Sudakov form factor from Markov property

Unitarity

$$P(\text{“some emission”}) + P(\text{“no emission”})$$
$$= P(0 < t \leq T) + \bar{P}(0 < t \leq T) = 1\ .$$

Multiplication law (no memory)

$$\bar{P}(0 < t \leq T) = \bar{P}(0 < t \leq t_1)\bar{P}(t_1 < t \leq T)$$

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Multiplication law (no memory)

$$\bar{P}(0 < t \le T) = \bar{P}(0 < t \le t_1)\bar{P}(t_1 < t \le T)$$

Then subdivide into n pieces: $t_i = \frac{i}{n}T, 0 \le i \le n$.

$$\bar{P}(0 < t \le T) = \lim_{n\to\infty} \prod_{i=0}^{n-1} \bar{P}(t_i < t \le t_{i+1}) = \lim_{n\to\infty} \prod_{i=0}^{n-1} \big(1 - P(t_i < t \le t_{i+1})\big)$$
$$= \exp\left(-\lim_{n\to\infty} \sum_{i=0}^{n-1} P(t_i < t \le t_{i+1})\right) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right) \ .$$

Sudakov form factor

Again, no–emission probability!

$$\bar{P}(0 < t \le T) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

So,

$$\begin{aligned}\mathrm{d}P(\text{first emission at } T) &= \mathrm{d}P(T)\bar{P}(0 < t \le T)\\ &= \mathrm{d}P(T)\exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)\end{aligned}$$

That's what we need for our parton shower! Probability density for next emission at t:

$$\mathrm{d}P(\text{next emission at } t) =$$
$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\,\exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Parton shower Monte Carlo

Probability density:

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Conveniently, the probability distribution is $\Delta(t)$ itself.

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Conveniently, the probability distribution is $\Delta(t)$ itself.
Hence, parton shower very roughly from (HERWIG):

1. Choose flat random number $0 \le \rho \le 1$.
2. If $\rho < \Delta(t_{\max})$: no resolbable emission, stop this branch.
3. Else solve $\rho = \Delta(t_{\max})/\Delta(t)$
   (= no emission between $t_{\max}$ and t) for t.
   Reset $t_{\max} = t$ and goto 1.

Determine z essentially according to integrand in front of exp.

Parton shower Monte Carlo

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$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Conveniently, the probability distribution is $\Delta(t)$ itself.

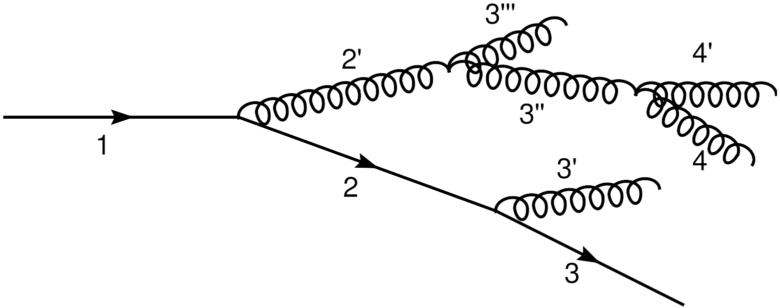
- That was old HERWIG variant. Relies on (numerical) integration/tabulation for $\Delta(t)$.
- Pythia, now also Herwig++, use the Veto Algorithm.
- Method to sample x from distribution of the type

$$\mathrm{d}P = F(x)\exp\left[-\int^{x}\mathrm{d}x'\,F(x')\right]\mathrm{d}x\ .$$

Simpler, more flexible, but slightly slower.

Parton cascade

Get tree structure, ordered in evolution variable t:



Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

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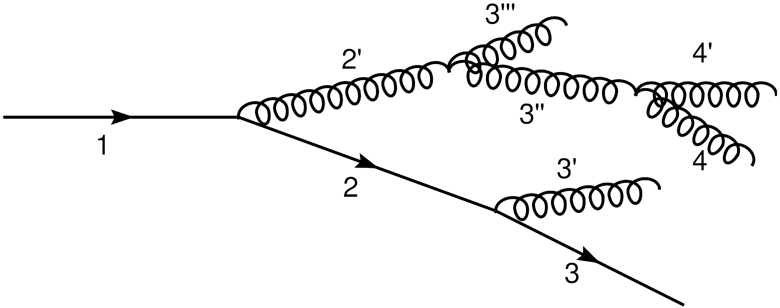
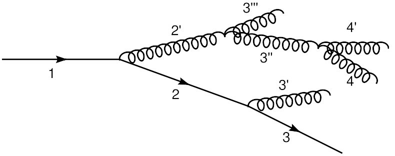


Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

Not at all unique!
Many (more or less clever) choices still to be made.

Parton cascade

Get tree structure, ordered in evolution variable t:



- t can be θ, Q^2, $p_\perp$, ...
- Choice of hard scale t_{max} not fixed. "Some hard scale".
- z can be light cone momentum fraction, energy fraction, ...
- Available parton shower phase space.
- Integration limits.
- Regularisation of soft singularities.
- ...

Good choices needed here to describe wealth of data!

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emissions

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- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emission: consider *eikonal factors*,
here for $q(p+q) \to q(p)g(q)$, soft g:

$$u(p) \not{\varepsilon} \frac{\not{p} + \not{q} + m}{(p+q)^2 - m^2} \longrightarrow u(p) \frac{p \cdot \varepsilon}{p \cdot q}$$

soft factorisation. Universal, *i.e.* independent of emitter.
In general:

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_S}{2\pi} \sum_{ij} C_{ij} W_{ij} \quad (\text{"QCD–Antenna"})$$

with

$$W_{ij} = \frac{1 - \cos\theta_{ij}}{(1 - \cos\theta_{iq})(1 - \cos\theta_{qj})} \ .$$

Soft emissions

We define

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})} \equiv W_{ij}^{(i)} + W_{ij}^{(j)}$$

with

$$W_{ij}^{(i)} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{qj}}\right) .$$

$W_{ij}^{(i)}$ is only collinear divergent if $q\|i$ etc .

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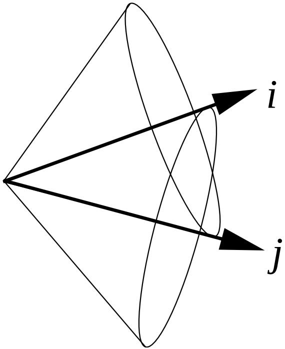
$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .
After integrating out the azimuthal angles, we find

$$\int \frac{d\phi_{iq}}{2\pi} W_{ij}^{(i)} = \begin{cases} \dfrac{1}{1-\cos\theta_{iq}} & (\theta_{iq} < \theta_{ij}) \\ 0 & \text{otherwise} \end{cases}$$

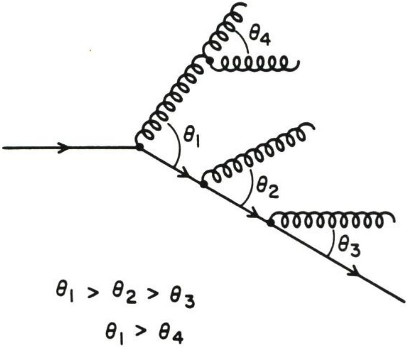
That's angular ordering.

Angular ordering

Radiation from parton *i* is bound to a cone, given by the colour partner parton *j*.



Results in angular ordered parton shower and suppresses soft gluons viz. hadrons in a jet.



Colour coherence from CDF

Events with 2 hard ($> 100\,\mathrm{GeV}$) jets and a soft 3rd jet ($\sim 10\,\mathrm{GeV}$)

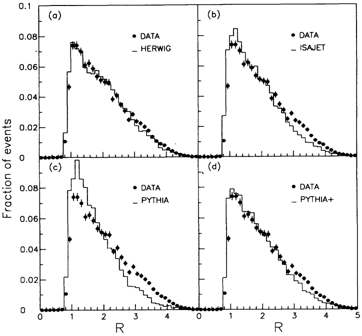


FIG. 14. Observed R distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

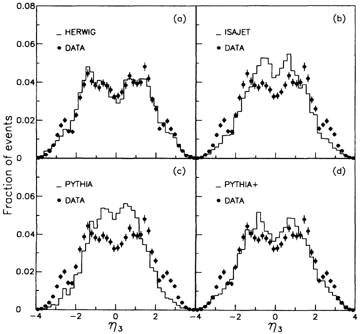


FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

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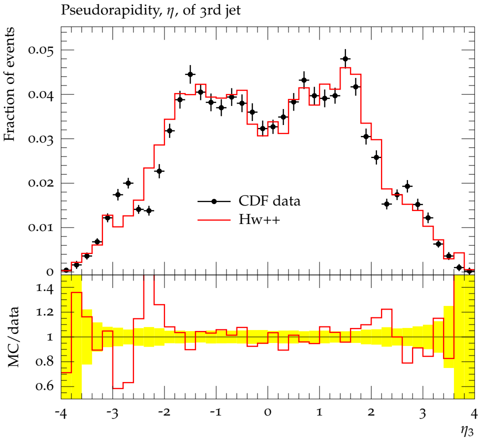

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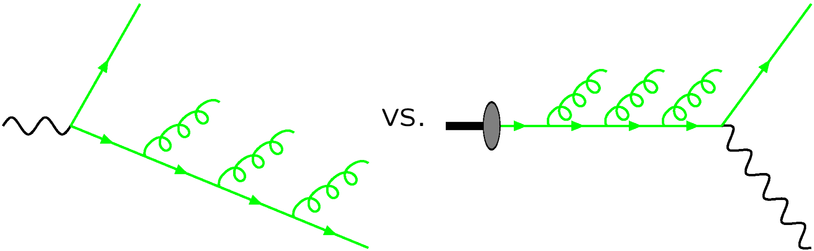


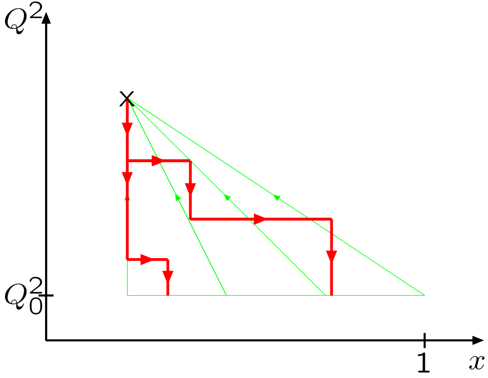
Similar to final state radiation. Sudakov form factor ($x' = x/z$)

$$\Delta(t, t_{\max}) = \exp\left[-\sum_b \int_t^{t_{\max}} \frac{\mathrm{d}t}{t} \int_{z_-}^{z_+} \mathrm{d}z \frac{\alpha_S(z,t)}{2\pi} \frac{x' f_b(x',t)}{x f_a(x,t)} \hat{P}_{ba}(z,t) \right]$$

Have to divide out the pdfs.

