Introduction to the Parton Model and Perturbative QCD

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I. The Parton Model and Deep-inelastic Scattering

II. From the Parton Model to QCD

III. Factorization and Evolution

As time allows . . .

This is an auspicious time for QCD, particle physics and whatever is beyond it.

Large Hadron Collider and Tevatron and much of RHIC phenomenology relies on essentially all that follows.

The Context of QCD: “Fundamental Interactions”

- Electromagnetic
- + Weak Interactions $\Rightarrow$ Electroweak
- + Strong Interactions (QCD) $\Rightarrow$ Standard Model
- + ... = Gravity and the rest?
- QCD: A theory “off to a good start”. Think of ...

  - $\vec{F}_{12} = -GM_1M_2\hat{r}/R^2$ $\Rightarrow$ elliptical orbits
    ... 3-body problem ...

  - $L_{\mathrm{QCD}} = \bar{q}\ \not{D} q - (1/4)F^2$ $\Rightarrow$ asymptotic freedom
    ... confinement ...

I. The Parton Model and Deep-inelastic Scattering

IA. Nucleons to Quarks

IB. DIS: Structure Functions and Scaling

IC. Getting at the Quark Distributions

ID. Classic Parton Model Extensions:
Fragmentation and Drell-Yan

Introduce concepts and results that predate QCD, led to QCD and were incorporated and explained by QCD.

IA. From Nucleons to Quarks

- **The pattern: nucleons, pions and isospin:**

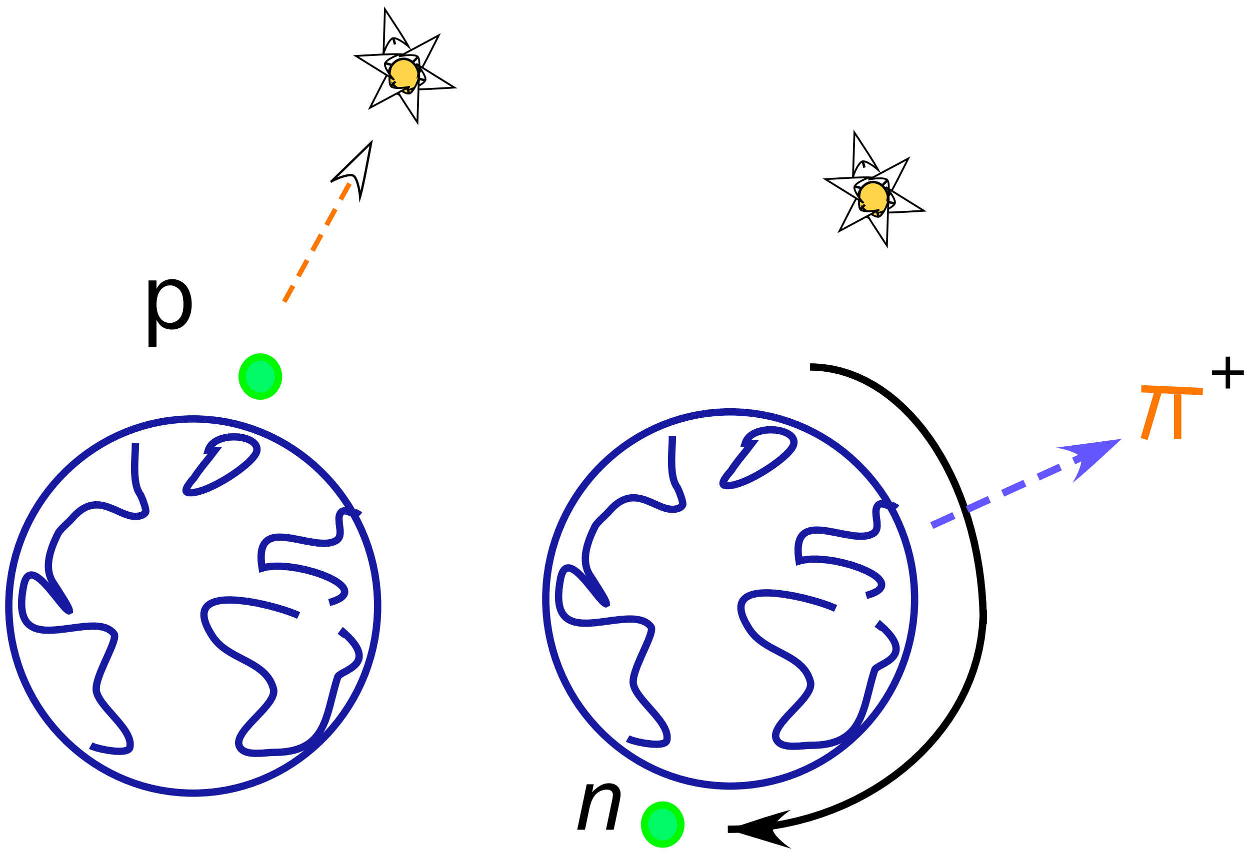
$$\begin{pmatrix} p \\ n \end{pmatrix}$$

  - **p: m=938.3 MeV,** $S = 1/2$, $I_3 = 1/2$
  - **n: m=939.6 MeV,** $S = 1/2$, $I_3 = -1/2$

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

  - $\pi^\pm$**: m=139.6 MeV,** $S = 0$, $I_3 = \pm 1$
  - π^0**: m=135.0 MeV,** $S = 0$, $I_3 = 0$

- **Isospin space . . .**
- **Globe with a "north star" set by electroweak interactions:**



Analog: the rotation group (more specifically, $SU(2)$).

- Explanation: π, N common substructure: quarks

  (Gell Mann, Zweig 1964)

- spin $S = 1/2$,
  $I = 1/2$ (u, d) & $I = 0$ (s)
  with approximately equal masses (s heavier);

$$\begin{pmatrix} u\ (Q = 2e/3, I_3 = 1/2) \\ d\ (Q = -e/3, I_3 = -1/2) \\ s\ (Q = -e/3, I_3 = 0) \end{pmatrix}$$

$$\pi^+ = (u\bar{d})\ , \quad \pi^- = (\bar{u}d)\ , \quad \pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)\ ,$$

$$p = (uud)\ , \quad n = (udd)\ , \quad K^+ = (u\bar{s}) \ldots$$

This is the quark model

- Quark model nucleon has symmetric spin/isospin wave function (return to this later)

- Early success: $\mu_p/\mu_n =$ -3/2 (good to %)

- And now, six: 3 ‘light’ (u, d, s), 3 ‘heavy’: (c, b, t)

- Of these all but t form bound states of quark model type.

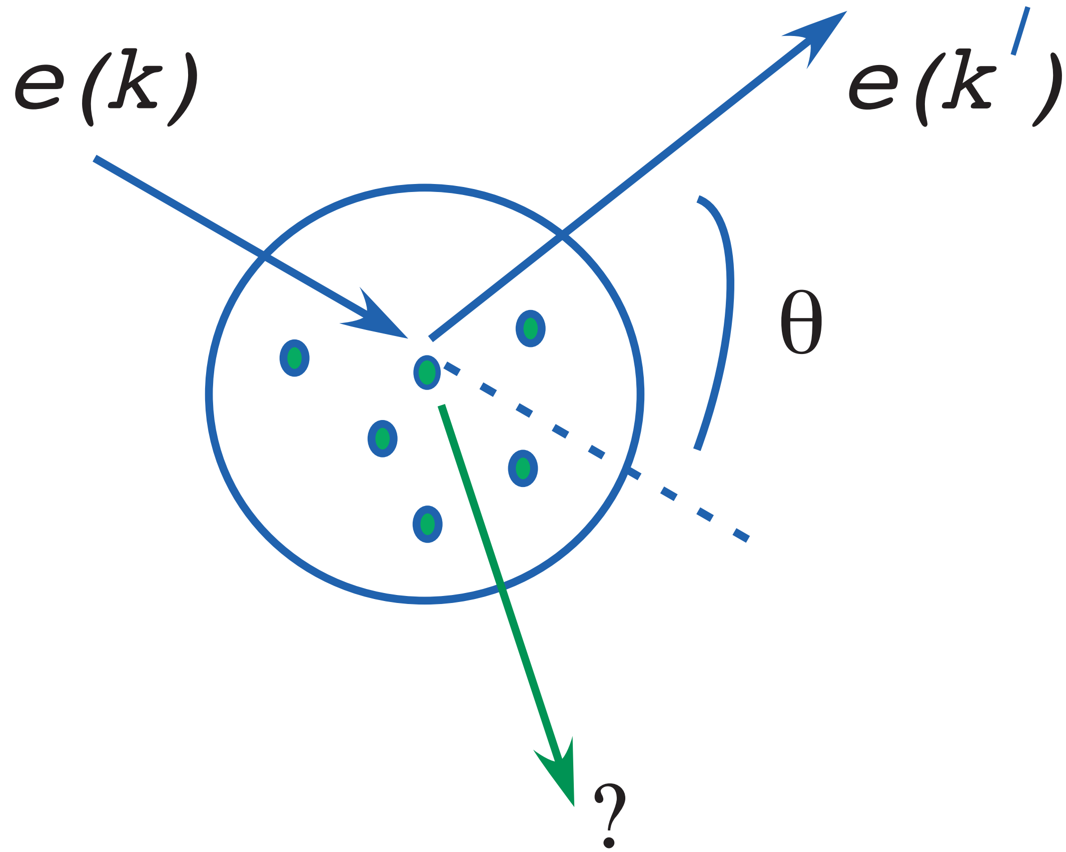
- **Quarks as Partons: "Seeing" Quarks.**

