Introduction to the Parton Model and Pertrubative QCD

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- III. Factorization and Evolution
- IV. Summary

- Challenge: use AF in observables
(cross sections (σ) (also some amplitudes . . .))
that are *not infrared safe*

- Possible *if:* σ has a short-distance subprocess.
Separate *IR Safe* from IR: this is factorization

- IR Safe part (short-distance) is calculable in pQCD

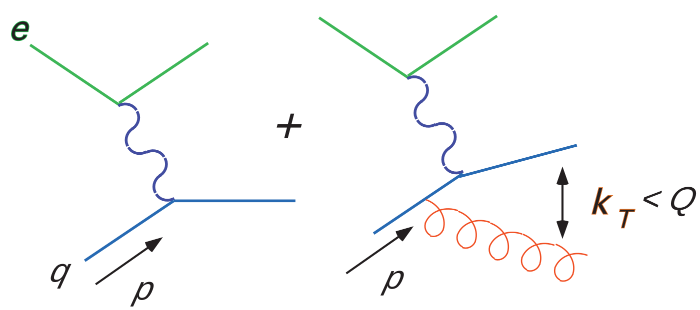
- Infrared part – example: parton distribution –
measureable and universal

- Infrared safety – insensitive to soft gluon emission
collinear rearrangements

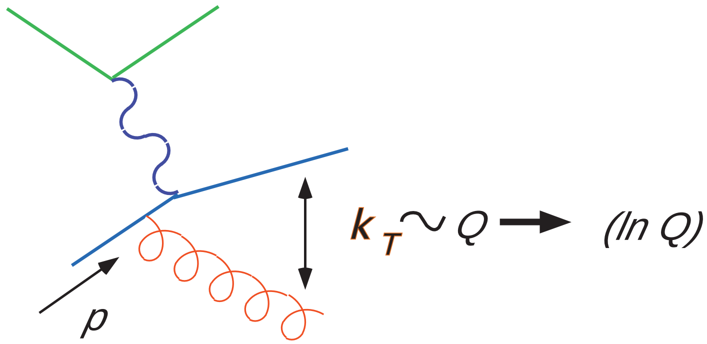
- Just like Parton Model except in Parton Model the infrared safe part is $\sigma_{\mathrm{Born}} \Rightarrow f(x)$ normalized uniquely

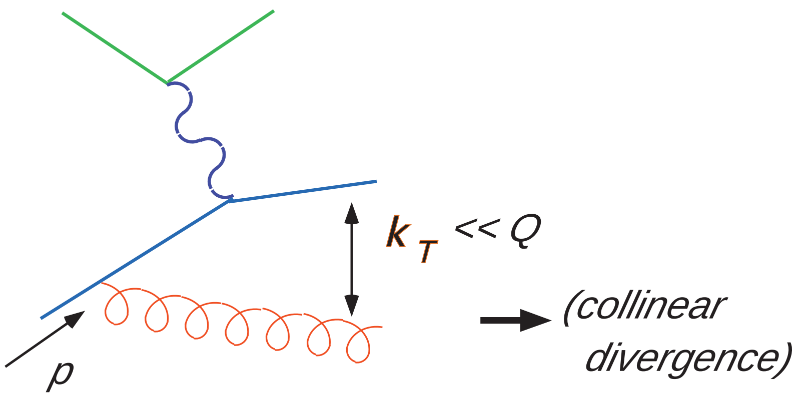
- In pQCD must define parton distributions more carefully: the factorization scheme

- Basic observation: virtual states not truly frozen. Some states fluctuate on scale $1/Q$ . . .



