Standard Model and Beyond

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Tao is Awesome!

- Some of you may actually know what I am supposed to look like, or more likely, already know who Tao is.
- Some very unfortunate circumstances (all my fault) have prevented me from being here today.
- This is bad for me, but good for you - you get someone even better to present these lectures. Thanks very much to Tao for filling in!
- Complaints or comments about the slides should go to me - its all my fault!
- I should be available for at least one of the recitation sessions next week, and you can please feel free to abuse me then...

Tao has my express permission to make fun of me as much as possible during these lectures! I hope he won't disappoint either of us....

Outline of the Lectures

- Lecture I: Introduction to the Standard Model
  - Structure of the SM
  - Successes and Predictions
- Lecture II: Visions for the Physics Beyond
  - Shortcomings of the Standard Model
  - Some of our favorite ways to address them.

Please feel free to stop ~~me~~ Tao with questions at any time!

Structure of the Standard Model

The Standard Model

- The Standard Model is the theory of "almost everything"!
- Its successes are many, frankly too many for me to even come close to doing justice. I'll try to cover some that I don't think will be much touched upon by the other lectures.
- In thinking about new physics, the SM:
  - Defines the problems we need to solve;
  - its successes constrain the solutions to those problems;
  - ...and is the background we need to understand before we can make discoveries!

My Definition of the SM

- Before getting into details, let's try to be precise as to how we define the Standard Model:
  - Renormalizable Quantum Field Theory
  - Lorentz invariant, locally invariant under: SU(3) x SU(2) x U(1) gauge transformations
  - Three generations of matter (defined below)
  - One Higgs doublet
- This definition is the most standard one, but variations exist. Some people include neutrino masses, others may not include the Higgs.
  - This version allows for self-consistent calculations, whose only ambiguities are the tree level parameters.

Renormalizable

- To some extent, the requirement that the SM be renormalizable is the most arbitrary of the list on the previous slide.
- Renormalizable theories are predictive, because the UV infinities we encounter can be absorbed into a finite set of parameters.
  - So once we define the tree level parameters, all loops will do is complicate how they are related to observables.
- Naive power counting requires that the Lagrangian contain couplings with only positive (or zero) mass dimension.
  - In terms of the possible products of fields (which I often refer to as 'operators' and which also have to be Lorentz and gauge invariant), this restricts us to terms with mass dimension four or less.
    - Canonically normalized, bosons are mass dimension 1.
    - Fermions are mass dimension 3/2.
    - The coefficient of an operator (its coupling) has mass dimension d_O-4.

Example: $\underbrace{g}_{\text{coupling}} \times \underbrace{\phi\, \bar{\Psi}\, \Psi}_{\text{operator}}$ $\quad d_O = 1 + 3/2 + 3/2 = 4 \quad d_g = 4 - 4 = 0$

g is dimensionless!

Gauge Invariance

- The gauge symmetries represent a redundancy of description.
  - Physical (measurable) quantities are gauge invariant.
  - This provides a check on calculations!
- Gauge invariance is important:
  - No ghost polarizations of vector particles.
  - Renormalizable.
  - Universal couplings.

Gauge Transformations

- The gauge symmetries (that we know of) realized in nature are mathematically the groups SU(3), SU(2), and U(1).

- Gauge fields themselves transform by shifting:

$$T^a V_\mu^a(x) \to U(x) \left(T^a V_\mu^a(x) + \frac{i}{g}\partial_\mu \right) U^\dagger(x) \qquad U(x) \equiv \mathrm{Exp}\ \left[i\alpha^a(x) T^a \right]$$

  - The T^a are the generators of the fundamental representation, and g is the (real) gauge coupling. The α define the transformation at every point x.

- The shift is a terrible nuisance in terms of building gauge invariant terms for a Lagrangian. It is much easier to deal with the (anti-symmetric) field strengths:

$$F_{\mu\nu}^c(x) \equiv \partial_\mu V_\nu^c(x) - \partial_\nu V_\mu^c(x) - igf^{abc} V_\mu^a(x) V_\nu^b(x) \qquad [T^a, T^b] = if^{abc}T^c$$

- Whose transformation properties can be simply expressed:

$$T^a F_{\mu\nu}^a(x) \to U(x) T^a F_{\mu\nu}^a(x) U^\dagger(x)$$

- Note that the field strength for a U(1) gauge field is itself gauge invariant.

Gauge Kinetic Terms

- In order for our vectors to propagate, we need to give them gauge invariant kinetic terms. This is easy to do using the field strengths:

$$\mathrm{Tr}\ \left[T^a F^a_{\mu\nu} T^b F^{b\ \mu\nu}\right] \to \mathrm{Tr}\ \left[U T^a F^a_{\mu\nu} U^\dagger U T^b F^{b\ \mu\nu} U^\dagger\right] = \mathrm{Tr}\ \left[T^a F^a_{\mu\nu} T^b F^{b\ \mu\nu}\right]$$

- The SM gauge kinetic terms are just the sum of one for each symmetry group:

$$-\frac{1}{4} G^a_{\mu\nu} G^{a\ \mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\ \mu\nu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

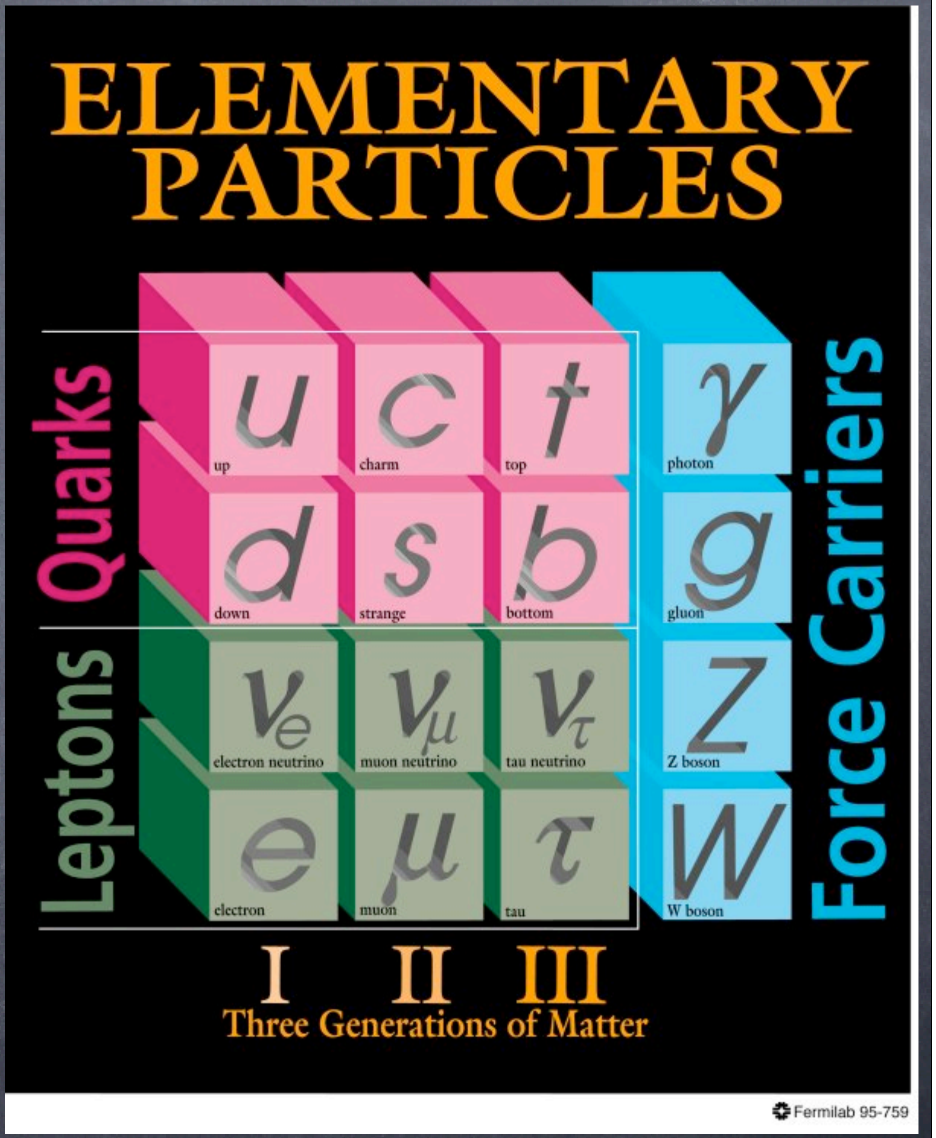
- These terms contain the familiar propagators (once we specify the gauge - which I am not going to do here). For non-Abelian groups, they also contain the three- and four-point interactions of the gauge fields among themselves.

