Introduction to Monte Carlo Event Generators

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CTEQ-Fermilab School on QCD and Electroweak Phenomenology

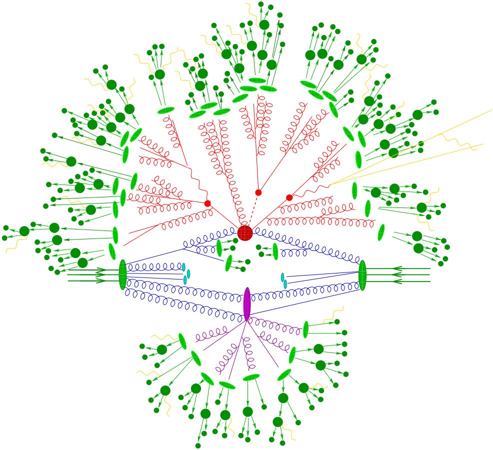
PUPC Lima, 07/30-08/09 2012

Outline

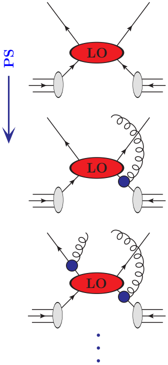
- Merging ME and PS
- Secondary hard interactions
- Hadronization
- Hadron decays

The structure of MC events

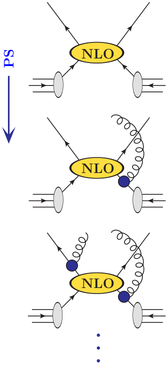
- Hard interaction
- QCD evolution
- Secondary hard interactions
- Hadronization
- Hadron decays
- Higher-order QED corrections



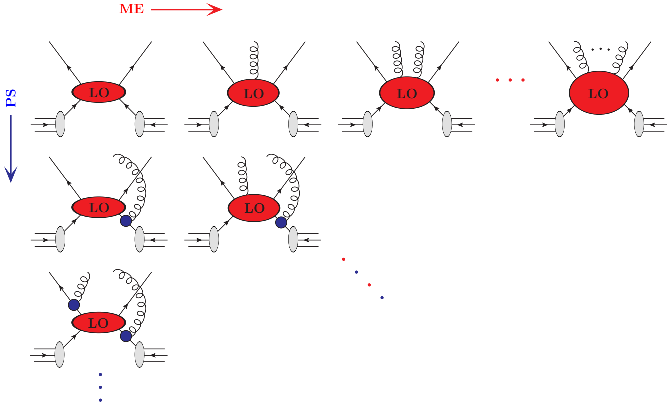
Parton showers



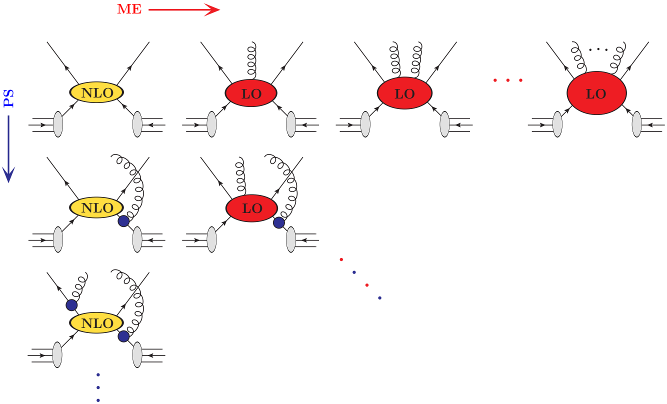
NLO PS matching



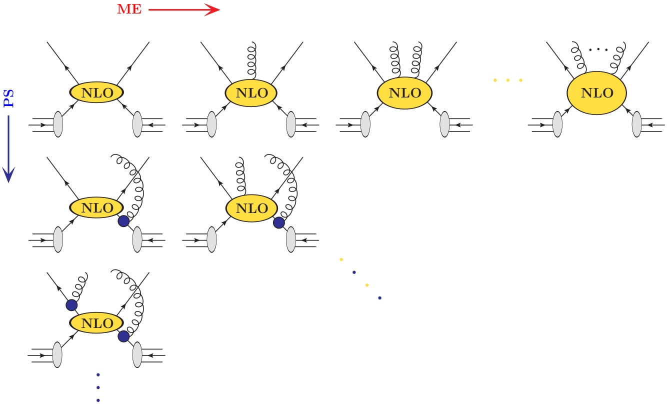
ME PS merging (MEPS)



MEPS combined with NLOPS (MENLOPS)

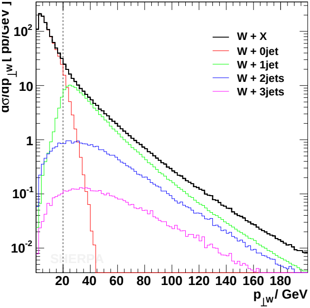


NLOPS merging with NLOPS (MEPS@NLO)



Basic idea of merging

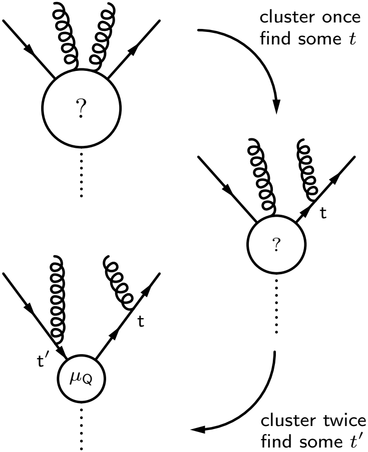
- Separate phase space into "hard" and "soft" region
- Matrix elements populate hard domain
- Parton shower populates soft domain
- Need criterion to define "hard" & "soft" → jet measure Q and corresponding cut, Q_{cut}