Long-lived states $\Rightarrow$ Collinear Logs (IR)





Short-lived states $\Rightarrow \ln(Q)$

RESULT: FACTORIZED DIS

$$F_2^{\gamma q}(x,Q^2) = \int_x^1 d\xi \, C_2^{\gamma q}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right) \times f_{q/q}(\xi,\mu_F,\alpha_s(\mu))$$

$$\equiv C_2^{\gamma q}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right) \otimes f_{q/q}(\xi,\mu_F,\alpha_s(\mu))$$

- f has $\ln(\mu_F/\Lambda_{\mathrm{QCD}})$ . . .
- C has $\ln(Q/\mu)$, $\ln(\mu_F/\mu)$
- Often pick $\mu = \mu_F$ and often pick $\mu_F = Q$. So often see:

$$F_2^{\gamma q}(x, Q^2) = C_2^{\gamma q}\left(\frac{x}{\xi}, \alpha_s(Q)\right) \otimes f_{q/q}(\xi, Q^2)$$

- But we still need to specify what we *really* mean by factorization: *scheme* as well as *scale*

- For this, compute $F_2^{\gamma q}(x, Q)$

- Keep $\mu = \mu_F$ for simplicity

- “Compute quark-photon scattering” – *What does this mean?*

- Must use an *IR-regulated* theory

- Extract the *IR Safe part* then take away the regularization

- Let’s see how it works . . .

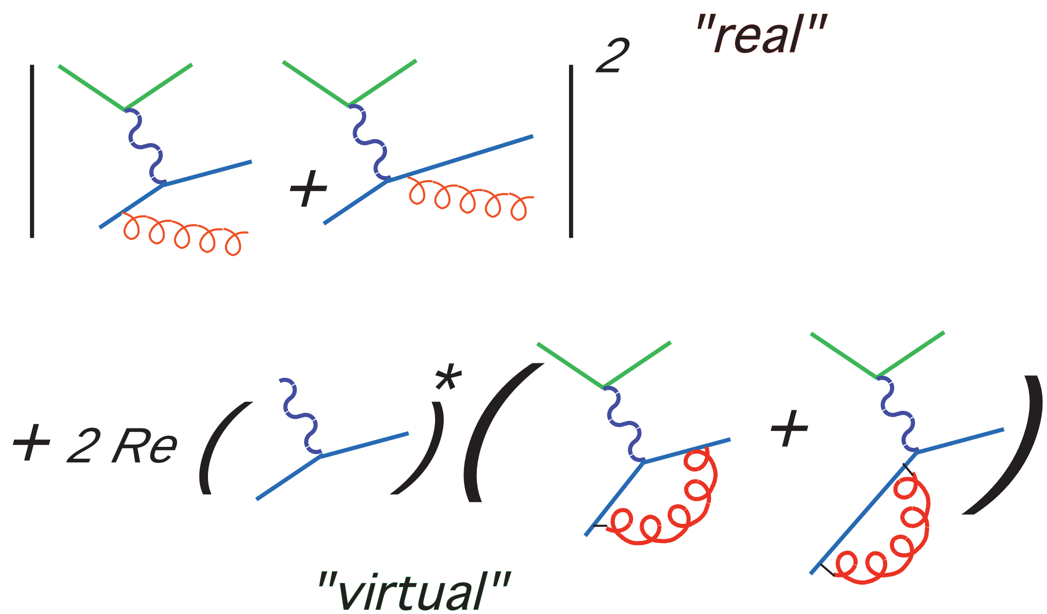
  - At *zeroth order – no interactions:*
  - $C^{\gamma q_f (0)} = Q_f^2 \; \delta(1 - x/\xi)$
    (Born cross section; parton model)
  - $f^{(0)}_{q_f/q_{f'}}(\xi) = \delta_{f f'} \, \delta(1 - \xi)$
    (at zeroth order, momentum fraction conserved)

$$
\begin{aligned}
F_2^{\gamma q_f\,(0)}(x,Q^2) &= \int_x^1 d\xi\; C_2^{\gamma q_f\,(0)}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right) \\
&\quad \times\; f^{(0)}_{q_f/q_f}(\xi,\mu_F,\alpha_s(\mu)) \\
&= Q_f^2\;\int_x^1 d\xi\;\delta(1-x/\xi)\;\delta(1-\xi) \\
&= Q_f^2\;x\,\delta(1-x)
\end{aligned}
$$

– On to one loop . . .