Initial state radiation

Evolve backwards from hard scale Q^2 *down* towards cutoff scale Q_0^2. Thereby increase x.

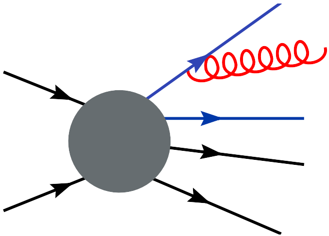


With parton shower we *undo* the DGLAP evolution of the pdfs.

Dipoles

Exact kinematics when recoil is taken by spectator(s).

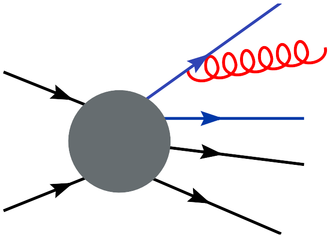
- Dipole showers.
- Ariadne.
- Recoils in Pythia.



Dipoles

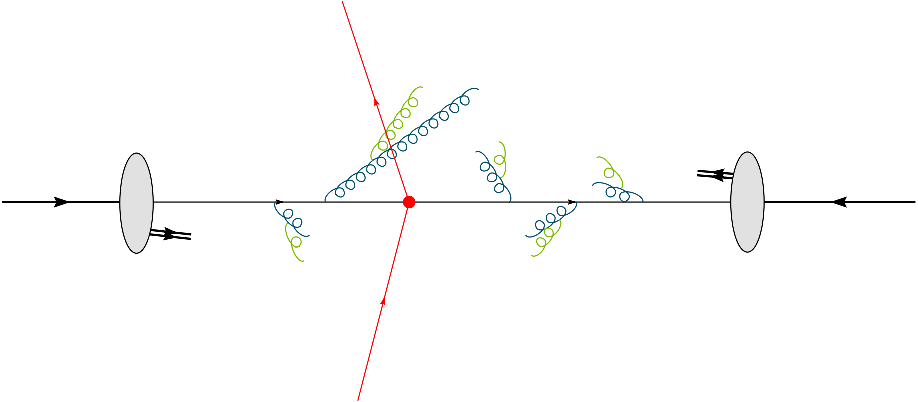
Exact kinematics when recoil is taken by spectator(s).

- Dipole showers.
- Ariadne.
- Recoils in Pythia.
- New dipole showers, based on
  - Catani Seymour dipoles.
  - QCD Antennae.
  - Goal: matching with NLO.
- Generalized to IS–IS, IS–FS.

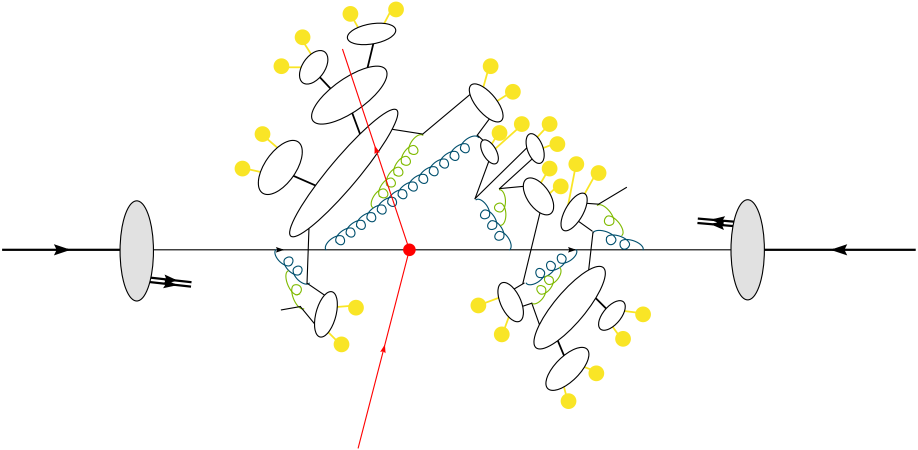


Hadronization

Parton shower



Parton shower $\longrightarrow$ hadrons

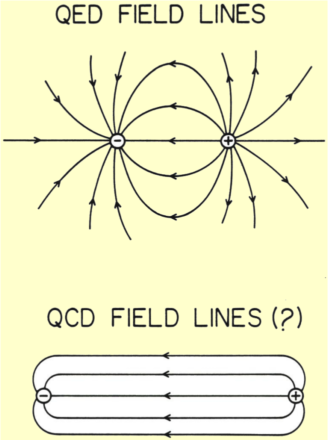


Parton shower $\longrightarrow$ hadrons

- Parton shower terminated at t_0 = lower end of PT.
- Can't measure quarks and gluons.
- Degrees of freedom in the detector are hadrons.
- Need a description of confinement.

Physical input

Self coupling of gluons
$\leftrightarrow$ "attractive field lines"



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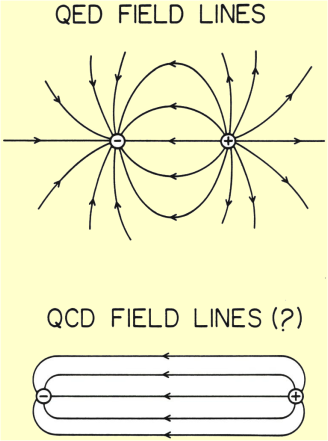
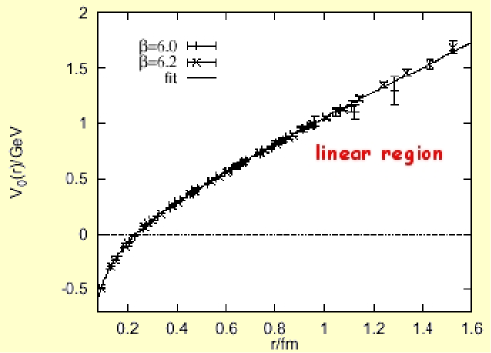


Linear static potential $V(r) \approx \kappa r$.



Supported by lattice QCD,
hadron spectroscopy.

Hadronization models

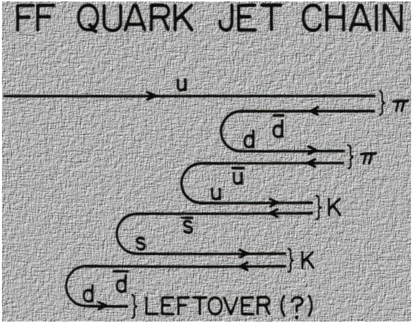
Older models:

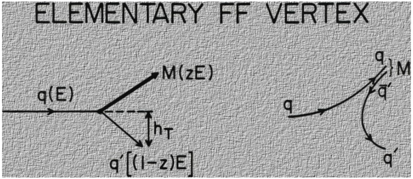
- Flux tube model.
- Independent fragmentation.

Today's models.

- Lund string model (Pythia).
- Cluster model (Herwig).

Independent fragmentation

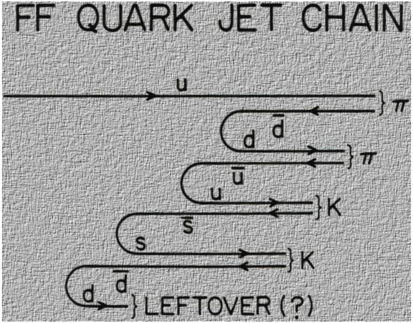


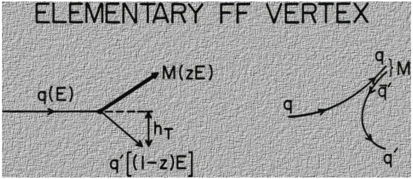


Feynman–Field fragmentation ('78).

- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q\to h}(z)$ = density of momentum fraction z carried away by hadron h from quark q.
- Gaussian $p_\perp$ distribution.

Independent fragmentation





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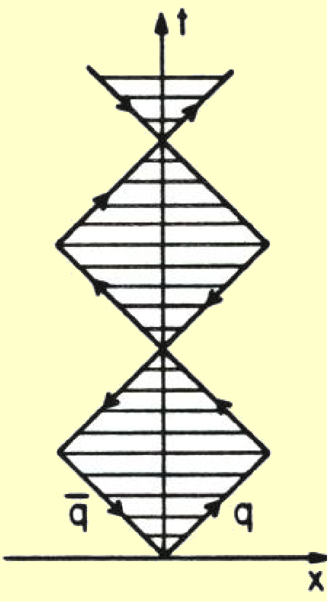
- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q\to h}(z)$ = density of momentum fraction z carried away by hadron h from quark q.
- Gaussian $p_\perp$ distribution.
- Problems:
  - "last quark".
  - not Lorentz invariant.
  - infrared safety.
  - …
- Good at that time.
- Still usefull for inclusive descriptions.

Lund string model

String model of mesons.
$L = 0$ mesons move in yoyo modes.
Area law: $m^2 \sim$ area.