- Note that the gauge invariance didn't make use of the Lorentz structure. So we can build related structures that are not Lorentz scalars, but are still gauge invariant:

examples: $F^a_{\mu\nu} F^a_{\alpha\beta}$ $\qquad$ $\epsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta} \equiv F^a_{\mu\nu} \widetilde{F}^{a\mu\nu}$

Matter

- The basic building block of matter in the SM is a 2-component (Weyl) fermion. Anticipating masses, we can further divide these into left- and right-handed fermions.
- One generation consists of:
  - a quark doublet (Q).
  - an up-type quark singlet (u).
  - a down-type quark singlet (d).
  - a lepton doublet (L).
  - a lepton singlet (e).
- Gauge interactions don't mix these, so who goes in which generation at this stage is a matter of convention.
- Gauge invariance itself doesn't tell us which representations to choose (well, more on this below).



Fermion Representations

Field \ Group	SU(3)	SU(2)	$U(1)_Y$
Q	3	2	+1/6
u	3	1	+2/3
d	3	1	-1/3
L	1	2	-1/2
e	1	1	-1

Quarks: Q, u, d

Leptons: L, e

Kinetic Terms

- The fermions transform either as singlets or fundamental representations of SU(3)xSU(2).

- Apparently nature isn't very sophisticated with group theory...

$$\psi \rightarrow \mathrm{Exp}\left[i\alpha^a(x)T^a_\psi\right]\,\mathrm{Exp}\left[i\alpha^i(x)t^i_\psi\right]\,\mathrm{Exp}\left[i\alpha(x)Y_\psi\right]\,\psi$$

  - T_ψ, t_ψ, Y_ψ are the generators of SU(3), SU(2), and U(1) for the rep of ψ.

- Since there are no gauge-invariant bilinears, fermion masses will have to wait for the Higgs mechanism.

- Derivative terms transform in a complicated way, because the derivatives can hit the transformation parameters. Once again, we can construct simple building blocks in the form of covariant derivatives:

$$D_\mu\psi \equiv \left[\partial_\mu\psi - ig_3 G^a_\mu T^a_\psi - ig_2 W^i_\mu t^a_\psi - ig_1 B_\mu Y_\psi\right]\psi \qquad D_\mu\psi \rightarrow U(x) D_\mu\psi$$

- In terms of the covariant derivatives, kinetic terms imply interactions:

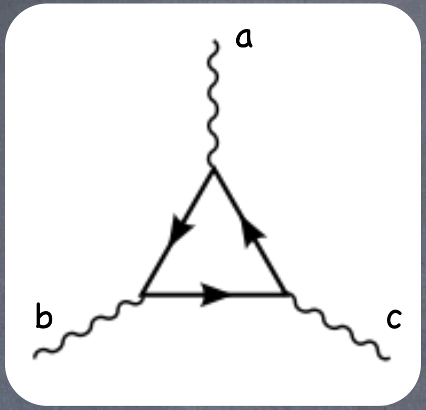
$$\bar{\psi}\gamma^\mu D_\mu\psi \rightarrow \bar{\psi}\gamma^\mu D_\mu\psi$$

Gauge Anomalies

- Gauge theories with chiral fermions are not automatically consistent - anomalies may spoil the gauge symmetries, rendering the theory inconsistent.

- The SM has potential for $SU(3)^3$, $SU(2)^3$, $U(1)^3$, $SU(3)^2$-$U(1)$, and $SU(2)^2$-$U(1)$ anomalies. The other combinations involve a single insertion of an SU(N) group, which vanishes since the generators of SU(N) are traceless.

- The cancellation of anomalies occurs non-trivially inside a single generation. This suggests that one SM generation is the "basic chiral unit".



$$\propto \sum_{\psi} \pm \mathrm{Tr}\left[T^a_\psi T^c_\psi T^b_\psi\right]$$

-/+: left/right-handed

Anomalies for **one** Generation

$SU(3)^3$	: 2 RH - 2 LH quarks = 0
$SU(2)^3$	: $\mathrm{Tr}[t^i\, t^j\, t^k] = 0$ (*)
$U(1)^3$	: $-6(1/6)^3+3(2/3)^3+3(-1/3)^3-2(-1/2)^3+(-1)^3 = 0$
$U(1)$-$SU(3)^2$	: $\delta^{ab}\,(-2\mathrm{x}1/6 + 2/3 - 1/3\,) = 0$
$U(1)$-$SU(2)^2$	: $\delta^{ij}\,(-3\mathrm{x}1/6 + 1/2\,) = 0$
$U(1)$-GR^2	: $-6(1/6)+3(2/3)+3(-1/3)-2(-1/2)+(-1) = 0$

We also have compatibility between U(1) and general covariance.

Symmetry Breaking

- To make contact with the real world, we need to break the electroweak symmetry.
  - So far everything is massless.
  - (Left-handed) electrons have exactly the same physics as neutrinos!
- To preserve the essential features of gauge invariance, we break the symmetry spontaneously with a single Higgs doublet.

Field \ Group	SU(3)	SU(2)	$U(1)_Y$
Φ	1	2	+1/2

- Again, having specified its representations, its gauge interactions are fixed by its kinetic term:

$$\left(D_\mu \Phi\right)^\dagger D^\mu \Phi$$

Spontaneous Symmetry Breaking

- The way to break the symmetry just enough to give masses to the particles without spoiling all the things we need the symmetry to do is to have the vacuum break the symmetry.

- An analogy is: 3d space is rotationally invariant. But a given configuration (say space with a table in it) may not be. The physics that gives rise to the existence of tables is invariant but nonetheless allows the existence of solutions like tables which don't themselves realize the symmetries.

  - So the laws of physics remain invariant under the symmetry, but the non-symmetric theatre nonetheless profoundly influences what happens there!

- The Higgs doublet carries electroweak charge. By giving it an expectation in the vacuum, we "fill the vacuum" with weak charge, spontaneously breaking the symmetry.

- As a scalar, the Higgs VEV preserves Lorentz invariance.

  - We have learned how to construct a relativistic form of ether...

