

The dominant contribution to σ_{tot}, if x is moderate

$$\sigma_{tot}^{NLO} \sim \left[1 + \frac{\alpha_s}{2\pi} C_F \left(1 + \frac{4\pi^2}{3}\right)\right] \sigma_{tot}^{LO}$$
$$\sim [1 + 3.005\alpha_s]\, \sigma_{tot}^{LO}$$

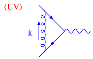
At $x \to 0$ or 1, $ln(x)$ or $ln^p(1-x)/(1-x)_+$ terms are enhanced; the NLO factor is not constant!

$2 \to 3$ contributions

Generate $Q_T \neq 0$, non-trivial θ_*, φ_* dependence

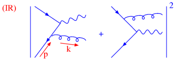
Immediate problems (Singularities)

- Ultraviolet singularity



$$\sim \int^{\infty} d^4k \frac{\not k \; \not k}{(k^2)\,(k^2)\,(k^2)} \to \infty$$

- Infrared singularities



$$\to \infty$$

as $k^\mu \to 0$ (soft divergence)

or $k^\mu \parallel p^\mu$ (collinear divergence)

- Solutions

Compute H_{ij} in pQCD in $n = 4 - 2\varepsilon$ dimensions (dimensional regularization)

(1) $n \neq 4 \Rightarrow$ UV & IR divergences appear as $\frac{1}{\varepsilon}$ poles in $\sigma_{ij}^{(1)}$ (Feynman diagram calculation)

(2) H_{ij} is IR safe $\Rightarrow$ no $\frac{1}{\varepsilon}$ in H_{ij} (H_{ij} is UV safe after "renormalization".)

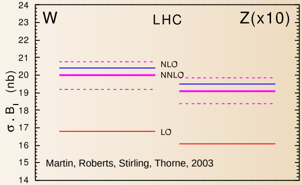
(Similar singularities also exist in virtual diagrams.)

- Treatment of collinear logarithms introduces dependence on the factorization scheme
- Residual soft logarithms in differential distributions may require resummation to all orders in α_s

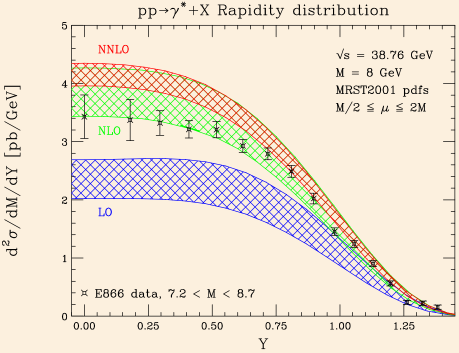
NNLO total section $\sigma_{tot}(AB \rightarrow W, Z)$

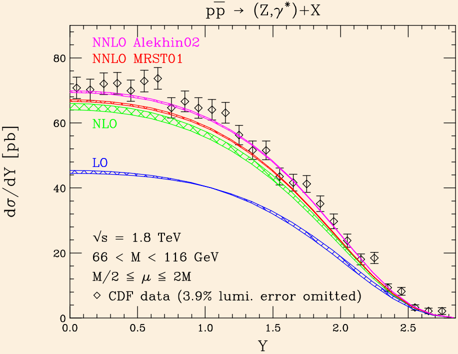
(Hamberg, van Neerven, Matsuura; Harlander, Kilgore)

- Scale dependence of order 1%
- NNLO K-factor is about 1.04 at the Tevatron and 0.98 at the LHC *(MRST'03)*



NNLO differential cross sections *(Anastasiou, Dixon, Melnikov, Petriello, 2003-05)*





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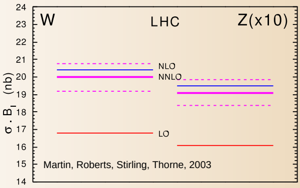

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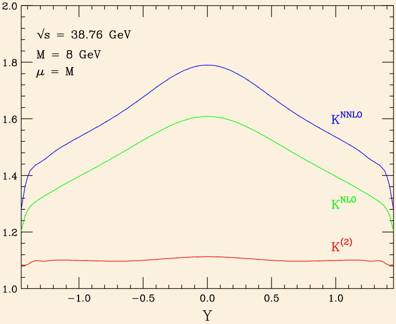

$K^{(N)/NLO}(y) = \sigma_{(N)NLO}/\sigma_{LO}$;

$K^{NLO} \approx 1.6\ (1.4)$ at $y = 0$ $(y = 1)$

Compare with $1 + 3\alpha_s(8) \approx 1.56$

$K^{(2)} = \sigma_{NNLO}/\sigma_{NLO}$ – uniform enhancement over NLO by $\sim 8\%$