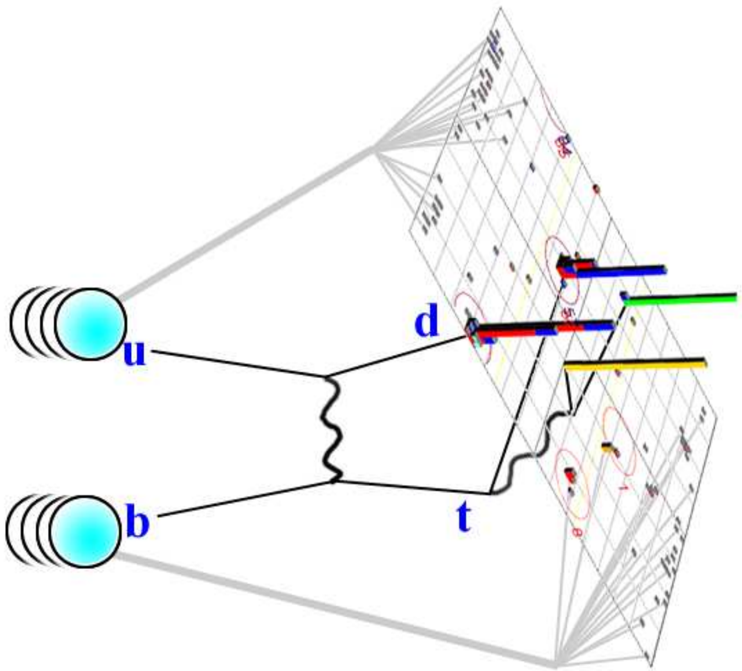
Single-top-quark production: a tool for understanding QCD



Zack Sullivan
Illinois Institute of Technology

Outline

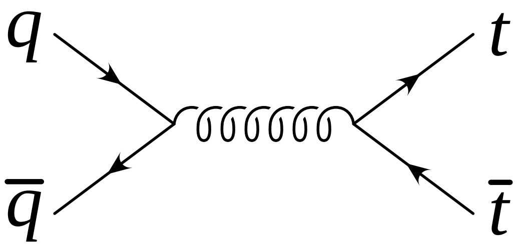
1. Understanding electroweak (EW) physics
   - What is single-top-quark production?
   - Why do we study it?
2. Understanding perturbative QCD
   - The *new* Drell-Yan (DY) and Deep Inelastic Scattering (DIS) (or, dealing with the lump we swept under the rug)
   - What we've had to learn about the cross section
3. Applied understanding
   - A new paradigm for interpreting higher-order calculations
   - Examining the connection between theory and experiment
   - The impact of angular correlations

What is single-top-quark production?

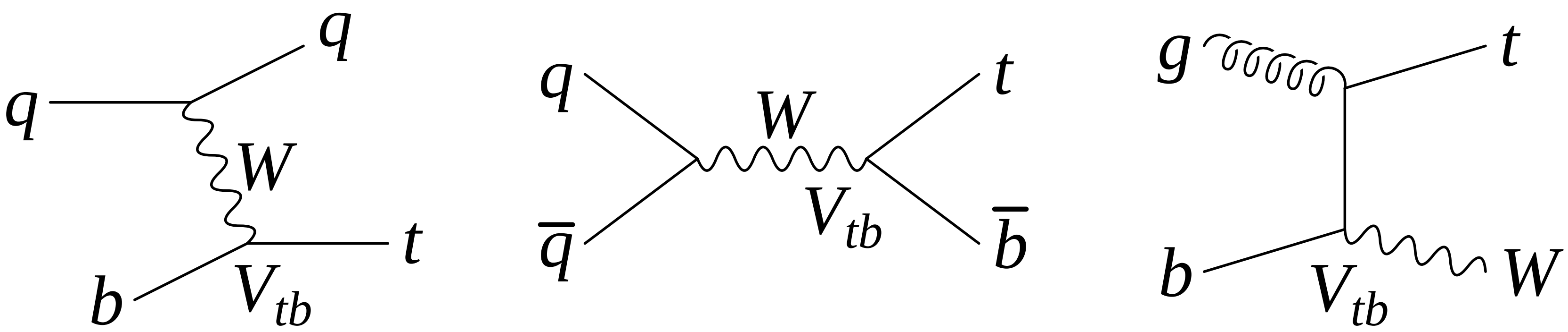
Why do we study it?

What is single-top-quark production?

Top quark pairs were discovered in 1995 via strong force production:



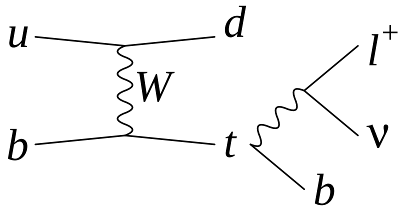
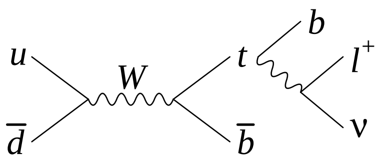
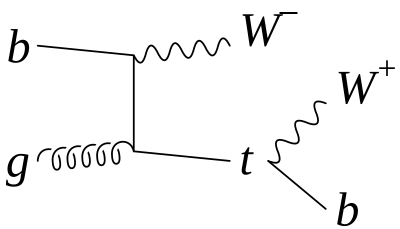
Single-top-quark production is an electroweak (EW) process.



t-channel s-channel Wt-associated

Signatures and NLO cross sections

Production modes are distinguished by the number of tagged b jets.

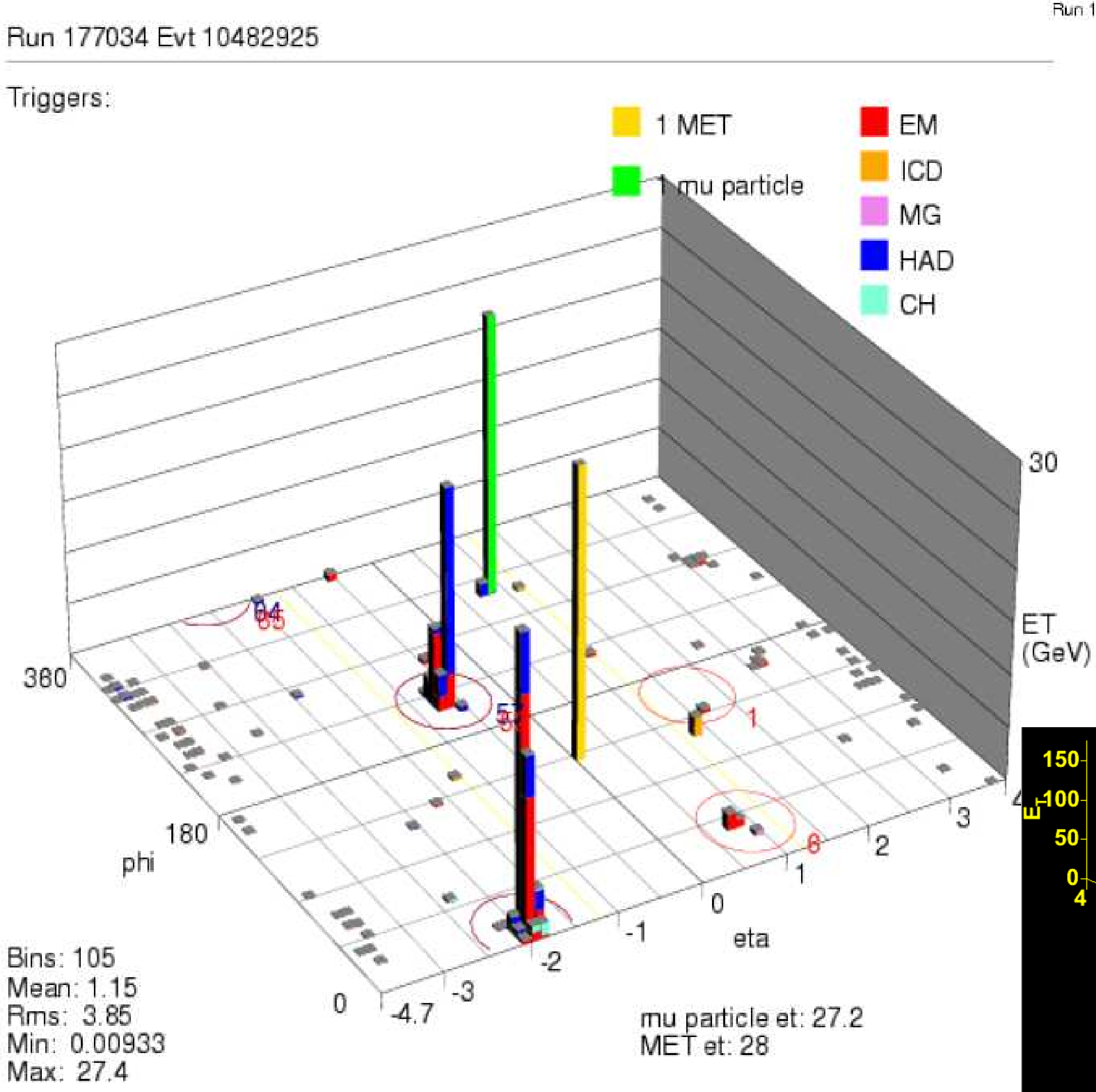
	Signature	NLO cross sections (pb) Tevatron$(t+\bar{t})$	LHC$(t/\bar{t})$
	$ebj\not{E}_T/\mu bj\not{E}_T$ (1 b-jet)	1.98 ± 0.2	$155.9/90.7 \pm 5\%$
	$ebb\not{E}_T/\mu bb\not{E}_T$ (2 b-jets)	0.88 ± 0.1	$6.6/4.1 \pm 10\%$
	W^+W^-b $(t\bar{t} - 1b$ jet)	~ 0.07	$\sim 33/33$

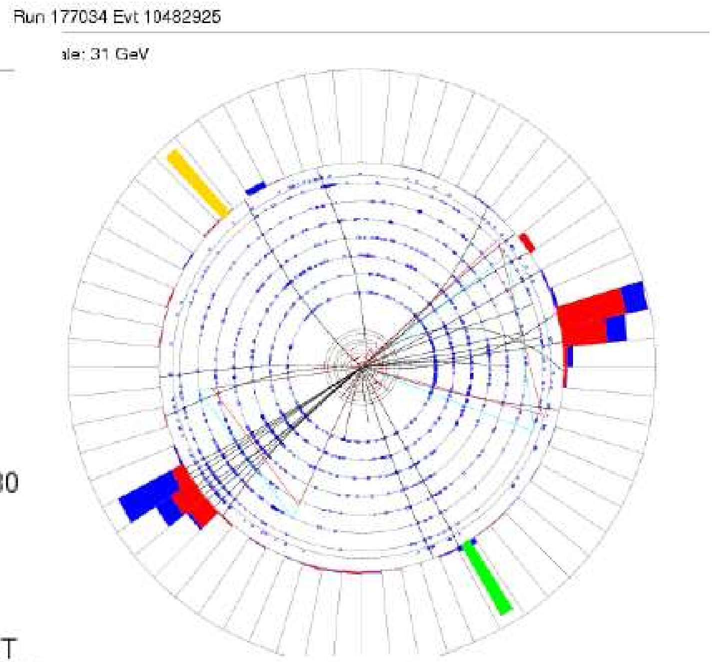
Z.S., PRD 70, 114012 (2004); J. Campbell, F. Tramontano, NPB 726, 109 (2005)

