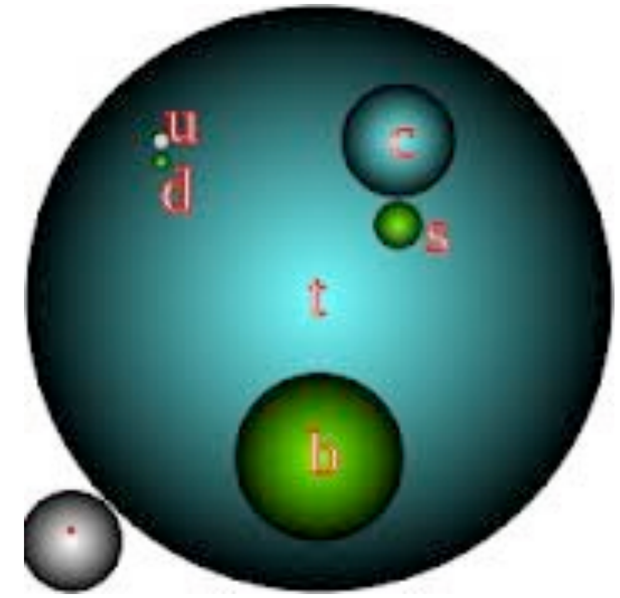
CTEQ Summer School 2013

Heavy Quarks



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Heavy?

Quarks are heavy if $m_Q \gg \Lambda_{\mathrm{QCD}}$

$$\frac{m_{\mathrm{pole}}}{\bar{m}(\bar{m})} = 1 + \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left(-1.0414 \ln(m_{\mathrm{pole}}^2/\bar{m}^2) + 13.4434\right) + \cdots$$

	Pole Mass	$\overline{\mathrm{MS}}$
Charm	~1.3 – 1.7 GeV	1.275 ± 0.025 GeV
Bottom	~4.5 – 5 GeV	4.18 ± 0.03 GeV
Top	173.5 ± 0.6 ± 0.8 GeV	160 ± 5 GeV

Measured from lepton+jets

Measured from measured cross section

See CMS paper today

Lecture 2

- Minimal Flavor Violation
- CP Violation
- Angles of CKM
- Rare *B* decays
- Little bit of top

New physics flavor problem?

$$\frac{(\bar{s}d)^2}{\Lambda^2} \Longrightarrow \Lambda \gtrsim 10^4 \text{ TeV} \qquad \frac{(\bar{b}d)^2}{\Lambda^2} \Longrightarrow \Lambda \gtrsim 10^3 \text{ TeV} \qquad \frac{(\bar{b}s)^2}{\Lambda^2} \Longrightarrow \Lambda \gtrsim 10^2 \text{ TeV}$$

Δm_K Δm_B Δm_{B_s}

TeV-scale NP models typically have new sources of flavor and CP violation

How do we protect flavor?

Back to flavor symmetry: $U(3)^3$ broken by Yukawa's

Extend SM by adding terms (local, Lorentz, gauge inv) that are invariant under $U(3)^3$ including spurions

$$\lambda_u, \lambda_d$$

Minimal Flavor Violation (MFV)

Fields transform as: $Q_L(3,1,1),\ u_R(1,3,1),\ d_R(1,1,3)$

Spurions transform as: $\lambda_u(3,\bar{3},1),\ \lambda_d(3,1,\bar{3})$

Go to basis $\lambda_d = \mathrm{diag}(y_d, y_s, y_b), \lambda_u = V^\dagger_{\mathrm{CKM}}\mathrm{diag}(y_u, y_c, y_t)$

EFT analysis possible - look at possible operators

To get non-diagonal terms (flavor changing), need at least:

$$\bar{Q}_L \lambda_u^\dagger \lambda_u Q_L,$$

$$d_r \lambda_d^\dagger \lambda_u \lambda_u^\dagger Q_L,$$

$$\bar{d}_r \lambda_d^\dagger \lambda_u \lambda_u^\dagger \lambda_d d_r$$

(or more insertions of λ)

Minimal Flavor Violation (MFV)

Extensions of SM with $U(3)^3$ breaking by $\lambda_{u,d}$ satisfy MFV

Examples

1. $B \to X_s \gamma$

$$\Delta\mathcal{L} = \frac{X}{\Lambda_{\rm NP}}\mathcal{O} = \frac{X}{\Lambda_{\rm NP}}(\bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R)$$

$\bar{s}_L b_R$ not invariant under $U(3)^3$

$\bar{Q}_L \lambda_d d_R$ is flavor diagonal

$$\bar{Q}_L \lambda_u \lambda_u^\dagger \lambda_d d_R \to \bar{s}_L V_{ts}^* V_{tb} y_t^2 y_b b_R$$

If MFV then $X \propto V_{ts}^* V_{tb} y_t^2 y_b$

Minimal Flavor Violation (MFV)

Extensions of SM with $U(3)^3$ breaking by $\lambda_{u,d}$ satisfy MFV

Examples

2. SUSY: Without SUSY breaking, MFV

$$\mathcal{L} = \int d^4\theta \left[\bar{Q}e^V Q + \bar{U}e^V U + \bar{D}e^V D\right] + \cdots + \left(\int d^2\theta W + \text{h.c.}\right)$$

$$W = H_1 U \lambda_u Q + H_2 D \lambda_d Q + \cdots$$

Add soft SUSY breaking

$$\Delta\mathcal{L}_{\text{SUSY}-\text{break}} = \phi_q^* M_q^2 \phi_q + \phi_u^* M_u^2 \phi_u + \phi_d^* M_d^2 \phi_d + (\phi_{h_1}\phi_u g_u \phi_q + \phi_{h_2}\phi_d g_d \phi_q + \text{h.c.})$$

Unless $M^2_{u,d,q} \propto \mathbb{I}$ and $g_{u,d} \propto \lambda_{u,d}$

large flavor-changing interactions

→ Motivation for gauge mediated SUSY breaking

Minimal Flavor Violation (MFV)

Predictions

1. Spectra: $y_{u,d,s,c} \ll 1$, so approx $U(2)^3$ remains

Ex: in gauge-mediated SUSY breaking,
first two generations of squarks near-degenerate

2. Mixing: Only source is $V_{\rm CKM}$

$$V_{\rm CKM}^{\rm (LHC)} \sim \begin{pmatrix} 1 & 0.2 & 0 \\ -0.2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

New particles decay to either
3rd or non-3rd generation (not both)

CP violation

In SM, due to phase of CKM. Need more.

Start with equal amounts of matter (X) and antimatter ($\bar{X}$)

X decays to A with baryon number N_A with probability p
and to B with baryon number N_B with probability $(1-p)$

$\bar{X}$ decays to $\bar{A}$ with baryon number $-N_A$ with probability $\bar{p}$
and to $\bar{B}$ with baryon number $-N_B$ with probability $(1-\bar{p})$

Leads to baryon asymmetry

$$\Delta N = N_A p + N_B(1-p) - N_A\bar{p} - N_B(1-\bar{p}) = (p-\bar{p})(N_A - N_B)$$

Magnitude?

$$\Delta N_B/N_\gamma = [N(\mathrm{baryon}) - N\mathrm{antibaryon})]/N_\gamma \sim 10^{-10}$$

CP violation

In SM, due to phase of CKM. Need more.

$$\Delta N_B/N_\gamma = [N(\text{baryon}) - N\text{antibaryon})]/N_\gamma \sim 10^{-10}$$

In SM, dimensionless, parametrization invariant measure of CP

$$(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)J/v^{12}$$

Need mass differences

Jarlskog

Order electroweak

Plugging in numbers $\sim 10^{-20}$

Need more CPV (leptogenesis? SUSY?)