Parton-shower histories

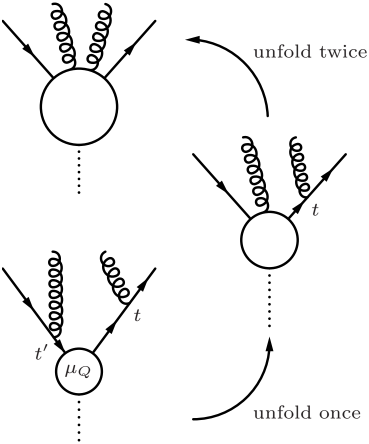
[André,Sjöstrand] hep-ph/9708390

- Start with some "core" process for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be reduced to core by clustering
- If we want to match ME & PS the correct clustering algorithm suggests itself
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until core process



Truncated & vetoed parton showers

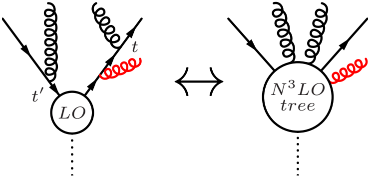
- If higher-multiplicity ME can be clustered back to core that means it is included in the inclusive cross section
- Must compute Sudakov suppression corresponding to no-decay probability of each intermediate parton
  → make inclusive ME exclusive
- Here the merging methods differ most
- Also need to use same scale for α_s as in parton shower



Truncated & vetoed parton showers

Efficient scheme to compute Sudakov suppression: Pseudo-showers

- Start PS from core process
- Evolve until predefined branching
  ↔ truncated parton shower
- Emissions that would produce
  additional hard jets
  lead to event rejection (veto)



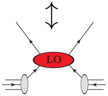
This corresponds to computing a Sudakov form factor given by

$$\Delta_n^{(\mathrm{PS})}(t,\mu_Q^2;>Q_{\mathrm{cut}}) \;=\; \exp\left\{-\int_t^{\mu_Q^2}\mathrm{d}\Phi_1\,K_n(\Phi_1)\,\Theta(Q-Q_{cut})\right\}$$

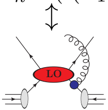
MEPS merging in MC@NLO notation

- Differential event rate to $\mathcal{O}(\alpha_s)$ given by

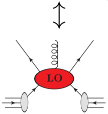
$$\mathrm{d}\sigma_{\mathrm{MEPS}} = \mathrm{d}\Phi_n \mathrm{B}_n(\Phi_n)\Bigg[\,\Delta_n^{(\mathrm{PS})}(t_c,\mu_Q^2)$$



$$+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1 \mathrm{K}_n(\Phi_1)\,\Delta_n^{(\mathrm{PS})}(t(\Phi_1),\mu_Q^2)\,\Theta(Q_{\mathrm{cut}}-Q)\Bigg]$$



$$+\,\mathrm{d}\Phi_n\int\mathrm{d}\Phi_1\,\mathrm{B}_{n+1}(\Phi_{n+1})\,\Delta_n^{(\mathrm{PS})}(t(\Phi_{n+1}),\mu_Q^2;>Q_{\mathrm{cut}})\,\Theta(Q-Q_{\mathrm{cut}})$$



- Jet veto in PS
- Jet cut on $n+1$-parton final state

CKKW-L and METS merging

Algorithms with

- Exact correspondence between clustering & PS evolution
- Sudakov form factors as defined in parton shower

[Lönnblad] hep-ph/0112284
[Lönnblad,Prestel] arXiv:1109.4829

CKKW-L (Pythia)

- Truncated showers generate suppression, but no emissions
- Jet criterion dynamically redefined during PS evolution

[SH,Krauss,Schumann,Siegert] arXiv:0903.1219
[Hamilton,Richardson,Tully] arXiv:0905.3072

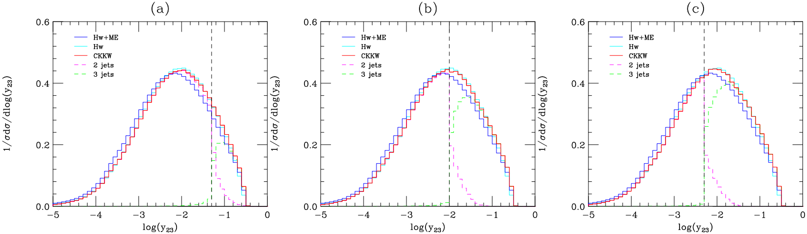
METS (Herwig, Sherpa)

- Truncated parton showers generate emissions and suppression
- Accounts for mismatch between jet criterion and evolution variable

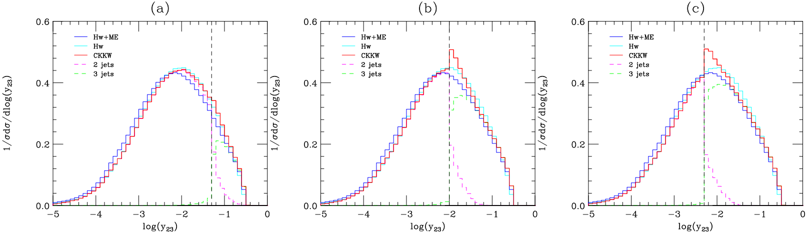
Effect of truncated showers in $e^+e^- \to$ hadrons

