Higher Order Tools and Techniques

(Mainly Fixed Order NLO)

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INTRODUCTION

Signals and Backgrounds, Need for NLO, Structure of NLO

FEYNMAN DIAGRAMS (THE TROUBLE WITH)

Integral Mess, Tensor Reduction, Stability, Complexity

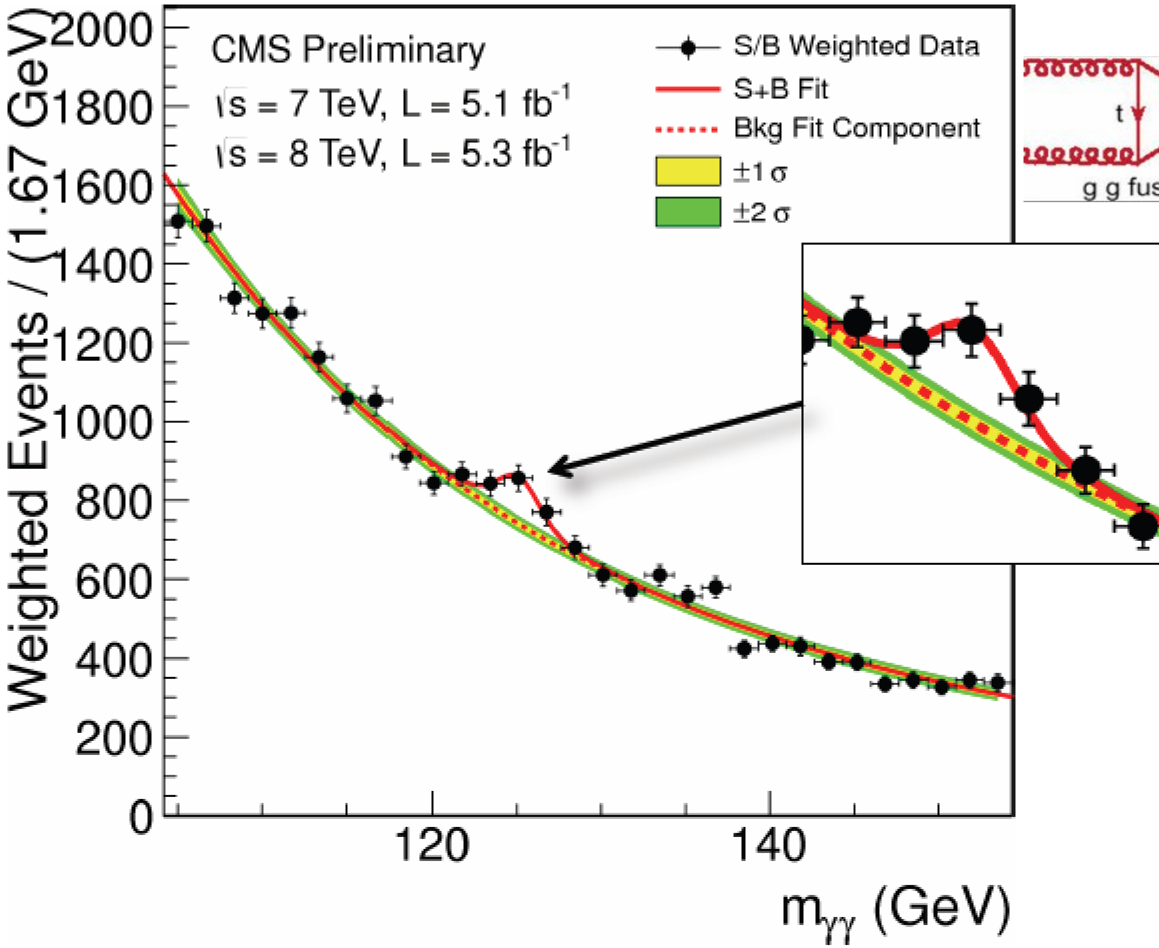
ON-SHELL AND UNITARITY TECHNIQUES

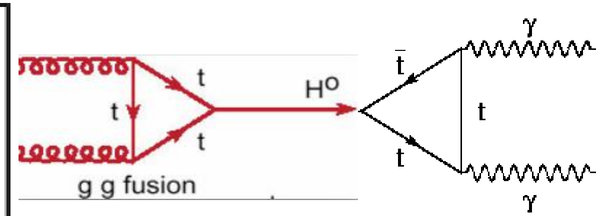
Tree Level Recursion, Loop Anatomy, Box Coefficient extraction

AUTOMATION AND TOOLS

NLO Programs, Automation, NTUPLES, Beyond NLO

A by now famous plot!



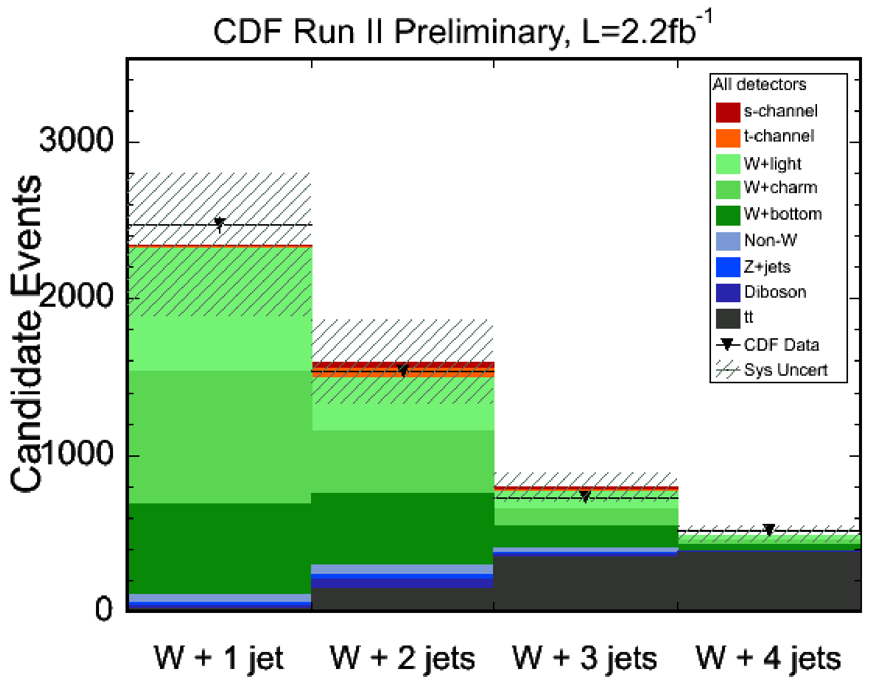


Clear signal over a continuous background!

But sometimes it isn't all that clean…

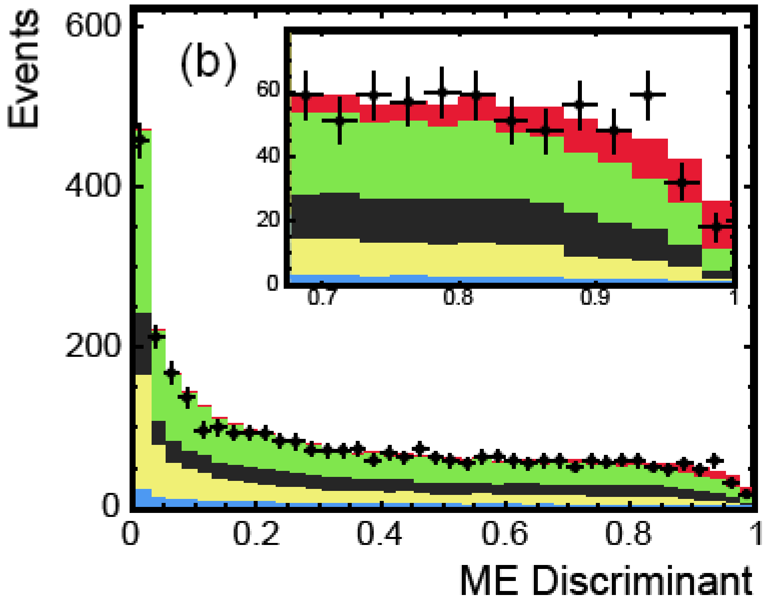
Tevatron: Single Top Production

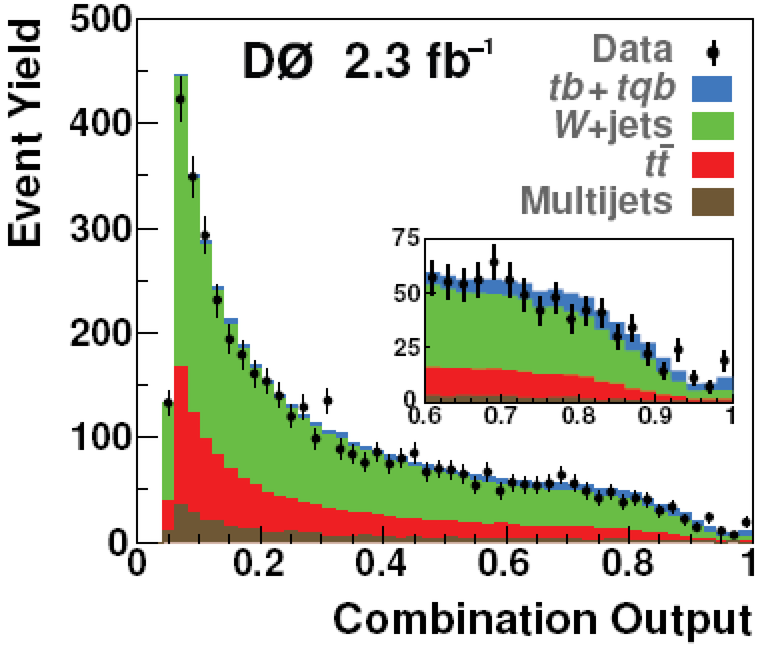
T. Aaltonen et al. [CDF Collaboration], arXiv:0809.2581



In these data sets, just by "counting", is not possible to extract a signal!

Tevatron: Single Top Production





CDF 5 sigma discovery!

D0 5 sigma discovery!

arXiv:0903.0885

arXiv:0903.0850

But exploiting the kinematical properties of the process signal and back ground can be disentangled! Now, it's clear that a precise knowledge of signal & background is needed.

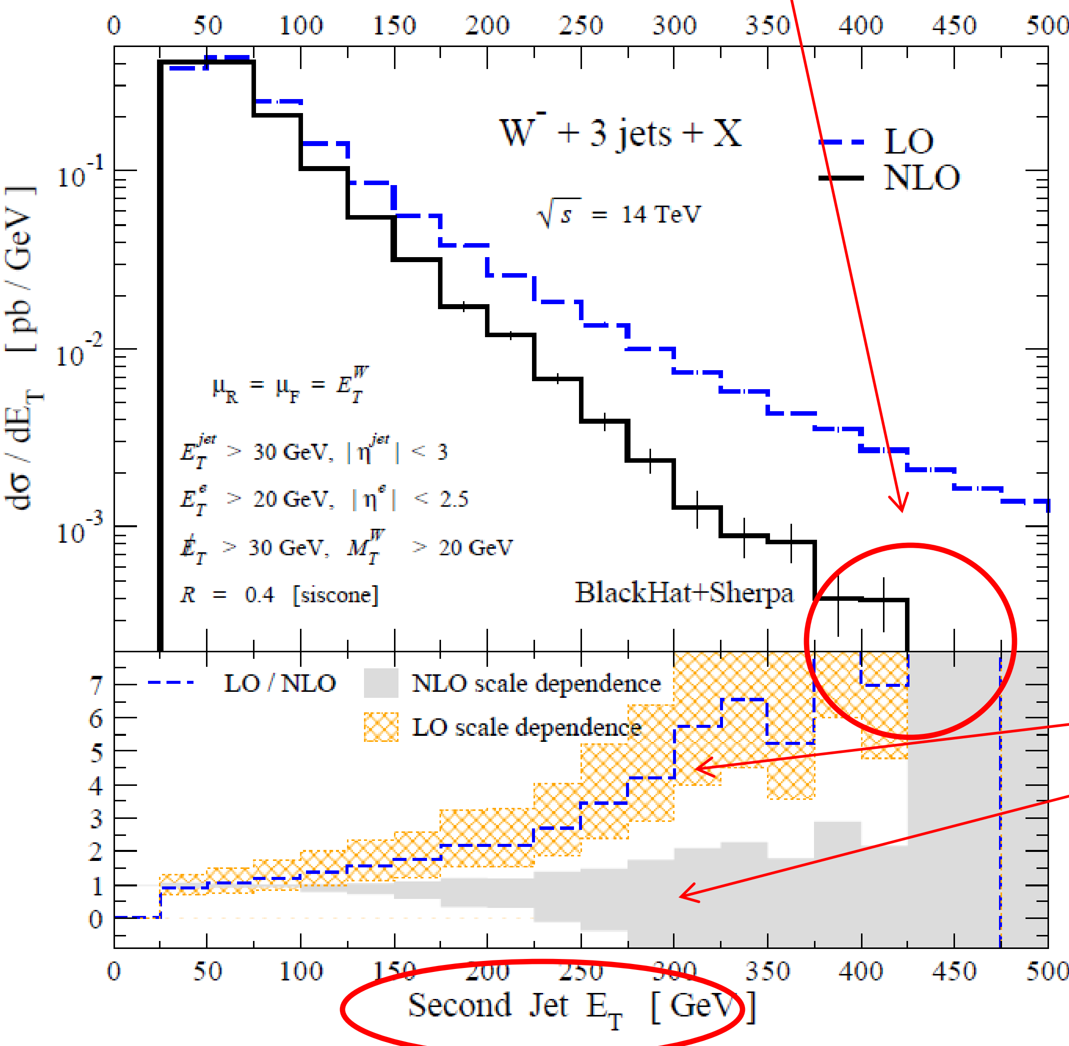
Need to go beyond LO QCD!

- Tree level (classical) predictions are only qualitative
- First quantitative reliable results appear with first order corrections in α_s
- Not only rates are well predicted, but also shapes of distributions
- Often relaxation of kinematical constrains and opening of production channels appear at NLO
- Unphysical renormalization/factorization scale dependence gets reduced with more terms of the perturbative series.

NLO as Indicator of Natural Scales

The renormalization scale common at Tevatron:
Turns out to be a bad choice at LHC.

$$\mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)}$$



Complicated processes have many scales.

LHC has a much greater dynamic range than Tevatron; M_W not characteristic scale.

Other signs of bad scale choice:

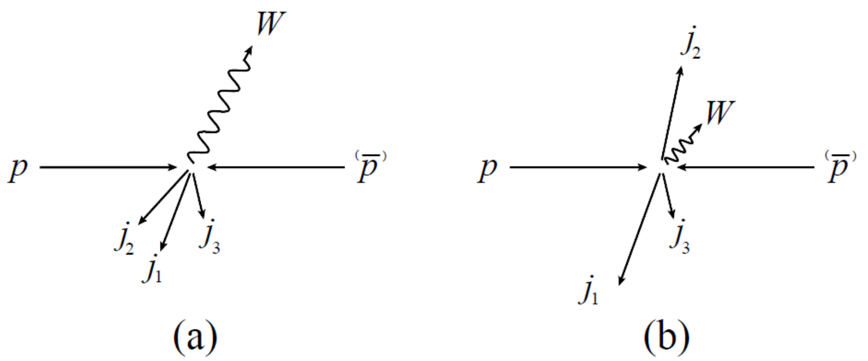
- Negative cross section.
- Large LO/NLO ratio.
- Rapid growth of scale bands with ET.