CP violation in decay

Need at least two amplitudes with nonzero weak and strong phases (show for HW)

$$A_f = \langle f|\mathcal{H}|B\rangle = \sum_k A_k e^{i\delta_k} e^{i\phi_k}$$

$$\bar{A}_{\bar{f}} = \langle \bar{f}|\mathcal{H}|\bar{B}\rangle = \sum_k A_k e^{i\delta_k} e^{-i\phi_k}$$

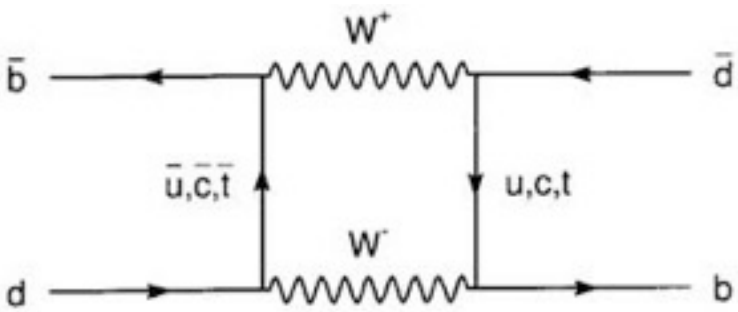
If $|\bar{A}_{\bar{f}}/A_f| \neq 1$, then *CP* is violated

Well established experimentally, i.e.

$$A_{K^-\pi^+} = \frac{\Gamma(\bar{B} \to K^-\pi^+) - \Gamma(B \to K^+\pi^-)}{\Gamma(\bar{B} \to K^-\pi^+) + \Gamma(B \to K^+\pi^-)}$$

$$= -0.097 \pm 0.012$$

Neutral Meson Mixing



$$i\frac{d}{dt}\begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} \quad i.e.,\ P^0 = B^0 = d\bar{b}$$

2x2 Hermitian

$$CPT \Longrightarrow M_{11} = M_{22},\ \Gamma_{11} = \Gamma_{22}$$

$$|P_{L,H}\rangle = p|P^0\rangle \pm q|\bar{P}^0\rangle$$ **with** $$|p|^2 + |q|^2 = 1$$

$$|P_{L,H}(t)\rangle = e^{-(im_{L,H}+\Gamma_{L,H}/2)t}|P_{L,H}\rangle$$

If $\frac{q}{p} = 1$, mass eigenstates are CP eigenstates

If $\frac{q}{p} \neq 1$, CP not conserved in time evolution

Neutral Meson Mixing

Some more definitions

$$m = \frac{m_H + m_L}{2}, \quad \Gamma = \frac{\Gamma_H + \Gamma_L}{2}$$

$$\Delta m = m_H - m_L, \quad \Delta\Gamma = \Gamma_H - \Gamma_L$$

Convention dependent!

$$m_{L,H} - \frac{i}{2}\Gamma_{L,H} = \left(M_{11} - \frac{i}{2}\Gamma_{11}\right) \pm \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)$$

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

Neutral Meson Mixing

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \qquad |P_{L,H}\rangle = p|P^0\rangle \pm q|\bar{P}^0\rangle$$

$$\Delta m = m_H - m_L, \quad \Delta\Gamma = \Gamma_H - \Gamma_L$$

- Δm (positive) depends on CKM and "bag" parameters (Lattice QCD) $\Delta m = |V_{tb}V_{td}^*|^2 f_B B_B \times \text{[known]}$
- Bag parameters cancel in $\Delta m_d/\Delta m_s \Longrightarrow |V_{td}/V_{ts}|^2$
- $\Delta\Gamma$ depends on widths into common states
- *CP* violation in mixing small $q/p \sim 1$

  $\arg(\Gamma_{12}/M_{12}) \to 0, M_{12} \ll \Gamma_{12}, M_{12} \gg \Gamma_{12}$

Neutral Meson Mixing

Solution simplifies in $\Gamma_{12} \ll M_{12}$ limit $(|\Delta\Gamma| \ll \Delta m)$

Good for B, B_s

$$\Delta m = 2|M_{12}| + \cdots$$

$$\Delta\Gamma = -2\text{Re}(M_{12}\Gamma_{12}^*)/|M_{12}| + \cdots = -2|\Gamma_{12}|\cos\phi_{12}$$

NP can only suppress $\Delta\Gamma_{B_s}$

$$\frac{q}{p} = -\frac{M_{12}^*}{M_{12}}\left(1 - \frac{1}{2}\text{Im}\frac{\Gamma_{12}}{M_{12}} + \cdots\right)$$

In the SM:

B_d $$\frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} = e^{-2i\beta} + \mathcal{O}(10^{-3})$$

B_s $$\frac{q}{p} = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*} = e^{-2i\beta_s}$$ $\beta_s \sim \lambda^2\beta \sim 1^\circ$

Neutral Meson Mixing

Results

	$\Delta m/\Gamma$	$\Delta\Gamma/2\Gamma$	$1-\|q/p\|^2$
D^0	$(0.63 \pm 0.19)\%$	$(0.75 \pm 0.12)\%$	$0.52^{+0.19}_{-0.24}$
B^0	0.770 ± 0.008	0.008 ± 0.009	-0.0003 ± 0.0021
B_s^0	26.49 ± 0.29	0.075 ± 0.010	-0.0109 ± 0.0040

Neutral Meson Mixing

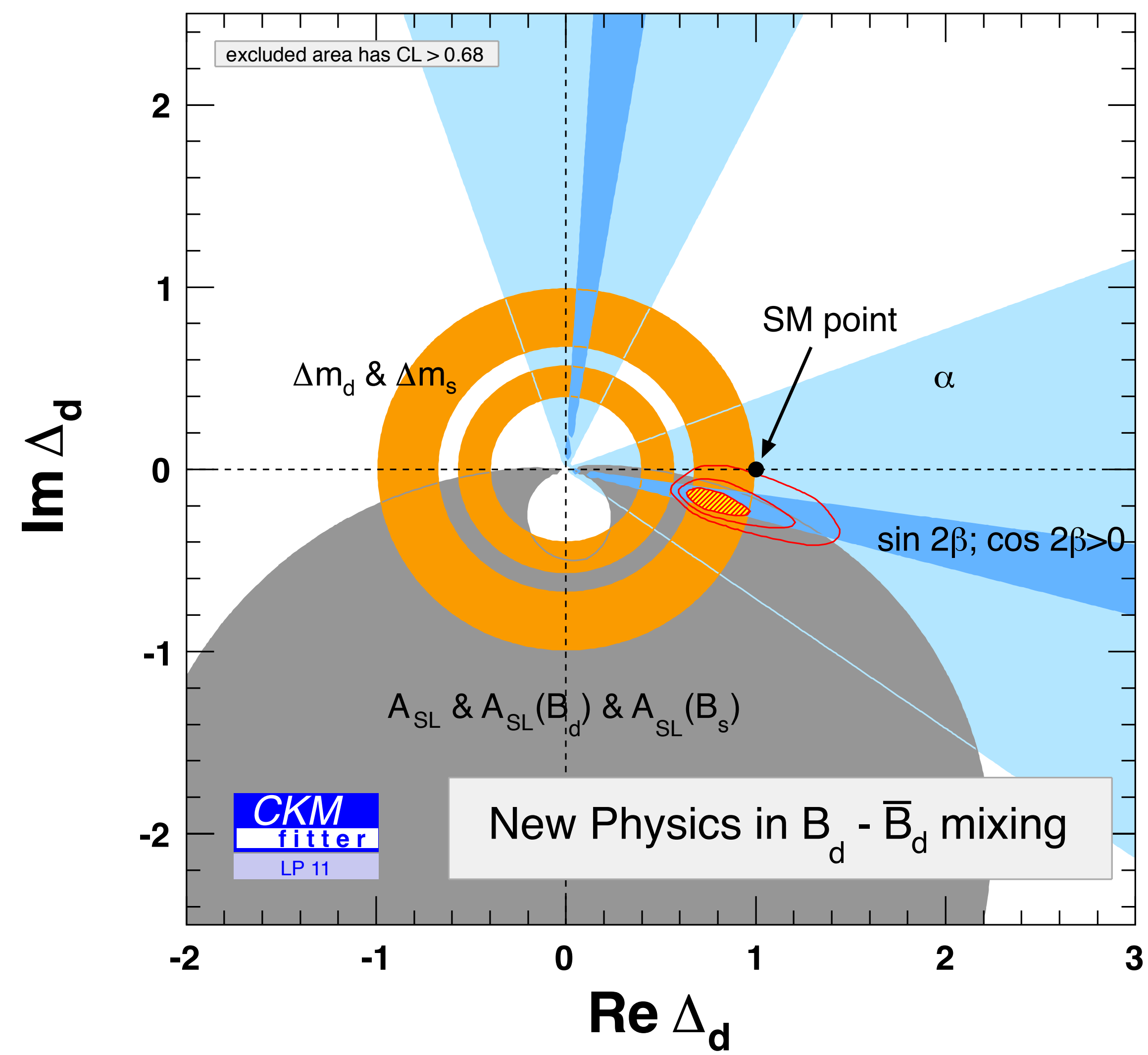
Room for NP?