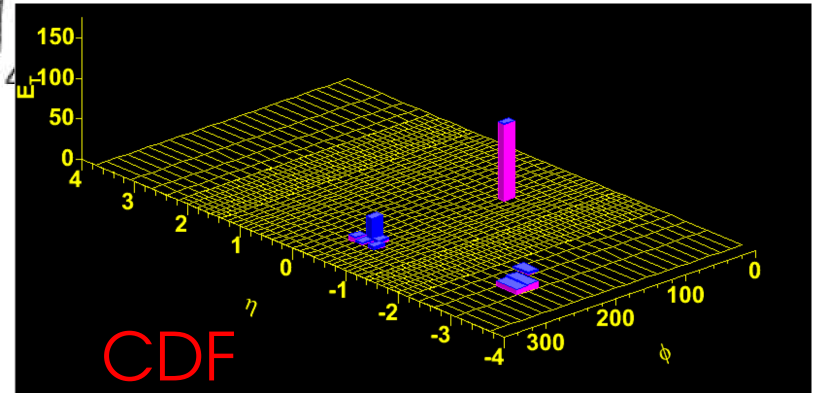
The Tevatron has produced ~ 20000 single-top-quark events (7 fb^{-1})
(vs. ~ 50000 $t\bar{t}$ events)

Candidate single-top-quark events

DØ

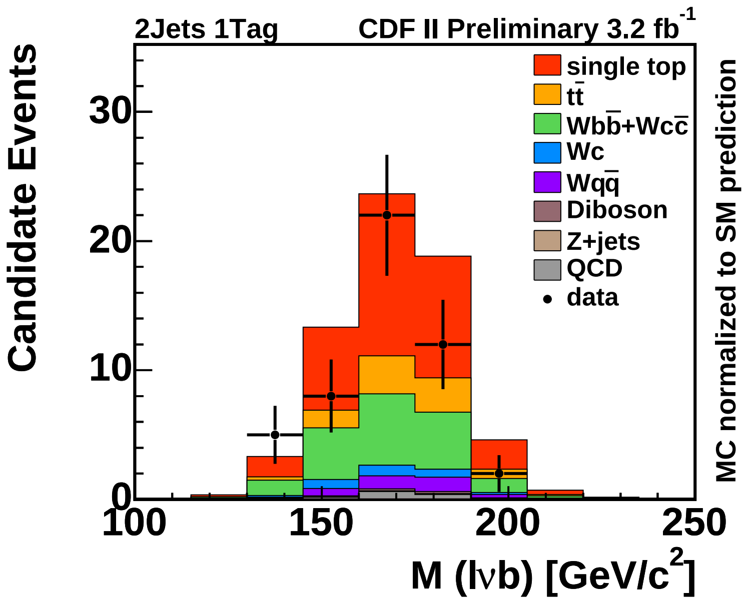


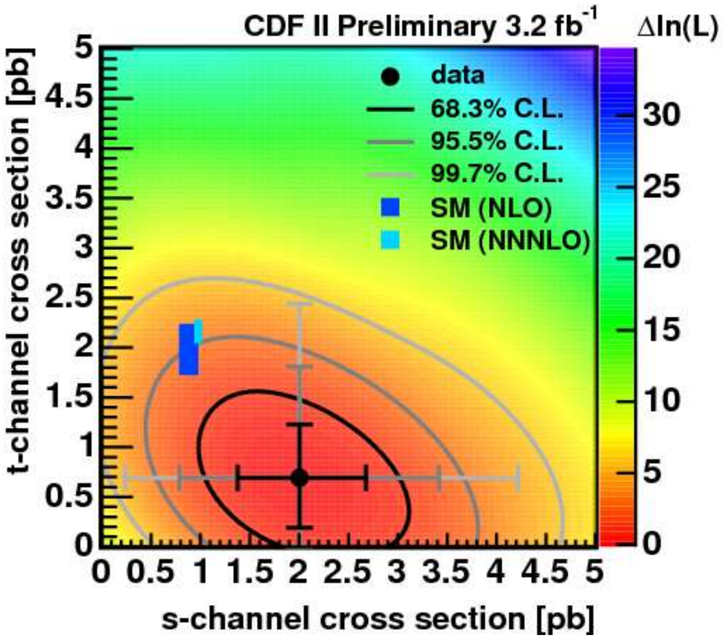


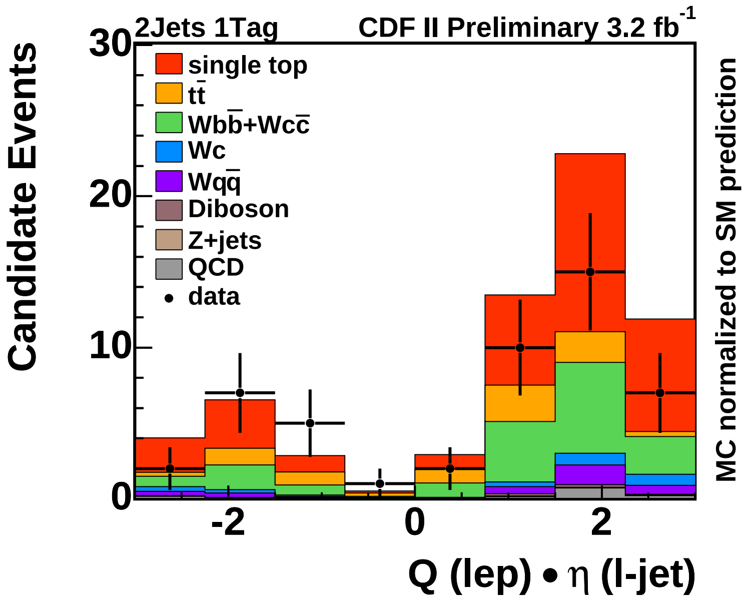


5σ discovery of single-top-quark production

This flagship measurement of the Fermilab Tevatron has been observed!







CDF: 5σ $s+t = 2.3^{+0.6}_{-0.5}$ pb

s only: 1.4 pb, t only: 1 pb

DØ: 5σ $s+t = 3.94 \pm 0.88$ pb

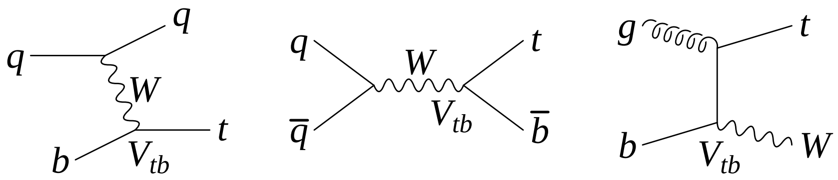
(Theory expects 2.86 ± 0.24 pb)

CDF hep-ex:0903.0885; PRL 101, 252001 (08);
DØ, hep-ex:0809:2581; PRL 98, 181802 (07)

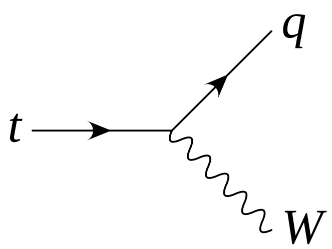
What is single-top-quark production?

Why do we study it?

Why we look at single-top-quark production



Weak interaction structure



$$-i \frac{g}{\sqrt{2}} V_{tq} \gamma^{\mu} \frac{1}{2}(1 - \gamma_5)$$

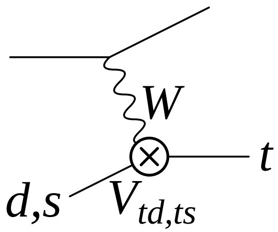
Goal: Determine the structure of the W-t-q vertex.

- Measure CKM couplings "direct measurement of V_{tb}"
- Measure Lorentz structure "spin correlations"

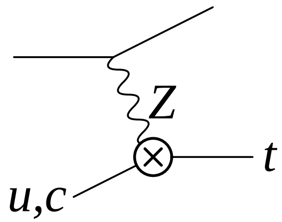
Direct or indirect new physics

New t-q couplings mostly affect t-channel measurement (Wbj).

- Larger V_{ts} or V_{td} give PDF enhancement to σ_t.



- FCNC production modes from, e.g. Z-t-c, increase σ_t.

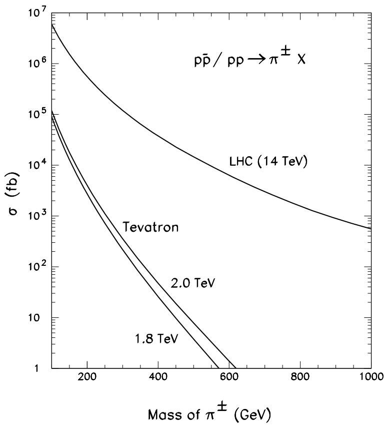


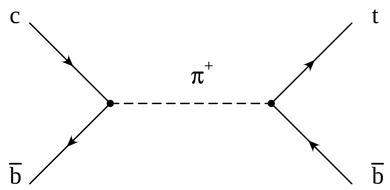
s-channel looks like t-channel, since distinguished by number of b-tags.

New physics in s-channel vs. t-channel

$t + b$ resonant production affects s-channel (Wbb)

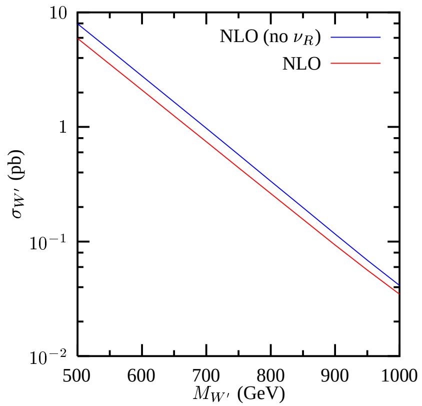
Charged scalars (spin-0)

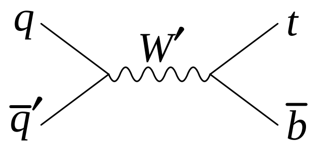




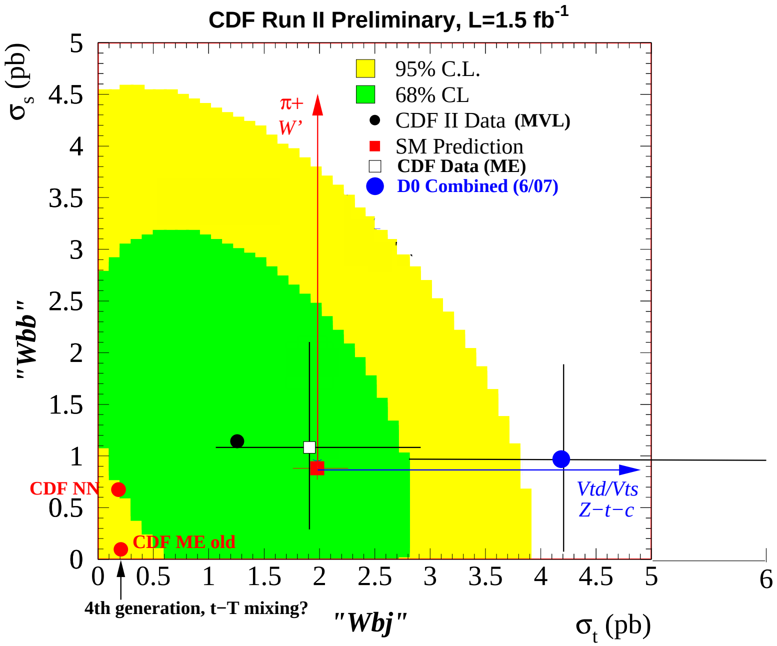
T. Tait, C.P. Yuan PRD 63, 014018 (2001)

W' bosons (spin-1)





Z.S., PRD 66, 075011 (2002)



Measuring both production cross sections provides strong constraints on many new physics scenarios.