$F^{\gamma q}$ AT ONE LOOP: FACTORIZATION SCHEMES

- Start with F_2 for a *quark*:



Have to combine final states with different phase space . . .

"Plus Distributions" :

$$\int_0^1 dx \, \frac{f(x)}{(1-x)_+} \quad \equiv \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)}$$

$$\int_0^1 dx \; f(x) \left(\frac{\ln(1-x)}{1-x} \right)_+ \quad \equiv \int_0^1 dx \; (\, f(x) - f(1) \,) \; \frac{\ln(1-x)}{(1-x)}$$

and so on . . . where

- $f(x)$ will be parton distributions
- $f(x)$ term: real gluon, with momentum fraction $1-x$
- $f(1)$ term: virtual, with elastic kinematics

A Special Distribution
DGLAP "evolution kernel" = "splitting function"

$$P_{qq}^{(1)}(x) = C_F\,\frac{\alpha_s}{\pi}\,\left[\frac{1+x^2}{1-x}\right]_+$$

- Will see: P_{qq} a probability per unit log k_T

Expansion and Result:

$$F_2^{\gamma q}(x, Q^2) \;=\; \int_x^1 d\xi\; C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \times f_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

$$F_2^{\gamma q_f}(x, Q^2) \;=\; C_2^{(0)}\; f^{(0)} + \frac{\alpha_s}{2\pi}\; C^{(1)}\; f^{(0)} + \frac{\alpha_s}{2\pi}\; C^{(0)}\; f^{(1)} + \ldots$$

$$
\begin{aligned}
F_2^{\gamma q_f}(x,Q^2) \;&=\; Q_f^2 \Bigg\{ x\,\delta(1-x) \\
&\quad + \frac{\alpha_s}{2\pi}\, C_F \left[\frac{1+x^2}{1-x}\left(\frac{\ln(1-x)}{x}\right) + \frac{1}{4}\,(9-5x)\right]_+ \\
&\quad + \frac{\alpha_s}{2\pi}\, C_F \int_0^{Q^2} \frac{dk_T^2}{k_T^2}\, \left[\frac{1+x^2}{1-x}\right]_+ \Bigg\} + \ldots
\end{aligned}
$$

$$
F_1^{\gamma q_f}(x,Q^2) \;=\; \frac{1}{2x}\,\left\{F_2^{\gamma q_f}(x,Q^2) - C_F\,\alpha\,\frac{\alpha_s}{\pi^2}\,2x\right\}
$$

Factorization Schemes

$\overline{\mathbf{MS}}$

$$f^{(1)}_{q/q}(x,\mu^2) = \frac{\alpha_s}{\pi^2}\; P_{qq}(x)\; \int_0^{\mu^2} \frac{dk_T^2}{k_T^2}$$

With k_T-integral "IR regulated".

Advantage: technical simplicity; not tied to process.

$C^{(1)}(x)_{\overline{\rm MS}} = (\alpha_s/2\pi)\; P_{qq}(x)\ln(Q^2/\mu^2) +$ μ-independent

DIS:

$$f_{q/q}(x,\mu^2) = \frac{\alpha_s}{\pi^2}\; F^{\gamma q_f}(x,\mu^2)$$

Absorbs all uncertainties in DIS into a PDF.

Closer to experiment for DIS.

$C^{(1)}(x)_{\overline{DIS}} = (\alpha_s/2\pi)\; P_{qq}(x)\ln(Q^2/\mu^2) + 0$

Using the Regulated Theory
and
Getting Parton Distributions for Real Hadrons

- IR-regulated QCD is not *REAL* QCD
- *BUT* it only differs at low momenta
- *THUS* we can use it for IR Safe functions: $C_2^{\gamma q}$, etc.
- This enables us to get PDFs for real hadrons . . .