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^d$$

Dimension 6 gives $\Lambda > 100\ \mathrm{TeV}$ (MFV?)

$$\langle P^0|\mathcal{H}_{\rm NP}|\bar{P}^0\rangle = \Delta \langle P^0|\mathcal{H}_{\rm SM}|\bar{P}^0\rangle$$

Assume no NP in tree amp



Neutral Meson Mixing

Room for NP?

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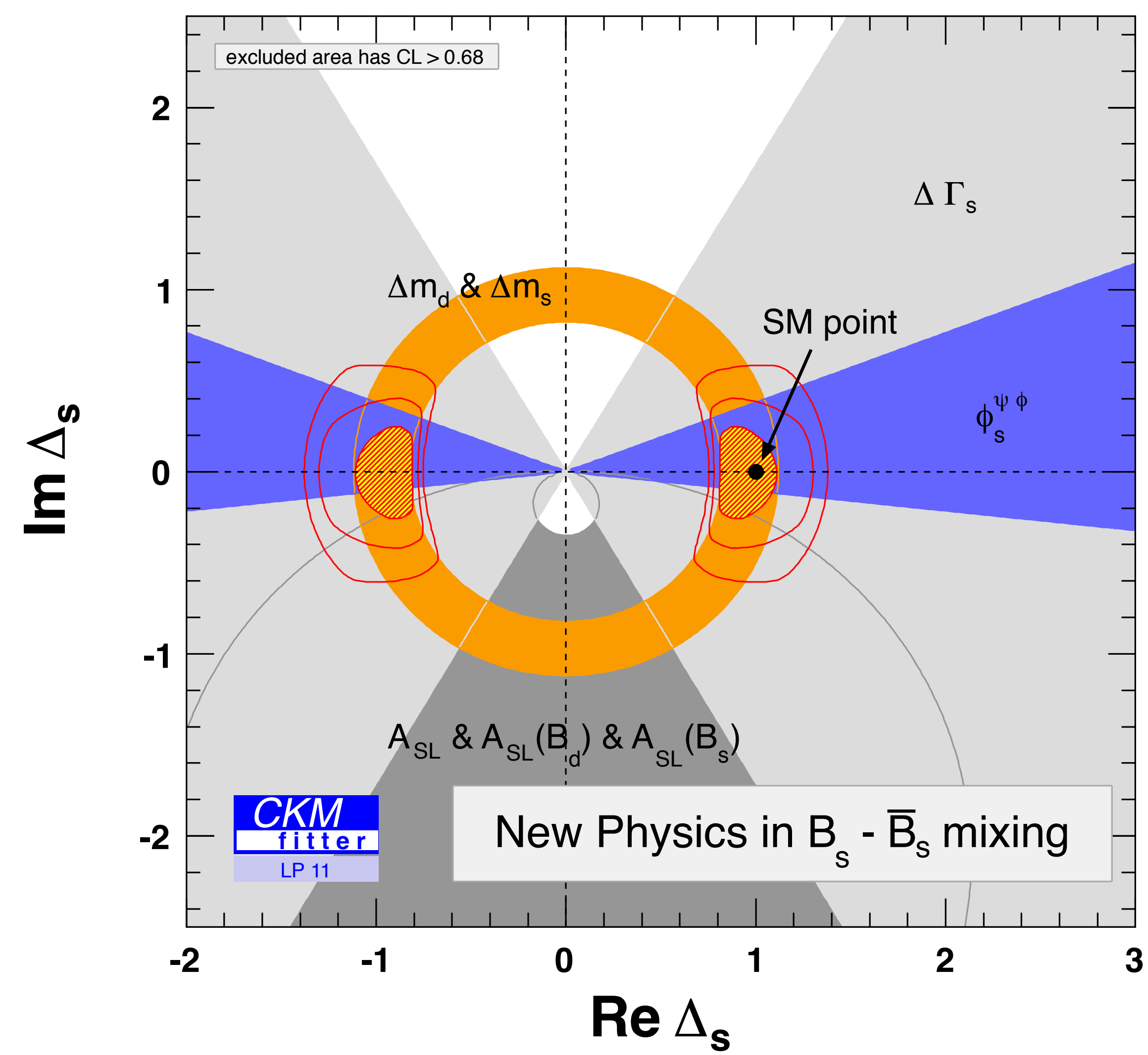
Dimension 6 gives $\Lambda > 100\ \text{TeV}$ (MFV?)

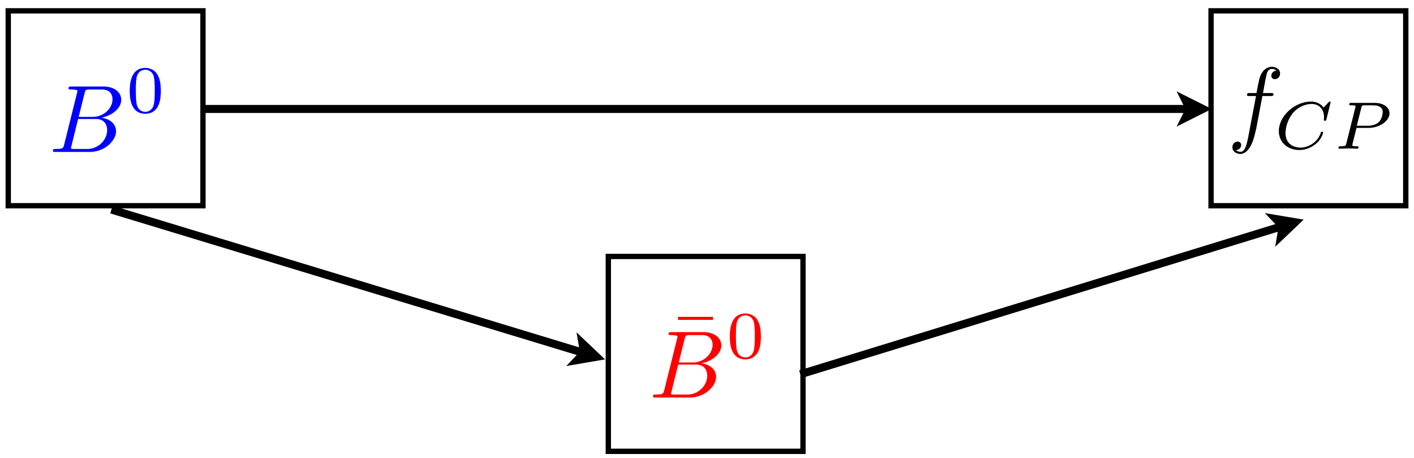
$$\langle P^0 | \mathcal{H}_{\text{NP}} | \bar{P}^0 \rangle = \Delta \langle P^0 | \mathcal{H}_{\text{SM}} | \bar{P}^0 \rangle$$

Assume no NP in tree amp



CP violation in interference between decay and mixing

For decays to *CP* eigenstate final states



$$\lambda_{f_{CP}} = \frac{q}{p}\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}}\frac{q}{p}\frac{\bar{A}_{f_{CP}}}{A_{\bar{f}_{CP}}}$$

$\eta_{f_{CP}} = \pm 1$

Measure from time dependent *CP* asymmetry

$$a_{f_{CP}} = \frac{\Gamma[\bar{B}^0(t) \to f] - \Gamma[B^0(t) \to f]}{\Gamma[\bar{B}^0(t) \to f] + \Gamma[B^0(t) \to f]}$$