The Trouble with E_T^W

See also: Mangano, Parke '90; Frixione '93; Arnold, Reno '89; Baur, Han, Ohnemus (9507336); Bozzi Jager, Oleari, Zeppenfeld (0701150)

Consider these 2 configurations:

- For (a) $\mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)}$ physical scale of interactions.
- For (b) E_T^W may be low and under-estimating the physical scale.



Looking at large E_T for the 2nd jet forces configuration (b).

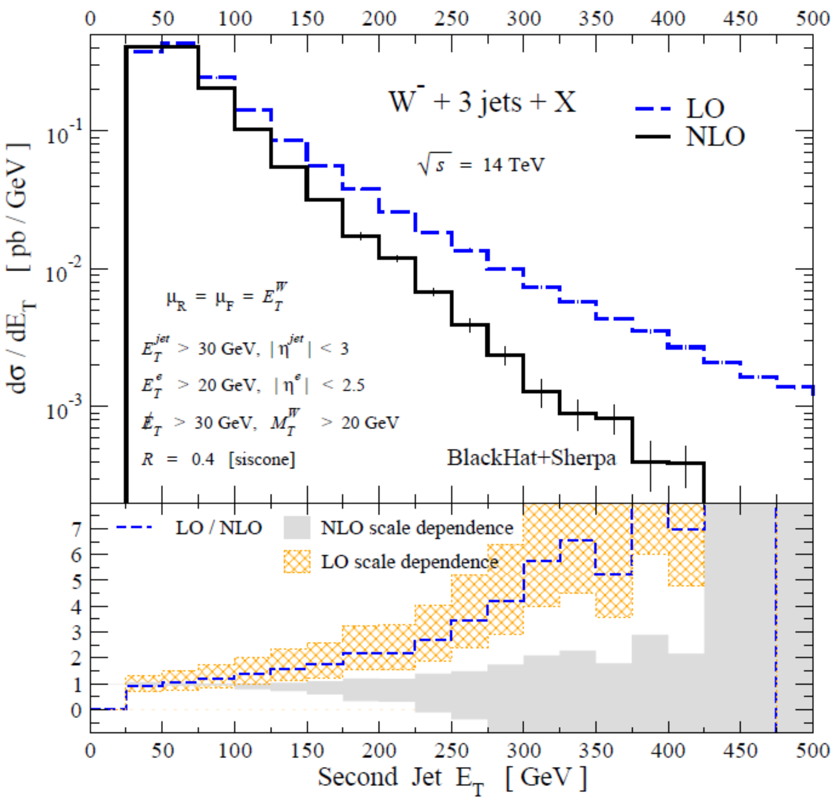
- The total (partonic) transverse energy is a better variable; gets large for both (a) and (b).

$$\hat{H}_T = \sum_p E_T^p + E_T^e + E_T^\nu$$

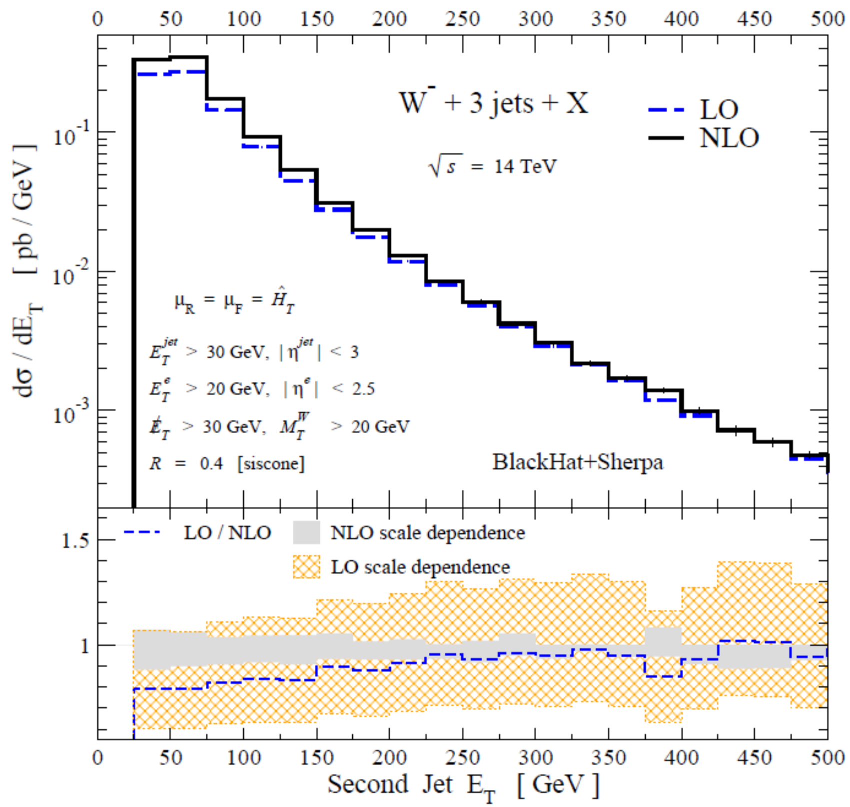
- Other reasonable scales are for example *invariant mass of the n jets* [Bauer, Lange arXiv:0905.4739] or *local scales* (at LO) inspired in CKKW reweighting [Melnikov, Zanderighi arXiv:0910.3671]

Compare Two Scale Choices

$\mu = E_T^W$



$\mu = \hat{H}_T$



Message: Do not use $\mu = E_T^W$

- LO/NLO ratio sensible
- NLO scale dependence under control

Must Match Experimental Needs!

An experimenter's wishlist

- Hadron collider cross-sections one would like to know at NLO

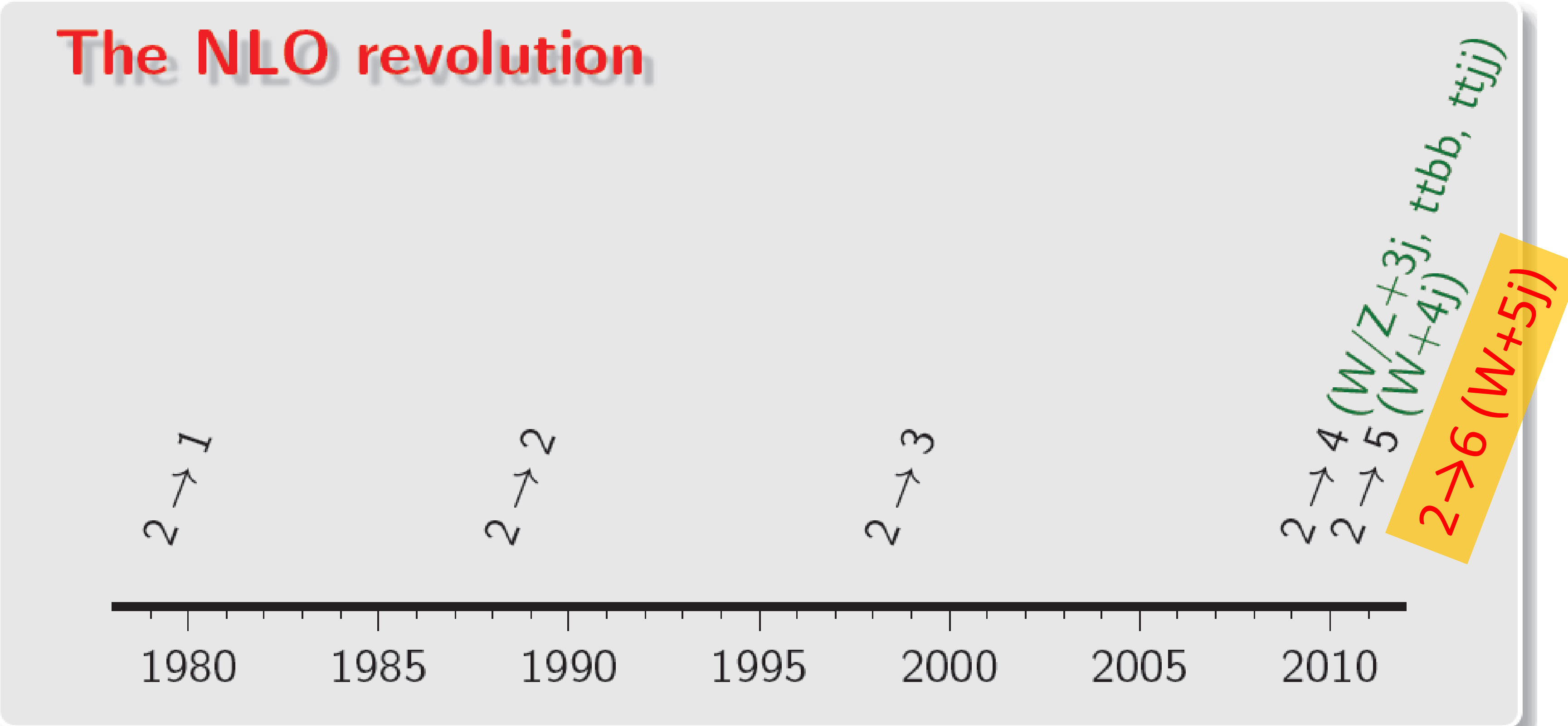
Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

LHC priority NLO wish list, Les Houches 2005/7

process	background	status - mostly from Feynman diagram approach
$pp \to VV + 1$ jet	WBF $H \to VV$	WWj (07)
$pp \to t\bar{t} + b\bar{b}$	$t\bar{t}H$	$q\bar{q} \to t\bar{t}b\bar{b}$ (08)
$pp \to t\bar{t} + 2$ jets	$t\bar{t}H$	$t\bar{t}j$ (07), $t\bar{t}Z$ (08)
$pp \to VV + b\bar{b}$	WBF $H \to VV$, $t\bar{t}H$, NP	
$pp \to VV + 2$ jets	WBF $H \to VV$	WBF $pp \to VVjj$ (07)
$pp \to V + 3$ jets	NP	$W + 3$ jets (09)
$pp \to VVV$	SUSY trilepton	ZZZ (07), WWZ (07), WWW (08), ZZW (08)
$pp \to b\bar{b}b\bar{b}^*$	Higgs and NP	

The NLO revolution



2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi] [unitarity]
2009: NLO $W+3j$ [BlackHat: Berger et al] [unitarity]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al] [traditional]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al] [traditional]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2010: NLO $Z+3j$ [BlackHat: Berger et al] [unitarity]

2010: NLO $W+4j$ [BlackHat: Berger et al] [unitarity]

General structure of a NLO calculation

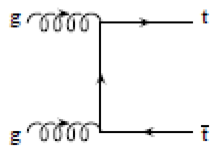
NLO cross-section:

$$d\sigma^{NLO}_{p\bar{p},pp} = \sum_{i,j} \int dx_1 dx_2 f_i^p(x_1,\mu) f_j^{\bar{p},p}(x_2,\mu) d\hat{\sigma}^{NLO}_{ij}(x_1,x_2,\mu)$$

Consider: $p\bar{p}, pp \to t\bar{t}$

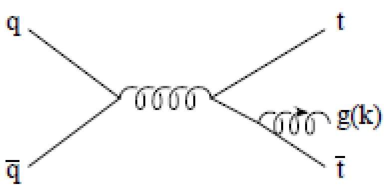
where

$$d\hat{\sigma}^{NLO}_{ij} = d\hat{\sigma}^{LO}_{ij} + \frac{\alpha_s}{4\pi}\delta d\hat{\sigma}^{NLO}_{ij}$$

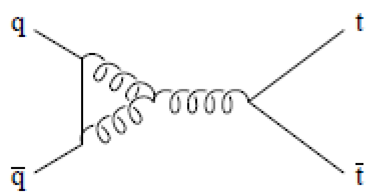


NLO corrections made of:

$$\delta d\hat{\sigma}^{NLO}_{ij} = d\hat{\sigma}^{virt}_{ij} + d\hat{\sigma}^{real}_{ij}$$



- $d\hat{\sigma}^{virt}_{ij}$: one loop virtual corrections.
- $d\hat{\sigma}^{real}_{ij}$: one gluon/quark real emission.
- use $\alpha_s^{NLO}(\mu)$ and match with NLO PDF's.



$\longrightarrow$ renormalize UV divergences ($d=4-2\epsilon_{UV}$)

$\longrightarrow$ cancel IR divergences in $d\hat{\sigma}^{virt}_{ij} + d\hat{\sigma}^{real}_{ij}$ + PDF's ($d=4-2\epsilon_{IR}$)

$\longrightarrow$ check μ-dependence of $d\sigma^{NLO}_{p\bar{p},pp}$ (μ_R,μ_F)

Real Piece: Subtraction Method

Subtract the singular behavior without introducing cutoffs. Schematically:

$$d\hat{\sigma}_{ij}^{NLO} = \left[d\hat{\sigma}_{ij}^{real} - d\hat{\sigma}_{ij}^{sub}\right]_{\epsilon\to 0} + \left[d\hat{\sigma}_{ij}^{virt} + d\hat{\sigma}_{ij}^{sub,CT}\right]_{\epsilon\to 0}$$

where

- $d\hat{\sigma}_{ij}^{sub}$ has the same singular behavior as $d\hat{\sigma}_{ij}^{real}$ at each phase space point (in d dimensions);
- $d\hat{\sigma}_{ij}^{sub}$ has to be analytically integrable over the singular one-parton phase space in d dimensions, such that we can define the subtraction "counterterm":

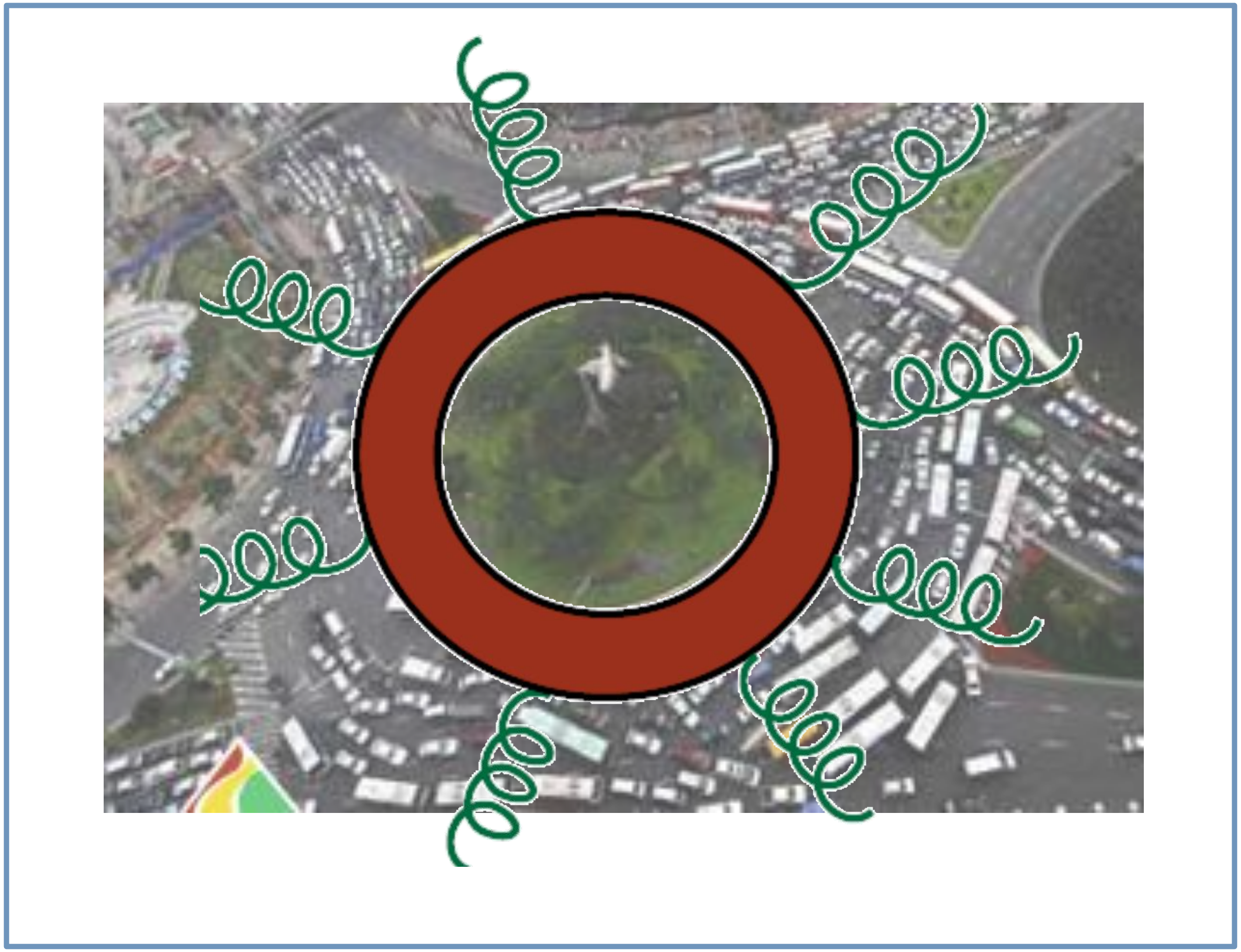
$$d\hat{\sigma}_{ij}^{sub,CT} = \int d(PS_g)\, d\hat{\sigma}_{ij}^{sub}$$

Nowadays we have several automated implementatios! (AMEGIC, COMIX, MadFKS...)