- Compute $F_2^{\gamma q}$, $F_2^{\gamma G}$ . . .

- Define factorization scheme; find IR Safe C's

- Use factorization in the full theory

$$F_2^{\gamma N} = \sum_{a=q_f,\bar{q}_f,G} C^{\gamma a} \otimes f_{a/N}$$

- Measure F_2; then use the known C's to derive $f_{a/N}$

- Multiple flavors and cross sections
  complicate technicalities; not logic (Global Fits)

$$\textit{NOW HAVE } f_{a/N}(\xi, \mu^2)$$

USE IT IN ANY OTHER PROCESS THAT FACTORIZES

EVOLUTION

- Q^2-dependence
- In general, Q^2/μ^2 dependence still in $C_a\left(x/\xi, Q^2/\mu^2, \alpha_s(\mu)\right)$
  Choose $\mu = Q$

$$F_2^{\gamma A}(x, Q^2) = \sum_a \int_x^1 d\xi \, C_2^{\gamma a}\left(\frac{x}{\xi}, 1, \alpha_s(Q)\right) \, f_{a/A}(\xi, \mu^2)$$

$Q \gg \Lambda_{\mathrm{QCD}} \rightarrow$ *compute C's in PT*.

$$C_2^{\gamma a}\left(\frac{x}{\xi}, 1, \alpha_s(Q)\right) = \sum_n \left(\frac{\alpha_s}{\pi}\right)^n C_2^{\gamma a(n)}\left(\frac{x}{\xi}\right)$$

But still need PDFs at $\mu = Q$: $f_{a/A}(\xi, Q^2)$

– Remarkable result: EVOLUTION

$Can\ use\ f_{a/A}(x,Q_0^2)$ to determine $f_{a/A}(x,Q^2)\ and\ hence\ F_{1,2,3}(x,Q^2)\ for\ any\ Q\ !$

So long at $\alpha_s(Q)$ is still small

– Illustrate by a ‘nonsinglet’ distribution

$$F_a^{\gamma \mathrm{NS}} = F_a^{\gamma p} - F_a^{\gamma n}$$

$$F_2^{\gamma \mathrm{NS}}(x, Q^2) = \sum_a \int_x^1 d\xi \; C_2^{\gamma \mathrm{NS}}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \alpha_s(\mu)\right) \; f_{\mathrm{NS}}(\xi, \mu^2)$$

Gluons, antiquarks cancel

At one loop: $C_2^{\mathrm{NS}} = C_2^{\gamma N}$

– 'Mellin' Moments and Anomalous Dimensions

$$\bar{f}(N) = \int_0^1 dx\ x^{N-1}\ f(x)$$

– Reduces convolution to a product

$$f(x) = \int_x^1 dy\ g\left(\frac{x}{y}\right)\ h(y) \rightarrow \bar{f}(N) = \bar{g}(N)\ \bar{h}(N+1)$$

– Moments applied to NS structure function:

$$\bar{F}_2^{\gamma\mathrm{NS}}(N,Q^2) = \bar{C}_2^{\gamma\mathrm{NS}}\left(N,\frac{Q}{\mu},\alpha_s(\mu)\right)\,\bar{f}_{\mathrm{NS}}(N,\mu^2)$$

(Note $f_{\mathrm{NS}}(N,\mu^2) \equiv \int_0^1 d\xi \xi^N f(\xi,\mu^2)$ here.)