  No isolated fractional charges seen ("confinement.")

  Can such a particle be detected? (SLAC 1969)

  Look closer: do high energy electrons bounce off anything hard? ('Rutherford-prime')

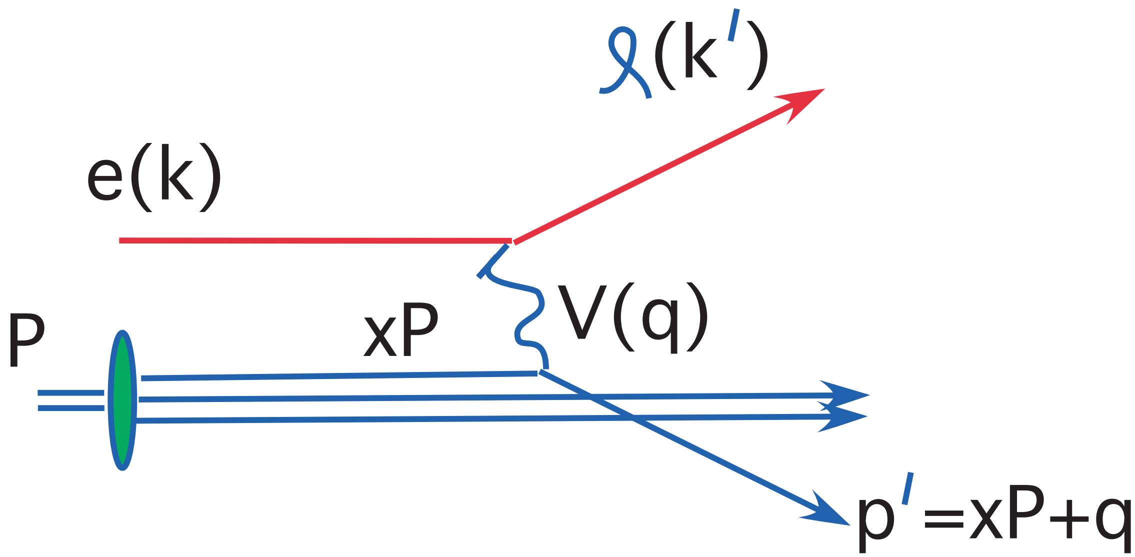
- **So look for:**



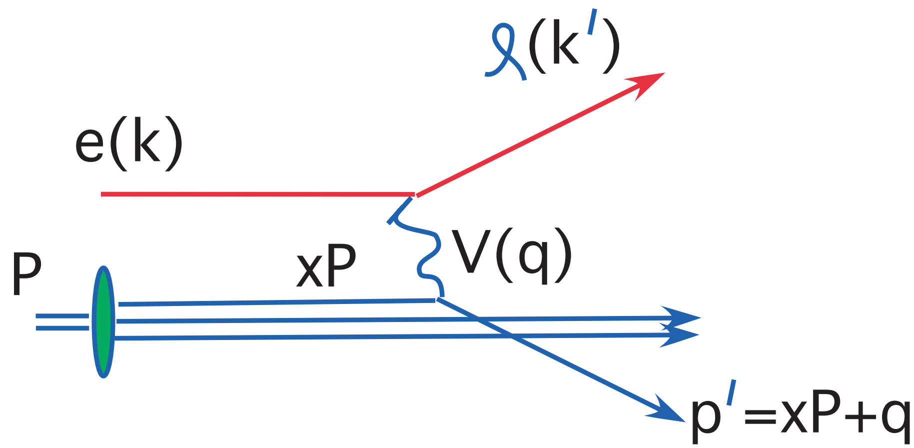
"Point-like' constituents.

The angular distribution gives information on the constituents.

Kinematics $(e + N(P) \to \ell + X)$



- $V = \gamma, Z_0 \Rightarrow \ell = e,\ \mu,$ **"neutral current" (NC).**
- $V = W^-(e^-, \nu_e)$, $V = W^+(e^+, \bar{\nu}_e)$, **or** $(e, \nu_e) \to (\mu, \nu_\mu)$ **"charged current" (CC).**
- $W^2 \equiv (p+q)^2 \gg m^2_{\mathrm{proton}}$: **Deep-inelastic scattering (DIS)**



$Q^2 = -q^2 = -(k - k')^2$ **momentum transfer.**

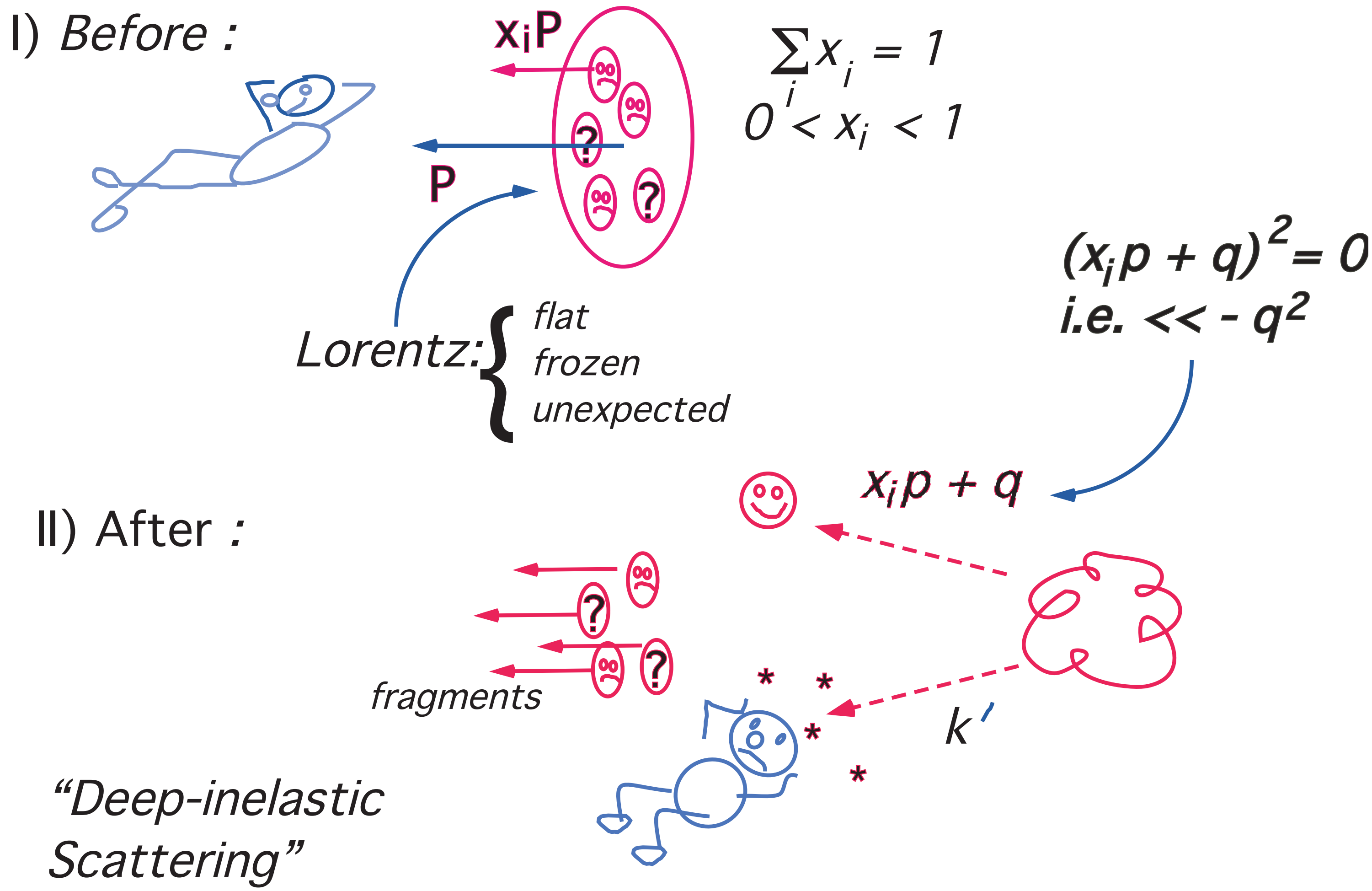
$x \equiv \frac{Q^2}{2p\cdot q}$ **momentum fraction (from** $p'^2 = (xp + q)^2 = 0$**).**