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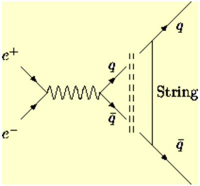
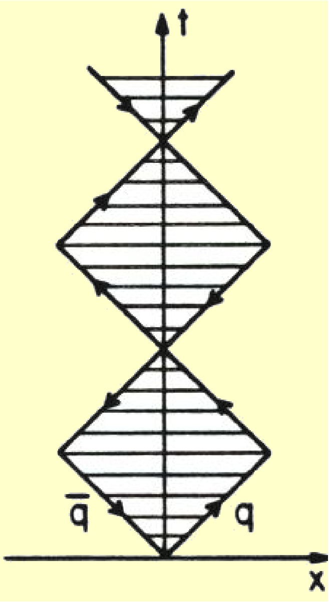
Simple model for particle production in e^+e^- annihilation:



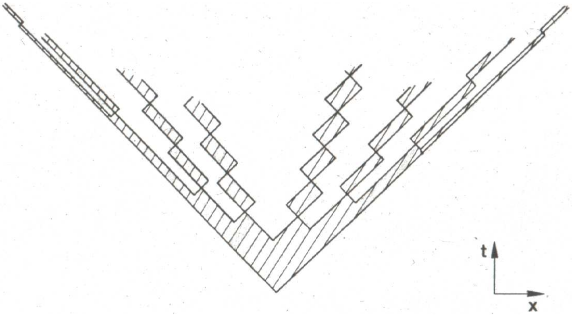
$q\bar{q}$ pair as pointlike source of string.



Lund string model

String energy $\sim$ intense chromomagnetic field.
$\longrightarrow$ Additional $q\bar{q}$ pairs created by QM tunneling.

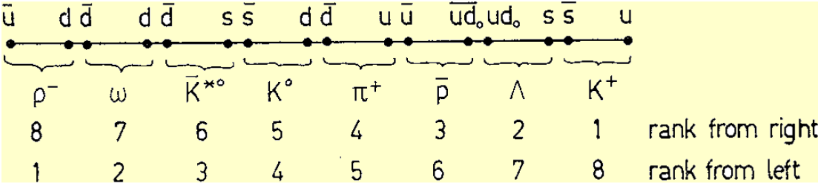
$$\frac{\mathrm{dProb}}{\mathrm{d}x\mathrm{d}t} \sim \exp\left(-\pi m_q^2/\kappa\right) \qquad \kappa \sim 1\,\mathrm{GeV}\,.$$



String breaking expected long before yoyo point.

Lund string model

Ajacent breaks form hadrons.



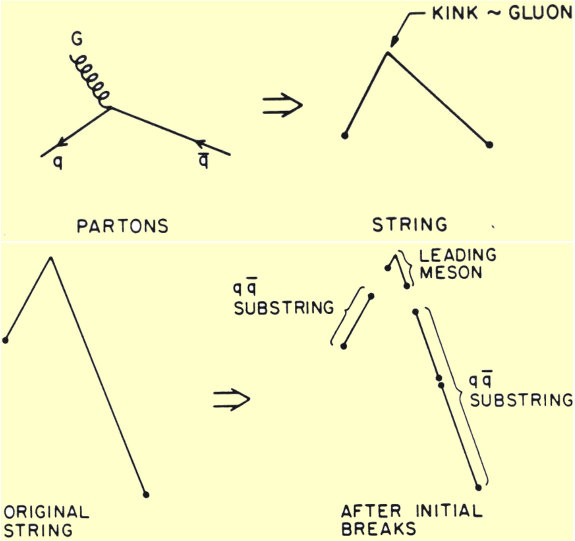
Works in both directions (symmetry).
Lund symmetric fragmentation function

$$f(z,p_\perp) \sim \frac{1}{z}(1-z)^a \exp\left(-\frac{b(m_h^2+p_\perp^2)}{z}\right)$$

a, b, m_h^2 main adjustable parameters.
Note: diquarks $\to$ baryons.

Lund string model

gluon = kink on string = motion pushed into the $q\bar{q}$ system.



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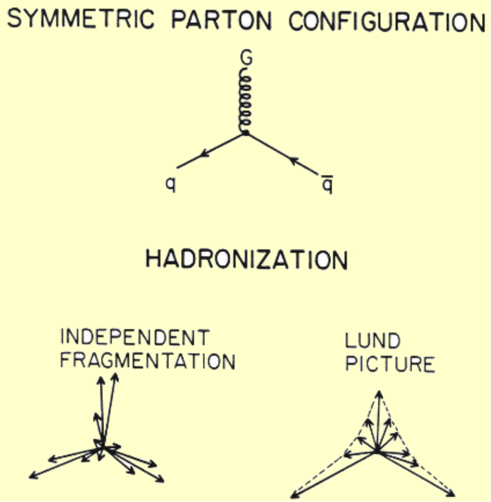

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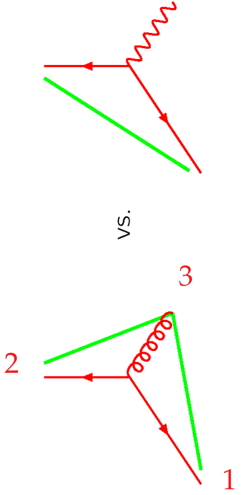
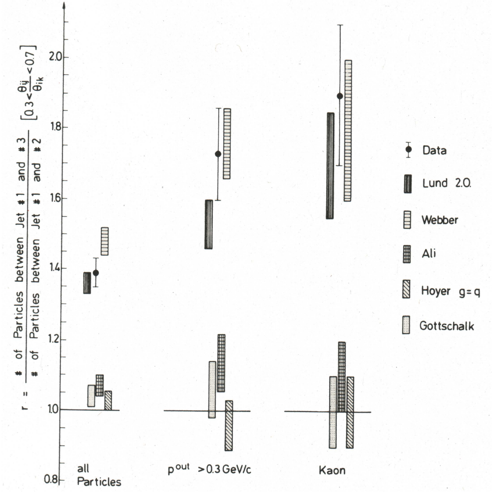


"String effect"



Lund string model

Some remarks:

- Originally invented without parton showers in mind.

Lund string model

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- Originally invented without parton showers in mind.
- Stong physical motivation.
- Very successful desription of data.
- Universal desription of data
  (fit at e^+e^-, transfer to hadron-hadron).
- Many parameters, ~ 1 per hadron.
- Too easy to hide errors in perturbative description?

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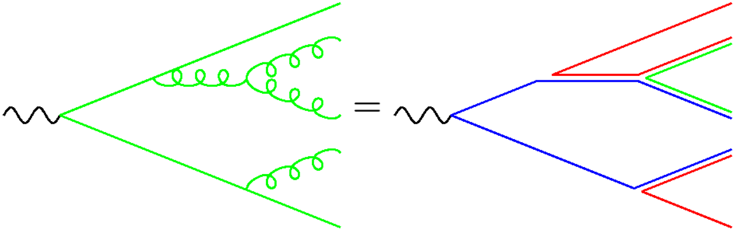
$\longrightarrow$ try to use more QCD information/intuition.

Colour preconfinement

Large N_C limit $\longrightarrow$ planar graphs dominate.
Gluon = colour — anticolourpair



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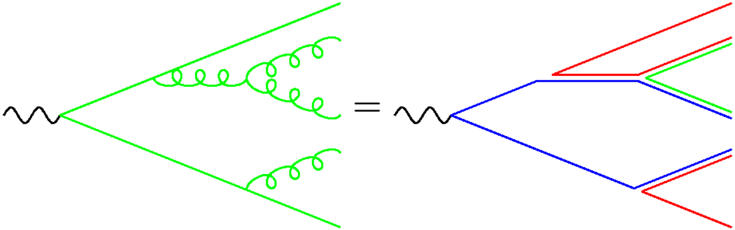
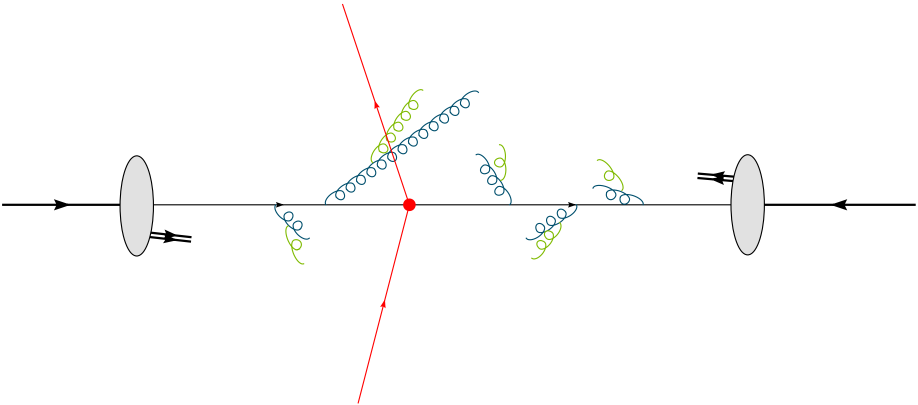


Parton shower organises partons in colour space. Colour partners (=colour singlet pairs) end up close in phase space.

