CTEQ-Fermilab School on QCD and Electroweak Phenomenology

CTEQ

Deeply Inelastic Scattering (DIS)

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Lima, Peru

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Fermilab

PONTIFICIA UNIVERSIDAD CATÓLICA DEL PERÚ

CONCYTEC

National Science Foundation
WHERE DISCOVERIES BEGIN

U.S. DEPARTMENT OF ENERGY

UNIVERSIDAD NACIONAL DE INGENIERIA
Scientia et Labor
136 años
Ciencia y Tecnología al Servicio del País

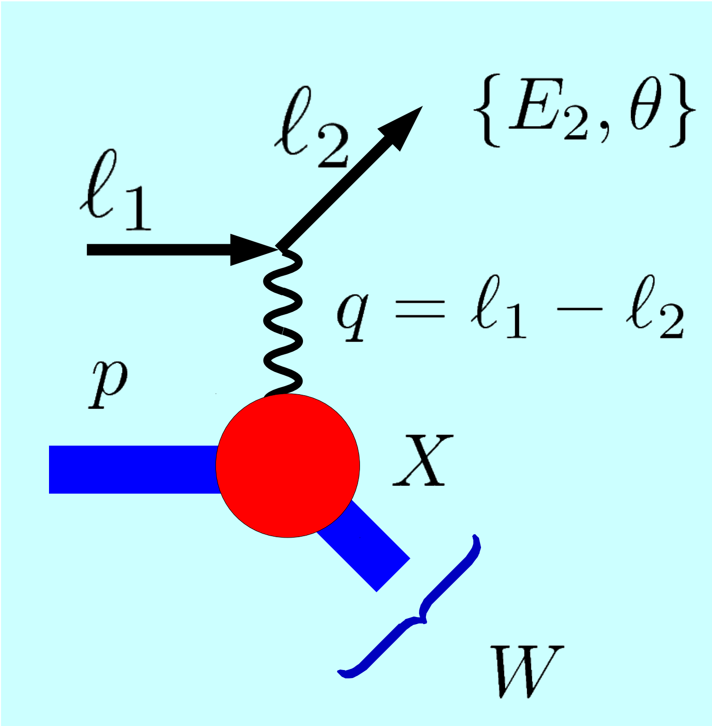
HOW TO CHARACTERIZE THE PROTON

Deeply Inelastic Scattering

(DIS)

... also see lectures by George Sterman

Inclusive Deeply Inelastic Scattering (DIS)

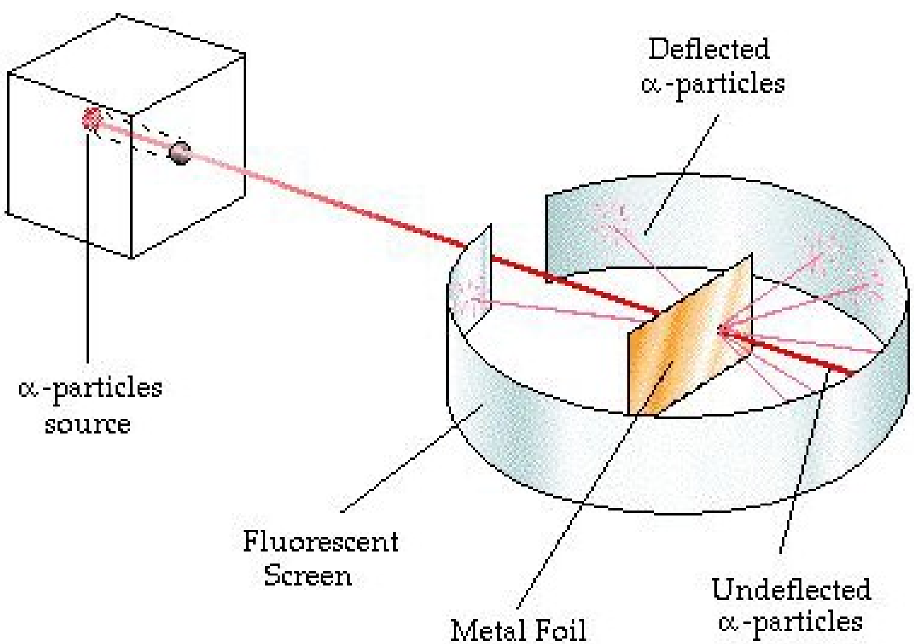


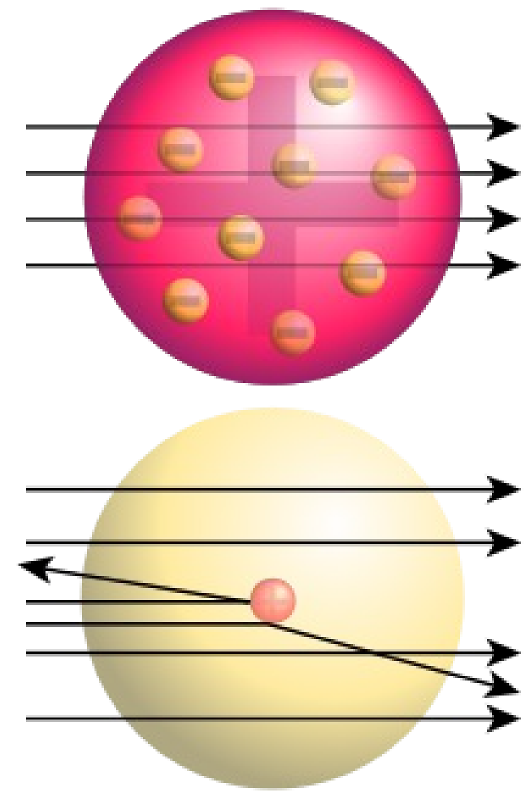
Measure $\{E_2, \theta\} \Leftrightarrow \{x, Q^2\}$ Inclusive

Deep: $Q^2 \geq 1 GeV^2$

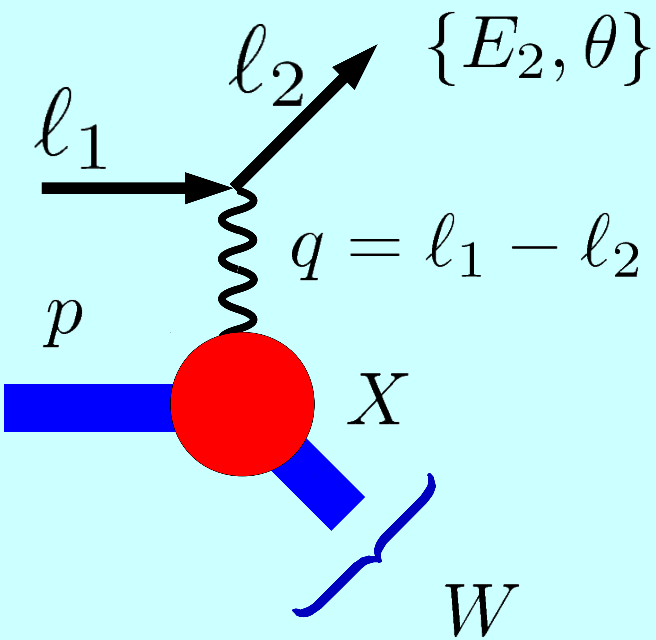
Inelastic: $W^2 \geq M_p^2$

Analogue of Rutherford scattering





Inclusive Deeply Inelastic Scattering (DIS)



Measure $\{E_2, \theta\} \Leftrightarrow \{x, Q^2\}$

$$Q^2 = -q^2 = 4E_1 E_2 \sin^2(\theta/2)$$

$$x = \frac{Q^2}{2p \cdot q} = \frac{2E_1 E_2 \sin^2(\theta/2)}{M(E_1 - E_2)}$$

x: partonic momentum fraction*
Q: characteristic energy scale

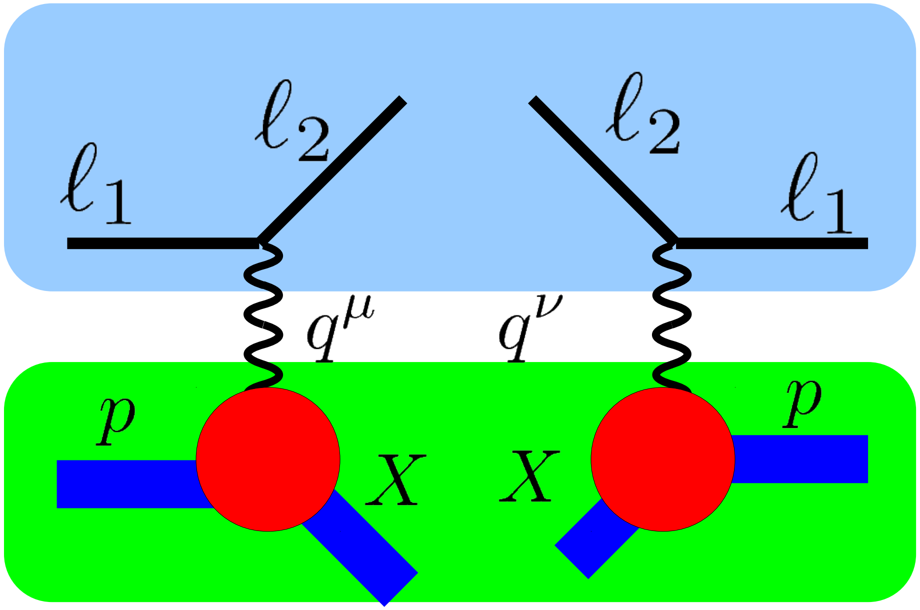
$$d\sigma \sim |A|^2$$

Other common DIS variables

$$\nu = \frac{p \cdot q}{p^2} = E_1 - E_2$$

$$y = \frac{\nu}{E_1} = \frac{Q^2}{2ME_2 x}$$

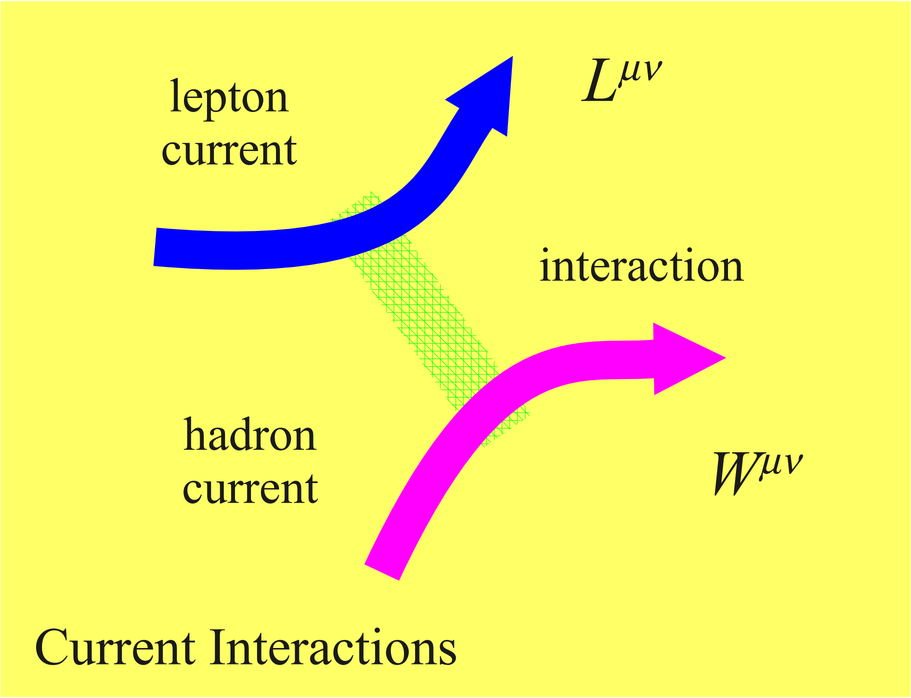
Lepton Tensor (L) and Hadronic Tensor (W)



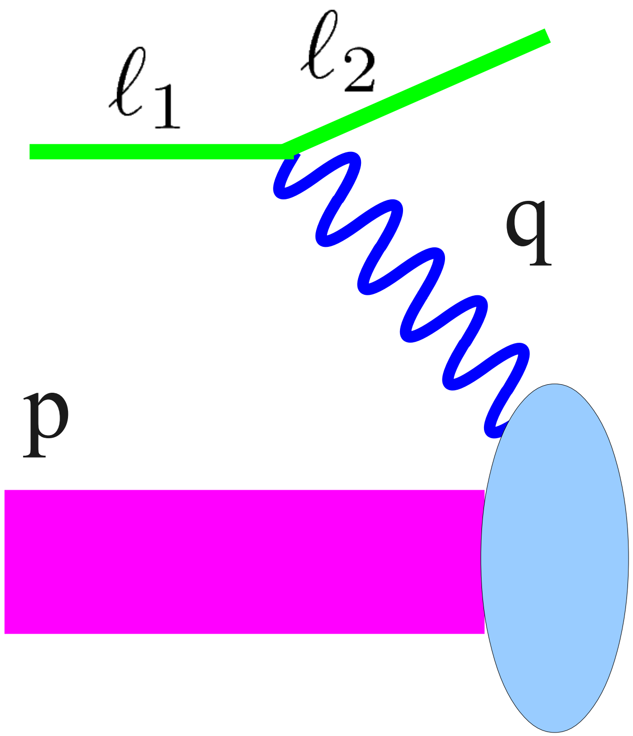
$L^{\mu\nu}$ Leptonic Tensor

$W_{\mu\nu}$ Hadronic Tensor

$$d\sigma \sim |A|^2 \sim L^{\mu\nu} W_{\mu\nu}$$



W and F Structure Functions



$$d\sigma \sim |A|^2 \sim L^{\mu\nu} W_{\mu\nu}$$

$$L^{\mu\nu} = L^{\mu\nu}(\ell_1, \ell_2)$$

$$W^{\mu\nu} = W^{\mu\nu}(p, q)$$

$$W^{\mu\nu} = -g^{\mu\nu} W_1 + \frac{p^\mu p^\nu}{M^2} W_2 - \frac{i\,\epsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma}{2M^2} W_3 + \ldots$$

Convert to “Scaling” Structure Functions

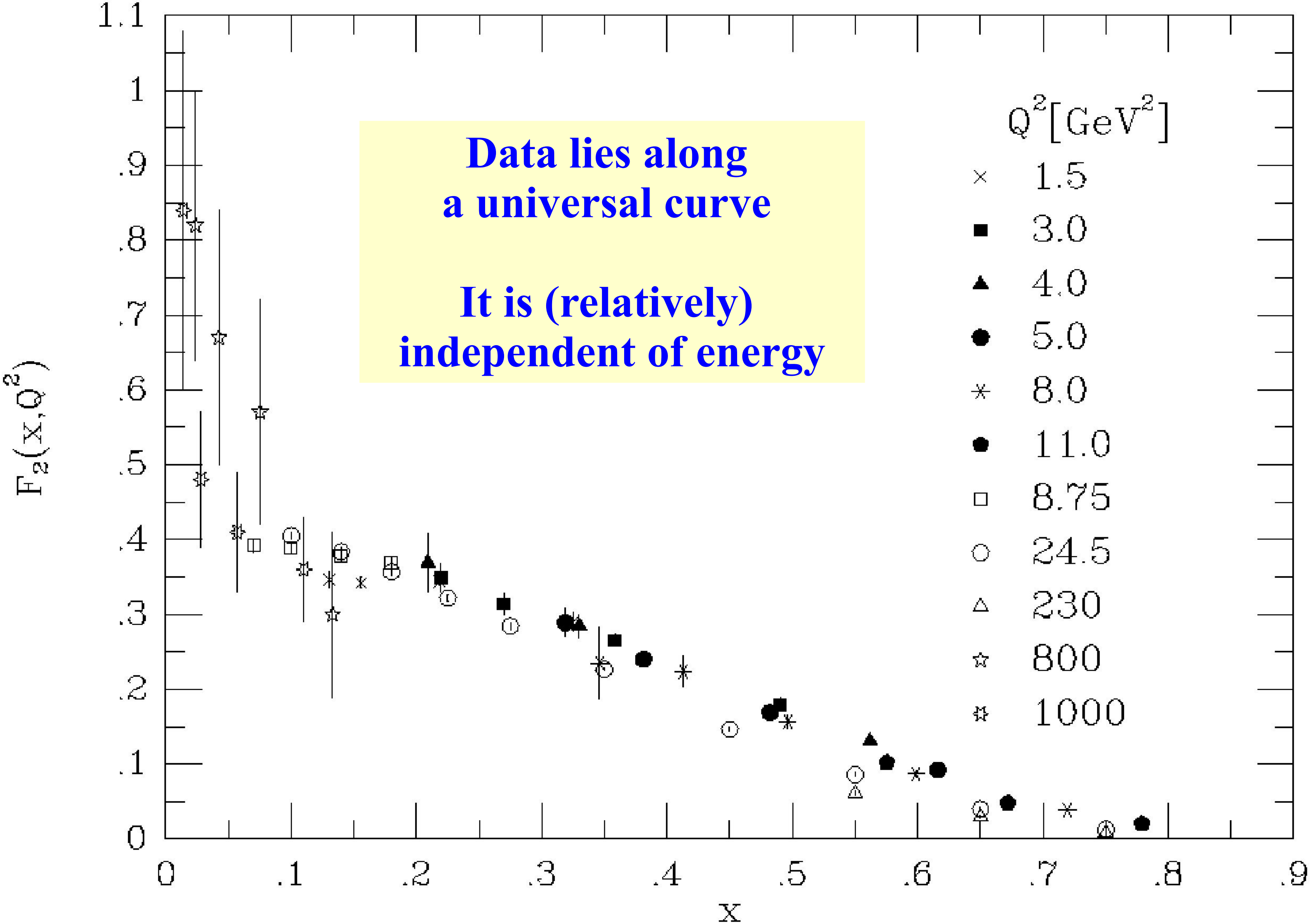
$$W_1 \to F_1 \qquad W_2 \to \tfrac{M}{\nu} F_2 \qquad W_3 \to \tfrac{M}{\nu} F_3$$

$$\frac{d\sigma}{dx\,dy} = N\left[xy^2 F_1 + (1 - y - \tfrac{Mxy}{2E_2})F_2 \pm y(1 - y/2)xF_3\right]$$

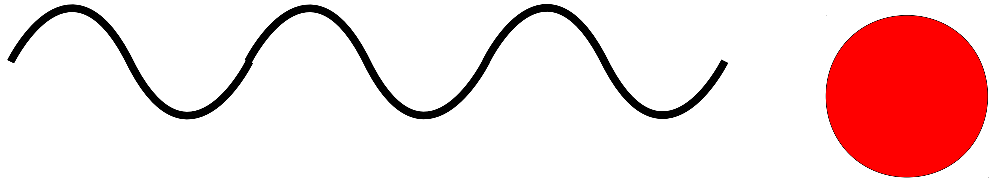
What's all this talk about

Scaling????

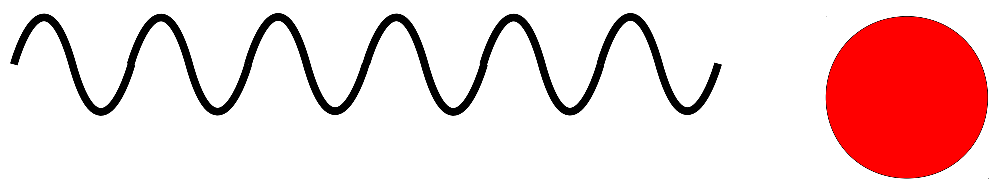
The Scaling of the Proton Structure Function



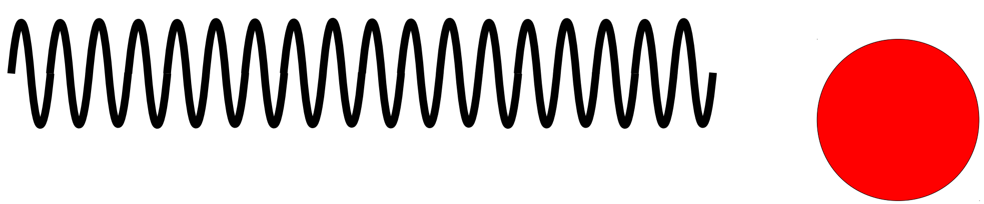
What do we expect for a point like particle



$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$



$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

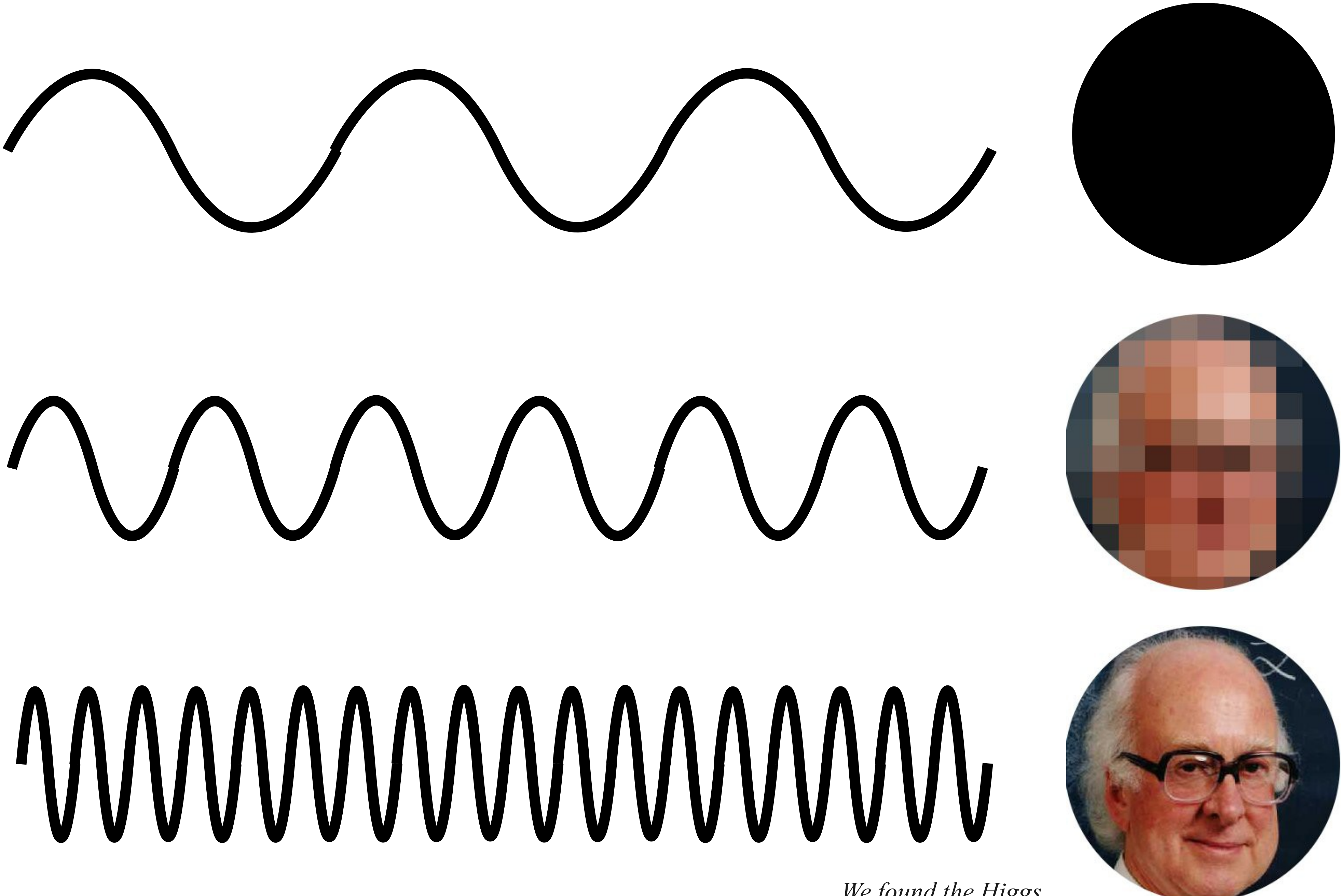


$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

Dimensional considerations

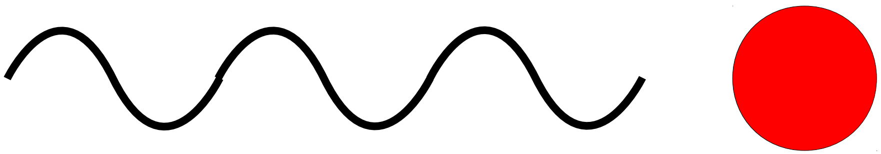
Structure Function

Is this a point like particle ???