$y = \frac{p\cdot q}{p\cdot k}$ **fractional energy transfer.**

$W^2 = (p + q)^2 = \frac{Q^2}{x}(1 - x)$ **squared final-state mass of hadrons.**

$$xy = \frac{Q^2}{S}$$

Parton Interpretation (Feynman 1969, 72)
Look in the electron's rest frame . . .



– Basic Parton Model Relation

$$\sigma_{\rm eh}(p,q) = \sum_{\text{partons } a} \int_0^1 d\xi\, \hat{\sigma}^{\rm el}_{ea}(\xi p, q)\ \phi_{a/h}(\xi)\,,$$

– **where**: σ_{eh} is the cross section for $e(k) + h(p) \to e(k' = k - q) + X(p+q)$

– **and** $\hat{\sigma}^{\rm el}_{ea}(xp,q)$ is the <u>elastic</u> cross section for $e(k) + a(\xi p) \to e(k' - q) + a(\xi p + q)$ which sets $(\xi p + q)^2 = 0 \to \xi = -q^2/2p\cdot q \equiv x$.

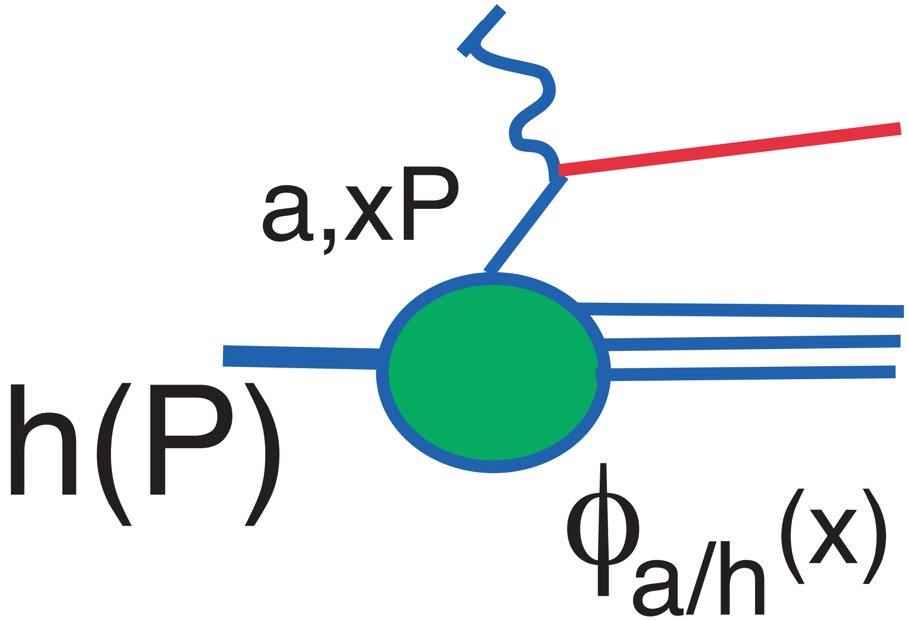
– **and** $\phi_{a/h}(x)$ is the distribution of parton a in hadron h, the "probability for a parton of type a to have momentum xp". It has a meaning independent of the details of the hard scattering – the hallmark of factorization.

– **in words: Hadronic INELASTIC cross section is the sum of convolutions of partonic ELASTIC cross sections with the hadron's parton distributions.**

– **The nontrivial assumption: quantum mechanical incoherence of large-q scattering and the partonic distributions. Multiply probabilities without adding amplitudes.**

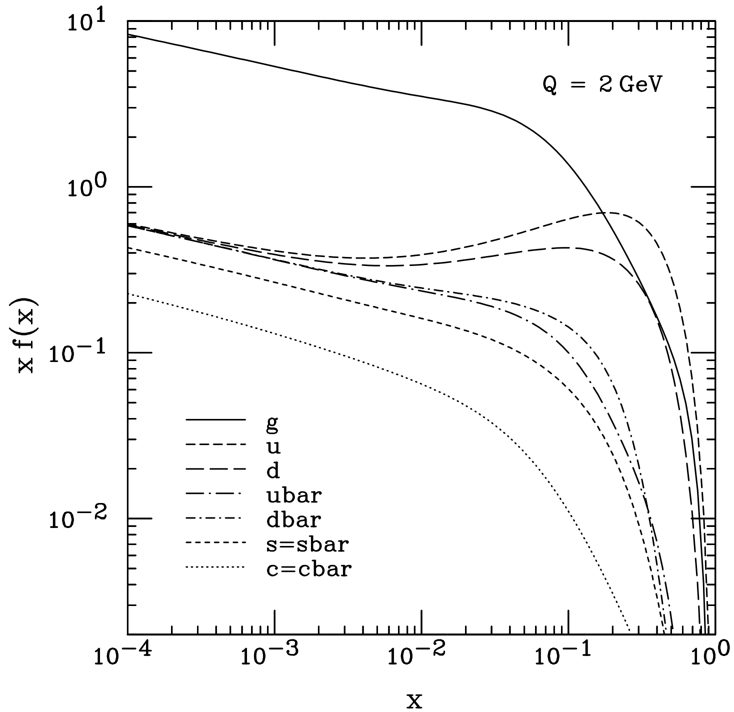
– **Heuristic justification: the binding of the nucleon involves long-time processes that do not interfere with the short-distance scattering. Later we'll see how this works in QCD.**

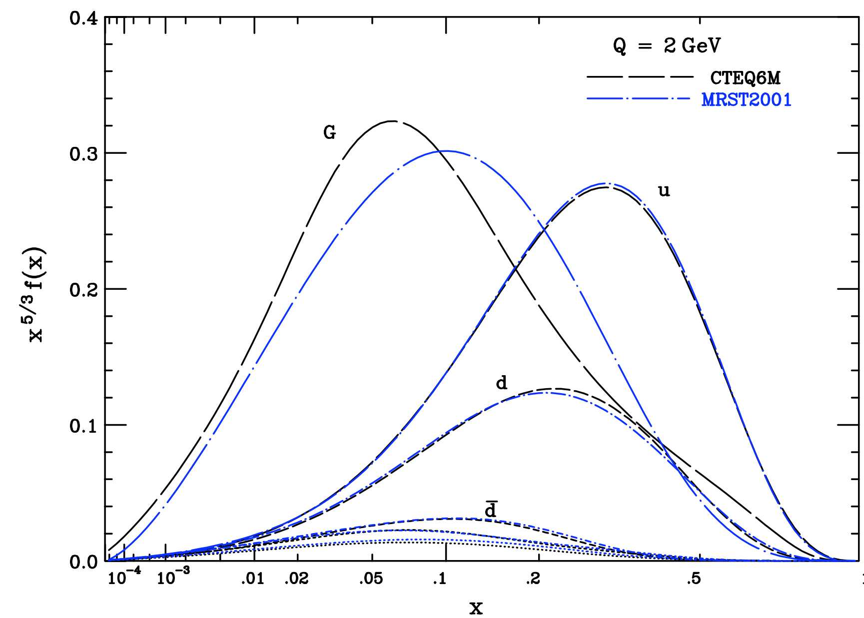
– **The familiar picture**



– **"QM incoherence" ⇔ no interactions between of the outgoing scattered quark and the rest.**

