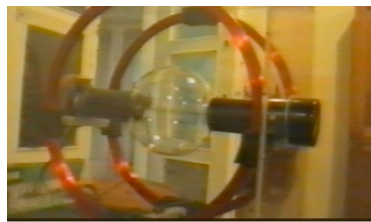


# Introduction to Deep Inelastic Scattering (DIS)

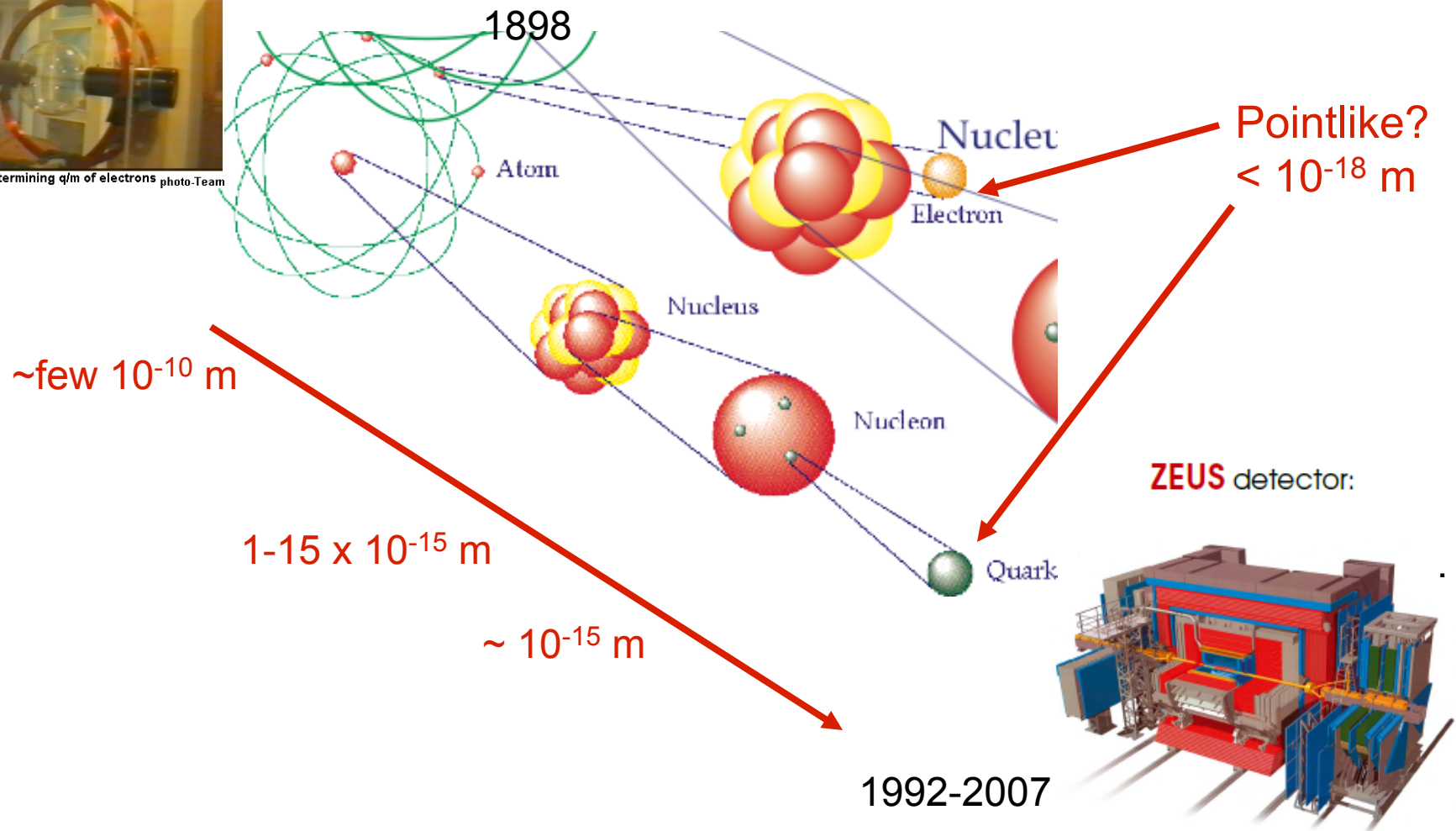
Stephen Magill  
Argonne National Laboratory  
2009 CTEQ Summer School  
Madison, WI

# Inelastic Scattering

## – Probing the Structure of Matter



Device for determining q/m of electrons photo-Team



# Quark-Parton Model (QPM) of DIS

Feynman's QPM explanation of DIS : the nucleon is made up of point-like, spin-1/2, non-interacting constituents – the quarks as partons. DIS is the *incoherent* sum of elastic scattering from these quarks.

Furthermore, the probability  $f(x)$  for the quark  $f$  to carry a fraction  $x$  of the nucleon momentum is an intrinsic property of the nucleon and is *process independent*.

We now know that QCD describes quark interactions with the addition of another “parton” - the gluon (QCD-improved QPM).

- Nucleons are just a “beam of partons” (*incoherent*).
- The  $f(x)$ s, the “beam parameters”, could be measured in some other process (*process independent*).

# Quarks and Gluons as Partons

$u(x)$  : up quark distribution

$\bar{u}(x)$  : up anti-quark distribution

etc. (d,s,c,b,t) and

$g(x)$  : gluon (spin-1)

Momentum has to add up to 1 (“momentum sum rule”) :

$$\int x[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)+s(x)+\bar{s}(x)+\dots+g(x)]dx = 1$$

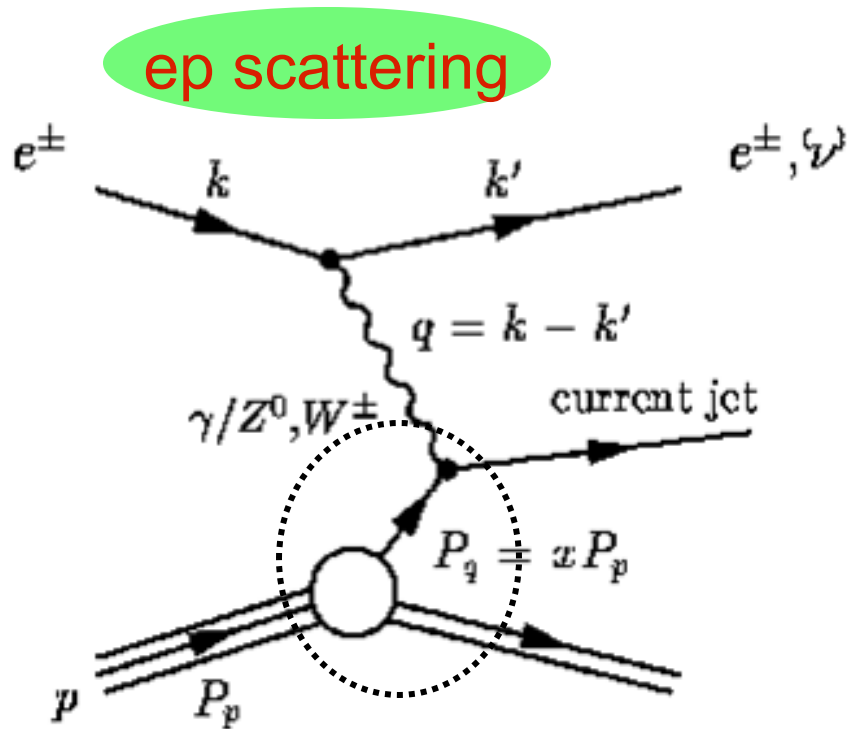
Quantum numbers of the nucleon have to be right :

So for a proton :

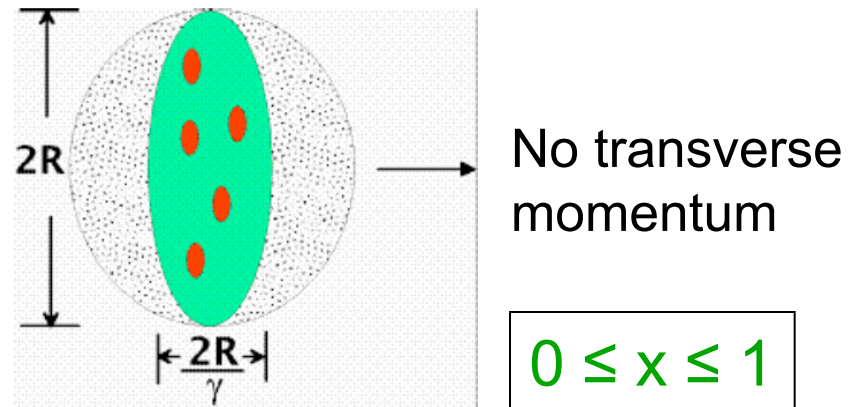
$$\int [u(x)-\bar{u}(x)]dx=2 \quad \# u_{\text{val}} \quad \int [d(x)-\bar{d}(x)]dx=1 \quad \# d_{\text{val}}$$

$$\int [s(x)-\bar{s}(x)+\dots]dx=0 \quad \text{“sea” quark contribution}$$

# DIS Kinematics – Scattering Variables



proton in “∞” momentum frame



$x$  = fractional longitudinal momentum carried by the struck parton

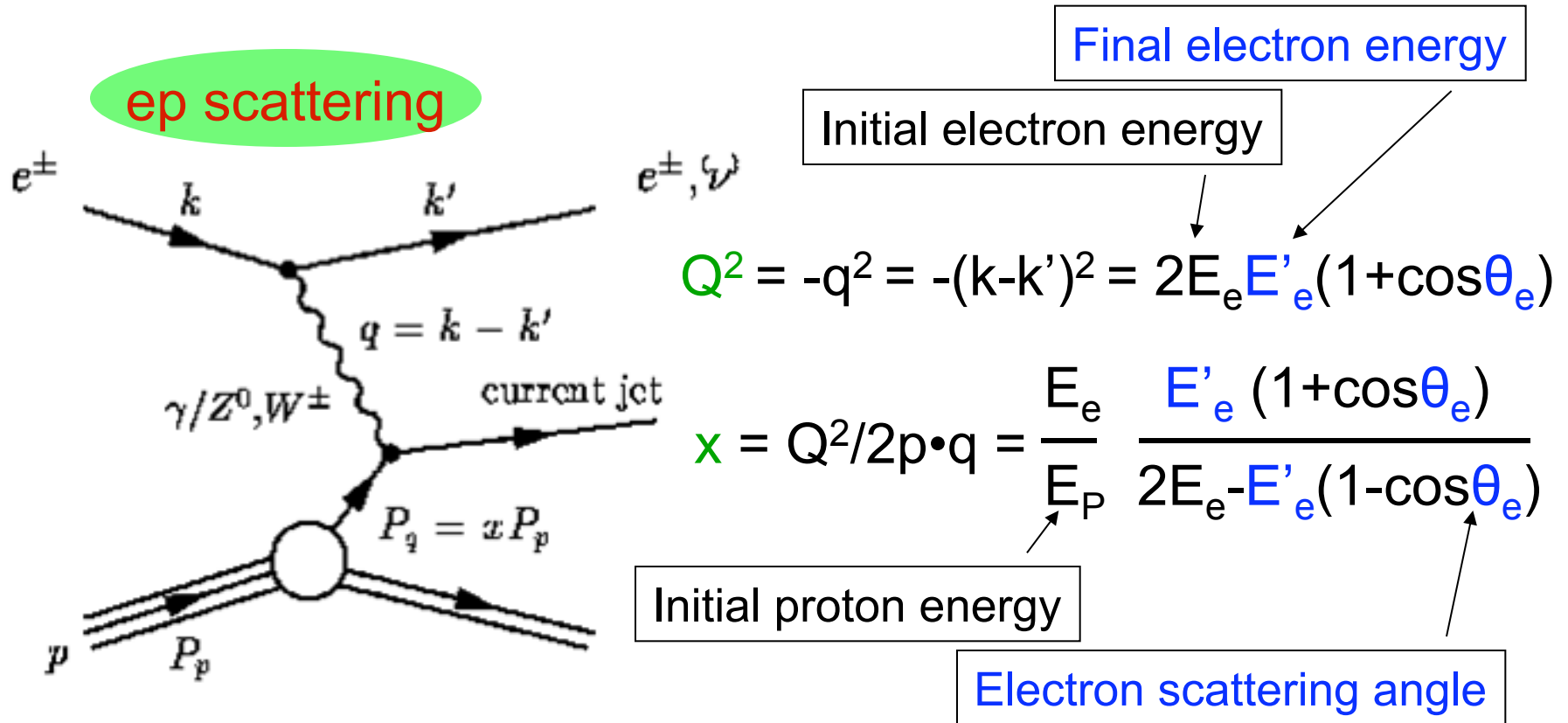
$y$  = fractional energy transfer

$$0 \leq y \leq 1$$

$\sqrt{s}$  = ep cms energy

$Q^2 = -q^2 = 4\text{-momentum transfer squared} = sxy$   
(or virtuality of the “photon”)

# DIS Kinematics – Experimental Variables

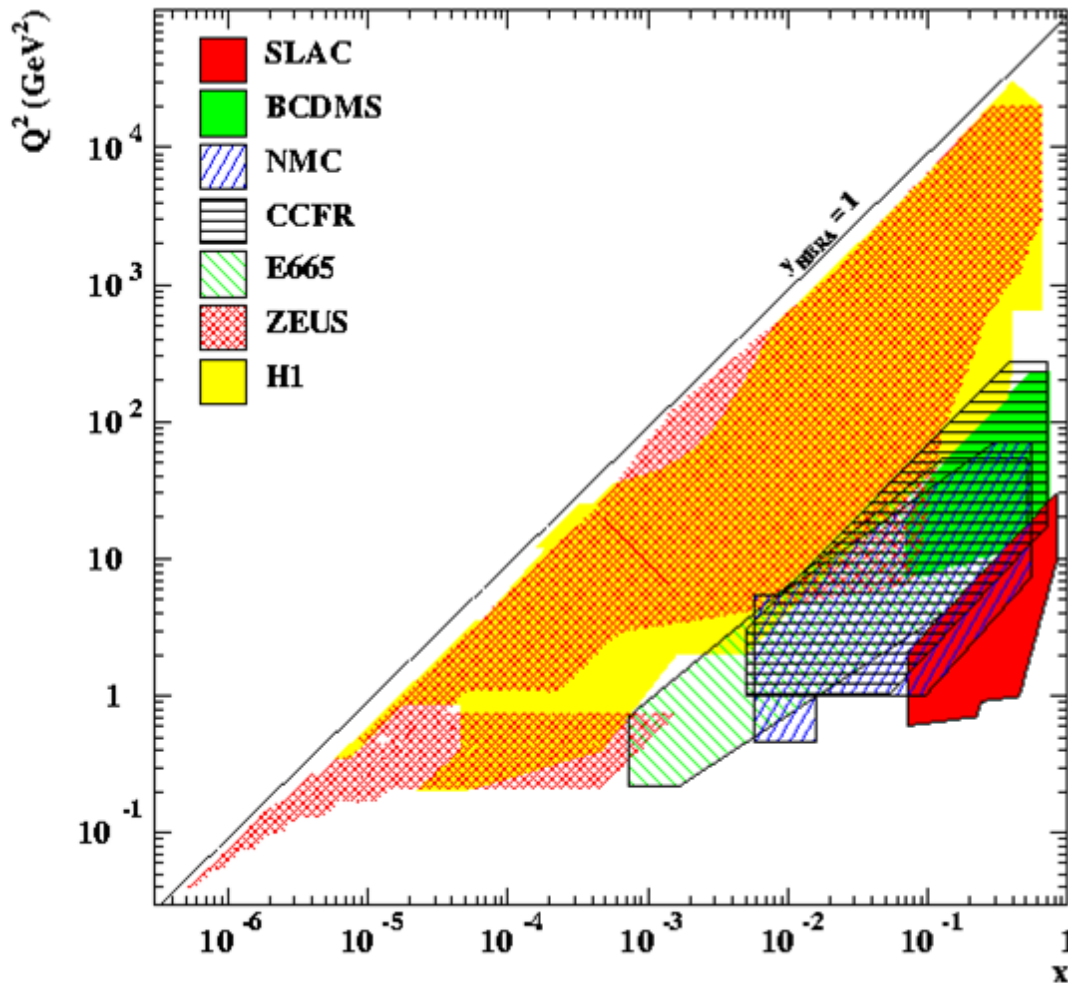


$E'_e, \theta_e$  : electron method

$E_h, \gamma_h$  : Jacquet-Blondel method (energy, angle of struck quark)

$\theta_e, \gamma_h$  : Double-Angle method (angles of scattered electron, struck quark)

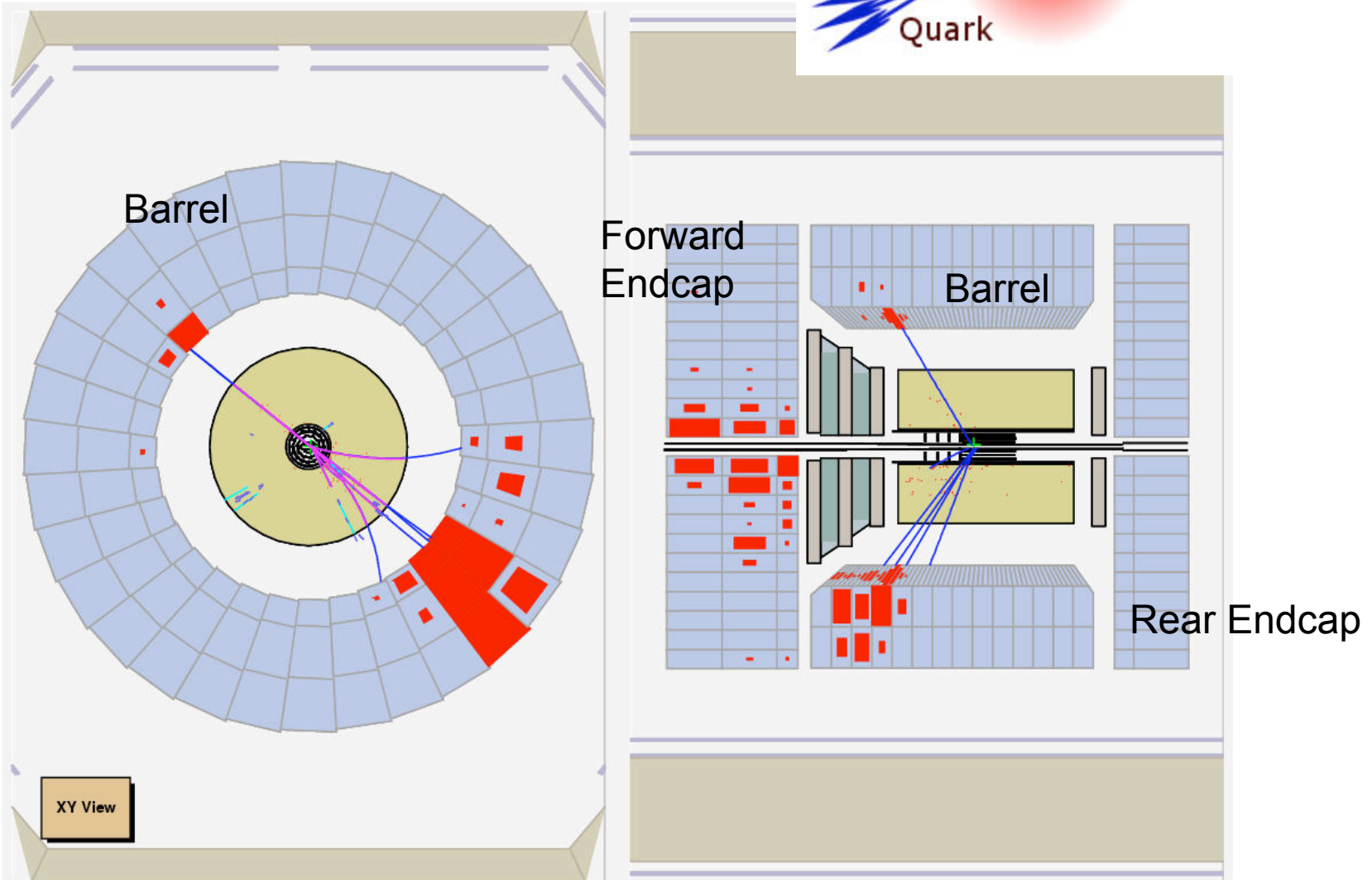
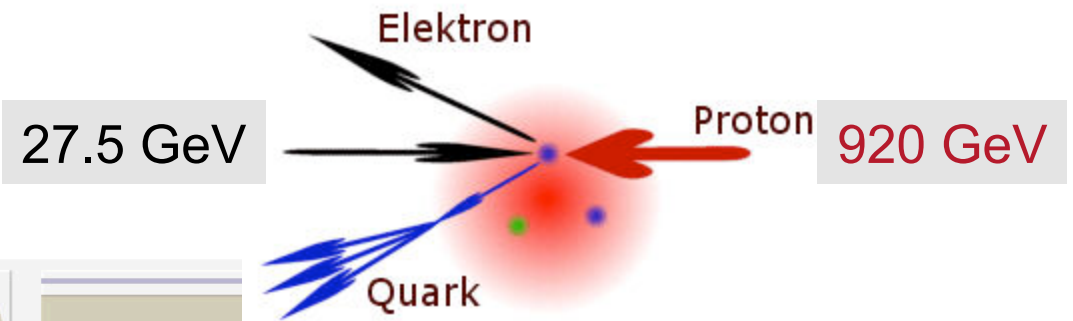
# Kinematics of DIS Experiments