CP violation in interference between decay and mixing

Measure from time dependent *CP* asymmetry

$$a_{f_{CP}} = \frac{\Gamma[\bar{B}^0(t) \to f] - \Gamma[B^0(t) \to f]}{\Gamma[\bar{B}^0(t) \to f] + \Gamma[B^0(t) \to f]}$$

$$= S_{f_{CP}} \sin(\Delta m t) - C_{f_{CP}} \cos(\Delta m t)$$

$$S_{f_{CP}} = \frac{2\mathrm{Im}\lambda_f}{1 + |\lambda_f|^2}, \; C_{f_{CP}} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \qquad \lambda_{f_{CP}} = \frac{q}{p}\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}} \frac{q}{p} \frac{\bar{A}_{f_{CP}}}{A_{\bar{f}_{CP}}}$$

If amplitudes with one weak phase dominates, then $a_{f_{CP}}$ measures phase in Lagrangian cleanly.

$$\Longrightarrow C_f = 0, a_{f_{CP}} = \mathrm{Im}\lambda_f \sin(\Delta m t)$$

$\arg \lambda_f$ phase difference of two decay paths

CP violation in interference between decay and mixing

Gold plated mode

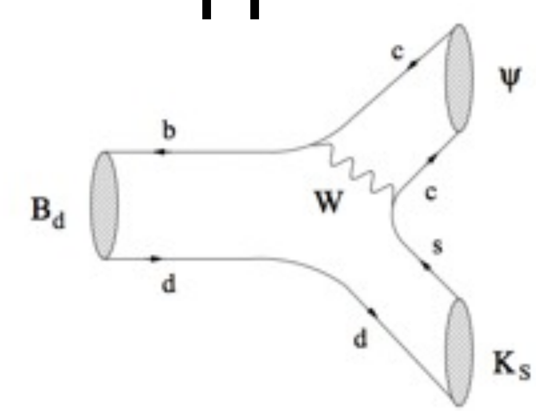
$$B \to J/\psi K_S$$

Interference between $\bar{B}^0 \to \psi \bar{K}^0 (b \to c\bar{c}s),\, B^0 \to \psi K^0 (\bar{b} \to c\bar{c}\bar{s})$

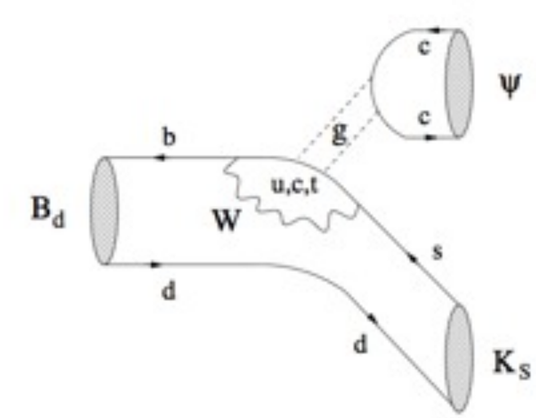
Penguin diagrams with weak phase different from tree suppressed

$$\bar{A}_{\psi K_S} = V_{cb} V_{cs}^* T + V_{ub} V_{us}^2 P$$

$\mathcal{O}(\lambda^2)$ $\mathcal{O}(\lambda^4)$



First term >> second term → theoretically clean



$$\arg \lambda_{\psi K_s} = (B\text{−mix} = 2\beta) + (\text{decay} = 0) + (K\text{−mix} = 0)$$

CP violation in interference between decay and mixing

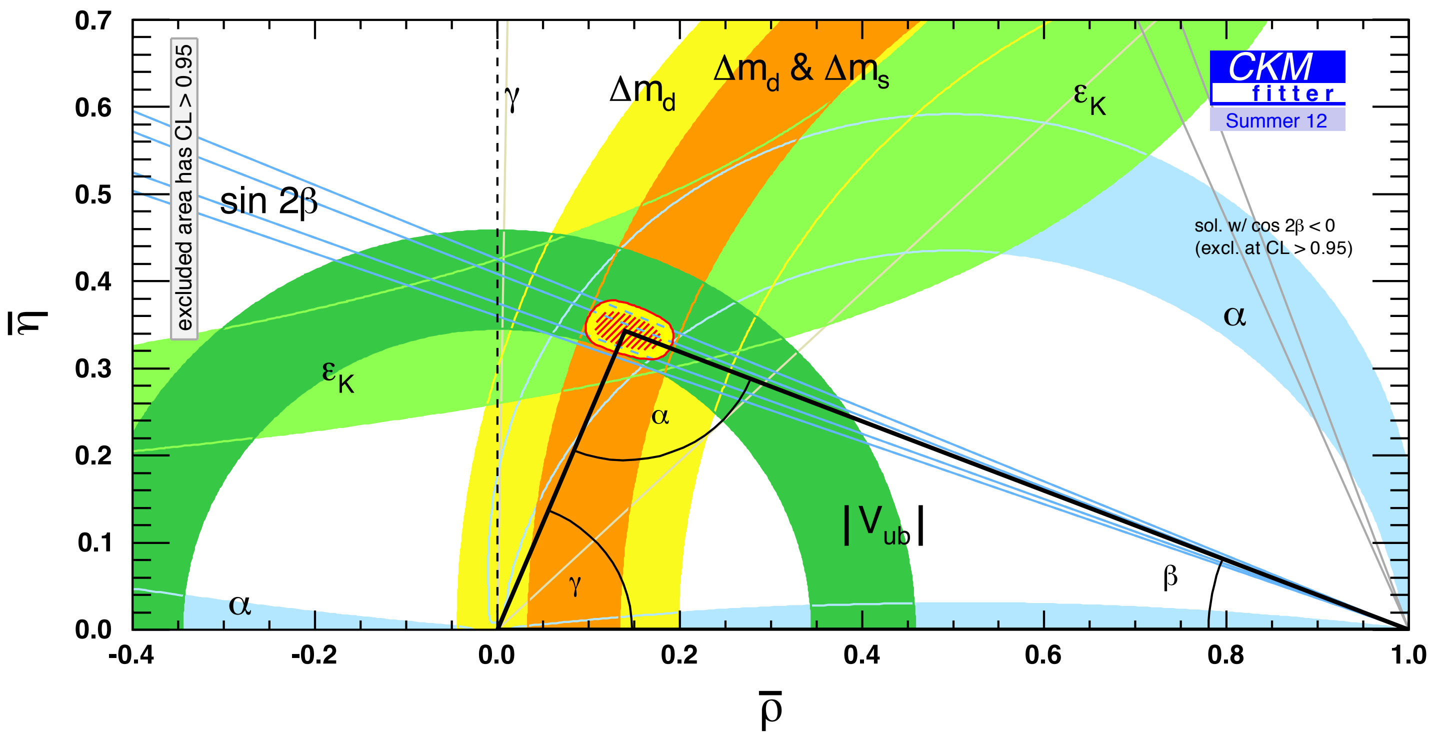
Gold plated mode

$$B \to J/\psi K_S$$

Expect (with usual phase convention)