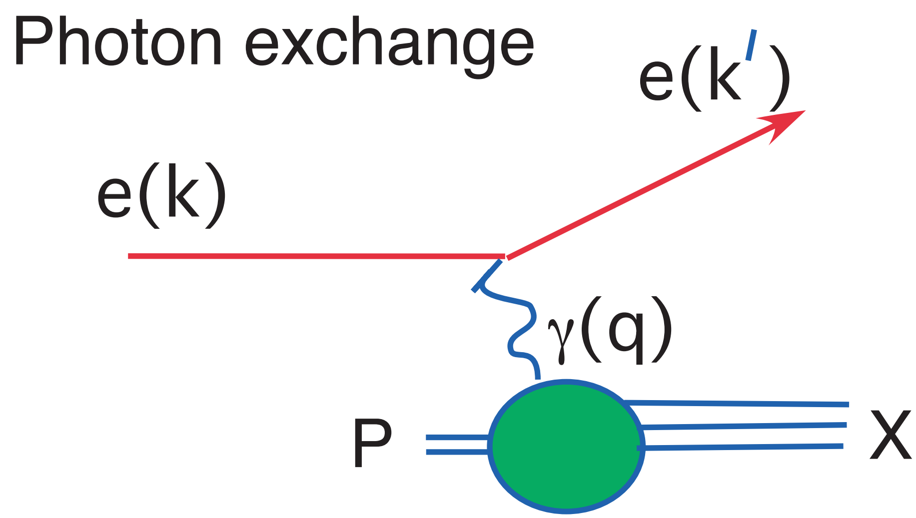
- **Two modern parton distribution sets at moderate momentum transfer (note different weightings with x):**





- **We'll see where these come from.**

IB. DIS: Structure Functions and Scaling



$$A_{e+N\to e+X}(\lambda,\lambda',\sigma;q) = \bar{u}_{\lambda'}(k')(-ie\gamma_\mu)u_\lambda(k)$$
$$\times \frac{-ig^{\mu\mu'}}{q^2}$$
$$\times \langle X|\, eJ^{\mathrm{EM}}_{\mu'}(0)\,|p,\sigma\rangle$$

- **Historically an assuption that the photon couples to hadrons by point-like current operator. Now, built into the Standard Model.**

- **The cross section:**

$$d\sigma_{\rm DIS} = \frac{1}{2^2}\frac{1}{2s}\frac{d^3k'}{(2\pi)^3 2\omega_{k'}} \sum_X \sum_{\lambda,\lambda',\sigma} |A|^2$$
$$\times\ (2\pi)^4\, \delta^4(p_X + k' - p - k)$$

In $|A|^2$, separate the known leptonic part from the "unknown" hadronic part: $|A|^2 = L^{\mu\nu} W_{\mu\nu}$.

- **The leptonic tensor:**

$$L^{\mu\nu} = \frac{e^2}{8\pi^2} \sum_{\lambda,\lambda'} \left(\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k)\right)^* \left(\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k)\right)$$
$$= \frac{e^2}{2\pi^2}\left(k^\mu k'^{\,\nu} + k'^{\,\mu} k^\nu - g^{\mu\nu} k\cdot k'\right)$$

- Leaves us with the hadronic tensor:

$$W_{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma,X} \langle X|J_\mu|p,\sigma\rangle^* \langle X|J_\nu|p,\sigma\rangle \\ \times\ (2\pi)^4 \delta^4(p_X - p - q)$$

- And the cross section becomes:

$$2\omega_{k'} \frac{d\sigma}{d^3k'} = \frac{1}{s(q^2)^2}\ L^{\mu\nu} W_{\mu\nu}$$

- $W_{\mu\nu}$ has sixteen components,
  but known properties of the strong interactions constrain $W_{\mu\nu}$ ...

- **An example: current conservation,**

$$
\begin{aligned}
&\partial^\mu J_\mu^{\mathrm{EM}}(x) = 0 \\
&\qquad \Rightarrow \langle X|\, \partial^\mu J_\mu^{\mathrm{EM}}(x)\, |p\rangle = 0 \\
&\qquad \Rightarrow (p_X - p)^\mu \langle X|\, J_\mu^{\mathrm{EM}}(x)\, |p\rangle = 0 \\
&\qquad \Rightarrow q^\mu W_{\mu\nu} = 0
\end{aligned}
$$

- **With parity, time-reversal, etc . . .**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(x, Q^2)$$
$$+ \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) W_2(x, Q^2)$$

- Often given in terms of the dimensionless structure functions,

$$F_1 = W_1 \qquad F_2 = p \cdot q W_2$$

- Note that if there is no other mass scale, the F's cannot depend on Q except indirectly through x.

- **Structure functions in the Parton Model: The Callan-Gross Relation**

  From the "basic parton model formula":

$$\frac{d\sigma_{eh}}{d^3k'} = \sum_{\text{quarks } f} \int d\xi \; \frac{d\sigma^{\text{el}}_{ef}(\xi p)}{d^3k'} \; \phi_{f/h}(\xi) \qquad (1)$$

Can treat a quark of "flavor" f just like any hadron and get

$$\omega_{k'} \, \frac{d\sigma^{\text{el}}_{ef}(\xi p)}{d^3k'} = \frac{1}{2(\xi s)Q^4} \, L^{\mu\nu} \, W^{ef}_{\mu\nu}(k + \xi p \to k' + p')$$

Let the charge of f be e_f.

Exercise 1: Compute $W_{\mu\nu}^{ef}$ to find:

$$W_{\mu\nu}^{ef} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\, \delta\left(1 - \frac{x}{\xi}\right) \frac{e_f^2}{2}$$
$$+ \left(\xi p_\mu - q_\mu\, \frac{\xi p \cdot q}{q^2}\right) \left(\xi p_\nu - q_\nu\, \frac{\xi p \cdot q}{q^2}\right)\, \delta\left(1 - \frac{x}{\xi}\right) \frac{e_f^2}{\xi p \cdot q}$$

Ex. 2: by substituting in (1), find the Callan-Gross relation,

$$F_2(x) = \sum_{\text{quarks} f} e_f^2 x\, \phi_{f/p}(x) = 2x F_1(x)$$

And Ex. 3: that this relation is quite different for scalar quarks.

- The Callan-Gross relation shows the compatibility of the quark and parton models.

- In addition: parton model structure functions are independent of Q^2, a property called “scaling”.

- With massless partons, there is no other massive scale. Then the F’s must be Q-independent; see above.

- Approximate properties of the kinematic region explored by SLAC in late 1960’s – early 1970’s.

- QCD “evolution” gives corrections to this picture.

IC. Getting at the Quark Distributions

- **Relating the parton distributions to experiment**
- **Simplifying assumptions that illustrate the general approach.**

$$\phi_{u/p} = \phi_{d/n} \qquad \phi_{d/p} = \phi_{u/n} \qquad \text{isospin}$$

$$\phi_{\bar{u}/p} = \phi_{\bar{u}/n} = \phi_{\bar{d}/p} = \phi_{\bar{d}/n} \qquad \text{symmetric sea}$$

$$\phi_{c/p} = \phi_{b/N} = \phi_{t/N} = 0 \qquad \text{no heavy quarks}$$

- **Adequate to early experiments, but no longer.**

- With assumptions above, find for e, ν and $\bar{\nu}$ scattering (see appendix)

$$F_2^{(eN)}(x) = 2xF_1^{(eN)}(x) = \sum_{f=u,d,s} e_F^2 x \phi_{f/N}(x)$$

$$F_2^{(W^+N)} = 2x\left(\sum_{D=d,s,b} \phi_{D/N}(x) + \sum_{U=u,c,t} \phi_{\bar{U}/N}(x)\right)$$

$$F_2^{(W^-N)} = 2x\left(\sum_D \phi_{\bar{D}/N}(x) + \sum_U \phi_{U/N}(x)\right)$$

$$F_3^{(W^+N)} = 2\left(\sum_D \phi_{D/N}(x) - \sum_U \phi_{\bar{U}/N}(x)\right)$$

$$F_3^{(W^-N)} = 2\left(- \sum_D \phi_{\bar{D}/N}(x) + \sum_U \phi_{U/N}(x)\right)$$

- Exercise: derive some of these for yourself.