Gauge Boson Masses

- Let's put aside for one moment the question as to why the Higgs should have a VEV in the first place. (We'll get there in a few slides).
- Instead, let's look at the consequences for the gauge bosons. The VEV ($v/\sqrt{2}$), when inserted into the covariant derivative, leads to mass terms for the gauge bosons.

$$(D_\mu \Phi)^\dagger D^\mu \Phi \to g_2^2 \begin{bmatrix} 0 & \frac{v}{\sqrt{2}} \end{bmatrix} t^i W_\mu^i t^j W^{j\mu} \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix} + g_1^2 v^2 B_\mu B^\mu$$

  - A combination of W_1 and W_2 gains a mass as a complex field:

$$\frac{g_2^2 v^2}{2} \left(W_1^\mu - i W_2^\mu \right) \left(W_m^1 u + i W_\mu^2 \right) \equiv M_W^2 W_+^\mu W_\mu^- \qquad M_W \sim 80 \text{ GeV}$$

  - Knowing g_2 we use the measured M_W to fix v = 246 GeV.
  - The VEV also mixes W_3 and B:

$$\frac{(g_1^2 + g_2^2) v^2}{2} \begin{bmatrix} B_\mu & W_\mu^3 \end{bmatrix} \begin{bmatrix} \frac{g_1^2}{g_1^2+g_2^2} & \frac{g_1 g_2}{g_1^2+g_2^2} \\ \frac{g_1 g_2}{g_1^2+g_2^2} & \frac{g_2^2}{g_1^2+g_2^2} \end{bmatrix} \begin{bmatrix} B^\mu \\ W_3^\mu \end{bmatrix}$$

Weak Mixing Angle

- The easy way to find the mass eigenstates is to define:

$$\sin\theta_W \equiv \frac{g_2}{\sqrt{g_1^2+g_2^2}} \sim \frac{1}{2} \qquad \cos\theta_W \equiv \frac{g_1}{\sqrt{g_1^2+g_2^2}} \sim \frac{\sqrt{3}}{2} \qquad e \equiv \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}} \sim 0.3$$

- Couplings e and sin θ are just another way to parameterize g_1 and g_2:

$$g_1 \equiv \frac{e}{\cos\theta_W} \qquad\qquad g_2 \equiv \frac{e}{\sin\theta_W}$$

- Now the matrix becomes,

$$\frac{e^2 v^2}{4\sin^2\theta_W\cos^2\theta_W}\begin{bmatrix} B_\mu & W_\mu^3 \end{bmatrix}\begin{bmatrix} \cos^2\theta_W & \sin\theta_W\cos\theta_W \\ \sin\theta_W\cos\theta_W & \sin^2\theta_W \end{bmatrix}\begin{bmatrix} B^\mu \\ W_3^\mu \end{bmatrix}$$

- With one zero eigenvalue (the photon) and one massive eigenstate (Z boson),

$$M_Z^2 \equiv \frac{e^2 v^2}{2\sin^2\theta_W\cos^2\theta_W} = \frac{M_W^2}{\cos^2\theta_W} \sim (90\ \mathrm{GeV})^2$$

- We can scale the W and Z masses together by adjusting v, but the (tree level) relationship between them is fixed by our choice of Higgs representations under SU(2) x U(1).

Neutral Currents

- The single massless photon still has a residual U(1) gauge symmetry. So we say the symmetry breaking has taken: $SU(2) \times U(1)_Y \to U(1)_{EM}$.
- θW specifies the orthogonal transformation to get to the mass eigenstates,

$$\begin{bmatrix} A^\mu \\ Z^\mu \end{bmatrix} = \begin{bmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{bmatrix} \begin{bmatrix} B^\mu \\ W_3^\mu \end{bmatrix}$$

- We can derive the couplings of the Z and γ fields to any particle as a function of their original SU(2) t^3 and hypercharge assignments:

$$g_1 Y B^\mu + g_2 t^3 W_3^\mu \quad \to \quad \frac{e}{\sin\theta_W \cos\theta_W}\left[t^3 - \sin^2\theta_W (t^3 + Y)\right] Z^\mu$$
$$+e\left[t^3 + Y\right] A^\mu$$

- From here we can derive the electric charges for our fermions:

Field	Q	u	d	L	e
Charge Q	$+2/3$ $-1/3$	$+2/3$	$-1/3$	0 -1	-1

Higgs Potential

- The Standard Model does better than just assign a VEV to the Higgs. It can actually generate one dynamically.

- In fact, under the rules we agreed to at the beginning, once we added the Higgs, we should have written down all of the Lorentz/gauge-invariant and renormalizable terms in the Lagrangian.

- The first class involve just the Higgs itself, and is called the Higgs potential:

$$-\mu^2 |\Phi|^2 - \frac{\lambda}{4} |\Phi|^4$$

- When the parameter $\mu^2 < 0$, the energy is minimized for a constant non-zero value of the Higgs field given by,

$$\Phi = \left[\begin{array}{c} \phi^+ \\ \frac{1}{\sqrt{2}} \left(\phi_0 + i \phi_3 \right) \end{array} \right] \qquad 2\phi^+\phi^- + \phi_0^2 + \phi_3^2 = 2v^2$$

- The original gauge symmetry allows me to put the VEV in any component I want to. My choice to put it in φ_0 is just a convention (though one that dictated how I assigned electric charges, so I actually made it implicitly earlier.

Higgs Mechanism

- The Higgs doublet originally contained four real fields. $\Phi = \begin{bmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left(\phi_0 + i\phi_3\right) \end{bmatrix}$
- Three of them would have been Goldstone bosons if the SU(2)xU(1) had been a global symmetry. Since it was a local symmetry, they are often called "would be"-Goldstone bosons.
  - They actually disappear as real degrees of freedom, appearing instead as the longitudinal degrees of freedom for the massive W and Z bosons.
  - I can "gauge them away".
  - In fact, in a general gauge they still appear. The gauge in which only physical particles appear is the "unitary" gauge.
- One real scalar remains after the symmetry-breaking and is physical. This is the infamous Higgs boson of the Standard Model. In terms of the parameters in the Higgs potential, its mass (and self-interactions) is given by:

$$M_h^2 = \lambda v^2$$

- So by adjusting λ, we can arrange for any Higgs mass that we want.

Yukawa Interactions

- There are also many gauge invariant operators we can construct linking the Higgs to our fermions:

$$\Phi\left(Y^d_{ij}\bar{Q}_i d_j + Y^e_{ij}\bar{L}_i e_j\right) + (i\sigma_2\Phi^*)Y^u_{ij}\bar{Q}_i u_j + H.c.$$

  - $(i\sigma_2\, \Phi^*)$ is just a way to write Φ^* as a 2 as opposed to a $\bar{2}$ of SU(2).

- Since nothing tells us how to put together the different fields, each set of couplings Y is actually a 3x3 matrix in flavor space.

- Replacing the Higgs by its VEV produces mass terms for up quarks, down quarks, and charged leptons (also 3x3 matrices, proportional to the Y's).

  - No neutrino masses are possible within the SM!

- To get to the mass basis, we apply unitary chiral transformations on our fields. Treating u, d, and e as 3 component vectors in family space:

$$u_L \to L_u u_L \quad u_R \to R_u u_R \qquad\qquad e_L \to L_e e_L \quad e_R \to R_e e_R$$

(and so on for d_L and d_R and ν_L) The L's and R's are 3x3 unitary matrices.

Fermion Masses

- We choose the rotations to diagonalize the fermion masses.

for example:

$$L_u \ (vY^u) \ R_u^\dagger \to D^u \equiv \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{bmatrix}$$

- D is the diagonal matrix of eigenvalues.