Understanding perturbative QCD through single-top-quark production

Structure of an observable cross section

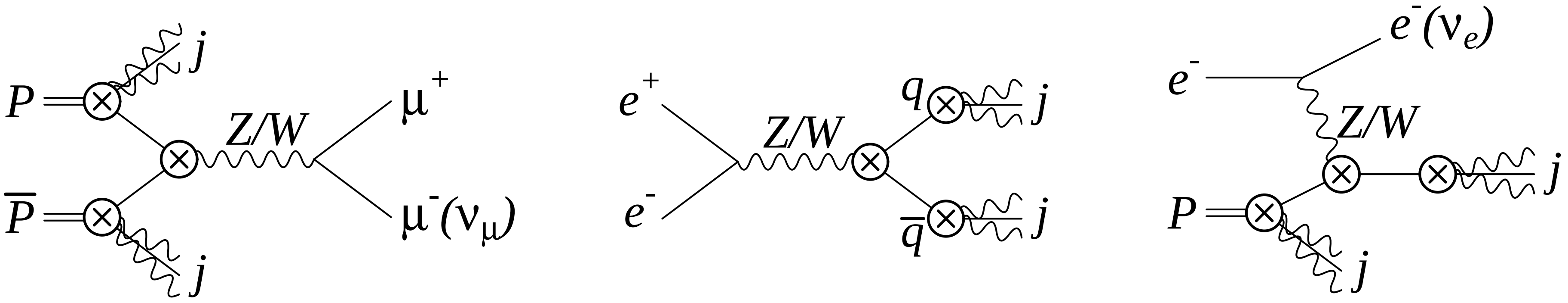
$$\sigma_{\mathrm{obs.}} = \int f_1(x_1,\mu_1) f_2(x_2,\mu_2) \otimes \overline{|M|^2} \otimes d\mathrm{P.S.} \otimes D_i(p_i) \ldots D_n(p_n)$$

Theorists factorize (break) the cross section into:

- Initial-state IR singularities swept into parton distribution "functions".
  These are not physical, but include scheme dependent finite terms:
  $\overline{\mathrm{MS}}$ — the current standard
  DIS — ill-defined in all modern PDF sets, could be fixed, but useless.
- A squared matrix element, which represents the bulk of the perturbative calculation effort.
- Phase space which you may not want to completely integrate out.
  $\Rightarrow$ Exclusive cross sections (jet counting), angular correlations
- Fragmentation functions or jet definitions.
  These provide the coarse graining to hide final-state IR singularities.

Drell-Yan and DIS

The traditional testbed of perturbative QCD have been restricted to Drell-Yan production, e^+e^- to jets, or deep inelastic scattering (DIS).

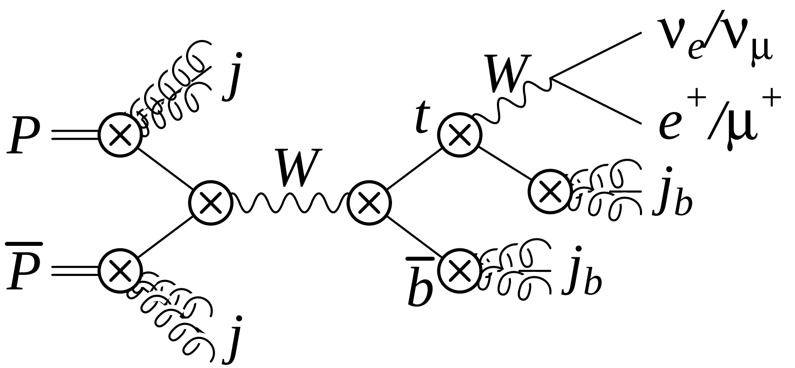


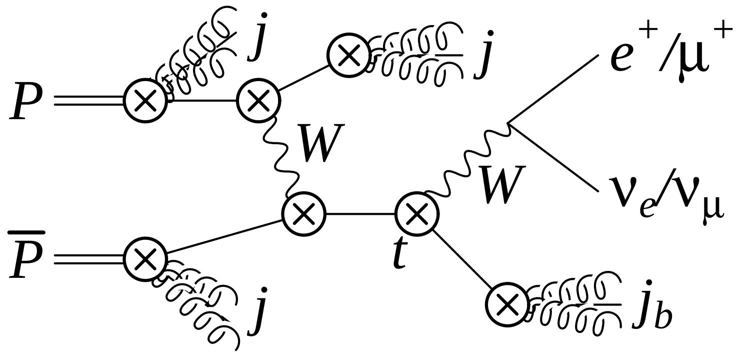
A key property that all three processes share is a complete factorization of QCD radiation between different parts of the diagrams.

- Drell-Yan → Initial-state (IS) QCD radiation only.
- e^+e^- →jets → Final-state (FS) QCD radiation only.
- DIS → Proton structure and fragmentation functions probed. Simple color flow.

s-/t-channel single-top-quark production (A generalized Drell-Yan and DIS)

A perfect factorization through next-to-leading order (NLO) makes single-top-quark production mathematically *identical*† to DY and DIS!





Generalized Drell-Yan.

IS/FS radiation are independent.

Double-DIS (DDIS) w/ 2 scales:

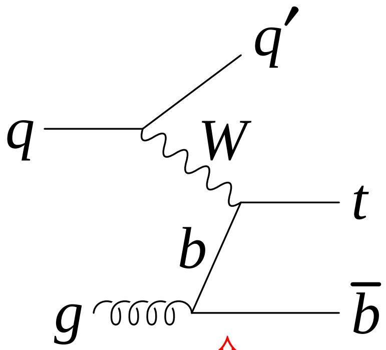
$\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$

Color conservation forbids the exchange of just 1 gluon between the independent fermion lines.

† Massive forms: m_t, m_b, and m_t/m_b are relevant.

Rethinking the initial state: W-gluon fusion $\to$ t-channel single-top

W-gluon fusion (circa 1996)



$\Uparrow$

$\sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right) + \mathcal{O}(\alpha_s)$

$m_t \approx 35 m_b$! $\alpha_s \ln \sim .7$-$.8$

Look at the internal b.

The propagator is

$$\frac{1}{(P_g - P_{\bar{b}})^2 - m_b^2} = \frac{1}{-2P_g \cdot P_{\bar{b}}}$$

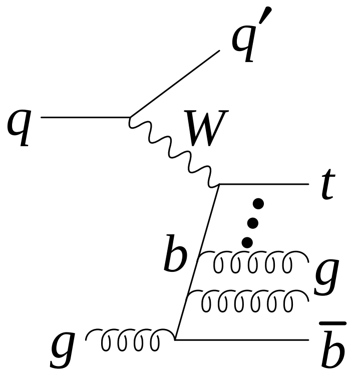
$$P_g = E_g(1,0,0,1),\ P_{\bar{b}} = (E_b, \vec{p}_T, p_z)$$

$$P_g \cdot P_{\bar{b}} = E_g\left(p_z\sqrt{1 + \frac{p_T^2+m_b^2}{p_z^2}} - p_z\right)$$

$$\approx E_g p_z\left(\frac{p_T^2+m_b^2}{2p_z^2}\right) \sim (p_T^2 + m_b^2)$$

$$\int_{p_{T\,\mathrm{cut}}} \frac{\mathrm{d}p_T^2}{p_T^2+m_b^2} \to \ln\left(\frac{1}{p_{T\,\mathrm{cut}}^2+m_b^2}\right)$$

The same procedure for the W leads to the massive formula for DIS.



Each additional order adds another

$$\frac{1}{n!}\left[\alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right)\right]^n$$

Looks bad for perturbative expansion...

Resummation of large logs and b PDF

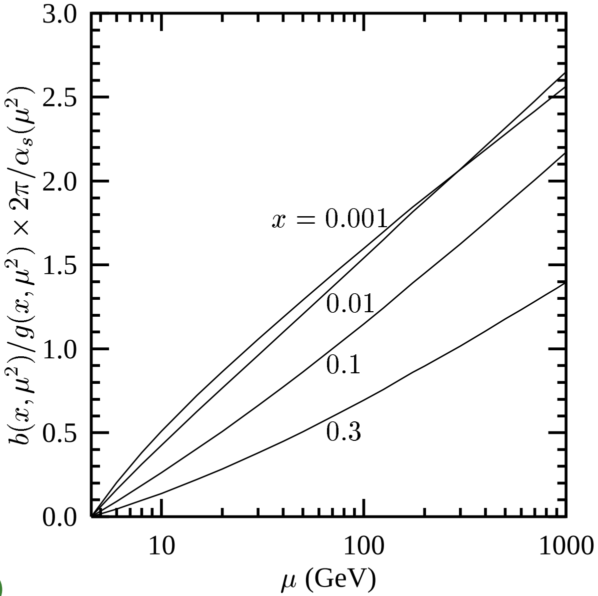
Use Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation to sum large logs due to (almost) collinear singularities in gluon splitting.

$b \propto \alpha_s \ln(\mu^2/m_b^2) \times g$

$$\frac{dQ(\mu^2)}{d\ln(\mu^2)} \approx \frac{\alpha_s}{2\pi} P_{Qg} \otimes g + \frac{\alpha_s}{2\pi} P_{QQ} \otimes Q; \; Q \ll g$$

$$P_{Qg}(z) = \frac{1}{2}[z^2 + (1-z)^2] \, .$$

$$Q(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_Q^2}\right) \int_x^1 \frac{dz}{z} P_{Qg}(z) g\left(\frac{x}{z},\mu^2\right)$$



Barnett, Haber, Soper, NPB 306, 697 (88)

Olness, Tung, NPB 308, 813 (88)

Aivazis, Collins, Olness, Tung, PRD 50, 3102 (94)

Stelzer, ZS, Willenbrock, PRD 56, 5919 (1997)

Aside: In the $\overline{\text{MS}}$ scheme, $b(\mu \leq m_b) \equiv 0$.

DIS scheme is not uniquely defined for heavy quarks.

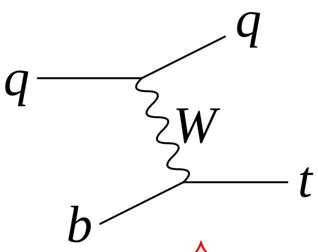
Do you choose $F_2 \equiv 0$ (traditional) or define w.r.t. $\overline{\text{MS}}$?

The first attempt to calculate single-top failed because the DIS scheme was used.

Bordes, van Eijk, NPB435, 23 (95)

New nomenclature and classification

New Leading Order



$(P_W^2 < 0)$

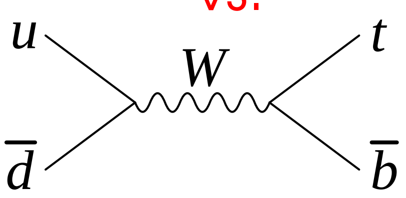
⇑

$b \sim \alpha_s \ln\left(\frac{\mu^2}{m_b^2}\right) \times g$

t-channel production

Named for the "t-channel" exchange of a W boson.

vs.