In this way:

- $[d\hat{\sigma}_{ij}^{real} - d\hat{\sigma}_{ij}^{sub}]$ is integrable over the entire phase space, and the limit $\epsilon \to 0$ can safely be taken;
- $[d\hat{\sigma}_{ij}^{virt} + d\hat{\sigma}_{ij}^{sub,CT}]$ is finite and integrable in $d=4$ because (modulus the IR singularities that are factored in the renormalized PDF's) $d\hat{\sigma}_{ij}^{sub,CT}$ contains all the IR poles of $d\hat{\sigma}_{ij}^{virt}$.

Loop Amplitudes: The Bottleneck!



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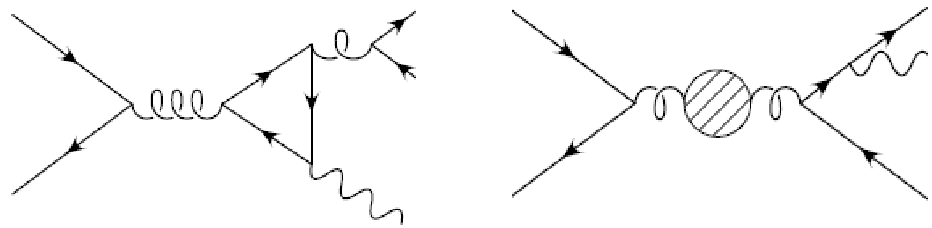
Feynman Diagrams

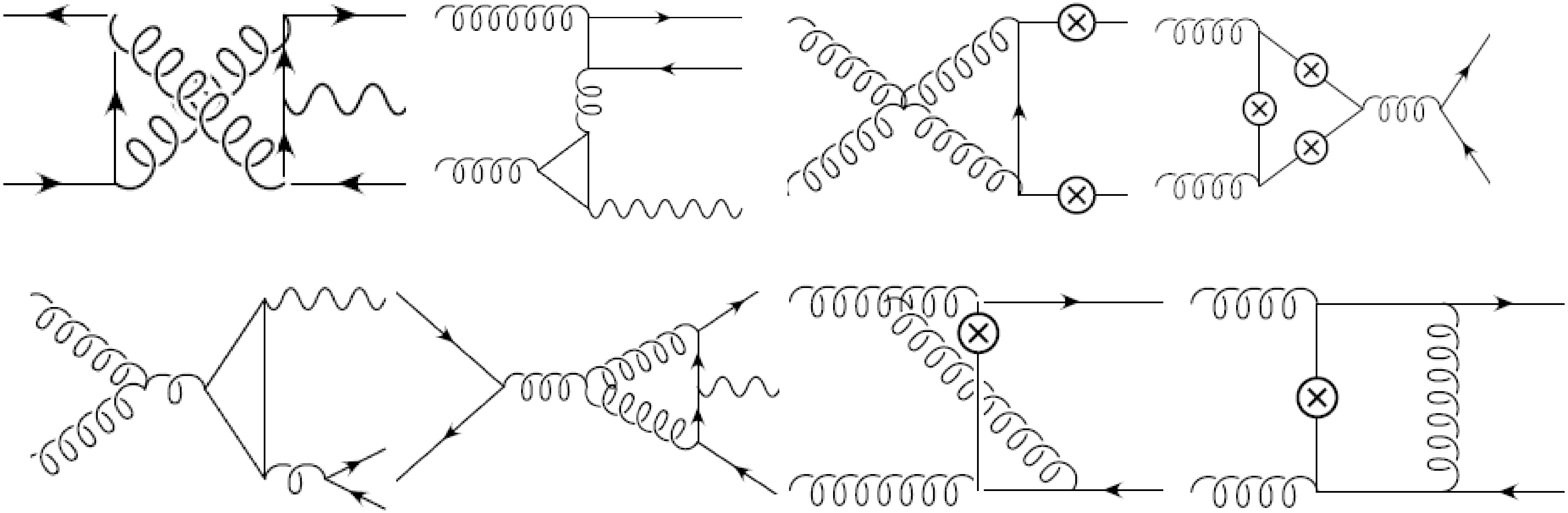
- Tool to compute amplitudes in Quantum Field Theories
- *Easy* to use
- In principle applies to all kind of processes and to all orders
- Tree level automation manageable (at least for up to 7/8 points in QCD)

- Complexity of calculations grow fast with number of legs and number of loops
- Introduces many non-physical degrees of freedom which cancel in final results
- Gauge invariance *hidden* in them

Loop Feynman Diagrams

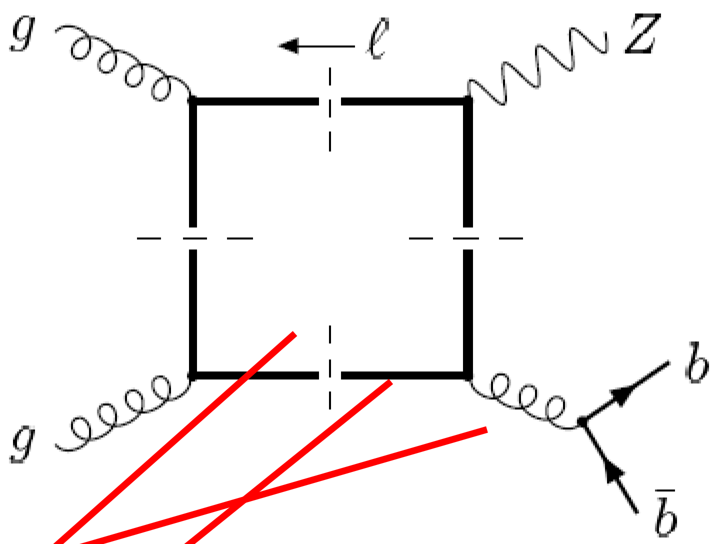
Consider: $pp \to Z\, b\bar{b}$





...

An Example…



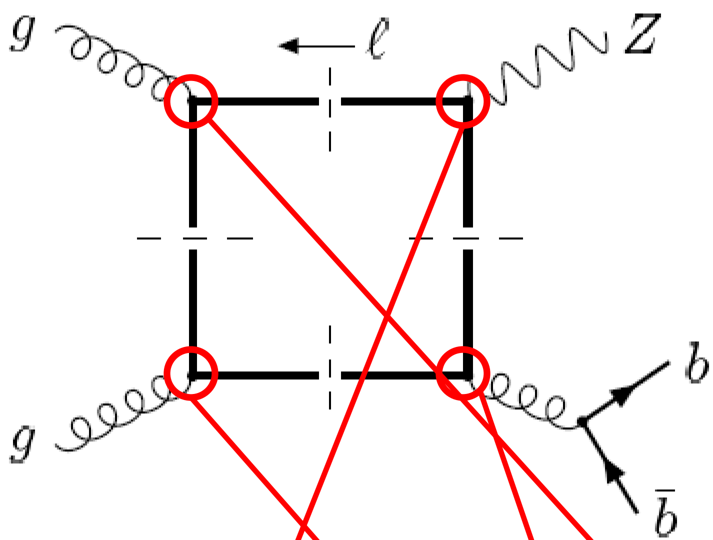
Not including couplings, polarization vectors, factors of "*i*", etc

$$\mathcal{A} \propto \frac{1}{m_{b\bar{b}}^2} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_t^2][(l+q_1)^2 - m_t^2][(l+q_1+q_2)^2 - m_t^2][(l-p_Z)^2 - m_t^2]}$$

$$Tr\Big[(\not{l} + m_t)\gamma_\rho(g_V^t + g_A^t\gamma_5)(\not{l} + \not{p}_Z + m_t)\gamma_\mu$$

$$(\not{l} + \not{q}_1 + \not{q}_2 + m_t)\gamma_\nu(\not{l} + \not{q}_1 + m_t)\gamma_\lambda\Big]$$

An Example…



Vertices

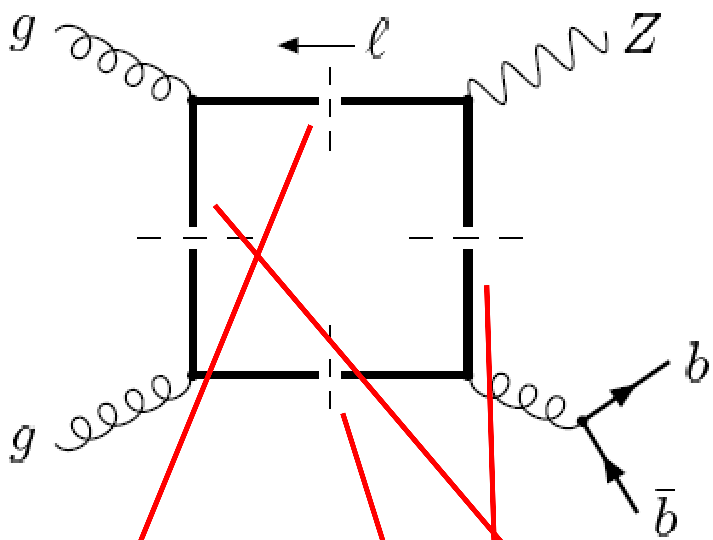
Not including couplings, polarization vectors, factors of "*i*", etc

$$\mathcal{A} \propto \frac{1}{m_{b\bar{b}}^2} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_t^2][(l+q_1)^2 - m_t^2][(l+q_1+q_2)^2 - m_t^2][(l-p_Z)^2 - m_t^2]}$$

$$Tr\Big[(\not{l} + m_t)\gamma_\rho(g_V^t + g_A^t\gamma_5)(\not{l} + \not{p}_Z + m_t)\gamma_\mu$$

$$(\not{l} + \not{q}_1 + \not{q}_2 + m_t)\gamma_\nu(\not{l} + \not{q}_1 + m_t)\gamma_\lambda\Big]$$

An Example...



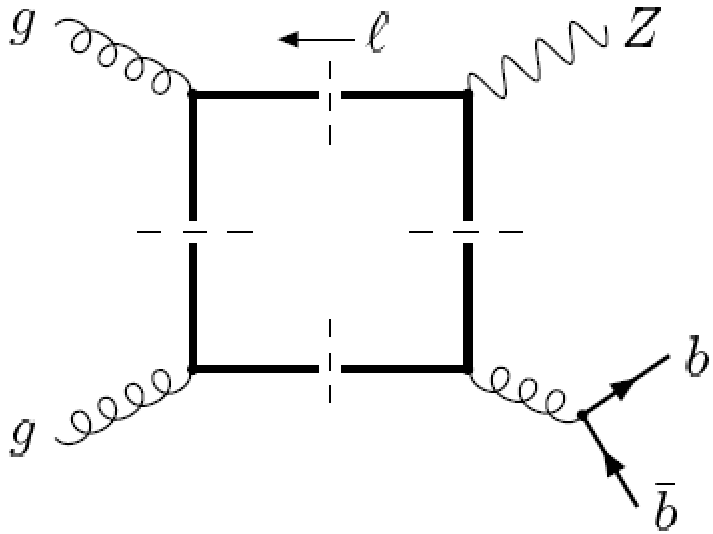
Propagators

Not including couplings, polarization vectors, factors of "*i*", etc

$$\mathcal{A} \propto \frac{1}{m_{b\bar{b}}^2} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_t^2][(l+q_1)^2 - m_t^2][(l+q_1+q_2)^2 - m_t^2][(l-p_Z)^2 - m_t^2]}$$

$$Tr\Big[(\not{l} + m_t)\gamma_\rho(g_V^t + g_A^t\gamma_5)(\not{l} + \not{p}_Z + m_t)\gamma_\mu$$

$$(\not{l} + \not{q}_1 + \not{q}_2 + m_t)\gamma_\nu(\not{l} + \not{q}_1 + m_t)\gamma_\lambda\Big]$$



$$\mathcal{A} \propto \frac{1}{m_{b\bar{b}}^2} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_t^2][(l+q_1)^2 - m_t^2][(l+q_1+q_2)^2 - m_t^2][(l-p_Z)^2 - m_t^2]}$$

$$Tr\Big[(\not{l} + m_t)\gamma_\rho(g_V^t + g_A^t\gamma_5)(\not{l} + \not{p}_Z + m_t)\gamma_\mu$$

This indeed is a complicated expression!