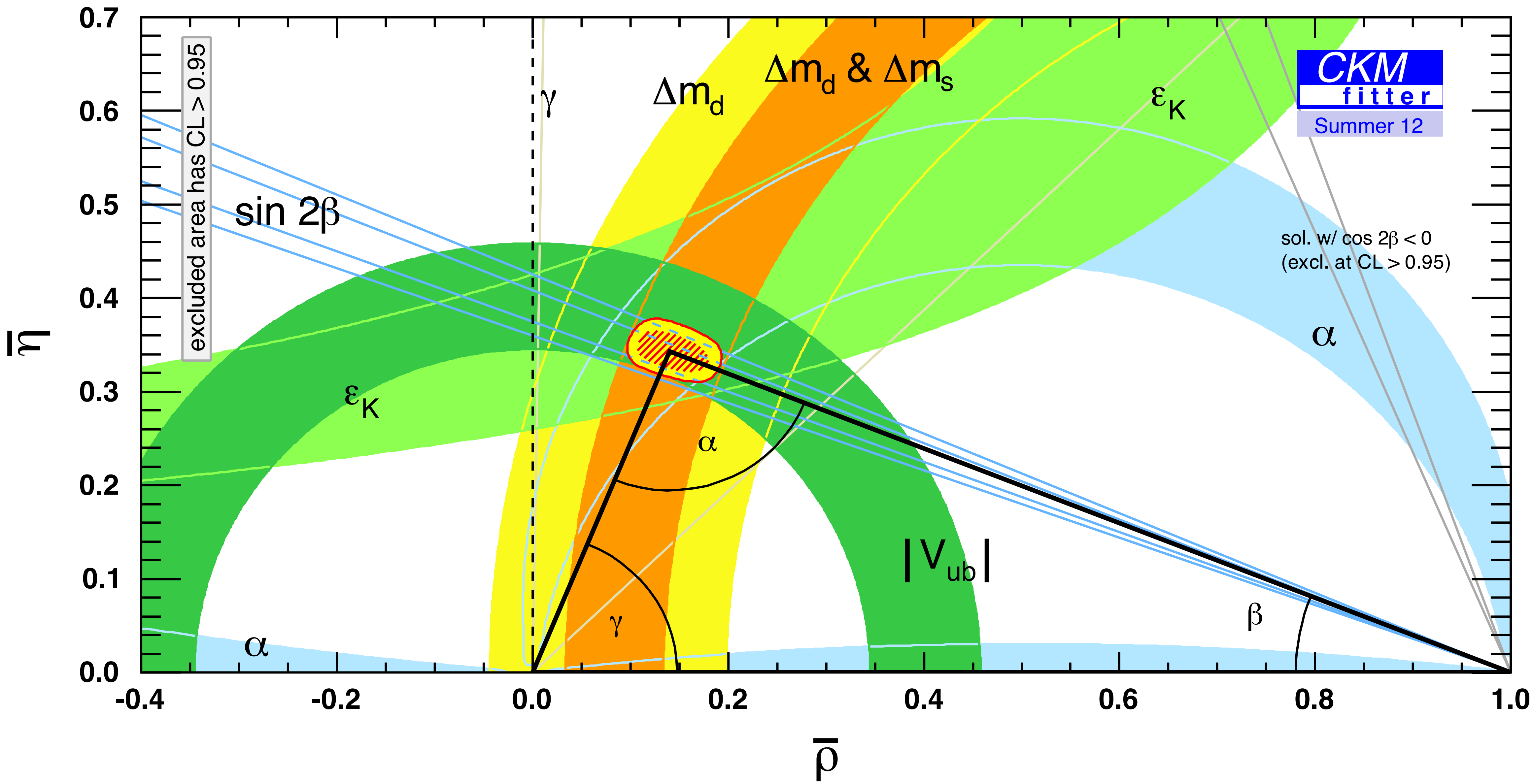
$$S_{\psi K} = \sin 2\beta, \quad C_{\psi K} = 0$$

$$\sin 2\beta = 0.671 \pm 0.022$$



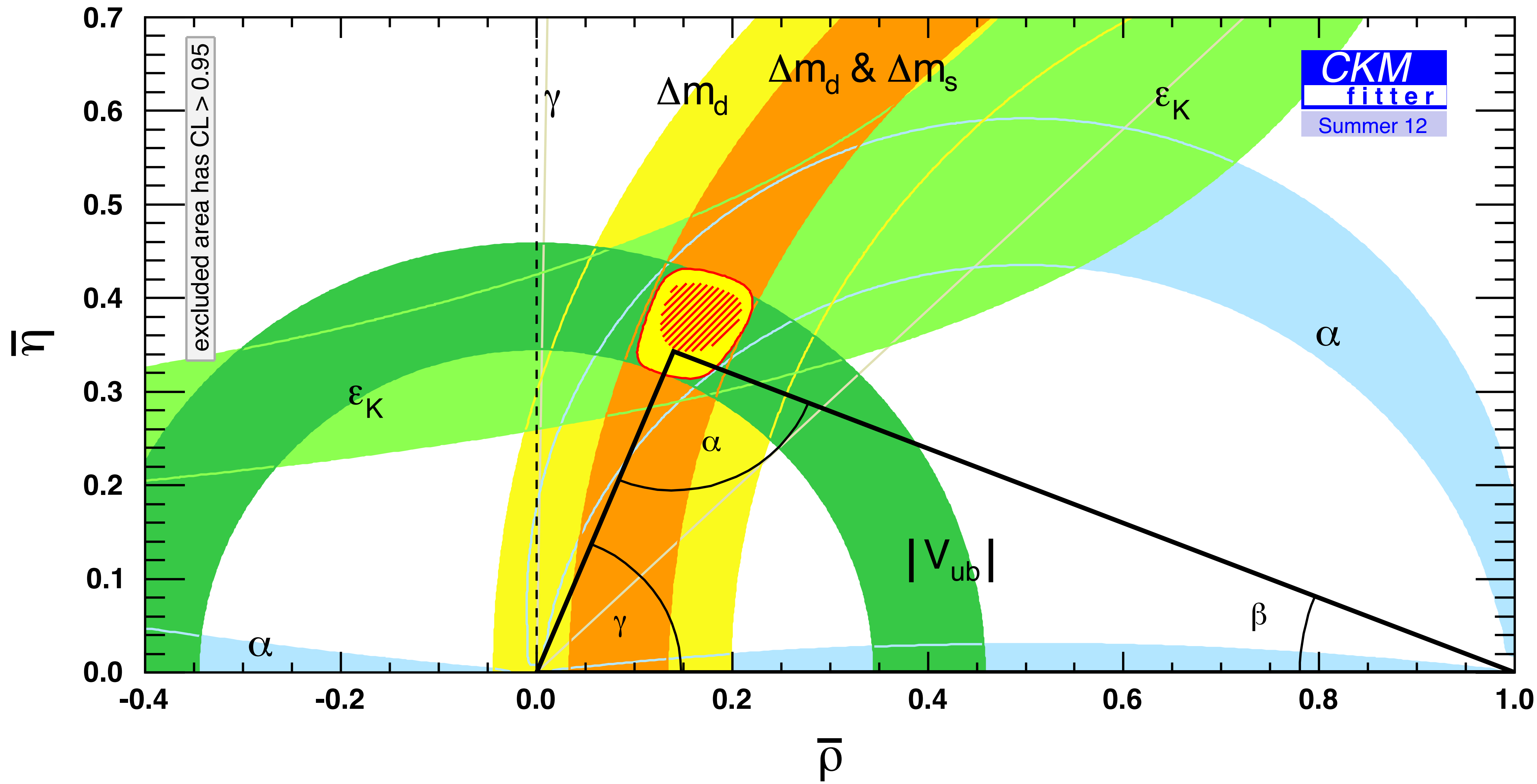
CP violation in interference between decay and mixing