– $\bar{F}_2^{\gamma\mathrm{NS}}(N,Q^2)$ is PHYSICAL

$$\Rightarrow \quad \mu\frac{d}{d\mu}\,\bar{F}_2^{\gamma\mathrm{NS}}(N,Q^2) = 0$$

– ‘Separation of variables’

$$\mu\frac{d}{d\mu}\ \ln \bar{f}_{\rm NS}(N,\mu^2) = -\gamma_{\rm NS}(N,\alpha_s(\mu))$$

$$\gamma_{\rm NS}(N,\alpha_s(\mu)) = \mu\frac{d}{d\mu}\ln \bar{C}_2^{\gamma^{\rm NS}}(N,\alpha_s(\mu))$$

– Because α_s is the only variable held in common!

$$\mu\frac{d}{d\mu}\ln \bar{f}_{\rm NS}(N,\mu^2) \;=\; -\gamma_{\rm NS}(N,\alpha_s(\mu))$$

$$\gamma_{\rm NS}(N,\alpha_s(\mu)) \;=\; \mu\frac{d}{d\mu}\ln \bar{C}_2^{\gamma {\rm NS}}(N,\alpha_s(\mu))$$

– Only need to know C's $\Rightarrow \gamma_n$ from IR regulated theory!

$\Downarrow$

Q-DEPENDENCE DETERMINED BY PT

EVOLUTION

THIS WAS HOW WE FOUND OUT QCD IS ‘RIGHT’

THIS IS HOW QCD PREDICTS PHYSICS
AT NEW SCALES

γ_{NS} AT ONE LOOP

Hint: $(1-x^2)/(1-x) = 1 + x \ldots (1-x^k)/(1-x) = \sum_{i=0}^{k-1} x^k$

$$
\begin{aligned}
\gamma_{\mathrm{NS}}(N,\alpha_s) &= \mu\frac{d}{d\mu}\,\ln \bar{C}_2^{\gamma^{\mathrm{NS}}}(N,\alpha_s(Q)) \\
&= \mu\frac{d}{d\mu}\,\left\{\,(\alpha_s/2\pi)\;\bar{P}_{qq}(N)\ln(Q^2/\mu^2)+\mu\ \text{indep.}\,\right\} \\
&= -\frac{\alpha_s}{\pi}\int_0^1 dx\; x^{N-1}\;P_{qq}(x) \\
&= -\frac{\alpha_s}{\pi}C_F\int_0^1 dx\;\left[\left(x^{N-1}-1\right)\,\frac{1+x^2}{1-x}\right] \\
&= -\frac{\alpha_s}{\pi}C_F\left[4\sum_{m=2}^{N}\frac{1}{m}-2\frac{2}{N(N+1)}+1\right] \\
&\equiv -\frac{\alpha_s}{\pi}\;\gamma_{\mathrm{NS}}^{(1)}
\end{aligned}
$$

SOLUTION: SCALE BREAKING

$$\mu \frac{d}{d\mu}\ \bar{f}_{\mathrm{NS}}(N, \mu^2) = -\gamma_{\mathrm{NS}}(N, \alpha_s(\mu))\ \bar{f}_{\mathrm{NS}}(N, \mu^2)$$

$$\bar{f}_{\mathrm{NS}}(N, \mu^2) = \bar{f}_{\mathrm{NS}}(N, \mu_0^2) \times \exp\left[-\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\ \gamma_{\mathrm{NS}}(N, \alpha_s(\mu)) \right]$$

$$\Downarrow$$

$$\bar{f}_{\mathrm{NS}}(N, Q^2)\ = \bar{f}_{\mathrm{NS}}(N, Q_0^2)\ \left(\frac{\ln(Q^2/\Lambda^2_{\mathrm{QCD}})}{\ln(Q_0^2/\Lambda^2_{\mathrm{QCD}})} \right)^{-2\gamma_N^{(1)}/\beta_0}$$

Hint:

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \, \ln(Q^2/\Lambda^2_{\rm QCD})}$$

So also:

$$\bar{f}_{\rm NS}(N, Q^2) \quad = \quad \bar{f}_{\rm NS}(N, Q_0^2) \, \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{-2\gamma_N^{(1)}/\beta^{(1)}}$$

$$\bar{f}_{\rm NS}(N,Q^2) \quad = \quad \bar{f}_{\rm NS}(N,Q_0^2)\,\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{-2\gamma_N^{(1)}/\beta^{(1)}}$$