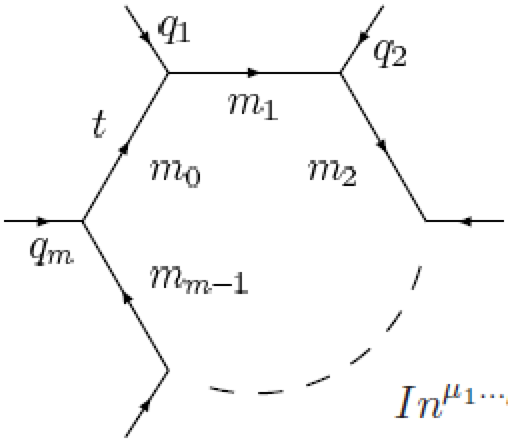
$$(\not{l} + \not{q}_1 + \not{q}_2 + m_t)\gamma_\nu(\not{l} + \not{q}_1 + m_t)\gamma_\lambda\Big]$$

You have to deal with Trace Technology and solve many integrals like this one:

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^{\mu_1} l^{\mu_2} l^{\mu_3} l^{\mu_4}}{[l^2 - m_t^2][(l+q_1)^2 - m_t^2][(l+q_1+q_2)^2 - m_t^2][(l-p_Z)^2 - m_t^2]}$$

$$\equiv D4(q_1, q_2, -p_Z + q_1 + q_2, m_t, m_t, m_t, m_t)$$

Dealing with Tensor Integrals



Associated with 1-loop topologies we deal with integrals like

$$In^{\mu_1 \ldots \mu_n}(q_1, \ldots, q_{m-1}, m_0, \ldots, m_{m-1}) = \int \frac{d^d t}{(2\pi)^d} \frac{t^{\mu_1} \ldots t^{\mu_n}}{[t^2 - m_0^2][(t+q_1)^2 - m_1^2] \cdots [(t+q_1+\cdots+q_{m-1})^2 - m_{m-1}^2]}$$

Notation: $I = A$ for one leg, $I = B$ for two legs, and so on.

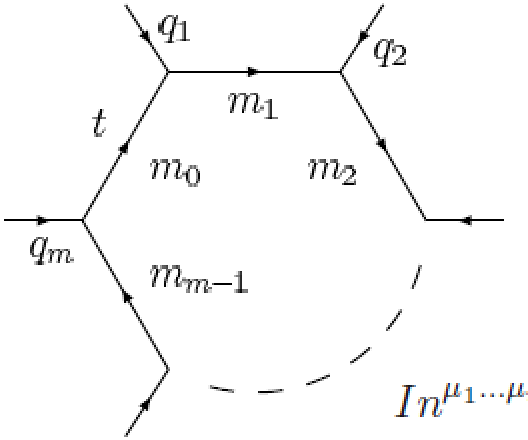
- Fully symmetric Lorentz tensor
- We can express it as linear combination of (tensor) Lorentz structures

And of course! We can only build this tensors out of the external momenta and the metric tensor (as long as the external momenta is not complete)!

$$\{q_i\} \quad g^{\mu\nu}$$

$$\Rightarrow (g^{\mu\nu} q_i^\rho q_i^\sigma + \text{perm})$$

Dealing with Tensor Integrals



Associated with 1-loop topologies we deal with integrals like

$$In^{\mu_1\cdots\mu_n}(q_1,\ldots,q_{m-1},m_0,\ldots,m_{m-1}) = \int \frac{d^dt}{(2\pi)^d}\frac{t^{\mu_1}\ldots t^{\mu_n}}{[t^2-m_0^2][(t+q_1)^2-m_1^2]\cdots[(t+q_1+\cdots+q_{m-1})^2-m_{m-1}^2]}$$

And example:

Direct Calculation

$$B1^\mu(q_1,m,m) = B^{(1)}(q_1,m,m)q_1^\mu = -\frac{1}{2}B0(q_1,m,m)q_1^\mu$$

Tensor coefficient ("PV coefficient")

Lorentz Structure

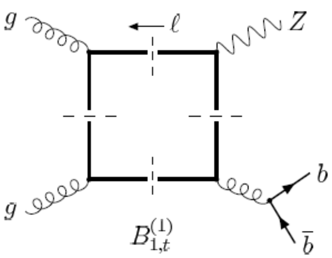
But it gets quite more involved with extra legs attached to the loop…

Dealing with Tensor Integrals

$$B1^{\mu}(q_1, m, m) = B^{(1)}(q_1, m, m)\, q_1^{\mu}$$

Tensor coefficient (“PV coefficient”)

Lorentz Structure



But for D4 functions we get:

$$
\begin{aligned}
&D4(q_1, q_2, q_3, m_0, m_1, m_2, m_3)^{\mu\nu\rho\sigma} = \\
&D^{(0000)}(g^{\mu\nu} g^{\rho\sigma} + \text{perm}) + \sum_{i\in\{1,2,3\}} D^{(00ii)}(g^{\mu\nu} q_i^{\rho} q_i^{\sigma} + \text{perm}) + \\
&\sum_{\substack{i,j\in\{1,2,3\}\\ i<j}} D^{(00ij)}(g^{\mu\nu}(q_i^{\rho} q_j^{\sigma} + q_j^{\rho} q_i^{\sigma}) + \text{perm}) + \sum_{i\in\{1,2,3\}} D^{(iiii)}(q_i^{\mu} q_i^{\nu} q_i^{\rho} q_i^{\sigma}) + \\
&\sum_{\substack{i,j\in\{1,2,3\}\\ i\neq j}} D^{(iiij)}(q_i^{\mu} q_i^{\nu} q_i^{\rho} q_j^{\sigma} + \text{perm}) + \sum_{\substack{i,j\in\{1,2,3\}\\ i<j}} D^{(iijj)}(q_i^{\mu} q_i^{\nu} q_j^{\rho} q_j^{\sigma} + \text{perm}) + \\
&\sum_{\substack{i,j,k\in\{1,2,3\}\\ i\neq j, k\neq i, j<k}} D^{(iijk)}(q_i^{\mu} q_i^{\nu} q_j^{\rho} q_k^{\sigma} + \text{perm})\,.
\end{aligned}
$$

We end up with 22 integral coefficients: Direct computation is a real challenge!

Tensor Integrals: The Passarino-Veltman Reduction

Take your tensor integral:

$$In^{\mu_1\cdots\mu_n}(q_1,\ldots,q_{m-1},m_0,\ldots,m_{m-1}) = \int \frac{d^d t}{(2\pi)^d} \frac{t^{\mu_1}\cdots t^{\mu_n}}{[t^2-m_0^2][(t+q_1)^2-m_1^2]\cdots[(t+q_1+\cdots+q_{m-1})^2-m_{m-1}^2]}$$

(1) Contract it with a given momentum and use relations like:

$$t\cdot q_1 = \frac{1}{2}\left[((t+q_1)^2-m_1^2)-(t^2-m_0^2)-(q_1^2+m_0^2-m_1^2)\right]$$

(2) Simplify and change variables to cast results in terms of lower rank/lower point integrals:

$$\begin{aligned} D4^{\mu\nu\rho\sigma}(q_1,q_2,q_3,m_0,m_1,m_2,m_3)\; q_{2\sigma} = \frac{1}{2}\Big[\\ C3^{\mu\nu\rho}(q_1,q_2+q_3,m_0,m_1,m_3) - C3^{\mu\nu\rho}(q_1+q_2,q_3,m_0,m_2,m_3) \\ -(q_2\cdot q_2+2q_2\cdot q_1+m_1^2-m_2^2)\;\; D3^{\mu\nu\rho}(q_1,q_2,q_3,m_0,m_1,m_2,m_3)\Big] \end{aligned}$$

(3) Contract also the expression with the Lorentz tensor structures

$$\begin{aligned} D4(q_1,q_2,q_3,m_0,m_1,m_2,m_3)^{\mu\nu\rho\sigma} = \\ D^{(0000)}(g^{\mu\nu}g^{\rho\sigma}+\mathrm{perm}) + \sum_{i\in\{1,2,3\}} D^{(00ii)}(g^{\mu\nu}q_i^\rho q_i^\sigma+\mathrm{perm}) + \ldots \end{aligned}$$

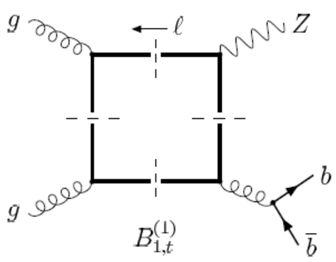
The comparison of the last two expressions gives you a (full) set of linear equations for the initial integrals in term of LOWER POINT and LOWER RANK integrals!

Tensor Integrals: The Passarino-Veltman Reduction

A recursive application of this procedure reduces our tensor integrals to Lorentz structures and scalar integrals (with a maximum of four internal propagators):

$$I0(q_1, \ldots, q_{m-1}; m_0, \ldots, m_{m-1}) = \mu^{4-d} \int \frac{d^d t}{(2\pi)^d} \frac{1}{[t^2 - m_0^2][(t+q_1)^2 - m_1^2] \cdots [(t + q_1 + \cdots + q_{m-1})^2 - m_{m-1}^2]}$$

All of which are known in the literature (See Ellis, Zanderighi arXiv:0712.1851)

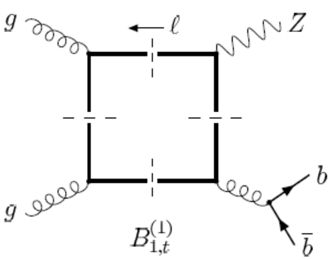


When solving the linear systems one encounters inverse powers of the Gram determinant:

$$\begin{pmatrix} q_1^2 & q_1 \cdot q_2 & q_1 \cdot q_3 \\ q_1 \cdot q_2 & q_2^2 & q_2 \cdot q_3 \\ q_1 \cdot q_3 & q_2 \cdot q_3 & q_3^2 \end{pmatrix}$$

Singularities associated with this determinant are non-physical and often are a source for numerical instabilities in the calculations!

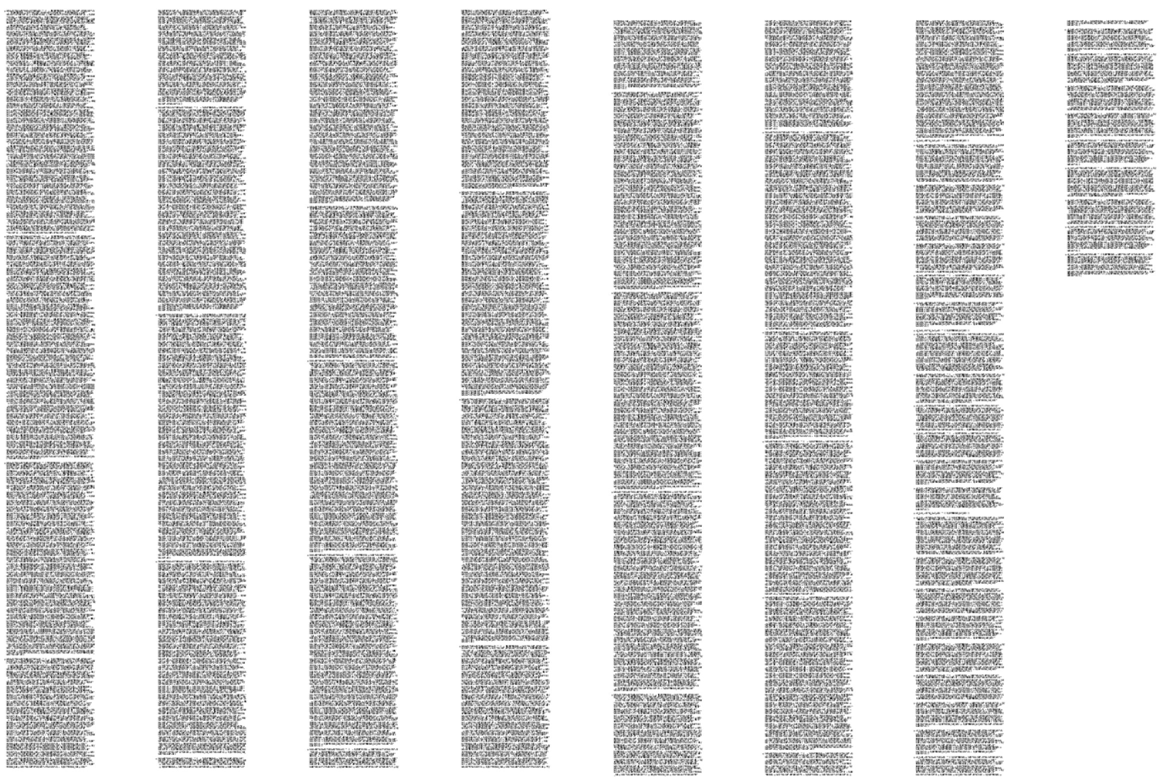
Tensor Integrals:
The Passarino-Veltman Reduction



When applying this procedure to our tensor integral of interest:

$$D4(q_1, q_2, -p_Z + q_1 + q_2, m_t, m_t, m_t, m_t)$$

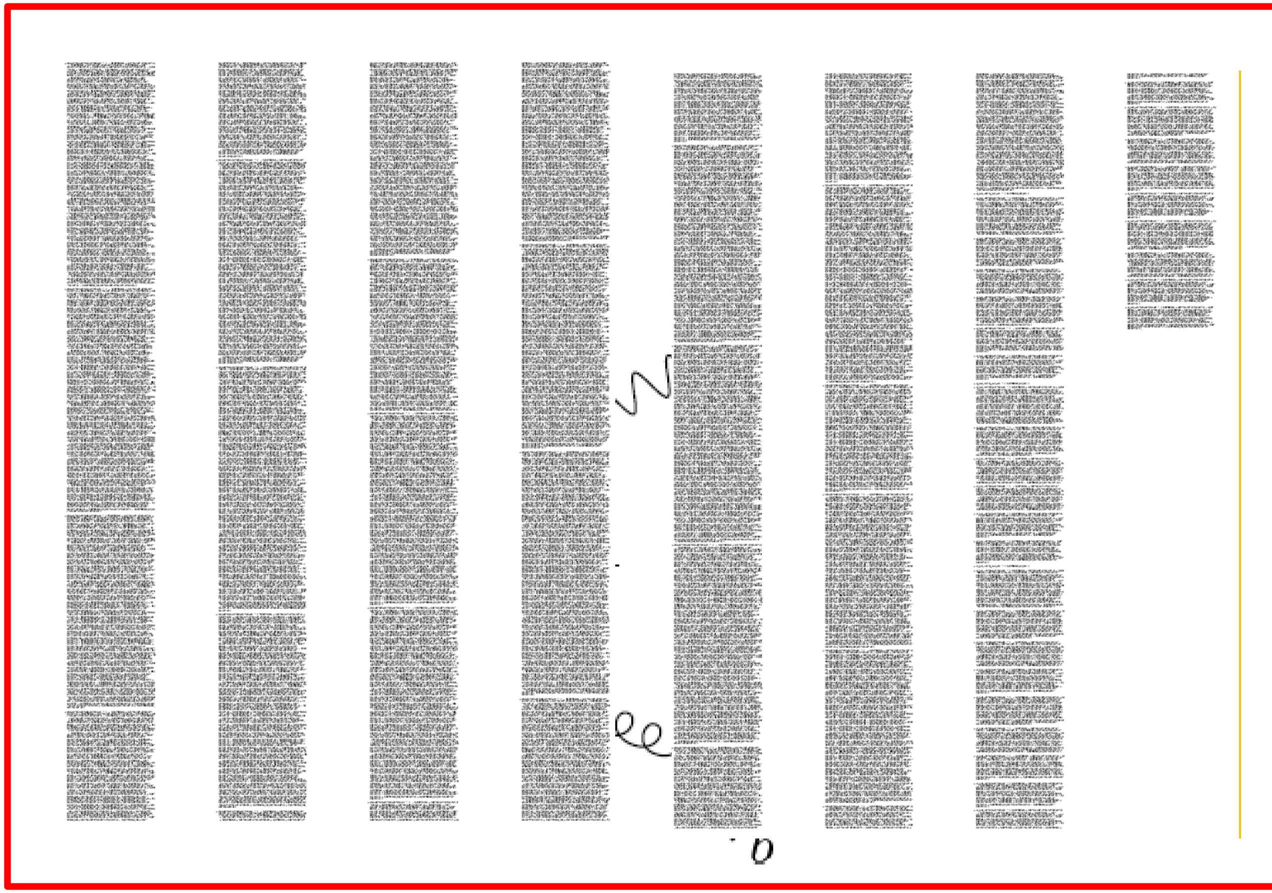
We find that ONLY the coefficient of the corresponding scalar box looks like:



Which is not only large and computer intensive, but suffers from strong numerical instabilities over PS!