HERA collider :  
H1 and ZEUS  
1992 – 2007

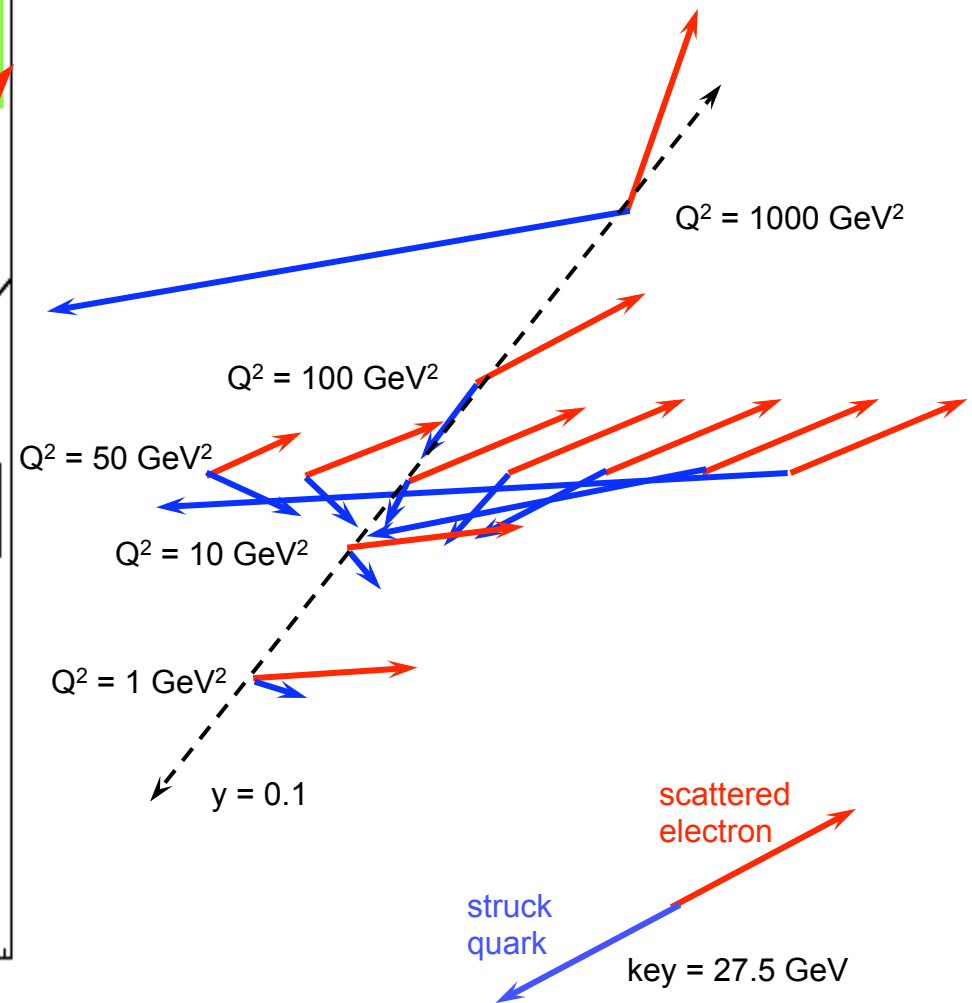
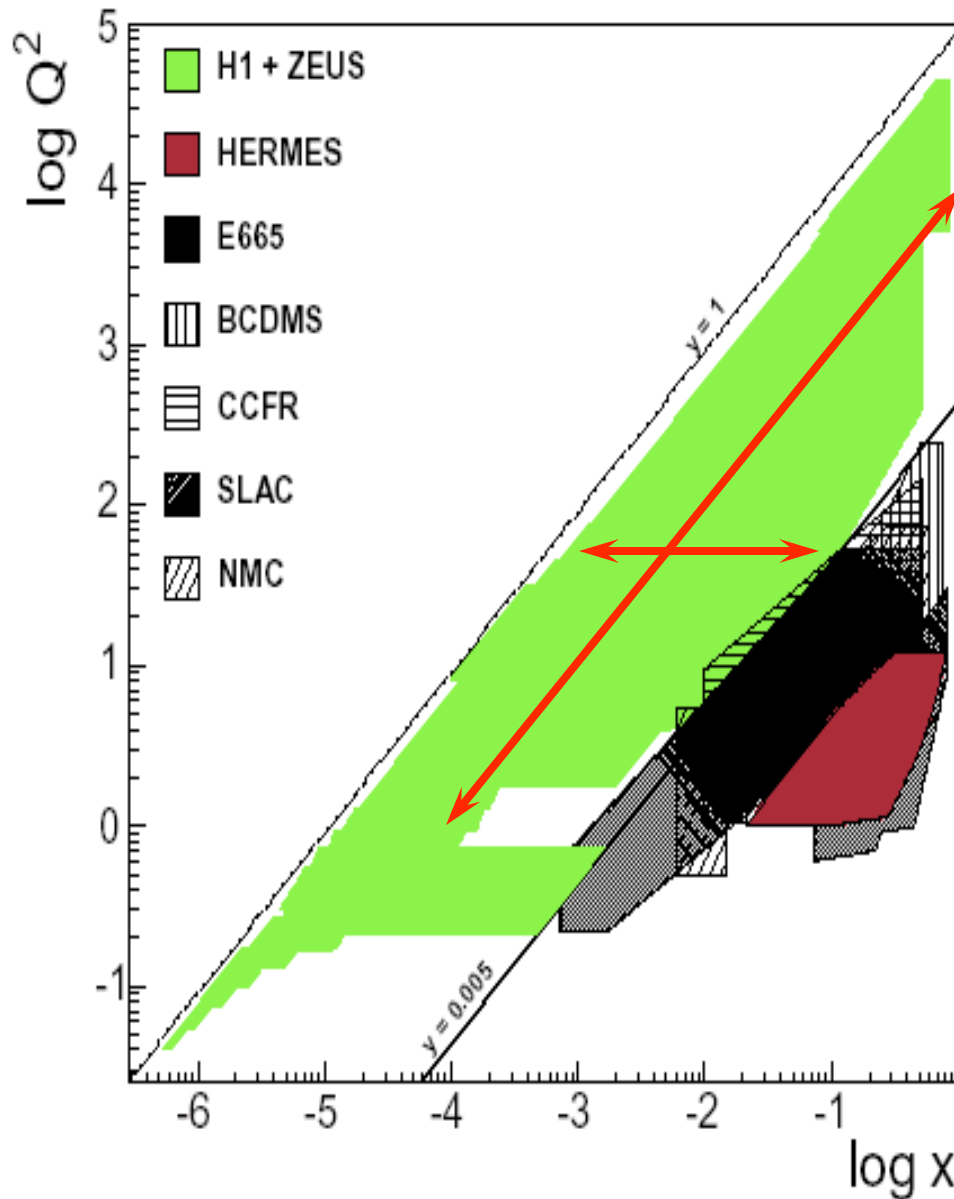
Fixed target :  
SLAC, FNAL and CERN  
completed ~10-20 years  
ago

# DIS at ZEUS





# Event Topology at the HERA Collider



109 CTEQ  
chool

# ep Neutral Current ( $\gamma^*, Z$ ) Cross Section

$$\frac{d\sigma_{e\pm p}^2}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} (Y_+ F_2 - y^2 F_L \mp Y_- x F_3)$$

$Y_{\pm} = (1 \pm (1-y)^2)$ , the inelasticity parameter

The structure functions of the proton are :

$$F_2(x, Q^2) = x \sum_q e_q^2 (q(x, Q^2) + \bar{q}(x, Q^2))$$

- the **sum** of the quark and anti-quark densities

$$xF_3(x, Q^2) = x \sum_q e_q^2 (q(x, Q^2) - \bar{q}(x, Q^2))$$

- the **difference** of the quark and anti-quark densities,  
small for  $Q^2 \ll M_Z^2$

$$F_L(x, Q^2) \sim F_2 - xg(x, Q^2)$$

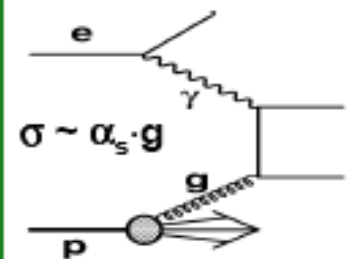
- the longitudinal structure function which **vanishes at LO in QCD** and is damped by  $y^2$  in the cross section

# HERA and PDFs: a rough guide

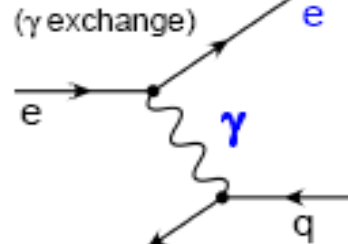
NC DIS:  $\sigma^{\pm} = \frac{d^2\sigma^{\pm}}{dx dQ^2} \frac{Q^4 x}{2\pi\alpha^2 Y_{\pm}} = \bar{F}_2^{\pm} \mp \frac{Y_{\pm}}{Y_{\pm}'} s \bar{F}_3^{\pm} - \frac{Y_{\pm}^2}{Y_{\pm}'} \bar{F}_L^{\pm}$

## Final States: (Jets, Charm, ...)

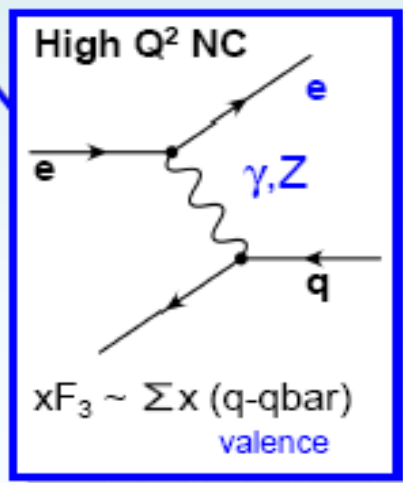
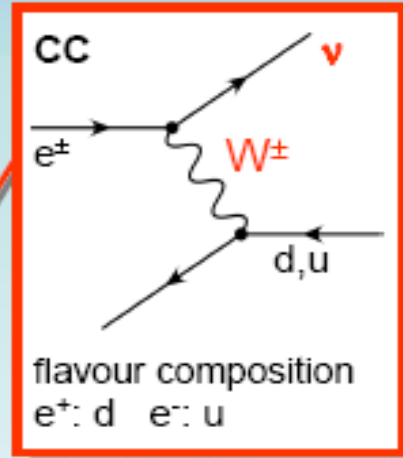
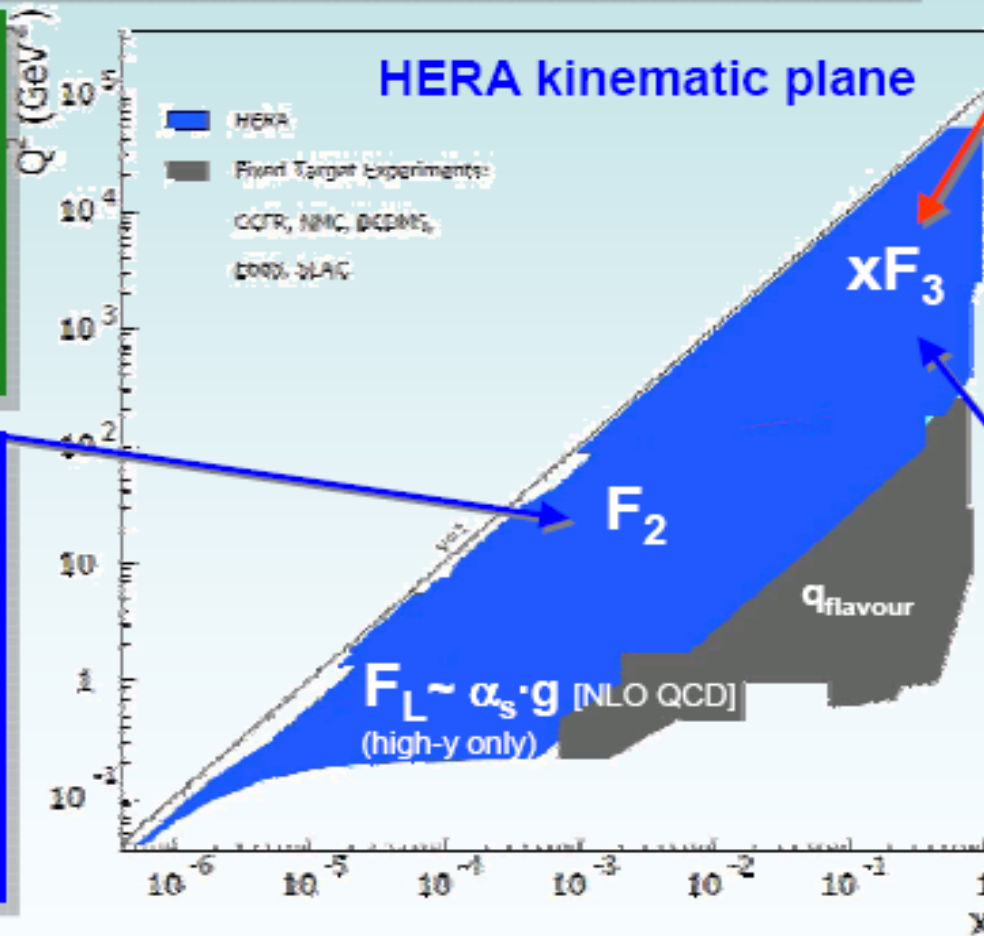
(Jets, Charm, ...)



## Low $Q^2$ NC ( $\gamma$ exchange)



$F_2 \sim \sum x (q + \bar{q})$   
 $dF_2/d\ln Q^2 \sim \alpha_s \cdot g$

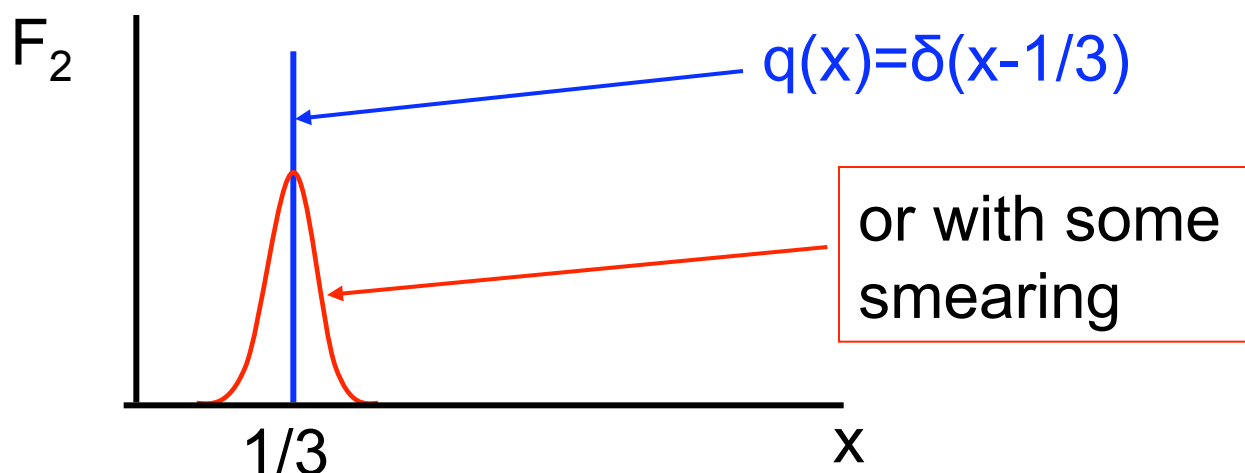


We will start with the structure function  $F_2$  :

If, proton was made of 3 quarks each with  $1/3$  of proton's momentum:

$$F_2 = x \sum_q e_q^2 (q(x) + \bar{q}(x))$$

no anti-quark!

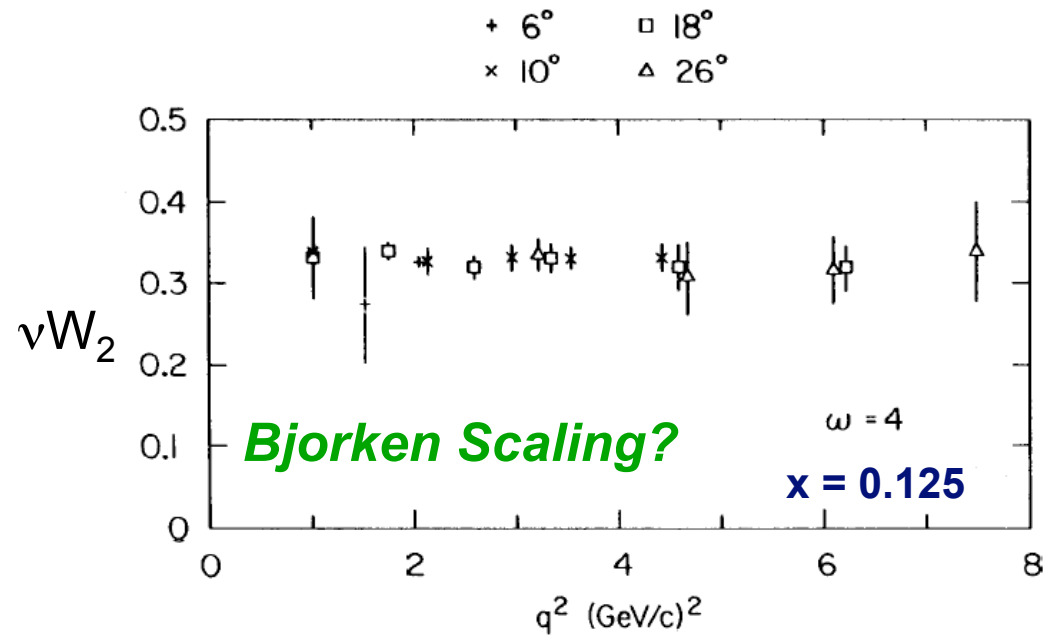
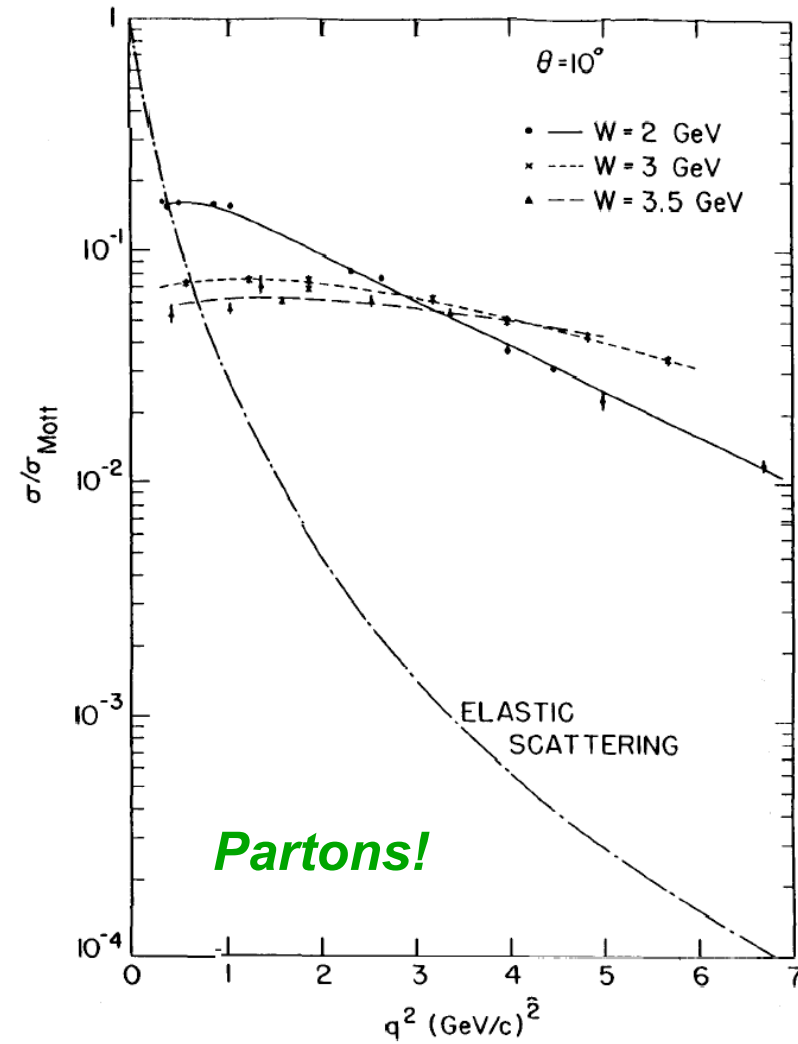


The partons are *point-like* and *incoherent* -  $F_2$  should be independent of  $Q^2$ .