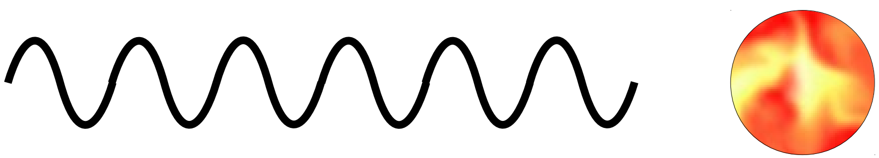


We found the Higgs

Structure of the Proton

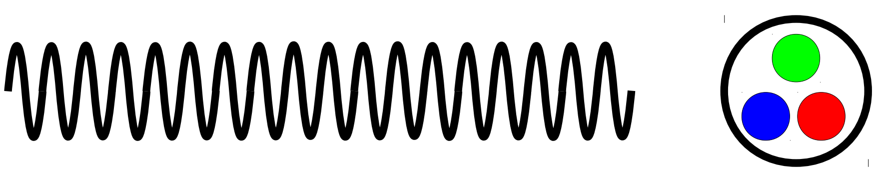


$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$



$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times F\left(\frac{Q^2}{\Lambda^2}\right)$$

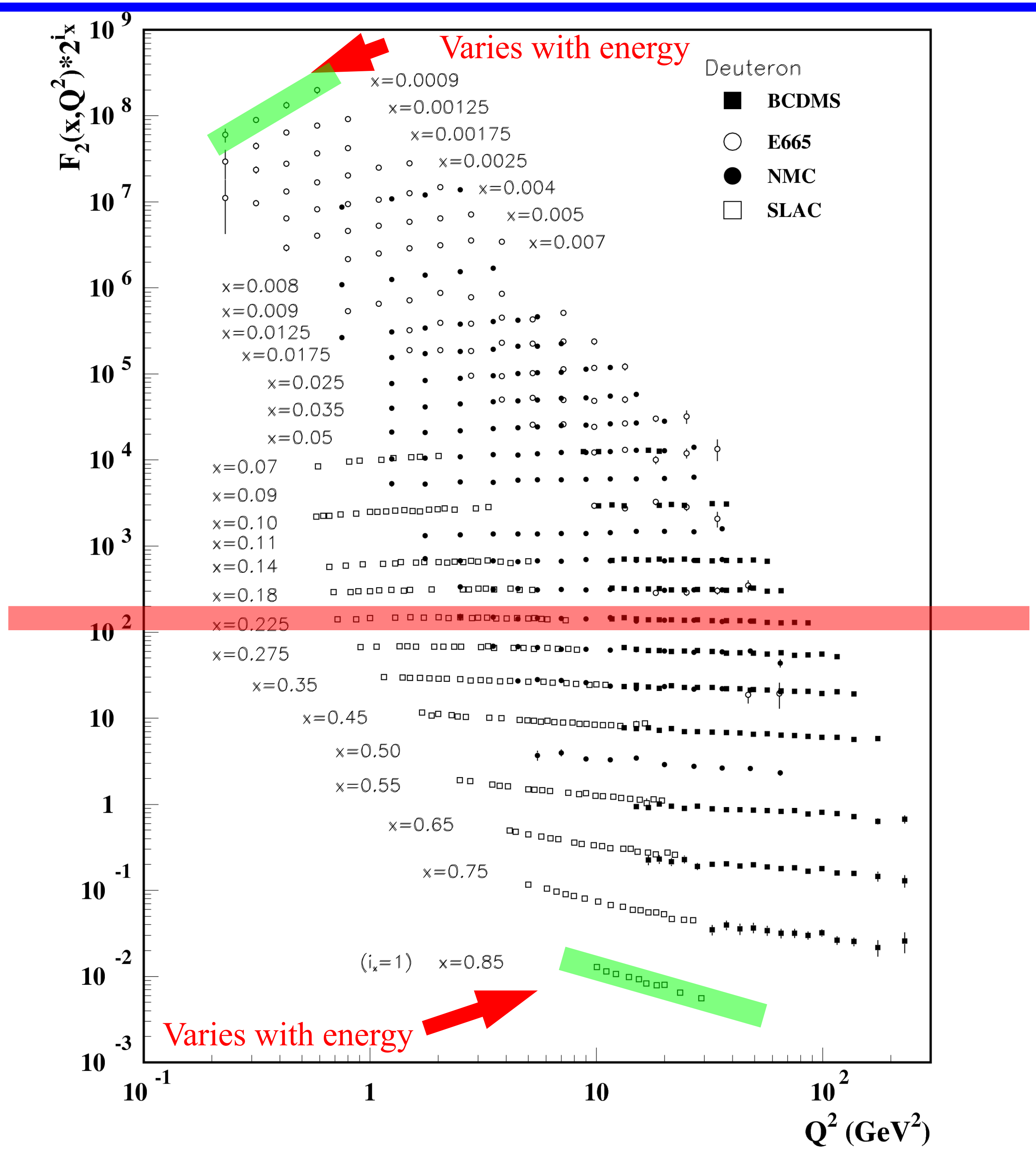
Λ of order of the proton mass scale



$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times \sum_i e_i^2$$

The Scaling of the Proton Structure Function

Data is (relatively) independent of energy



Scaling Violations observed at extreme x values

… not yet at the

Parton Model

Use Helicity Basis

$$\frac{d\sigma}{dx\,dy} = N\left[xy^2F_1 + (1 - y - \tfrac{Mxy}{2E_2})F_2 \pm y(1-y/2)xF_3\right]$$

Taking the limit $M \to 0$ for neutrino DIS

$$\frac{d\sigma^\nu}{dx\,dy} = N\left[(1-y)^2F_+ + 2(1-y)F_0 + F_-\right]$$

For $\bar{\nu}$, $F_+ \Leftrightarrow F_-$

$$\begin{aligned} F_1 &= \tfrac{1}{2}(F_- + F_+) \\ F_2 &= x(F_- + F_+ + 2F_0) \\ F_3 &= (F_- - F_+) \end{aligned}$$

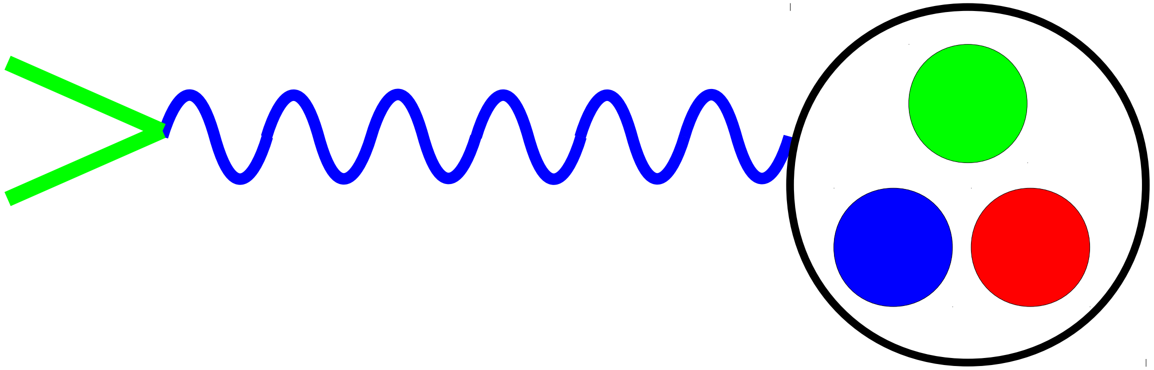
$$\begin{aligned} F_+ &= F_1 - \tfrac{1}{2}F_3 \\ F_- &= F_1 + \tfrac{1}{2}F_3 \\ F_0 &= \tfrac{1}{2x}F_2 - F_1 \end{aligned}$$

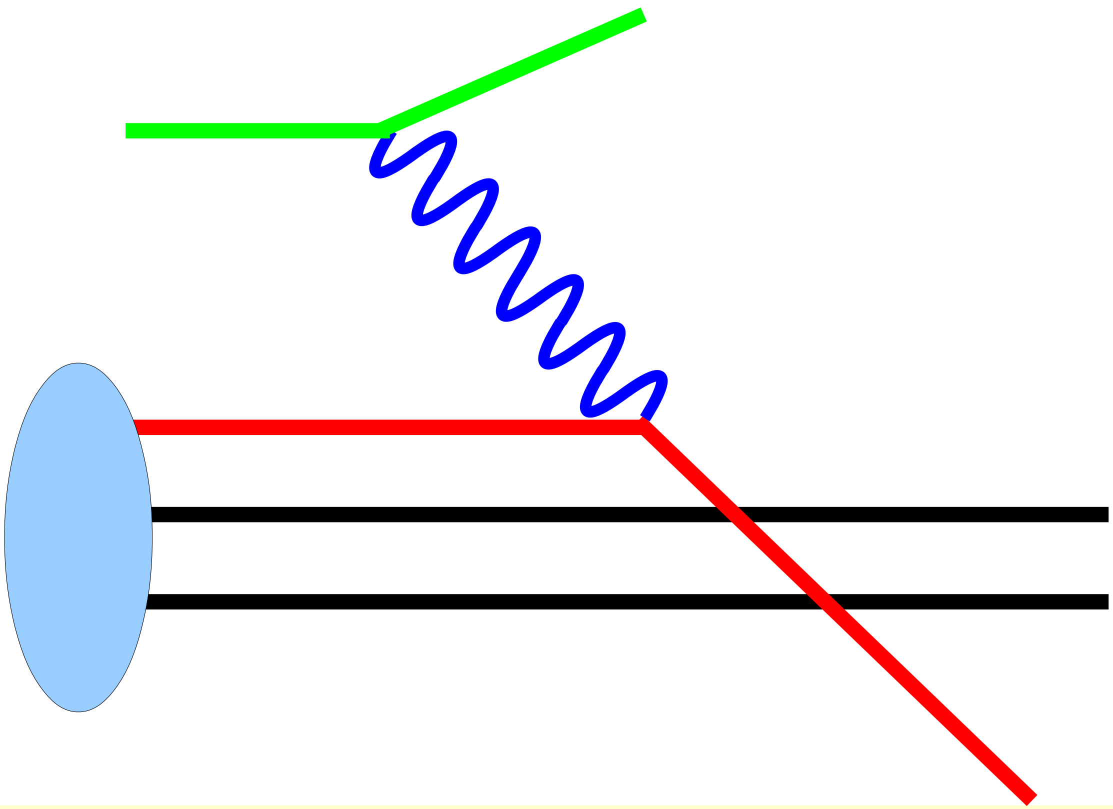
I have not yet mentioned the parton model!!!

A Review of Target Mass Corrections.
Ingo Schienbein et al.
J.Phys.G35:053101,2008.

Parton Model

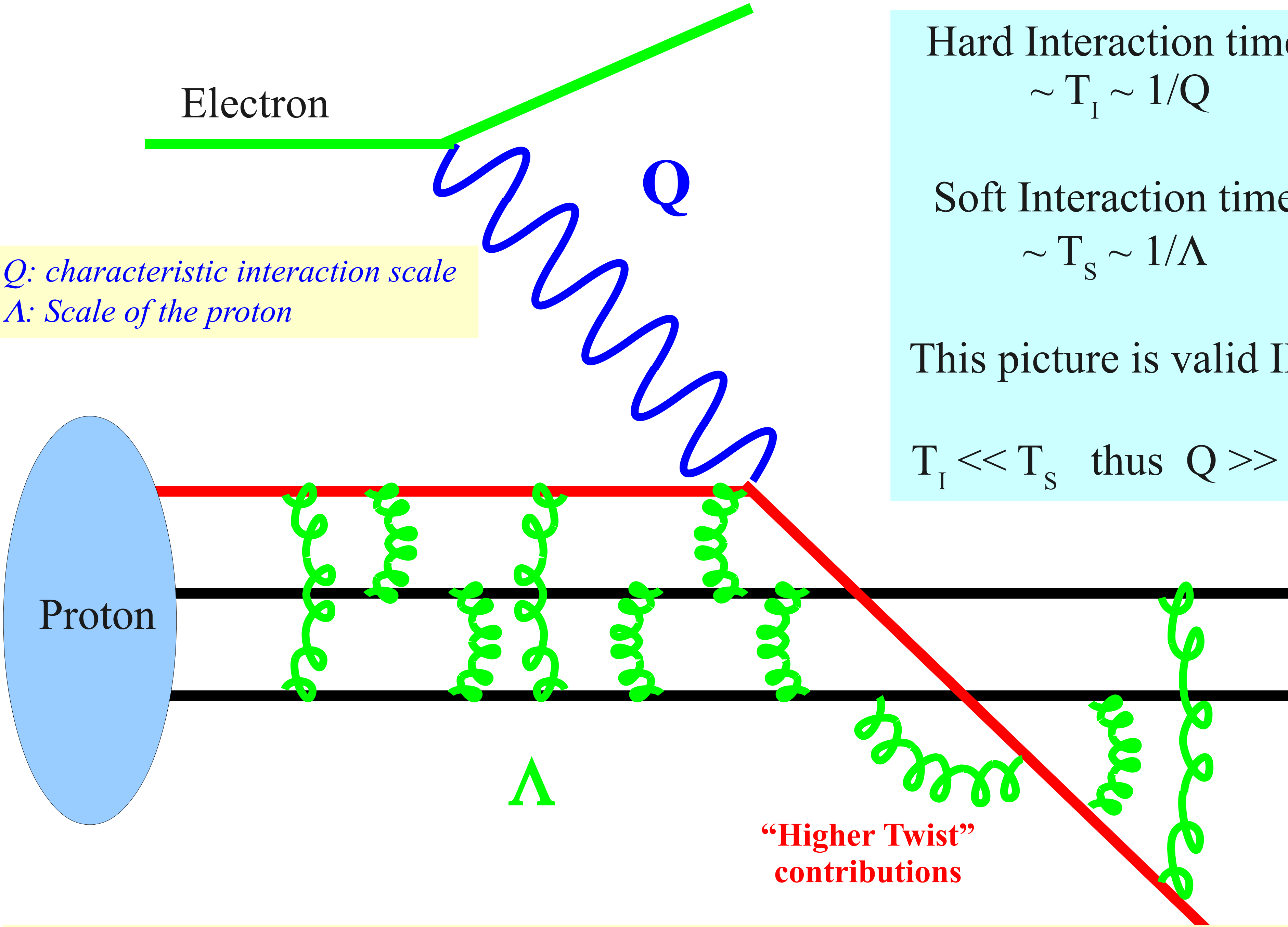
Proton as a bag of free Quarks





$$f(x,Q) = u(x,Q) + d(x,Q) = 2\ \delta(x - \tfrac{1}{3}) + 1\ \delta(x - \tfrac{1}{3})$$

Quarks are not quite free



Q: characteristic interaction scale
Λ: Scale of the proton

Hard Interaction time $\sim T_I \sim 1/Q$

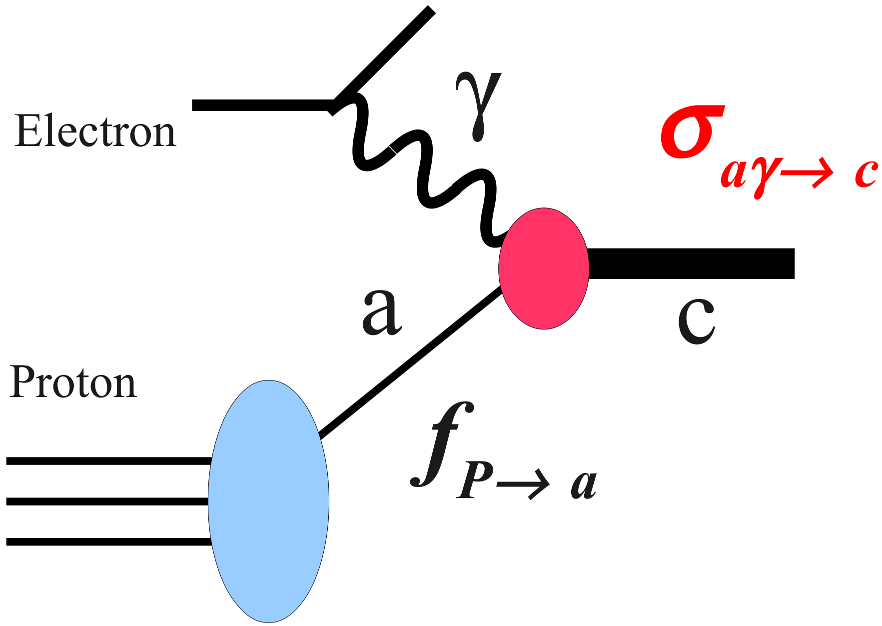
Soft Interaction time $\sim T_S \sim 1/\Lambda$

This picture is valid IF:

$T_I \ll T_S$ thus $Q \gg \Lambda$

Corrections to this picture (non-factorizable/ higher twist) terms are suppressed by powers of Λ/Q

The Parton Model and Factorization



Parton Distribution Functions

(PDFs) $f_{P\to a}$

are the key to calculations

involving hadrons!!!

must extract from experiment

calculable from theoretical model

$$\sigma_{P\gamma\to c} = f_{P\to a} \otimes \hat{\sigma}_{a\gamma\to c}$$

Corrections of order (Λ^2/Q^2)

Quark *Anti-Quark* *Scalar*

$$f(x) = \sum q(x) + \bar{q}(x) + \phi(x) + \ldots = u(x) + d(x) + \ldots$$

Homework Problem: Convolutions

Part 1) Show these 3 definitions are equivalent; work out the limits of integration.

$$f \otimes g = \int_0^1 \int_0^1 f(x) g(y) \delta(z - x * y) \, dx \, dy$$

$$f \otimes g = \int f(x) g\left(\frac{z}{x}\right) \frac{dx}{x}$$

$$f \otimes g = \int f\left(\frac{z}{y}\right) g(y) \frac{dy}{y}$$

Part 2) Show convolutions are the ``natural'' way to multiply probabilities.

If f represents the heads/tails probability distribution for a single coin flip, show that the distribution of 2 coins is $f \oplus f$ and 3 coins is: $f \oplus f \oplus f$.

$$f \oplus g = \int f(x) g(y) \delta(z - (x + y)) \, dx \, dy$$

$$f(x) = \frac{1}{2} (\delta(1 - x) + \delta(1 + x))$$

Careful: convolutions involve + and *

***BONUS**: How many processes can you think of that don't factorize?*

Structure Function & PDF Correspondence at Leading Order

$$\frac{d\sigma^\nu}{dx\,dy} = N\left[(1-y)^2 F_+ + 2(1-y)F_0 + F_-\right]$$

Compute with Hadronic Tensor

$$\frac{d\sigma^\nu}{dx\,dy} = N\left[(1-y)^2 (2\bar{q}) + 2(1-y)(\phi) + (2q)\right]$$

Compute in Parton Model

Scalar

$$\begin{aligned} F_+ &= 2\bar{q} \qquad & F_+ &= F_1 - \tfrac{1}{2}F_3 \\ F_- &= 2q \qquad & F_- &= F_1 + \tfrac{1}{2}F_3 \\ F_0 &= \phi \qquad & F_0 &= \tfrac{1}{2x}F_2 - F_1 \end{aligned}$$

Scalar

$$F_L = 0 = F_0$$

$$F_2 = 2xF_1$$

Callan-Gross Relation

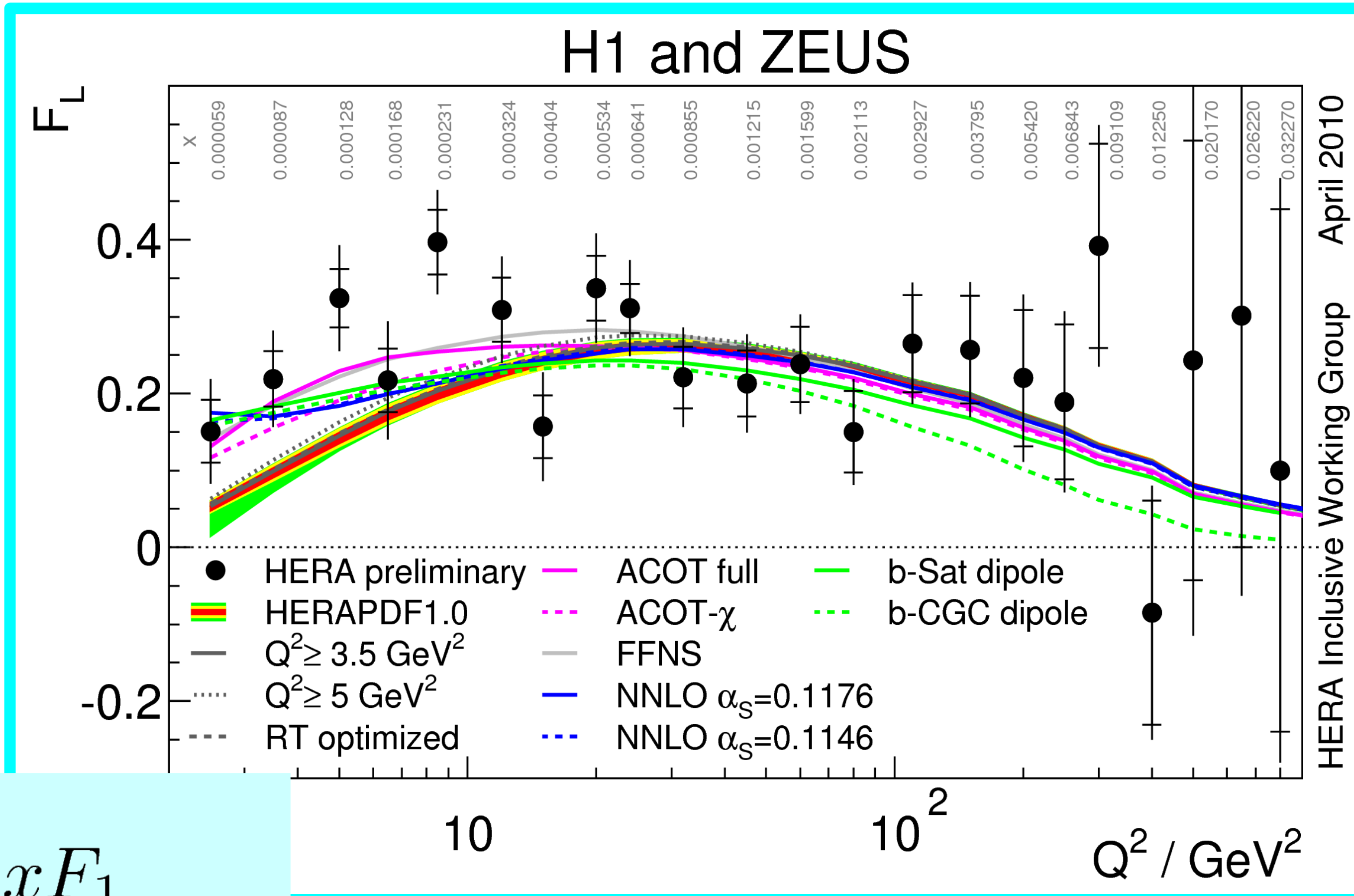
$$F_L = 2xF_0$$

F_L

and

Callan-Gross

Why is F_L special ???