[Hamilton, Richardson, Tully] arXiv:0905.3072

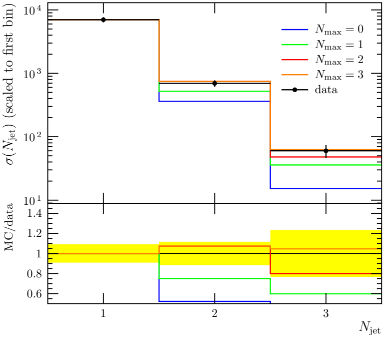
truncated shower on

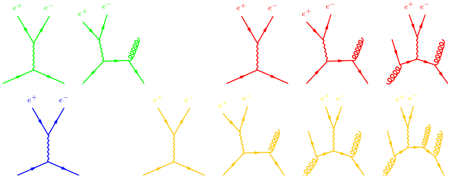


truncated shower off



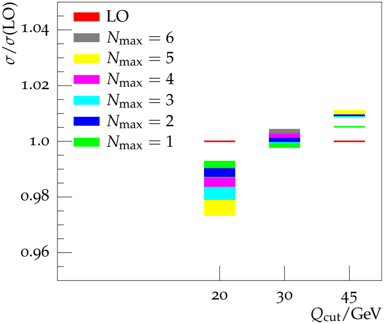
Z+jets at the Tevatron





- MC predictions for exclusive n-jet rates match data well as long as corresponding final states are described by matrix elements

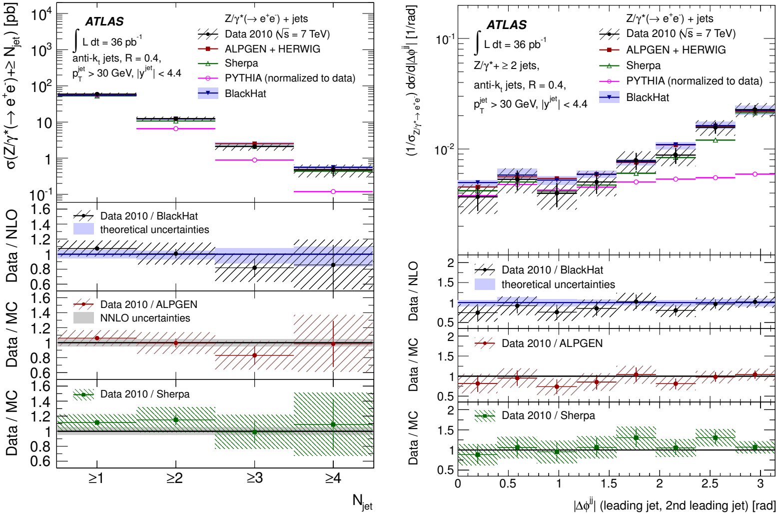
Z+jets at the Tevatron



- MEPS effectively replaces splitting kernels of the parton shower with ratios of LO matrix elements for the emission terms
- We have not corrected the Sudakov form factors, hence there is a mismatch between emission- and no-emission probability
- The inclusive cross section changes, but corrections are small

Z+jets at the LHC

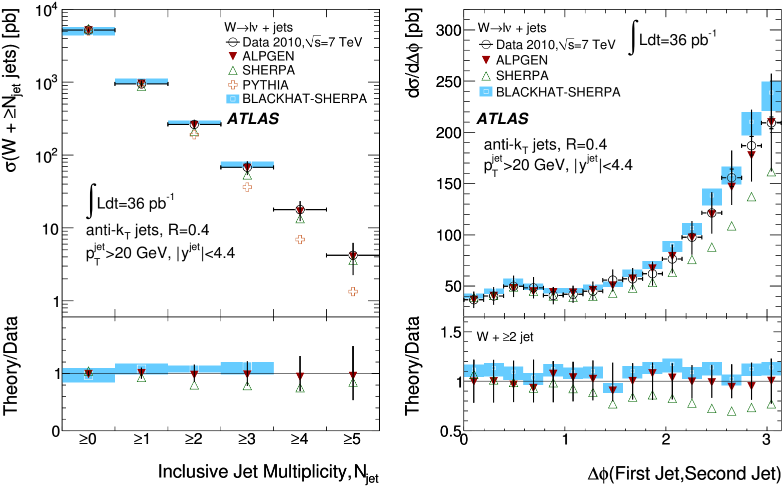
[ATLAS] arXiv:1111.2690



- Good agreement with both ALPGEN (MLM) and Sherpa
- PS alone fails for $n_{\text{jet}} \geq 2$

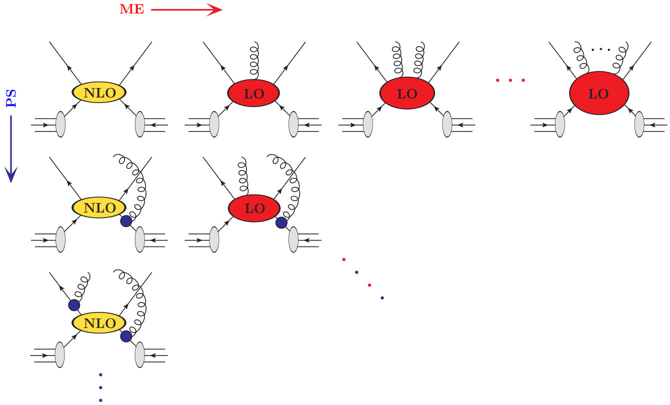
W+jets at the LHC

[ATLAS] arXiv:1201.1267



- Good agreement with ALPGEN (MLM), not so good with Sherpa

MEPS combined with NLOPS (MENLOPS)

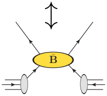


MENLOPS

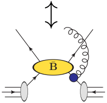
[Hamilton,Nason] arXiv:1004.1764
[SH,Krauss,Schönherr,Siegert] arXiv:1009.1127