$\longrightarrow$ Cluster hadronization model

Cluster hadronization



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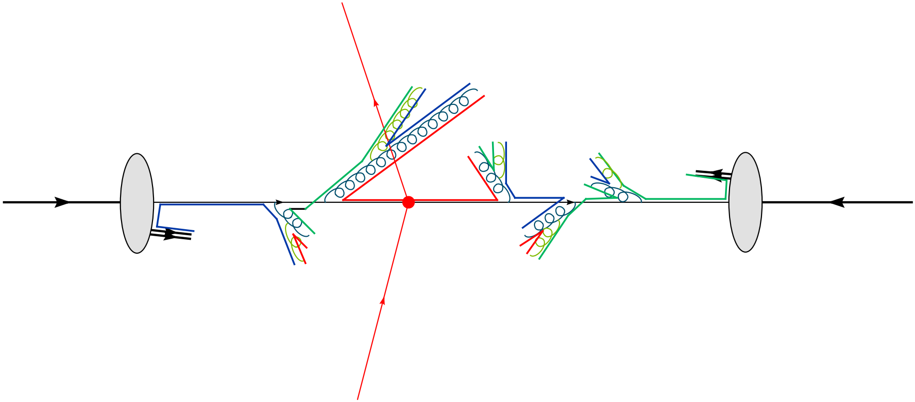

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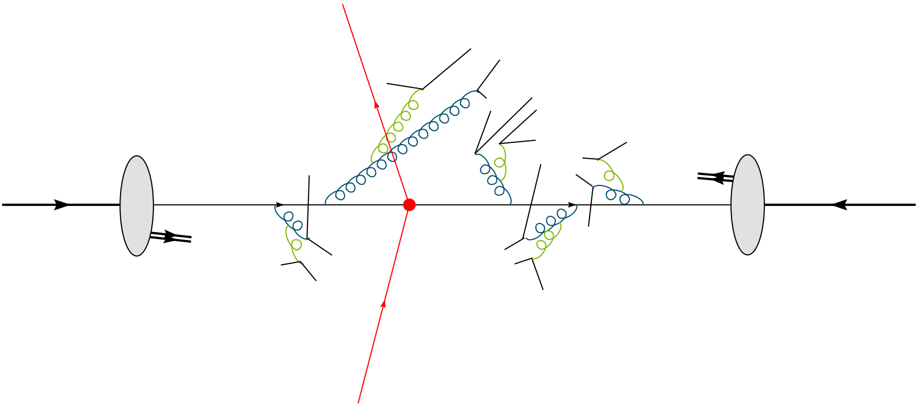

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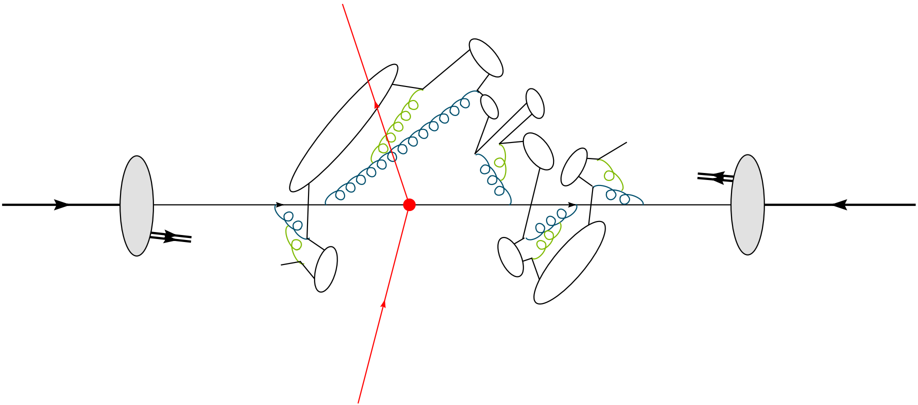
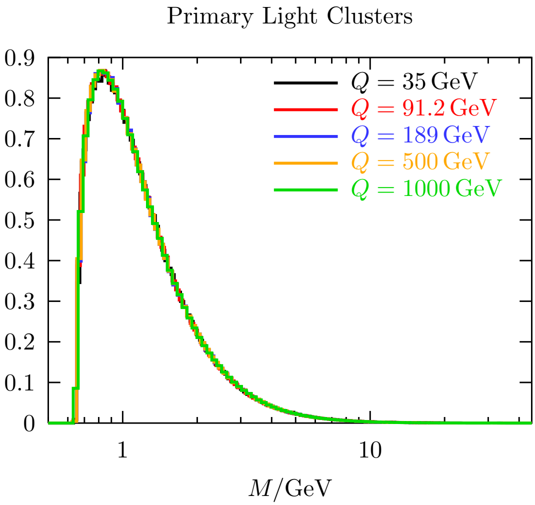


Primary cluster mass spectrum independent of production mechanism. Peaked at some low mass.



Cluster hadronization

Primary cluster mass spectrum independent of production mechanism. Peaked at some low mass.

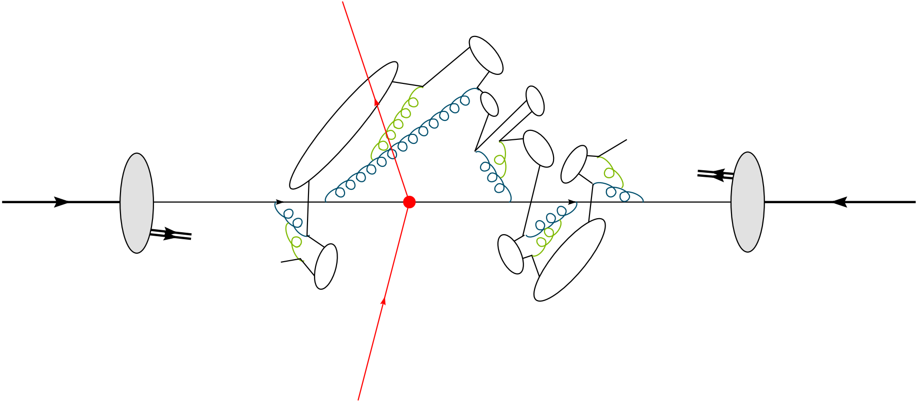
Cluster = continuum of high mass resonances.
Decay into well–known lighter mass resonances
= discrete spectrum of hadrons.

No spin information carried over, i.e. only phase space.

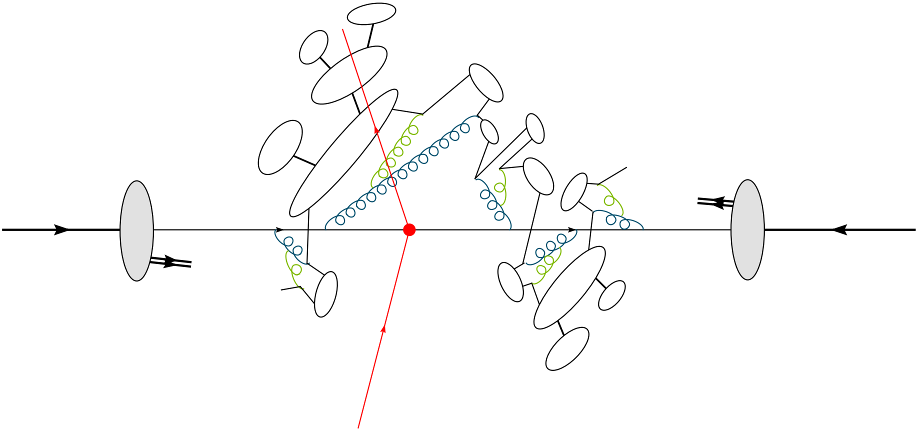
Suppression of heavier particles
(particularly baryons, can be problematic).

Cluster spectrum determined entirely by parton shower, i.e. perturbation theory. Hence, t_0 crucial parameter.

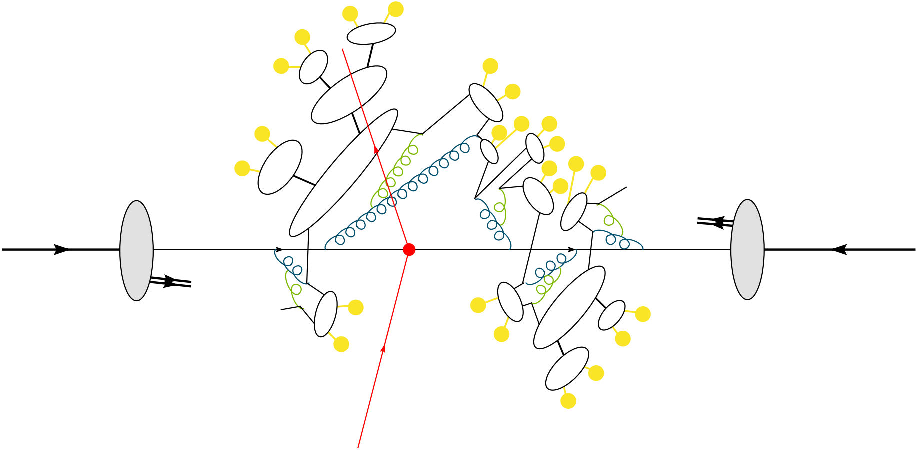
Cluster hadronization



Cluster hadronization



Cluster hadronization

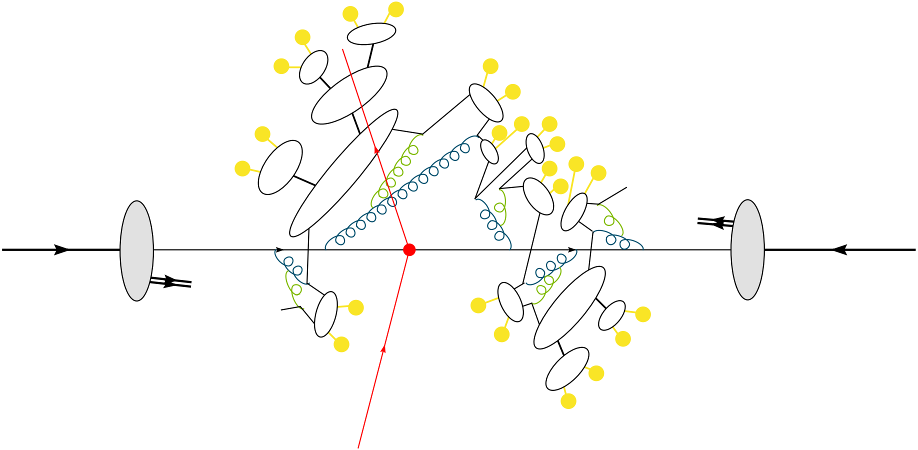


Hadronization

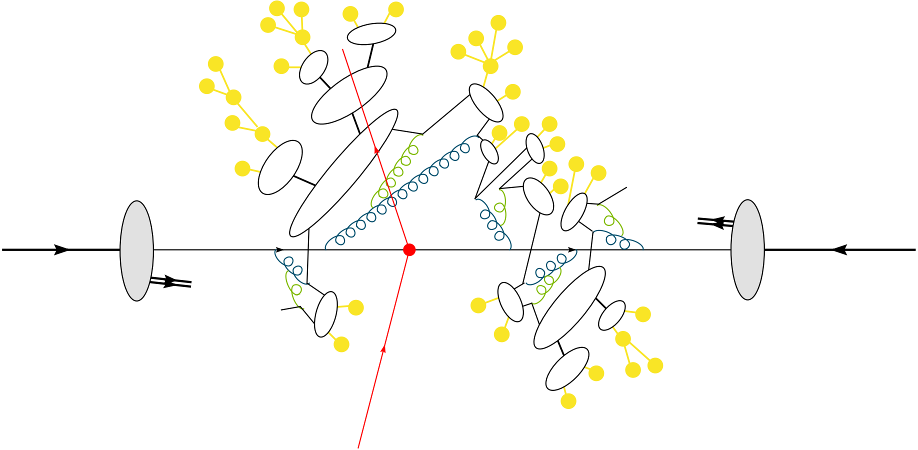
- Only string and cluster models used in recent MC programs.
  Independent fragmentation only for inclusive observables.
- Strings started non–perturbatively,
  improved by parton shower.
- Cluster model started mostly on perturbative side,
  improved by string like cluster fission.