- Technical aside: for a fixed matrix Y we can determine L and R and the masses by solving the ordinary eigenvalue equation for Hermitean matrices:

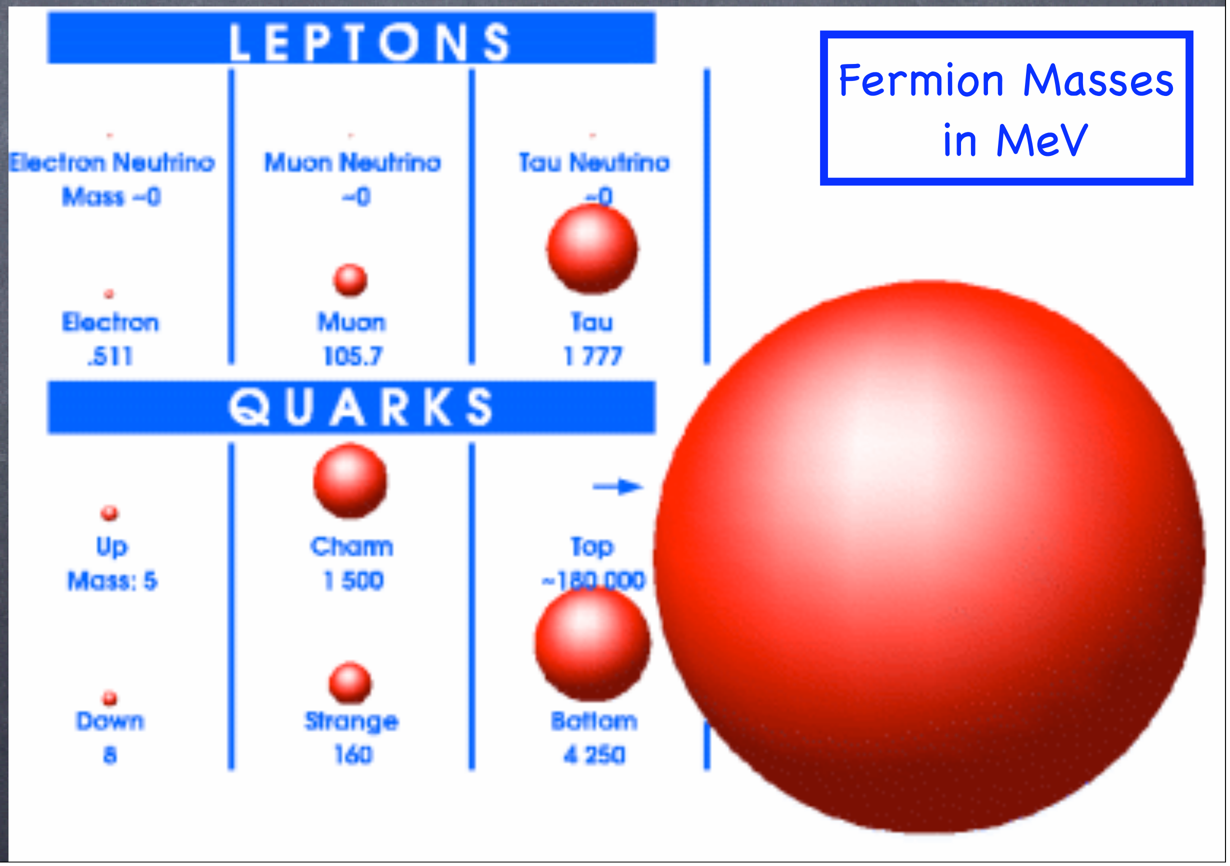
$$L_u Y Y^\dagger L_u^\dagger = D_u^2 \qquad\qquad R_u Y^\dagger Y R_u^\dagger = D_u^2$$

- Having diagonalized the masses, the physics is parameterized by the mass of each fermion, and the mixing matrices that were needed.

- Since the original (undiagonal) mass matrices were proportional to the Y matrices, this automatically diagonalizes the couplings to the Higgs as well.

Fermion Spectroscopy

- By measuring the fermion masses, we are thus fixing (at least at tree level) the Yukawa couplings we need in the mass basis.
- The SM doesn't explain why we observe the masses that we do, but it can describe them.
- Masses above about a TeV would stop making sense, because the Yukawa coupling needed would be so strong that the theory would not make sense in perturbation theory.
  - But no worries...
- Yukawa couplings range from ~10^{-5} (electron) to about ~1 (top quark).



No Flavor-Changing Neutral Currents!

(at tree Level)

- We should also work out what the rotations in family space do to the gauge interactions as well. In the case of the neutral currents, gauge invariance has already guaranteed that in the original basis, the interactions were universal.

for example: $$\bar{u}_L \not{Z} u_L \rightarrow \bar{u}_L L_u^\dagger L_u \not{Z} u_L = \bar{u}_L \not{Z} u_L$$

- Since gauge interactions don't mix left- and right-chiral fermions, the rotations are always compensated, and disappear from the interactions. This happens separately for each of the quarks and leptons.

- As a result, the Z, γ, and gluon interactions remain diagonal in the mass basis, just as they were in the flavor basis.

- So at tree level, none of the neutral gauge bosons can change the flavor of a quark or lepton. This is a striking result of the structure of the SM.

CKM Matrix

- The charged currents are more interesting, because while they are still universal, they involve an up-type quark and a down-type quark.

- Since up- and down-type quarks have different rotations, this leaves behind a physical effect in the couplings of the W boson:

$$\bar{d}\, \not{W}^- u + H.c \rightarrow \bar{d} L_d^\dagger L_u\, \not{W}^- u + H.c \equiv \bar{d} V\, \not{W}^- u + H.c$$

- The combination $V = (L^\dagger_d L_u)$ has physical consequences, and is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

- Something similar would have happened for the leptons too, but in that case since there are no neutrino masses to screw up (in the SM), I can just cancel the L_e rotation with an L_ν rotation, which otherwise has no effect.

  - This stops working when we have neutrino masses, but we often to use this basis anyway and leave the neutrino masses undiagonalized.

The Parameters of Flavor

- So out of the many parameters in the rotation matrices, only one 3x3 unitary matrix actually had any physical consequences at the end of the day.
- The CKM matrix naively is parameterized by 8 real parameters. By rephasing the quarks I can reduce this to 4 real mixing angles and one complex phase.
- The "Standard parameterization" uses the 3 Euler angles θ_{12} θ_{23} θ_{13} and one complex phase δ to specify the matrix:

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}.$$

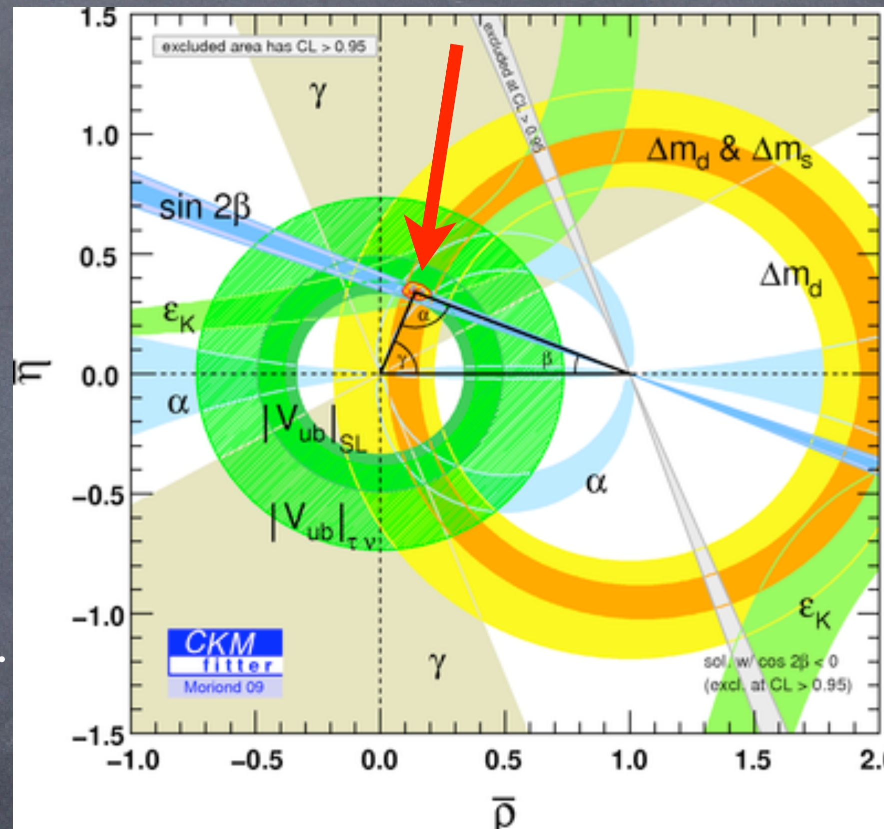
- Measurements from flavor physics and the unitarity of the matrix itself allow us to measure these parameters:

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{bmatrix}.$$

Measurements of CKM

- One also often sees the Wolfenstein parameterization:

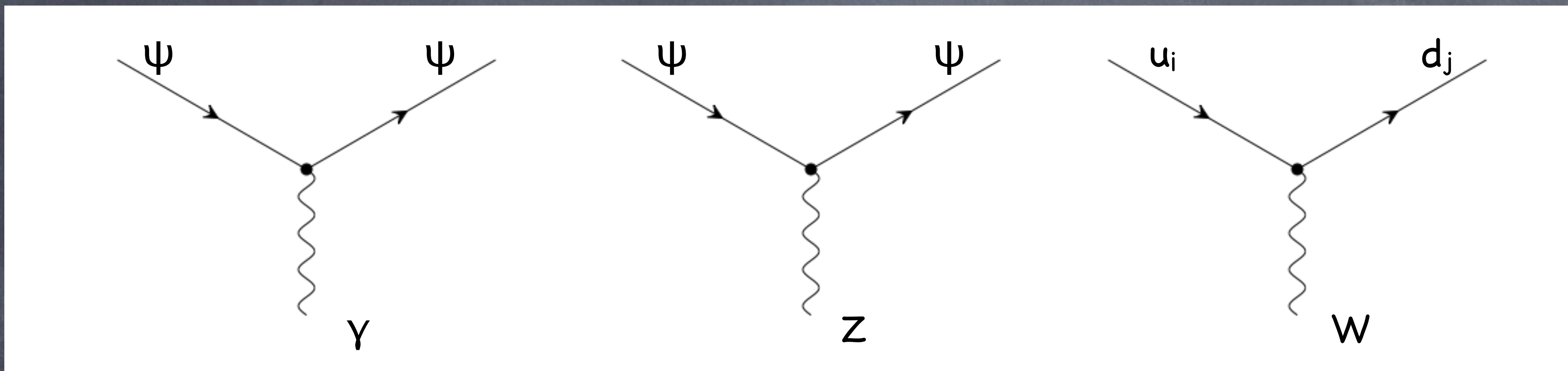
$$\begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}.$$



- At this point, there are many precise measurements, and global fits to several observables provide the clearest picture.

- While there are some anomalies which persist at the few σ level, so far the SM has explained all measurements, leading to a non-trivial test of its description of flavor and CP violation.

Feynman Rules for Fermions

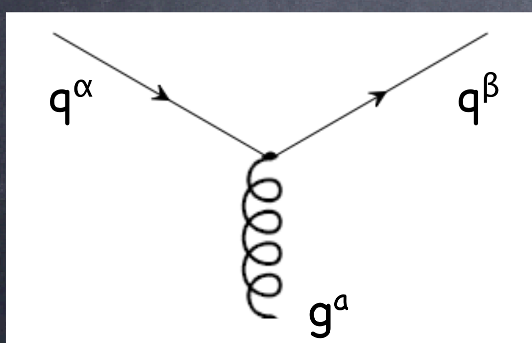


$$-ieQ_\psi \gamma^\mu$$

$$-i\frac{e}{\sin\theta\cos\theta}\gamma^\mu \times \left[\left(t^3_\psi - Q_\psi \sin^2\theta\right) P_L - Q_\psi \sin^2\theta P_R\right]$$

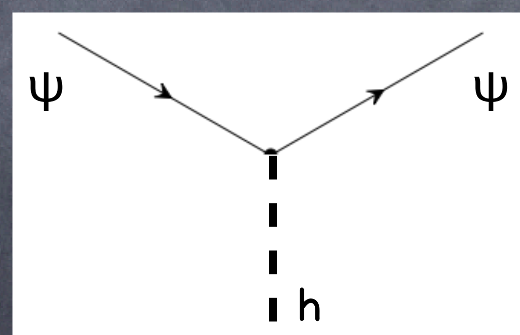
$$-i\frac{e}{\sin\theta} V_{ij}\gamma^\mu P_L$$

i and j are flavors



$$-ig_3 T^a_{\beta\alpha}\gamma^\mu$$

a, α, and β are colors



$$-i\left(\frac{m_\psi}{v}\right)$$

Physics is fun!

Recap of Parameters

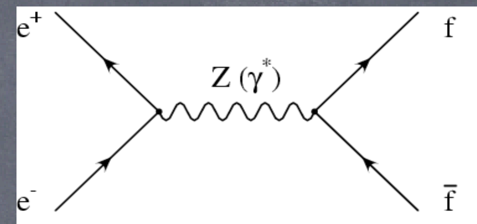
- Let's have a recap of the parameters we have introduced so far:
  - Three gauge couplings which we rewrote after EWSB as: e, $\sin\theta$, g_3.
    - They determine gauge boson self-interactions and interactions with the fermions and Higgs.
  - Two Higgs potential parameters which we expressed as v and M_h.
  - Six quark masses.
  - Three charged lepton masses.
  - Three CKM angles and one CP-violating phase.
- For a grand total of 18 parameters.
  - (I cheated slightly here, but if there is time, I will 'come clean' briefly in the next session).

(A few) Successes of the Standard Model

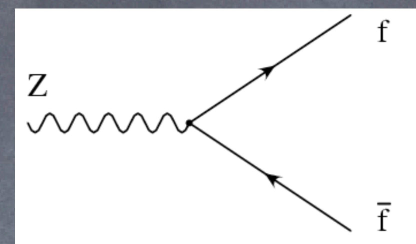
These are very very incomplete, and mostly have been chosen because they help illustrate results in the second part of the lecture...

Electroweak Fit

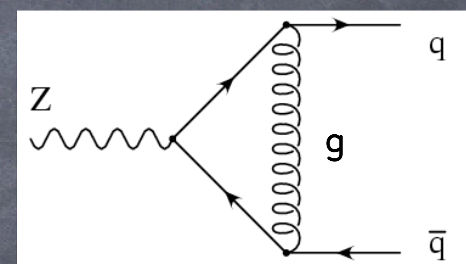
- Precision measurements of the Z boson have provided some of the most stringent tests of the SM.



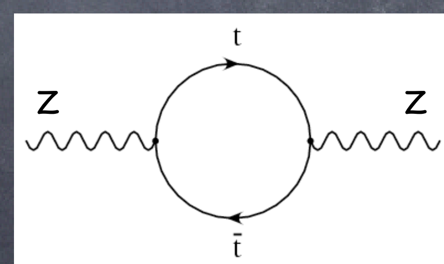
- The LEP and SLC experiments have made measurements of Z boson couplings at the per mil level.