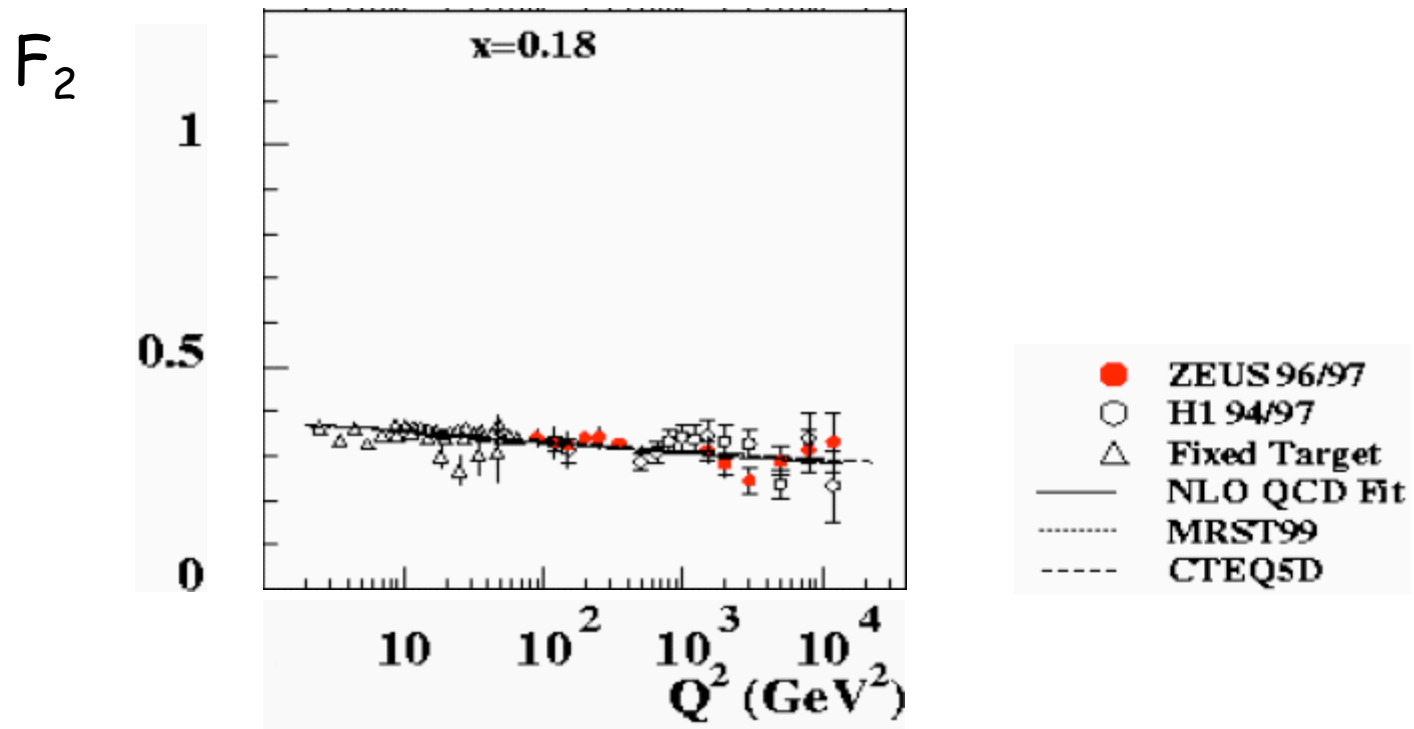
→ **Bjorken scaling** :  $F_2$  has no  $Q^2$  dependence.

**Does the data support this? →**

# SLAC-MIT Results (1969)



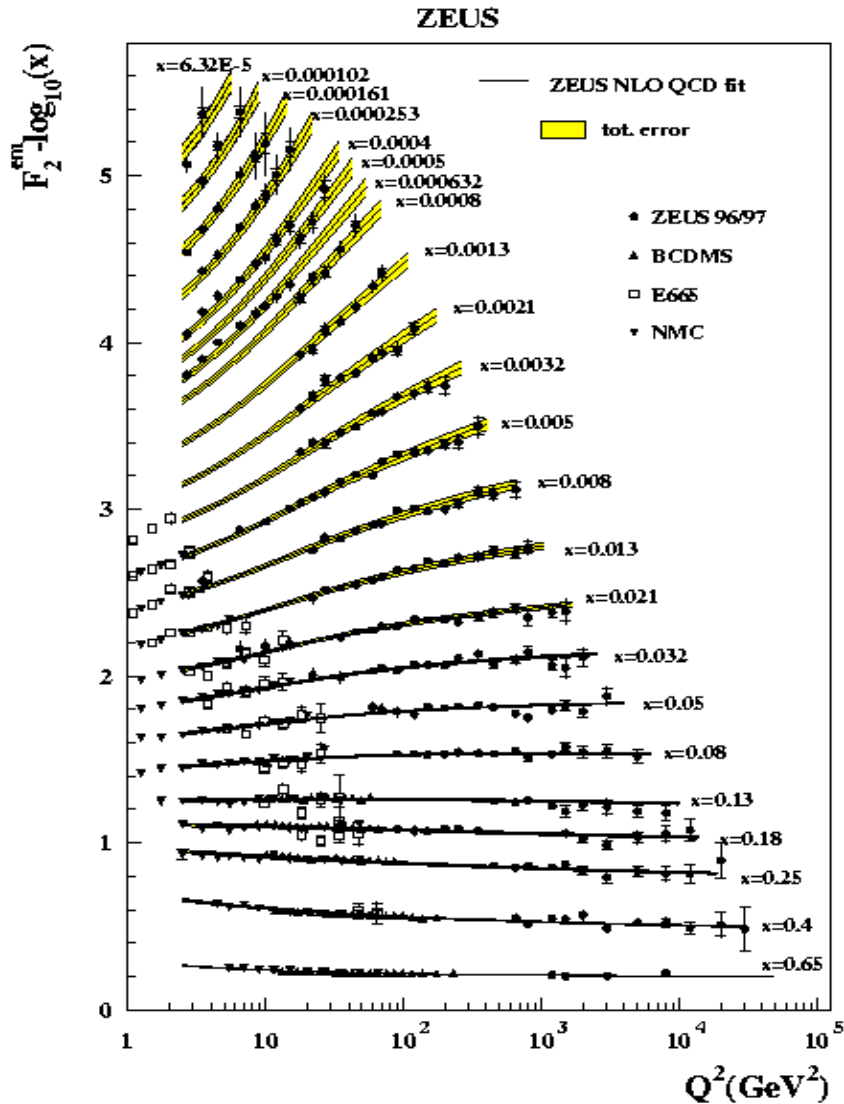
# Proton Structure Function $F_2$



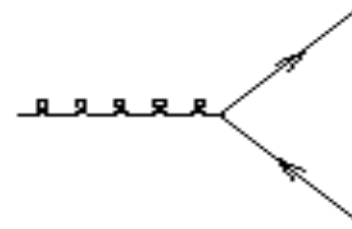
Bjorken scaling is .... **NOT seen at all  $x$ !**

# QCD – $F_2$ violates Bjorken Scaling

$$\frac{\partial F_2}{\partial \ln Q^2} \sim \alpha_s x g$$

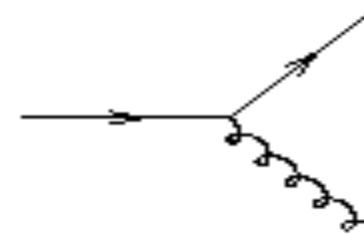


At low  $x$  :



Gluon splitting enhances quark density  
 $\rightarrow F_2$  rises with  $Q^2$

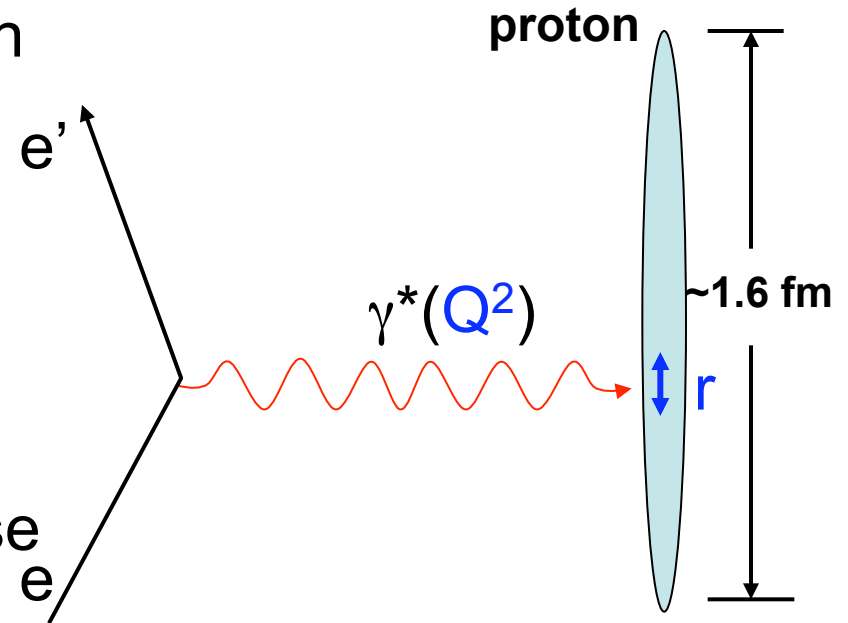
At high  $x$  :



Gluon radiation shifts quark to lower  $x$   
 $\rightarrow F_2$  falls with  $Q^2$

# Proton probe with a photon of virtuality $Q^2$

- Distance scale  $r$  at which proton is probed :  
 $r \approx hc/Q = 0.2\text{fm}/Q[\text{GeV}]$
- Because the virtual photon is absorbed in a time much shorter than the characteristic time of parton-parton interactions (Impulse Approximation), the DIS cross section factorizes as :

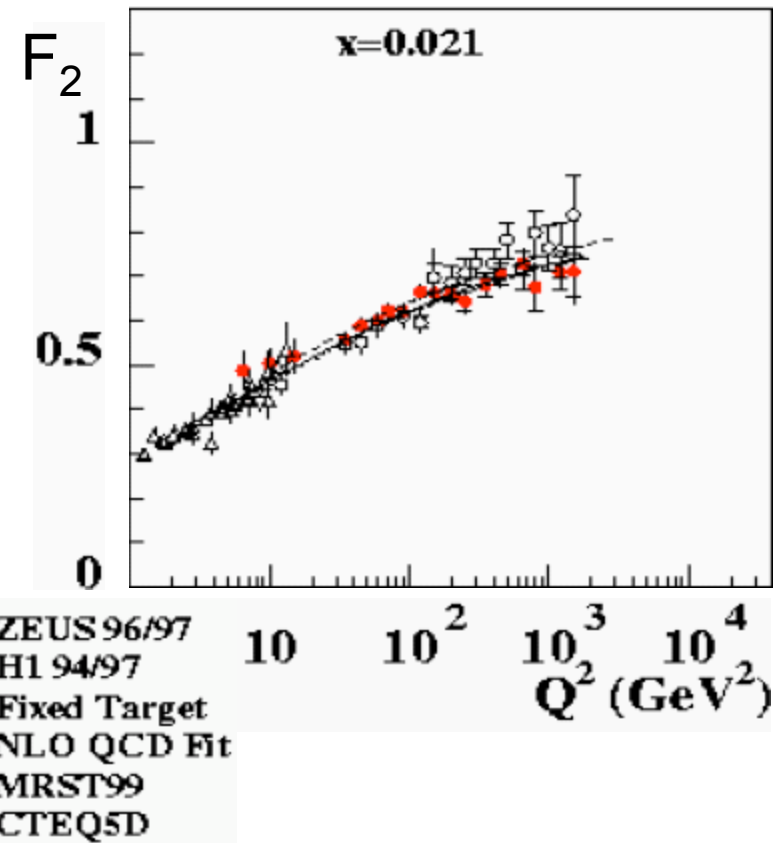
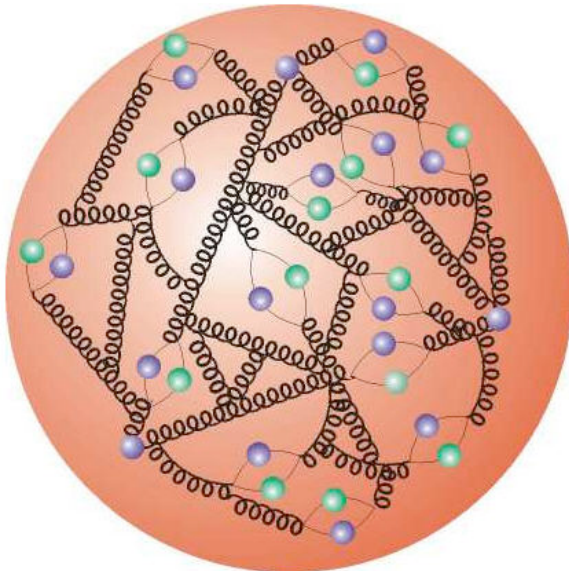


$$\sigma_{\text{DIS}} \sim f_p(\mathbf{x}) \otimes \sigma$$

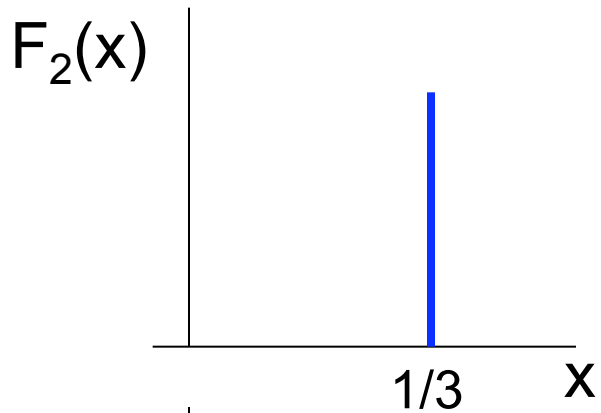
$f_p(\mathbf{x})$  : (universal) parton density functions in the proton  
 $\sigma$  : hard scattering partonic cross section  $\rightarrow$  pQCD



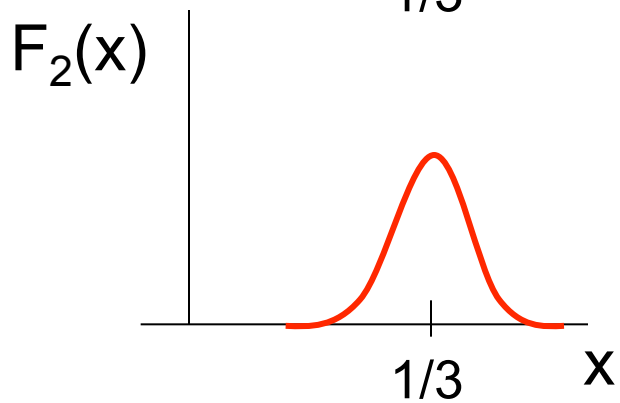
Higher the resolution (i.e. higher the  $Q^2$ ) more branchings to lower  $x$  we “see”.



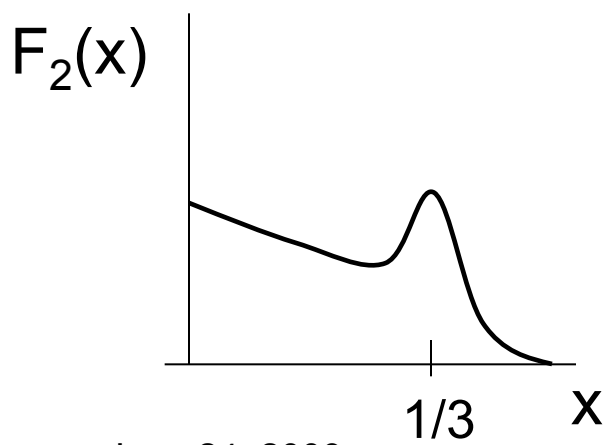
So what do we expect  $F_2$  as a function of  $x$  at a fixed  $Q^2$  to look like?



Three quarks with  $1/3$  of total proton momentum each.



Three quarks with some momentum smearing.

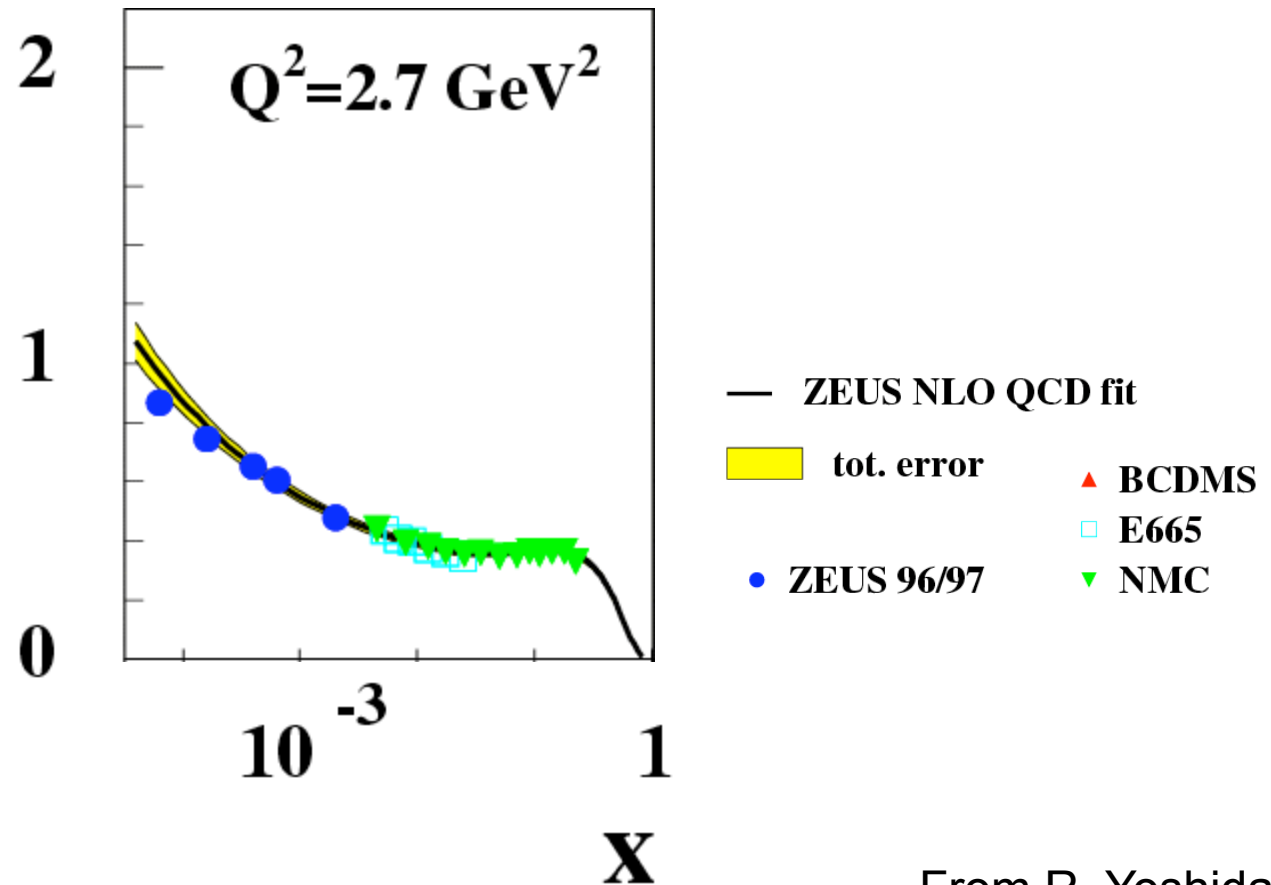


The three quarks radiate partons at low  $x$ .