Hadronic Decays

Hadronic decays



Hadronic decays



Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \rightarrow \gamma B^0 \\
\hookrightarrow \bar{B}^0 \\
\hookrightarrow e^- \bar{\nu}_e D^{*+} \\
\hookrightarrow \pi^+ D^0 \\
\hookrightarrow K^- \rho^+ \\
\hookrightarrow \pi^+ \pi^0 \\
\hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
$$\hookrightarrow \bar{B}^0$$
$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$
$$\hookrightarrow \pi^+ D^0$$
$$\hookrightarrow K^- \rho^+$$
$$\hookrightarrow \pi^+ \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

EM decay.

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
$$\hookrightarrow \bar{B}^0$$
$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$
$$\hookrightarrow \pi^+ D^0$$
$$\hookrightarrow K^- \rho^+$$
$$\hookrightarrow \pi^+ \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

Weak mixing.

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \quad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\quad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\quad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Weak decay.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \quad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\quad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\quad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Strong decay.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \quad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad \hookrightarrow \pi^+ {\color{red}D^0} \\
& \qquad\quad {\color{red}\hookrightarrow K^- \rho^+} \\
& \qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\quad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Weak decay, ρ^+ mass smeared.

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
$$\hookrightarrow \bar{B}^0$$
$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$
$$\hookrightarrow \pi^+ D^0$$
$$\hookrightarrow K^- \rho^+$$
$$\hookrightarrow \pi^+ \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

ρ^+ polarized, angular correlations.

Many aspects:

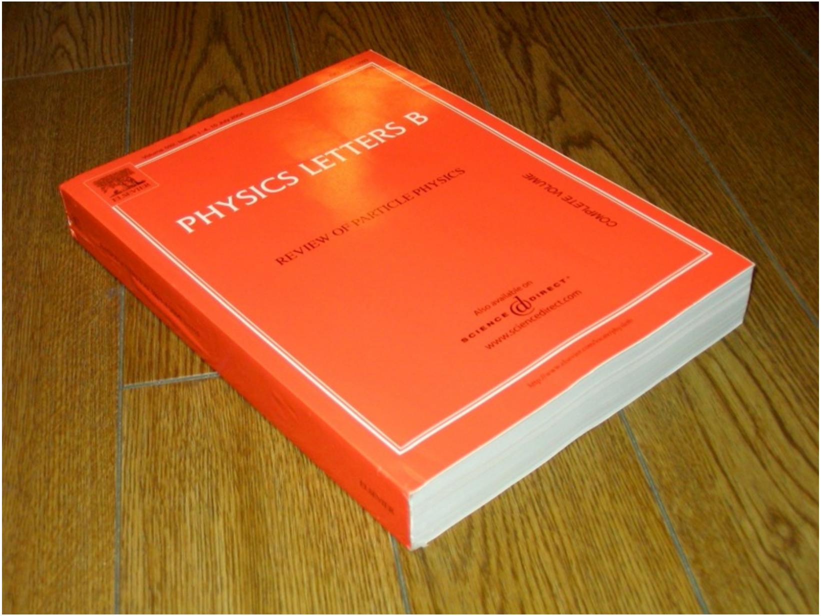
$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \qquad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad\qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\qquad\qquad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad\qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\qquad\qquad\qquad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Dalitz decay, m_{ee} peaked.

Hadronic decays

Tedious.
100s of different particles, 1000s of decay modes, phenomenological matrix elements with parametrized form factors…

Hadronic decays



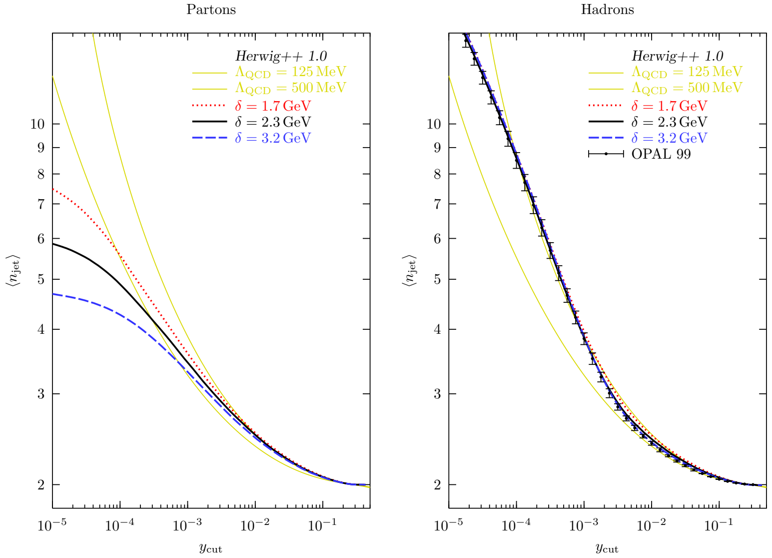
A few plots

How well does it work?

- $e^+e^- \to$ hadrons, mostly at LEP.
- Jet shapes, jet rates, event shapes, identified particles. . .
- 'Tuning' of parameters.
- Want to get *everything* right with *one* parameter set.
- Compare to literally 100s of plots.

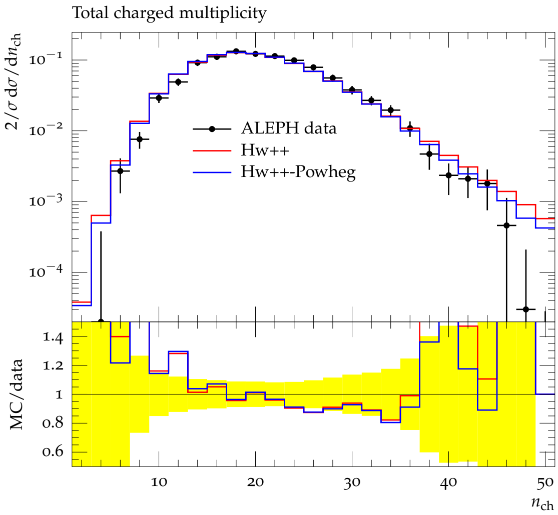
How well does it work?

Smooth interplay between shower and hadronization.



How well does it work?

$N_{\rm ch}$ at LEP. Crucial for t_0 (Herwig++ 2.5.2)

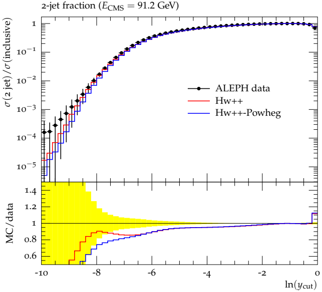


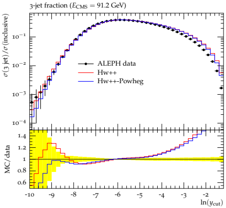
How well does it work?

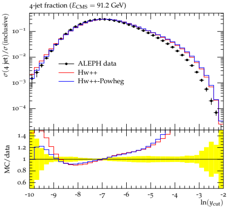
Jet rates at LEP.

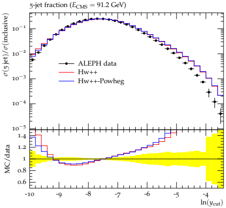
$$R_n = \sigma(n\text{–jets})/\sigma(\text{jets})$$
$$R_6 = \sigma(>5\text{–jets})/\sigma(\text{jets})$$

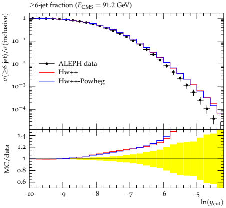
(Herwig++ 2.5.2)





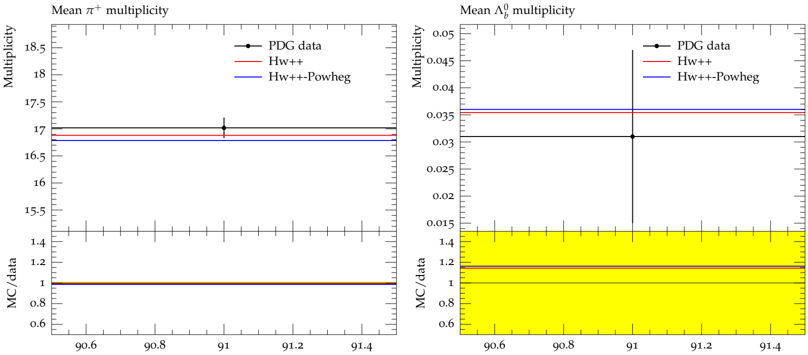






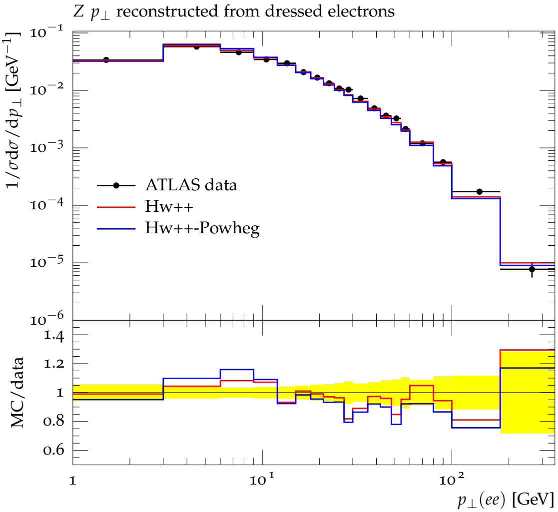
How well does it work?