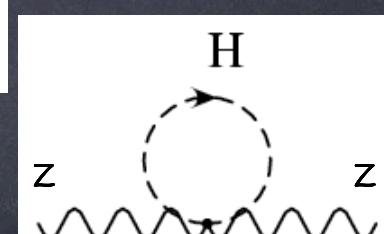


- We've already seen at tree level that such measurements constrain e, sin θ and v.

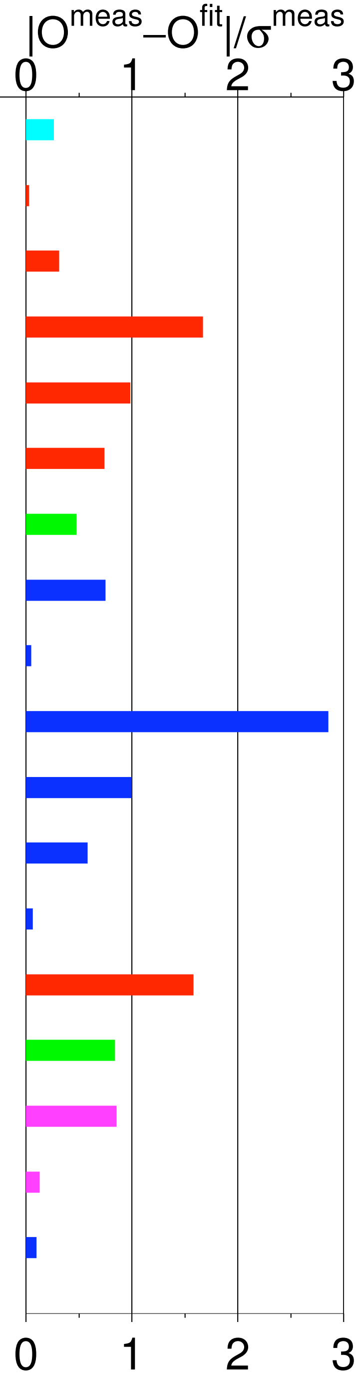


- Such precision is enough that one-loop corrections contribute to the theory predictions.
    - In the SM, this implies dependence on m_t, α_S, and m_H.
    - We can learn about the Higgs even if it is too heavy to produce!





	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^{0} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.399 ± 0.025	80.378
Γ_W [GeV]	2.098 ± 0.048	2.092
m_t [GeV]	173.1 ± 1.3	173.2

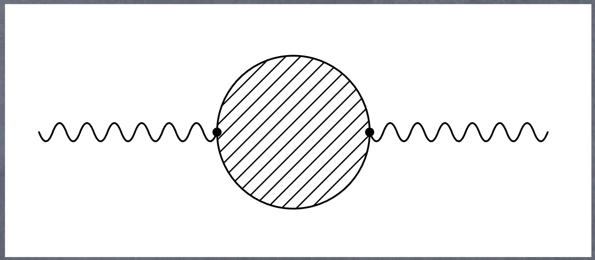


March 2009

Fantastic accuracy, and fantastic agreement with the Standard Model.

Oblique Corrections

- We can characterize the effects of some physics on the properties of the W and Z bosons as oblique or non-oblique.
- Oblique corrections are independent of the fermion species, and can be written as modifications of the propagators.
- Because the Higgs couples very weakly to light fermions, to good approximation it is oblique.
- Top is slightly less so, because it corrects the Z-b-b vertex at one loop. But still to good approximation, it contributes obliquely.
- New physics, if it contributes obliquely, can also be parameterized by the oblique parameters.



Peskin-Takeuchi / Altarelli Parameters

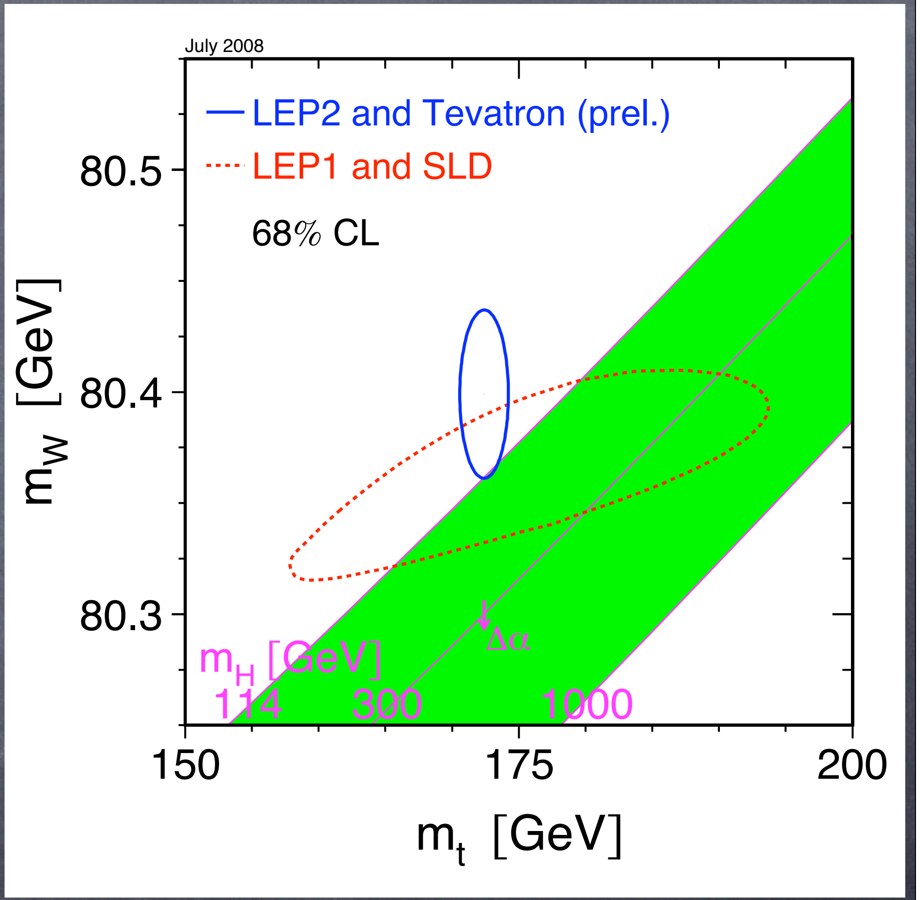
ε_2 or S: Measure of the correction to the Z couplings (on-shell). In practice, measures the amount of chiral matter.

ε_1 or T: Measure of the difference between W and Z propagators at zero momentum.