- **The distributions are actually overdetermined with these assumptions, which checks the consistency of the picture.**
- **Further consistency checks: Sum Rules.**

$$N_{u/p} = \int_0^1 dx \left[\phi_{u/p}(x) - \phi_{\bar{u}/p}(x) \right] = 2$$

etc. for $N_{d/p} = 1$**.**

The most famous ones make predictions for structure functions ...

- **The Adler Sum Rule:**

$$1 = N_{u/p} - N_{d/p}$$

$$= \int_0^1 dx \left[\phi_{d/n}(x) - \phi_{\bar{u}/p}(x) - \Big(\phi_{d/p}(x) - \phi_{\bar{u}/n}(x) \Big) \right]$$

$$= \int_0^1 dx \left[\sum_D \phi_{D/n}(x) + \sum_U \phi_{\bar{U}/n}(x) \right]$$

$$- \int_0^1 dx \left[\sum_D \phi_{D/p}(x) + \sum_U \phi_{\bar{U}/p}(x) \right]$$

$$= \int_0^1 dx \, \frac{1}{2x} \left[F_2^{(\nu n)} - F_2^{(\nu p)} \right]$$

In the second equality, we've used isospin invariance, in the third, all the extra terms from heavy quarks $D = s, b$, $U = c, t$ cancel.

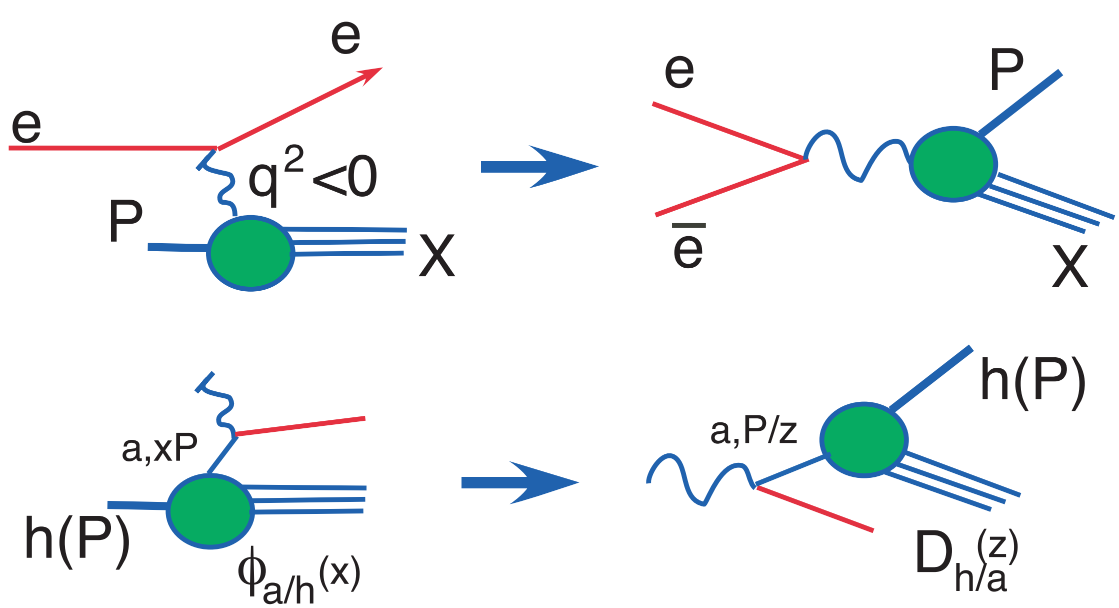
- **And similarly, the Gross-Llewellyn-Smith Sum Rule:**

$$3 = N_{u/p} + N_{d/p} = \int_0^1 dx \, \frac{1}{2x} \left[xF_3^{(\nu n)} + xF_3^{(\nu p)} \right]$$

ID. Classic Parton Model Extensions: Fragmentation and Drell Yan

- **Fragmentation functions**
- **"Crossing" applied to DIS: "Single-particle inclusive" (1PI) From scattering to pair annihilation.**

Parton distributions become "fragmentation functions".



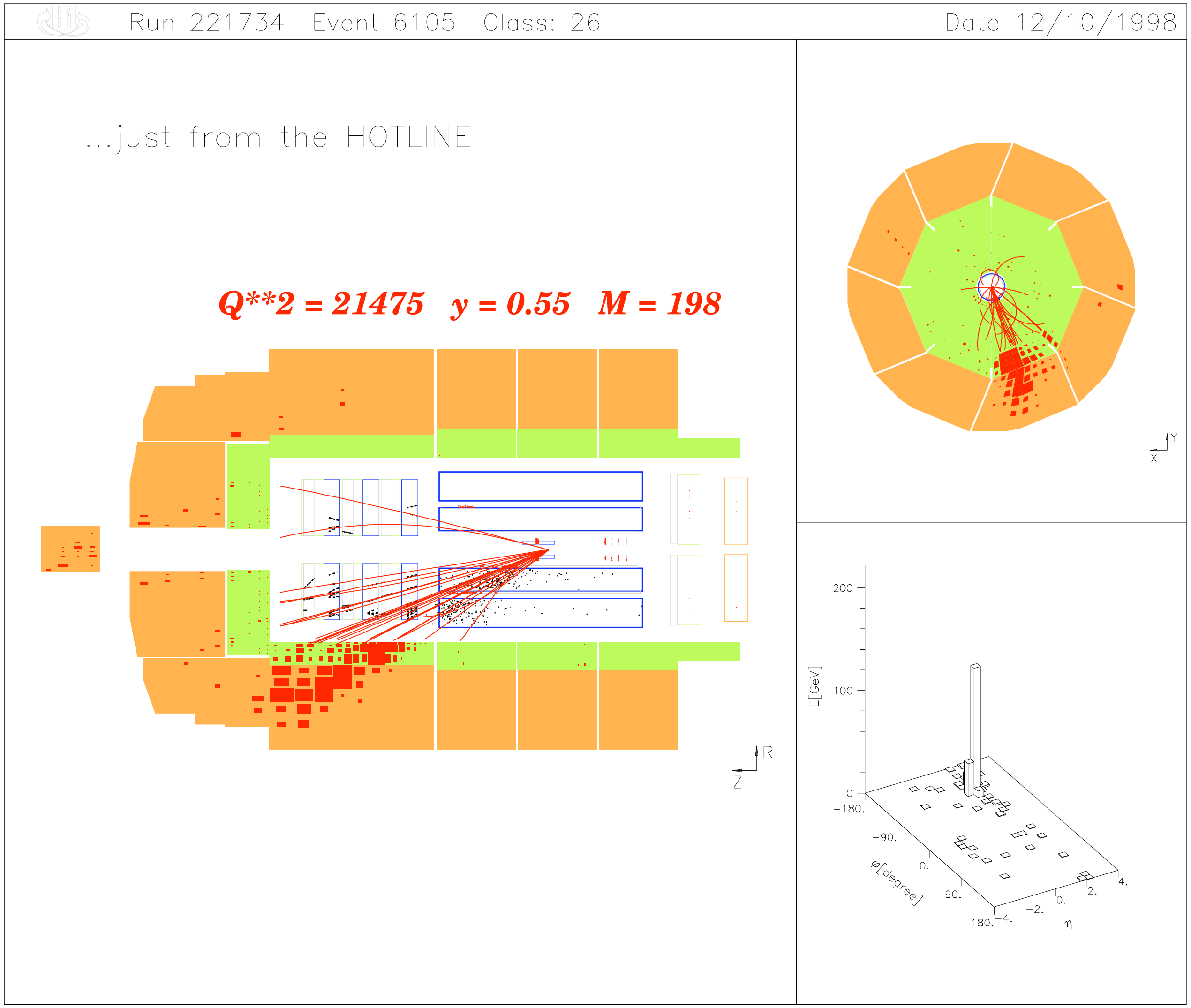
- Parton model relation for 1PI cross sections

$$\sigma_h(P,q) = \sum_a \int_0^1 dz\ \hat{\sigma}_a(P/z,q)\ D_{h/a}(z)$$

- Heuristic justification: Formation of hadron C from parton a takes a time τ_0 in the rest frame of a, but much longer in the CM frame – this "fragmentation" thus decouples from $\hat{\sigma}_a$, and is independent of q (scaling).