$$F_L = 2xF_0 = F_2 - 2xF_1$$

$$F_L = 0 \quad \Longrightarrow \quad F_2 = 2xF_1$$

Callan-Gross

$$F_L \sim \frac{m^2}{Q^2}\, q(x) + \alpha_S \left\{ c_g \otimes g(x) + c_q \otimes q(x) \right\}$$

Masses are important

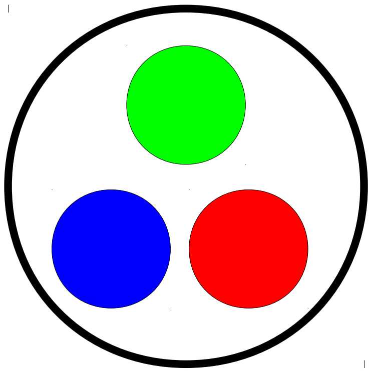
Higher orders are important

TOY

PDFs

Proton as a bag of free Quarks: Part 2

$$f(x,Q) = u(x,Q) + d(x,Q) = 2\ \delta(x - \tfrac{1}{3}) + 1\ \delta(x - \tfrac{1}{3})$$



$$u(x,Q) = 2\ \delta(x - \tfrac{1}{3})$$

$$d(x,Q) = 1\ \delta(x - \tfrac{1}{3})$$

Perfect Scaling PDFs
Q independent

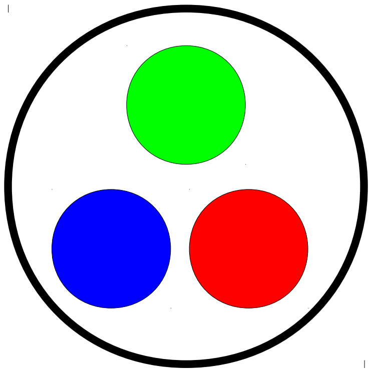
Quark Number Sum Rule

$$\langle q \rangle = \int_0^1 dx\, q(x) \qquad \langle u \rangle = 2 \quad \langle d \rangle = 1 \quad \langle s \rangle = 0$$

Quark Momentum Sum Rule

$$\langle x\, q \rangle = \int_0^1 dx\, x\, q(x) \qquad \langle x\, u \rangle = \tfrac{2}{3} \quad \langle x\, d \rangle = \tfrac{1}{3}$$

Problem #1: The proton does not add up???



$$\begin{aligned} F_+ &= 2\bar{q} \\ F_- &= 2q \\ F_L &= \phi \end{aligned}$$

$$q + \bar{q} = \frac{F_+ + F_-}{2}$$

Momentum Sum Rule

$$\sum_i \langle x\, q_i \rangle = \int_0^1 dx \sum x\, [q_i(x) + \bar{q}_i(x)] = 50\% \neq 100\%$$

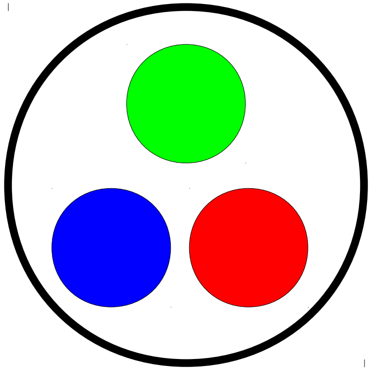
Substitute F

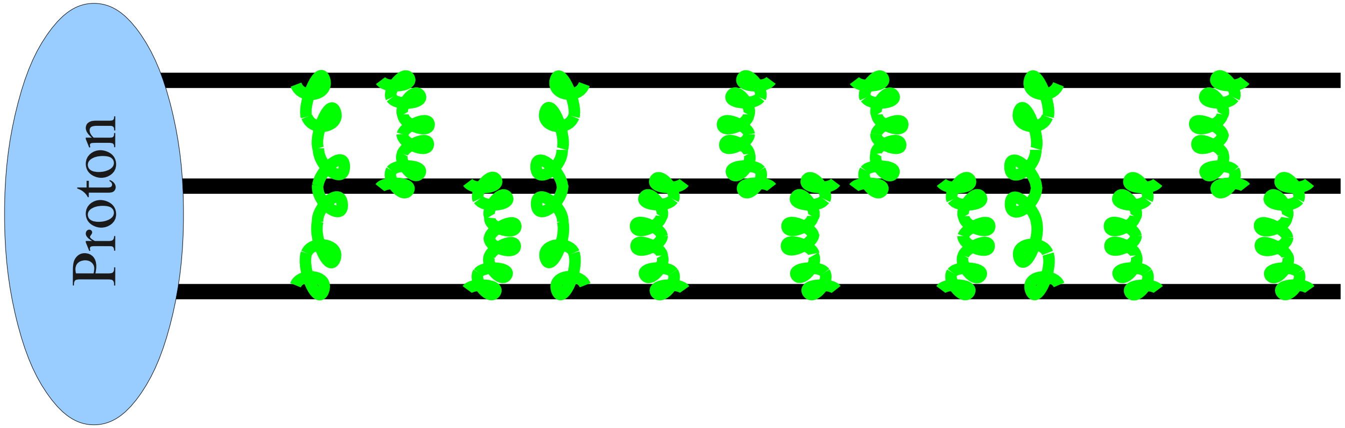
SOLUTION:

Gluons carry half the momentum,

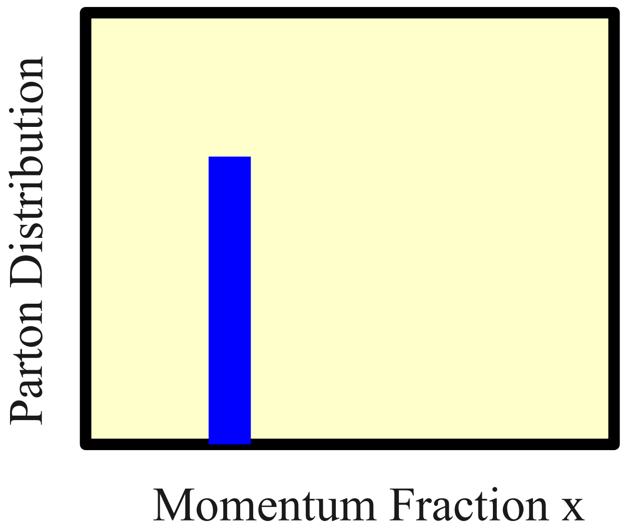
but don't couple to the photons

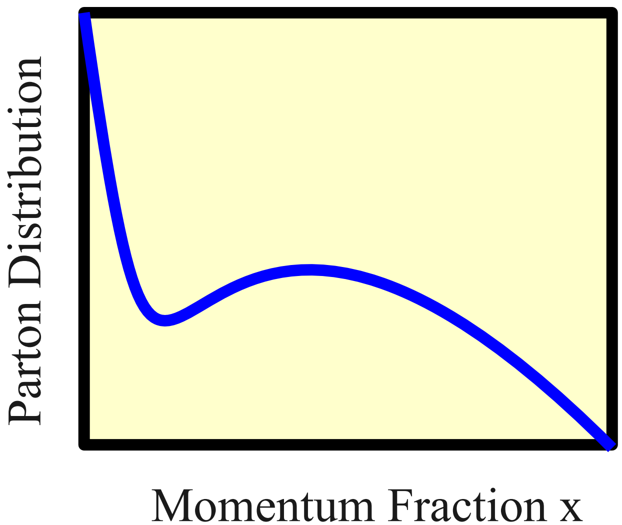
Gluons smear out PDF momentum





Gluons allow partons to exchange momentum fraction

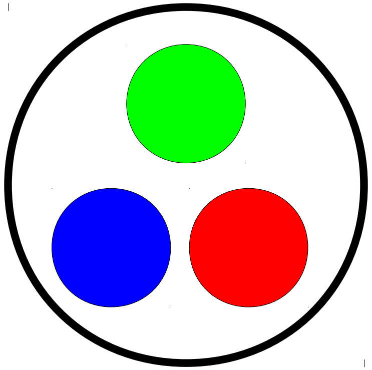




$$f(x,Q) = u(x,Q) + d(x,Q) = 2\ \delta(x - \tfrac{1}{3}) + 1\ \delta(x - \tfrac{1}{3})$$

α_s is large at low Q, so it is easy to emit soft gluons

Problem #2: Infinitely many quarks

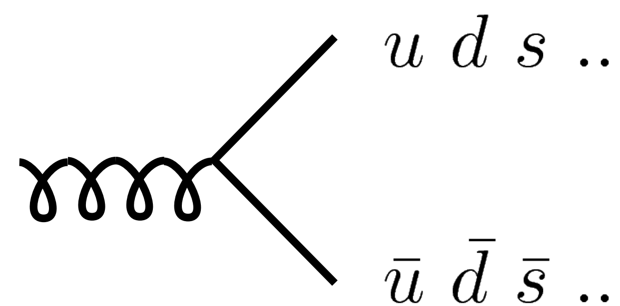


Reconsider the Quark Number Sum Rule

$$\langle u, d \rangle = \infty \qquad \langle q \rangle = \int_0^1 dx\, q(x)$$

Quark Number Sum Rule: More Precisely

$$q(x) \sim 1/x^{1.5}$$



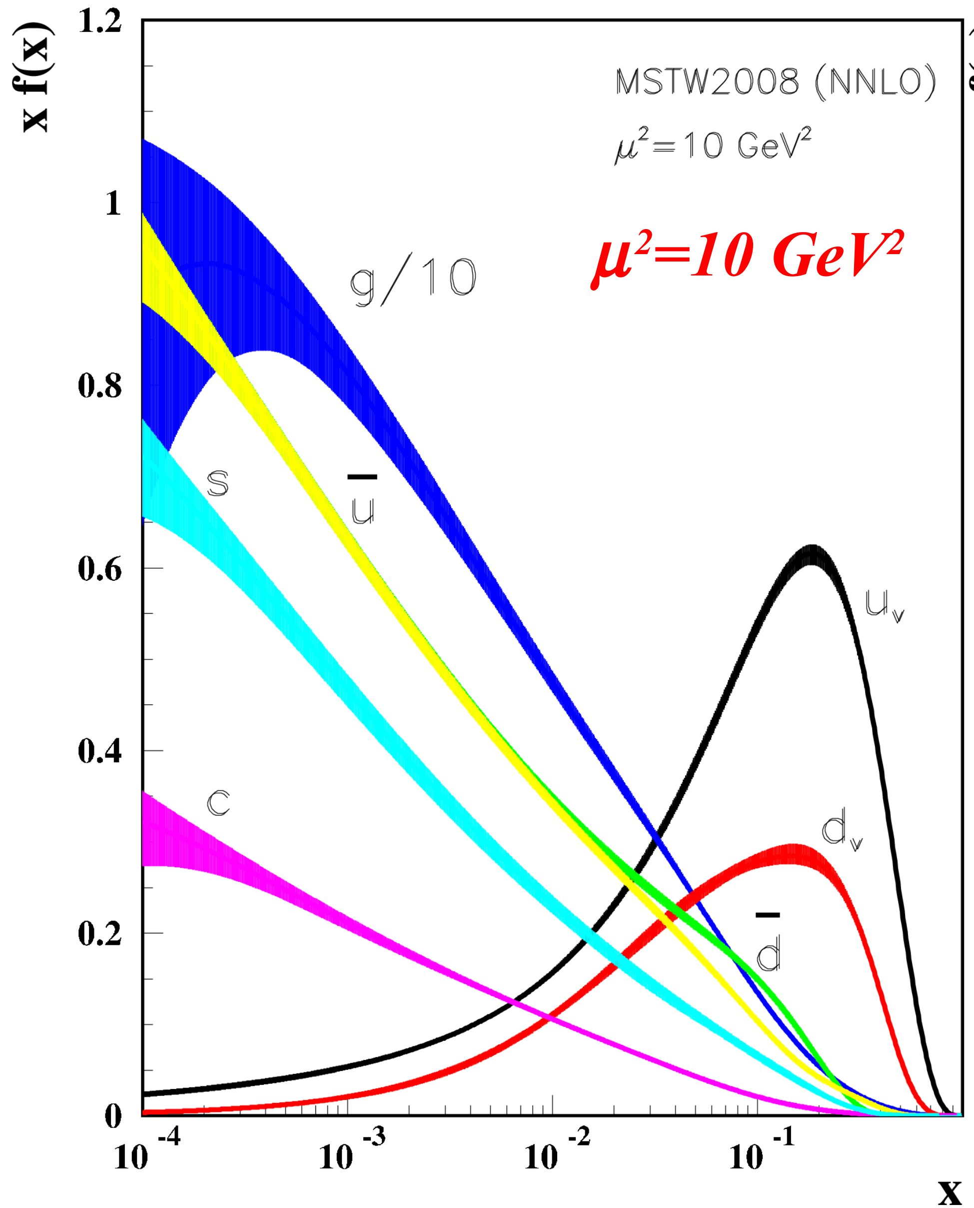
$$\langle u - \bar{u} \rangle = 2 \qquad \langle d - \bar{d} \rangle = 1 \qquad \langle s - \bar{s} \rangle = 0$$

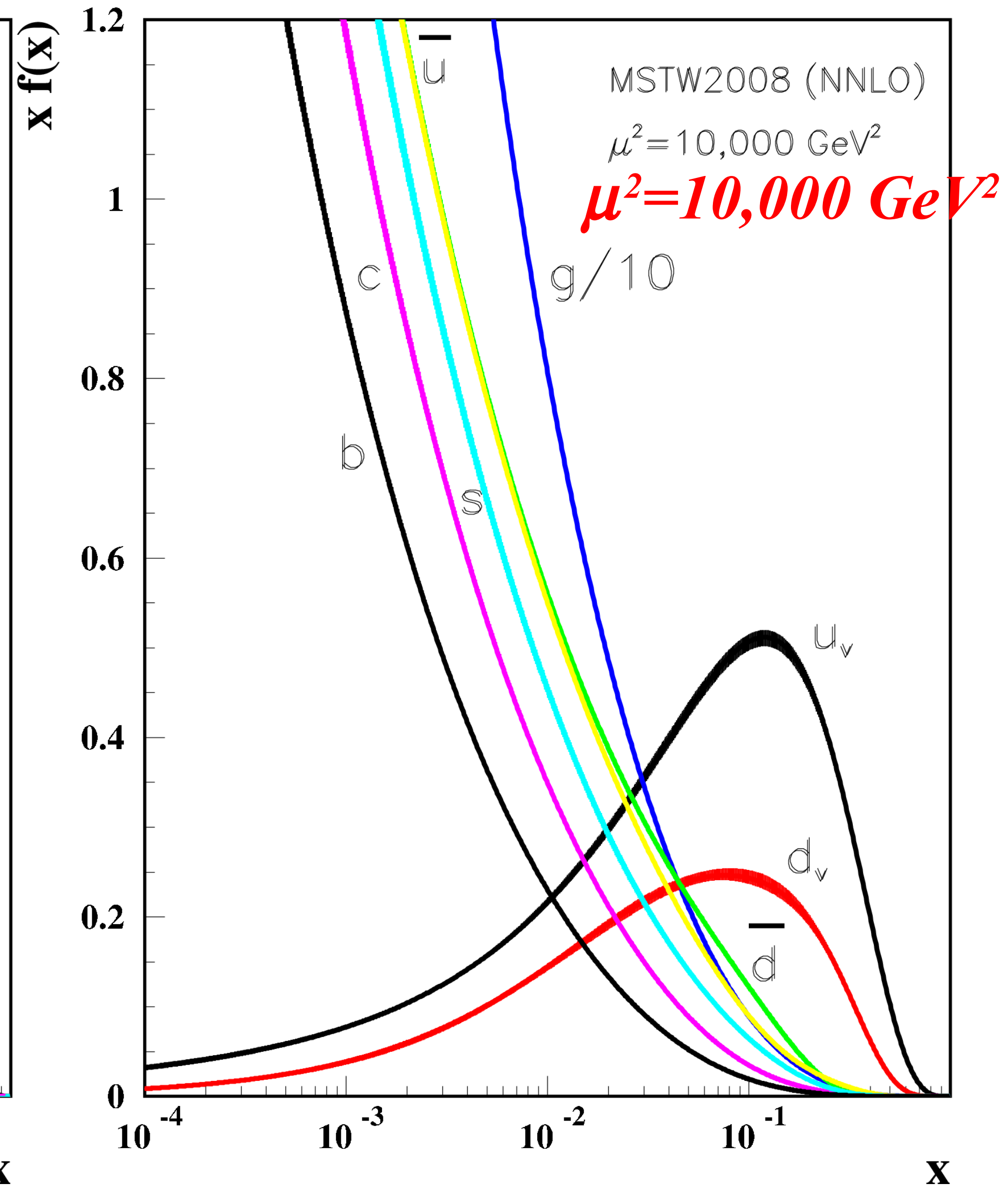
SOLUTION: Infinite number of u quarks in proton, because they can be pair produced:
(We neglect saturation ....)

PDFs

cf., lectures by Dan Stump

Sample PDFs: The rich structure of the proton

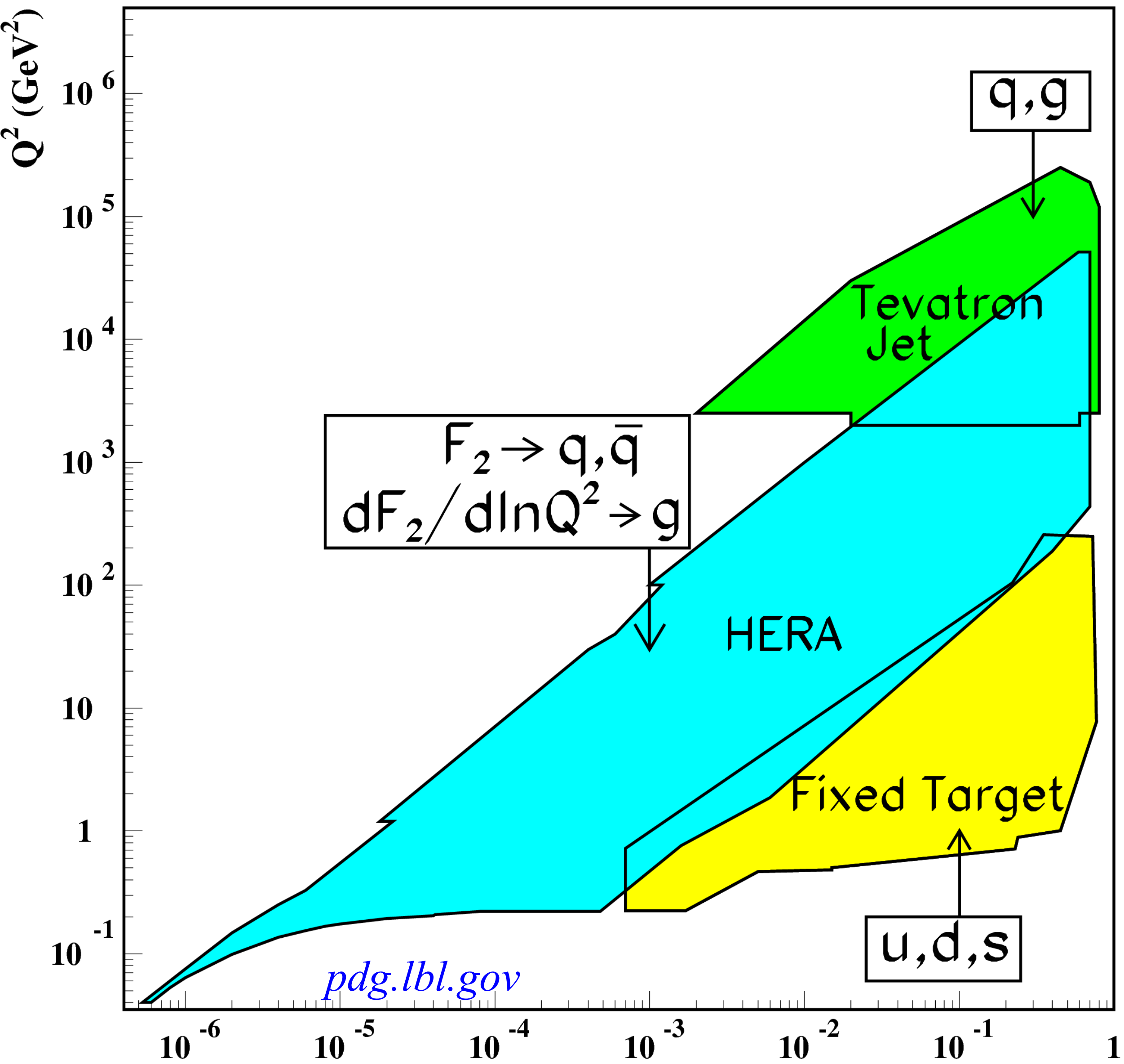




Scaling violations are essential feature of PDFs

Where do PDFs come from???? Universality!!!