And this is only a piece of a single tensor integral that appears in a single Feyman diagram…

This is the coefficient of the box scalar diagram of one of the integrals in the amplitude…



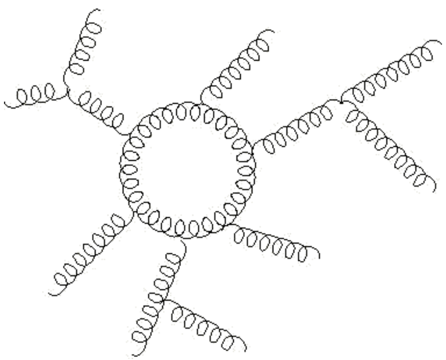
$$\mathcal{A} \propto \frac{1}{m_{b\bar{b}}^2} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_t^2][(l+q_1)^2 - m_t^2][(l+q_1+q_2)^2 - m_t^2][(l-p_Z)^2 - m_t^2]}$$

$$Tr\Big[(\not{l} + m_t)\gamma_\rho(g_V^t + g_A^t\gamma_5)(\not{l} + \not{p}_Z + m_t)\gamma_\mu$$

$$(\not{l} + \not{q}_1 + \not{q}_2 + m_t)\gamma_\nu(\not{l} + \not{q}_1 + m_t)\gamma_\lambda\Big]$$

But, it gets worse! With the number of legs...

- Consider scattering of pure gluon QCD:



6 gluons	**~10,000** diagrams
7 gluons	**~150,000** diagrams
8 gluons	~ 3,000,000 diagrams

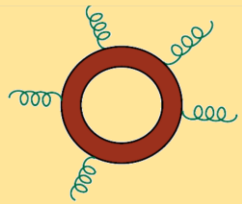
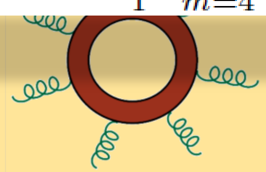
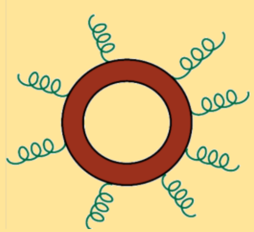
Gauge dependant quantities, large cancellations between terms

Each one of them having an exponentially growing complexity!

- A Factorial growth in the number of terms, particularly bad for large number of partons.

Are there alternative ways to this Feynman diagrams MESS?!

THERE MIGHT BE!

# of jets	# 1-loop Feynman diagrams	
3		810
4		
5		168,925
6		3,017,490

[Bern, Dixon, Dunbar, Kosower **hep-ph/9409265**]

$$\frac{c_\Gamma (\mu^2)^\epsilon A^{\mathrm{tree}}_{1\,2}(1,\ldots,n)}{2}\left\{ \left(\mathrm{K}_0(t_2^{[2]}) + \mathrm{K}_0(t_n^{[2]}) \right) - \frac{1}{t_1^{[2]}} \sum_{m=4}^{n-1} \frac{\mathrm{L}_0\left(-t_2^{[m-2]}/(-t_2^{[m-1]})\right)}{t_2^{[m-1]}} \left(\mathrm{tr}_+[\not{k}_1 \not{k}_2 \not{k}_m \not{q}_{m,1}] - \mathrm{tr}_+[\not{k}_1 \not{k}_2 \not{q}_{m,1} \not{k}_m] \right) \right\}$$

definitively looks like magic...

INTRODUCTION

Signals and Backgrounds, Need for NLO, Structure of NLO

FEYNMAN DIAGRAMS (THE TROUBLE WITH)

Integral Mess, Tensor Reduction, Stability, Complexity

ON-SHELL AND UNITARITY TECHNIQUES

Tree Level Recursion, Loop Anatomy, Box Coefficient extraction

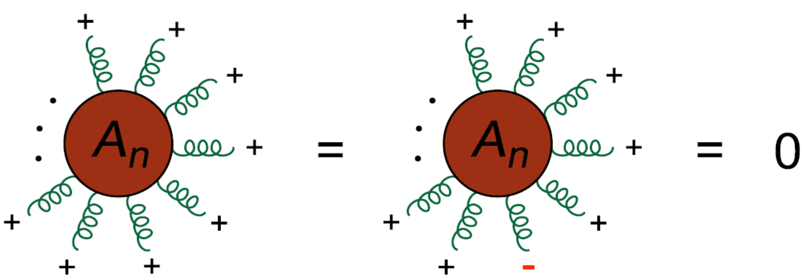
AUTOMATION AND TOOLS

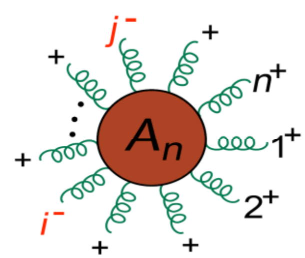
NLO Programs, Automation, NTUPLES, Beyond NLO

On-shell simplifications.

- Calculated ON-SHELL, amplitudes much simpler than expected.

- For example: some tree level all-multiplicity gluon amplitudes can fit on a page:



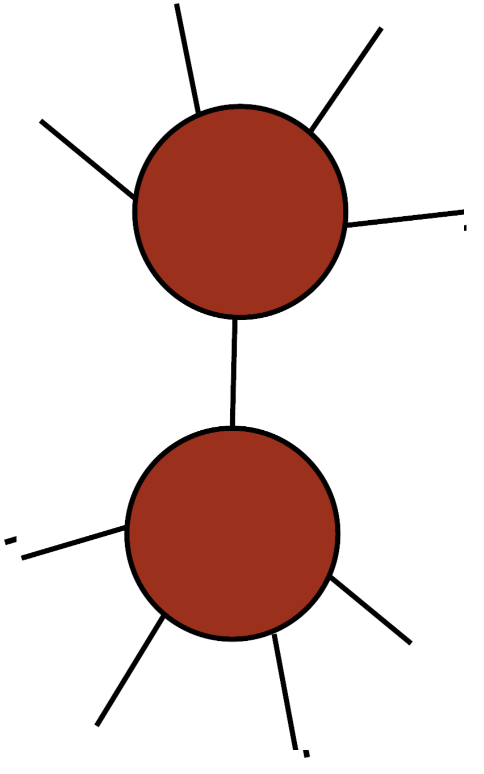


$$= \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\cdots\langle n\, 1\rangle}$$

Park, Taylor

Factorization

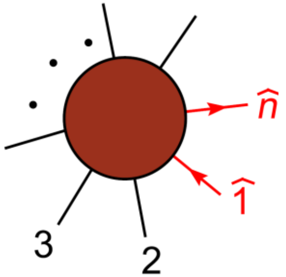
How amplitudes "fall apart" into simpler ones in special limits



Explore limits in complex plane

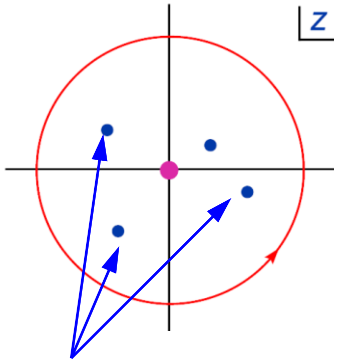
Britto, Cachazo, Feng, Witten, hep-th/0501052

Inject complex momentum at leg 1, remove it at leg *n*.



$$k_1(z) + k_n(z) = k_1 + k_n \qquad \Rightarrow \qquad A(0) \rightarrow A(z)$$
$$k_1^2(z) = k_n^2(z) = 0$$

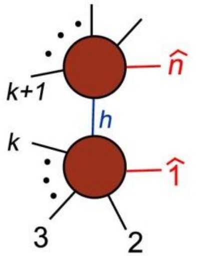
special limits ⟺ poles in *z*



Cauchy: If $A(\infty) = 0$ then

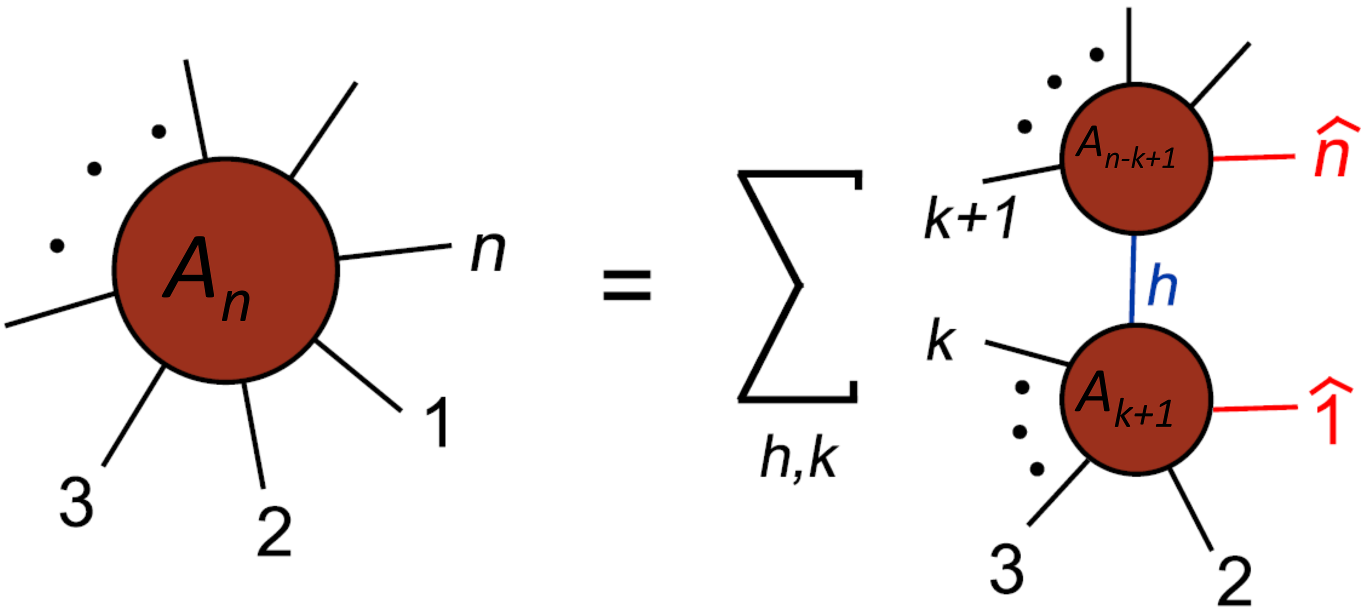
$$0 = \frac{1}{2\pi i} \oint dz \frac{A(z)}{z} = A(0) + \sum_k \mathrm{Res}\left[\frac{A(z)}{z}\right]\Big|_{z=z_k}$$

residue at z_k = [k^{th} factorization limit] =



→ BCFW (on-shell) recursion relations

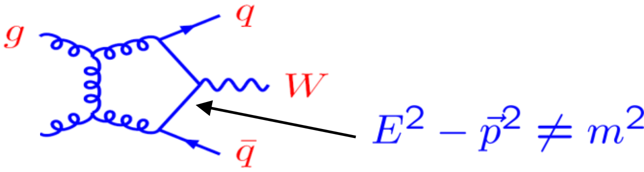
Britto, Cachazo, Feng, hep-th/0412308



A_{k+1} and A_{n-k+1} are on-shell tree amplitudes with **fewer** legs,
and with momenta shifted by a **complex** amount

Trees recycled into trees!

Think off-shell, work on-shell!



- Vertices and propagators involve unphysical gauge-dependent off-shell states.

Would like to reconstruct amplitude using only on-shell information.

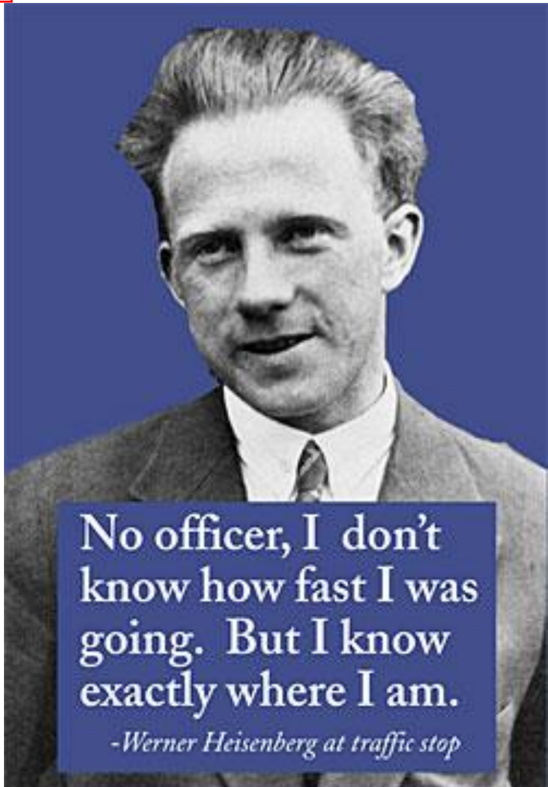
$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- Feynman diagram loops have to be off-shell because they encode the uncertainty principle.

Fact: Off-shellness is essential for getting the correct answer.

- Keep particles on-shell in intermediate steps of calculation, not in final results.

Bern, Dixon, Dunbar, Kosower



The result: one-loop basis.

See Bern, Dixon, Dunbar, Kosower, hep-ph/9212308.

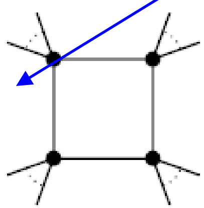
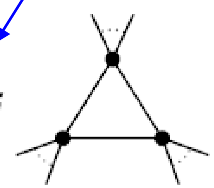
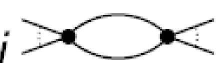
All external momenta in *D=4*, loop momenta in *D=4-2ε* (dimensional regularization).

Rational part

Cut part

Process dependent D=4 rational integral coefficients

$$A = R + C$$

$$C = \sum_i b_i$$  $$+ \sum_i c_i$$  $$+ \sum_i d_i$$ 

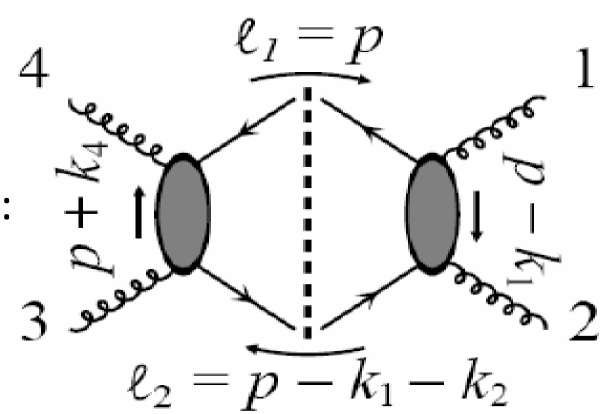
- Cut Part from unitarity cuts in 4 dimensions.
- Rational part from on-shell recurrence relations.