# Proton Structure Function $F_2 - Q^2$ Evolution

$Q^2$  dependence quantitatively described by :

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations



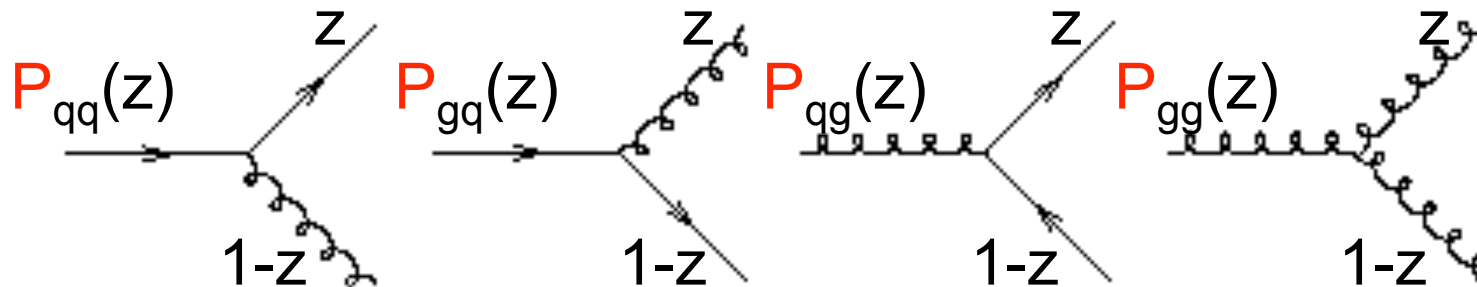
From R. Yoshida

# DGLAP Equations

The evolution of the parton densities with  $Q^2$  is given by the DGLAP equations :

$$\partial f_p / \partial \ln Q^2 \sim f_p \otimes P$$

First,  $P$  represents the four “splitting functions” :



$P_{ba}(z)$  : probability that parton  $a$  will radiate a parton  $b$  with the fraction  $z$  of the original momentum carried by  $a$ .

So, the DGLAP equations,  $\partial f_p / \partial \ln Q^2 \sim f_p \otimes P$  for quarks and gluons are :

$$\frac{\partial}{\partial \ln Q^2} \Sigma(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} ([\Sigma \otimes P_{qq}] + [g \otimes 2n_f P_{qg}])$$

where  $\Sigma(x, Q^2) = \sum_i (q_i(x, Q^2) + \bar{q}_i(x, Q^2))$  is the quark density summed over all (active) flavors

And for the gluon :

$$\frac{\partial}{\partial \ln Q^2} g(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} ([\Sigma \otimes P_{gq}] + [g \otimes P_{gg}])$$

# Parton Density Functions (PDFs) Extraction

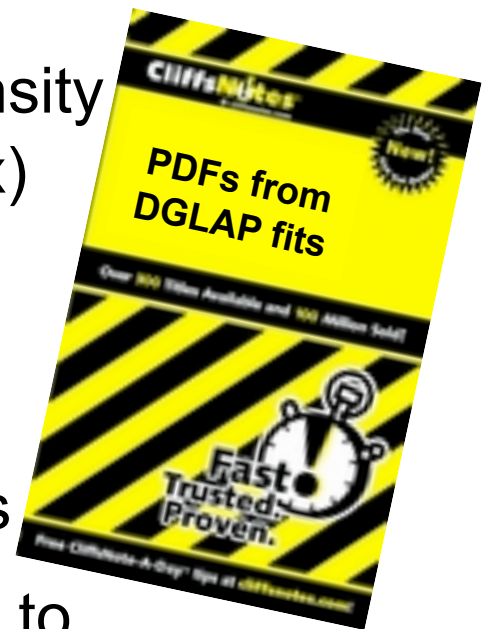
**DGLAP fit** (or QCD fit) extracts the parton distributions from measurements.

(Lectures on Friday and Saturday by Pavel Nadolsky)

The Cliffs Notes version :

*Step 1:* parametrize the parton momentum density  $f(x)$  at some  $Q^2 \rightarrow f(x) = p_1 x^{p_2} (1-x)^{p_3} (1 + p_4 \sqrt{x} + p_5 x)$

$u_v(x)$	u-valence	}	“The original three quarks”
$d_v(x)$	d-valence		
$g(x)$	gluon		
$S(x)$	sum of all “sea” (non valence) quarks		

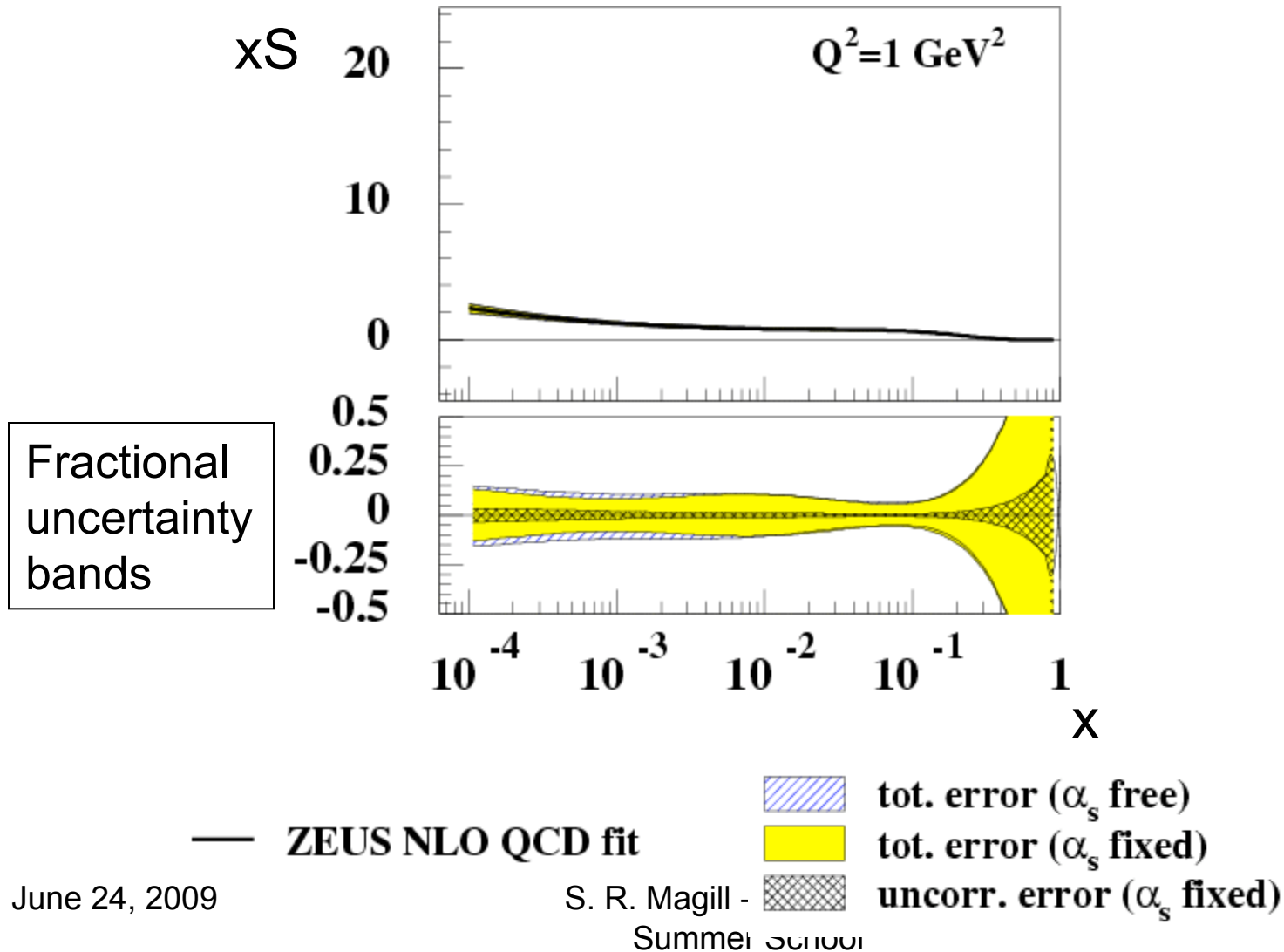


*Step 2:* find the parameters ( $p_1 \rightarrow p_5$ ) by fitting to DIS (and other) data using the DGLAP equations to evolve  $f(x)$  in  $Q^2$ .

At  $x \ll 1/3$ , quarks and (anti-quarks) are all “sea”.

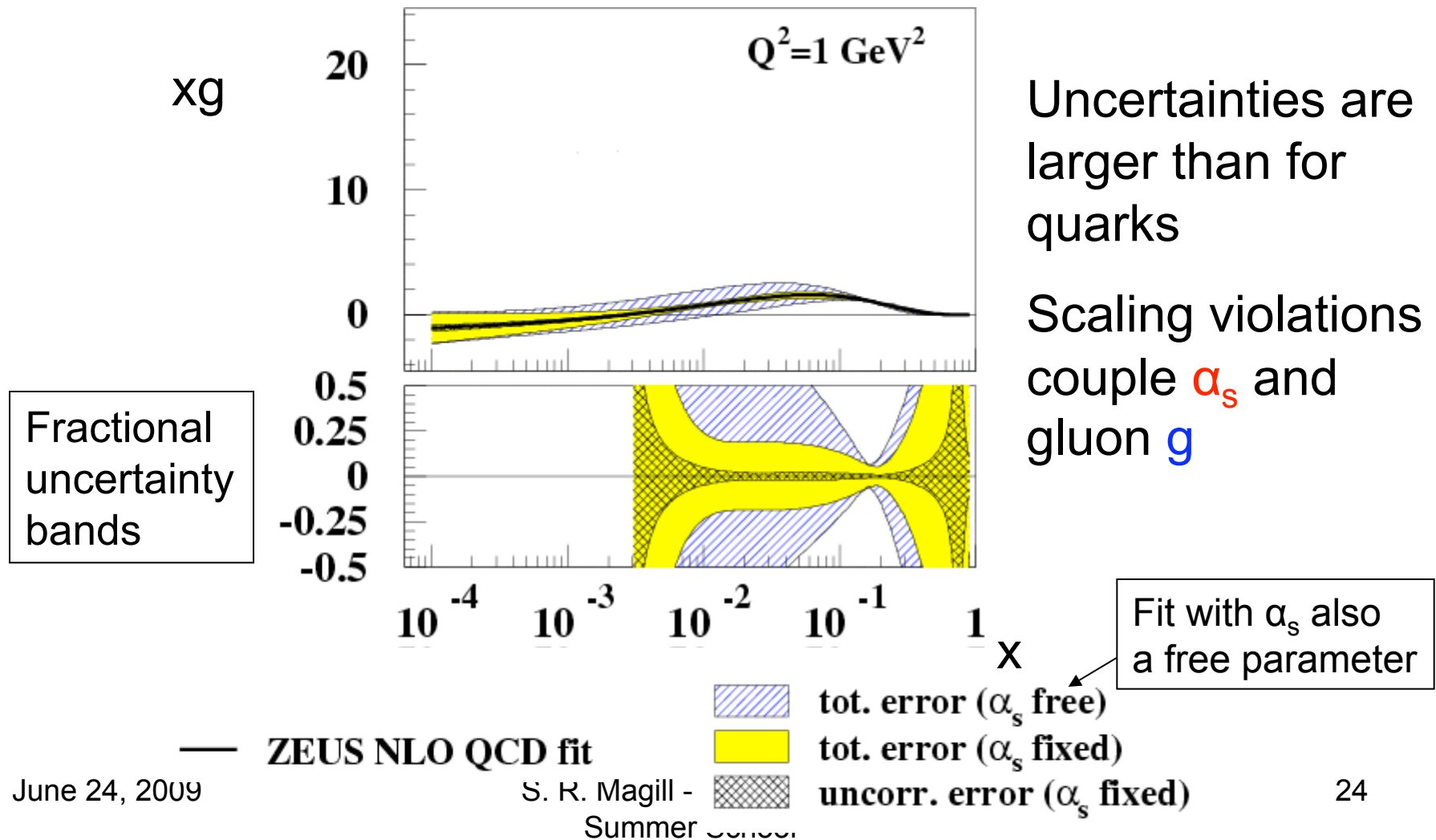
Since  $F_2 = e_q^2 \sum x(q + \bar{q})$ ,  $xS$  is very much like  $F_2$

### Sea PDF



The gluon pdf is determined from scaling violations,  $dF_2/d\ln Q^2$  via the DGLAP equations.

## Gluon PDF





Summarizing so far:

$$F_2 \sim \sum(q+\bar{q}) \approx S \text{ (sea quarks)} \quad \text{measured directly in NC DIS}$$

Scaling violations

$$dF_2/d\ln Q^2 \sim \alpha_s \cdot g \quad \text{Scaling violations gives gluons (times } \alpha_s \text{). DGLAP equations.}$$

What about valence quarks?

$$\sum(q-\bar{q}) = u_v + d_v \quad \text{can we determine them separately?}$$

Can we decouple  $\alpha_s$  and  $g$  ?

# Proton Structure Function $xF_3$

Back to the NC cross section :

$$\frac{d\sigma_{e\pm p}^2}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} (Y_+ F_2 - y^2 F_L \mp Y_- xF_3)$$

$Y_{\pm} = (1 \pm (1-y)^2)$ , the inelasticity parameter

$$xF_3 = \sum_i (q_i(x, Q^2) - \bar{q}_i(x, Q^2)) \times B_q \quad \sim \text{The valence quarks!}$$

$$B_q = \underbrace{-2e_q a_q a_e x_Z}_{\gamma\text{-Z interference}} + \underbrace{4v_q a_q v_e a_e x_Z^2}_{\text{Z-exchange}}$$

$$x_Z \propto Q^2 / (M_Z^2 + Q^2)$$

->  $xF_3$  small if  $Q < M_Z$

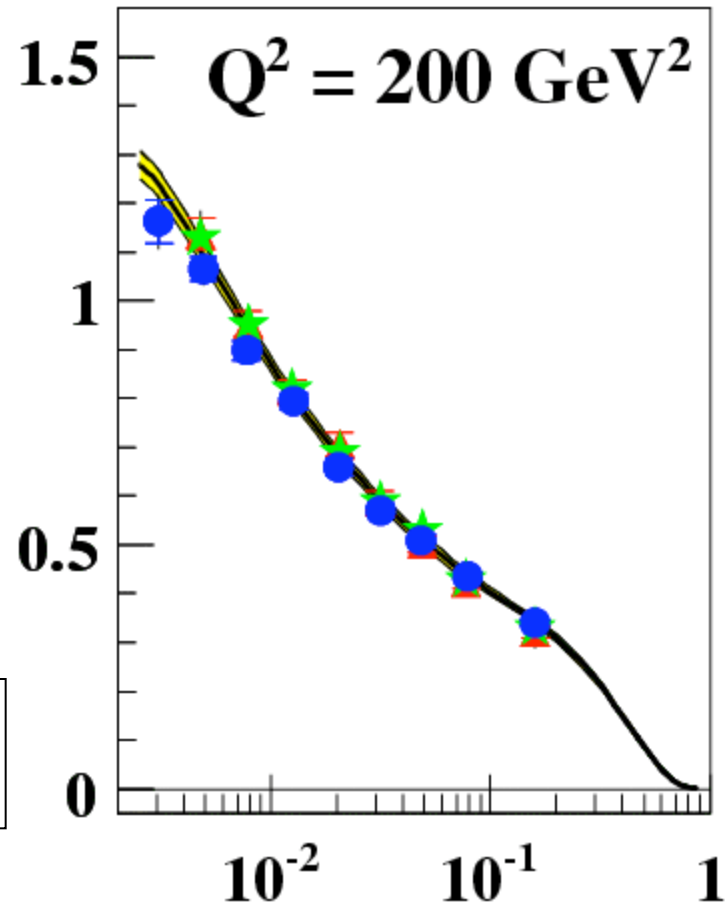
$e_q$ : electric charge of a quark

$a_q v_q$ : axial-vector and vector couplings of a quark

$a_e v_e$ : axial-vector and vector couplings of an electron

# Reduced Neutral Current Cross-section

$\sigma^{\text{NC}\pm}$



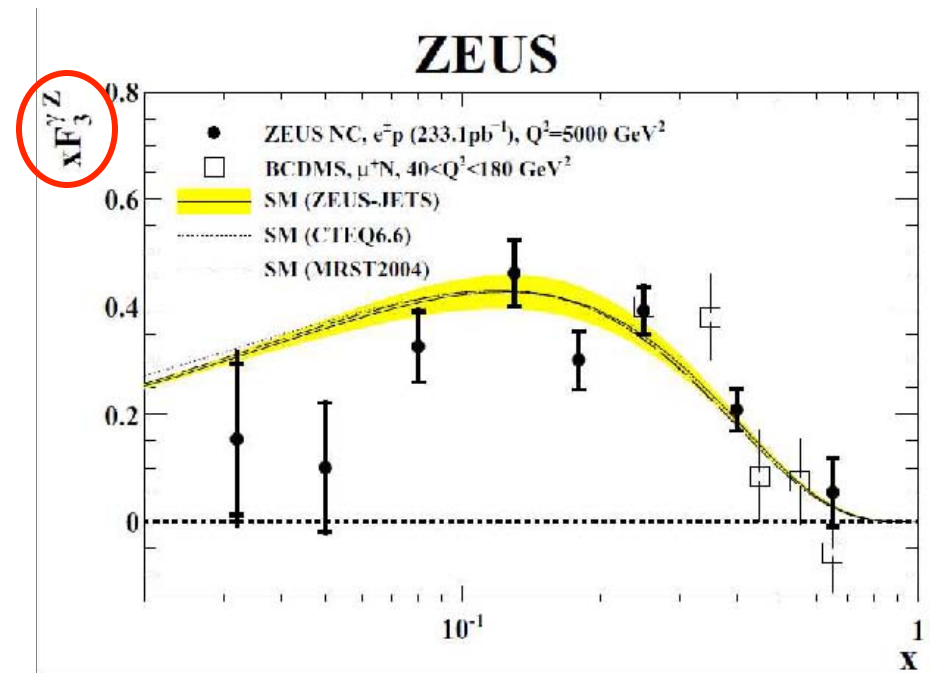
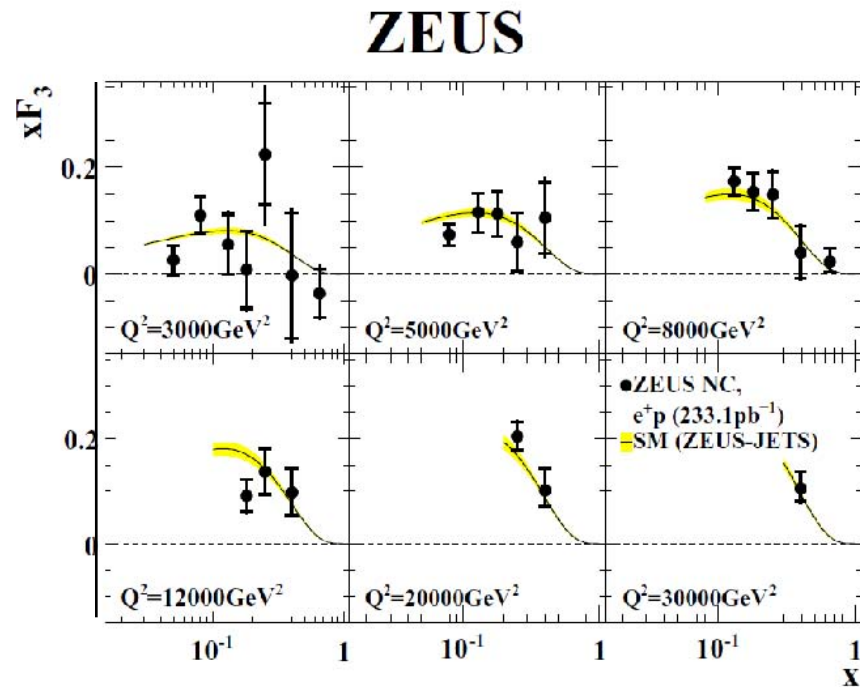
— NLO QCD fit