$$\sigma_{P\gamma \to c} = f_{P \to a} \otimes \sigma_{a\gamma \to c}$$

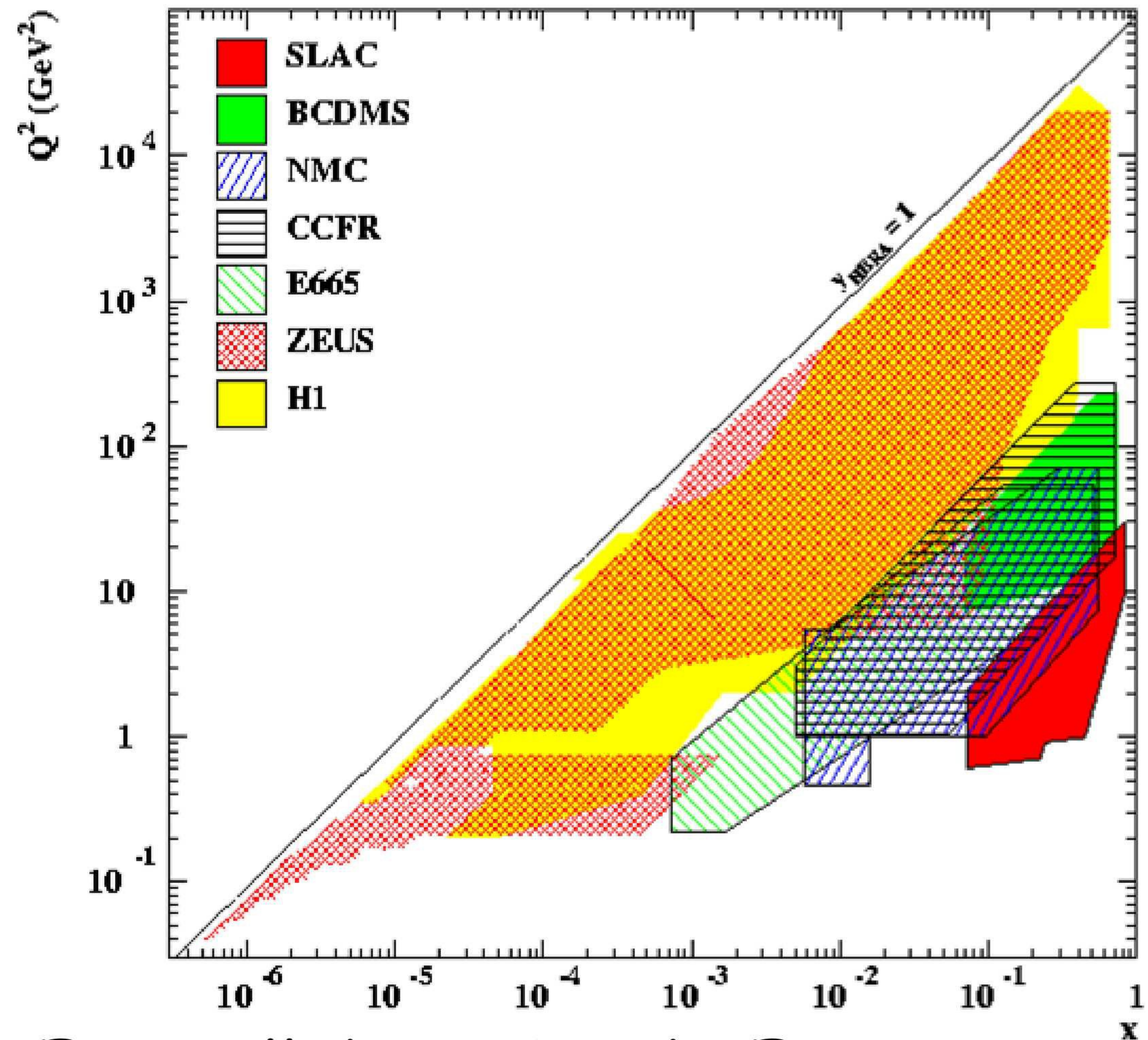


Calculable from theoretical model

Must extract from experiment

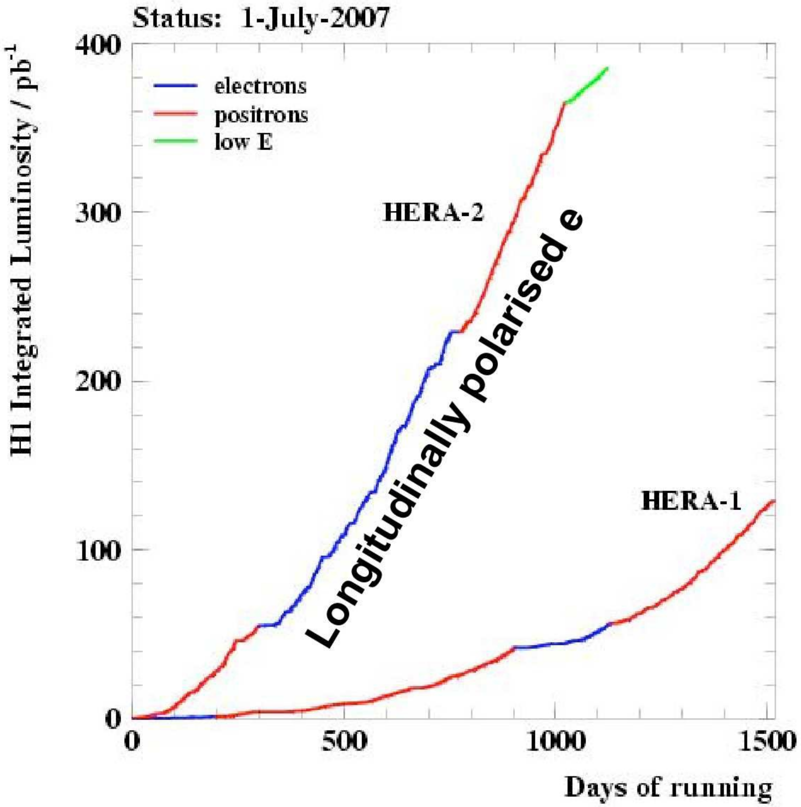
Note we can combine different experiments.
FACTORIZATION!!!

Deep Inelastic Scattering experiments



HERA collider: H1 and ZEUS experiments
1992 – 2007 (data analysis ongoing)

HERA *ep* Collider: 1992-2007



Two colliding beam experiments: H1 and ZEUS
~0.5 fb^{-1} collected per experiment
approximately same amount of collisions with
electrons and positrons of
Left- and right-handed polarisation

E_e = 27.5GeV, E_p = 920 GeV

dedicated low Ep runs
Ep = 460GeV,575 GeV

H1 & ZEUS Collaborations

Collaborations of 300-400 Physicists, at ~40 Institutes of ~15 Countries

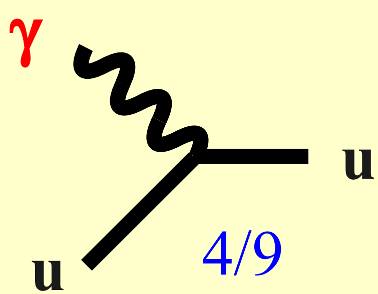
How do we distinguish flavors???

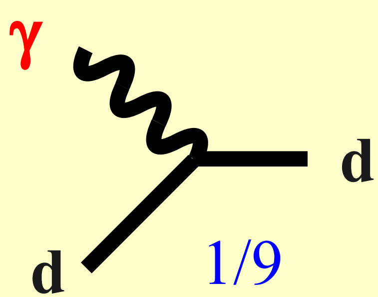
How to Distinguish Flavors???

Use different boson probes

PHOTON

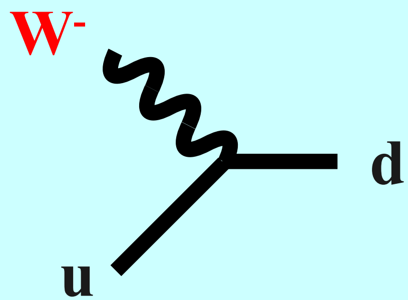
$$ie_q\gamma^\mu$$

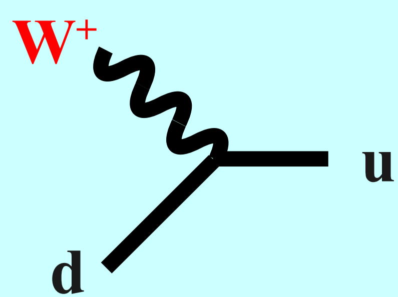




W-BOSON

$$\frac{-ig}{\sqrt{2}}\,\frac{\gamma^\mu(1-\gamma_5)}{2}$$



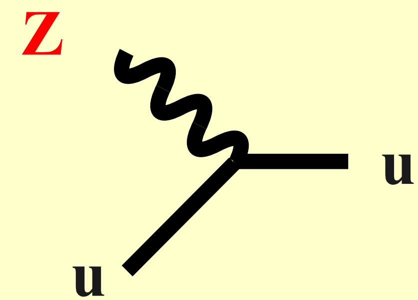


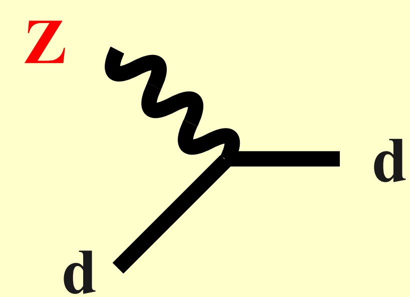
Note: equal weight

Take Cabibbo angle =0

Z-BOSON

$$\frac{-ig}{\cos\theta_W}\,\gamma^\mu(c_V-c_A\gamma_5)$$





HOMEWORK

Sum Rules
&
Structure Functions

$$F_2^{ep} = \tfrac{4}{9}x\left[u+\bar{u}+c+\bar{c}\right] + \tfrac{1}{9}x\left[d+\bar{d}+s+\bar{s}\right]$$

$$F_2^{en} = \tfrac{4}{9}x\left[d+\bar{d}+c+\bar{c}\right] + \tfrac{1}{9}x\left[u+\bar{u}+s+\bar{s}\right]$$

$$F_2^{\nu p} = 2x\left[d+s+\bar{u}+\bar{c}\right]$$

$$F_2^{\nu n} = 2x\left[u+s+\bar{d}+\bar{c}\right]$$

$$F_2^{\bar{\nu} p} = 2x\left[u+c+\bar{d}+\bar{s}\right]$$

$$F_2^{\bar{\nu} n} = 2x\left[d+c+\bar{u}+\bar{s}\right]$$

$$F_3^{\nu p} = 2\left[d+s-\bar{u}-\bar{c}\right]$$

$$F_3^{\nu n} = 2\left[u+s-\bar{d}-\bar{c}\right]$$

$$F_3^{\bar{\nu} p} = 2\left[u+c-\bar{d}-\bar{s}\right]$$

$$F_3^{\bar{\nu} n} = 2\left[d+c-\bar{u}-\bar{s}\right]$$

Verify:

i.e., Check for typos ...

We use these different observables to dis-entangle the flavor structure of the PDFs

See talks by
Alberto Gago (Neutrinos)
&
Dan Stump (PDFs)

In the limit

$$\theta_{Cabibbo} = 0$$

$$m_c = 0$$

Homework: Part 2 Sum Rules

Verify:

i.e., Check for typos ...

Adler (1966)

$$\int_0^1 \frac{dx}{2x} \left[F_2^{\nu n} - F_2^{\nu p}\right] = 1$$

Bjorken (1967)

$$\int_0^1 \frac{dx}{2x} \left[F_2^{\bar{\nu} p} - F_2^{\nu p}\right] = 1$$

Gross Llewellyn-Smith (1969)

$$\int_0^1 dx \left[F_3^{\nu p} + F_3^{\bar{\nu} p}\right] = 6$$

Gottfried (1967)

if $\bar{u} = \bar{d}$ $$\int_0^1 dx \left[F_2^{ep} - F_2^{en}\right] = \frac{1}{3}$$

Homework (19??)

$$\frac{5}{18} F_2^{\nu N} - F_2^{eN} = ?$$

Before the parton model was invented, these relations were observed. Can you understand them in the context of the parton model?

This one has been particularly important/controversial

How do we distinguish flavors???

Different Targets

Possible Targets:

p Proton
n Neutron
N Nucleon ~ Z p + (A-Z) n

… for isoscalar: N= ½ (p+n)

Relate p and n

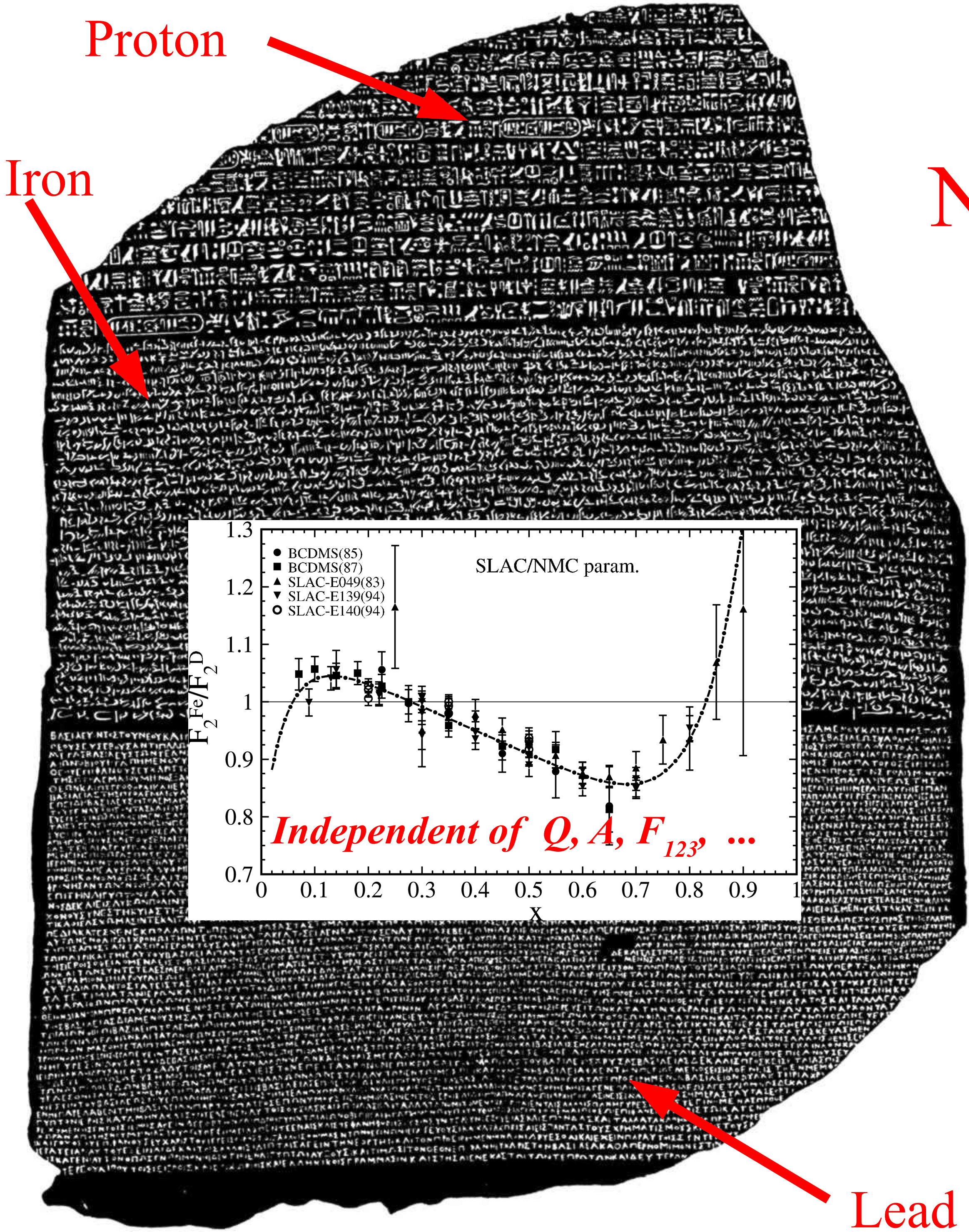
Proton		Neutron
u_p	=	d_n
d_p	=	u_n
q_p	=	q_n

Complications

$$N \neq Z\,p + (A-Z)\,n$$

Nuclear Corrections

???



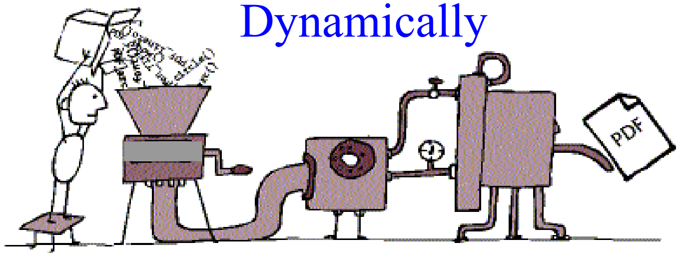
Where do Nuclear Corrections come from ???

carved in stone

Discovered by the French in 1799 at Rosetta, a harbor on the Mediterranean coast in Egypt. Comparative translation of the stone assisted in understanding many previously undecipherable examples of hieroglyphics.

Traditional Input to Global PDF Fit

Include Nuclear Dimension Dynamically



Start with Neutrino data on heavy targets (Fe, Pb, ...)

Nuclear Correction

"Corrected" data for isoscalar

Isospin Symmetry

"Corrected" data for proton

Nuclear Correction "Frozen"

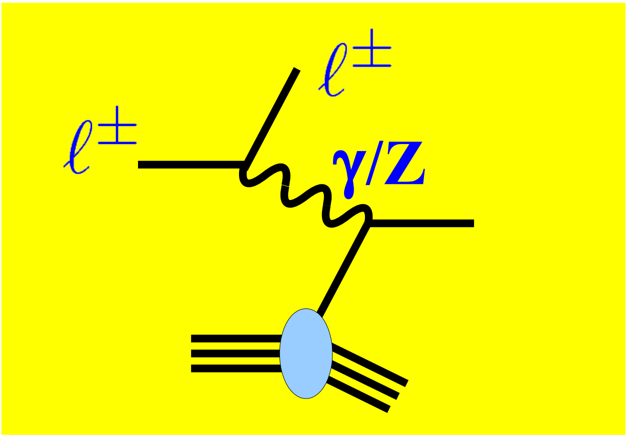
Global PDF Fit

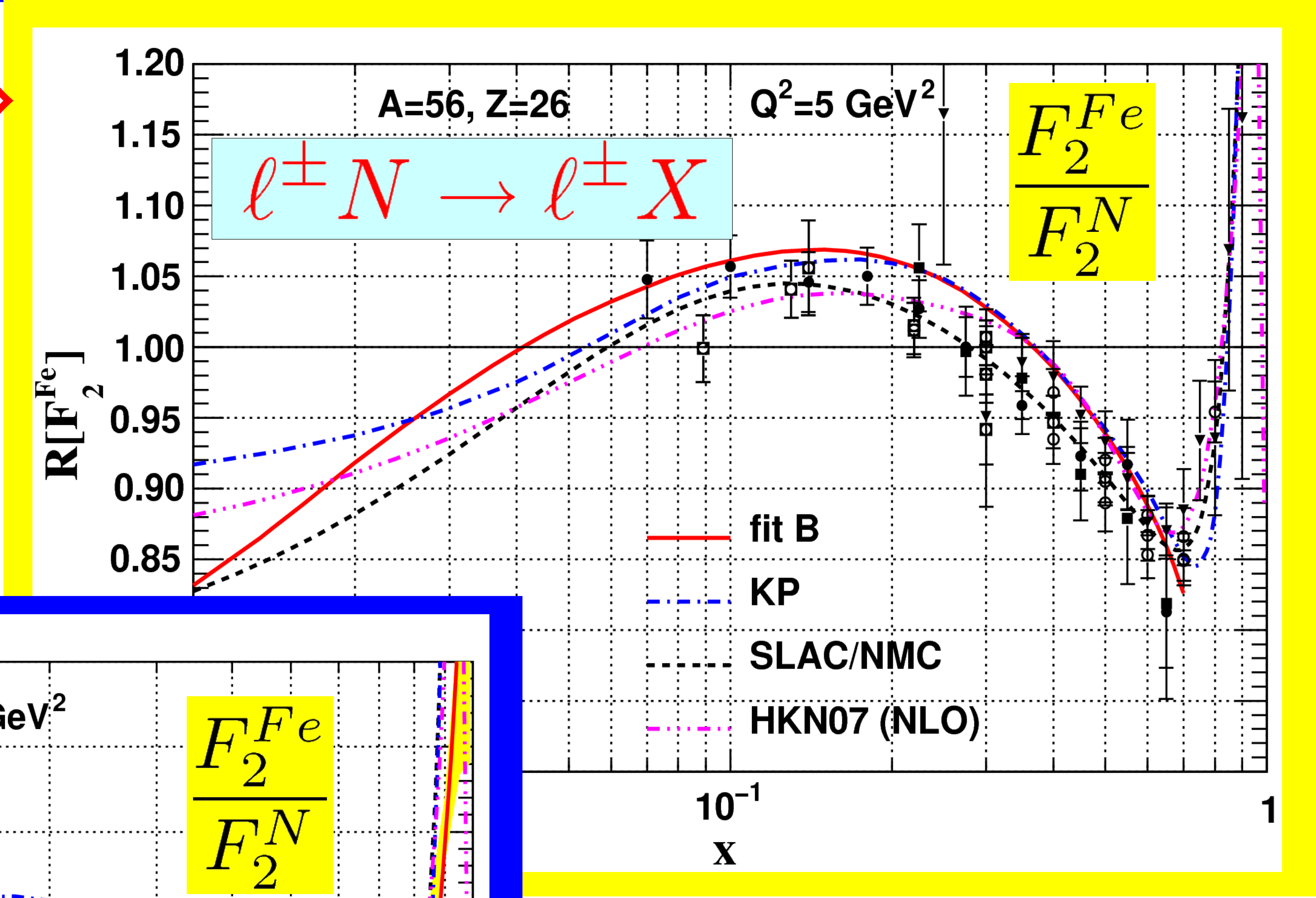
No Feedback!!!