– Mild' scale breaking

– For $\alpha_s \to \alpha_0 \neq 0$, get a power Q-dependence:

$$\left(Q^2\right)^{\gamma^{(1)}\frac{\alpha_s}{2\pi}}$$

– QCD's consistency with the Parton Model (73-74)

$$\mu \frac{d}{d\mu}\ \bar{f}_{\mathrm{NS}}(N,\mu^2) = -\gamma_N(\alpha_s(\mu))\ \bar{f}_{\mathrm{NS}}(N,\mu^2)$$

$$\Downarrow$$

$$\mu \frac{d}{d\mu}\ \bar{f}_{\mathrm{NS}}(N,\mu^2) = \int_x^1 \frac{d\xi}{\xi}\ P_{\mathrm{NS}}(\xi,\alpha_s(\mu))\ \bar{f}_{\mathrm{NS}}(\xi,\mu^2)$$

Splitting function $\leftrightarrow$ Moments

$$\int_0^1 dx\ x^{N-1}\ P_{qq}(x,\alpha_s) = \gamma_{qq}(N,\alpha_s)$$

BEYOND NONSINGLET COUPLED EVOLUTION

$$\mu \frac{d}{d\mu}\ \bar{f}_{b/A}(N, \mu^2) = \sum_{b=q,\bar{q},G} \int_x^1 \frac{d\xi}{\xi}\ P_{ab}(\xi, \alpha_s(\mu))\ \bar{f}_{b/A}(\xi, \mu^2)$$

Physical Contxt of Evolution

– Parton Model: $f_{a/A}(x)$ density of parton a with momentum fraction x, assumed independent of Q

– PQCD: $f_{a/A}(x, \mu)$: same density, but with transverse momentum $\leq \mu$

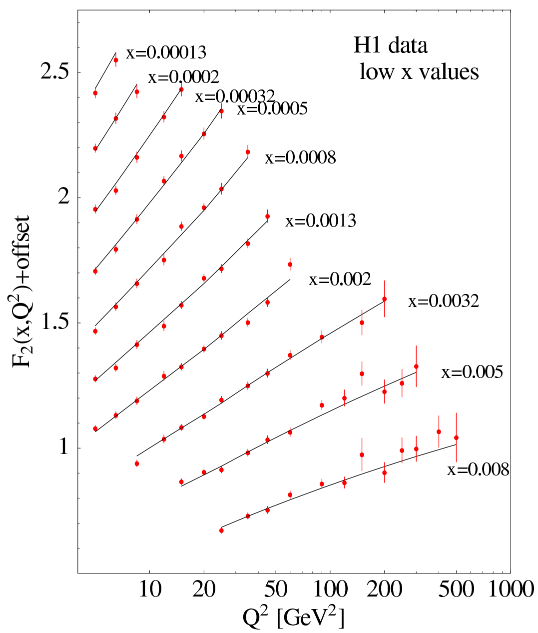
– If there $were$ a maximum transverse momentum Q_0, $f(x, Q_0)$ would freeze for $\mu \geq Q_0$

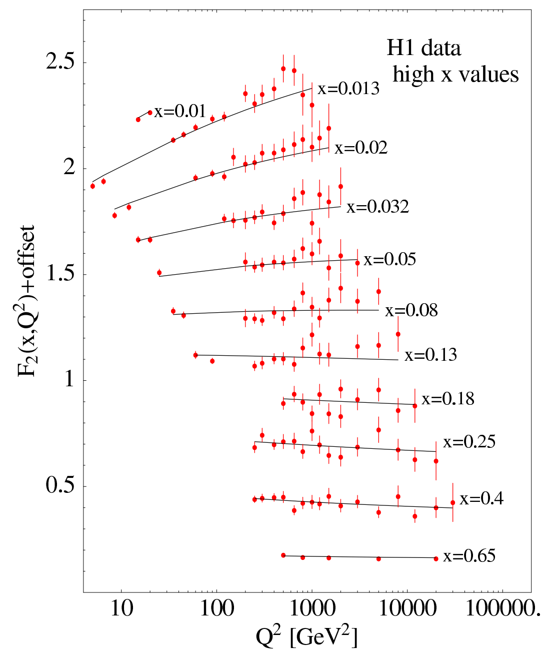
– $Not\ so$ in renormalized PT

– Scale breaking measures the change in the density as maximum transverse momentum increases