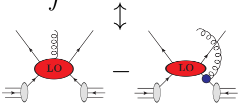
- Increase accuracy below $Q_{\rm cut}$ to full NLO

$$\mathrm{d}\sigma = \mathrm{d}\Phi_n \bar{\mathrm{B}}_n^{(\mathrm{A})}(\Phi_n) \Big[\, \Delta_n^{(\mathrm{A})}(t_c, \mu_Q^2)$$

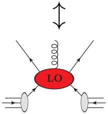


$$+ \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \frac{\mathrm{D}_n^{(\mathrm{A})}(\Phi_{n+1})}{\mathrm{B}_n(\Phi_n)} \Delta_n^{(\mathrm{A})}(t, \mu_Q^2) \Theta(Q_{\rm cut} - Q) \Big] + \mathrm{d}\Phi_n \int \mathrm{d}\Phi_1 \, \mathrm{H}_n^{(\mathrm{K})}(\Phi_{n+1}) \Theta(Q_{\rm cut} - Q)$$





$$+ \mathrm{d}\Phi_n \int \mathrm{d}\Phi_1 \, \mathrm{k}_n^{(\mathrm{A})}(\Phi_{n+1}) \mathrm{B}_{n+1}(\Phi_{n+1}) \, \Delta_n^{(\mathrm{PS})}(t, \mu_Q^2; > Q_{\rm cut}) \, \Theta(Q - Q_{\rm cut})$$



- Local K-factor for smooth merging

MENLOPS

- Local K-factor for POWHEG

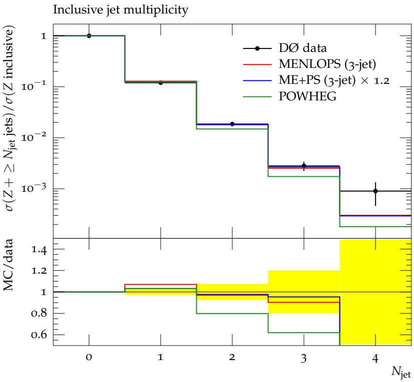
$$\mathrm{k}_n^{(\mathrm{R})}(\Phi_n) = \frac{\bar{\mathrm{B}}_n^{(\mathrm{R})}(\Phi_n)}{\mathrm{B}_n(\Phi_n)}$$

- Local K-factor for MC@NLO

$$\mathrm{k}_n^{(\mathrm{K})}(\Phi_{n+1}) = \frac{\bar{\mathrm{B}}_n^{(\mathrm{K})}(\Phi_n)}{\mathrm{B}_n(\Phi_n)}\left(1 - \frac{\mathrm{H}_n^{(\mathrm{K})}(\Phi_{n+1})}{\mathrm{R}_n(\Phi_{n+1})}\right) + \frac{\mathrm{H}_n^{(\mathrm{K})}(\Phi_{n+1})}{\mathrm{R}_n(\Phi_{n+1})}$$

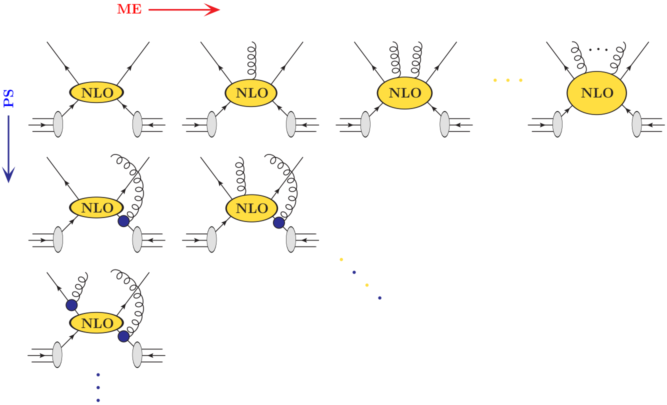
- Amounts to rescaling higher-multiplicity ME such that their contribution to MENLOPS event sample is the same as their original contribution to the MEPS event sample

Z+jets at Tevatron



- Jet rates in MENLOPS improved over NLOPS
- Total cross section in MENLOPS improved over MEPS

NLOPS merging with NLOPS (MEPS@NLO)



NL^{3}SP

[Lavesson,Lönnblad] arXiv:0811.2912
[Lönnblad,Prestel] ICHEP'12

- Ingredients: Rescaled CKKW-L plus POWHEG sample
  Global K-factor for CKKW-L $\to K = 1 + \sum \alpha_s^i(\mu_R^2) k_i$
- Change NLO parton-level scales to CKKW-L scheme
  - Renormalization scale $\to$ use 1-loop running of α_s

$$\alpha_s(\mu_R^2) \to \alpha_s(\mu_R^2)\left(1 - \frac{\alpha_s(\mu_R^2)}{2\pi}\beta_0 \log\frac{t}{\mu_R^2}\right)$$

  - Factorization scale $\to$ use DGLAP evolution of PDFs

$$f_a(x,Q^2) \to f_a(x,Q^2) + \frac{\alpha_s(\mu_R^2)}{2\pi}\log\frac{t}{\mu_F^2}\sum_{b=q,g}\int_x^1 \frac{\mathrm{d}z}{z}\,P_{ab}(z)\,f_b(x/z,\mu_F^2)$$

- Remove $1 + \mathcal{O}(\alpha_s)$ term present in NLO-scaled CKKW-L sample

$$K\Delta_n^{(\mathrm{PS})}(t_c,\mu_Q^2;>Q_{\mathrm{cut}}) - \Big(1 + \alpha_s(\mu_R^2)\,k_1\Big) + \int\limits_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\,\frac{\alpha_s(\mu_R^2)}{\alpha_s(t)}\mathrm{K}_n\,\Theta(Q_{n+1} - Q_{\mathrm{cut}})$$