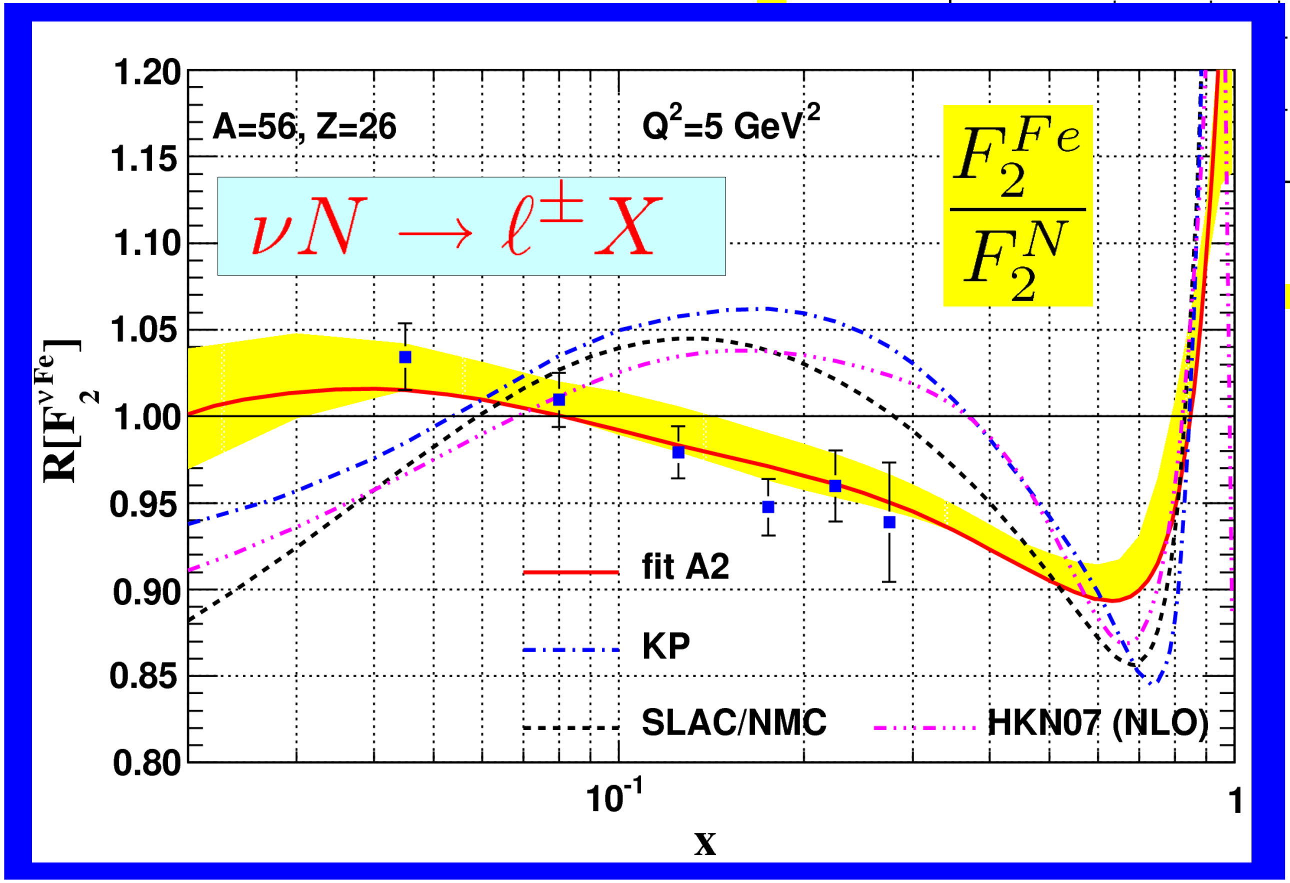
Oooooops!

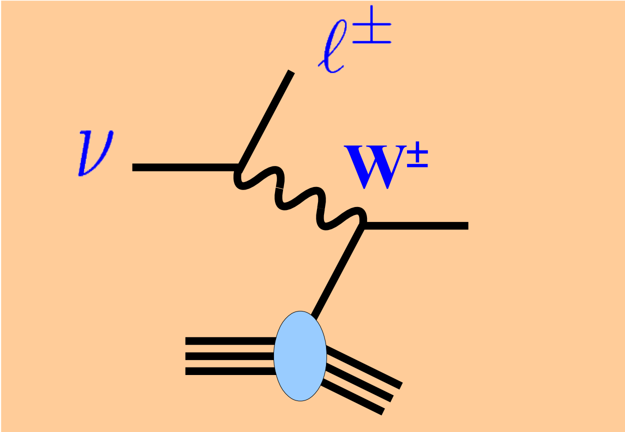
Nuclear Corrections: Compare Neutrino and Charged Lepton DIS

Charged Lepton DIS ⇒









⇐ Neutrino DIS

nCTEQ Nuclear PDF's

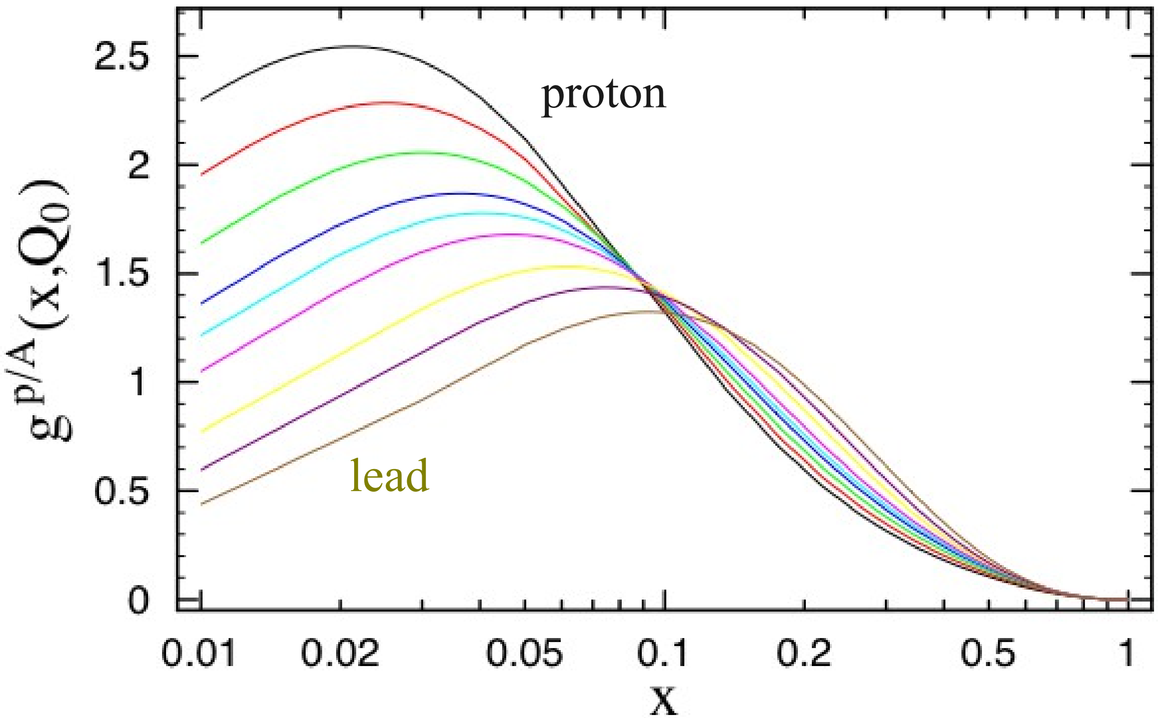
- ✔ CTEQ style global fit extended
  handle various nuclear targets
- ✔ CTEQ Data + nuclear DIS & DY
  [~15 targets; ~2000+ data]
- ✔ A-dependence modeled;
  NLO fits work well

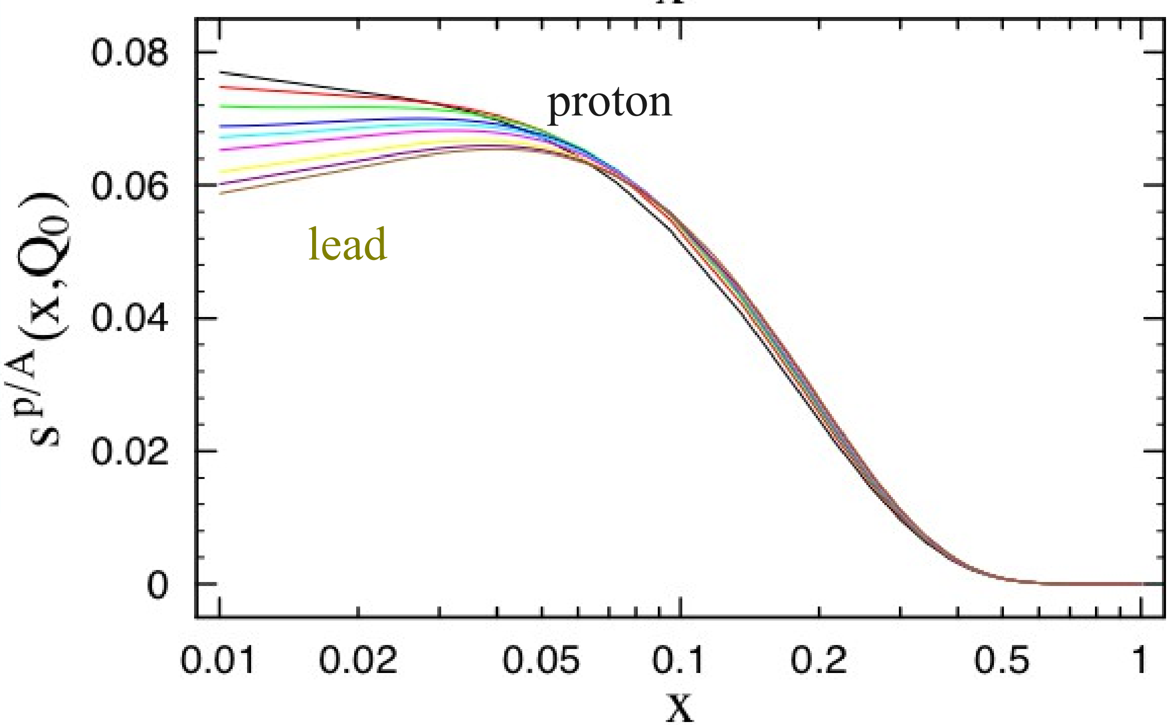
A-Dependent PDFs

$$xf(x) = x^{a_1}(1-x)^{a_2} e^{a_3 x}(1+e^{a_4}x)^{a_5}$$

$$a_i \rightarrow a_i(A)$$

$$a_k = a_{k,0} + a_{k,1}\left(1 - A^{-a_{k,2}}\right)$$





Nuclear PDFs from neutrino deep inelastic scattering.
I. Schienbein, J.Y. Yu, C. Keppel, J.G. Morfin,
F. Olness, J.F. Owens. Phys.Rev.D77:054013,2008.

Nuclear CTEQ parton distribution functions - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://projects.hepforge.org/ncteq/

The CTEQ Meta-Page | Nuclear CTEQ parton distri...

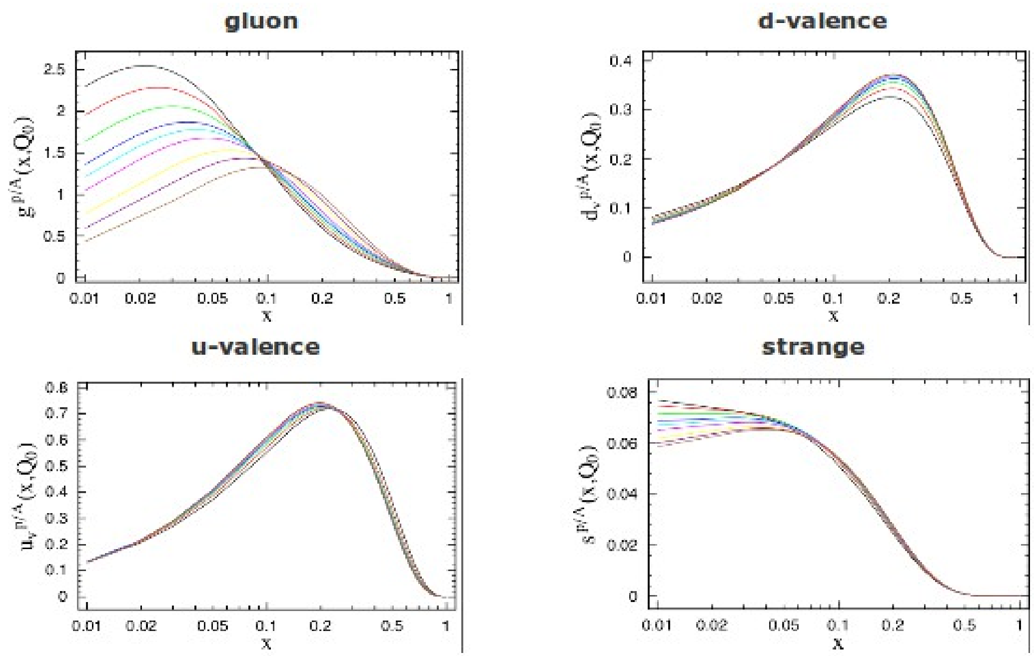
nCTEQ Go! hosted by CEDAR HepForge

nCTEQ

nuclear parton distribution functions

- Home
- PDF grids & code
- Papers & Talks
- Subversion
- Tracker
- Wiki

nCTEQ project is an extension of the CTEQ collaborative effort to determine parton distribution functions inside of a free proton. It generalizes the free-proton PDF framework to determine densities of partons in bound protons (hence nCTEQ which stands for nuclear CTEQ). More details on the framework and the first results can be found in arXiv:09072357 [hep-ph].
The effects of the nuclear environement on the parton densities can be shown as modified parton densities



where all black curves stand for free proton PDF and red, green, blue, cyan, pink, yellow, magenta and brown curves show PDF in protons bound in nuclei - from deuterium (red) to lead (brown).

K Kovarik,
I. Schienbein,
J.Y. Yu,
T. Stavreva,
T Jezo,
C. Keppel,
J.G. Morfin,
F. Olness,
J.F. Owens.

Done

MINERνA

Per ton per 10^20 protons

DIS Comparisons:

Charged Current

Experiment	ν DIS events	anti-ν DIS events
CCFR	1.03 M	0.179 M
NuTeV	1.3 M	0.4 M

MINERνA Comparisons

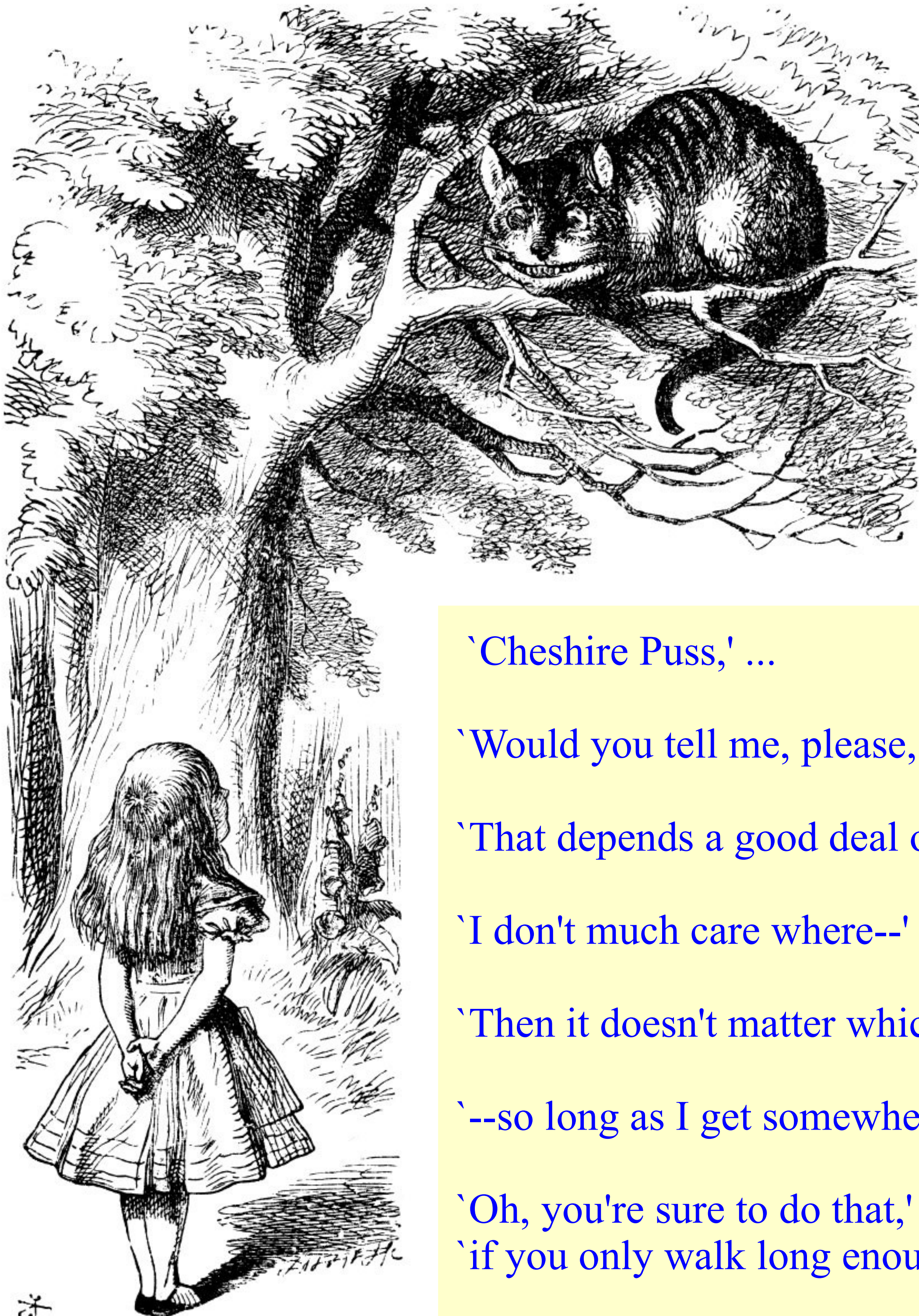
Target	Mass *(Tons)*	Events
Fe	0.70	2 M
Pb	0.85	2.5 M
He	0.40	600 K
C	0.15	430 K
CH	3	9 M

Process	CC/ton	NC/ton
Quasi-Elastic	270 K	90 K
Resonance	530 K	165 K
Transition	670 K	210 K
DIS	1370 K	400 K
Coherent	28 K	14 K
Total (ν)	2870 K	880 K

Per ton per w/ 4-year run

Evolution

What does the proton look like???



The answer is dependent upon the question

`Cheshire Puss,' ...

`Would you tell me, please, which way I ought to go from here?'

`That depends a good deal on where you want to get to,' said the Cat.

`I don't much care where--' said Alice.

`Then it doesn't matter which way you go,' said the Cat.

`--so long as I get somewhere,' Alice added as an explanation.

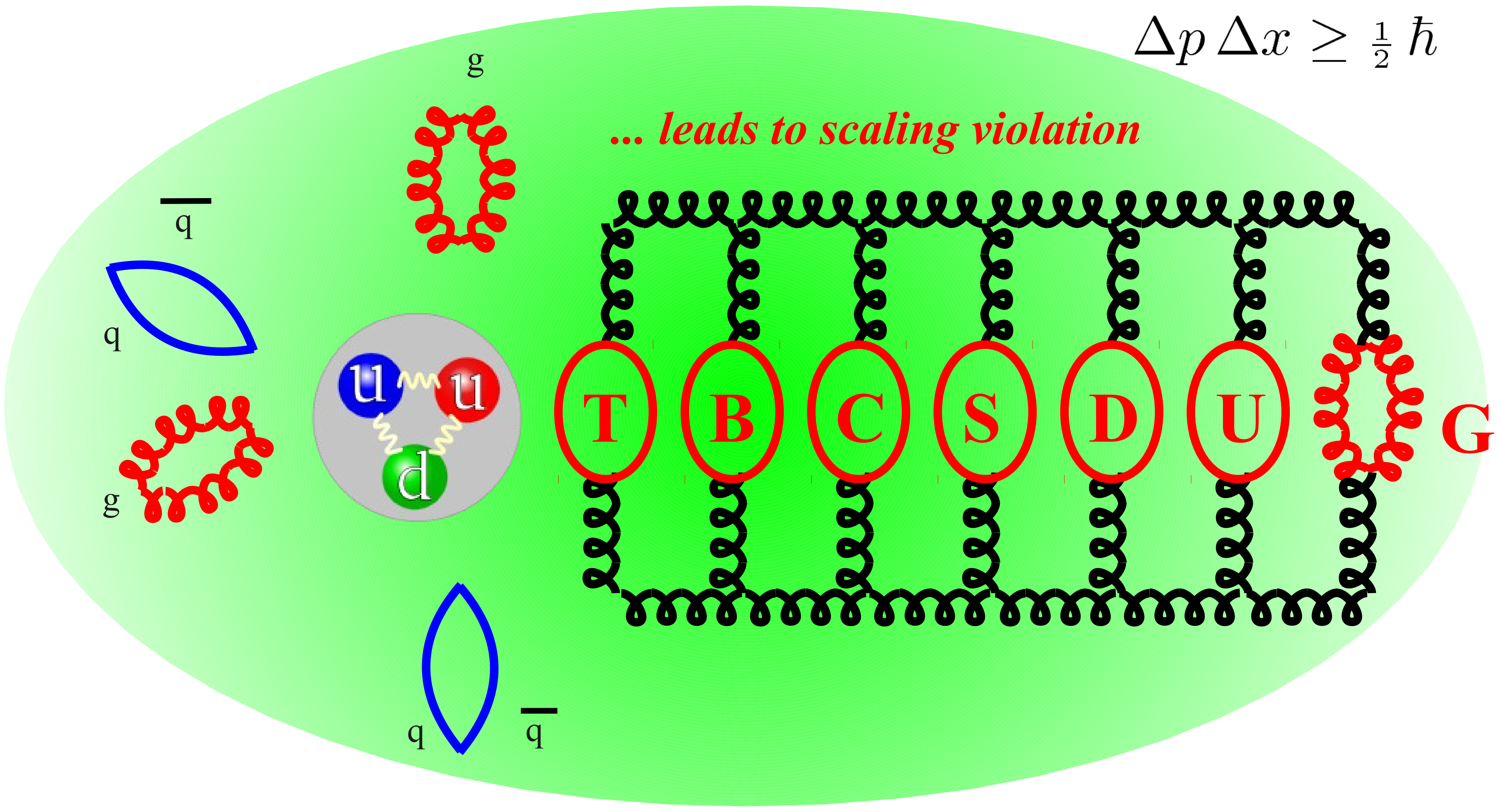
`Oh, you're sure to do that,' said the Cat,
`if you only walk long enough.'