- ★ ZEUS NC  $e^+p$  99-00
- ▲ ZEUS NC  $e^-p$  98-99
- ZEUS NC  $e^+p$  96-97

Note the change of sign from  $e^+p$  to  $e^-p$

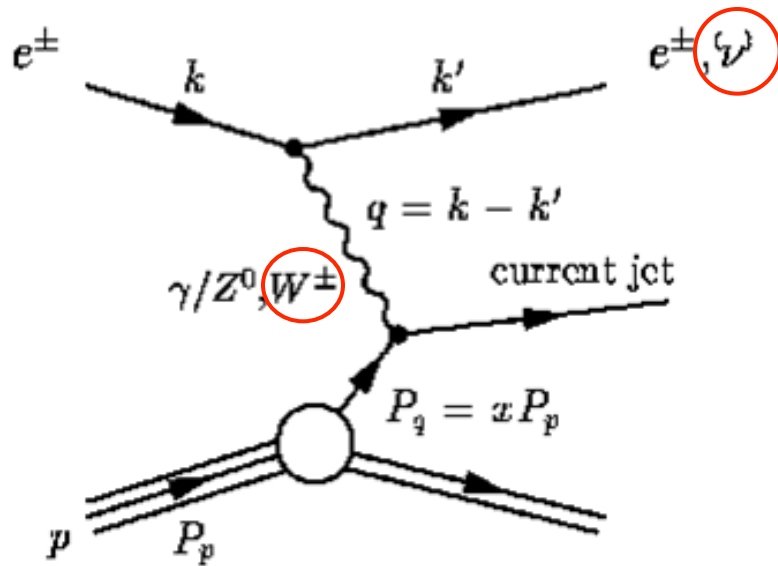
$$\sigma^{\text{NC}\pm} = F_2(x, Q^2) \mp (Y_-/Y_+) x F_3(x, Q^2)$$

# Recent $xF_3$ (DIS 09) Results from HERA



$\gamma Z$  interference term larger than Z exchange

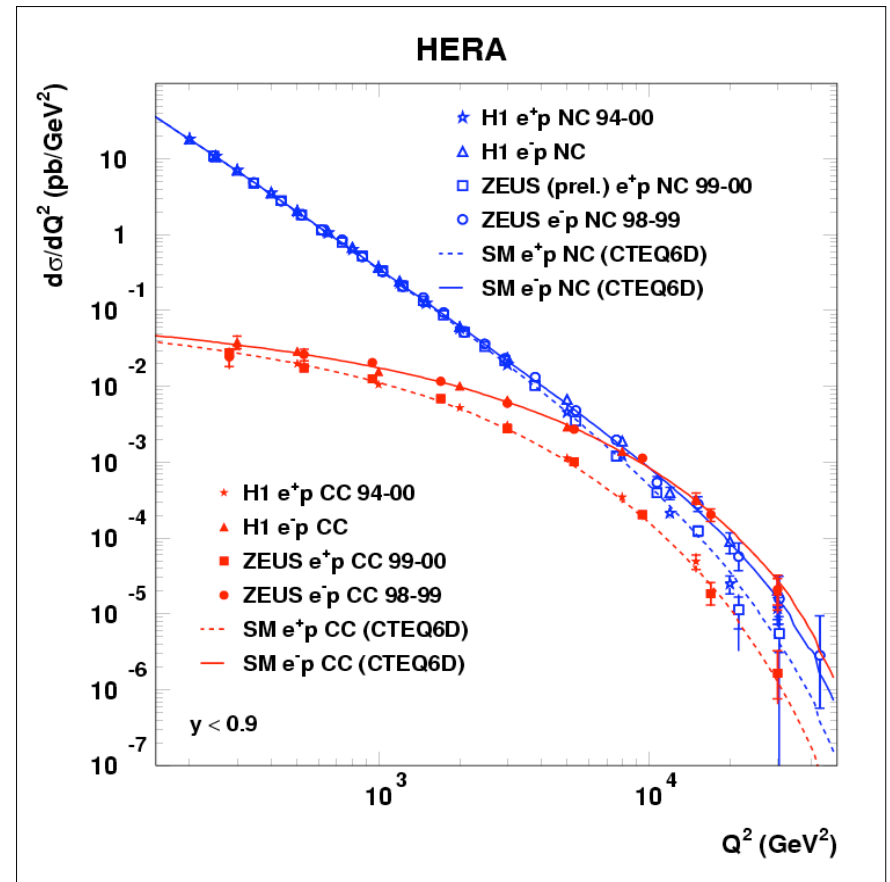
# Neutral and Charged Current Cross-Sections



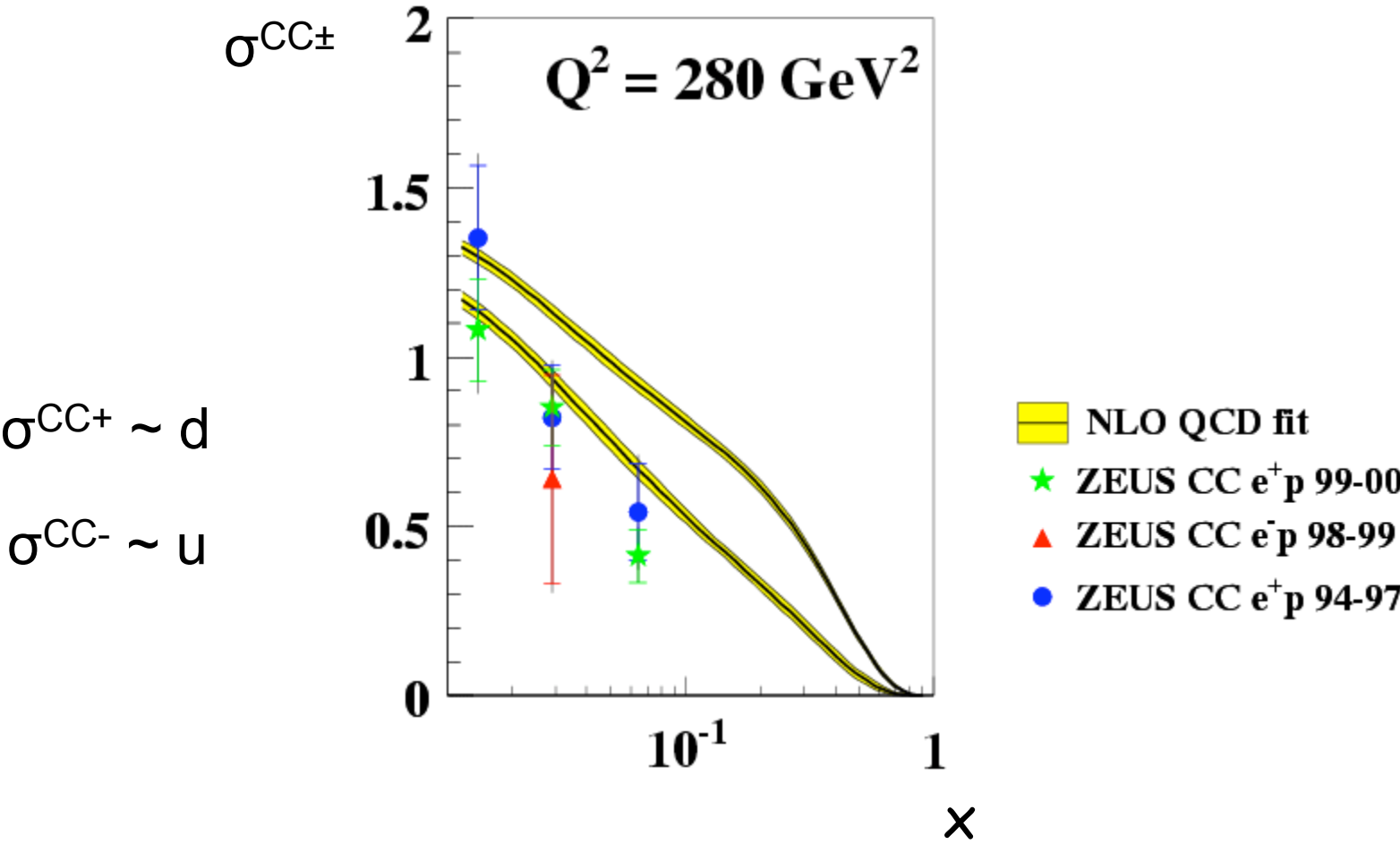
$$\frac{d\sigma^{CC}(e^\pm p)}{dx dQ^2} = \frac{G_F^2}{2\pi x} \left[ \frac{M_W^2}{M_W^2 + Q^2} \right]^2 \sigma^{CC\pm}$$

$$\sigma^{CC+} = x [\bar{u} + \bar{c} + (1-y)^2(d + s)] \sim d$$

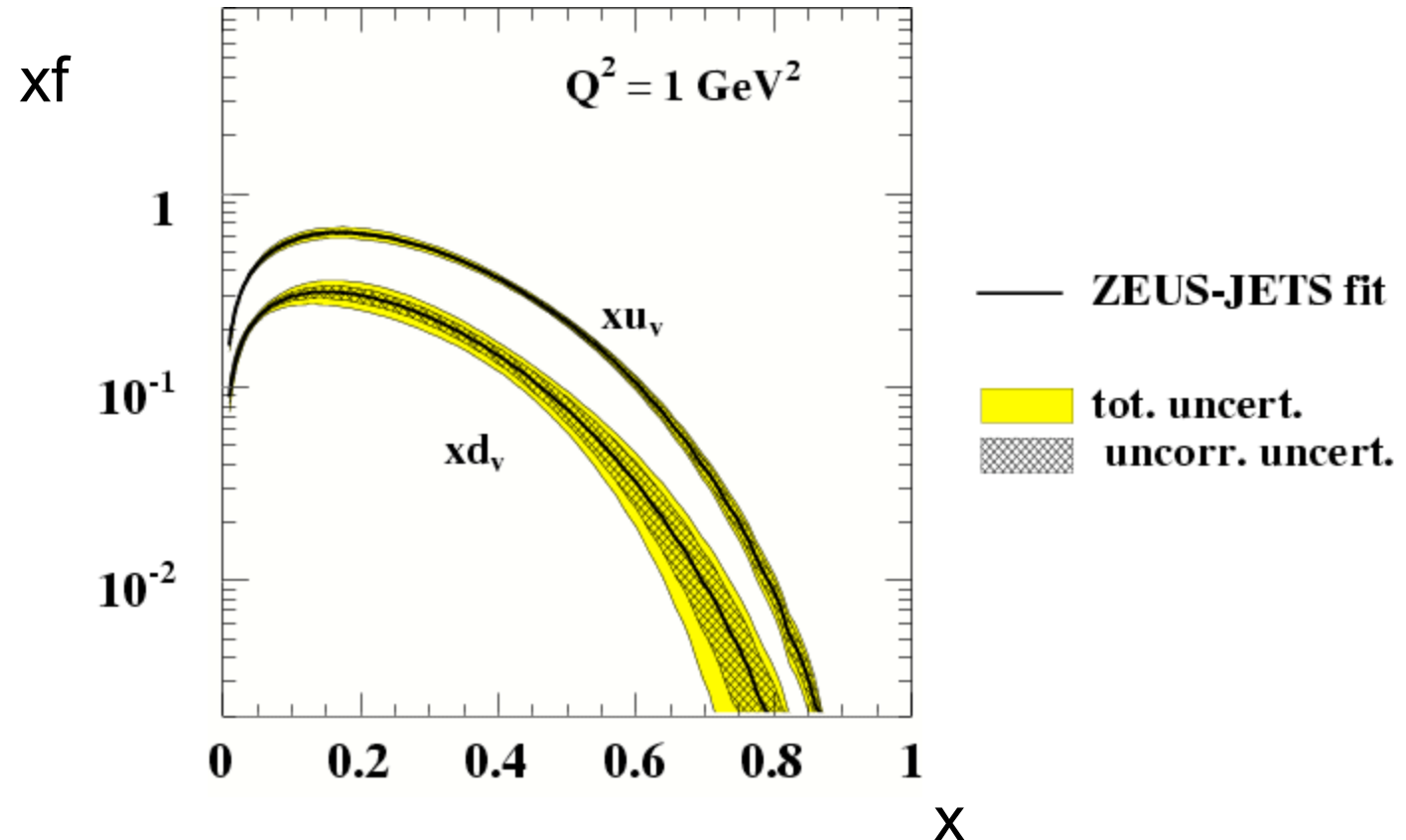
$$\sigma^{CC-} = x [u + \underbrace{c}_{\text{charm}} + (1-y)^2(\bar{d} + \bar{s})] \sim u$$



# Reduced Charged Current Cross-Section



# Valence PDFs from QCD Fit

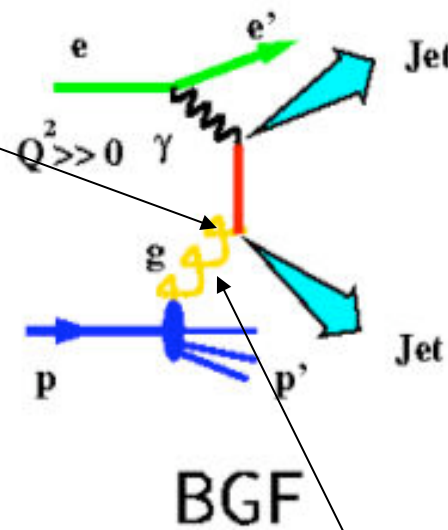
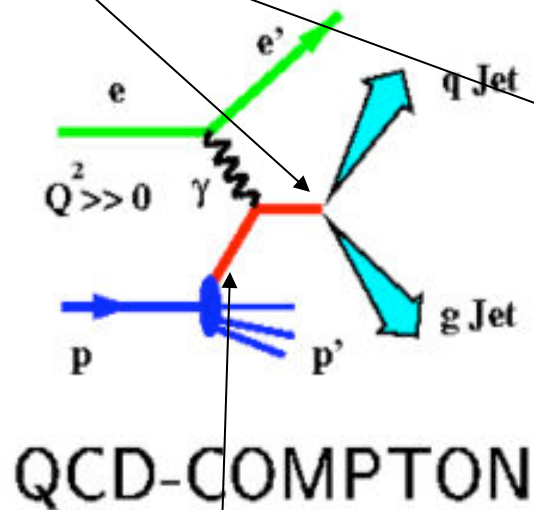


The momenta from valence quarks are producing gluons and sea quarks at low  $x$

# Jet production in DIS (HERA)

Sensitive to  $\alpha_s$

$$\sigma_{\text{jet}} \sim \alpha_s \cdot f(x)$$



Sensitive to quarks  
 $\sim 10^{-2} < x < \sim 10^{-1}$

Sensitive to gluon  
 $\sim 10^{-3} < x < \sim 10^{-2}$

Same range as NC and CC

complementary  
 to gluon from  $F_2$



# Jet Measurements in the Breit Frame

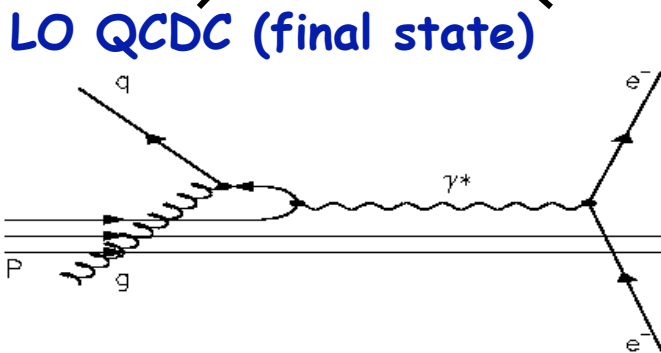
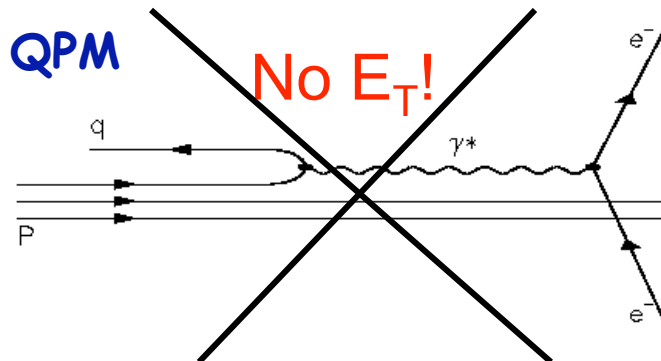
Some definitions :

+z from IP in proton direction - *Target Region*

-z from IP in  $\gamma^*$  direction - *Current Region*

$\gamma^*$  has 4-vector  $q = (0,0,0,-Q)$

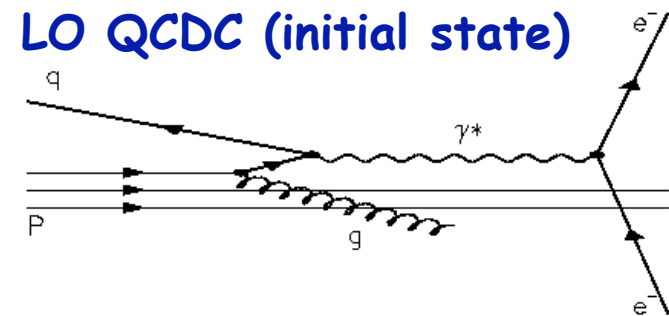
Struck quark in QPM carries away momentum  $-Q/2$