Unitarity: an on-shell method of calculation.

Bern, Dixon, Kosower

$$-i(T - T^\dagger) = T^\dagger T\,.$$

Cutting loops = sewing trees:

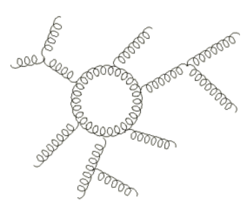
Sewing: 

$$\mathrm{Im}\, T^{1-\mathrm{loop}} = \sum_{j \in B} c_j \,\mathrm{Cut}\, \mathcal{I}_j\,.$$

Cutting: 2x $\dfrac{i}{p^2 + i\varepsilon} \longrightarrow 2\pi\, \delta^{(+)}(p^2)$

Equation:

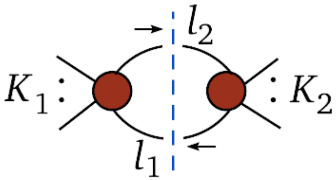
$$\sum_{j \in B} c_j \,\mathrm{Cut}\, \mathcal{I}_j = \int \frac{dp^4}{(2\pi)^4} 2\pi\delta^{(+)}(\ell_1^2 - m^2) 2\pi\delta^{(+)}(\ell_2^2 - m^2)$$
$$A_4^{\mathrm{tree}}(-\ell_1, 1, 2, \ell_2) A_4^{\mathrm{tree}}(-\ell_2, 3, 4, \ell_1)\,.$$

And NOT: $A = \int \frac{dp^4}{(2\pi)^4} \sum_{\text{Number of diags.}}$ 

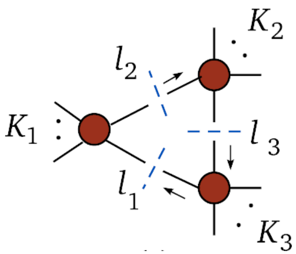
Generalized Unitarity: isolate the leading discontinuity.

Cutting: n x $\dfrac{i}{p^2 + i\varepsilon} \longrightarrow 2\pi\, \delta^{(+)}(p^2)$

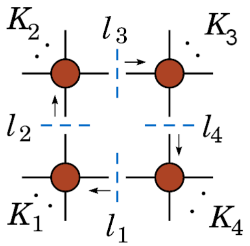
More cuts, more trees, less algebra:



- Two-particle cut: product of trees contains subset of box-, triangle- and bubble-integrals. (Bern, Dixon, Kosower, Dunbar)

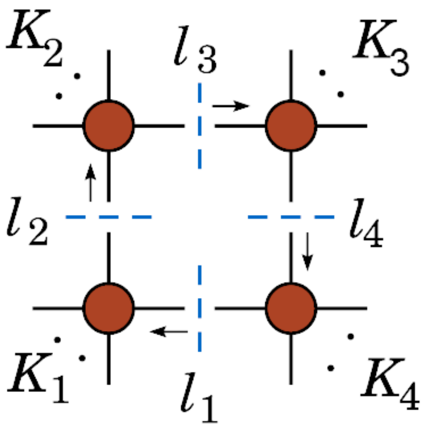


- Triple-cut: product of three trees contains triangle- and box-integrals. (Bern, Dixon, Kosower)



- Quadruple-cut: read out single box coefficient. (Britto, Cachazo, Feng)

Boxes: the simplest cuts.



Berger, Bern, Dixon, FFC, Forde,Ita, Kosower, Maitre 0803.4180; Risager 0804.3310.

$$d_i = \frac{1}{2}\sum_{\sigma=\pm} d_i^\sigma \,,$$

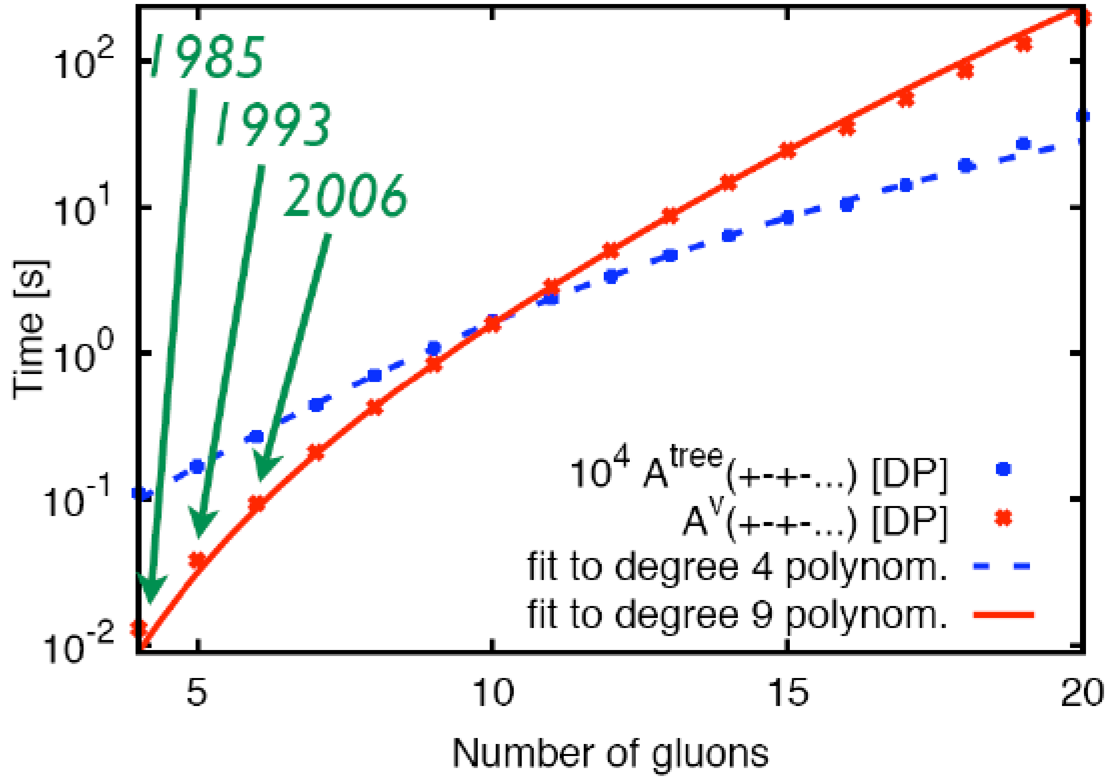
$$d_i^\sigma = A^{\rm tree}_{(1)} A^{\rm tree}_{(2)} A^{\rm tree}_{(3)} A^{\rm tree}_{(4)} \Big|_{l_i=l_i^{(\sigma)}}$$

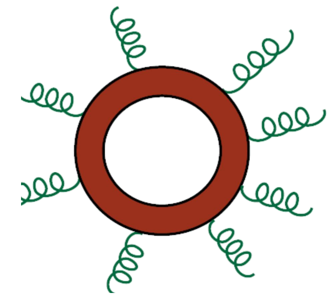
$$(l_1^{(\pm)})^\mu = \frac{\langle 1^\mp|\, \not{K}_2 \not{K}_3 \not{K}_4 \gamma^\mu \,|1^\pm\rangle}{2\,\langle 1^\mp|\, \not{K}_2 \not{K}_4 \,|1^\pm\rangle}\,, \qquad (l_2^{(\pm)})^\mu = -\frac{\langle 1^\mp|\, \gamma^\mu \not{K}_2 \not{K}_3 \not{K}_4 \,|1^\pm\rangle}{2\,\langle 1^\mp|\, \not{K}_2 \not{K}_4 \,|1^\pm\rangle}\,,$$

$$(l_3^{(\pm)})^\mu = \frac{\langle 1^\mp|\, \not{K}_2 \gamma^\mu \not{K}_3 \not{K}_4 \,|1^\pm\rangle}{2\,\langle 1^\mp|\, \not{K}_2 \not{K}_4 \,|1^\pm\rangle}\,, \qquad (l_4^{(\pm)})^\mu = -\frac{\langle 1^\mp|\, \not{K}_2 \not{K}_3 \gamma^\mu \not{K}_4 \,|1^\pm\rangle}{2\,\langle 1^\mp|\, \not{K}_2 \not{K}_4 \,|1^\pm\rangle}\,.$$

Un-physical (=spurious) singularities from parameterization.
Have to cancel eventually: role of rational term R.

A Powerful Technique!





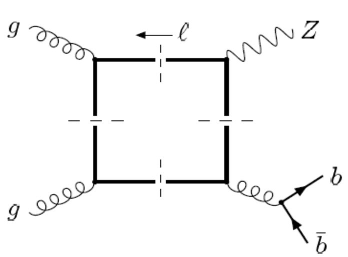
[Giele, Zanderighi arXiv:0806.2152]

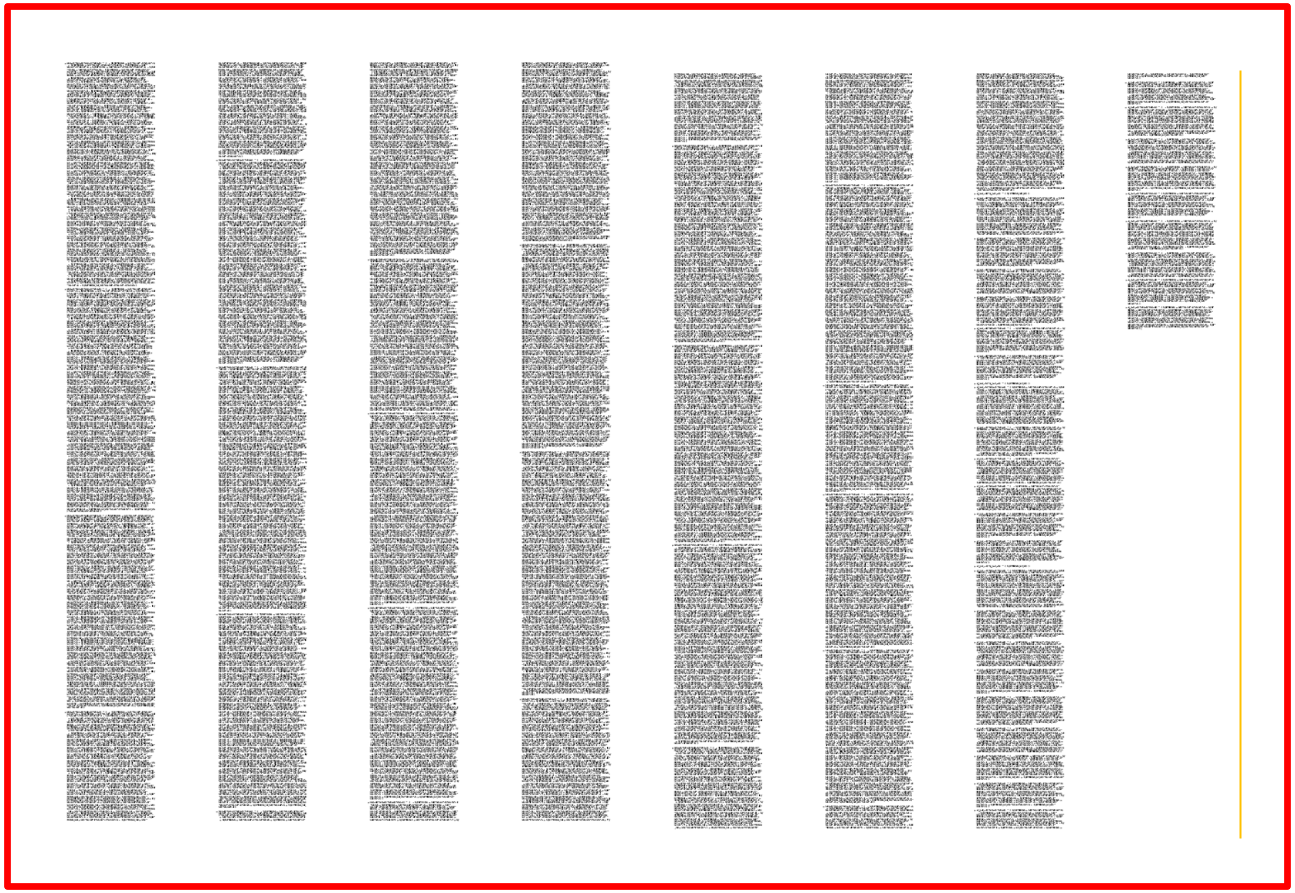
BUT STILL VERY COMPUTER INTENSIVE

[BlackHat + Sherpa]

NTUPLES: STORE THE MORE INFORMATION YOU CAN DURING YOUR COMPUTATION!

This is the coefficient of the box scalar diagram of one of the integrals in the amplitude…



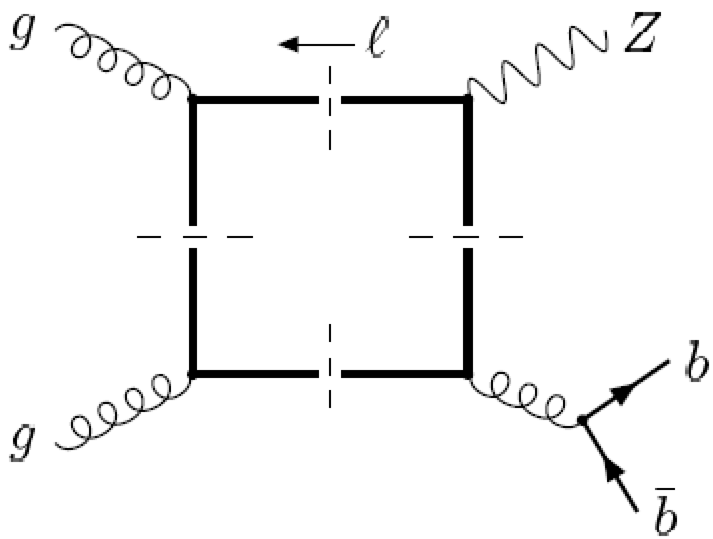


$$\mathcal{A} \propto \frac{1}{m_{b\bar{b}}^2} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_t^2][(l+q_1)^2 - m_t^2][(l+q_1+q_2)^2 - m_t^2][(l-p_Z)^2 - m_t^2]}$$

$$Tr\Big[(\not{l} + m_t)\gamma_\rho(g_V^t + g_A^t\gamma_5)(\not{l} + \not{p}_Z + m_t)\gamma_\mu$$

$$(\not{l} + \not{q}_1 + \not{q}_2 + m_t)\gamma_\nu(\not{l} + \not{q}_1 + m_t)\gamma_\lambda\Big]$$

Now, use unitarity! The Quad Cut!