NL^{3}SP

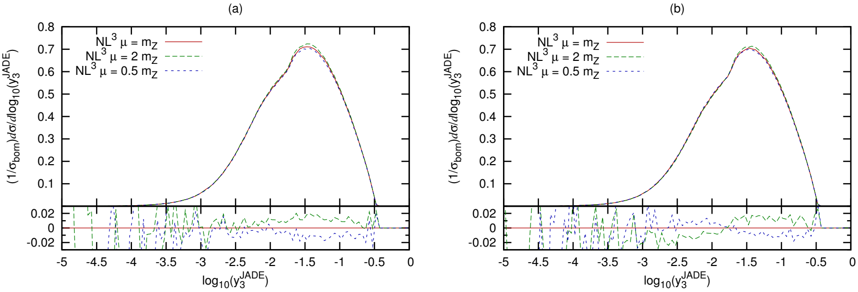
- Differential event rate for exclusive $n+k$-jet events assuming all scales already chosen correct in POWHEG

$$\begin{aligned}
\mathrm{d}\sigma^{n+k,\mathrm{excl}}_{\mathrm{NL^3SP}} = {}& \mathrm{d}\Phi_{n+k}\,\Theta(Q_{n+k}-Q_{\mathrm{cut}})\prod_{i=n}^{n+k-1}\Delta_i^{(\mathrm{PS})}(t_{i+1},t_i;<Q_{\mathrm{cut}})\\
&\times\Bigg\{\bar{\mathrm{B}}^{(\mathrm{R})}_{n+k}\Bigg[\Delta^{(\mathrm{R})}_{n+k}(t_c,t_{n+k})+\int_{t_c}^{t_{n+k}}\mathrm{d}\Phi_1\,\frac{\mathrm{R}_{n+k}}{\mathrm{B}_{n+k}}\,\Delta^{(\mathrm{R})}_{n+k}(t,t_n)\,\Theta(Q_{\mathrm{cut}}-Q_{n+k+1})\Bigg]\\
&\quad+\mathrm{B}_{n+k}\Bigg(K\prod_{i=n}^{n+k-1}\Delta_i^{(\mathrm{PS})}(t_{i+1},t_i;>Q_{\mathrm{cut}})\\
&\qquad-\Big(1+\alpha_s(\mu_R^2)\,k_1\Big)+\sum_{i=n}^{n+k}\int_{i+1}^{i}\mathrm{d}\Phi_1\,\mathrm{K}_i\,\Theta(Q_{i+1}-Q_{\mathrm{cut}})\Bigg)\Bigg\}
\end{aligned}$$

- If emission above t_{n+k}, but below Q_{cut}, reject event → PS domain
- Subtraction needed only in ME / PS overlap region

e^+e^- →hadrons at LEP

[Lavesson,Lönnblad] arXiv:0811.2912



- Scale variations around 2%
- Agreement between 1- and 2-loop
  but no further reduction of uncertainty

MEPS@NLO

[SH,Krauss,Schönherr,Siegert] arXiv:1207.5030
[Gehrmann,SH,Krauss,Schönherr,Siegert] arXiv:1207.5031

- Define compound evolution kernel

$$\begin{aligned}\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) &= \mathrm{D}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1})\,\Theta(t_{n+k}-t_{n+k+1})\\ &\quad+\mathrm{B}_{n+k}(\Phi_{n+k})\sum_{i=n}^{n+k-1}\mathrm{K}_i(\Phi_i)\,\Theta(t_i-t_{n+k+1})\Theta(t_{n+k+1}-t_{i+1})\end{aligned}$$

- Extend MC@NLO modified subtraction

$$\begin{aligned}\tilde{\mathrm{B}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k}) &= \Big[\mathrm{B}_{n+k}(\Phi_{n+k})+\tilde{\mathrm{V}}_{n+k}(\Phi_{n+k})+\mathrm{I}_{n+k}(\Phi_{n+k})\Big]\\ &\quad+\int\mathrm{d}\Phi_1\Big[\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1})-\mathrm{S}_{n+k}(\Phi_{n+k+1})\Big]\\ \tilde{\mathrm{H}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) &= \mathrm{R}_{n+k}(\Phi_{n+k+1})-\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1})\end{aligned}$$

- Scales of NLO calculation chosen in accordance with MEPS

MEPS@NLO

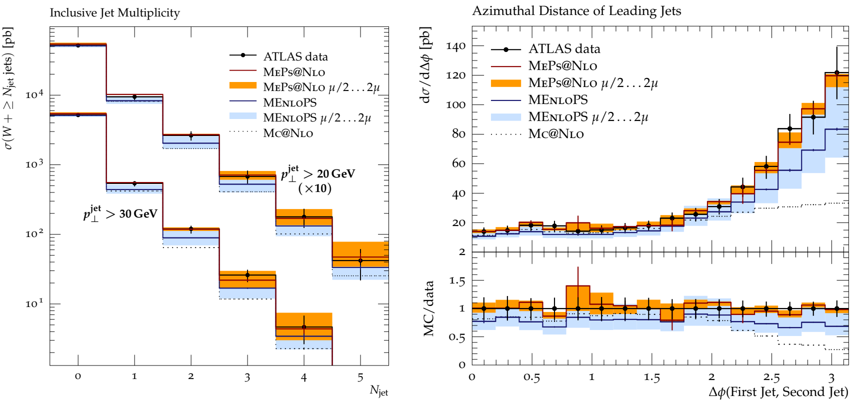
- Differential event rate for exclusive $n+k$-jet events