Evolution: What you see depends upon what you ask

Proton is a complex object

$$\Delta E\, \Delta t \geq \tfrac{1}{2}\, \hbar$$

$$\Delta p\, \Delta x \geq \tfrac{1}{2}\, \hbar$$



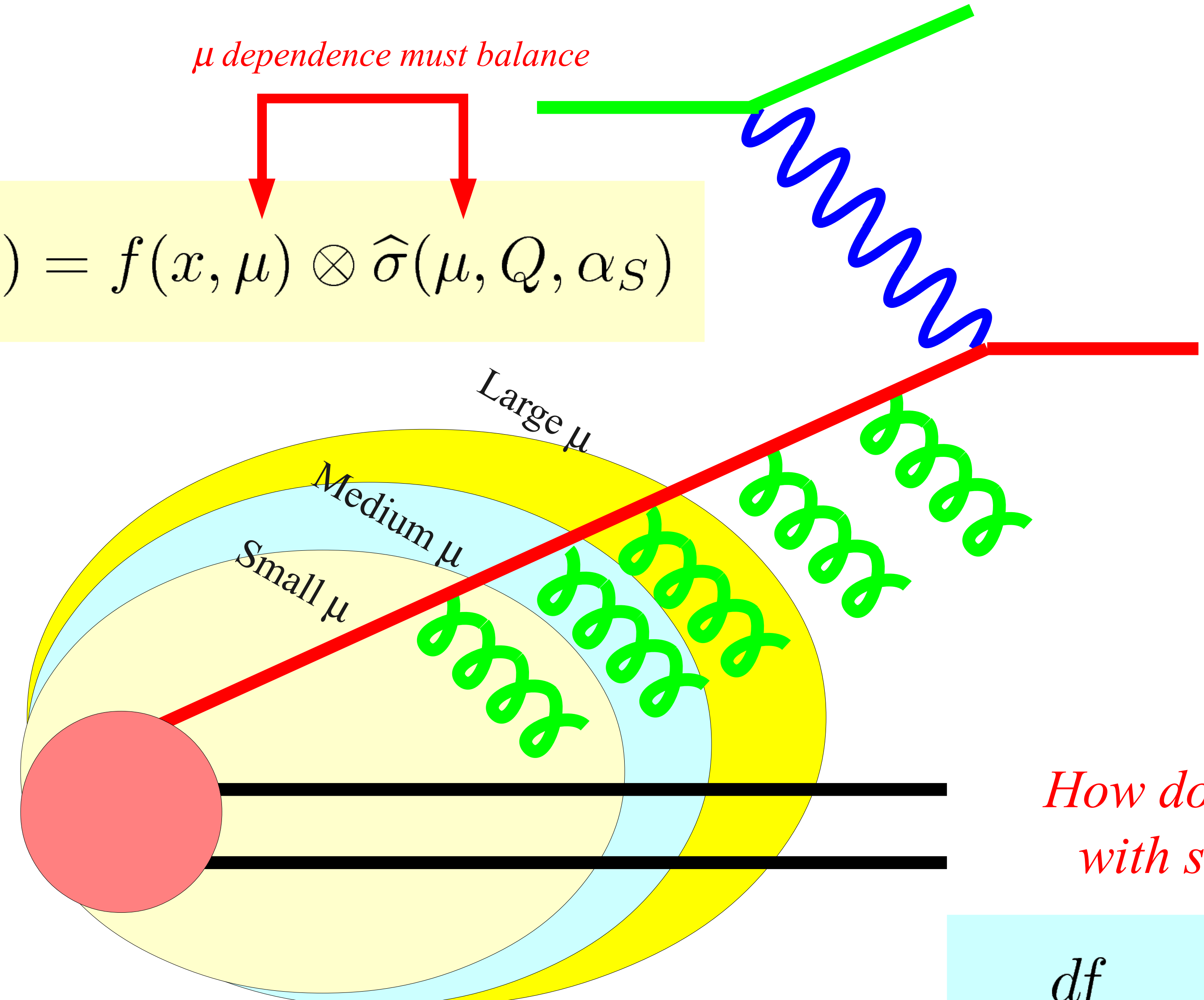
m_t	m_b	m_c	m_s	m_d	m_u	m_g
175	4.5	1.3	0.3	0.00?	0.00?	0

$$\Lambda_{QCD} \sim 200\,\mathrm{MeV}$$

Evolution of the PDFs

μ dependence must balance

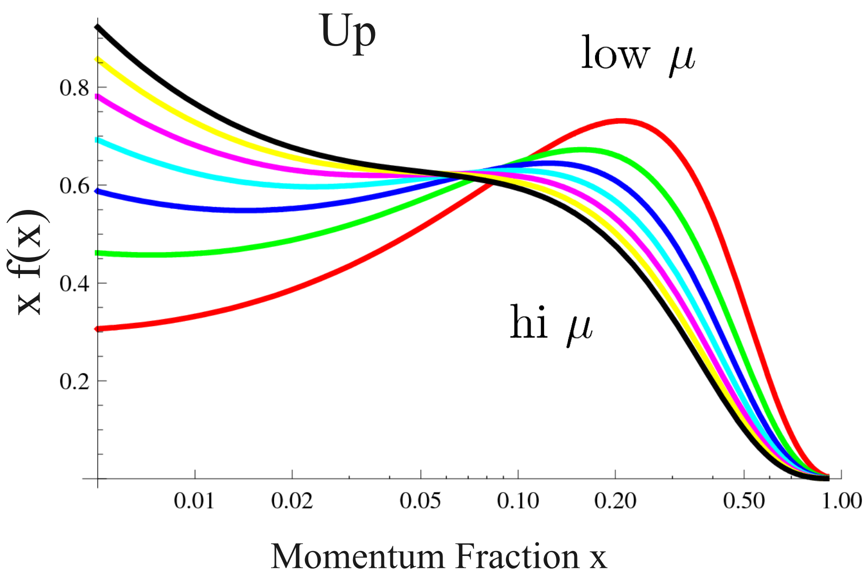
$$\sigma(Q,x) = f(x,\mu) \otimes \widehat{\sigma}(\mu, Q, \alpha_S)$$

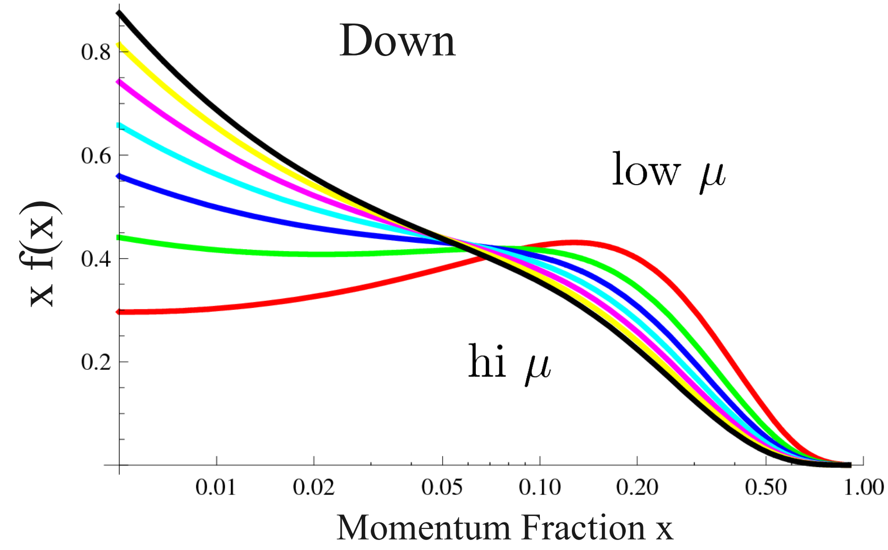


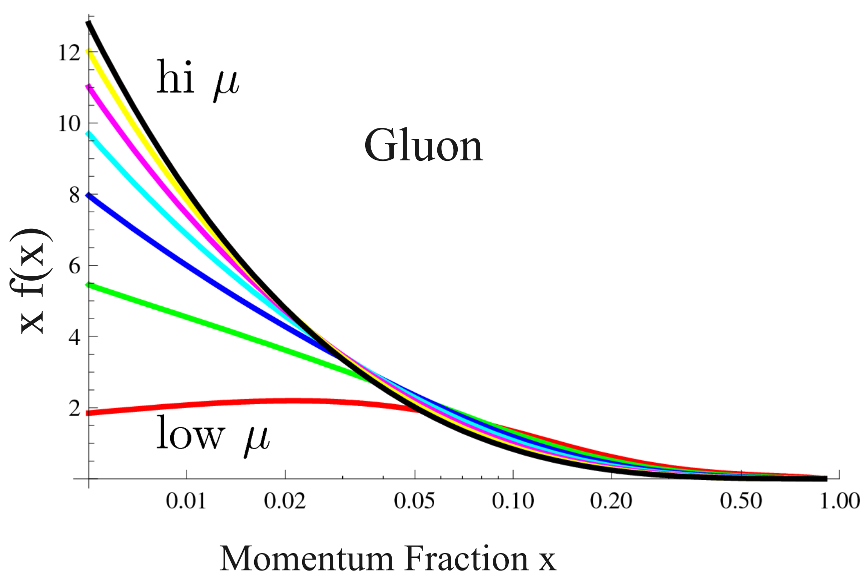
How does f change with scale μ???

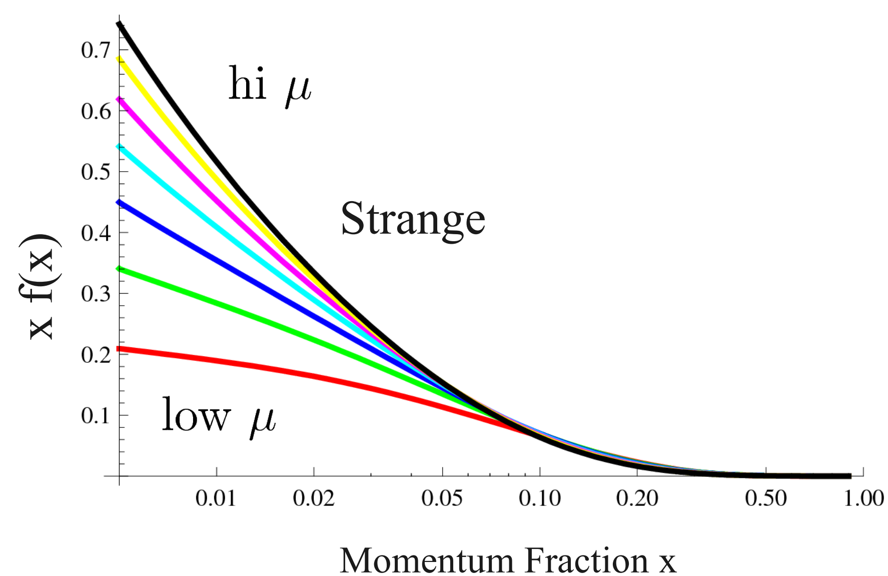
$$\frac{df}{d\ \ln[\mu]} = ???$$

Evolution of the PDFs



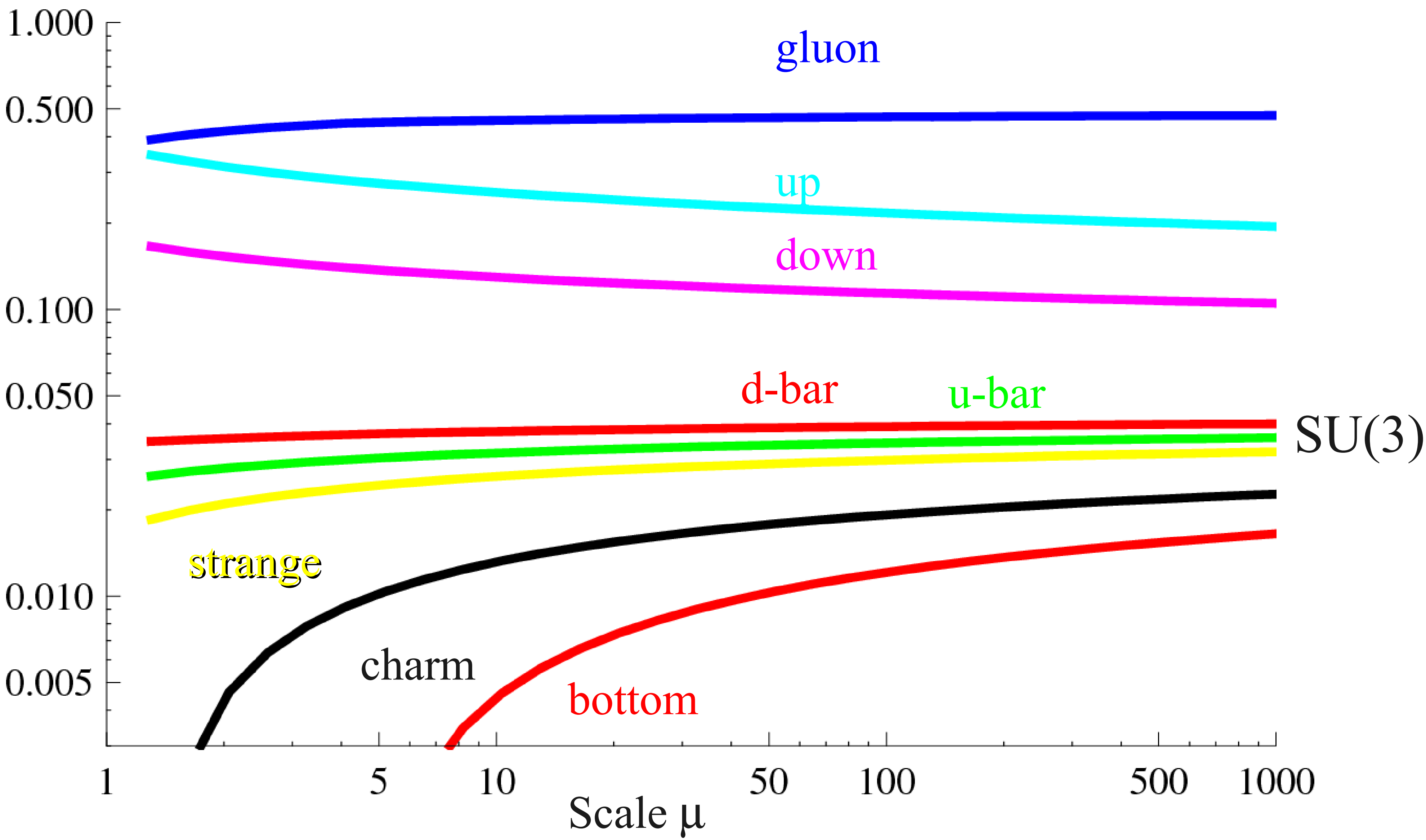






PDF Momentum Fractions vs. scale μ

Momentum Fraction



Scaling violations are essential feature of PDFs

Renormalization Group Equation

Parton Model

$$\sigma = f \otimes \omega$$

ω or $\hat{\sigma}$
Not physical!
Poor notation

Renormalization Group Equation

$$\frac{d\sigma}{d\mu} = 0 = \frac{d\tilde{f}}{d\mu}\,\tilde{\omega} + \tilde{f}\,\frac{d\tilde{\omega}}{d\mu}$$

Take Mellin Transform

Separation of variables

$$\frac{1}{\tilde{f}}\,\frac{d\tilde{f}}{d\ln[\mu]} = -\gamma = -\frac{1}{\tilde{\omega}}\,\frac{d\tilde{\omega}}{d\ln[\mu]}$$

DGLAP Equation

DGLAP

$$\frac{d\tilde{f}}{d\ln[\mu]} = -\tilde{f}\,\gamma \qquad \frac{df}{d\ln[\mu]} = P \otimes f$$

$$\tilde{f} \sim \mu^{-\gamma}$$

Anomalous Dimension

If "f" scaled, γ would vanish

It is the dimension of the mass scaling

DETAILS

Evolution and such

Homework: Mellin Transform

$$\widetilde{f}(n) = \int_0^1 dx\, x^{n-1}\, f(x)$$

$$\sigma = f \otimes \omega$$

$$f(x) = \frac{1}{2\pi i}\int_C dn\, x^{-n}\, \widetilde{f}(n)$$

$$\widetilde{\sigma} = \widetilde{f}\, \widetilde{\omega}$$

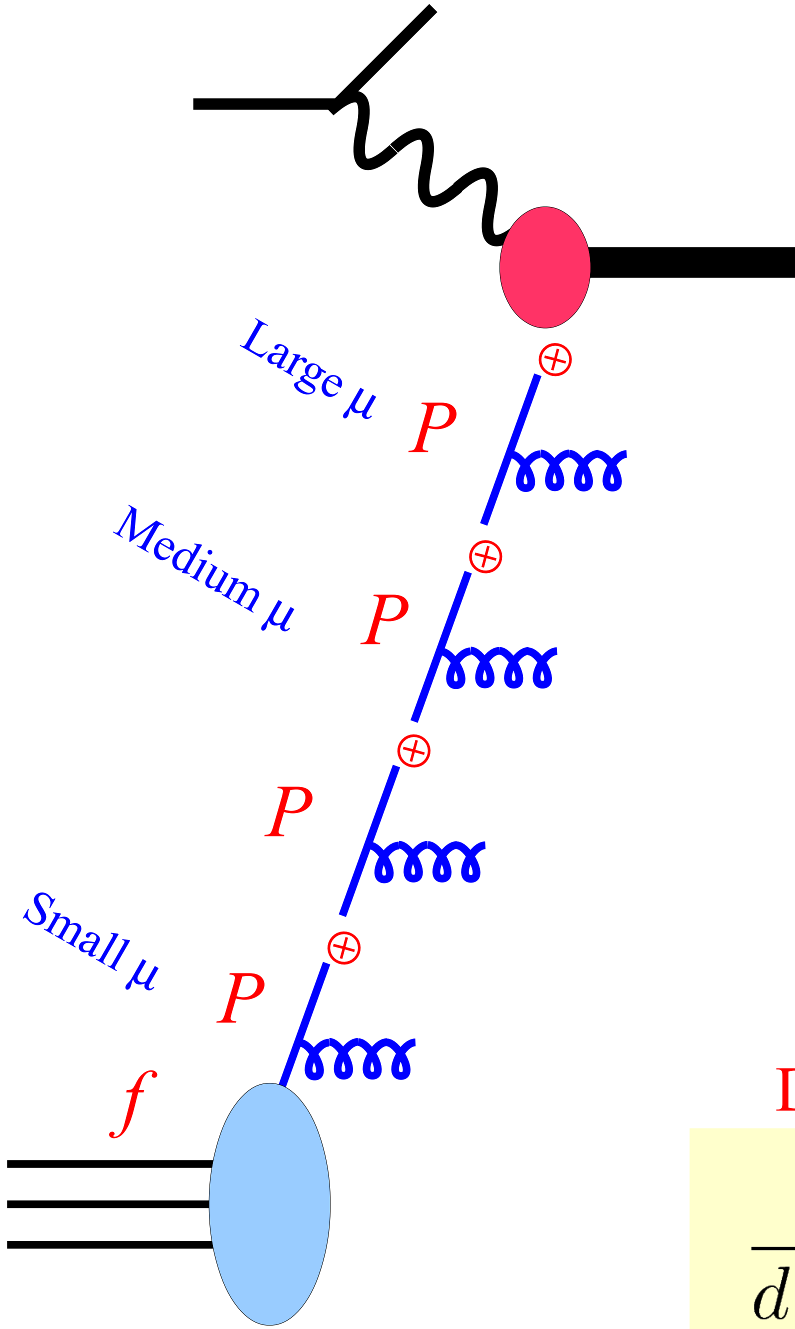
C is parallel to the imaginary axis, and to the right of all singularities

1) Take the Mellin transform of $f(x) = \sum_{m=1}^{\infty} a_m x^m$, and verify the inverse transform of $\widetilde{f}$ regenerates $f(x)$

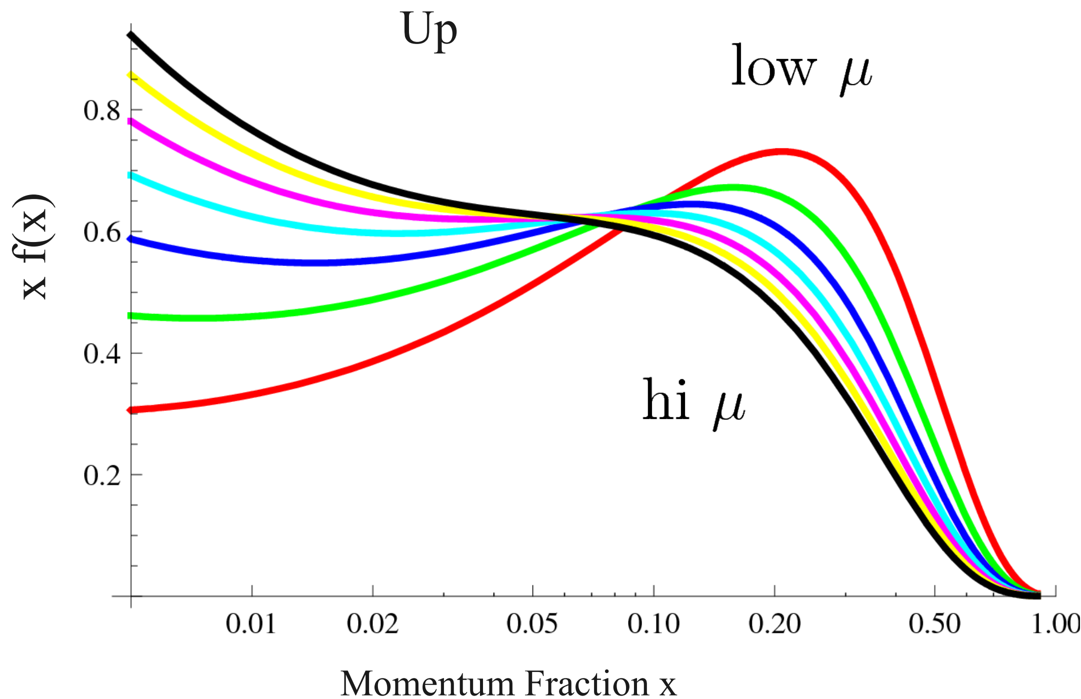
2) Take the Mellin transform of $\sigma = f \otimes \omega$ to demonstrate that the Mellin transform separates a convolution yields $\widetilde{\sigma} = \widetilde{f}\,\widetilde{\omega}$.

A useful reference:
Courant, Richard and Hilbert, David. Methods of Mathematical Physics, Vol. 1. New York: Wiley, 1989. 561 p.