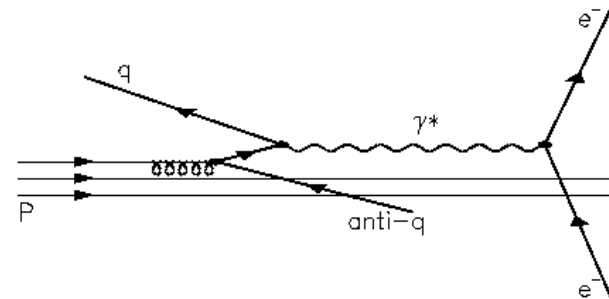
**Hard Scale  $Q = E_+$**

June 24, 2009

$\rightarrow$  *High  $Q^2$*



**LO BGF**

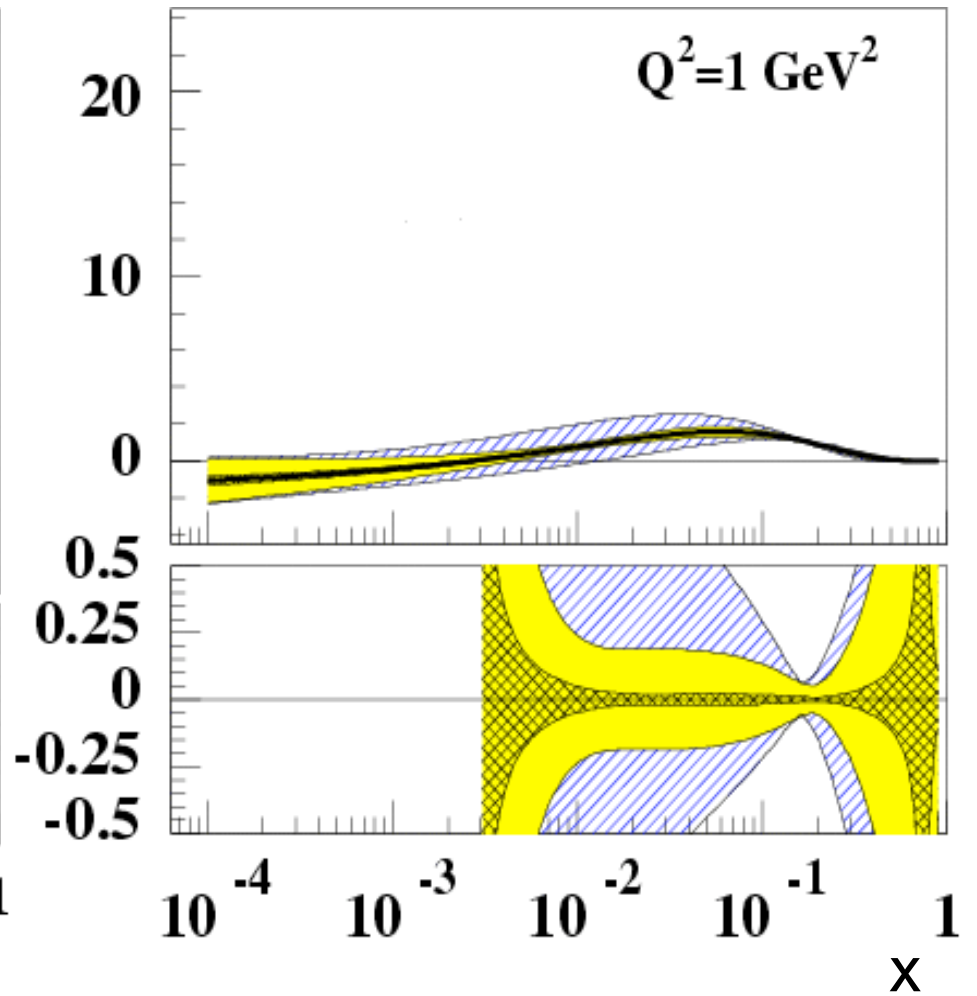
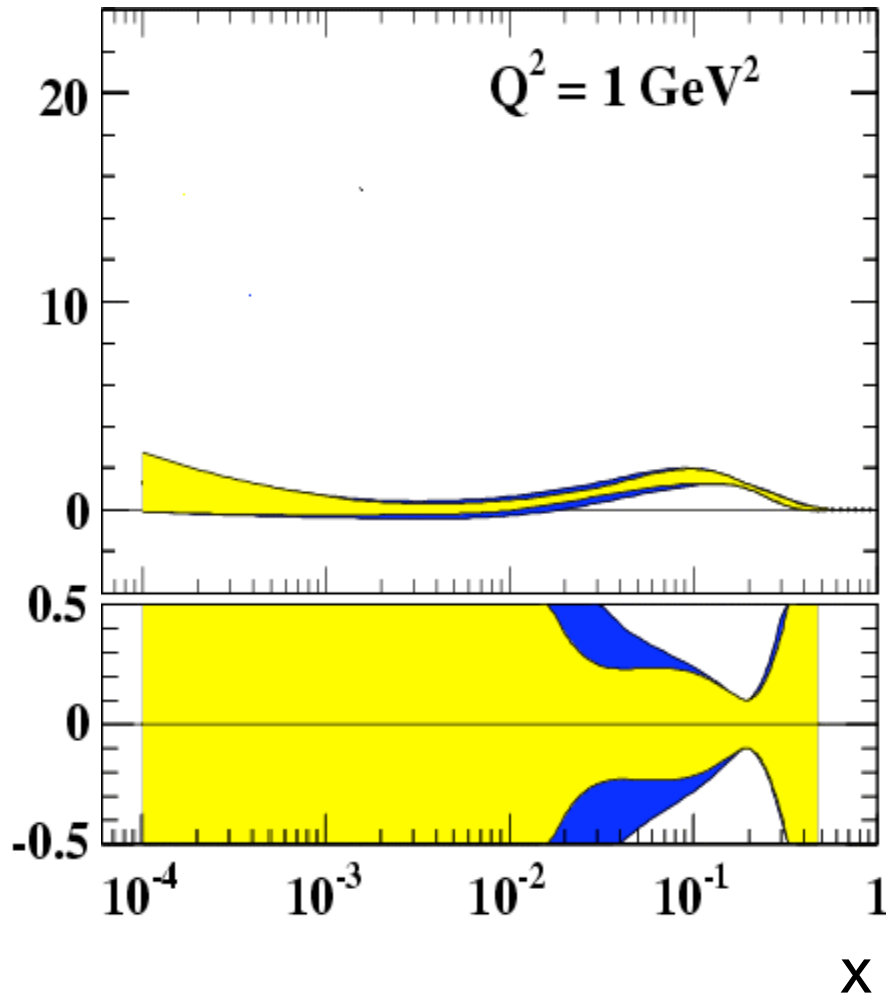


**Hard Scale  $E_+ (>Q)$**

33

$\rightarrow$  *Low  $Q^2$*

# Gluon distributions



Using only HERA (ZEUS) data including NC, CC and jets

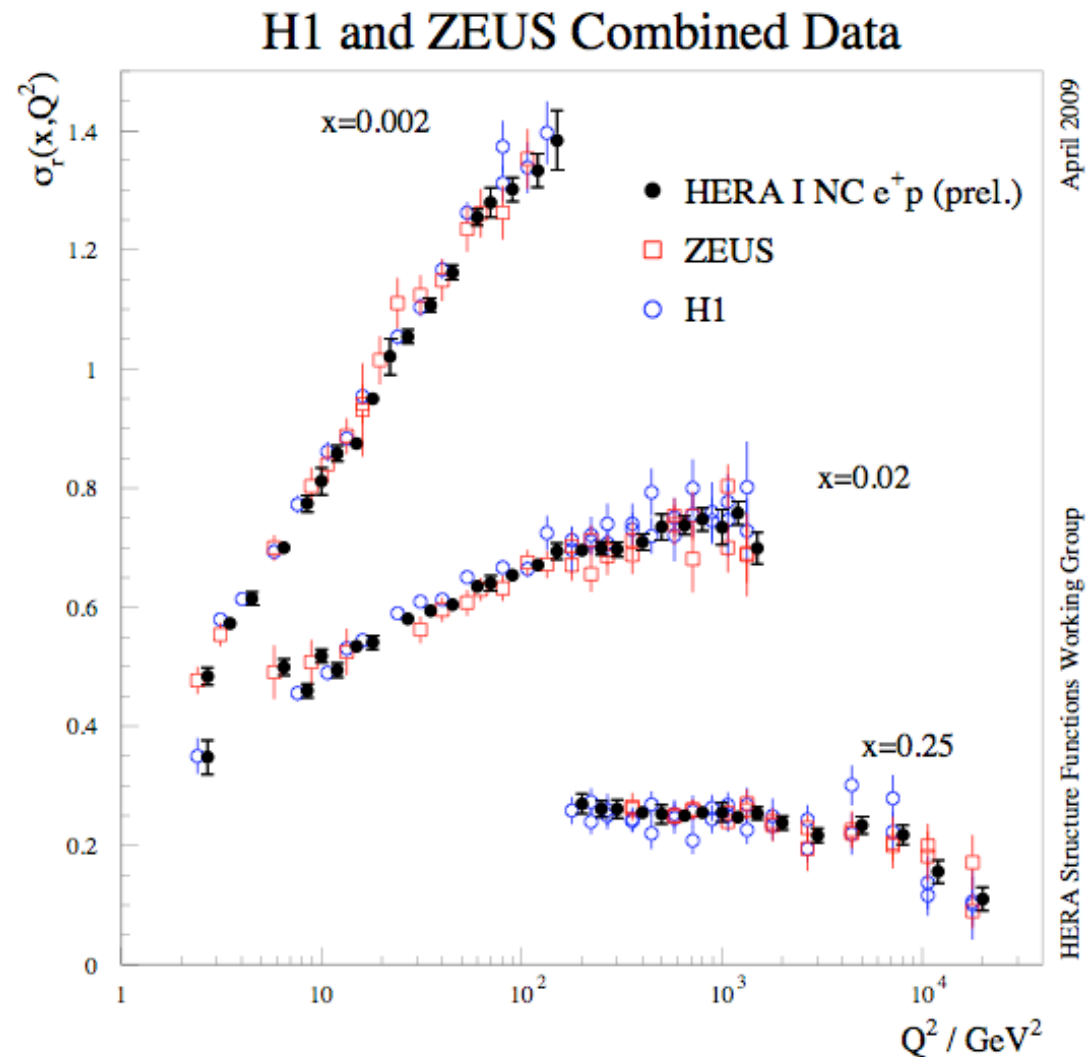
Using HERA (ZEUS)  $F_2$  data and FNAL, CERN fixed tgt

# Combined PDFs from HERA – DIS Precision

Combine the measured H1 and ZEUS cross sections. Double statistics and take advantage of complementary measurement techniques which result in reduced systematic uncertainties.

Sample of NC  $e^+p$  data showing the ZEUS and H1 data and the combined data as a result of the averaging procedure

Statistical errors shown

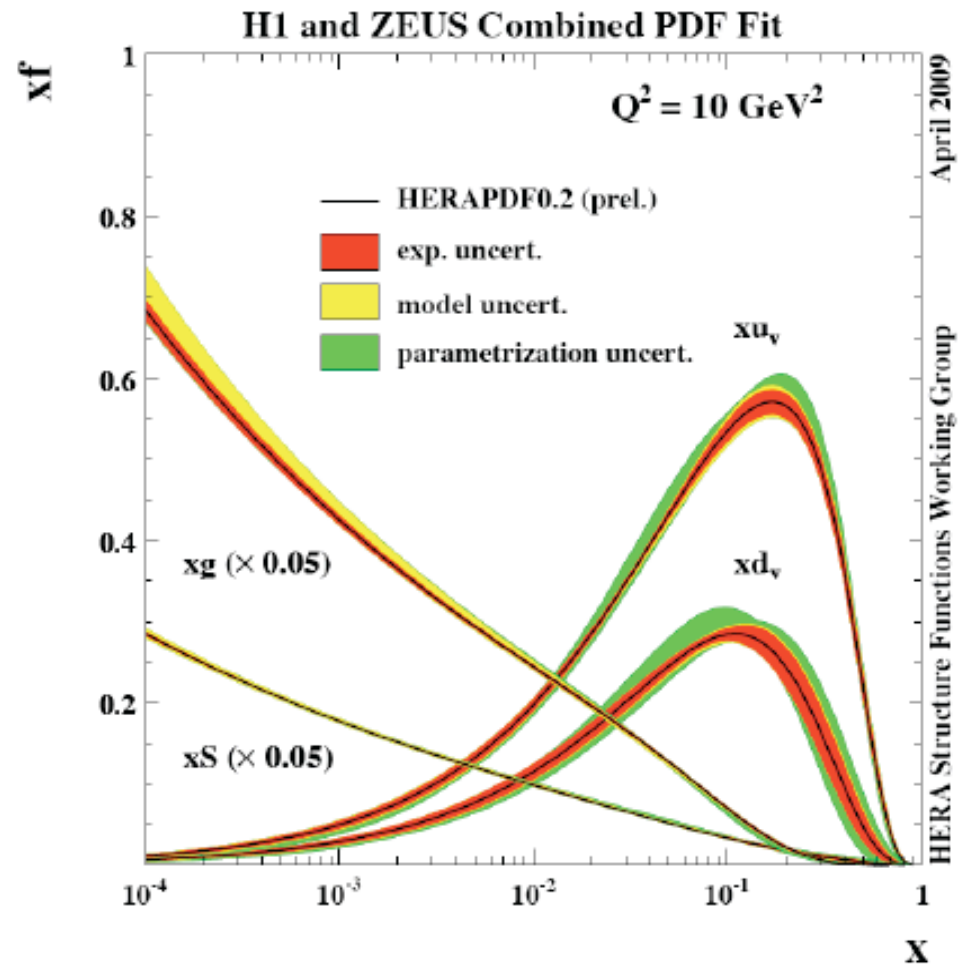


# HERAPDF0.2

- **Red:** experimental uncertainties
- **Yellow:** model uncertainties
- **Green:** pdf parametrization uncertainties

## Observations:

- ▶ High- $x$  and valence are mostly affected by the PDF parametrisation
- ▶ The procedure to estimate PDF parametrisation uncertainty addresses the high- $x$  region
- ▶ Low- $x$  region interesting to investigate

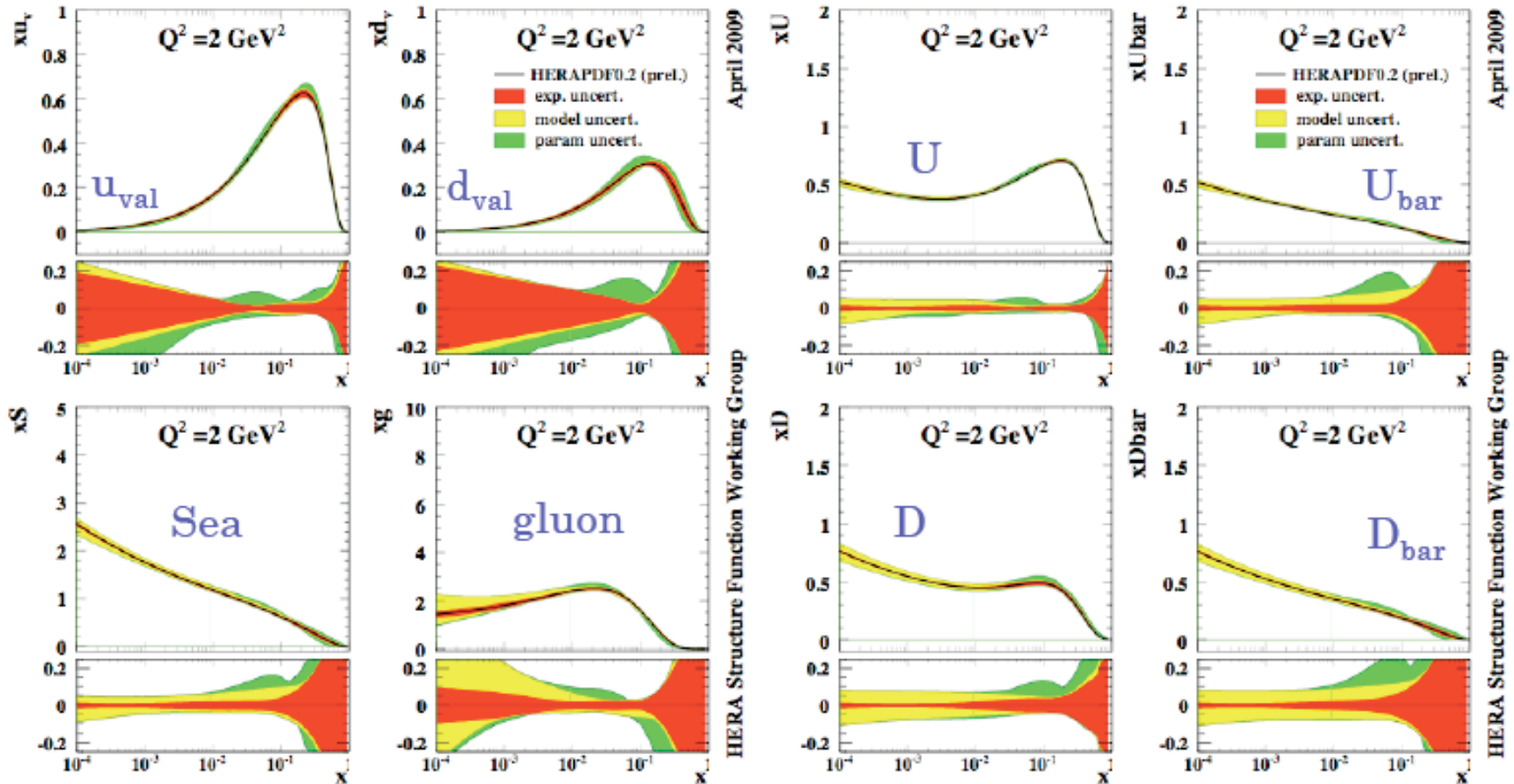


# HERAPDF0.2 at $Q^2=2 \text{ GeV}^2$

- At the starting scale gluon is valence like
- $Q_0^2, Q_{\text{min}}^2$  dominate the model uncertainty of gluon and valence PDFs
- PDF parametrisation uncertainty dominates valence PDFs and high x region

H1 and ZEUS Combined PDF Fit

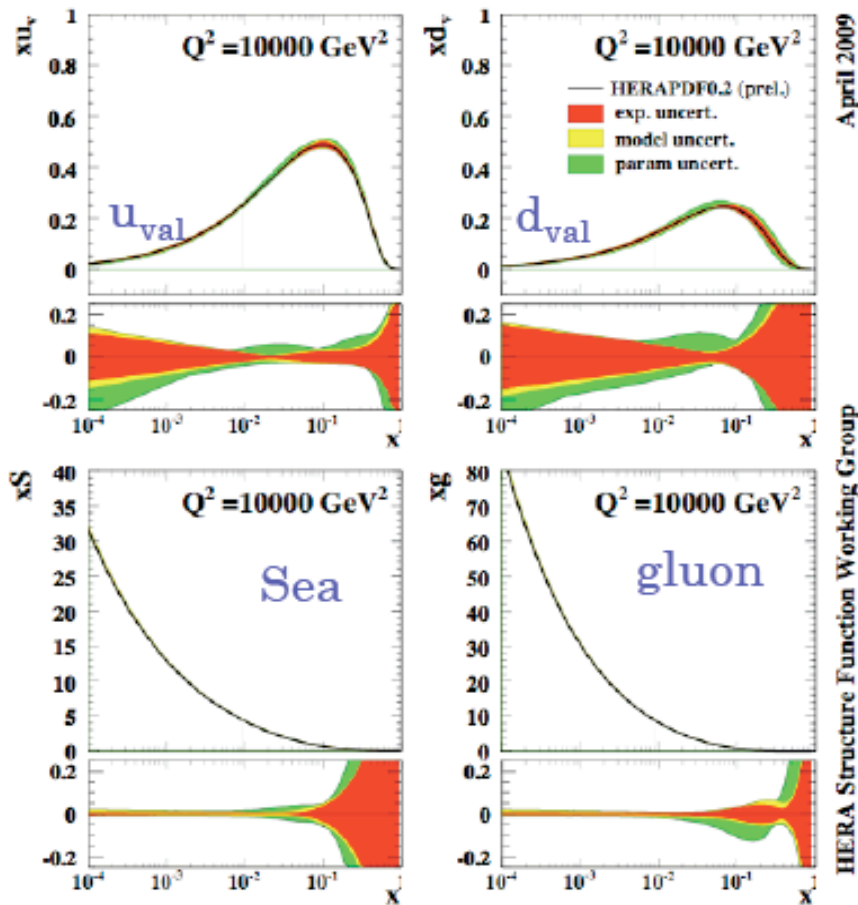
H1 and ZEUS Combined PDF Fit



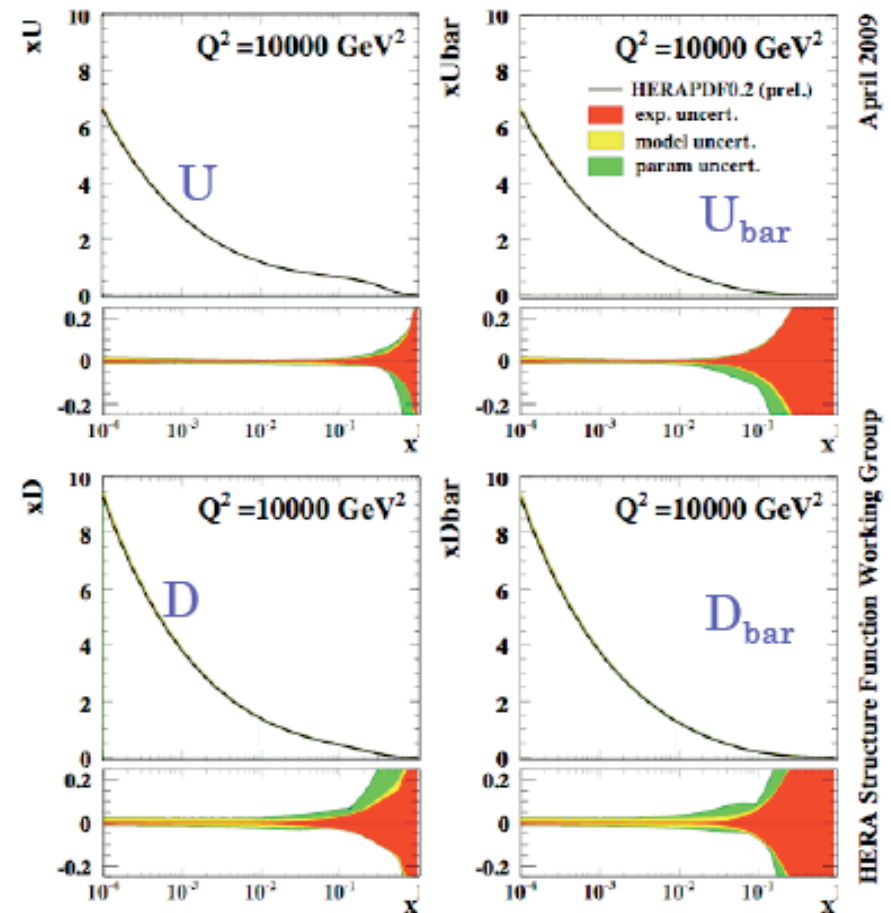
# HERAPDF0.2 at $Q^2=10000 \text{ GeV}^2$