$$\begin{aligned}\mathrm{d}\sigma^{n+k,\mathrm{excl}}_{\mathrm{MEPS@NLO}} &= \mathrm{d}\Phi_{n+k}\,\Theta(Q(\Phi_{n+k})-Q_{\mathrm{cut}})\,\tilde{\mathrm{B}}^{(\mathsf{A})}_{n+k}\\ &\quad\times\left[\tilde{\Delta}^{(\mathsf{A})}_{n+k}(t_c,\mu_Q^2)+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\,\frac{\tilde{\mathrm{D}}^{(\mathsf{A})}_{n+k}}{\mathrm{B}_{n+k}}\,\tilde{\Delta}^{(\mathsf{A})}_{n+k}(t,\mu_Q^2)\,\Theta(Q_{\mathsf{cut}}-Q_{n+k+1})\right]\\ &\quad+\int\mathrm{d}\Phi_{n+k+1}\,\tilde{\mathrm{H}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1})\,\tilde{\Delta}^{(\mathrm{PS})}_{n+k}(t_{n+k+1},\mu_Q^2;>Q_{\mathrm{cut}})\,\Theta(Q_{\mathsf{cut}}-Q(\Phi_{n+k+1}))\end{aligned}$$

- Structurally equivalent to MENLOPS
- Truncated PS contributes at $\mathcal{O}(\alpha_s)$

W+jets at LHC

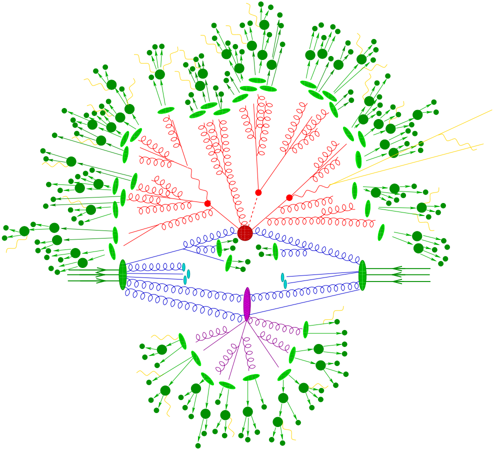
[SH,Krauss,Schönherr,Siegert] arXiv:1207.5030



- MEPS@NLO with 0,1&2 jet PL at NLO plus 3&4 jet PL at LO
- MENLOPS with 1-4 jet PL at LO

The structure of MC events

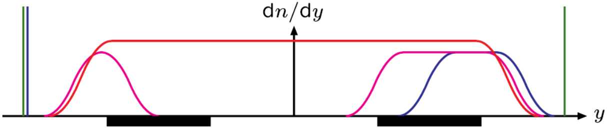
- Hard interaction
- QCD evolution
- Secondary hard interactions
- Hadronization
- Hadron decays
- Higher-order QED corrections



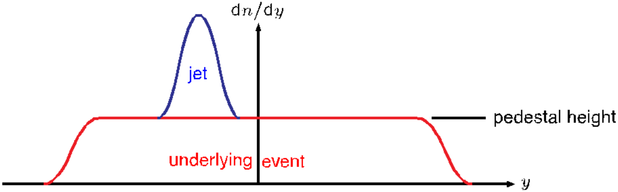
What is what

- Soft inclusive collision

$$\sigma_{\text{tot}} = \sigma_{\text{elastic}} + \sigma_{\text{single diffractive}} + \sigma_{\text{double diffractive}} + \sigma_{\text{non-diffractive}}$$

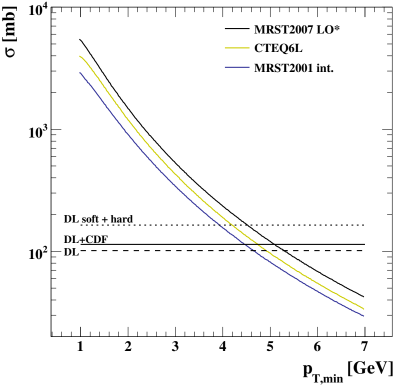


- Underlying event



Modeling the pedestal

[Sjöstrand,Zijl] PRD36(1987)2019

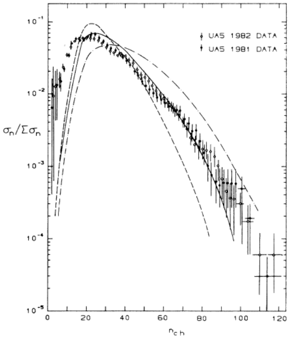


- Partonic cross sections diverge roughly like $\mathrm{d}p_T^2/p_T^4$
- Total cross section at LHC exceeded for $\mathrm{p_T} \approx$ 2-5 GeV
- Interpretation as possibility for multiple hard scatters with

$$\langle n \rangle = \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$

- Main free parameter is $p_{T,\mathrm{min}}$
  Determines size of σ_{hard}

Modeling the pedestal



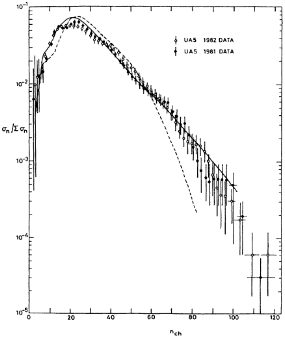
- Despite MPI wrong charged multi distribution
  Impact parameter dependent model needed
- Various hadron shape models in b-space
  (Exponential, Gaussian, double Gaussian)

$$\langle n \rangle = \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$

$$\downarrow$$

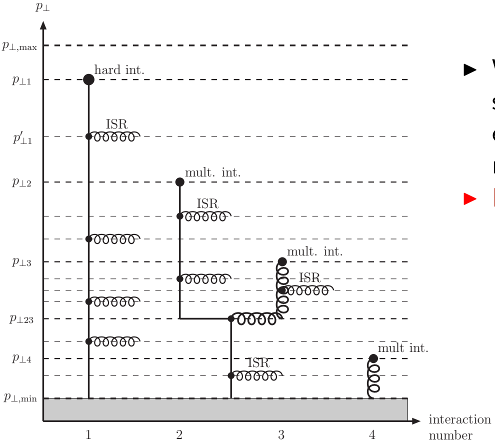
$$\langle \tilde{n}(b) \rangle = f_c f(b) \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$