Evolution of the PDFs

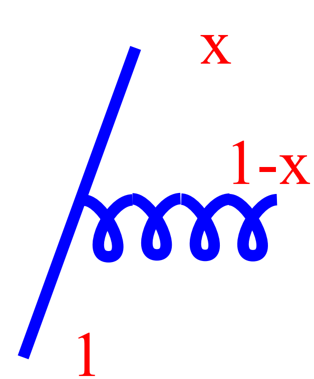


Evolution (generally) shifts partons from hi-x to low-x

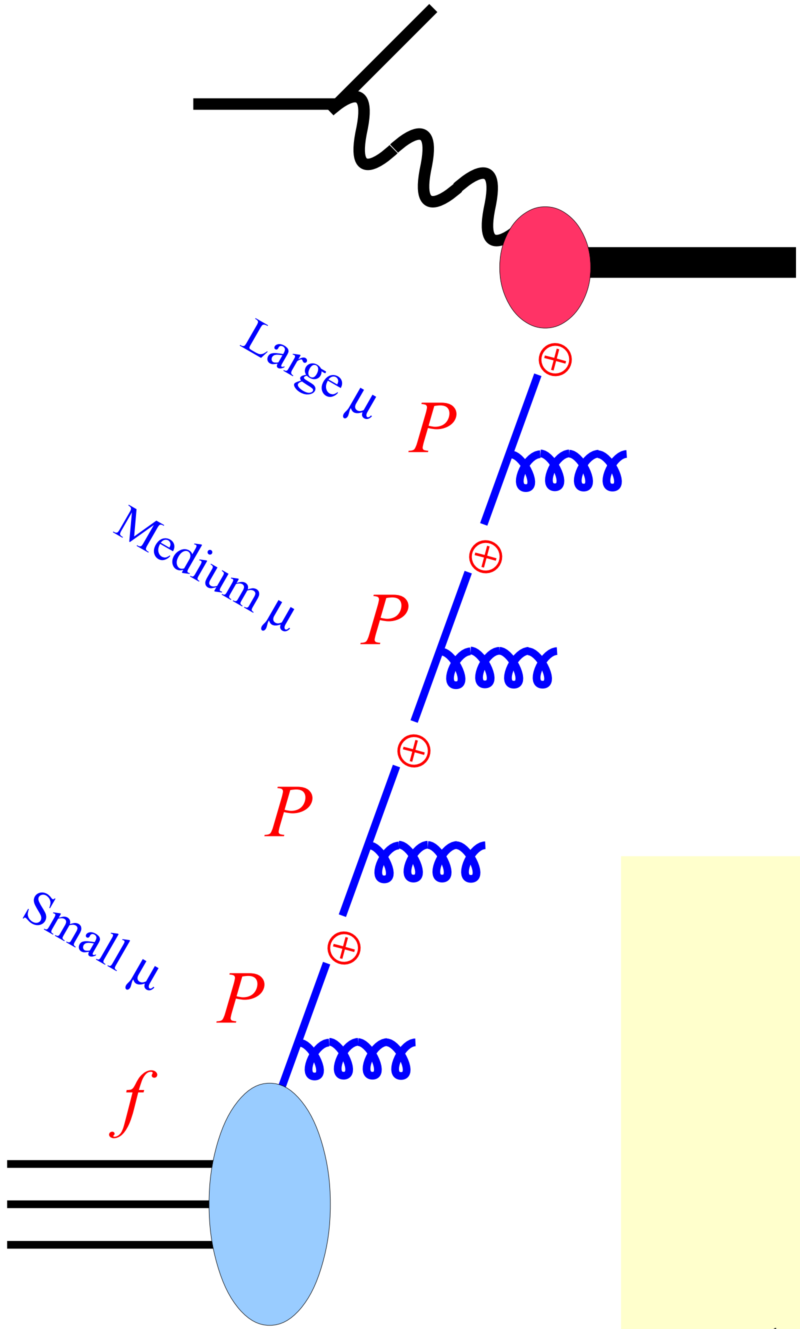


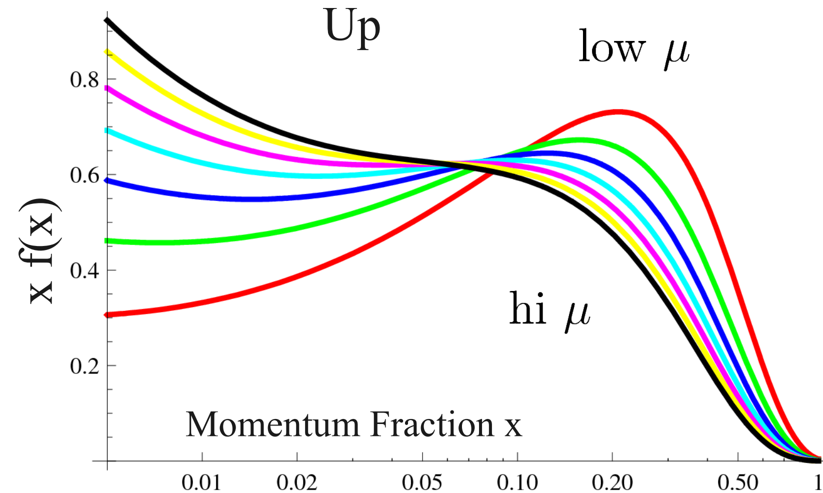
DGLAP Equation

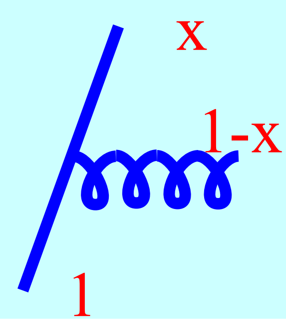
$$\frac{d\tilde{f}}{d\ln[\mu]} = P \otimes f$$



Evolution of the PDFs







$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x} \right]_+$$

$$\frac{df}{d\ \ln[\mu]} = P \otimes f \ \simeq \ \frac{\alpha_S}{2\pi} P^{(1)} \otimes f$$

$$P \simeq \delta + \tfrac{\alpha_S}{2\pi} P^{(1)} + \left(\tfrac{\alpha_S}{2\pi}\right)^2 P^{(2)} + \ldots$$

$$f_a(x,\mu_1) \sim f_a(x,\mu_0) + \frac{\alpha_S}{2\pi}\ P_{ab}^{(1)} \otimes f_b \ln\left(\frac{\mu_1^2}{\mu_2^0}\right)$$

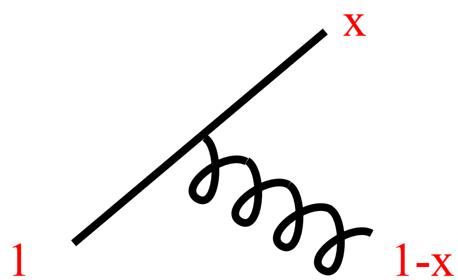
HOMEWORK

Sum Rules
&
Structure Functions

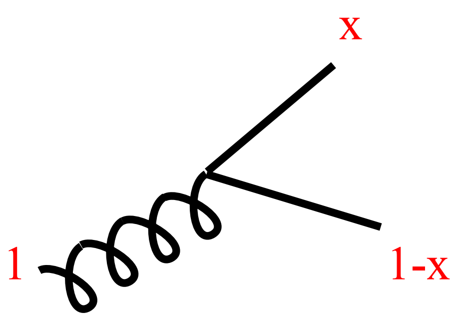
The Splitting Functions:

Read backwards

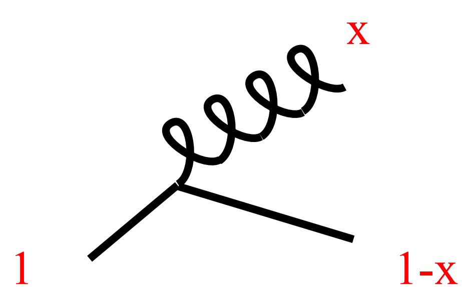
Note singularities



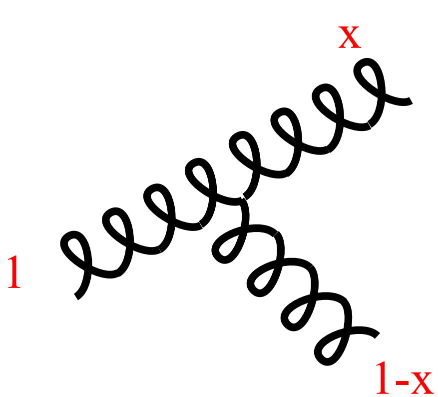
$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x} \right]_+$$



$$P_{qg}^{(1)}(x) = T_F \left[(1-x)^2 + x^2 \right]$$



$$P_{gq}^{(1)}(x) = C_F \left[\frac{(1-x)^2 + 1}{x} \right]$$



$$P_{gg}^{(1)}(x) = 2C_F \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \left[\frac{11}{6} C_A - \frac{2}{3} T_F N_F \right] \delta(1-x)$$

Definition of the Plus prescription:

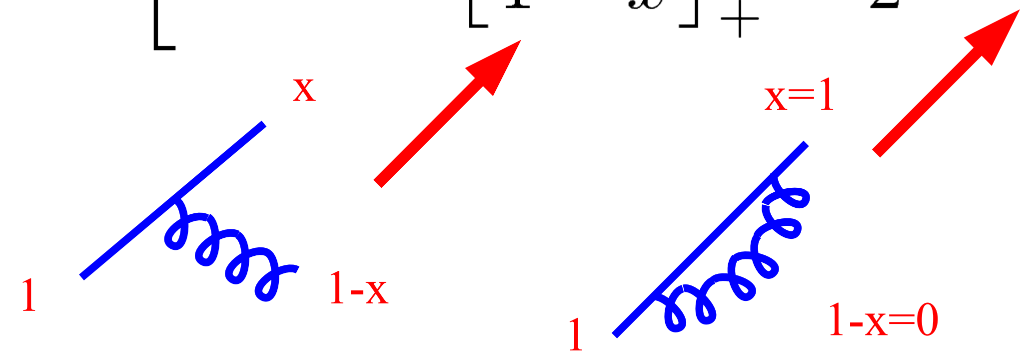
$$\int_0^1 dx \, \frac{f(x)}{(1-x)_+} = \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)}$$

1) Compute: $$\int_a^1 dx \, \frac{f(x)}{(1-x)_+} = ???$$

Homework: Part 2 P(q←q)

2) Verify:

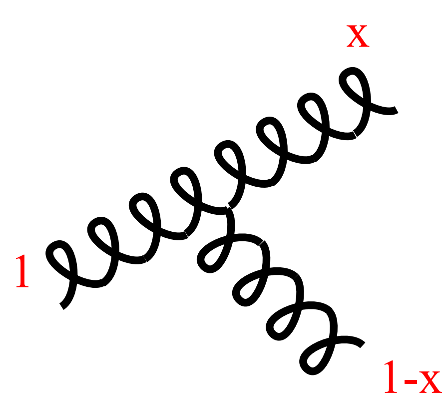
$$P^{(1)}_{qq}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+ \equiv C_F \left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\delta(1-x)\right]$$

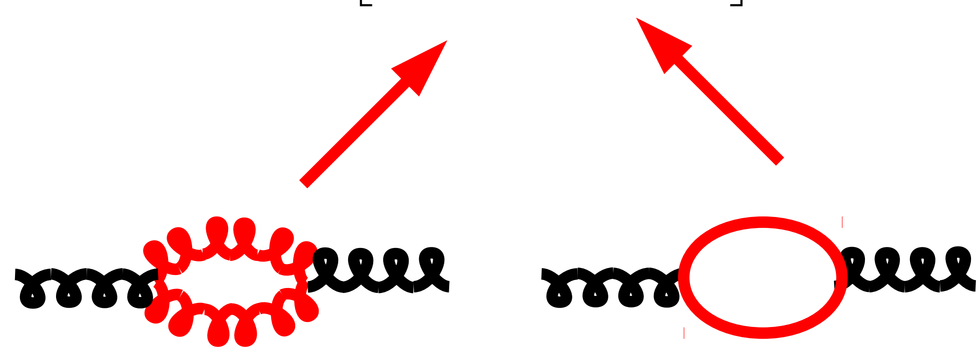


Observe

$$P^{(1)}_{gg}(x) = 2C_F \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right]$$

$$+ \left[\frac{11}{6}C_A - \frac{2}{3}T_F N_F\right] \delta(1-x)$$

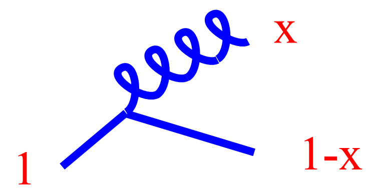


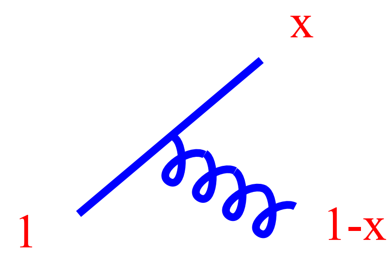


Verify the following relation among the regular parts (from the real graphs)

For the regular part show:

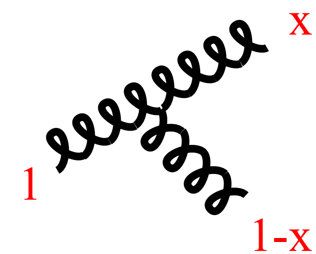
$$P_{gq}^{(1)}(x) = P_{gq}^{(1)}(1-x)$$





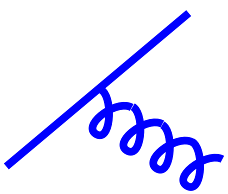
For the regular part show:

$$P_{gg}^{(1)}(x) = P_{gg}^{(1)}(1-x)$$

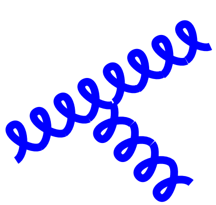


Verify, in the soft limit:

$$P_{qq}^{(1)}(x) \underset{x \to 1}{\longrightarrow} 2C_F \frac{1}{(1-x)_+}$$



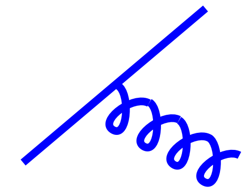
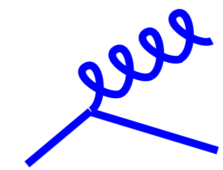
$$P_{gg}^{(1)}(x) \underset{x \to 1}{\longrightarrow} 2C_F \frac{1}{(1-x)_+}$$



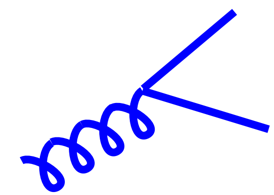
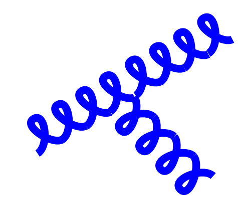
HOMEWORK: Part 4: Conservation Rules

Verify conservation of momentum fraction

$$\int_0^1 dx\, x\, \left[P_{qq}(x) + P_{gq}(x)\right] = 0$$

$$\int_0^1 dx\, x\, \left[P_{qg}(x) + P_{gg}(x)\right] = 0$$

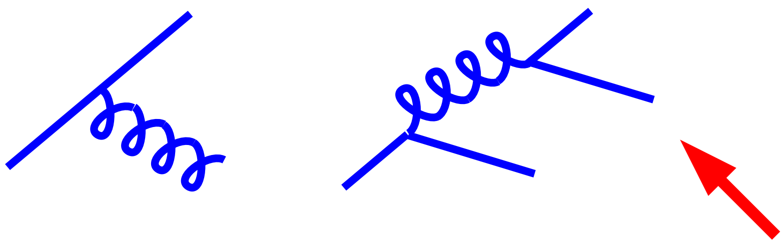
Verify conservation of fermion number

$$\int_0^1 dx \quad \left[P_{qq}(x) - P_{q\bar{q}}(x)\right] = 0$$

Homework: Part 5: Using the Real to guess the Virtual

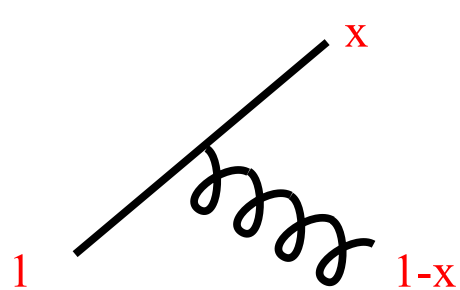
Use conservation of fermion number to compute
the delta function term in P(q←q)

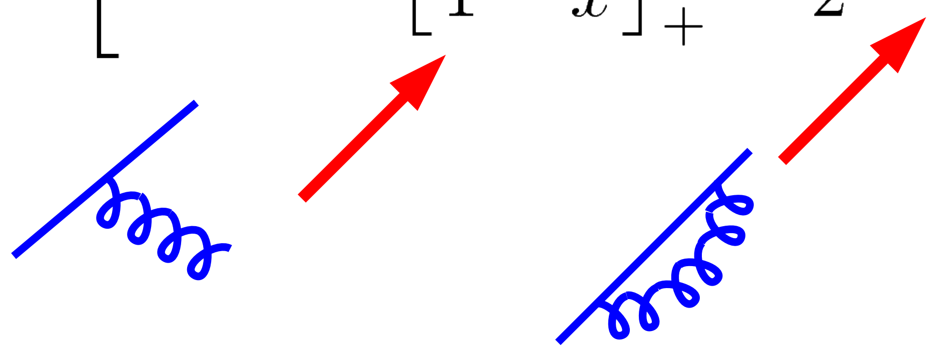
$$\int_0^1 dx \quad [P_{qq}(x) - P_{q\bar{q}}(x)] = 0$$



This term only starts at NNLO

$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+ \equiv C_F \left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\,\delta(1-x)\right]$$



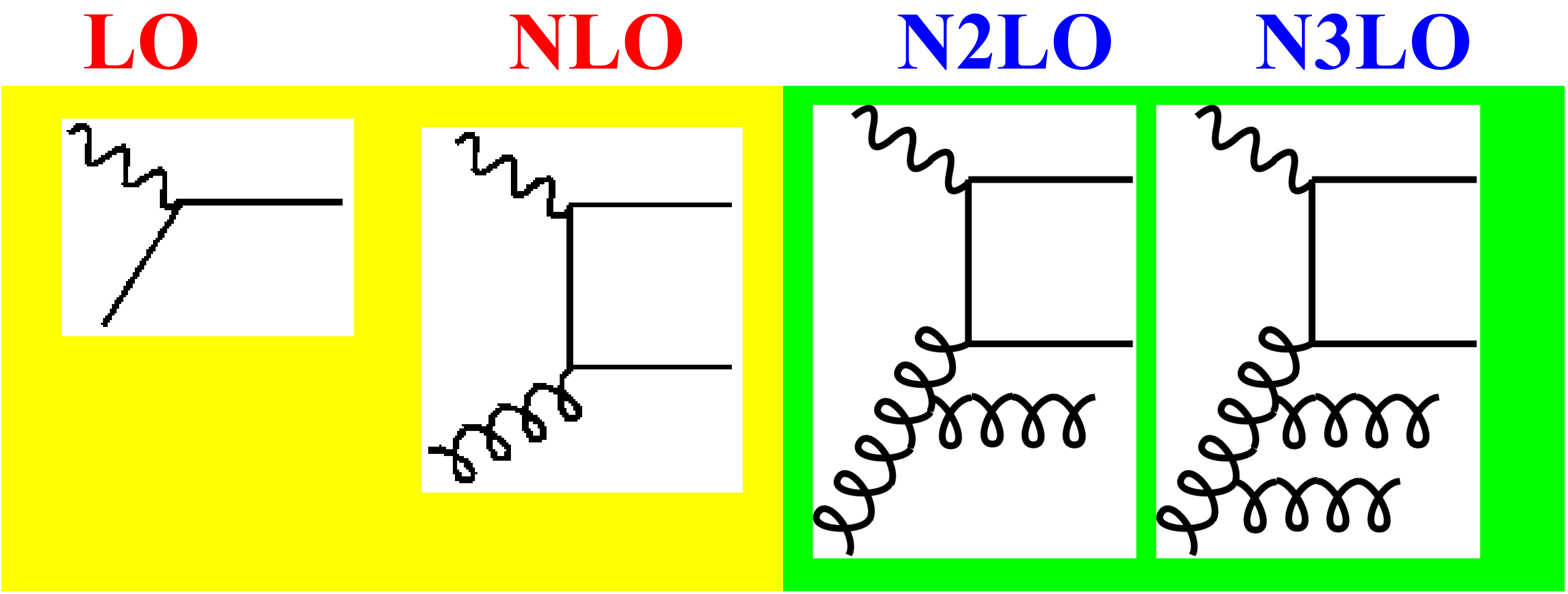


Powerful tool: Since we know real and virtual must balance, we can use to our advantage!!!

Recap so far

- Rutherford Scattering $\Rightarrow$ Deeply Inelastic Scattering (DIS)
  - Works for protons as well as nuclei
- Compute Lepton-Hadron Scattering 2 ways
  - Use Leptonic/Hadronic Tensors to extract Structure Functions
  - Use Parton Model; relate PDFs to F_{123}
- Parton Model Factorizes Problem:
  - PDFs are independent of process
  - Thus, we can combine different experiments. ESSENTIAL!!!
- PDFs are not truly scale invariant; they evolve
  - We use evolution to “resum” an important set of graphs

N^3LO



Full ACOT

Extensible to any order

$$\sigma = f(\xi(x, m_{ps}), Q) \otimes \hat{\sigma}(m_{dyn})$$

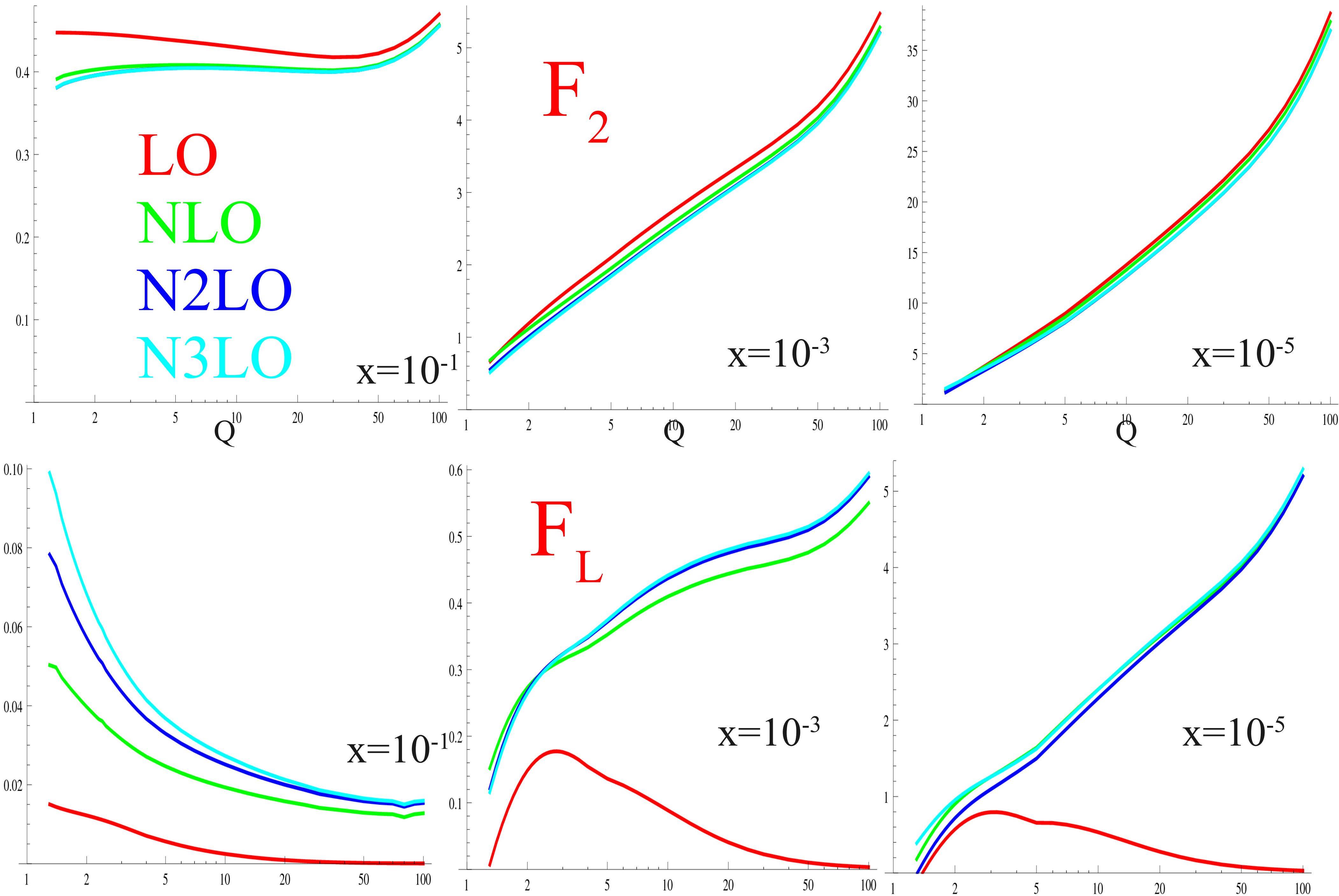
$$\xi(x, m_{ps}) = x \left(1 + \left[\frac{n\, m_{ps}}{Q} \right]^2 \right)$$