- Fragmentation picture suggests that hadrons are aligned along parton direction $\Rightarrow$ jets. And this is what happens.

• **For DIS:**



Run 221734 Event 6105 Class: 26
Date 12/10/1998
...just from the HOTLINE
Q**2 = 21475 y = 0.55 M = 198
R
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Y
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E[GeV]
200
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−180.
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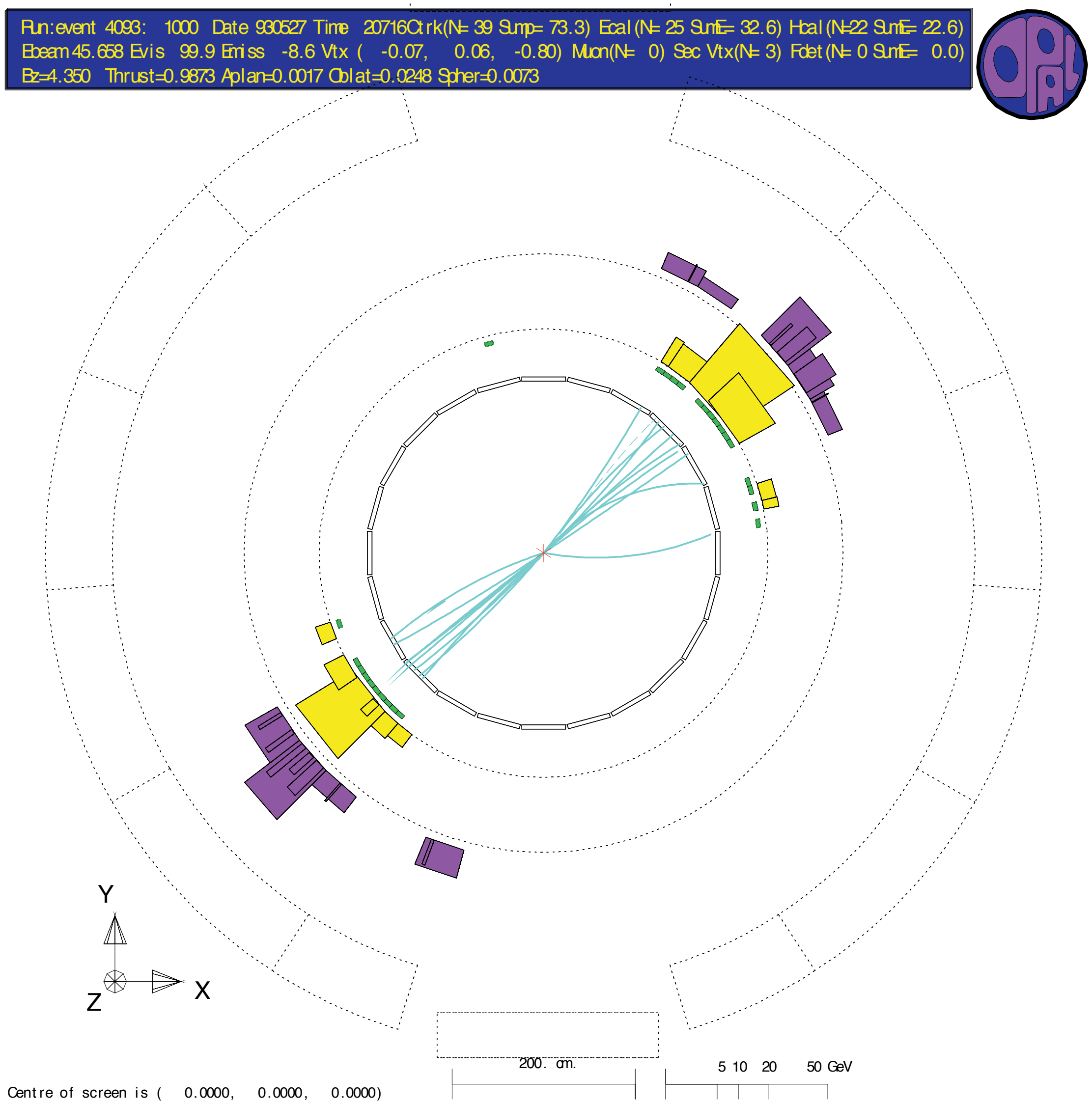
- **For** e^+e^-**:**



Run:event 4093: 1000 Date 930527 Time 20716 Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SumE= 32.6) Hcal(N=22 SumE= 22.6)
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
Y
X
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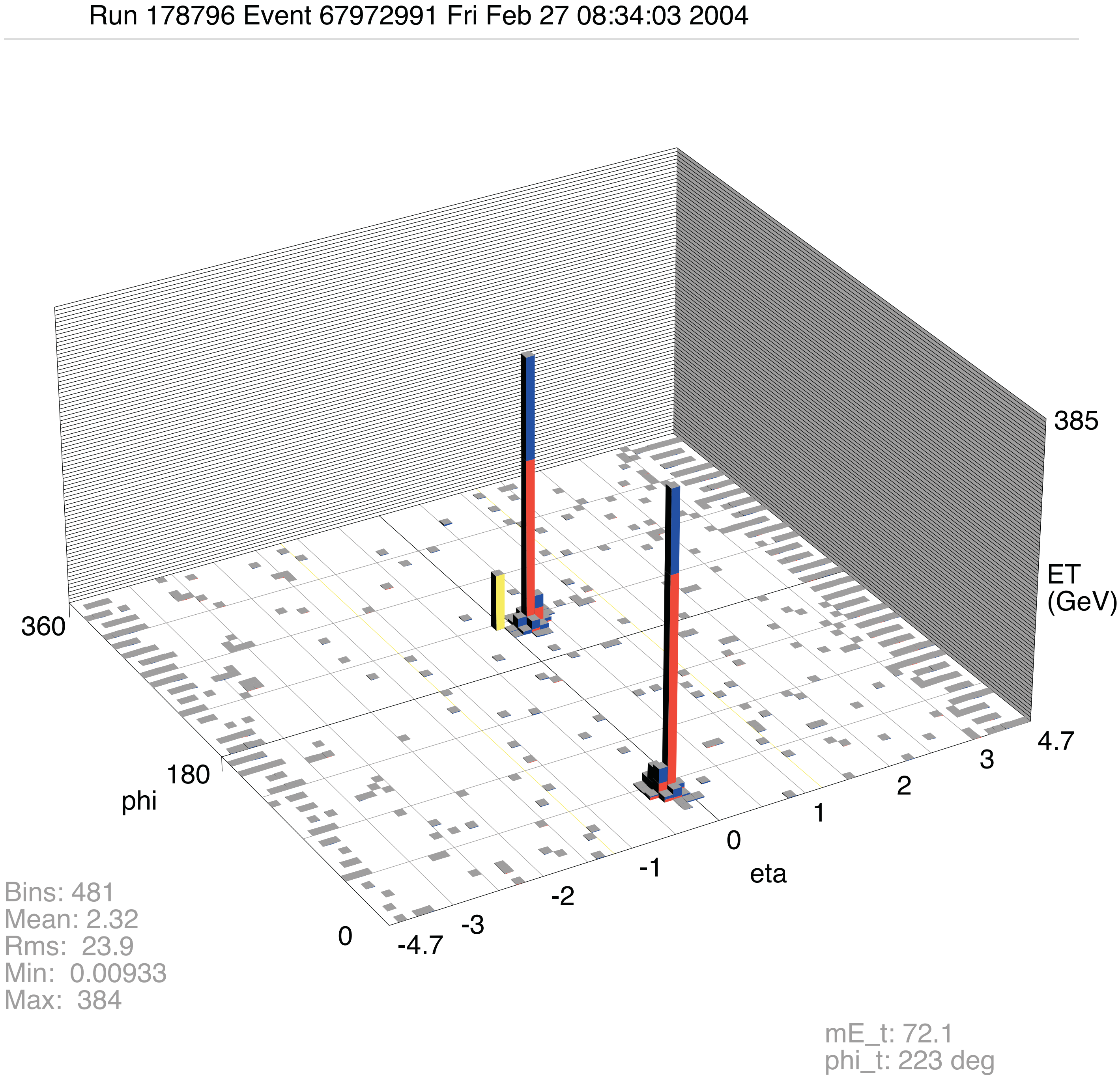
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)