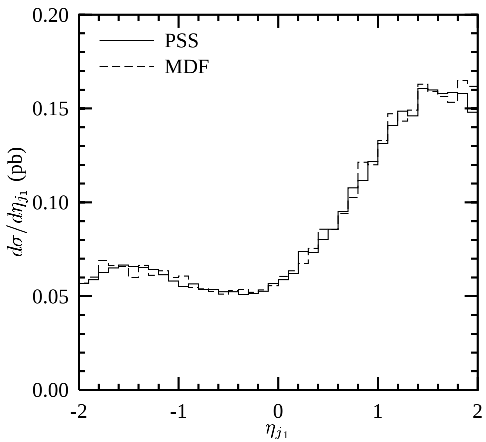
$(P_W^2 > 0)$

s-channel production

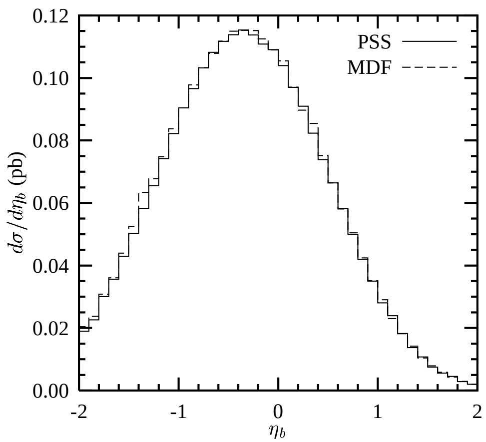
Named for the "s-channel" exchange of a W boson.

Classifying processes by analytical structure leads directly to kinematic insight:

Jets from t-channel processes are more forward than those from s-channel.

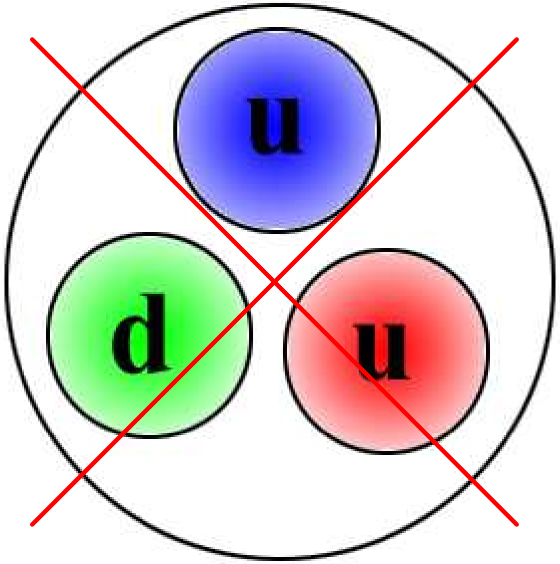


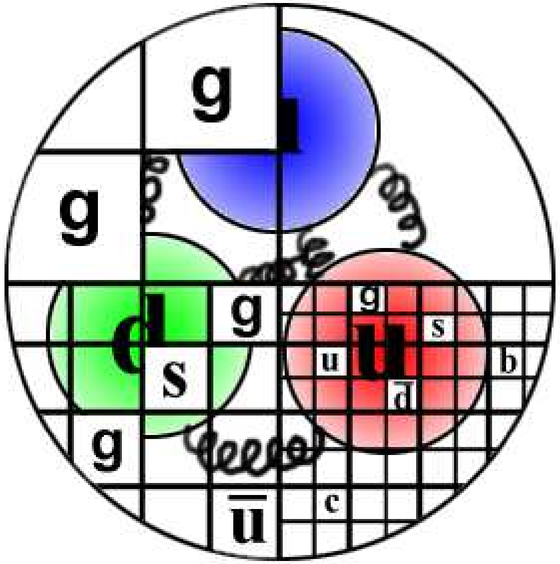
jet from t-channel



b jet from s-channel

Rethinking the proton



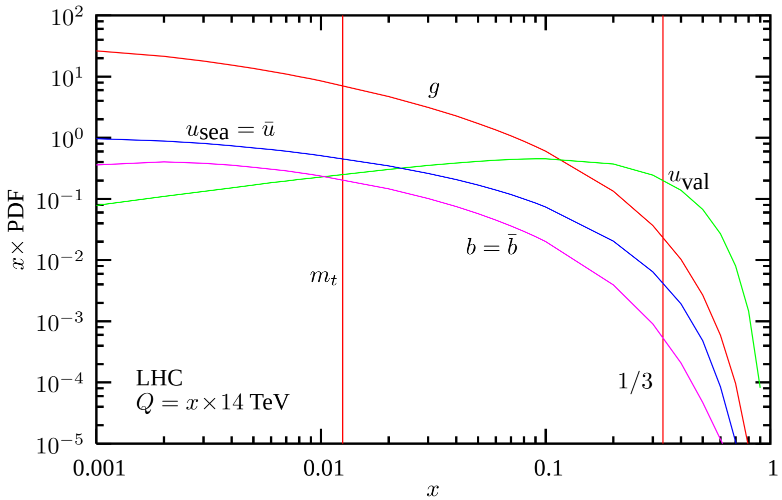


Using DGLAP was NOT just a math trick!

This "valence" picture of the proton is not complete.

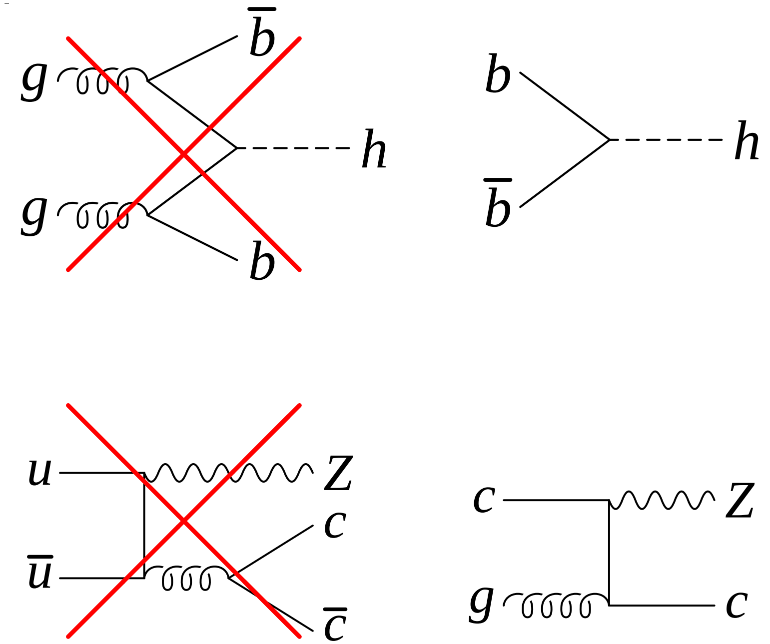
Larger energies resolve smaller structures.

The probability of finding a particle inside the proton is given by PDFs (Parton Distribution Functions)



b (and c) quarks are full-fledged members of the proton structure.

Rethinking several physical processes

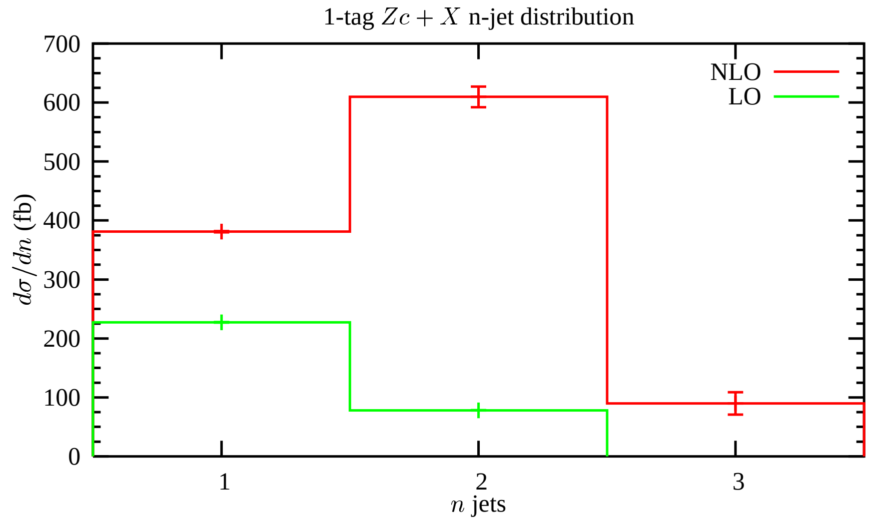


Starting with a c/b gives us:

$b\bar{b} \to h$	Largest SUSY Higgs cross section
Zb/Zc	Affects LHC luminosity monitor
Zbj/Zcj	Higgs background
Wbj	Largest single-top background
etc.	

Why is this important?

Zc at Tevatron



Parton luminosity can be more important than counting powers of α_s!

This is exaggerated at LHC:

$$Z \approx Z + 1\ \text{jet} \approx Z + 2\ \text{jets!}$$

(True of $W + X$ as well!)

Is jet counting poorly-defined (theoretically) at LHC?

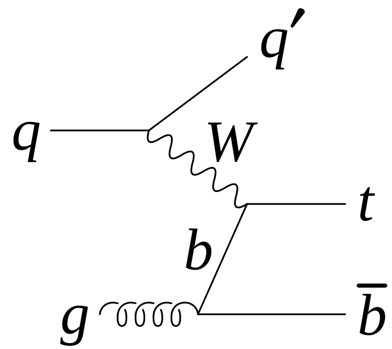
Rethinking the matrix element: A practical problem for experiments

The same large logs that lead to a reordered perturbation for t-channel single-top, implied a potentially large uncertainty in measurable cross sections when cuts were applied.

Recall: t-channel and s-channel are distinguished by the number of b-jets.

A problem: About 20% of the time, the extra $\bar{b}$-jet from the t-channel process is hard and central.

Real problem: Is the b contamination 20%, 30%, 10%?



Another problem: To distinguish from $t\bar{t}$, the cross section in the $W + 2$ jet bin has to be known.

Counting jets is IDENTICAL to performing a jet veto.

Inclusive cross sections are not enough, we need to calculate exclusive cross sections

Fully Differential NLO Techniques

- In 2001, there were few matrix-element techniques or calculations that could deal IR singularities in processes with massive particles.
- Experiments were mostly stuck using LO matrix elements to predict semi-inclusive or exclusive final states.
- We needed methods to provide the 4-vectors, spins, and corresponding weights of exclusives final-state configurations.

These needs led to work on 3 techniques:

- Phase space slicing method with 2 cutoffs.

  L.J. Bergmann, Ph.D. Thesis, FSU (89)
  cf. H. Baer, J. Ohnemus, J.F. Owens, PRD 40, 2844 (89)
  B.W. Harris, J.F. Owens, PRD 65, 094032 (02)

- Phase space slicing method with 1 cutoff.

  W.T. Giele, E.W.N. Glover, PRD 46, 1980 (92)
  cf. W.T. Giele, E.W.N. Glover, D.A. Kosower, NPB 403, 633 (93)
  E. Laenen, S. Keller, PRD 59, 114004 (99)

- Massive dipole formalism (a subtraction method) coupled with a helicity-spinor calculation. Invented to solve single-top production.

  cf. L. Phaf, S. Weinzierl, JHEP 0104, 006 (01)
  S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi, NPB 627,189 (02)

Massive Dipole Formalism (subtraction)

$$\sigma_{NLO} = \int_{n+1} d\sigma^{\mathrm{Real}} + \int_{n} d\sigma^{\mathrm{Virtual}}$$
$$= \int_{n+1} \left(d\sigma^{R} - d\sigma^{A}\right) + \int_{n} \left(d\sigma^{V} + \int_{1} d\sigma^{A}\right)$$

- $d\sigma^A$ is a sum of color-ordered dipole terms.
  - $d\sigma^A$ must have the same point-wise singular behavior in D dimensions as $d\sigma^R$.
    $\Rightarrow$ $d\sigma^A$ is a local counterterm for $d\sigma^R$.
  - $\int_1 d\sigma^A$ is analytic in D dimensions, and reproduces the soft and collinear divergences of $d\sigma^R$.
- Some advantages over Phase Space Slicing are:
  - You can easily project out spin eigenstates.
    $\Rightarrow$ Explicitly test different spin bases at NLO after cuts.
  - Event generators use color-ordered matrix elements.
- Both methods have some contribution to n-body final states from $n+1$ phase-space. Hence, you must do 2 separate integrations.