$$n = \{0, 1, 2\}$$

Distinguish:
"phase space" mass
"dynamic" mass

Demonstrate:
1) PS mass dominates
2) Estimated Error small

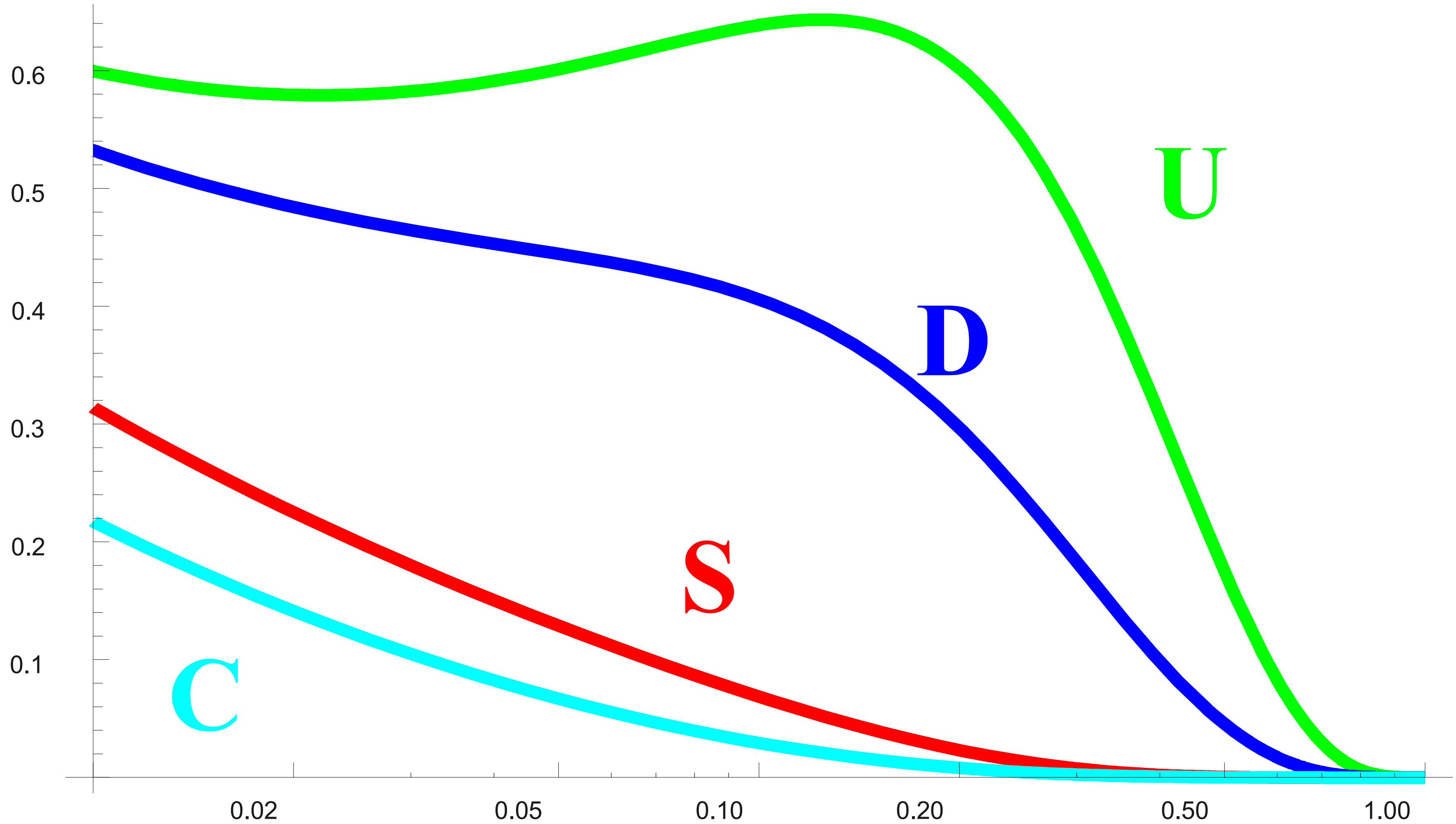
$F_{2,L}$ @ N3LO



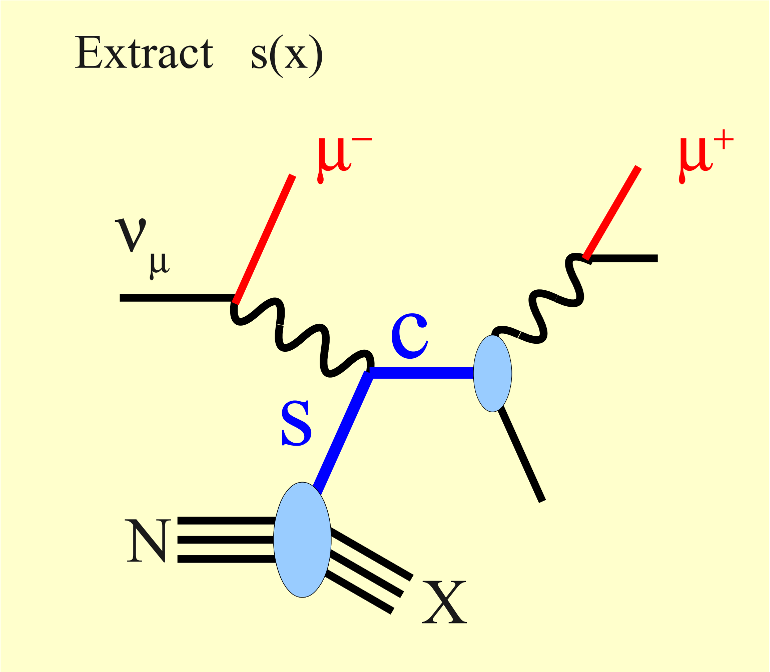
Strange Production

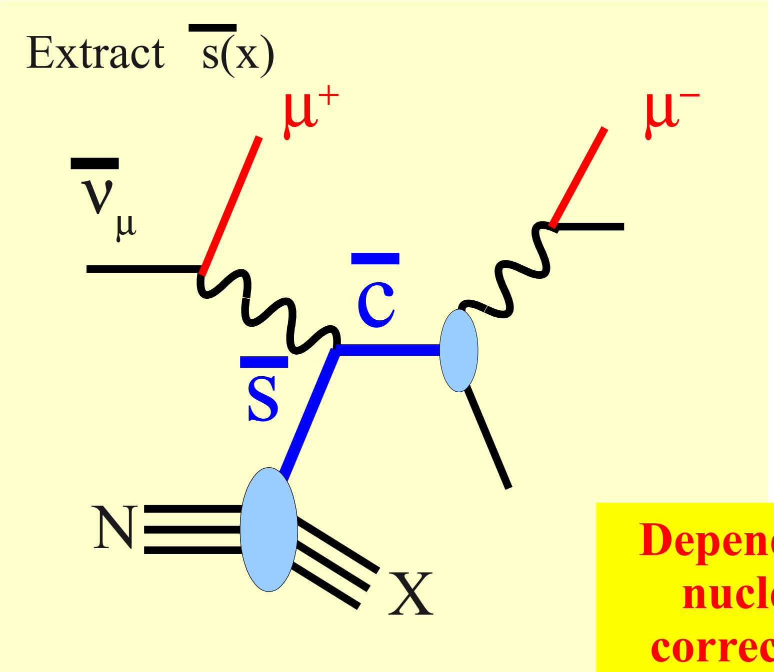
the LHC and PDFs

What constrains the Strange???



Di-muon production ⇒ Extract s(x) Parton Distribution



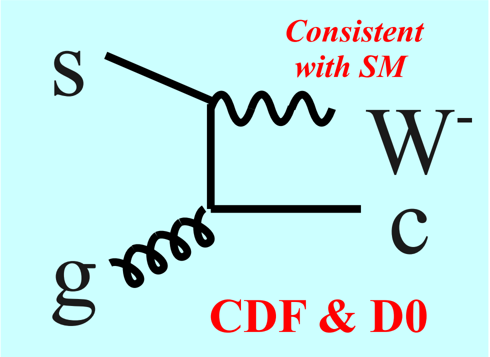


Depends on nuclear corrections

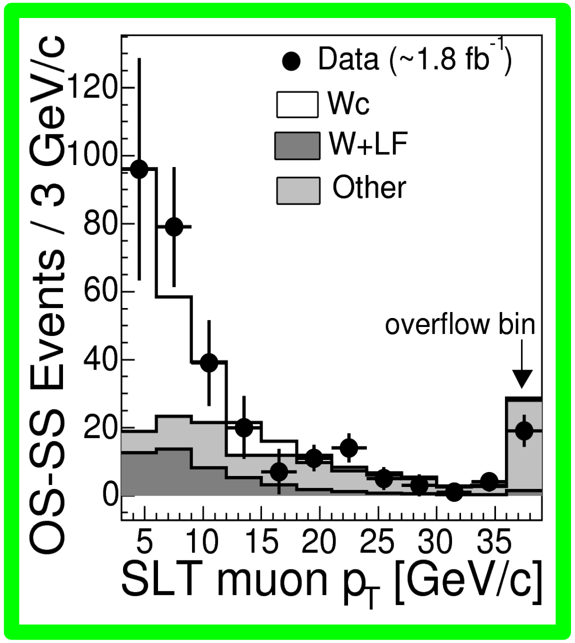
s(x) and $\bar{s}$(x) are essential in extraction of $\sin\theta_W$

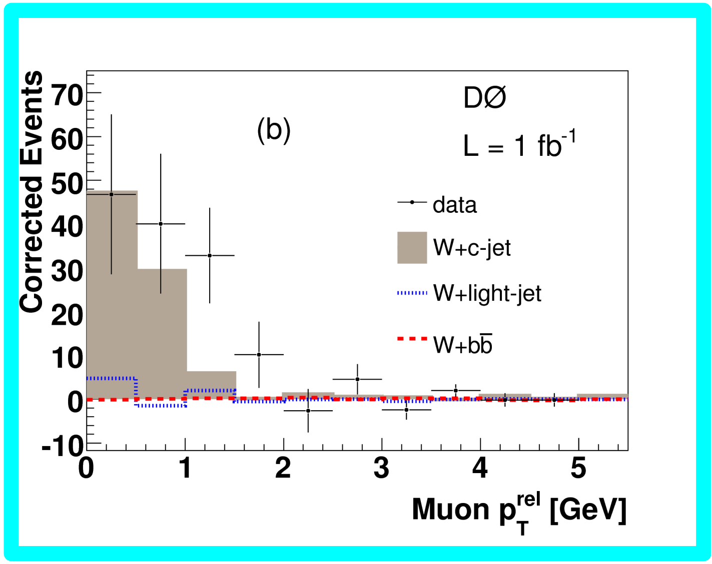
Used in CTEQ6 Fits

s g→Wc at the Tevatron



CDF: PRL 100:091803,2008.
D0: PLB666:23,2008.

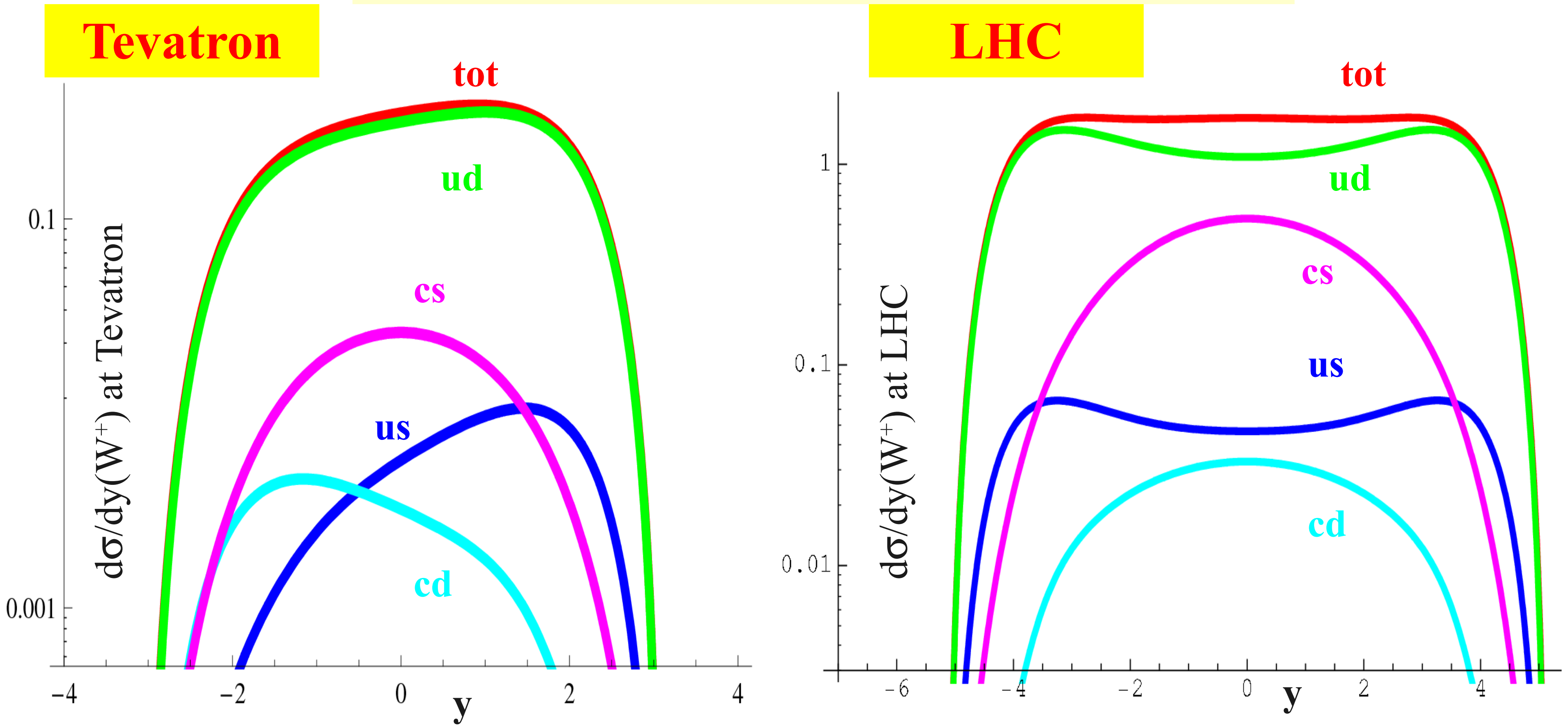




Also a challenge at LHC

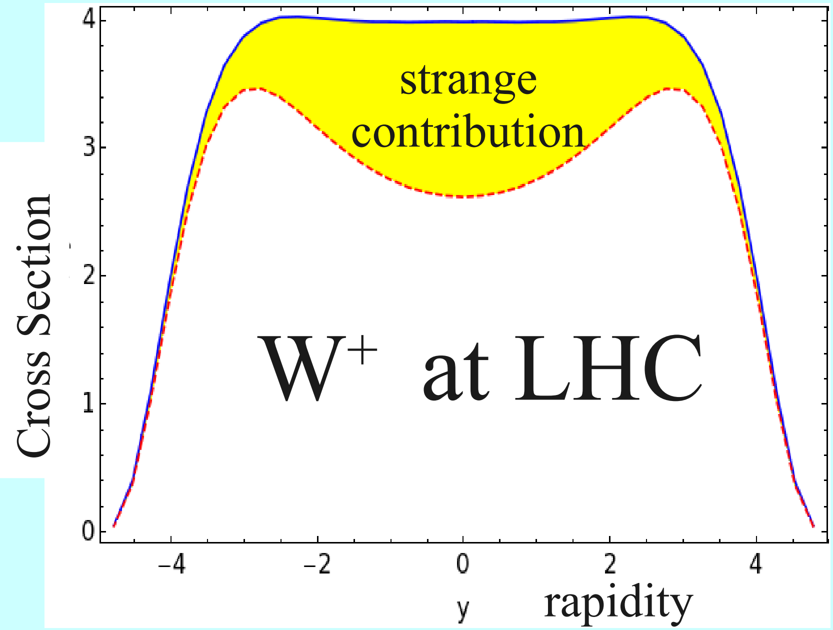
W Production at LHC: A Benchmark Cross Section

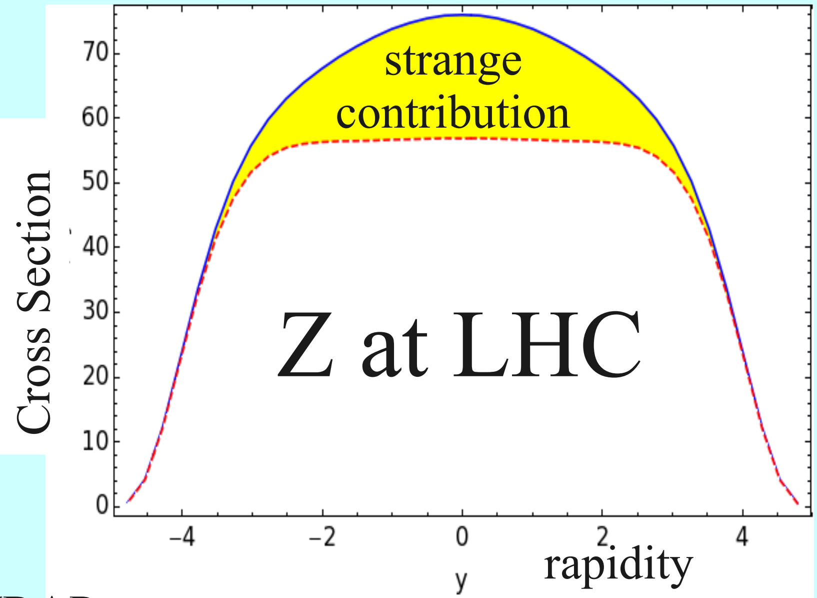
Heavy quark PDFs are essential ingredient



Heavy Quark components play an increasingly important role at the LHC

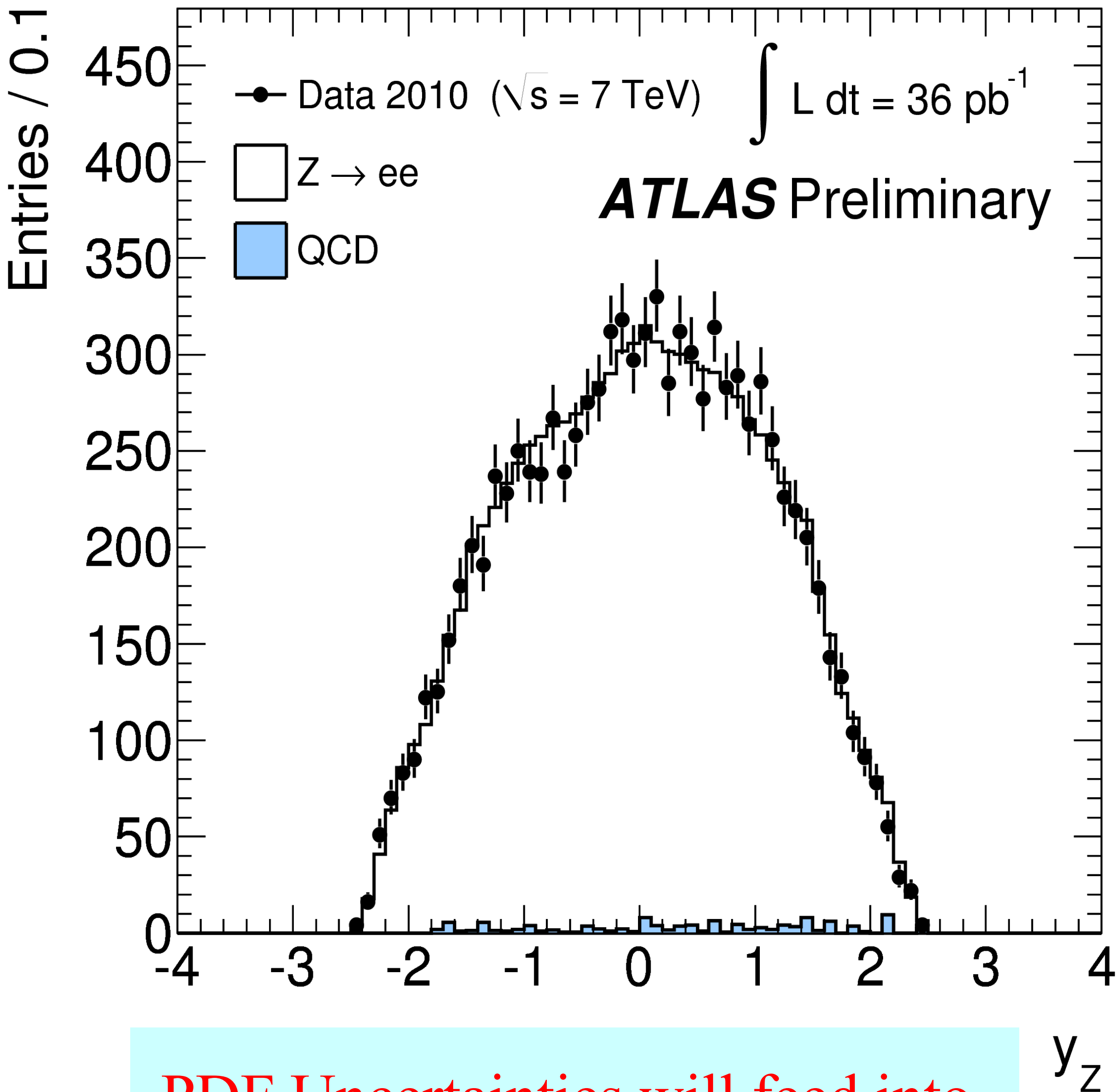
PDF Uncertainties ⇒ S(x) PDF ⇒ W/Z at LHC





VRAP Code

Anastasiou, Dixon, Melnikov, Petriello, Phys.Rev.D69:094008,2004.



PDF Uncertainties will feed into LHC "Benchmark" processes

Comparison with new NNPDF sets: Les Houches 2009

SUMMARY

- Rutherford Scattering ⇒ Deeply Inelastic Scattering (DIS)
  - Works for protons as well as nuclei
- Compute Lepton-Hadron Scattering 2 ways
  - Use Leptonic/Hadronic Tensors to extract Structure Functions
  - Use Parton Model; relate PDFs to F123
- Parton Model Factorizes Problem:
  - PDFs are independent of process
  - Thus, we can combine different experiments. ESSENTIAL!!!
- PDFs are not truly scale invariant; they evolve
  - We use evolution to “resum” an important set of graphs
- NLO Calculations, and beyond
- New Era: Strong constraints on PDF from LHC

END OF LECTURE