- **And in nucleon-nucleon collisions:**



- Finally: the Drell-Yan process

- **In the parton model (1970).**
  Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q ... any electroweak boson in NN scattering.

$$
\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim
$$

$$
\int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a}\bar{\mathrm{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}
$$

$$
\times(\text{probability to find parton a}(\xi_1)\text{ in }N)
$$

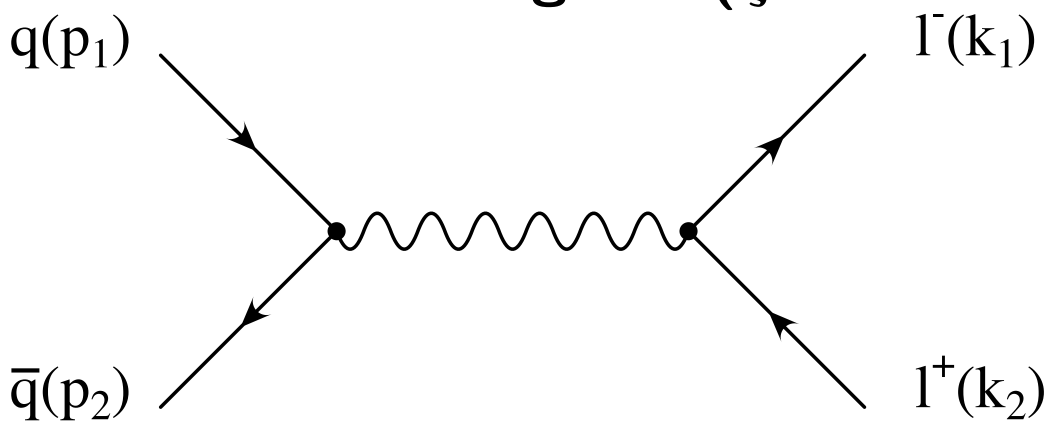
$$
\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)
$$

The probabilities are $\phi_{q/N}(\xi_i)$'s from DIS!

How it works (with colored quarks) . . .

- **The Born cross section**

$\boldsymbol{\sigma}^{\mathrm{EW,Born}}$ **is all from this diagram (ξ's set to unity):**



With this matrix element:

$$M = e_q \frac{e^2}{Q^2} \overline{u}(k_1)\gamma_\mu v(k_2) \overline{v}(p_2)\gamma^\mu u(p_1)$$

- **First square and sum/average M. Then evaluate phase space.**

- Total cross section at pair mass Q

$$\sigma^{\mathrm{EW,\,Born}}_{q\bar{q}\to\mu\bar{\mu}}(x_1p_1, x_2p_2) = \frac{1}{2\hat{s}}\int\frac{d\Omega}{32\pi^2}\frac{e_q^2e^4}{3}(1+\cos^2\theta)$$
$$= \frac{4\pi\alpha^2}{9Q^2}\sum_q e_q^2 \equiv \sigma_0(M)$$

With Q the pair mass and 3 for color average.

- **And measured rapidity:**

  Pair mass (Q) and rapidity

$$\eta \equiv (1/2)\ln\left(\frac{Q^+}{Q^-}\right) = (1/2)\ln\left(\frac{Q^0+Q^3}{Q^0-Q^3}\right)$$

- **ξ's are overdetermined $\rightarrow$ delta functions in the Born cross section**

$$\frac{d\sigma^{(PM)}_{NN\rightarrow\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2d\eta} =$$
$$\int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \sigma^{\mathrm{EW,\,Born}}_{a\bar{a}\rightarrow\mu\bar{\mu}}(\xi_1 p_1, \xi_2 p_2)$$
$$\times\delta\left(Q^2-\xi_1\xi_2 S\right)\,\delta\left(\eta-\frac{1}{2}\ln\left(\frac{\xi_1}{\xi_2}\right)\right)$$
$$\times\phi_{a/N}(\xi_1)\,\phi_{\bar{a}/N}(\xi_2)$$

• and integrating over rapidity, back to $d\sigma/dQ^2$,

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha_{\mathrm{EM}}^2}{9Q^4}\right) \int_0^1 d\xi_1\, d\xi_2\, \delta\left(\xi_1\xi_2 - \tau\right)$$
$$\times \sum_a \lambda_a^2\, \phi_{a/N}(\xi_1)\, \phi_{\bar{a}/N}(\xi_s)$$

Found by Drell and Yan in 1970 (aside from 1/3 for color).

Analog of DIS scaling in x is DY scaling in $\tau = Q^2/S$.

• Template for all hard hadron-hadron scattering

• Next, the quantum field theory of all this ... QCD

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- **Appendix I: Quarks in the Standard Model**
  Electroweak interactions of quarks: SU(2)$_L$ $\times$ U(1). Their non-QCD interactions.

- **Quark and lepton fields: L(eft) and R(ight)**
  - $\psi = \psi^{(L)} + \psi^{(R)} = \frac{1}{2}(1-\gamma_5)\psi + \frac{1}{2}(1+\gamma_5)\psi$; $\psi = q, \ell$
  - **Helicity: spin along $\vec{p}$ (R=right handed) or opposite (L=left handed) in solutions to Dirac equation**
  - $\psi^{(L)}$: **expanded only in L particle solutions to Dirac eqn. R antiparticle solutions**
  - $\psi^{(R)}$: **only R particle solutions, L antiparticle**
  - **An essential feature: L and R have different interactions in general!**

– L quarks come in "weak $SU(2)$" = "weak isospin" pairs:

$$q_i^{(L)} = \begin{pmatrix} u_i \\ d_i' = V_{ij} dj \end{pmatrix} \qquad u_i^{(R)},\ d_i^{(R)}$$

$$(u, d') \qquad (c, s') \qquad (t, b')$$

$$\ell_i^{(L)} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \qquad e_i^{(R)},\ \nu_i^{(R)}$$

$$(\nu_e, e) \qquad (\nu_\mu, \mu) \qquad (\nu_\tau, \tau)$$

(We've neglected neutrino masses.)

– V_{ij} is the "CKM" matrix.

– The electroweak interactions distinguish L and R.