Hadron Multiplicities at LEP (e.g. π^+, Λ_b^0).

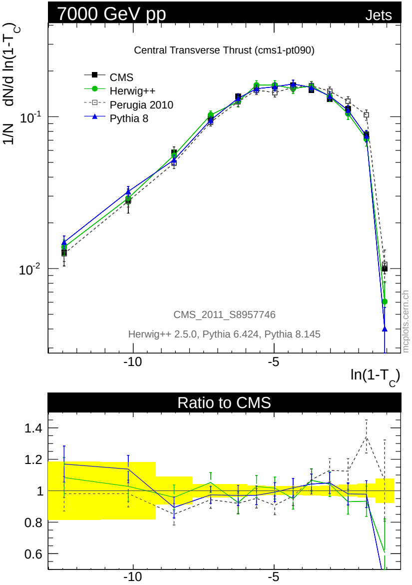


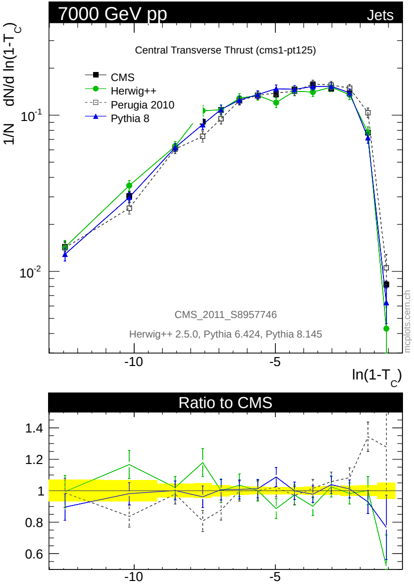
How well does it work?

$p_\perp(Z^0) \to$ intrinsic $k_\perp$ (LHC 7 TeV).



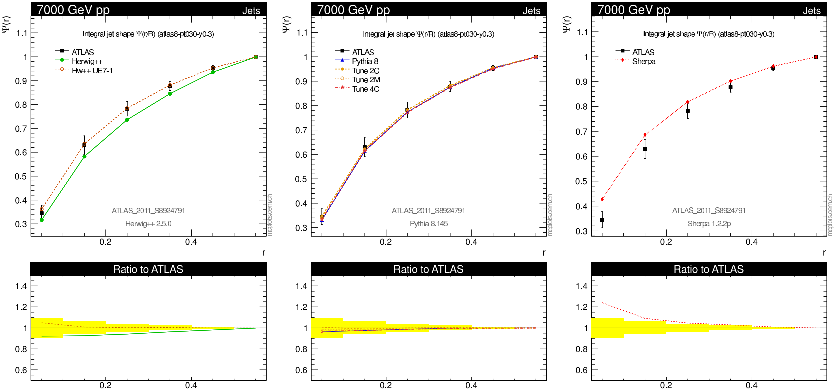
Transverse thrust





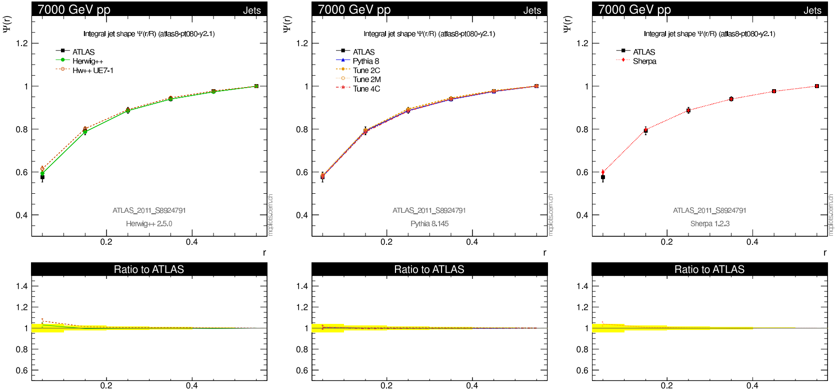
Integral jet shapes

not too hard, central ($30 < p_T/\mathrm{GeV} < 40; 0 < |y| < 0.3$)

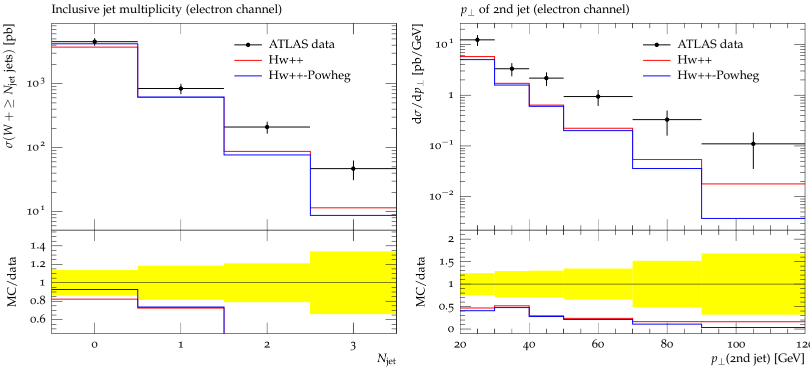


Integral jet shapes

harder, more forward $(80 < p_T/\text{GeV} < 110; 1.2 < |y| < 2.1)$



$W + \text{jets}$, LHC 7 TeV.

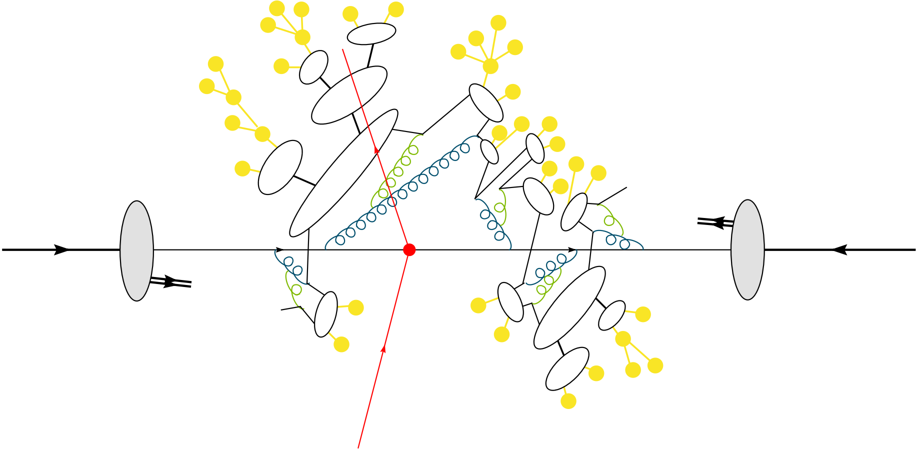


Higher jets not covered by parton shower only $\rightarrow$ matching.

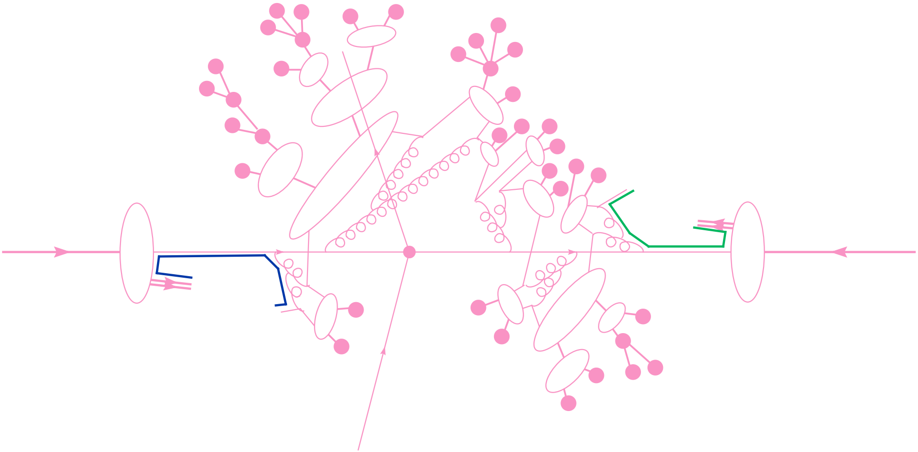
Multiple Partonic Interactions

(very sketchy)

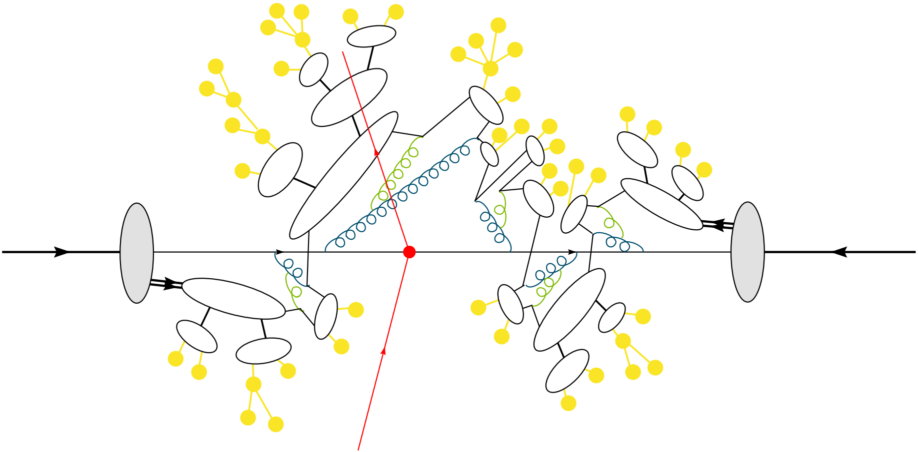
What about the remnants?