- PDF parametrisation uncertainty dominates valence PDFs and high x region
- Impressive precision at the scale relevant to LHC

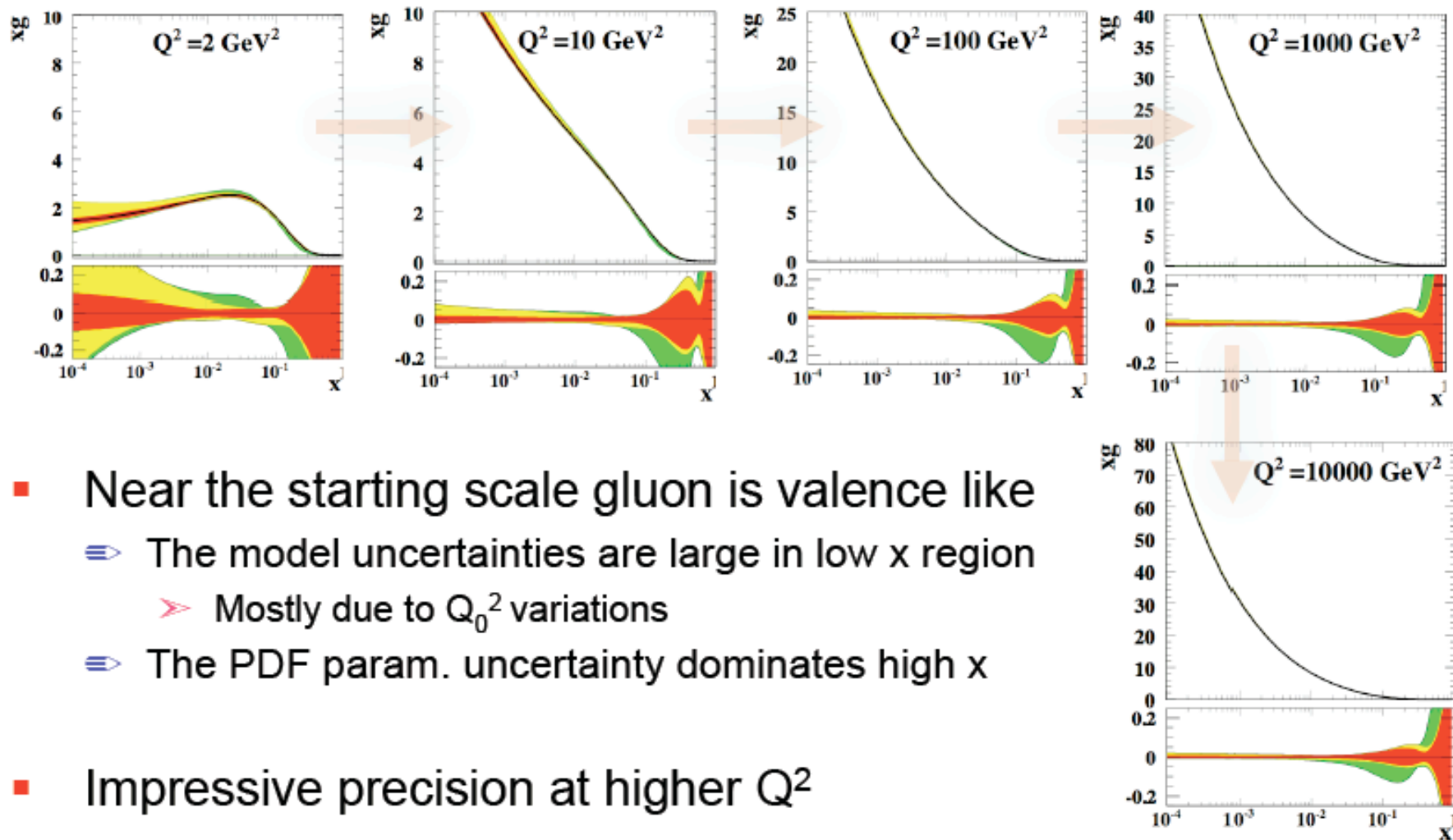
H1 and ZEUS Combined PDF Fit



H1 and ZEUS Combined PDF Fit

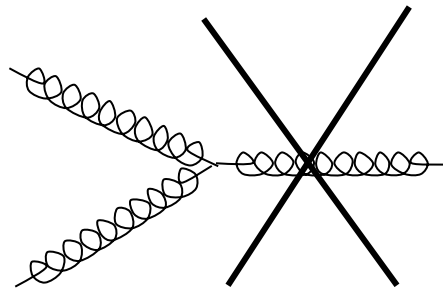


# Gluon Evolution



## Understanding DGLAP equations – pdf evolution :

The “**incoherence**” of the original parton model is preserved. i.e. a parton doesn't know anything about its neighbor.



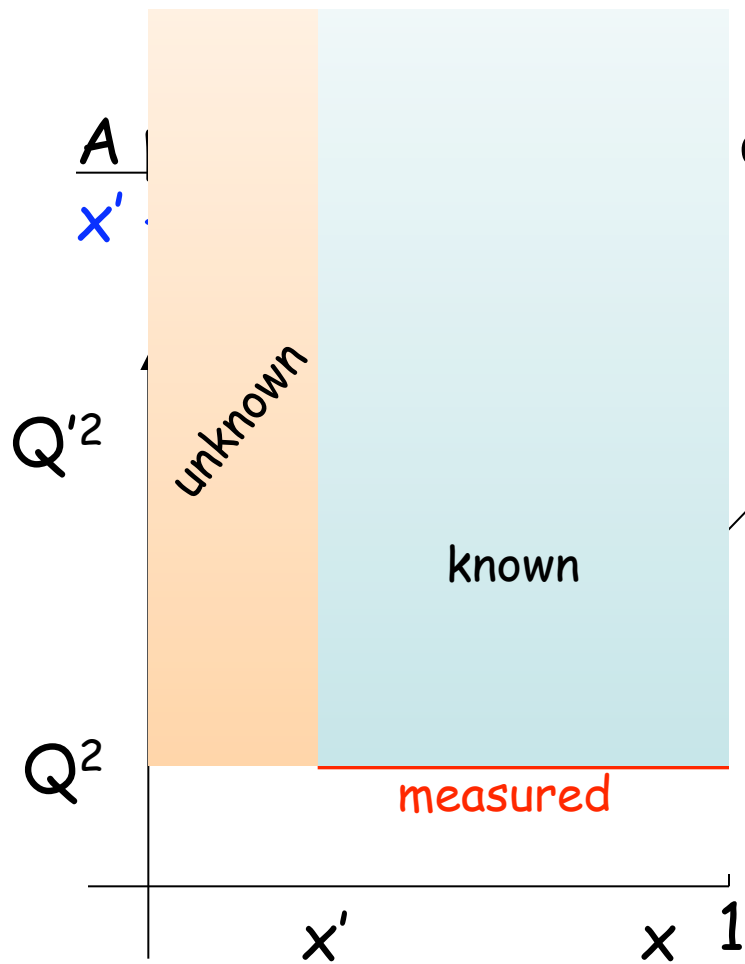
never happens

The “**process independent**” partons also survive.

But now parton densities must be “evolved” in  $Q^2$ .

An example for future analyses →





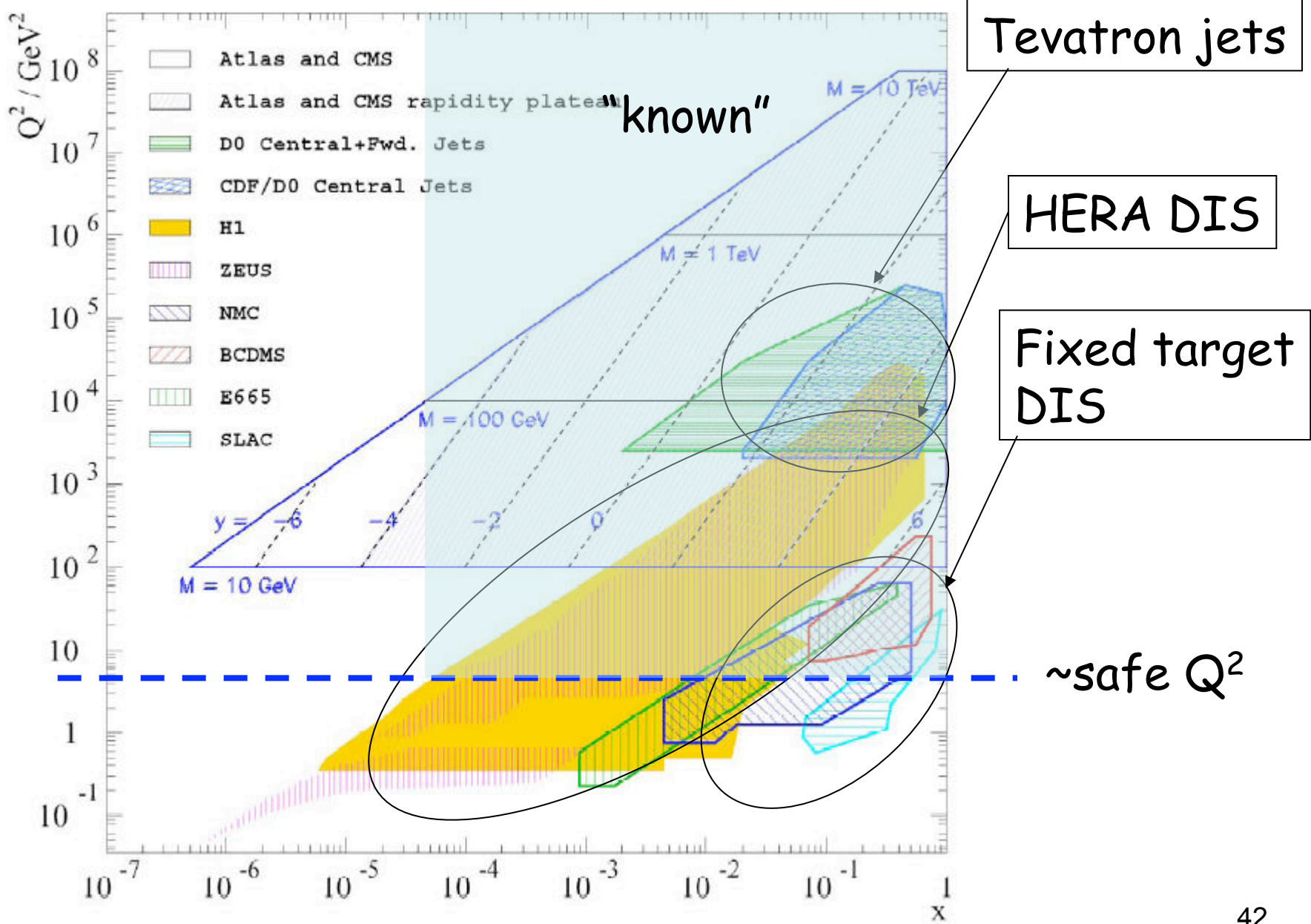
a source of partons at

In fact, any parton at  $x > x'$  at  $Q^2$  is a source.

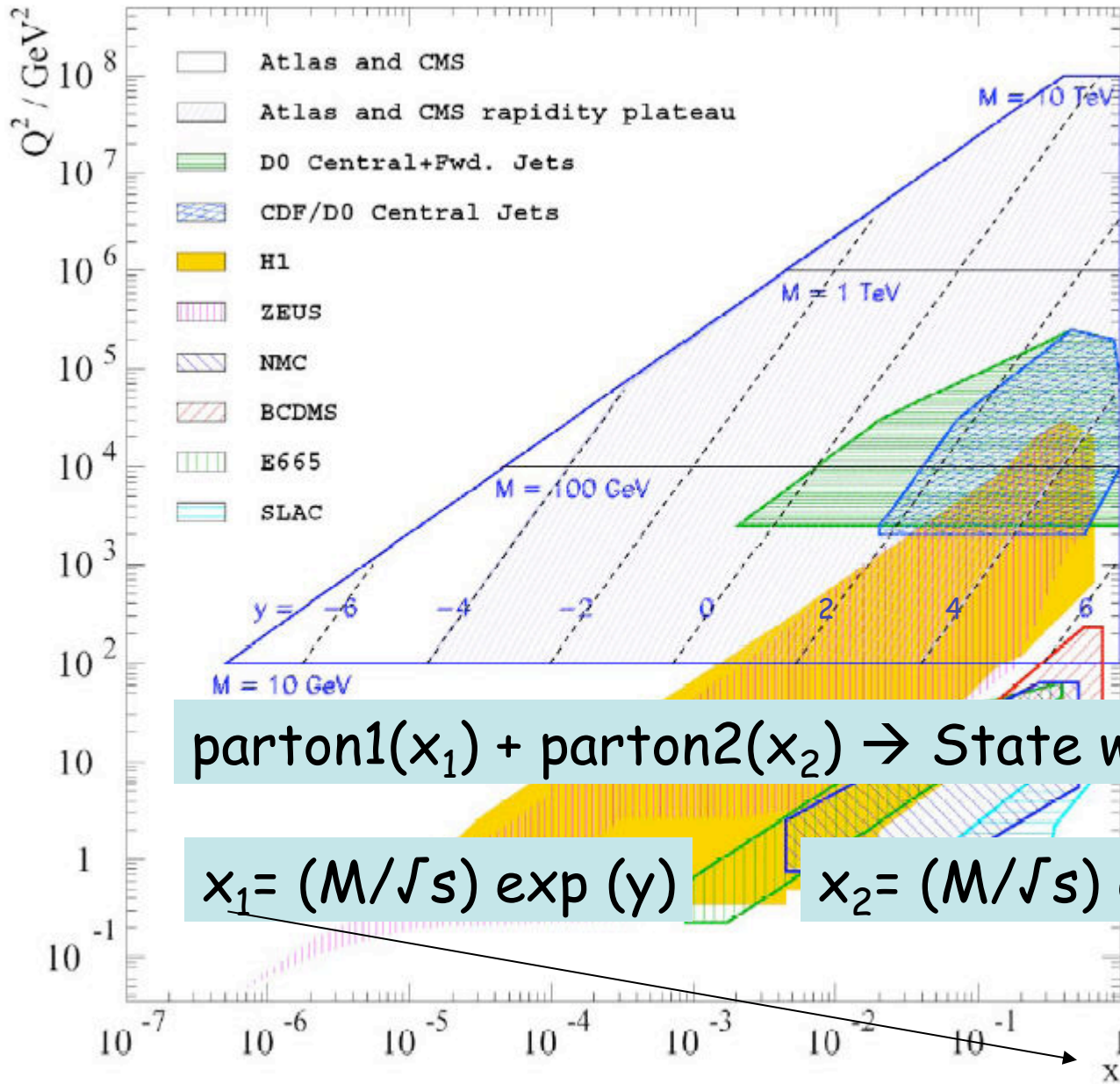
To know the parton density at  $x', Q'^2$  it's necessary (and sufficient) to know the parton density in the range:  $x' \leq x \leq 1$

at some lower  $Q^2$ .  
 If you know the partons in range  $x \leq x \leq 1$  at some  $Q^2$ , then you know the partons in the range  $x' \leq x \leq 1$  for all  $Q'^2 > Q^2$ .