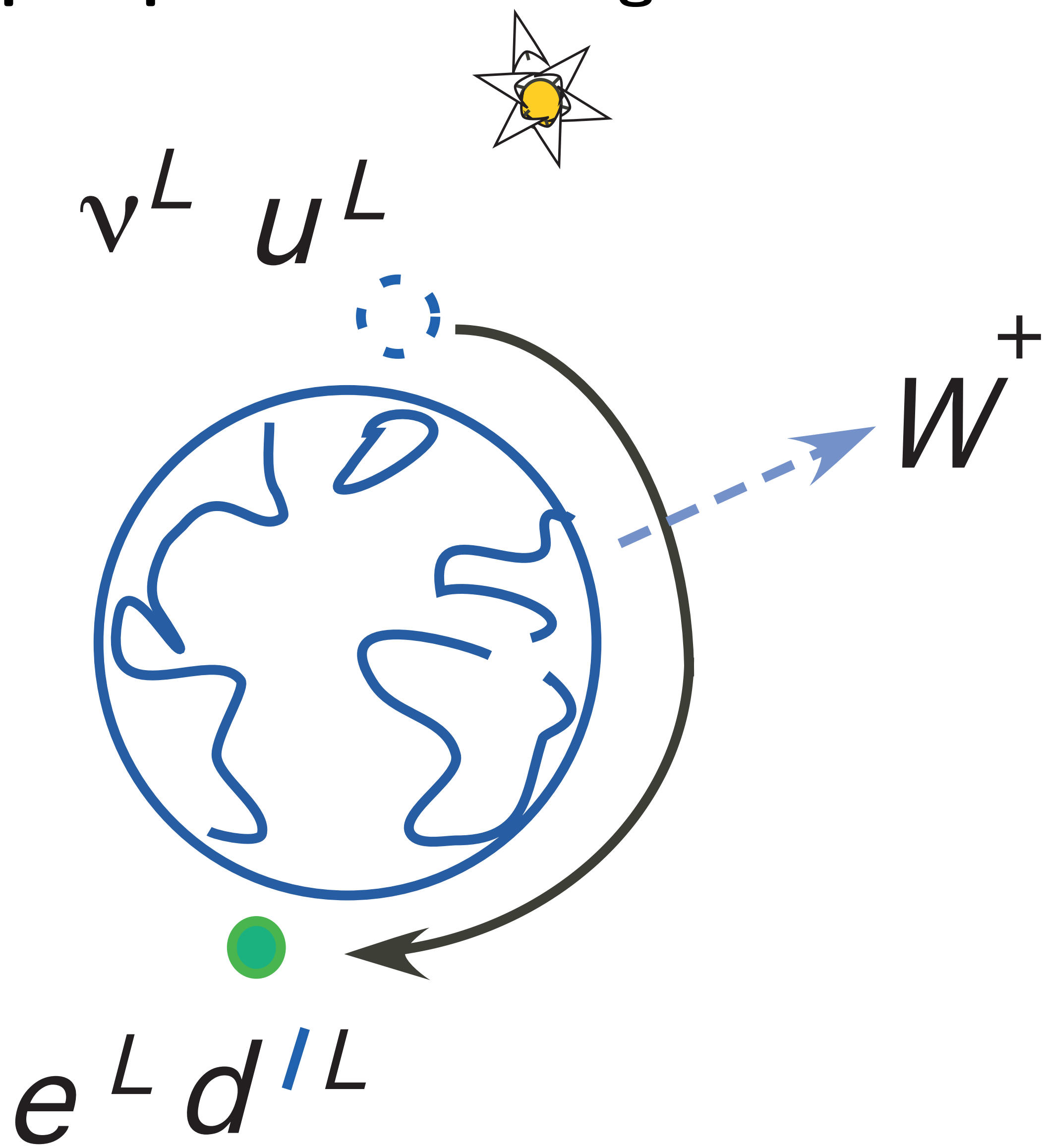
- **Weak vector bosons: electroweak gauge groups**
  - **SU(2): three vector bosons B_i with coupling g**
  - **U(1); one vector boson C with coupling g'**
  - **The physical bosons:**

$$W^{\pm} = B_1 \pm iB_2$$
$$Z = -C\sin\theta_W + B_3\cos\theta_W$$
$$\gamma \equiv A = C\cos\theta_W + B_3\sin\theta_W$$

$$\sin\theta_W = g'/\sqrt{g^2+g'^2} \qquad M_W = M_Z/\cos\theta_W$$

$$e = gg'/\sqrt{g^2+g'^2} \qquad M_W \sim g/\sqrt{G_F}$$

- **Weak isospin space: connecting u with d'**



- **Only left handed fields move around this globe.**

– **The interactions of quarks and leptons with the photon, W, Z**

$$\mathcal{L}_{\mathrm{EW}}^{(fermion)} = \sum_{\text{all }\psi} \bar{\psi}\left(i\not{\partial} - e\lambda_\psi \not{A} - (g m_\psi 2M_W)h\right)\psi$$
$$-(g/\sqrt{2}) \sum_{q_i,e_i} \bar{\psi}^{(L)}\left(\sigma^+ \not{W}^+ + \sigma^- \not{W}^-\right)\psi^{(L)}$$
$$-(g/2\cos\theta_W) \sum_{\text{all }\psi} \bar{\psi}\left(v_f - a_f\gamma_5\right)\not{Z}\,\psi$$

– **Interactions with W are through ψ_L's only.**

– **Neutrino Z exchange depends on $\sin^2\theta_W$ even at low energy.**

– **This observation made it clear by early 1970's that $M_W \sim g/\sqrt{G_F}$ is large $\rightarrow$ a need for colliders.**

– **Coupling to the Higgs $h \propto$ mass (special status of t).**

- Symmetry violations in the standard model:

  - W's interact through $\psi^{(L)}$ only, $\psi = q, \ell$.

  - These are left-handed quarks & leptons; right-handed antiquarks, antileptons.

  - Parity (P) exchanges L/R; Charge conjugation (C) exchanges particles, antiparticles.

  - CP combination OK ($L \to_P R \to_C L$) if all else equal, but it's not (quite) . . .

    Complex phases in CKM V result in CP violation.

- **Appendix II: Structure Functions and Photon Polarizations**

In the P rest frame can take

$$q^\mu = \left(\nu; 0, 0, \sqrt{Q^2+\nu^2}\right)\ , \qquad \nu \equiv \frac{p\cdot q}{m_p}$$

In this frame, the possible photon polarizations ($\epsilon \cdot q = 0$):

$$\epsilon_R(q) = \frac{1}{\sqrt{2}}\,(0; 1, -i, 0)$$
$$\epsilon_L(q) = \frac{1}{\sqrt{2}}\,(0; 1, i, 0)$$
$$\epsilon_{\rm long}(q) = \frac{1}{Q}\left(\sqrt{Q^2+\nu^2}, 0, 0, \nu\right)$$

- **Alternative Expansion**

$$W^{\mu\nu} = \sum_{\lambda=L,R,long} \epsilon_\lambda^{\mu *}(q)\epsilon_\lambda^{\nu}(q)\, F_\lambda(x,Q^2)$$

- **For photon exchange (Exercise 4):**

$$F^{\gamma e}_{L,R} = F_1$$
$$F_{\text{long}} = \frac{F_2}{2x} - F_1$$

- **So F_{long} vanishes in the parton model by the C-G relation.**

- **Generalizations: neutrinos and polarization**

- **Neutrinos: flavor of the “struck” quark is changed when a $W^{\pm}$ is exchanged. For W^{+}, a d is transformed into a linear combination of u, c, t, determined by CKM matrix (and momentum conservation).**

- **Z exchange leaves flavor unchanged but still violates parity.**

• **The Vh structure functions for $= W^+, W^-, Z$:**

$$W_{\mu\nu}^{(Vh)} - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{(Vh)}(x, Q^2)$$
$$+ \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) \frac{1}{m_h^2} W_2(x, Q^2)$$
$$- i\epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \frac{1}{m_h^2} W_3^{(Vh)}(x, Q^2)$$

• **with dimensionless structure functions:**

$$F_1 = W_1\,, \qquad F_2 = \frac{p\cdot q}{m_h^2} W_2\,, \qquad F_3 = \frac{p\cdot q}{m_h^2} W_3$$

• $F_i^{(\nu h)}$ **gives** $W^+ h$ **scattering,** $F_i^{(\bar\nu h)}$ **gives** $W^- h$

• **And with spin (for the photon).**

$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4z\, e^{iq\cdot z} \; \langle h(P,S) \mid J^\mu(z) J^\nu(0) \mid h(P,S)\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \; F_1(x,Q^2)$$

$$+ \left(P^\mu - q^\mu \frac{P\cdot q}{q^2}\right)\left(P^\nu - q^\nu \frac{P\cdot q}{q^2}\right) F_2(x,Q^2)$$

$$+ \, i m_h \, \epsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P\cdot q} g_1(x,Q^2) + \frac{S_\sigma (P\cdot q) - P_\sigma (S\cdot q)}{(P\cdot q)^2} g_2(x,Q^2)\right.$$

- **Parton model structure functions:**

$$F_2^{(eh)}(x) = \sum_f e_f^2\, x\, \phi_{f/h}(x)$$
$$g_1^{(eh)}(x) = \frac{1}{2} \sum_f e_f^2 \left(\Delta\phi_{f/n}(x) + \Delta\bar{\phi}_{f/h}(x)\right)$$

- **Notation:** $\Delta\phi_{f/h} = \phi^{+}_{f/h} - \phi^{-}_{f/h}$ **with** $\phi^{\pm}_{f/h}(x)$ **probability for struck quark** f **to have momentum fraction** x **and helicity with (+) or against (−)** h**'s helicity.**