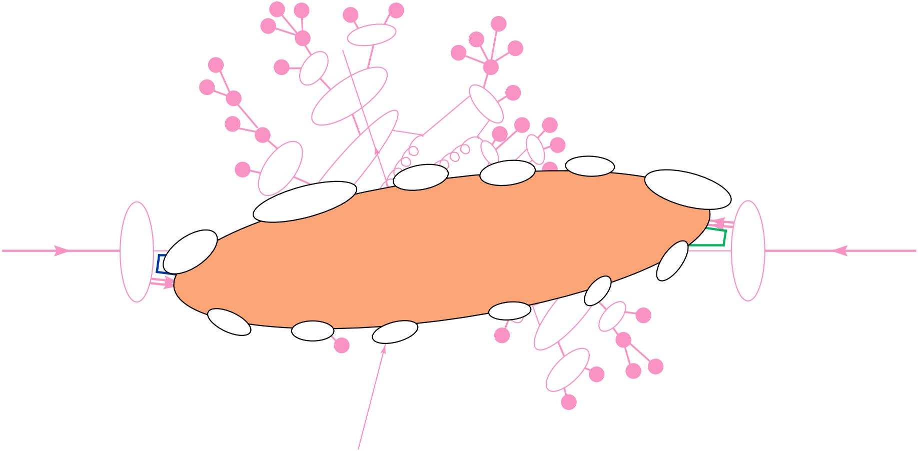
What about the remnants?



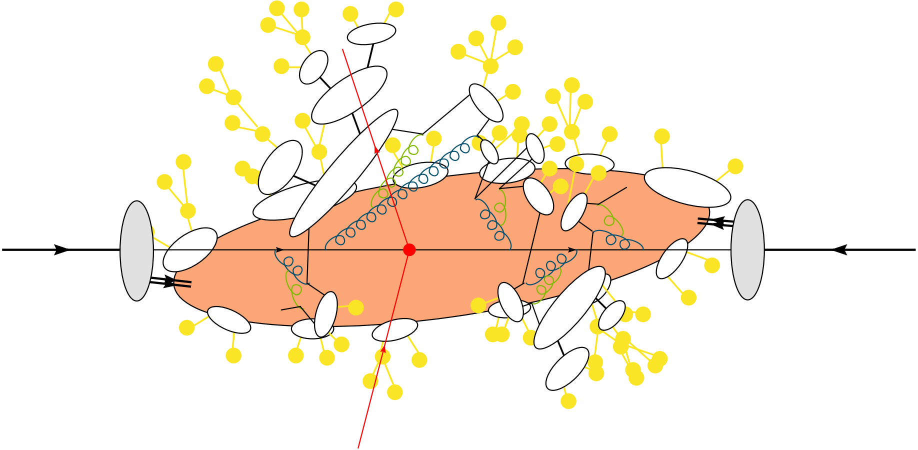
What about the remnants?



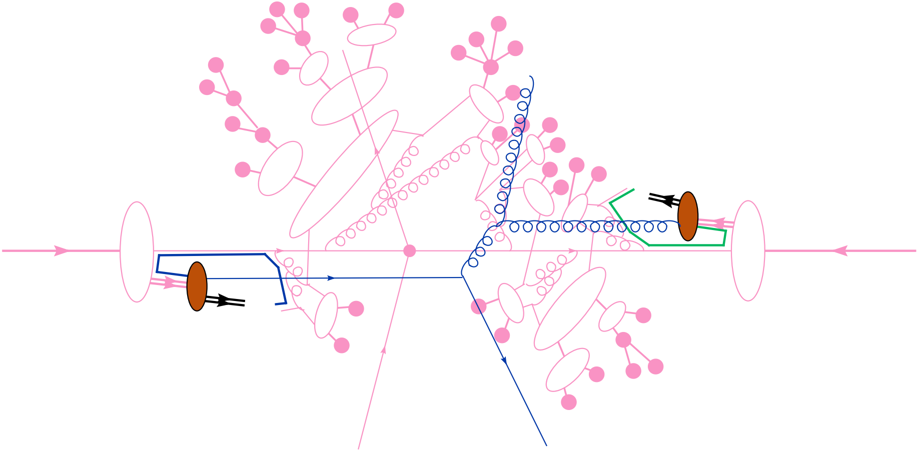
What about the remnants?



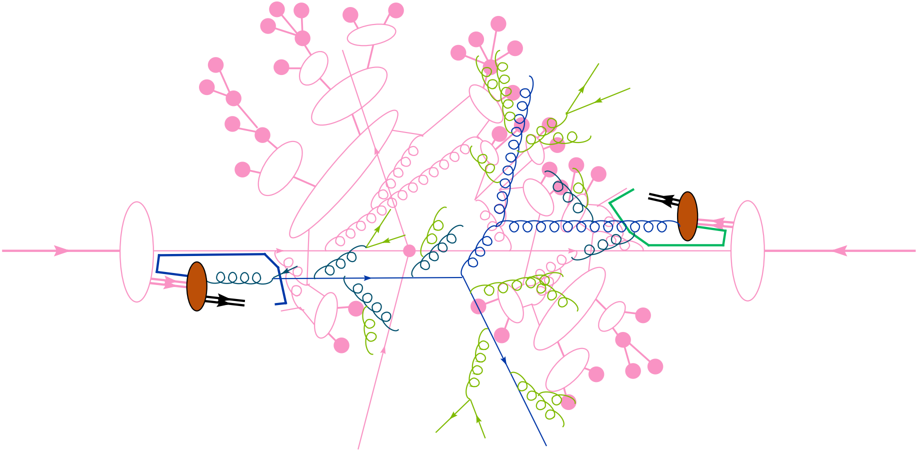
What about the remnants?



What about the remnants?



What about the remnants?



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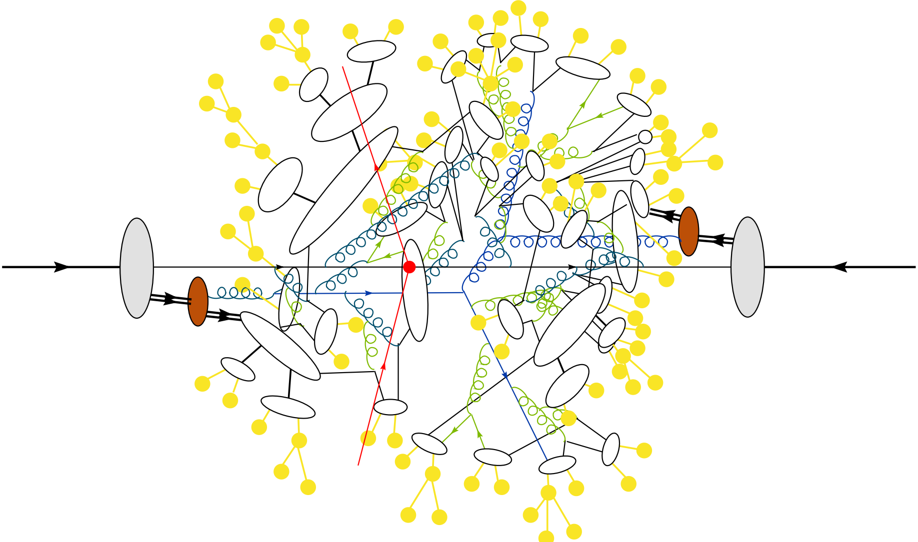

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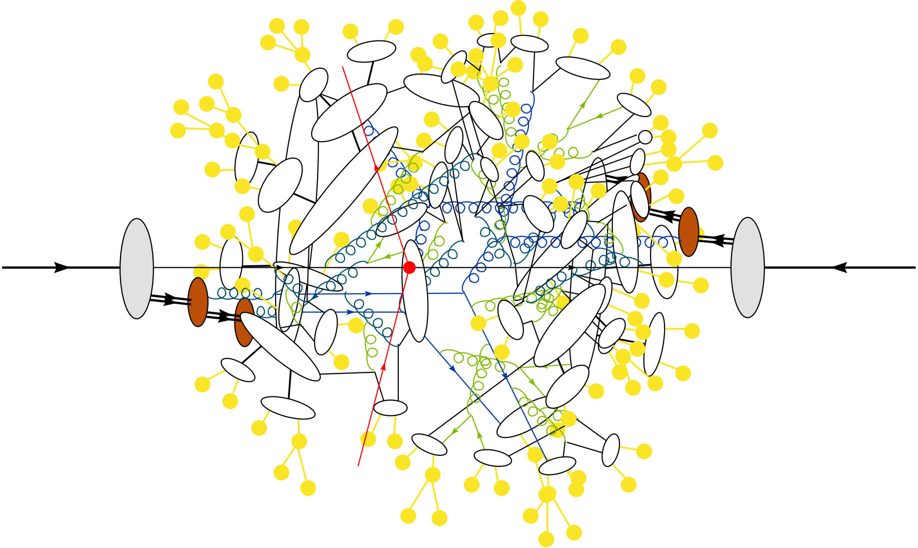
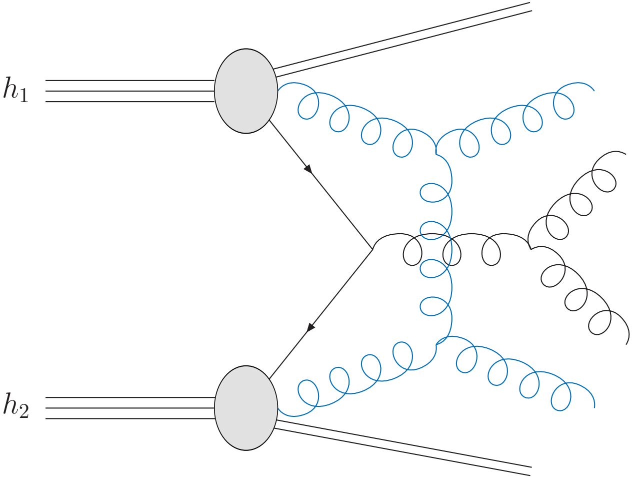