Phase Space Slicing Method (2 cutoffs)

B.W. Harris, J.F. Owens, PRD 65, 094032 (02)

The essential challenge of NLO differential calculations is dealing with initial- and final-state soft or collinear IR divergences.

$$\sigma_{\text{obs.}} \sim \int \frac{1}{s_{ij}} \sim \int \frac{dE_i dE_j d\cos\theta_{ij}}{E_i E_j (1-\cos\theta_{ij})}$$

If $E_{i,j} \to 0$ "soft" singularity

If $\theta_{ij} \to 0$ "collinear" singularity

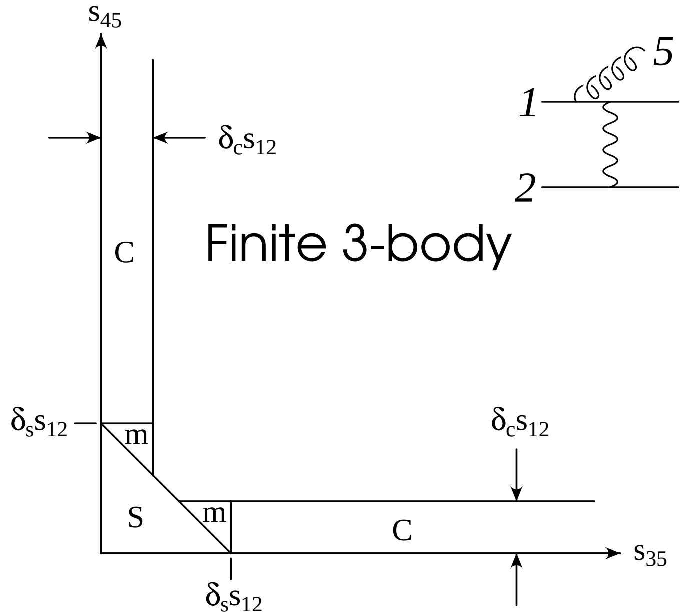
IDEA: Introduce arbitrary cutoffs (δ_s, δ_c) to remove the singular regions. . .

We traded dependence on physical observables (energy, angles) for logarithmic dependence on arbitrary parameters $(\ln\delta_s, \ln\delta_c)$

Divide phase space into 3 regions:

1. soft: $E_g \leq \delta_s\sqrt{\hat{s}}/2$ gluons only
2. collinear: $\hat{s}_{35}, \hat{s}_{45}, \ldots < \delta_c \hat{s}$;
3. hard non-collinear: (finite, particles well separated, $E > 0$)

Phase space plane (s_{35}, s_{45})



Subtraction vs. phase space slicing

In practical terms, the difference in methods is in how to integrate in the presence of infrared singularities.

$$I = \lim_{\epsilon \to 0^+} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\}$$

Subtraction: Add and subtract $F(0)$ under the integral

$$\begin{aligned} I &= \lim_{\epsilon \to 0^+} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon \left[F(x) - F(0) + F(0) \right] - \frac{1}{\epsilon} F(0) \right\} \\ &= \int_0^1 \frac{dx}{x} \left[F(x) - F(0) \right], \quad \text{finite up to machine precision} \end{aligned}$$

PSS: Integration region divided into two parts $0 < x < \delta$ and $\delta < x < 1$, with $\delta \ll 1$. A Maclaurin expansion of $F(x)$ yields

$$\begin{aligned} I &= \lim_{\epsilon \to 0^+} \left\{ \int_0^\delta \frac{dx}{x} x^\epsilon F(x) + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\} \\ &= \int_\delta^1 \frac{dx}{x} F(x) + F(0) \ln \delta + \mathcal{O}(\delta), \quad \text{take } \lim_{\delta \to 0} \text{ numerically} \end{aligned}$$

Remaining $\ln \delta$ singularities removed by summing all integrals I_i.

Explicit t-channel calculation (soft)

Soft region: $0 \leq E_g \leq \delta_s \frac{\sqrt{s}}{2}$

$$
\begin{aligned}
d\sigma_t^{(S)} &= d\sigma_t^{(0)} \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s} \right)^{\epsilon} \right] \left(\frac{A_2^t}{\epsilon^2} + \frac{A_1^t}{\epsilon} + A_0^t \right) \\
A_2^t &= 3C_F \\
A_1^t &= C_F \left[1 - 6\ln\delta_s - 2\ln\left(\frac{-t}{s\beta}\right) - \ln\left(\frac{(m^2-t)^2}{m^2 s}\right) \right] \\
A_0^t &= C_F \left[6\ln^2\delta_s - 2\ln\delta_s + 4\ln\delta_s \ln\left(\frac{-t}{s\beta}\right) \right. \\
&\quad + 2\ln\delta_s \ln\left(\frac{(m^2-t)^2}{m^2 s}\right) + \frac{s+m^2}{s-m^2}\ln\left(\frac{s}{m^2}\right) + \ln^2\left(\frac{-t}{s\beta}\right) + 2\mathrm{Li}_2\left(1+\frac{t}{s\beta}\right) \\
&\quad \left. - \frac{1}{2}\ln^2\left(\frac{s}{m^2}\right) + \ln^2\left(\frac{m^2}{m^2-t}\right) + 2\mathrm{Li}_2\left(\frac{t}{m^2}\right) - 2\mathrm{Li}_2\left(\frac{u}{s+u}\right) \right],
\end{aligned}
$$

where the top-quark mass is denoted as m, and $\beta = 1 - m^2/s$.

t-channel (collinear)

Hard region divided into hard collinear (HC) and hard-noncollinear (H$\overline{\text{C}}$)

- H$\overline{\text{C}}$ computed numerically in 4 dimensions.
- HC where invariants $s_{ij} = (p_i + p_j)^2$ or $t_{ij} = (p_i - p_j)^2$ in the denominator become smaller in magnitude than $\delta_c s$.

Singular regions from FS radiation give:

$$d\sigma_p^{(HC,FS)} = d\sigma_p^{(0)} \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s} \right)^\epsilon \right] \left(\frac{A_1}{\epsilon} + A_0 \right)$$

$$A_1 = C_F \left(2\ln\delta_s + \frac{3}{2} - 2\ln\beta \right)$$

$$A_0 = C_F \left[\frac{7}{2} - \frac{\pi^2}{3} - \ln^2\delta_s - \ln^2\beta + 2\ln\delta_s \ln\beta - \ln\delta_c \left(2\ln\delta_s + \frac{3}{2} - 2\ln\beta \right) \right]$$

Singular regions from IS radiation give:

$$d\sigma_{p,C}^{ij} = d\sigma_p^{(0)} \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s} \right)^\epsilon \right] \left[\widetilde{f}_j^H(z,\mu_F) + \left(\frac{A_1^{sc}}{\epsilon} + A_0^{sc} \right) f_j^H(z,\mu_F) \right]$$

$$A_1^{sc} = C_F \left(2\ln\delta_s + \frac{3}{2} \right) \qquad A_0^{sc} = C_F \left(2\ln\delta_s + \frac{3}{2} \right) \ln\left(\frac{s}{\mu_F^2} \right)$$

$\widetilde{f}_j^H(z,\mu_F)$ is a universal modified PDF.

t-channel (virtual)

Virtual contribution has two pieces. One $\propto$ Born, one not:

$$d\sigma_p^{(V)} = d\sigma_p^{(0)} \left[\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{s}\right)^{\epsilon}\right]\left(\frac{A_2^V}{\epsilon^2}+\frac{A_1^V}{\epsilon}+A_0^V\right)+\left(\frac{\alpha_s}{2\pi}\right)d\tilde{\sigma}_p^{(V)}$$

$$A_2^V = C_F\left\{[-2]-[1]\right\}$$

Note: $\lambda = t/(t-m^2)$

$$A_1^V = C_F\left\{\left[-3-2\ln\left(\frac{s}{-q^2}\right)\right]+\left[-\frac{5}{2}-2\ln(1-\lambda)-\ln\left(\frac{s}{m^2}\right)\right]\right\}$$

$$\begin{aligned} A_0^V = & \; C_F\left\{\left[-\ln^2\left(\frac{s}{-q^2}\right)-3\ln\left(\frac{s}{-q^2}\right)-8-\frac{\pi^2}{3}\right]\right. \\ & + \left[-\frac{1}{2}\ln^2\left(\frac{s}{m^2}\right)-\frac{5}{2}\ln\left(\frac{s}{m^2}\right)-2\ln(1-\lambda)\ln\left(\frac{s}{m^2}\right)-6\right. \\ & - \left.\left.\frac{1}{\lambda}\ln(1-\lambda)-\ln^2(1-\lambda)-2\ln(1-\lambda)+2\mathrm{Li}_2(\lambda)-\frac{\pi^2}{3}\right]\right\} \end{aligned}$$

$$d\tilde{\sigma}_t^{(V)} = \frac{1}{2s}\frac{1}{4}g^4|V_{ud}|^2|V_{tb}|^2 C_F\frac{m^2su}{t}\ln\left(\frac{m^2}{m^2-t}\right)\left(\frac{1}{t-M_W^2}\right)^2 d\Gamma_2\,,$$

We can keep track of light- and heavy-quark contributions separately.

t-channel (summing it up)

We now see cancellation of singularities:

$\frac{1}{\epsilon^2}$: $A_2^t + A_2^V = 3C_F + (-3C_F) = 0$

$\frac{1}{\epsilon}$: $A_1^t + A_1^V + A_1 + 2A_1^{sc} = 0$ (e.g., $C_F[1 + 3/2 + (-3 - 5/2) + 2(3/2)] = 0$)

Final 2-to-2 result

$$\sigma^{(2)} = \left(\frac{\alpha_s}{2\pi}\right) \sum_{a,b} \int dx_1 dx_2 \left\{ f_a^{H_1}(x_1, \mu_F) f_b^{H_2}(x_2, \mu_F) \times \right.$$
$$\left[d\sigma_p^{(0)} \left(A_0^p + A_0^V + A_0 + 2A_0^{sc} \right) + d\tilde{\sigma}_p^{(V)} \right]$$
$$\left. + \, d\sigma_p^{(0)} \left[f_a^{H_1}(x_1, \mu_F) \widetilde{f}_b^{H_2}(x_2, \mu_F) + \widetilde{f}_a^{H_1}(x_1, \mu_F) f_b^{H_2}(x_2, \mu_F) \right] + (x_1 \leftrightarrow x_2) \right\}$$

Final 2-to-3 result

$$\sigma^{(3)} = \sum_{a,b} \int dx_1 dx_2 \frac{1}{2s} \int_{H\overline{C}} g^4 |V_{ud}|^2 |V_{tb}|^2 \overline{\Psi}(p_i) d\Gamma_3$$