With $\sin 2\beta$



CP violation in interference between decay and mixing

Without $\sin 2\beta$



Other angles harder

Need more measurements to remove hadronic physics

$$\alpha \text{ from } B \to \pi\pi, B \to \rho\rho, B \to \rho\pi$$

Need to do isospin analysis

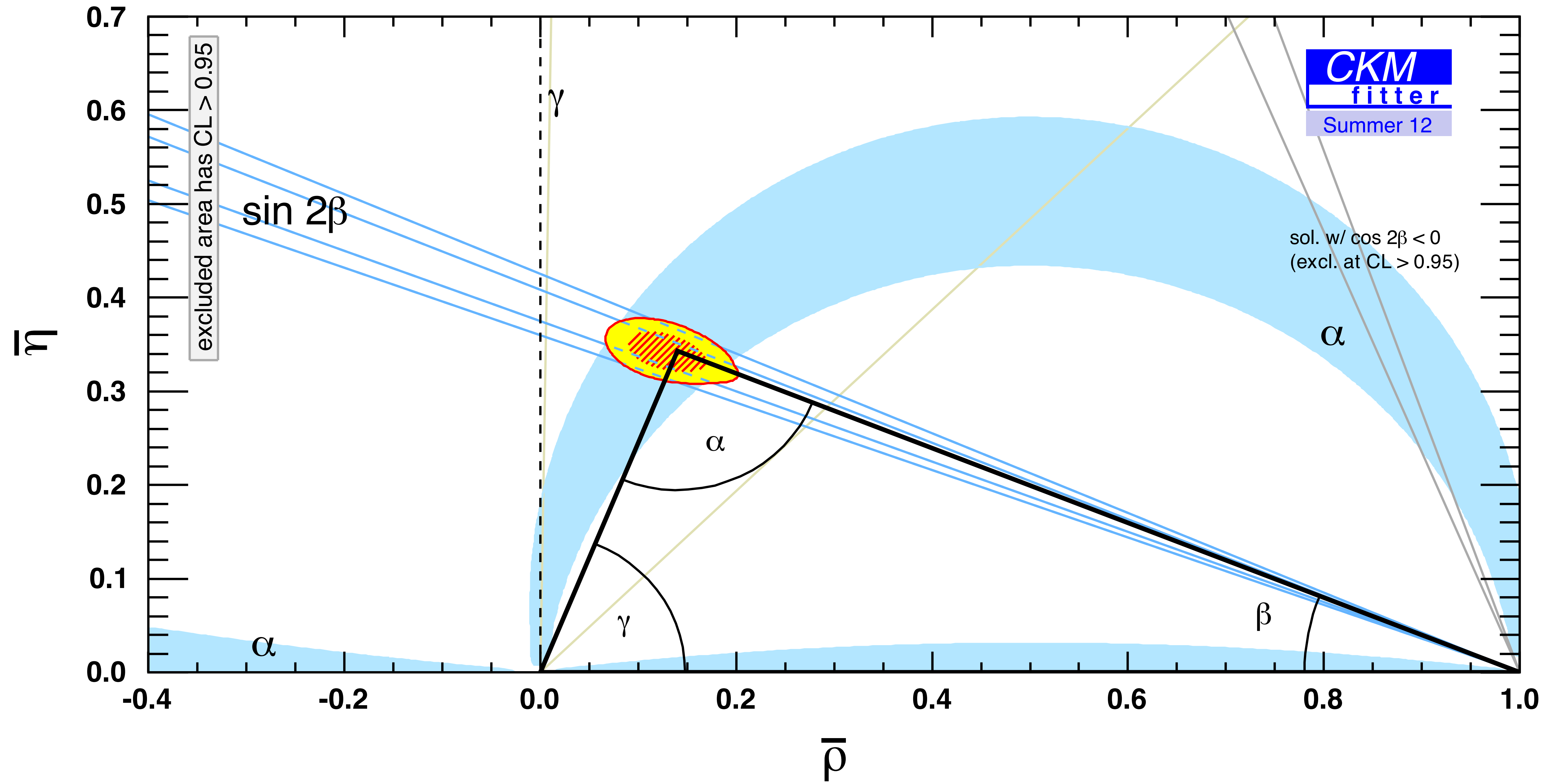
$$\alpha = (89.0^{+4.4}_{-4.2})^\circ$$

$$\gamma \text{ from tree-level decays}$$

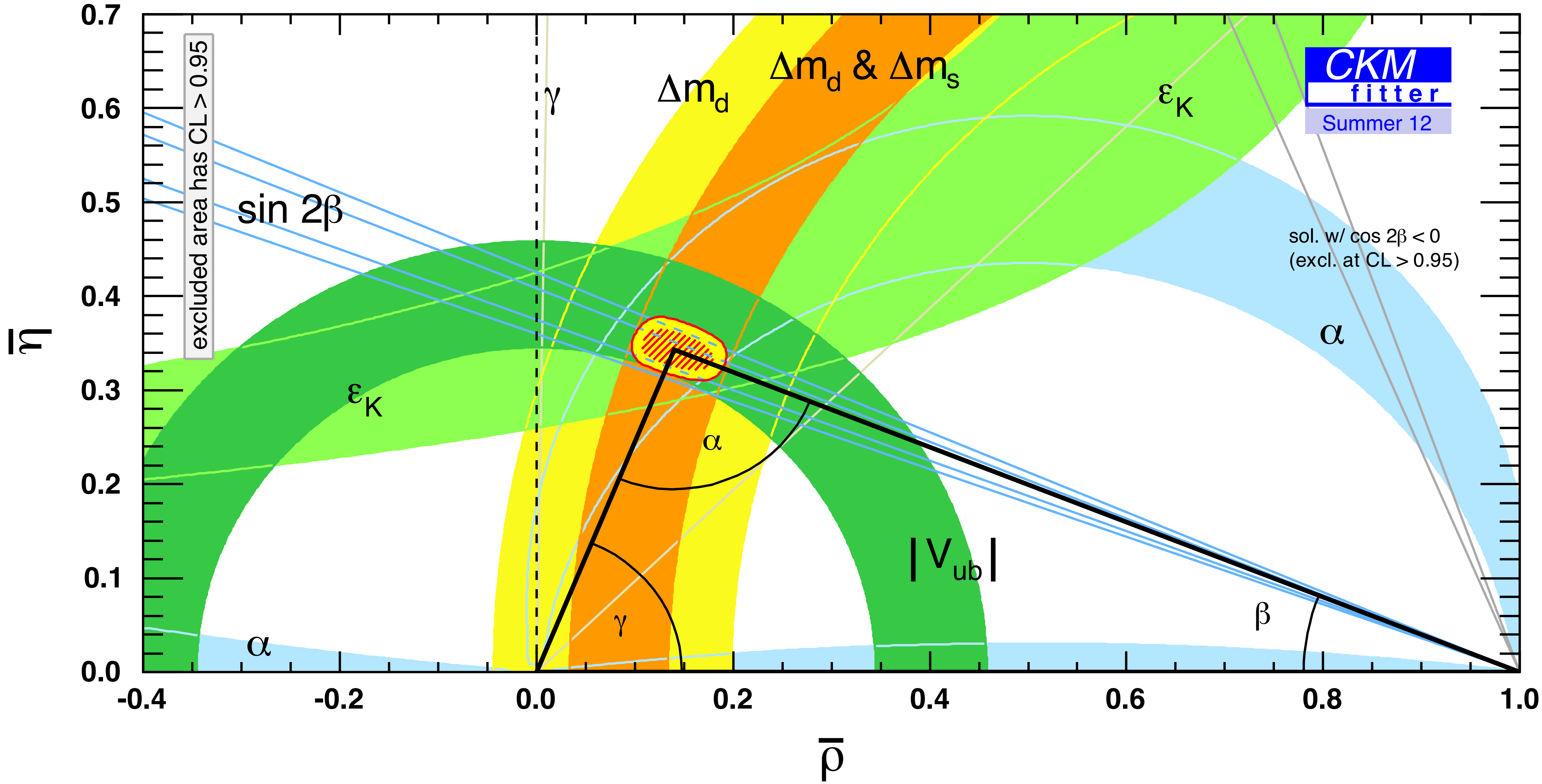
$$B^- \to D^0 K^- \; (b \to c\bar{u}s) \text{ and } B^- \to \bar{D}^0 K^- \; (b \to u\bar{c}s)$$