– Cross sections we compute still depend on our choice of μ through uncomputed "higher orders" in C and evolution

– Evolution in DIS (with CTEQ6 fits)





IV. SUMMARY

- Specific problems for perturbation theory in QCD

1. Confinement

$$\int e^{-iq\cdot x} \langle 0|\, T[f_a(x)\ldots]\,|0\rangle$$

   has no $q^2 = m^2$ pole for f_a that
   transforms nontrivially under color (confinement)

2. The pole at $p^2 = m_\pi^2$

$$\int e^{-iq\cdot x} \langle 0|\, T[\pi(x)\ldots]\,|0\rangle$$

   is not accessible to perturbation theory

- Response: use infrared safety & **asymptotic freedom**:

$$
\begin{aligned}
Q^2\, \hat{\sigma}_{\rm SD}(Q^2, \mu^2, \alpha_s(\mu)) &= \sum_n c_n(Q^2/\mu^2)\, {\alpha_s}^n(\mu) + \mathcal{O}\left(1/Q^p\right) \\
&= \sum_n c_n(1)\, {\alpha_s}^n(Q) + \mathcal{O}\left(1/Q^p\right)
\end{aligned}
$$

- What can we really calculate? PT for color singlet operators
  - $\int \mathrm{e}^{-iq\cdot x}\langle 0|\, T[J(x)J(0)\ldots]\,|0\rangle$ for color singlet currents

    $\mathrm{e}^+\mathrm{e}^-$ total . . . no QCD in initial state

– Jet cross sections are from matrix elements also:

$$\lim_{R\to\infty} \int dx_0 \int d\hat{n}\, S(\hat{n})\, \mathrm{e}^{-iq\cdot y} \langle 0|\, J(0) T[\hat{n}_i \Theta_{0i}(x_0, R\hat{n}) J(y)]\, |0\rangle$$

Where the operator Θ_{0i} measures momentum flow

"Weight" $S(\hat{n})$ introduces no new dimensional scale

Short-distance dominated if all $d^k S/d\hat{n}^k$ bounded

Individual final states have IR divergences, but these cancel in sum over collinear splitting/merging and soft parton emission because they respect energy flow

But what of the initial state? (viz. parton model)

- **Factorization**

$$Q^2\sigma_{\rm phys}(Q,m) = \omega_{\rm SD}(Q/\mu,\alpha_s(\mu)) \otimes f_{\rm LD}(\mu,m) + \mathcal{O}\left(1/Q^p\right)$$

  - $\mu =$ **factorization scale**; $m=$ **IR scale** (m *may* be perturbative)
  - **New physics** in $\omega_{\rm SD}$; $f_{\rm LD} = f$ and/or D "universal"
  - ep DIS inclusive, $\rm pp \rightarrow$ jets, $Q\bar{Q}$, $\pi(p_T)$ . . .
  - Exclusive limits: $\rm e^+e^- \rightarrow$ JJ as $m_J \rightarrow 0$

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q,m)$$

$$\mu \frac{d \ln(f \text{ or } D)}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