- Hardness of the collision determines overlap
  Collisions with large overlap in turn
  have more secondary interactions



Combination with the parton shower

[Sjöstrand,Skands] hep-ph/0408302

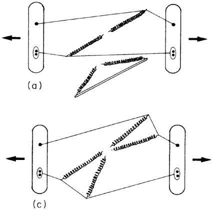


- When attaching IS shower to secondary scattering can ask at each point whether emission or new interaction is more likely
- New evolution equation

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}p_T} = \left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{MI}}}{\mathrm{d}p_T} + \frac{\mathrm{d}\mathcal{P}_{\mathrm{ISR}}}{\mathrm{d}p_T}\right) \times \exp\left\{-\int_{p_T} \mathrm{d}p'_T \left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{MI}}}{\mathrm{d}p'_T} + \frac{\mathrm{d}\mathcal{P}_{\mathrm{ISR}}}{\mathrm{d}p'_T}\right)\right\}$$

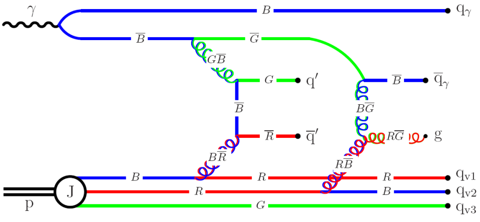
Color connections and beam remnants

[Sjöstrand,Skands] hep-ph/0402078



- Secondary scatterings need to be color-connected to something
- Simplest model would decouple them from proton remnants
- Next-to-simplest model would put all scatters on one color string

- New models embed scatters into existing color topology
- Three different options for string drawing
  - At random
  - Rapidity ordered
  - String length optimized



A model for minimum bias collisions

[Butterworth,Forshaw,Seymour] hep-ph/9601371
[Borozan,Seymour] hep-ph/0207283

- Assume parton distribution within beam hadron is

$$\frac{\mathrm{d}n_a(x,\mathbf{b})}{\mathrm{d}^2\mathbf{b}\mathrm{d}x} = f_a(x)\,G(\mathbf{b})$$

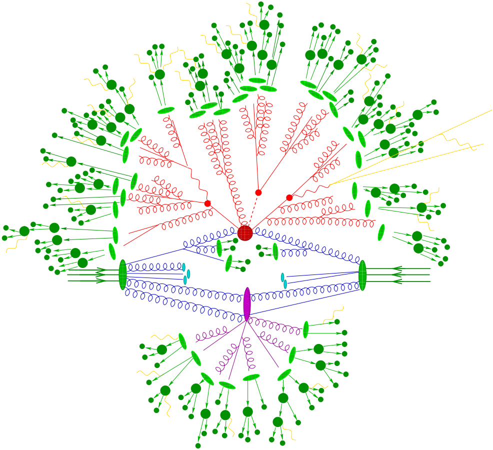
- Use electromagnetic form factor

$$G(\mathbf{b}) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\,\frac{\exp(\mathbf{k}\cdot\mathbf{b})}{(1+\mathbf{k^2}/\mu^\mathbf{2})^\mathbf{2}}$$

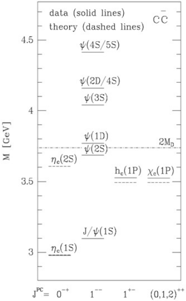
- EM measurements indicate $\mu_P = 0.71$ GeV
  μ is however left free in model $\rightarrow$ tuning
- Continue model below $p_{T,\mathrm{min}}$ with same b-space parametrization
  but cross section as Gaussian in p_T $\rightarrow$ inclusive non-diffractive events

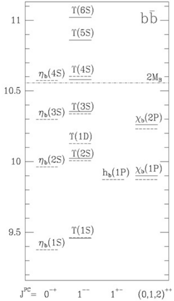
The structure of MC events

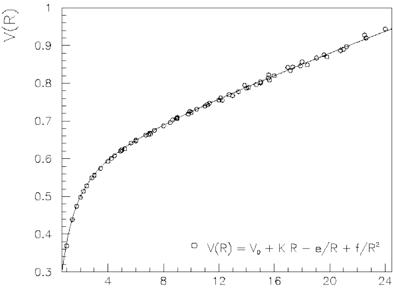
- Hard interaction
- QCD evolution
- Secondary hard interactions
- Hadronization
- Hadron decays
- Higher-order QED corrections



The inter-quark potential



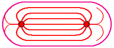




- Measure QCD potential from quarkonia masses
- Or compute using lattice QCD
- Approximately linear potential ↔ QCD flux tube

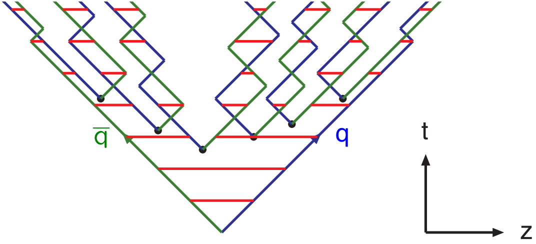
The Lund string model

[Andersson,Gustafson,Ingelman,Sjöstrand] PR97(1983)31

- Start with example $e^+e^- \to q\bar{q}$
- QCD flux tube with constant energy per unit rapidity ↔ 
- New $q\bar{q}$-pairs created by tunneling (κ - string tension)