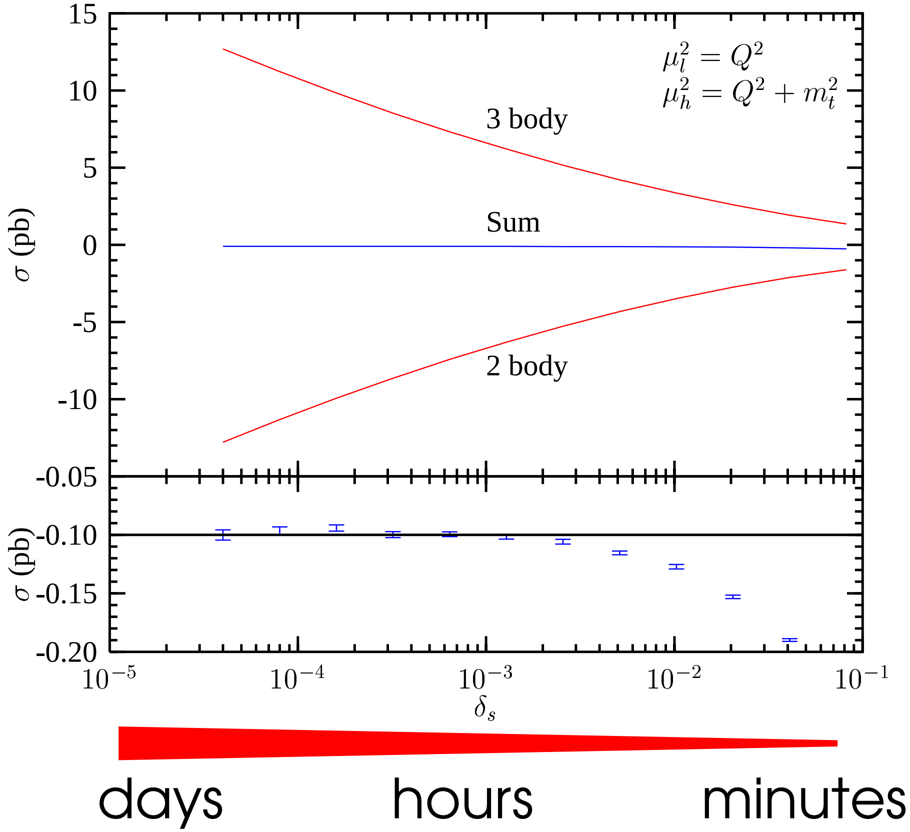
$\alpha_{s\, l(h)}$ and the luminosity functions $L_{l(h)} = f_a^{H_1}(x_1, \mu_{F\, l(h)}) f_b^{H_2}(x_2, \mu_{F\, l(h)})$ are evaluated using the scales at the light(heavy)-quark lines, respectively.

$\sigma^{\text{final}} = \sigma^{(2)} + \sigma^{(3)}$ is cutoff independent

Cut-off dependence of NLO correction

Each term is logarithmically divergent for small δ_s, δ_c

Logarithmic dependence on the cutoffs cancels in any IR-safe observable at the histogramming stage.



At the end we take δ_s and δ_c to zero via numerical computation.
This can take a long time. . .

Rethinking jet definitions and phase space: Experiments need exclusive $t+1$ jet at NLO

ZTOP, Z.S., PRD 70, 114012 (2004) (hep-ph/0408049)

	# b-jets	tj (Wbj)	tjj $(Wbjj)$
s-channel	= 2	0.620 pb $^{+13}_{-11}\%$	0.168 pb $^{+24}_{-19}\%$
	= 1	0.022 pb $^{+24}_{-19}\%$	(NNLO)
t-channel	= 1	0.950 pb $^{+16}_{-15}\%$	0.152 pb $^{+17}_{-14}\%$
	= 2	0.146 pb $^{+21}_{-16}\%$	0.278 pb $^{+21}_{-16}\%$

Every number here, even the concept of t-channel single-top, required a new or revised understanding of QCD.

Cuts: $p_{Tj} > 15$ GeV, $|\eta_j| < 2.5$, no cuts on t

Jet definition: $\Delta R_{k_T} < 1.0$ $(\approx \Delta R_{\rm cone} < 0.74)$

Breakdown of *shape-independent* uncertainties

Process		$\times\delta m_t$(GeV)	$\mu/2$–2μ	PDF	b mass	$\alpha_s(\delta_{\rm NLO})$
s-channel	$p\bar{p}$	$^{-2.33}_{+2.71}\%$	$^{+5.7}_{-5.0}\%$	$^{+4.7}_{-3.9}\%$	$< 0.5\%$	$\pm1.4\%$
	pp	$^{-1.97}_{+2.26}\%$	$\pm2\%$	$^{+3.3}_{-3.9}\%$	$< 0.4\%$	$\pm1.2\%$
t-channel	$p\bar{p}$	$^{-1.6}_{+1.75}\%$	$\pm4\%$	$^{+11.3}_{-8.1}\%$	$< 1\%$	$\pm0.01\%$
	pp	$^{-0.73}_{+0.78}\%$	$\pm3\%$	$^{+1.3}_{-2.2}\%$	$< 1\%$	$\pm0.1\%$

- b PDFs $\rightarrow$ t-channel
- PDF uncertainties
- multiple scales: m_t/m_b
- 2 expansions: α_s, $1/\ln$
- Fully differential NLO jet calculations
- . . .

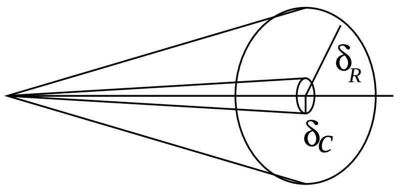
Applied Understanding

Jet calculations
Theory vs. experiment

Phase Space Slicing ⇒ physical picture

Let's return to the construction of phase space slicing:

Whenever 2 massless particles get too close ($\theta_{12} \to 0$), we draw a cone around them.



Physically you can think of phase space slicing as forming a "pre-jet" that is much smaller than any measurable jet of radius R. ($\delta_c \ll \delta_R$)

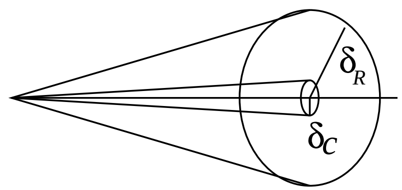
Unlike inclusive NLO calculations, exclusive NLO calculations are only well-defined in the presence of a jet definition or hadronization function.

⇒ The mathematics of quantum field theory tells us we cannot resolve the quarks inside of these jets!

How do we interpret exclusive NLO calculations?

“Paradigm of jet calculations”

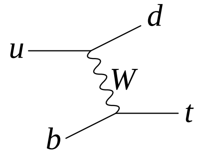
- We are calculating extensive objects, i.e., jets not “improved quarks.”
  — NLO calculations are not well defined w/o a jet definition.



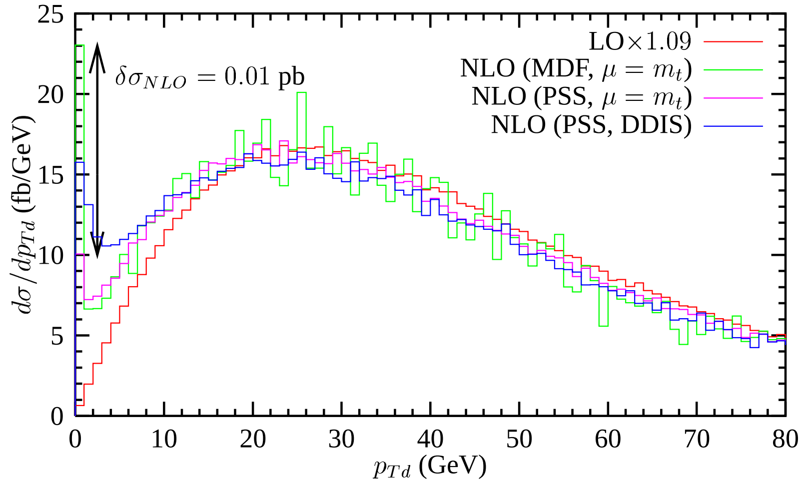
- “Bad things” happen if you treat jets as partons. . .

Transverse momenta distributions at NLO

At LO, a d-quark recoils against the top quark in t-channel.

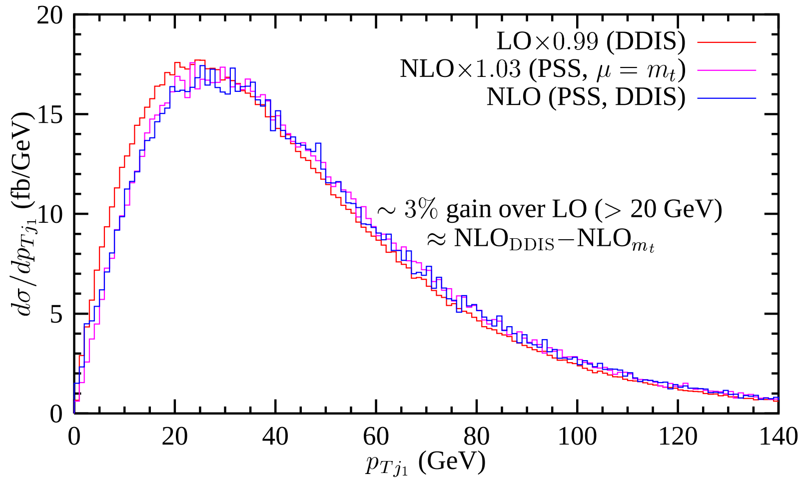


NLO "d-jet" (no cuts)



- Perturbation theory is not terribly stable at low p_{Td} (or even high p_{Td}).
- This is not what we want.
  Be careful what you ask for!

We measure the highest E_T jet

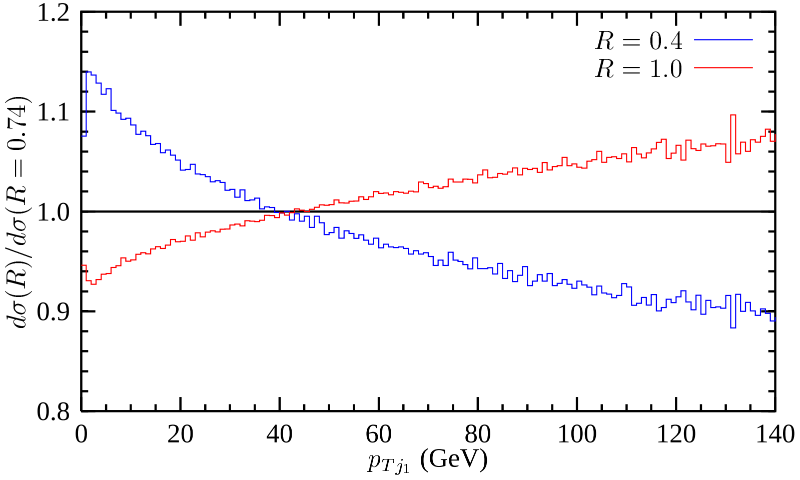


The highest E_T jet recoils against the top. The measurable change in shape is comparable to the scale uncertainty.

Jet distributions depend on jet definition

Just like the experimentalists, theorists must study the effect jet algorithms with different cone sizes R will have on measurable properties.