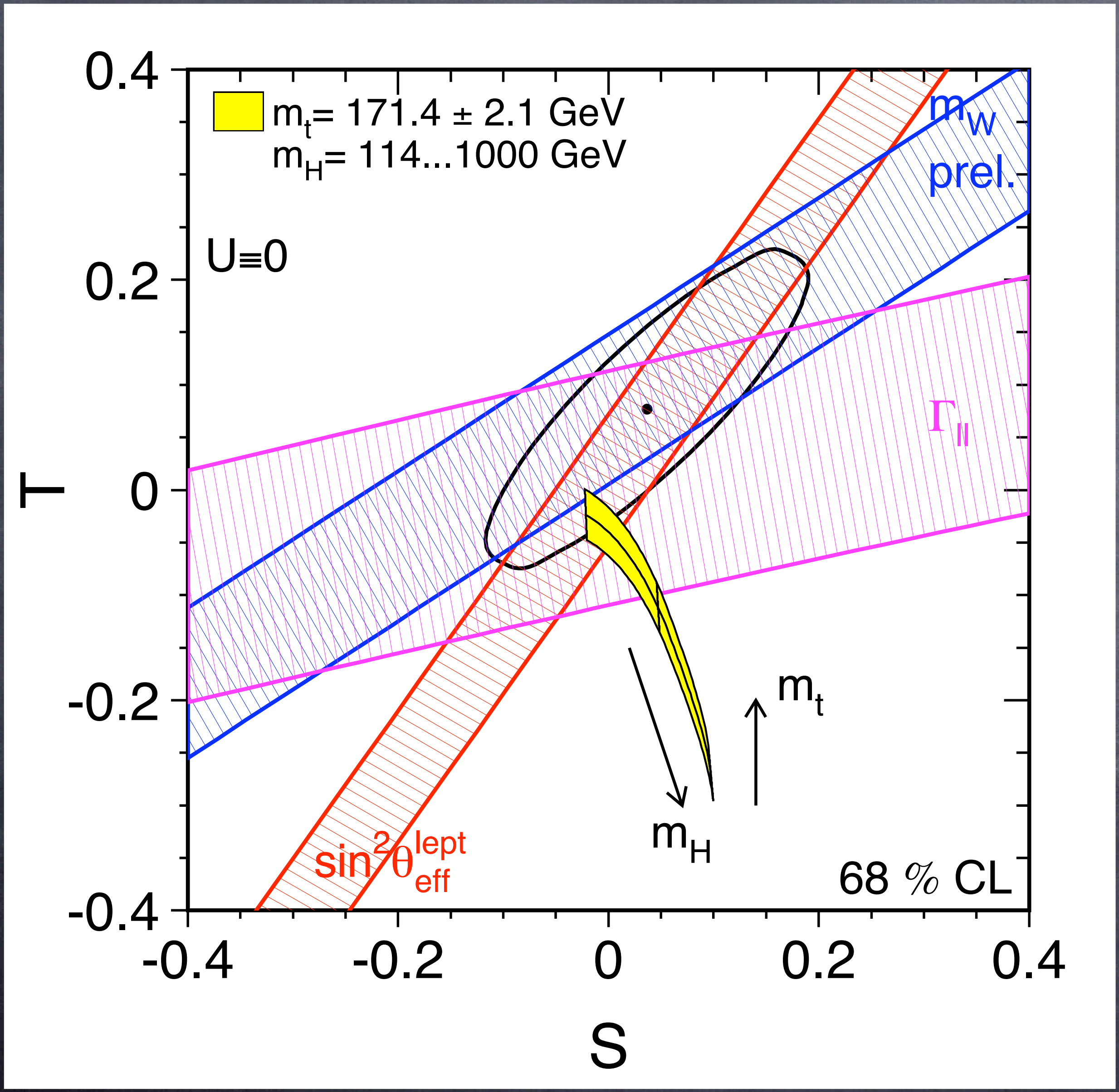
ε_3 or U: Measure of the difference between the W and Z couplings on shell. In practice less important than S and T.

Top Mass in the SM

- In the Standard Model, we can compute S and T (and U) at one loop, and see the dependence on the Higgs and top masses.
- The correction to T grows quadratically in m_t and like the log of m_h.
    - The fit is VERY sensitive to m_t!
- Corrections to S are as a log of both parameters.
- LEP/SLC data was precise enough that the value of m_t could be inferred before the top quark was actually discovered at the Tevatron!
- However, the true power of the fit came about when m_t was indepdently measured - at that point, the fit starts telling us about m_h...



Fit to S and T



In the SM, the fit to S and T make it clear how the fit to the Higgs mass depend crucially on the value of the top mass.

In a theory of physics beyond the SM, the fit to S and T shows how new physics may change the preferred range of the Higgs masses.

EW Fit: Higgs Mass

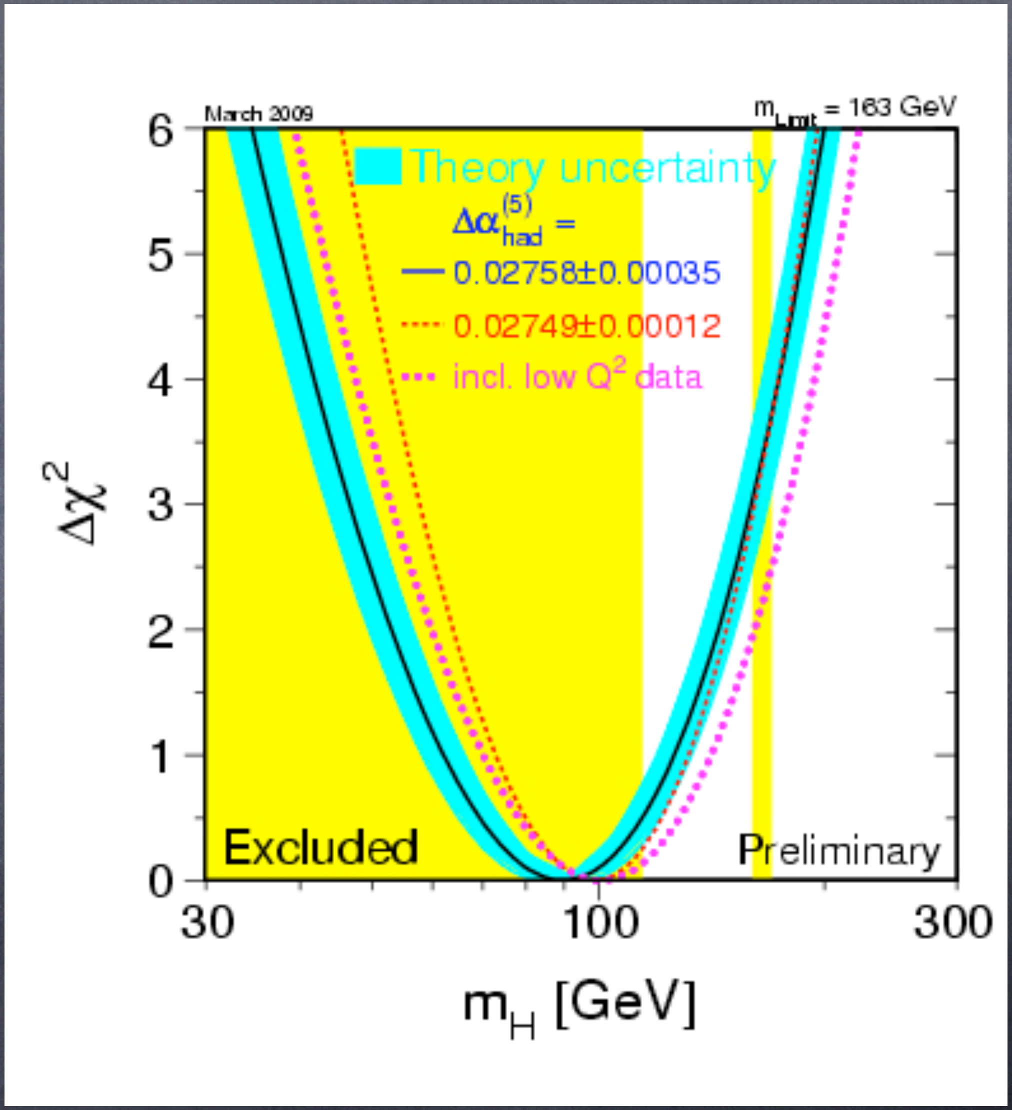
Within the SM, and using the top mass from the Tevatron, the fit becomes a prediction for the Higgs mass of the SM.