$$\gamma = (68^{+10}_{-11})^\circ$$

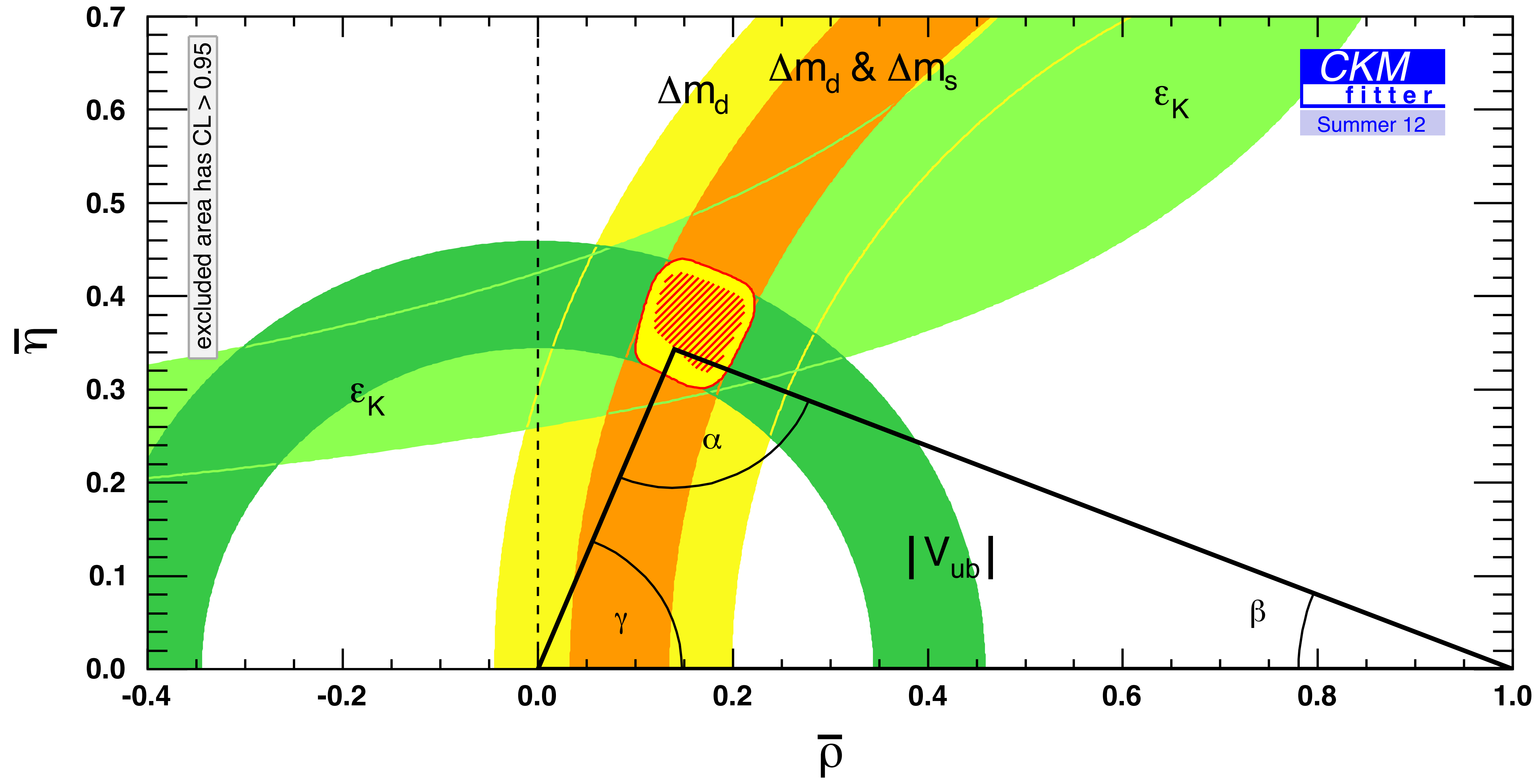
Constraint on CKM using angles only



Constraint on CKM using angles only



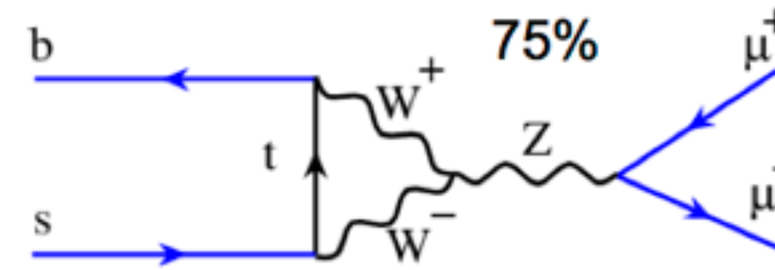
Constraint on CKM using sides only

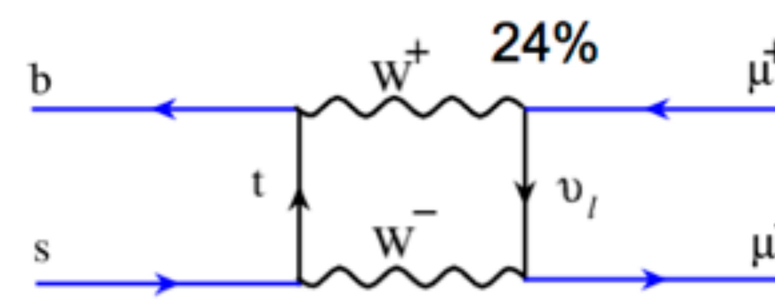


Rare decays

$B_s^0 \to \mu^+ \mu^-$ highly suppressed in SM

- Absence of tree-level FCNC
- V-A structure of weak interaction
  → helicity suppression of leptonic decay
- Hierarch of CKM matrix elements





Why interesting?

NP does not (necessarily) respect these
In SUSY $\propto \tan^6 \beta$

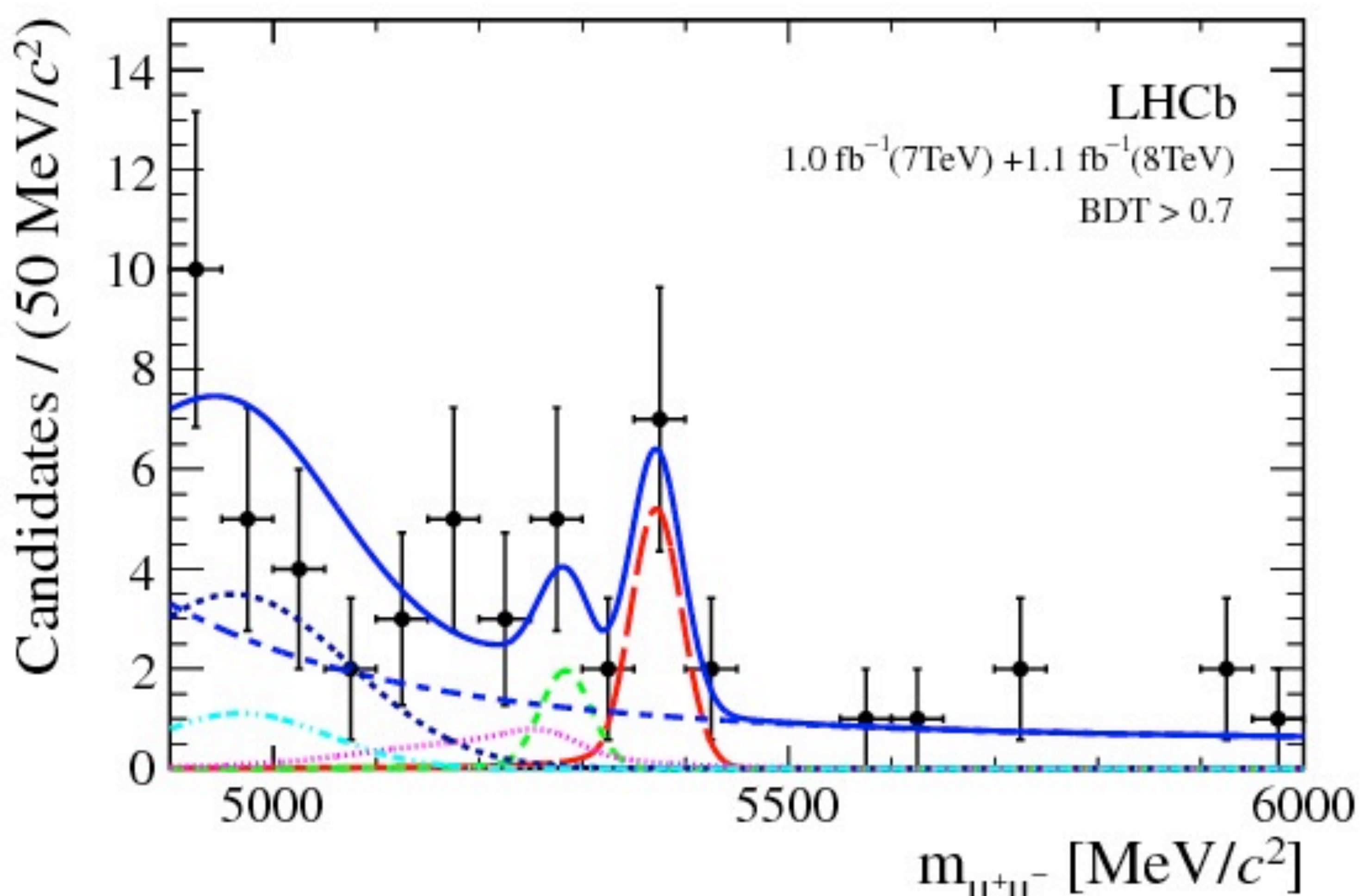
Rare decays

$B_s^0 \to \mu^+\mu^-$ highly suppressed in SM

SM prediction

$$\mathrm{Br}(B_s^0 \to \mu^+\mu^-)_{\mathrm{SM}} = (3.25 \pm 0.17) \times 10^{-9}$$