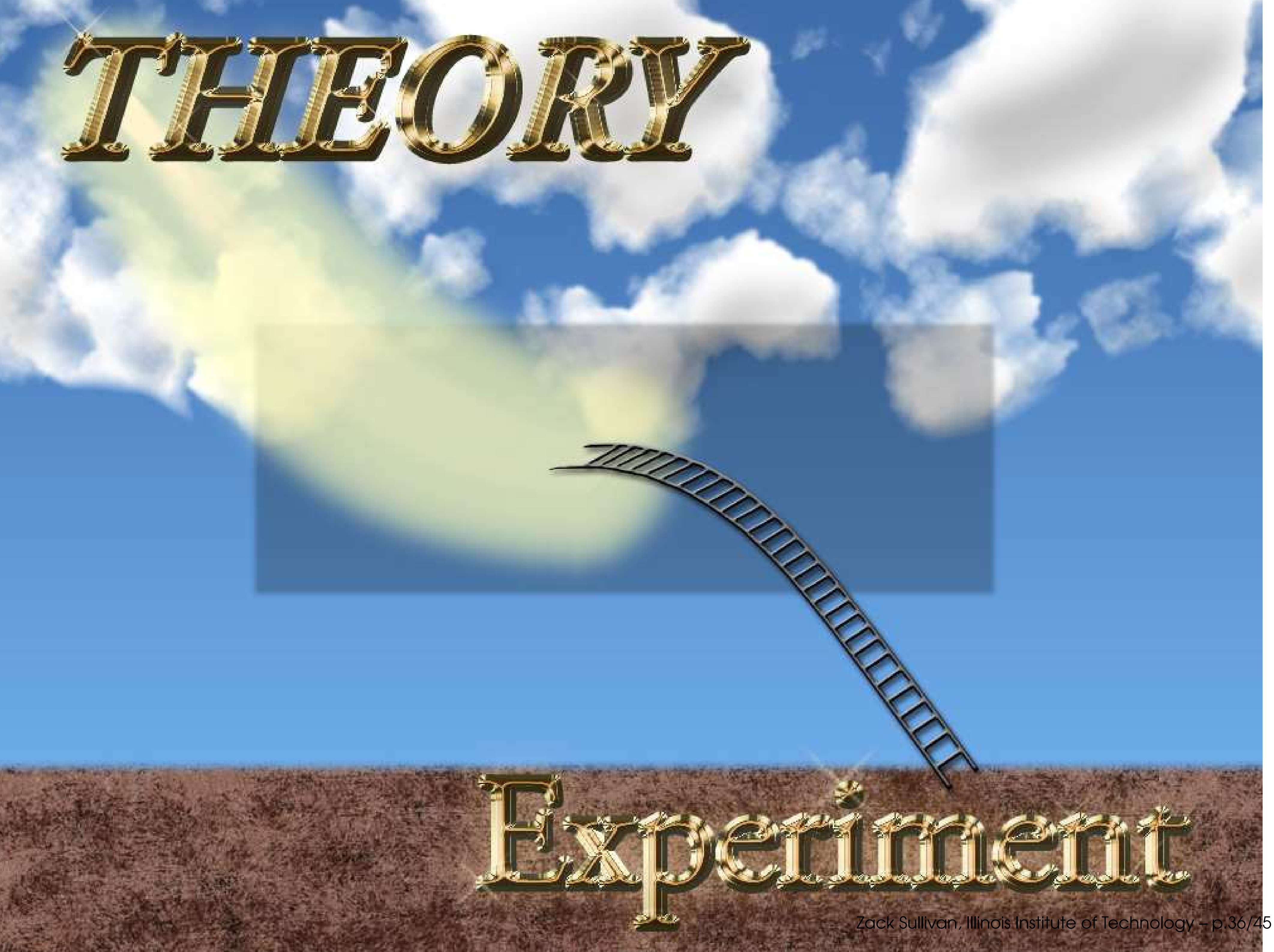
Ratio of $d\sigma(R)/dp_{Tj}$ to $d\sigma(R=0.74)/dp_{Tj}$



For "reasonable" values of R the variation is $< 10\%$, but this must be checked for all observables. (Note: theoretical uncertainty $< 5\%$)

Upshot: NLO exclusive calculations give jets not partons.

Without some thought, mismatches between theory and experiment can be larger than the theory error alone would indicate.

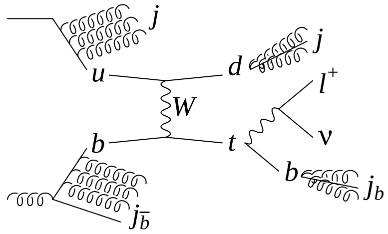

THEORY
Experiment

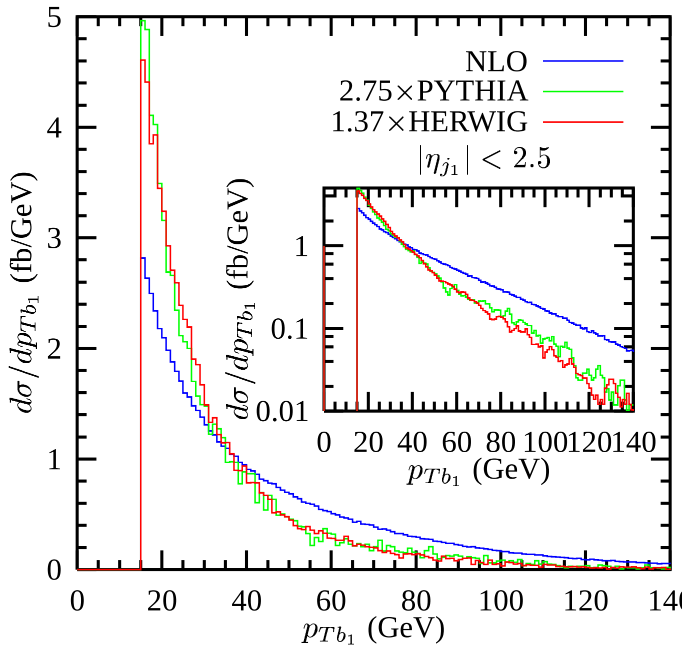
Event generators vs. NLO t-channel $t\bar{b}$ $(Wb\bar{b})$

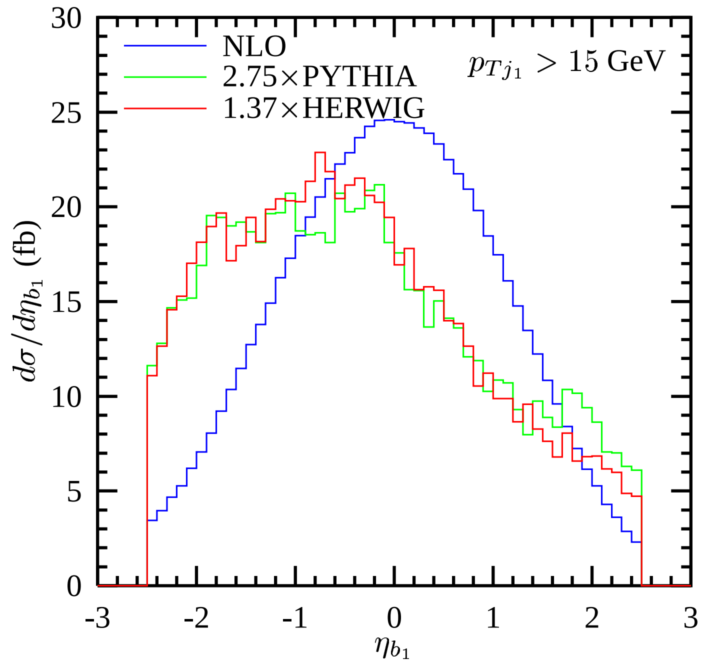
Z.S., PRD 70, 114012 (04)

Initial-state radiation (ISR) is generated by backward evolution of angular-ordered showers.

$\Rightarrow$ The jet containing the extra $\bar{b}$ comes from soft ISR.





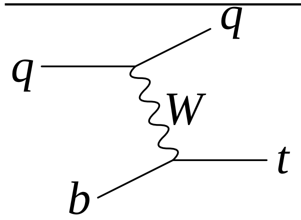
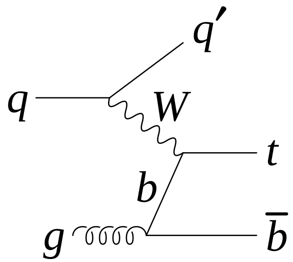


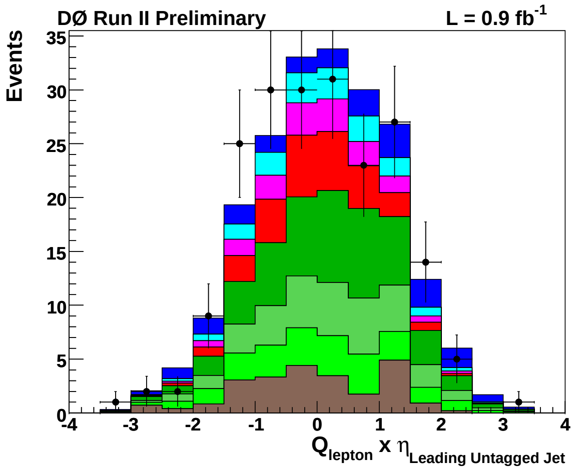
- PYTHIA/HERWIG completely underestimate the $Wb\bar{b}$ final state.
- The background to $WH \to Wb\bar{b}$ is much larger than we thought!
- Lesson: n-jets+showers $\neq n+1$ jets. $\Rightarrow$ Need NLO matching.

How do we solve this conundrum?
We return to the lessons of single-top. . .

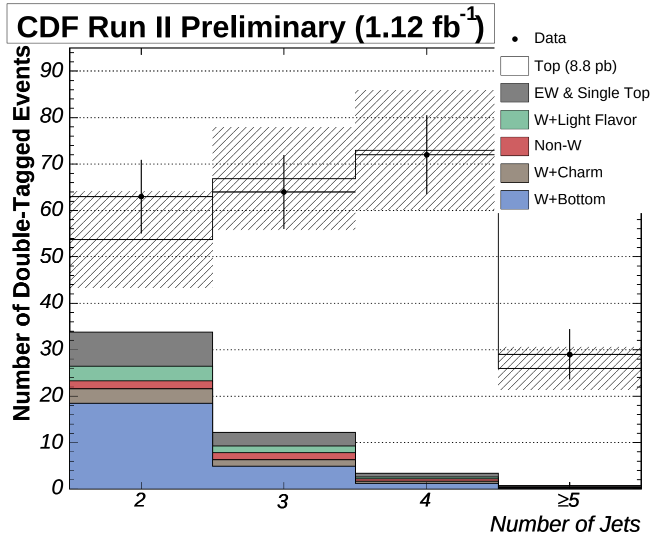
A simple prescription (variants used by CDF and DØ)

Classifying processes by analytical structure yields kinematic shapes:

You observe	You simulate
Wbj	
Wbb	



Normalize to the exclusive cross sections: Wbj, Wbb, $Wbjj$, $Wbbj$



This procedure is reproduces NLO by construction, but is tedious.