From Unitarity, our coefficient can be obtained from:

Loop integral frozen!

$$\rightarrow \quad \sum_{\ell=\ell_\pm} \left(\ell^2 - m_t^2\right)\left((\ell+q_1)^2 - m_t^2\right)\left((\ell+q_1+q_2)^2 - m_t^2\right)\left((\ell+p_Z)^2 - m_t^2\right) \cdot \mathcal{A}\Big|_\ell$$

Where the sum is over the two solutions of the (*simple*) algebraic on-shell conditions

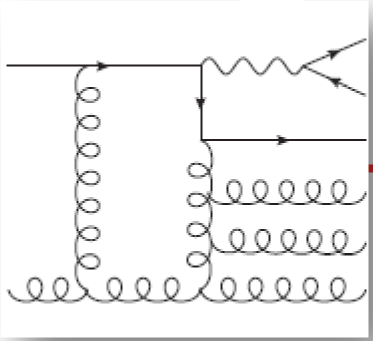
$$\{\ell \mid \ell^2 = m_t^2,\ \ (\ell+q_1)^2 = m_t^2,\ \ (\ell+q_1+q_2)^2 = m_t^2,\ \ (\ell+p_Z)^2 = m_t^2\}$$

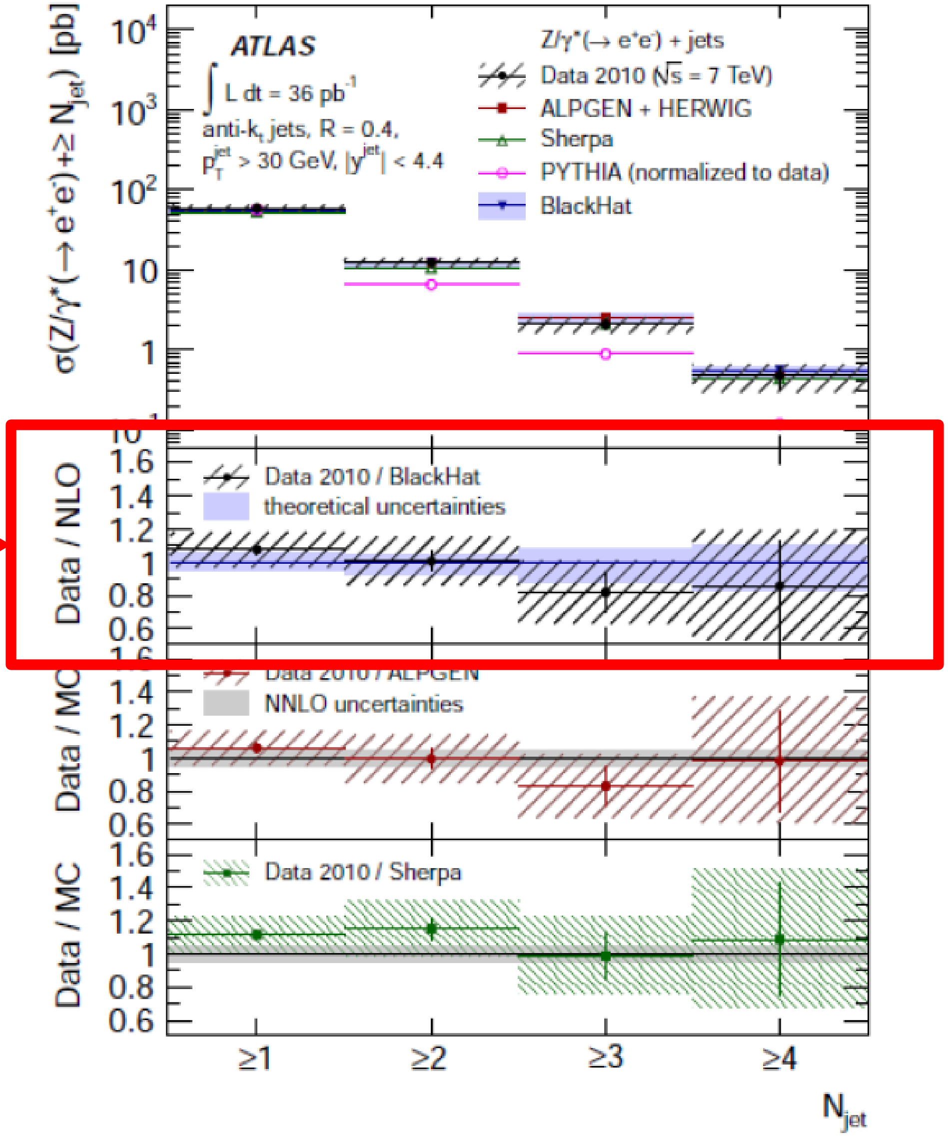
¡¡¡!!!

On-Shell Techniques in action @ LHC!

Z+Jets at the LHC

- 36 pb-1
- Inclusive cross section for each Multiplicity
- Good agreement of NLO with the data all the way to four jets





These calculations made within an automated framework (BlackHat+SHERPA) based on On-Shell/Unitarity techniques!

ArXiv:1111.2690

INTRODUCTION

Signals and Backgrounds, Need for NLO, Structure of NLO

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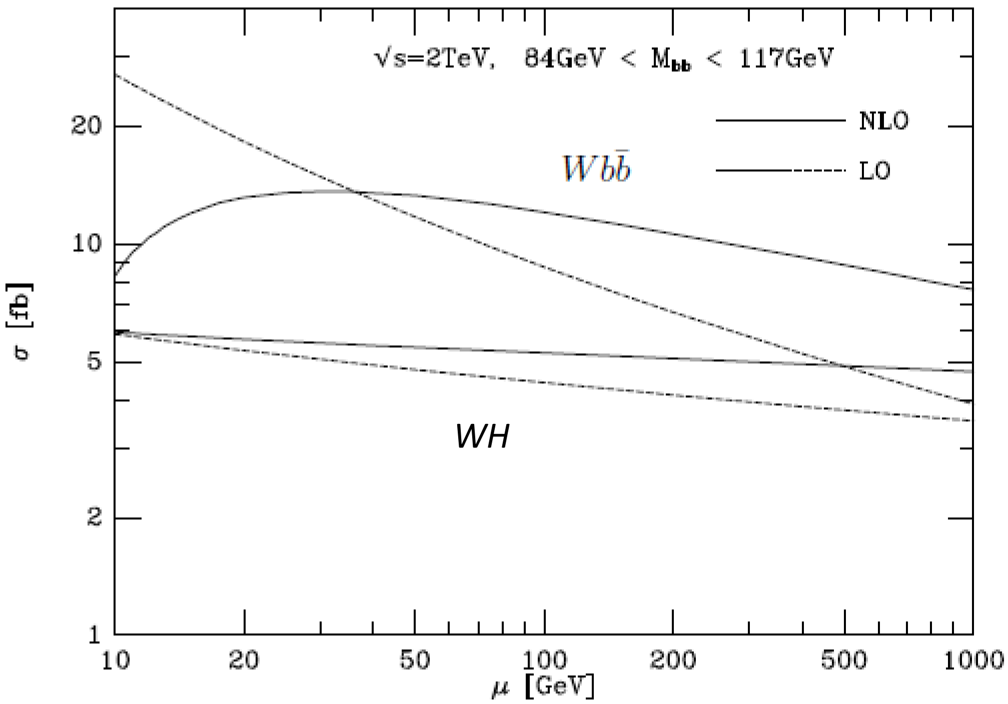
MCFM v1

John Campbell, Keith Ellis

→ FORTRAN based Parton Level NLO Montecarlo
→ First released in 2000, with a compilation of analytically computed NLO QCD corrections
→ Originally included a handful of processes (W/Z production, W/Z+jet, W/Z+bb, Weak Vector Boson Pairs and Higgstrahlung processes)
→ Meant to make available important calculations to the larger experimental and theory community
→ Easy access to multiple observables

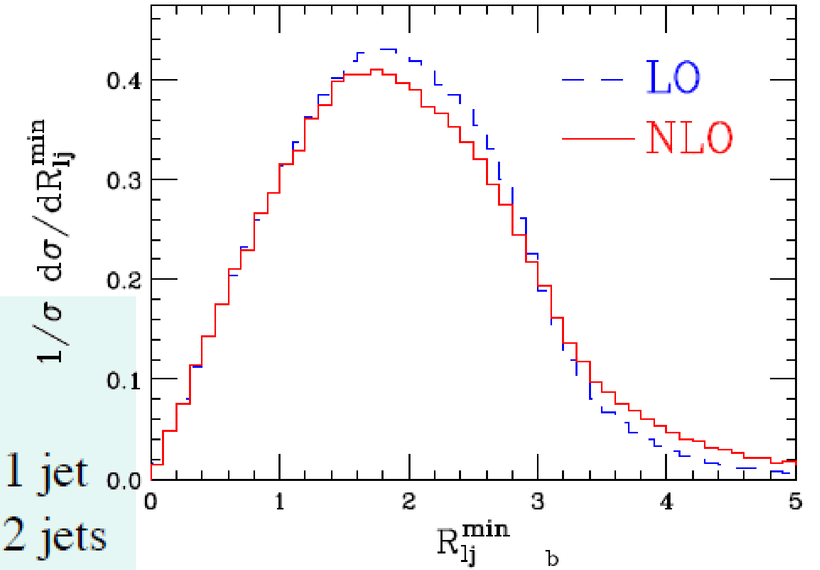


MCFM v6.2

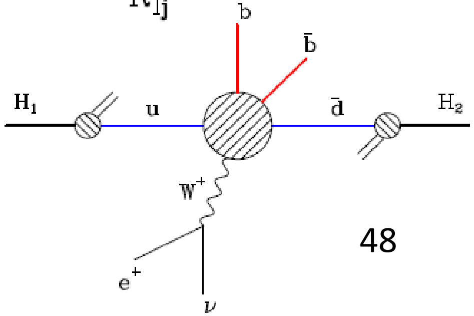
John Campbell, Keith Ellis, Ciaran Williams

→ Widely used by experimental collaborations and theorist
→ Instrumental in the computation of recent state of the art calculations (like W+3 jets with Rocket)
→ Large amount of procceses included. Still analytical *handmade* calculations

http://mcfm.fnal.gov/
arXiv:1107.5569 [hep-ph], arXiv:1105.0020 [hep-ph], arXiv:1011.6647 [hep-ph] ...



- pp → W/Z
- pp → W+Z, WW, ZZ
- pp → W/Z + 1 jet
- pp → W/Z +2 jets
- pp → t W
- pp → tX (s&t channel)
- pp → tt
- pp → W/Z+H
- pp (gg) → H
- pp→(gg) → H + 1 jet
- pp→(gg) → H + 2 jets
- pp(VV) → H +2 jets
- pp → W/Z +b , W+c
- pp → W/Z +bb



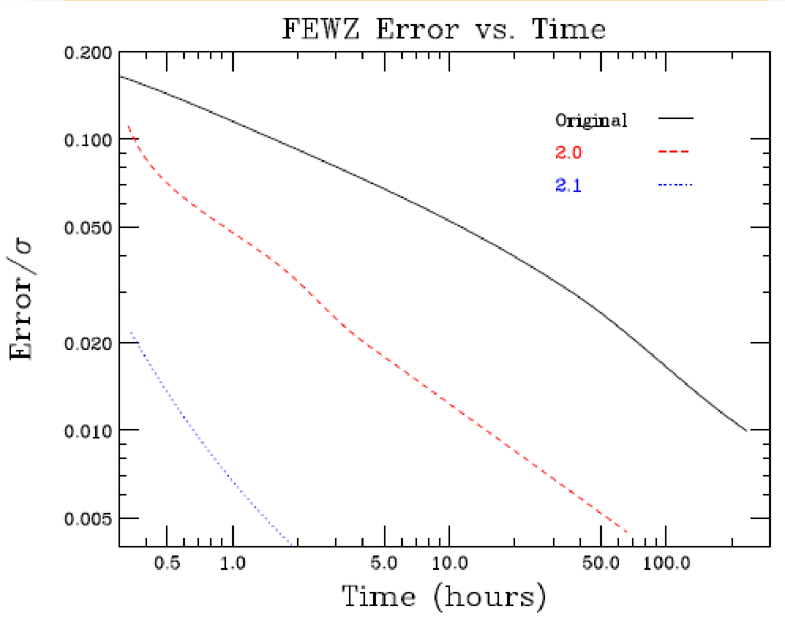
and more...

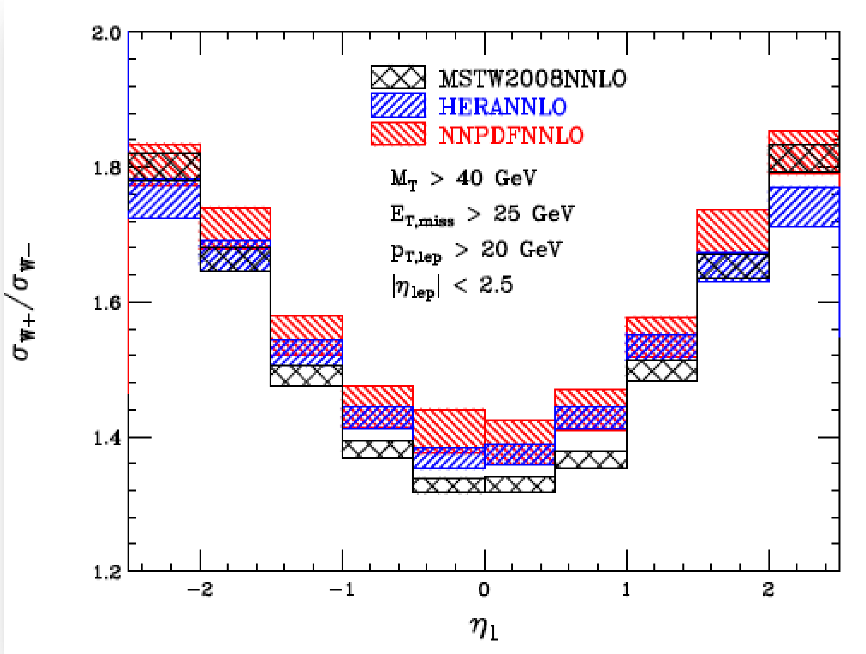
FEWZ v2.1

→ Parton Level Montecarlo of fully exclusive NNLO QCD calculation of W/Z production (including decaying products)
→ Reference for Drell-Yan studies at Hadron Colliders
→ Important recent improvements on convergence of numerical integration for observables

Frank Petriello, Seth Quackenbush, Ryan Gavin, Ye Li

http://gate.hep.anl.gov/fpetriello/FEWZ.html
arXiv:1201.5896 [hep-ph] arXiv:1011.3540 [hep-ph]
…





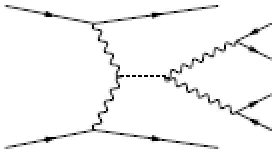
Recently Catani, Cieri, Ferrara, de Florian and Grazzini have presented a similar/alternative code (see for example arXiv:0903.2120 [hep-ph]) which should be made public soon.