We hope to someday discover the Higgs, and if its mass falls within the predicted range, the fit is restricting how new physics can contribute to S & T.

If the mass falls outside the preferred range, S & T are telling us to look for new physics!

Yellow regions are excluded by direct searches for the Higgs.

Errors on the top mass control the width of the $\Delta\chi^2$ distribution.

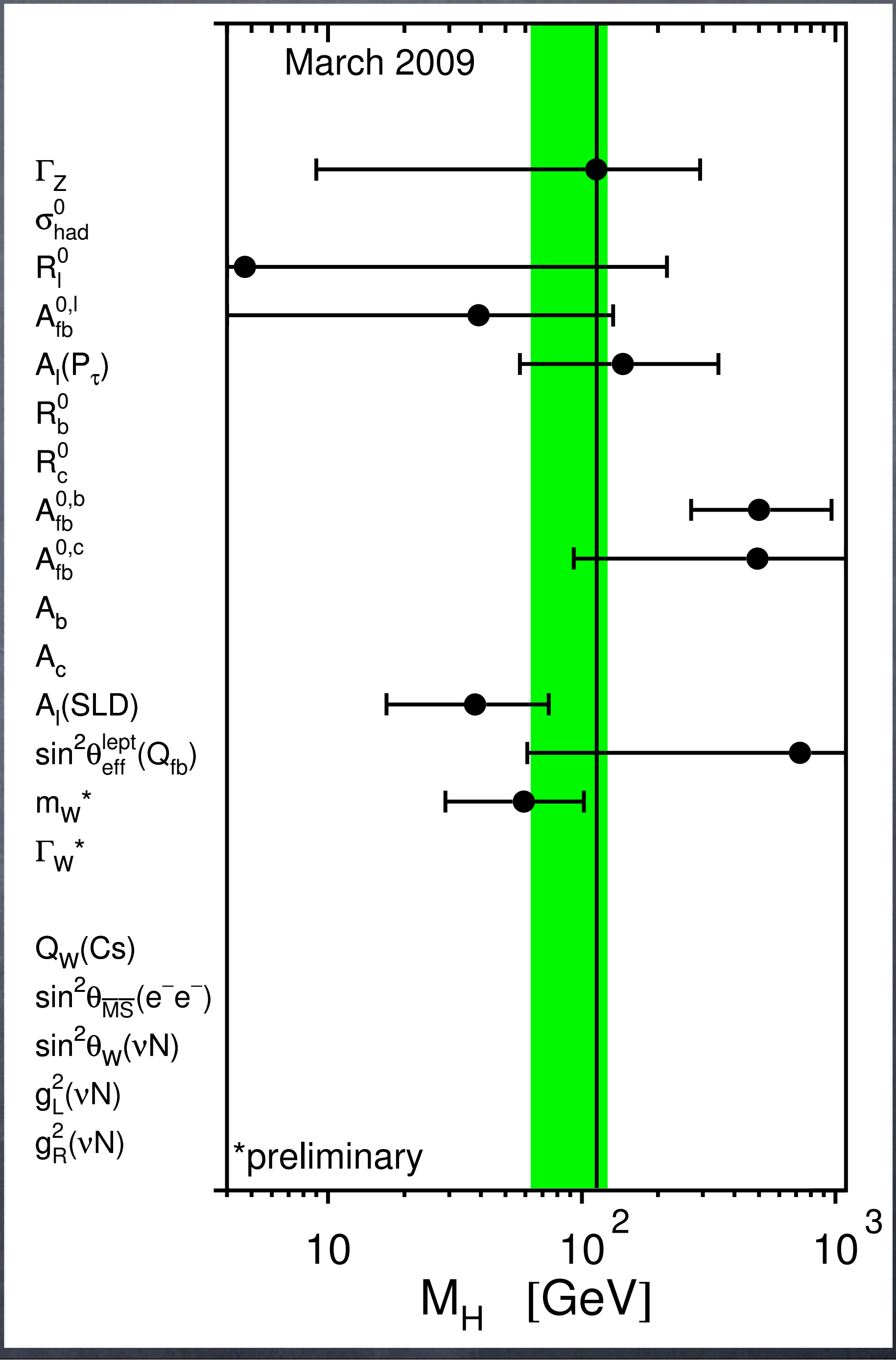


A Tug of War

We can also look at which value of the Higgs mass each observable prefers. In the SM, this makes no sense, but if new (non-oblique!) physics corrects some of them differently from others, it tells us something about which Higgs mass is really preferred by the data.

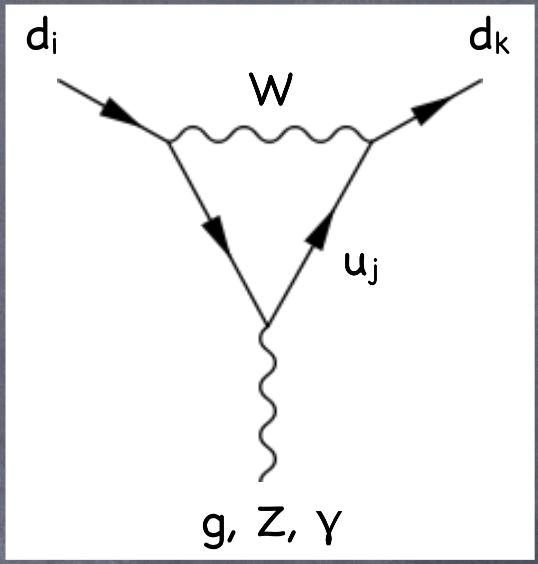
Interestingly, the fit to the Higgs mass is something of a "tug of war" between the lepton observables R_l, A_l and A^l_{FB}, and the hadronic asymmetries, A^b_{FB} and A^c_{FB}.

Does this mean something or is it just statistical fluctuation? It's not clear at the moment!



A Second Success: GIM

- We already saw that the SM's structure forbade the appearance of tree level flavor-changing neutral currents.
- The charged current interactions are flavor-changing, so at the loop level they can induce flavor change in the neutral currents too.
- The SM also has a built-in mechanism to minimize contributions to FCNCs at loop level the Glashow-Illiopoulos-Maiani (GIM) mechanism.
- A loop-induced FCNC like the one shown here must be proportional to two CKM elements summed over the intermediate quark.
- The loop is typically dominated by momenta of order M_W. Since all quarks but top have masses much less than this, the loop function F becomes approximately independent of m_j (unless j=t).
- So largest effects come from the top quark, balanced by small CKM elements!



$$\sim \sum_j V^\dagger_{kj} V_{ji}\; F\left(Q^2, m_i, m_k; m_j\right)$$

$$\to F(Q^2) \sum_j V^\dagger_{kj} V_{ji}$$

$$\to F(Q^2)\delta_{ij}$$

Not flavor-violating!

In the SM V is unitary, so:

$$\sum V^\dagger_{kj} V_{ji} = \delta_{ki}$$

Outlook: A Recipe for the Standard Model

- To conclude this session, a quick recap is in order.
- To construct the Standard Model:
  - Start with local SU(3) x SU(2) x U(1) invariance.
  - Stir in three generations of chiral matter: Q, u, d, L, e.
  - Toss in all renormalizable, Lorentz invariant terms.
  - Break SU(2) x U(1) -> U(1) with a single Higgs doublet, setting aside the Higgs boson to discover later.
  - Season by adjusting parameters to match experiments.
- **Bake for 30+ years, working all the while to understand all of its consequences and searching for some sign of physics inconsistent with it!**