What does this mean for the LHC? →



# LHC (or hadron-hadron) parton kinematics



rapidity:

$$y = \frac{1}{2} \ln \left( \frac{E + P_z}{E - P_z} \right)$$

pseudo-rapidity:

$$\eta = -\ln \tan(\theta/2)$$

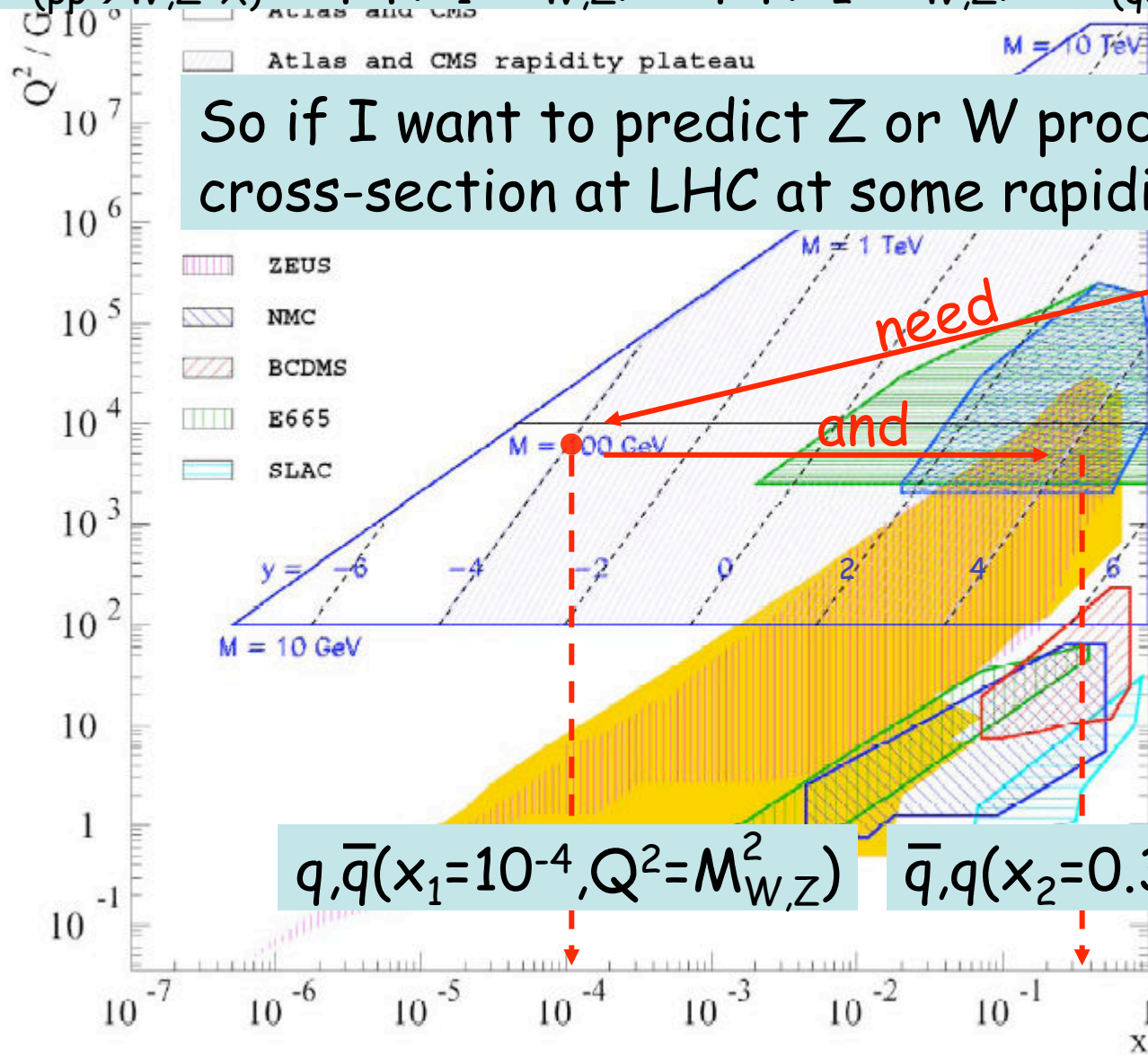
angle wrt beam

parton1( $x_1$ ) + parton2( $x_2$ )  $\rightarrow$  State with mass  $M$

$$x_1 = (M/\sqrt{s}) \exp(y)$$

$$x_2 = (M/\sqrt{s}) \exp(-y)$$

$$\sigma_{(pp \rightarrow W,Z+X)} \sim q, \bar{q}(x_1, M_{W,Z}^2) \times \bar{q}, q(x_2, M_{W,Z}^2) \times \sigma_{(q\bar{q} \rightarrow W,Z)}$$



So if I want to predict Z or W production cross-section at LHC at some rapidity  $y$ , say,  $-4$ :

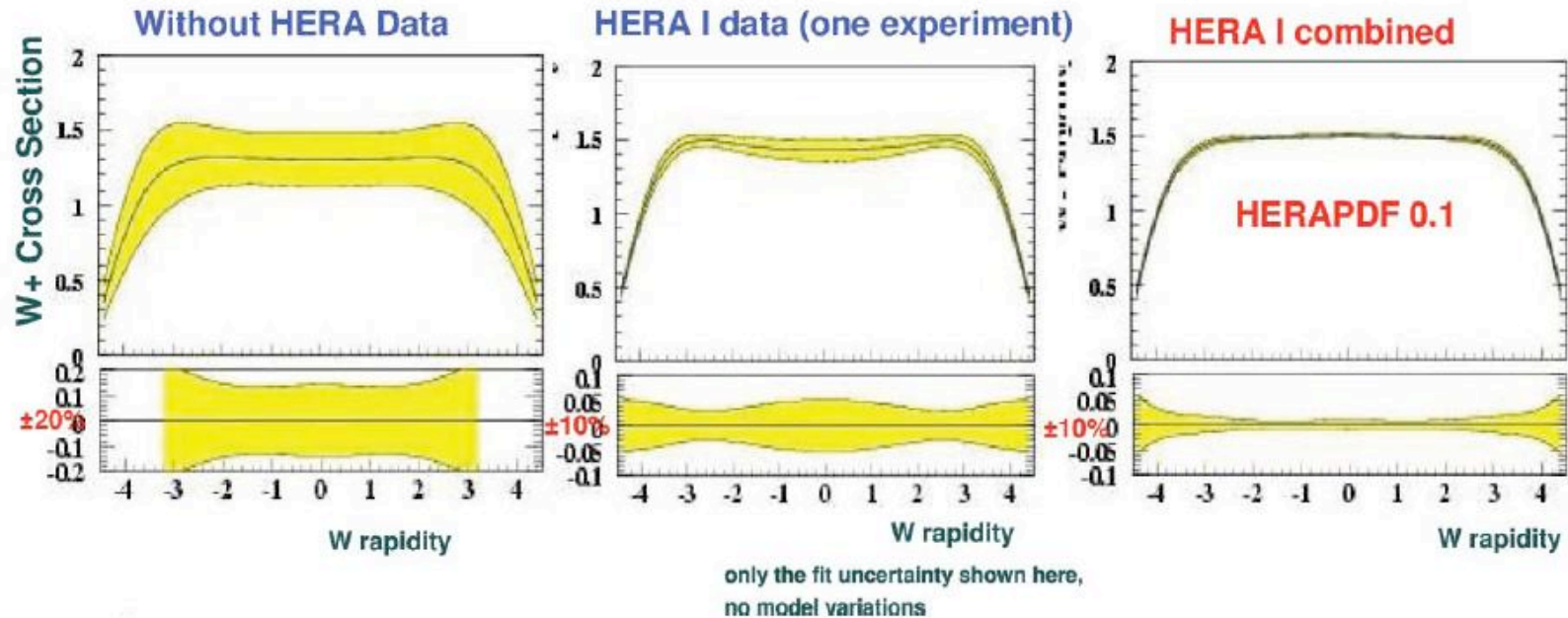
need

and

$$q, \bar{q}(x_1 = 10^{-4}, Q^2 = M_{W,Z}^2)$$

$$\bar{q}, q(x_2 = 0.3, Q^2 = M_{W,Z}^2)$$

# Parton densities from combined data



A test on a standard candle process at LHC

HERA PDFs 0.1 available in LHAPDF

HERAPDF0.2 has factor of 2 smaller uncertainty than 0.1 (more data) at low  $x$   
Available soon in LHAPDF

With DIS data up to now :

$$F_2 \sim \sum(q+\bar{q}) \approx S \text{ (sea quarks)} \quad \text{measured in NC DIS}$$

From scaling violations in  $F_2$

$$dF_2/d\ln Q^2 \sim \alpha_s \cdot g \quad \text{sensitive to gluons (times } \alpha_s)$$

$$xF_3 \sim \sum(q-\bar{q}) = u_v + d_v \quad \text{valence quarks}$$

Use jet cross sections to decouple  $\alpha_s$  and  $g$

**DGLAP fits** with all of the above -> precise predictions at LHC

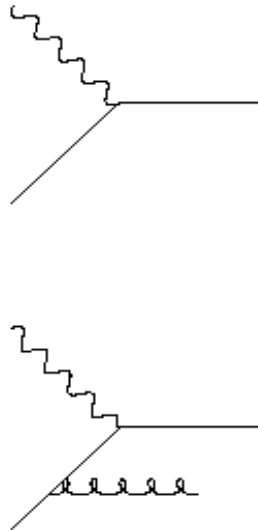
Now, for the last piece – the longitudinal structure function  $F_L$  to give us direct access to the gluons ->

# Longitudinal Structure Function $F_L$

- $F_L$  corresponds to absorption of longitudinally polarized virtual photon.

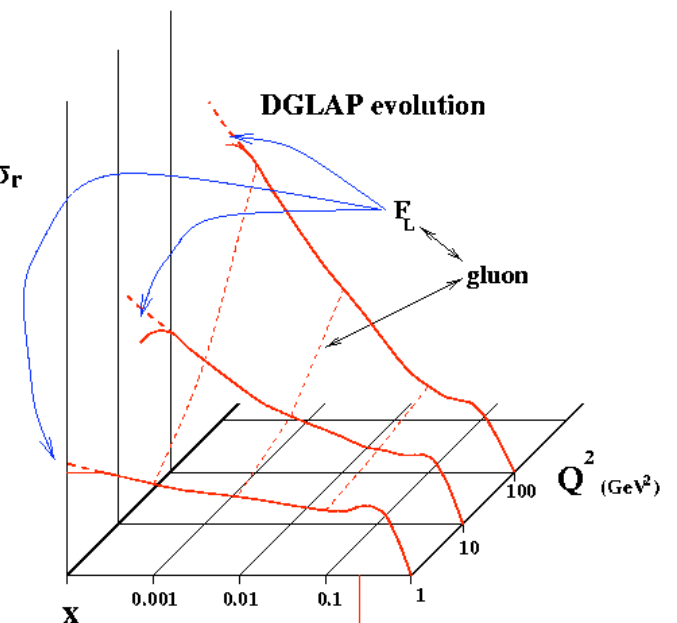
$$F_L = (Q^2/4\pi^2\alpha) \sigma_L$$

- Spin 1/2 quarks (with no transverse momentum) cannot absorb a longitudinally polarized boson.



LO:  $k_T=0, F_L=0, \sigma_r$

NLO:  $k_T \neq 0, F_L \neq 0$



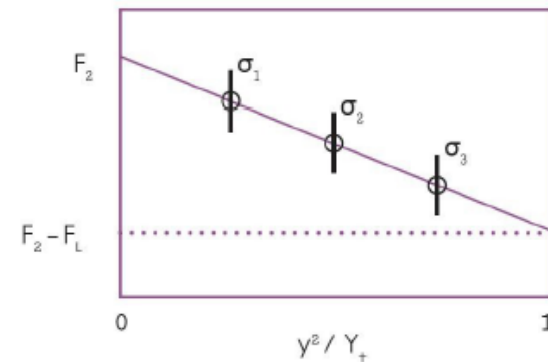
$$F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3} \left[ \frac{16}{3} F_2 + 8 \sum e_q^2 \left(1 - \frac{x}{z}\right) zg \right]$$

## Analysis strategy

- Direct  $F_L$  measurement requires measurement of the reduced cross sections at **same  $x$  and  $Q^2$  but different  $y$** :

$$\sigma_r(x, Q^2, y) = F_2(x, Q^2) - \frac{y^2}{Y_+} \cdot F_L(x, Q^2)$$

- Larger difference in  $y \rightarrow$  better sensitivity to  $F_L$  (bigger “lever arm”)
- **$Q^2 = xys$** : different  $y \rightarrow$  different  $s \rightarrow$  different beam energies
- Direct  $F_L$  measurement only possible if HERA operates with different proton beam energies

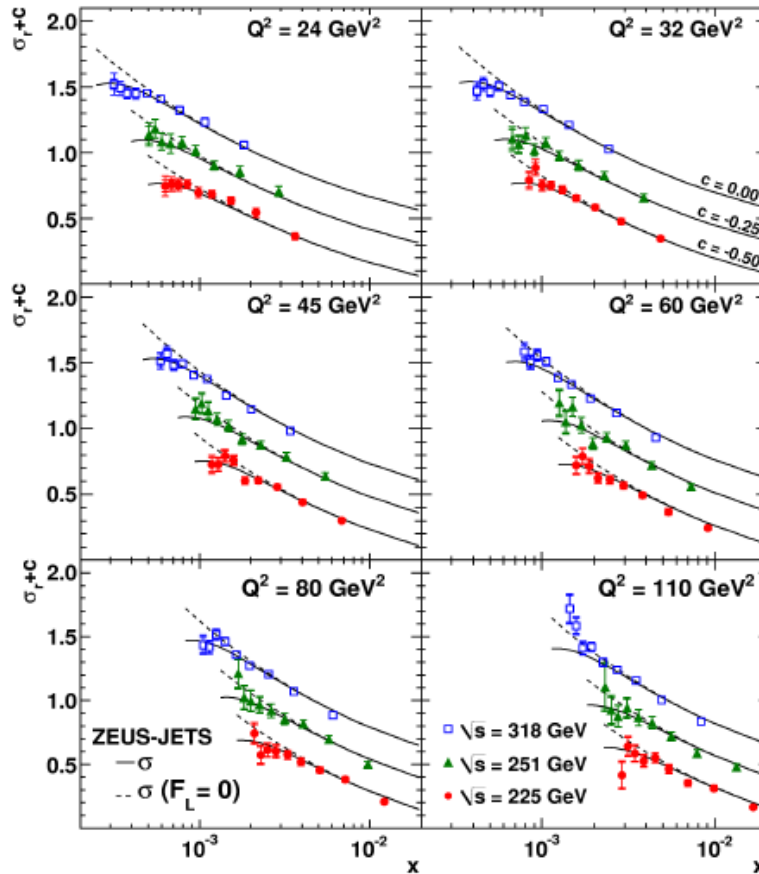


At given  $x$  and  $Q^2$ :  
 $\rightarrow F_2$  is the intercept at  $y$ -axis  
 $\rightarrow F_L$  is the negative slope



## Measured reduced cross sections

### ZEUS



Kinematic region:

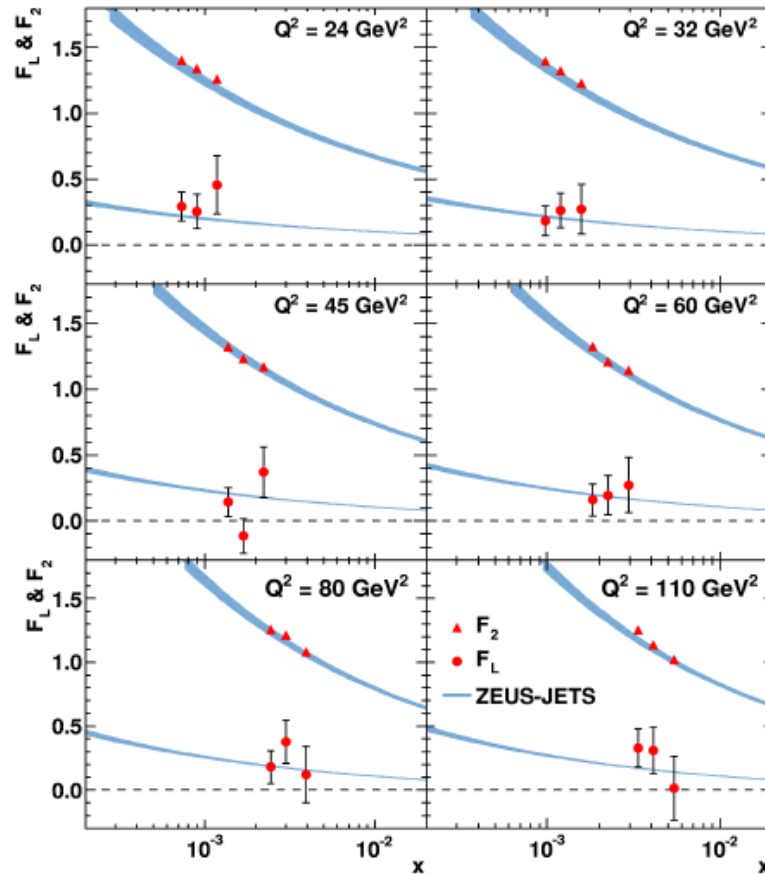
$$20 \text{ GeV}^2 < Q^2 < 130 \text{ GeV}^2$$

$$5 \cdot 10^{-4} < x < 7 \cdot 10^{-3}$$

- First ZEUS  $F_L$  publication available
- Most precise cross section measurement from ZEUS in the kinematic region studied
- Measured cross sections are published and available for fits
- Measured cross sections compared to ZEUS-JETS with and without  $F_L$
- Turnover at low  $x$  small but visible

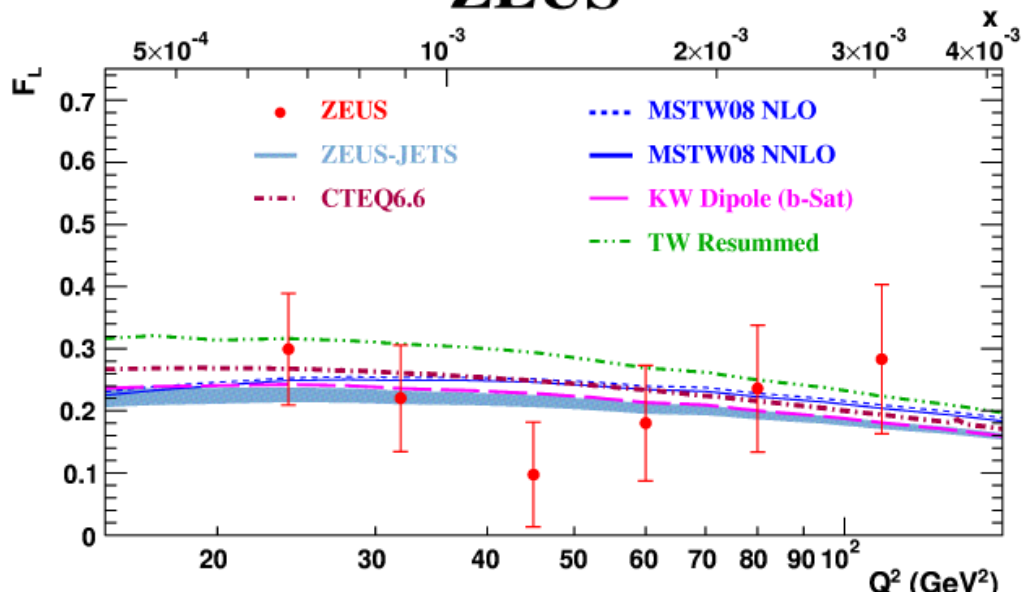
## Extracted $F_L$ and $F_2$

### ZEUS

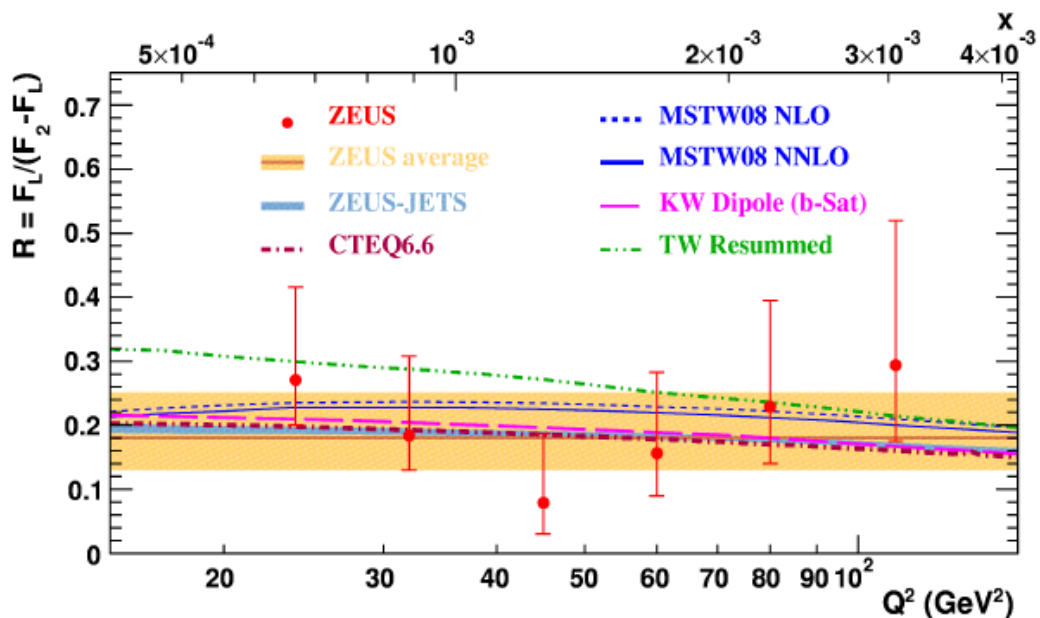


- Most precise  $F_2$  measurement from ZEUS at kinematic region studied
- **First  $F_2$  measurement without assumptions on  $F_L$**
- Data support a non-zero  $F_L$
- Predictions for  $F_2$  and  $F_L$  are consistent with data

# ZEUS



- Data support a non-zero  $F_L$
- Predictions are consistent with data



Ratio R is fitted:

$$R = F_L / (F_2 + F_L)$$

Average R from all data:

$$R = 0.18^{+0.07}_{-0.05}$$

## PDF fits with $F_L$ data included

- Measured cross sections for 3 data sets (HER, LER, MER) are included in ZEUS PDF fits
- Data has impact on the low  $x$ :
  - Steeper rise of gluon at low  $x$
  - Sea and gluon uncertainty reduced

→ For more details see talk of A. Cooper-Sarkar