Recently, evidence at LHCb, with 3.5σ significance



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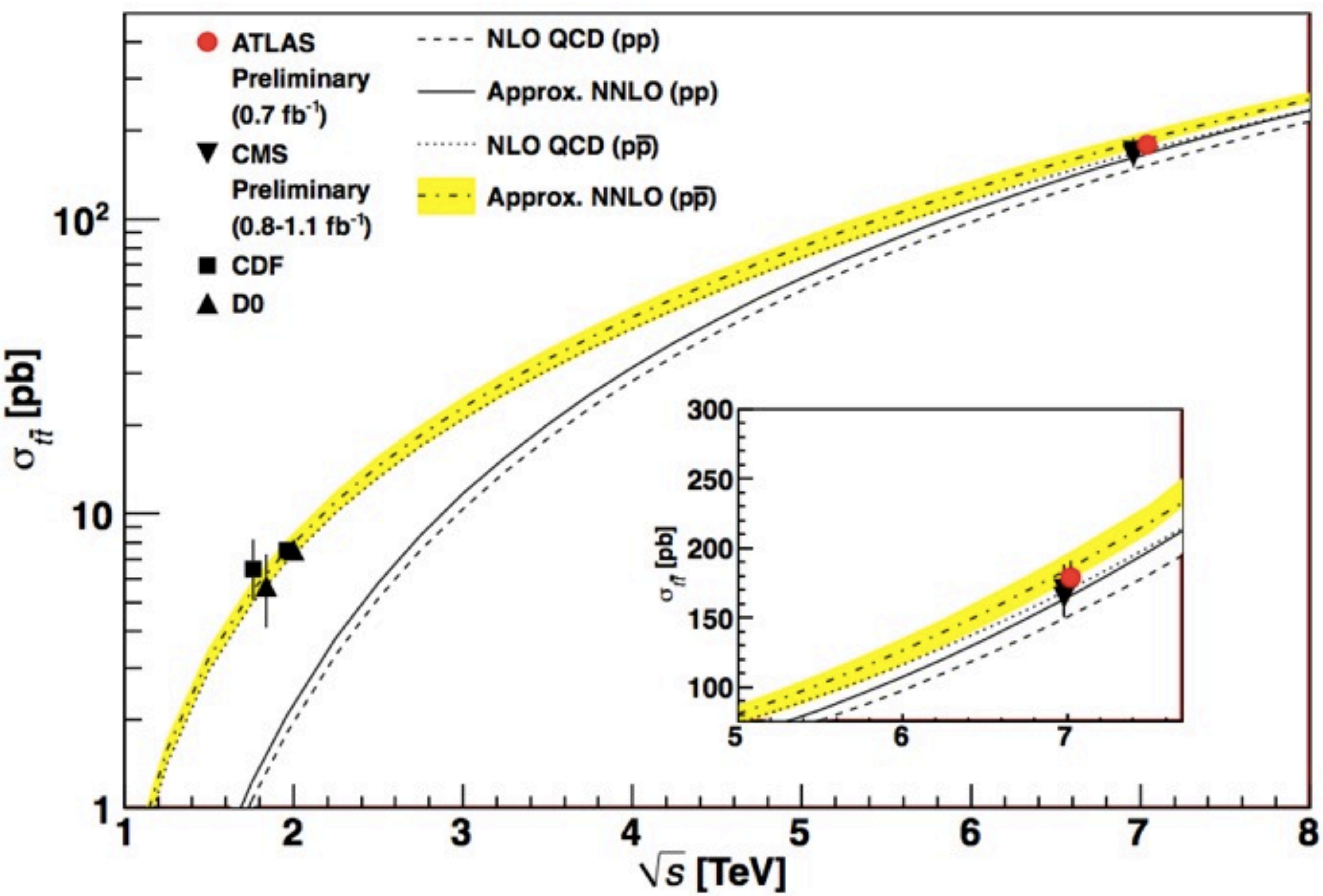
Recently, evidence at LHCb, with 3.5σ significance

$$\mathrm{Br}(B_s^0 \to \mu^+\mu^-)_{\mathrm{exp}} = (3.2^{+1.4}_{-1.2}{}^{+0.5}_{-0.3}) \times 10^{-9}$$

Top

Lots of interesting physics

$t\bar{t}$ production cross section

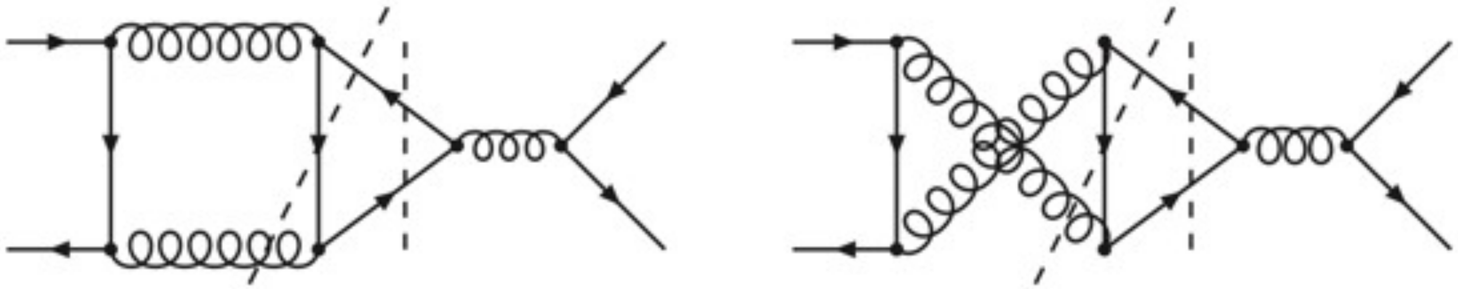


Top

Lots of interesting physics

Focus on forward-backward asymmetry

$$A_{t\bar{t}}(Y) = \frac{N(y_t > y_{\bar{t}}) - N(y_{\bar{t}} > y_t)}{N(y_t > y_{\bar{t}}) + N(y_{\bar{t}} > y_t)}$$



$$A_{t\bar{t}}(Y)_{\mathrm{SM@NLO}} = (5.0 \pm 1.5)\%$$

$$A_{t\bar{t}}(Y)_{\mathrm{D\emptyset}} = (19.6 \pm 6.5)\%$$

$$A_{t\bar{t}}(Y)_{\mathrm{CDF}} = (20.1 \pm 6.7)\%$$

Summary

- MFV → constraints on new physics (but may not be true)
- CP Violation gives lots of information about CKM, constrains NP
- Rare *B* decays, also constrains NP
- Top forward-backward asymmetry may be interesting...

Quick SM review

Quark matter content (and Higgs)

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L, \ u_R, \ d_R; \ H: \ \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

q_L: $(3,2)_{1/6}$; u_R: $(3,1)_{2/3}$; d_R: $(3,1)_{-1/3}$; H: $(1,2)_{-1/2}$

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \sum_\psi \bar{\psi} i \not{D} \psi - [\lambda^u_{ij} \tilde{H} \bar{u}^i_R q^j_L + \lambda^d_{ij} H \bar{d}^i_R q^j_L + \text{h.c.}]$$

$$D_\mu = \partial_\mu + i g_3 A^a_\mu T^a + i g_2 W^a_\mu \frac{\sigma^a}{2} + i g_1 B_\mu Y$$

$$\text{If } \lambda = 0, \text{ then } U(3)^3 \text{ symmetry } (q_L \to U_q q_L, \text{ etc})$$

Keep track of breaking using spurions

$\tilde{H} \bar{u}_R \lambda^u q_L \to \tilde{H} \bar{u}_R U^\dagger_u \lambda^u U_q q_L$ so $\lambda^u \to U_u \lambda^u U^\dagger_q$

$$\bar{u}_R \lambda^u q_L \to \bar{u}_R U^\dagger_u (U_u \lambda^u U^\dagger_q) U_q q_L = \bar{u}_R \lambda^u q_L$$