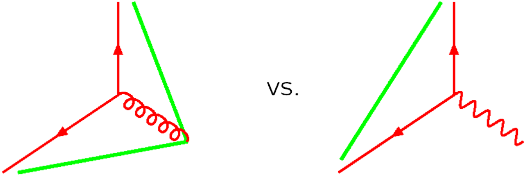
$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}x\mathrm{d}t} = \exp\left\{-\frac{\pi^2 m_q^2}{\kappa}\right\}$$

- Expanding string breaks into hadrons, then yo-yo modes
- Baryons modeled as quark-diquark pairs



The Lund string model

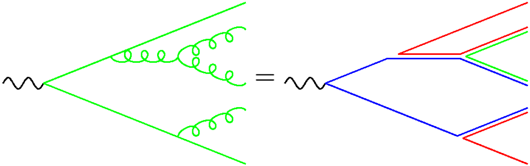
- String model very well motivated, but many parameters
- But also gives genuine prediction of “string effect”
- Gluons are kinks on string
  String accelerated in direction of gluon
- Infrared safe matching to parton showers
  Gluons with $k_T \lesssim 1/\kappa$ irrelevant



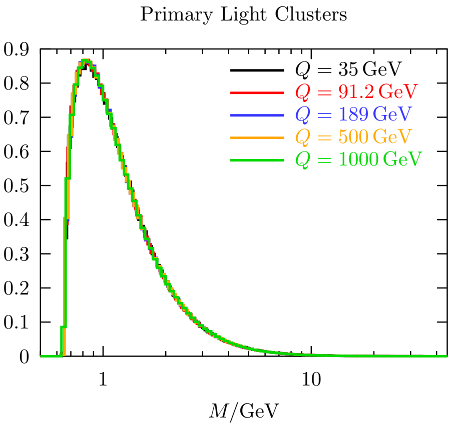
The cluster model

[Webber] NPB238(1984)492

- Underlying idea: Preconfinement
- Follow color structure of parton showers:
  color singlets end up close in phase space
- Mass of color singlets peaked at low scales ($\approx t_c$)



The cluster model



- Mass spectrum of primordial clusters independent of cm energy

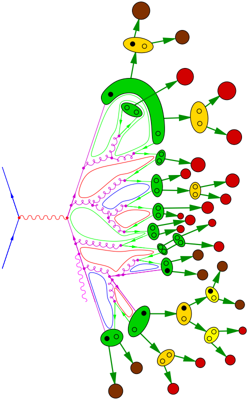
The cluster model

Naïve model

- Split gluons into $q\bar{q}$-pairs
- Color-adjacent pairs form primordial clusters
- Clusters decay into hadrons according to phase space → baryon & heavy quark production suppressed

Improved model

- Heavy clusters decay into lighter ones
- Three options: C→CC, C→CH & C→HH
- Leading particle effects



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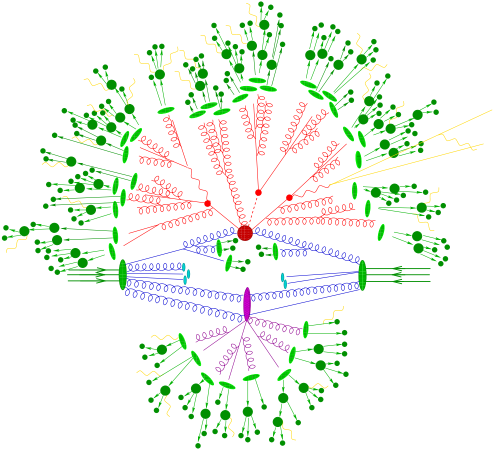
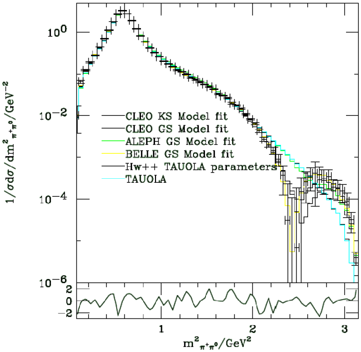


Secondary particle decays

- String and clusters decay to some stable hadrons but main outcome are unstable resonances
- These decay further according to the PDG decay tables
- Many hadron decays according to phase space but also a large variety of form factors known
- Not all branching ratios known precisely plus many BR's in PDG tables do not add up to one
- Significant effect on hadronization yields, hadronization corrections to event shapes, etc.

Secondary particle decays

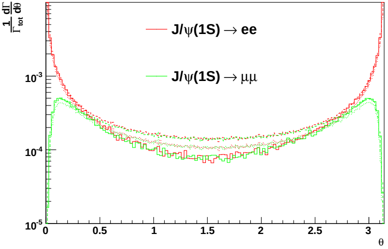
- Previous generations of generators relied on external decay packages Tauola (τ-decays) & EvtGen (B-decays)
- New generation programs Herwig++ & Sherpa contain at least as complete a description
- Spin correlations and B-mixing built in
- No interfacing issues



Photon radiation

[Yennie,Frautschi,Suura] AP13(1961)379

- Previous generations of generators relied on external package Photos to simulate QED radiation
- New generation programs Herwig++ & Sherpa have simulation of QED radiation built in



Angular radiation pattern in ℓ^+-ℓ^- frame

Summary

- Parton showers can be merged with matrix elements at LO or NLO to improve the description of multiple exclusive observables at once
- Secondary hard interactions are modeled, inspired by QCD but currently least well understood and weakly constrained
- Hadronization process modeled, but better understood largely constrained by data, extrapolation to LHC ok
- Some event generators have B- and τ-decay as well as photon radiation simulations built in