Mulitple hard interactions



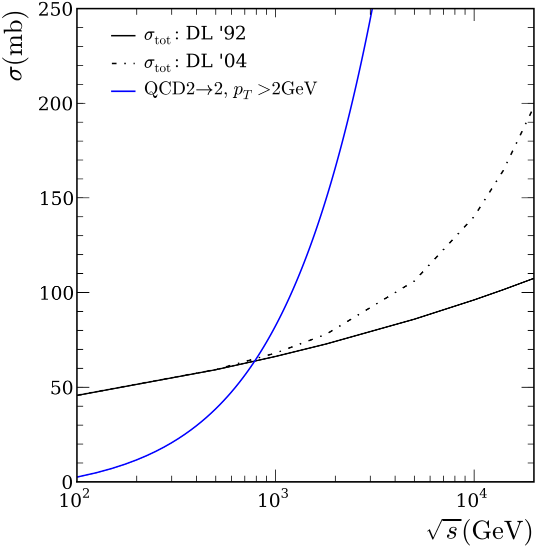
Eikonal model basics

Starting point: hard inclusive jet cross section.

$$\sigma^{\mathrm{inc}}(s;p_t^{\mathrm{min}}) = \sum_{i,j} \int_{p_t^{\mathrm{min}\,2}} \mathrm{d}p_t^2 f_{i/h_1}(x_1,\mu^2) \otimes \frac{\mathrm{d}\hat{\sigma}_{i,j}}{\mathrm{d}p_t^2} \otimes f_{j/h_2}(x_2,\mu^2)\,,$$

$\sigma^{\mathrm{inc}} > \sigma_{\mathrm{tot}}$ eventually (for moderately small p_t^{min}).

Eikonal model basics



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$\sigma^{\rm inc} > \sigma_{\rm tot}$ eventually (for moderately small $p_t^{\rm min}$).

Interpretation: $\sigma^{\rm inc}$ counts *all* partonic scatters that happen during a single *pp* collision $\Rightarrow$ more than a single interaction.

$$\sigma^{\rm inc} = \bar{n}\sigma_{\rm inel}\,.$$

Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number m of additional scatters,

$$P_m(\vec{b},s) = \frac{\bar{n}(\vec{b},s)^m}{m!}\mathrm{e}^{-\bar{n}(\vec{b},s)} \ .$$

Then we get σ_{inel}:

$$\sigma_{\mathrm{inel}} = \int \mathrm{d}^2\vec{b} \sum_{m=1}^{\infty} P_m(\vec{b},s) = \int \mathrm{d}^2\vec{b} \left(1 - \mathrm{e}^{-\bar{n}(\vec{b},s)}\right) \ .$$

Eikonal model basics

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Cf. σ_{inel} from scattering theory in eikonal approx. with scattering amplitude $a(\vec{b},s) = \frac{1}{2i}(\mathrm{e}^{-\chi(\vec{b},s)} - 1)$

$$\sigma_{\mathrm{inel}} = \int \mathrm{d}^2\vec{b} \left(1 - \mathrm{e}^{-2\chi(\vec{b},s)}\right) \qquad \Rightarrow \qquad \chi(\vec{b},s) = \tfrac{1}{2}\bar{n}(\vec{b},s) \ .$$

$\chi(\vec{b},s)$ is called *eikonal* function.

Eikonal model basics

Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$\bar{n}(\vec{b},s) = L_{\text{partons}}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

$$= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

$$\times D_{i/A}(x_1,p_t^2,|\vec{b}'|) D_{j/B}(x_2,p_t^2,|\vec{b}-\vec{b}'|)$$

Eikonal model basics

Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$\begin{aligned}
\bar{n}(\vec{b},s) &= L_{\text{partons}}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&\quad \times D_{i/A}(x_1,p_t^2,|\vec{b}'|) D_{j/B}(x_2,p_t^2,|\vec{b}-\vec{b}'|) \\
&= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&\quad \times f_{i/A}(x_1,p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2,p_t^2) G_B(|\vec{b}-\vec{b}'|) \\
&= A(\vec{b}) \sigma^{\text{inc}}(s;p_t^{\text{min}}) \,.
\end{aligned}$$

Eikonal model basics

Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$\begin{aligned}
\bar{n}(\vec{b},s) &= L_{\text{partons}}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
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&= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&\qquad \times f_{i/A}(x_1,p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2,p_t^2) G_B(|\vec{b}-\vec{b}'|) \\
&= A(\vec{b}) \sigma^{\text{inc}}(s;p_t^{\text{min}}) \ .
\end{aligned}$$

$$\Rightarrow \quad \chi(\vec{b},s) = \tfrac{1}{2}\bar{n}(\vec{b},s) = \tfrac{1}{2}A(\vec{b})\sigma^{\text{inc}}(s;p_t^{\text{min}}) \ .$$

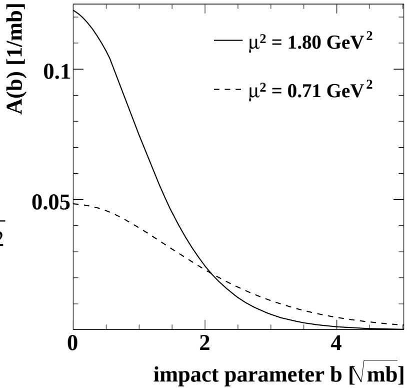
Overlap function

$$A(b) = \int \mathrm{d}^2\vec{b}' G_A(|\vec{b}'|) G_B(|\vec{b} - \vec{b}'|)$$

$G(\vec{b})$ from electromagnetic FF:

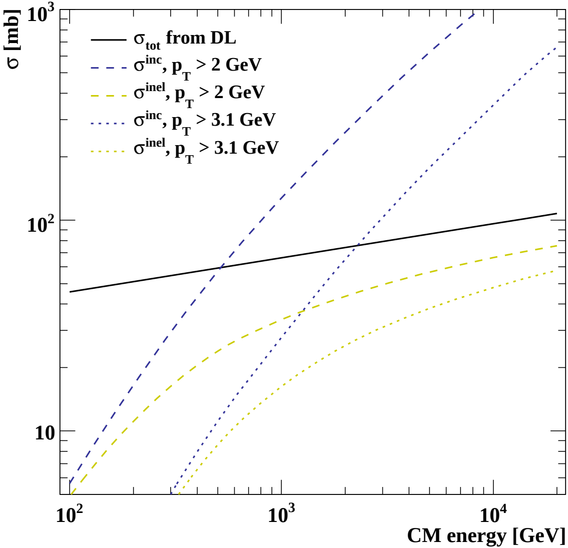
$$G_p(\vec{b}) = G_{\bar{p}}(\vec{b}) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2} \frac{\mathrm{e}^{i\vec{k}\cdot\vec{b}}}{(1 + \vec{k}^2/\mu^2)^2}$$

But μ^2 *not fixed* to the *electromagnetic* 0.71 GeV^2.
Free for colour charges.

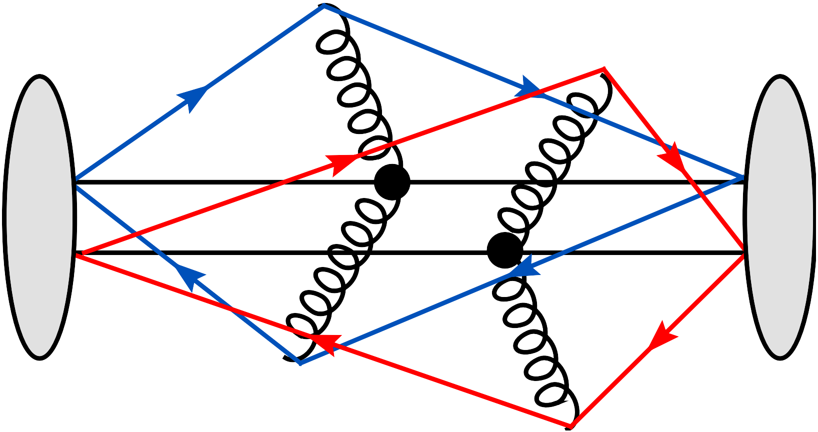


$\Rightarrow$ Two main parameters: $\mu^2, p_t^{\min}$.

Unitarized cross sections

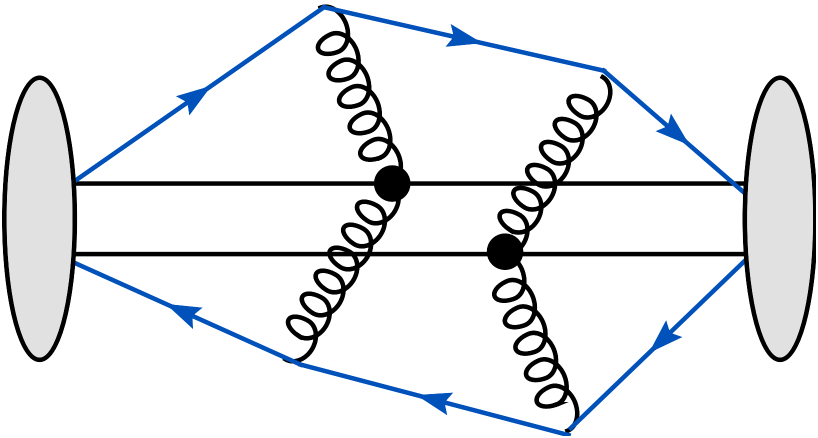


Colour reconnection at hadron colliders



- Colour preconfinement
- Shorten colour string/lower mass clusters.

Colour reconnection at hadron colliders



- Colour preconfinement
- Shorten colour string/lower mass clusters.

Monte Carlo

training studentships

3-6 month fully funded studentships for current PhD students at one of the MCnet nodes. An excellent opportunity to really understand the Monte Carlos you use!

Application rounds every 3 months.

MCnet

funded by: MARIE CURIE ACTIONS

for details go to:
www.montecarlonet.org