Several groups have begun to incorporate NLO matching into event generators (MLM, CKKW, MC@NLO)

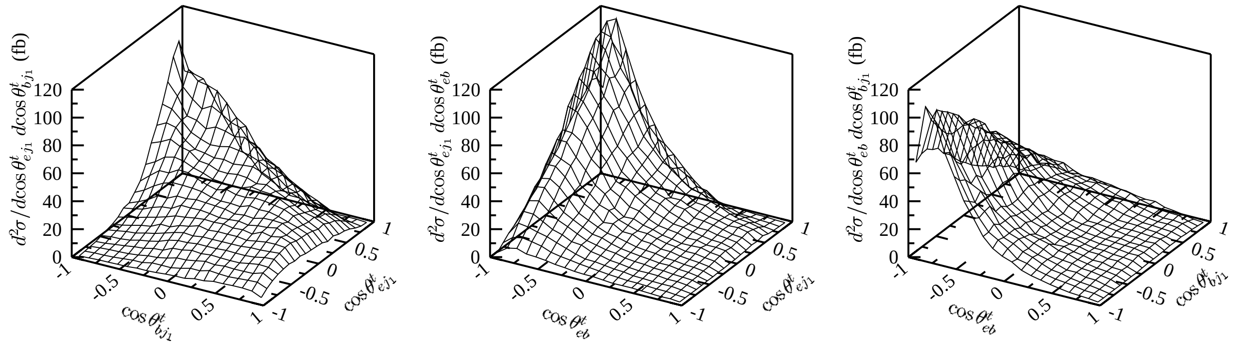
Significant work remains to verify these new methods.

What is the next step?

Because single-top-quark production is a weak-interaction process, there are strong angular correlations between <u>all</u> final state leptons and jets.

These correlations are enhanced by coupling between particle spins, momenta, and boosts given by the structure of the proton.

$\cos\theta^t_{ej_1}$ vs. $\cos\theta^t_{eb}$ vs. $\cos\theta^t_{bj_1}$



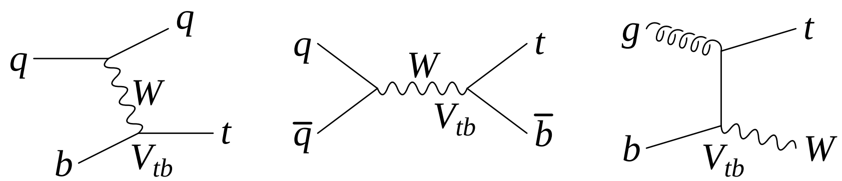
Use of these correlations will improve single-top-quark measurements.

At LHC, single-top-quark will be one of the largest backgrounds to supersymmetry, Higgs, W', etc.!

Strong angular cuts are typically required in these difficult analyses.

One theme that will dominate many LHC analyses is our understanding of angular correlations, and the stability of our theoretical predictions of measurable quantities.

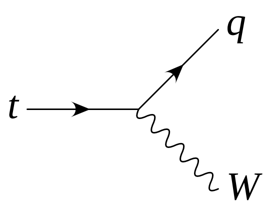
Beginnings of understanding



$\sigma_{\text{tot}} = 3.94 \pm 0.88$ pb (DØ)
2.3 ± 0.6 pb (CDF)
2.86 ± 0.24 (theory)

Single-top-quark production forces us to reconsider our intuitions and develop new technologies that push the frontiers of perturbative physics:

1. Understanding electroweak physics
   - We have a first measurement of weak interaction structure.



$$-i\frac{g}{\sqrt{2}}V_{tq}\gamma^{\mu}\frac{1}{2}(1-\gamma_5)$$

$V_{tb} = 1.07 \pm 0.12$ (DØ)
$0.91 \pm 0.11 \pm 0.07$ (CDF)

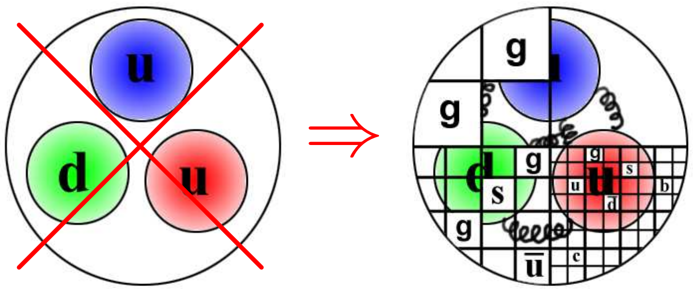
Angular correlations will play an important role in improving S/B

   - Anything that modifies the effective coupling of t to anything effects single-top
   - Any new charged current (W') is observable up to 5.5 TeV at LHC

Beginnings of understanding

2. Single-top-quark production is the new Drell-Yan and DIS

$$\sigma_{\rm obs.} = \int f_1(x_1,\mu_1) f_2(x_2,\mu_2) \otimes \overline{|M|^2} \otimes d\mathrm{P.S.} \otimes D_i(p_i) \ldots D_n(p_n)$$



— b/c are inside the proton

— Analytic structure gives direct kinematical insight

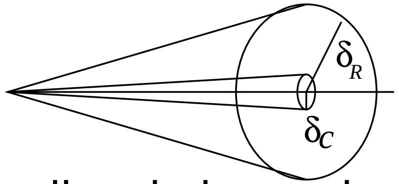
⇒ New processes & new questions:

- How do we reliably calculate $Zb+X$, $Zc+X$, $Wb+X$, etc.?
  Is jet counting well-defined at the LHC?
- There are 3 new mathematical techniques to calculate exclusive jet observables: MDF, PSS1, PSS2
- Single-top was the first process for which all theoretical uncertainties were studied:

  Heavy-quark parton distribution uncertainties
  Kinematic uncertainties and stability of an exclusive final states

Beginnings of understanding

3. Single-top-quark production has emboldened us to reexamine our interpretations of QCD



- The "paradigm of jet calculations"

  Exclusive NLO calculations intrinsically describe jets, not quarks. This introduces a new layer of subtlety when comparing experimental results to theoretical predictions.

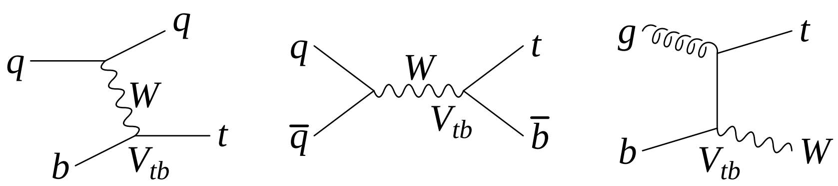
- There are now many matching schemes to improve the coupling of theory and experiment: MLM, CKKW, MC@NLO — significantly better understanding is needed

The story of single-top-quark production has been one of success engendered by a close interplay between theory and experiment.

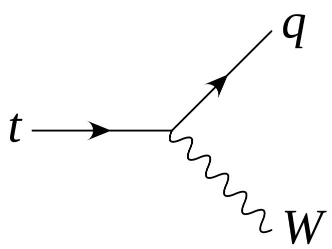
The next decade is your opportunity expand our understanding in a new age of precision quantum chromodynamics.

THANK YOU

Why we look at single-top-quark production



Weak interaction structure



$$-i\frac{g}{\sqrt{2}}V_{tq}\gamma^{\mu}\frac{1}{2}(1-\gamma_5)$$

Goal: Determine the structure of the W-t-q vertex.

- Measure CKM couplings "direct measurement of V_{tb}"
- Measure Lorentz structure "spin correlations"

Don't we know V_{tb}?

If we assume 3 generations, unitarity tells us $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9739 - 0.9751 & 0.221 - 0.227 & 0.0029 - 0.0045 \\ 0.221 - 0.227 & 0.9730 - 0.9744 & 0.039 - 0.044 \\ 0.0048 - 0.014 & 0.037 - 0.043 & 0.9990 - 0.9992 \end{pmatrix}$$

PDG, PLB 592, 1 (2004)

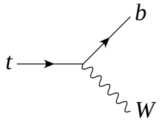
Relaxing the assumption of 3 generations, V_{tb} is barely constrained.

$$\Rightarrow \begin{pmatrix} 0.9730 - 0.9746 & 0.2174 - 0.2241 & 0.0030 - 0.0044 & \dots \\ 0.213 - 0.226 & 0.968 - 0.975 & 0.039 - 0.044 & \dots \\ 0 - 0.08 & 0 - 0.11 & 0.07 - 0.9993 & \dots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

Other new physics (e.g., SUSY) can also invalidate the indirect constraints

Any measurement of $V_{tb} \neq 0.9991$ is proof of new physics.

Measurement of V_{tb}

CDF/DØ look at t decays 

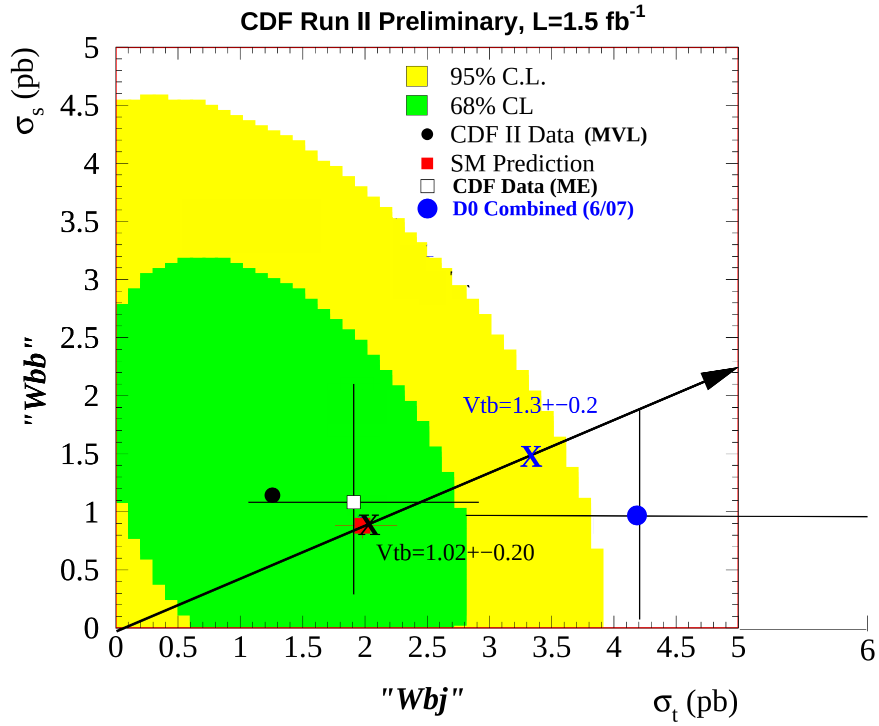
$$\frac{\mathrm{BR}(t\to Wb)}{\mathrm{BR}(t\to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2+|V_{ts}|^2+|V_{tb}|^2} = 0.97\pm0.09$$

$\Rightarrow |V_{tb}| > 0.89$ at 95% C.L.

DØ, ex0801.1326; CDF, PRL 95, 102002 (05)

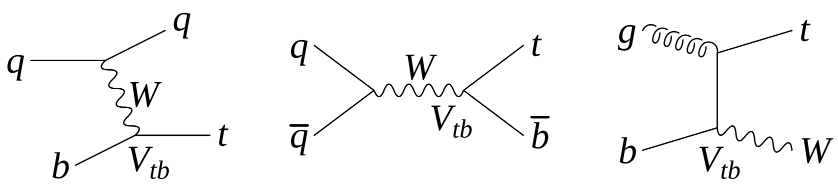
They assume unitarity to extract V_{tb}

You really need to measure the full and partial widths at a linear collider.



Single-top-quark production cross section is proportional to $|V_{tb}|^2$.

Measure $\mathrm{BR}(t\to Wb)$ in $t\bar{t}$, extract $|V_{tb}|$ from σ_t with an error $\sim\delta\sigma_t/2$.



ΔV_{tb} falls along the black line.

Winter 2009 results:

DØ: $V_{tb} = 1.07 \pm 0.12\ (s+t)$

CDF: $V_{tb} = 0.91 \pm 0.11 \pm 0.07\ (s+t)$
s only: $V_{tb} \approx 1.2$, t only: $V_{tb} \approx 1.0$

$\delta V_{tb} \sim 0.1$ is pushing theory error!