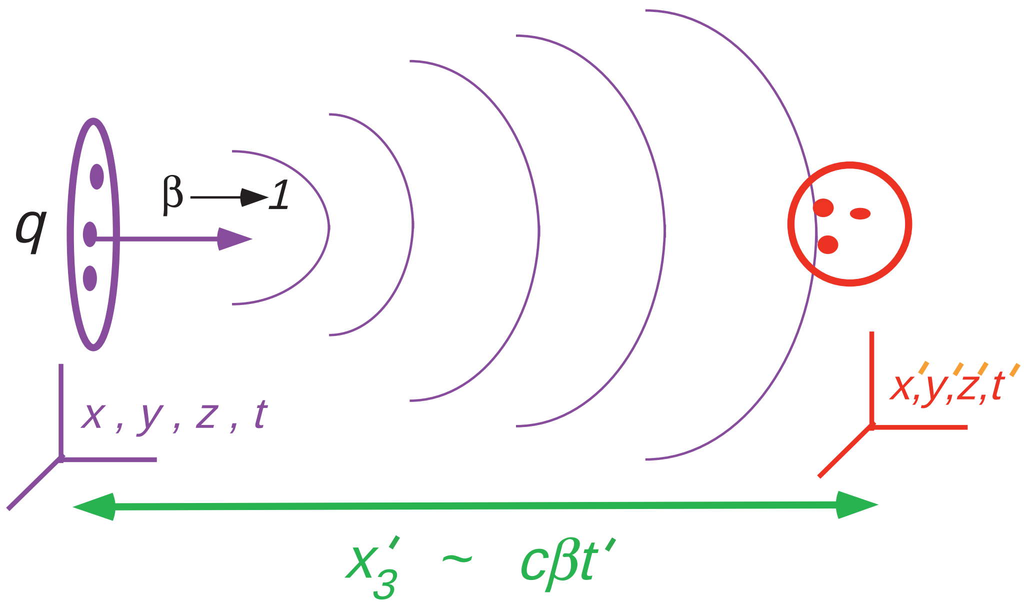
PDF f or Fragmentation D

- **Wherever there is evolution there is resummation**

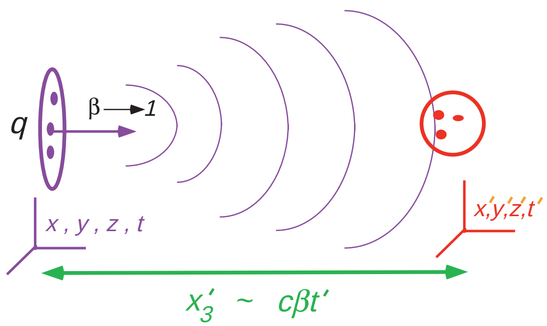
$$\ln \sigma_{\rm phys}(Q,m) = \exp \left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

- Basis of Factorization proofs:
  - (1) $\omega_{\rm SD}$ **incoherent** with long-distance dynamics
  - (2) Mutual incoherence when $v_{\rm rel} = c$:
    Jet-jet factorization.
  - (3) Wide-angle soft radiation sees only total color flow:
    jet-soft factorization.
  - (4) Dimensionless coupling and renormalizability
    $\Leftrightarrow$ no worse that logarithmic divergence in the IR:
    suppression even by a fractional power $\Rightarrow$ finiteness

– Why Factorization? Heuristic, classical argument:



$$\Delta \equiv \beta c t' - x_3'$$



field	x frame	x' frame
scalar	$\frac{q}{\lvert\vec{x}\rvert}$	$\frac{q}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
gauge $(-)$	$A^-(x) = \frac{q}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{q\gamma(1+\beta)\Delta}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
field strength	$E_3(x) = \frac{q}{\lvert\vec{x}\rvert^2}$	$E_3'(x') = \frac{-q\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$

- Classical: Lorentz contracted fields of incident particles don't overlap until the moment of the scattering, creation of heavy particle, etc.!
- Initial-state interactions decouple from the hard process
- Summarized by multiplicative factors: **parton distributions**
- Evolution of partons to jets/hadrons too late to know details of the hard scattering
- Summarized by multiplicative factors: **fragmentation functions**
- "Left-over" cross section for hard scattering is IR safe