VBFNLO v2.6.0

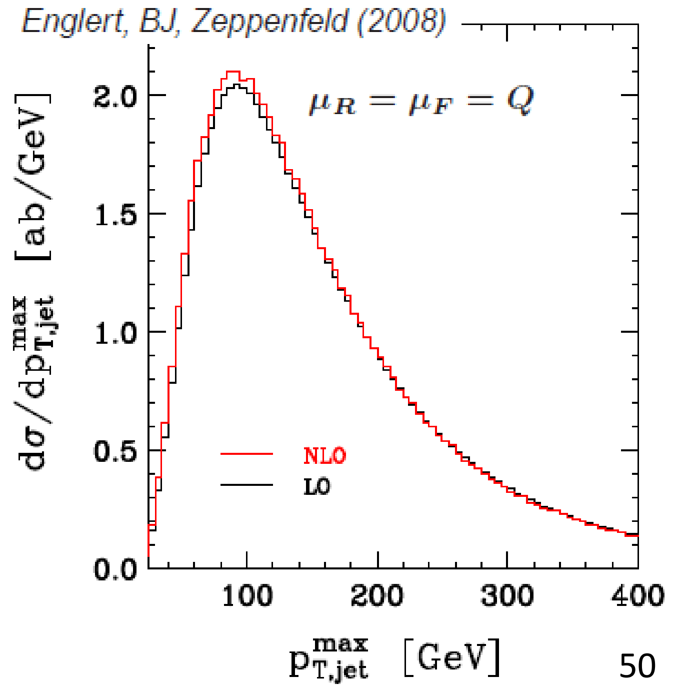
Arnold, Bellm, Bozzi, Campanario, Englert, Feigl, Frank, Figy, Jäger, Kerner, Kubocz, Oleari, Palmer, Rauch, Rzehak, Schissler, Schlimpert, Spannowsky, Zeppenfeld

→Flexible Parton Level Montecarlo at NLO-QCD
→ Meant for processes with EW bosons
→ Includes calculations for CP-odd and CP-even Higgs boson production

http://www-itp.particle.uni-karlsruhe.de/~vbfnloweb/
arXiv:1207.4975 [hep-ph] arXiv:1107.3149 [hep-ph]
arXiv:1106.4009 [hep-ph] …



EW $VVjj$ production



it can simulate:

- various weak vector boson fusion processes
- double and triple weak boson production processes
- double weak boson production processes in association with a hard jet
- Higgs production via gluon fusion in association with two jets

BlackHat + SHERPA

Bern, Dixon, FFC, Hoeche, Ita, Kosower, Maitre, Ozeren

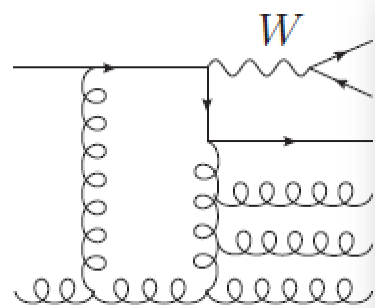
→Automated implementation of on-shell and unitarity techniques to NLO QCD computations

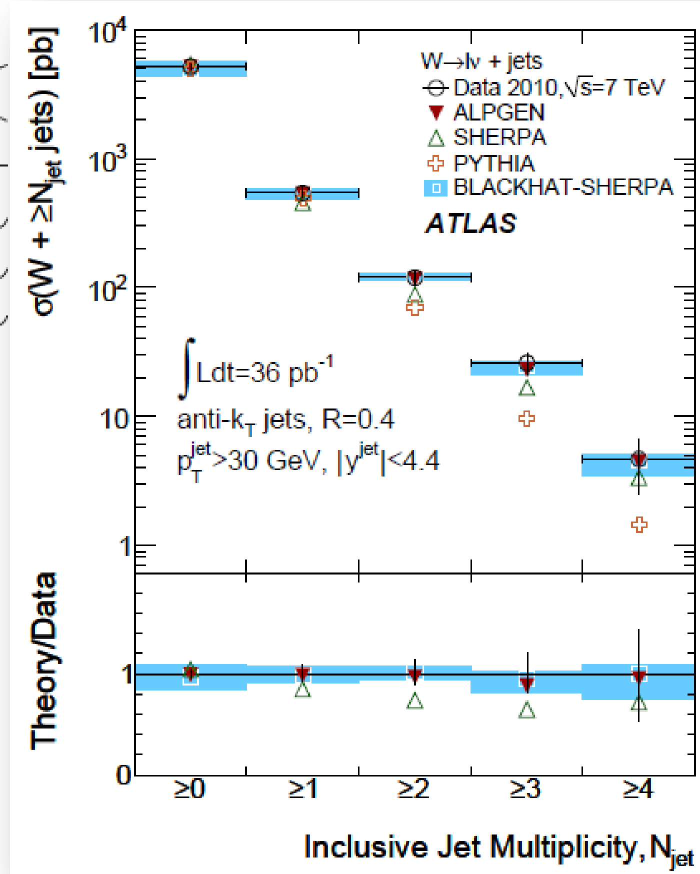
→ Focus on state of the art processes with large amount of jets (V+1,2,3,4,5 jets, pure QCD 2,3,4 jet production)

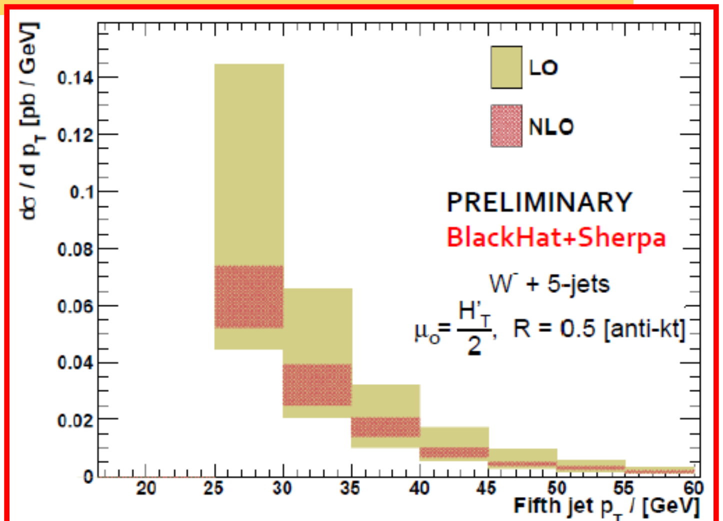
→ Access to calculations through **NTUPLES**: Flexible to allow user defined scale variations, change of PDFs, extract any IR safe observable, etc

http://blackhat.hepforge.org/ (not public yet, ntuples available) http://sherpa.hepforge.org/trac/wiki

arXiv:1206.6064 [hep-ph], arXiv:1112.3940 [hep-ph], arXiv:1108.2229 [hep-ph] …





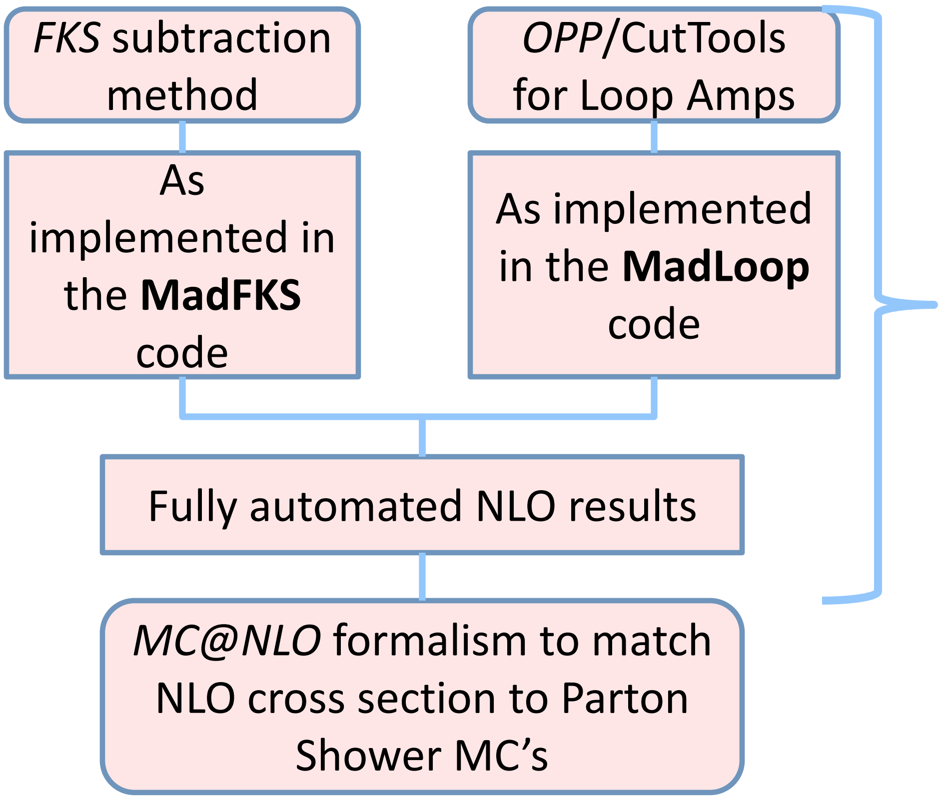


The aMC@NLO Framework

→ Collaborative Project for public automated MC tools for event generators with NLO precision for the LHC (built around MadGraph)

Alwall, Artoisenet, Frederix, Frixione, Fuks, Hirschi, Maltoni, Mattelaer, Pittau, Serret, Stelzer, Torrielli, Zaro

http://amcatnlo.web.cern.ch/ arXiv:1110.5502 [hep-ph] arXiv:1010.0568 [hep-ph] …



NLO all in *one go*…

	Process	μ	n_{lf}	Cross section (pb) LO	Cross section (pb) NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

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FKS subtraction method

As implemented in the **MadFKS** code

OPP/CutTools for Loop Amps

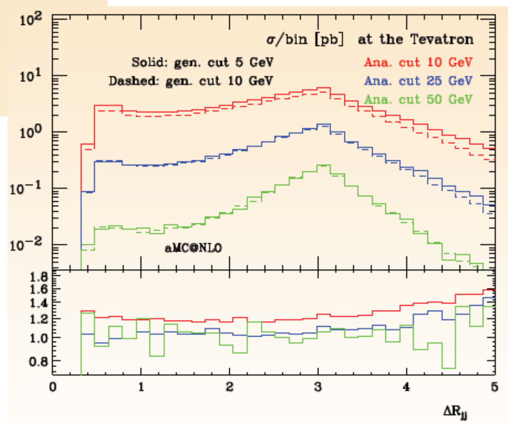
As implemented in the **MadLoop** code

Fully automated NLO results

MC@NLO formalism to match NLO cross section to Parton Shower MC's

NLO+PS also all in *one go*…

PP → HTT/ATT
PP → WBB/ZBB
PP → ZZ → 4L
PP → WJ
PP → WJJ



Watch out for similar progress within the **POWHEG BOX** and **SHERPA** frameworks!!! The later already includes matching of loop ME's for several jet multiplicities (see arXiv:1207.5030 [hep-ph])

And much (much) more…

→ **HRes** (de Florian, Ferrera, Grazzini, Tommasini) *NNLO and NNLL gg fusion production of Higgs (with decay modes!)*

→ **NLOJET++** (Nagy) *C++ library to compute jet cross sections in lepton colliders, DIS and hadron colliders*

→ **FastNLO** (Kluge, Rabbertz, Wobisch) *provides computer codes and tables of pre-computed perturbative coefficients for various observables at hadron colliders*

→ **The PHOX family** (Aurenche, Binoth, Fontannaz, Guillet, Heinrich, Pilon, Werlen) *provides NLO corrections to processes involving Photons, hadrons and jets*

→ …

→ **ROCKET** (Ellis, Melnikov, Zanderighi) *Private F90 program for automated loop calculations using D-dimensional Unitarity*

→ **GOLEM95** (Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers) *Feynman based Fortran 95 program for automated computation of loop diagrams with up to six external legs*

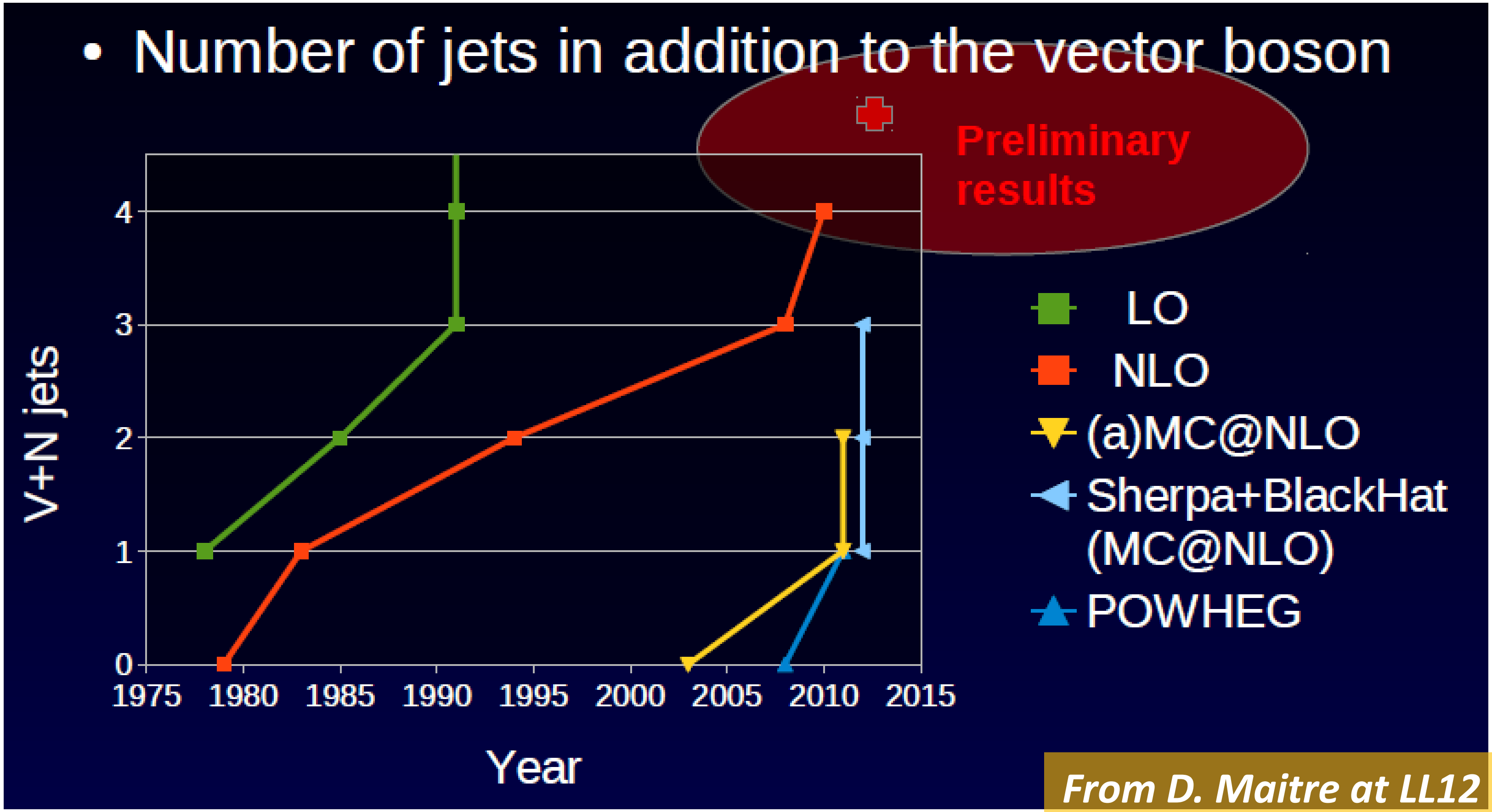
→ **SAMURAI** (Mastrolia, Ossola, Reiter, Tramontano) *Automated implementation to compute loop multi-leg amplitudes within the D-dimensional Unitarity approach*

→ **CutTools** (Pittau) *Automated approach to loop amps/integrals using OPP algorithm*

→ …

CHECK OUT http://www.hepforge.org/downloads/ for a large amount of available programs for High Energy Physics!

What to expect in the near future?



From D. Maitre at LL12

NLO Montecarlo for Standard Experimental Analyses…