

# Jet substructure

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Background material with help from Zoltan Nagy and Michael Spannowsky

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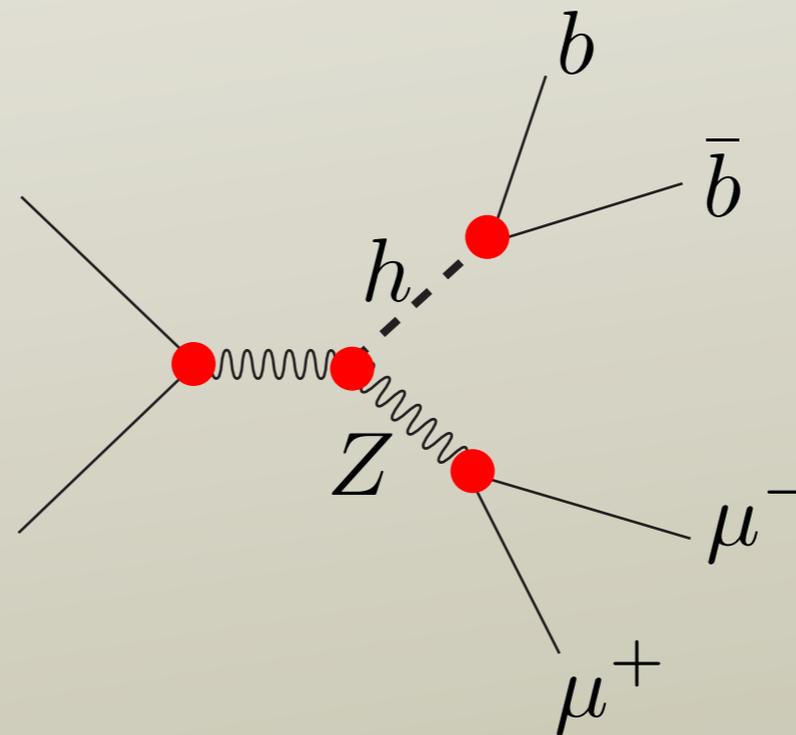
- There are several methods that use jet substructure to find signals of interest in LHC data.
- This talk is about some of the techniques that these methods use.
- It is also about the structure in nature that these methods try to take advantage of.

# Signals and backgrounds at the LHC

# Signal events

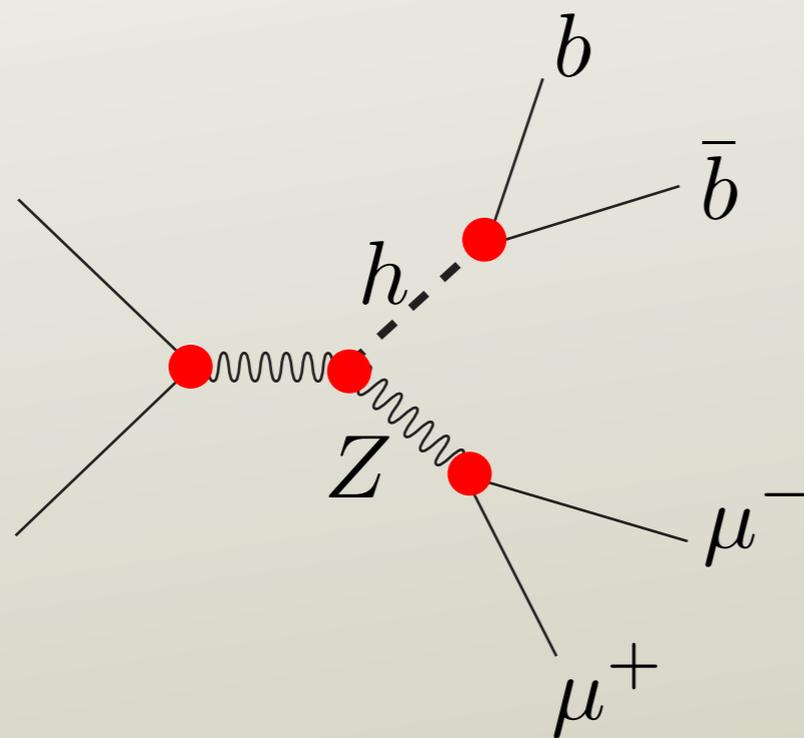
- I take a signal event to be one in which one or more new heavy particles is created and decays.
- Sometimes there is a chain of decays.
- Also, top quarks could be part of both signal and background events for some signals.

A Higgs boson  
signal event

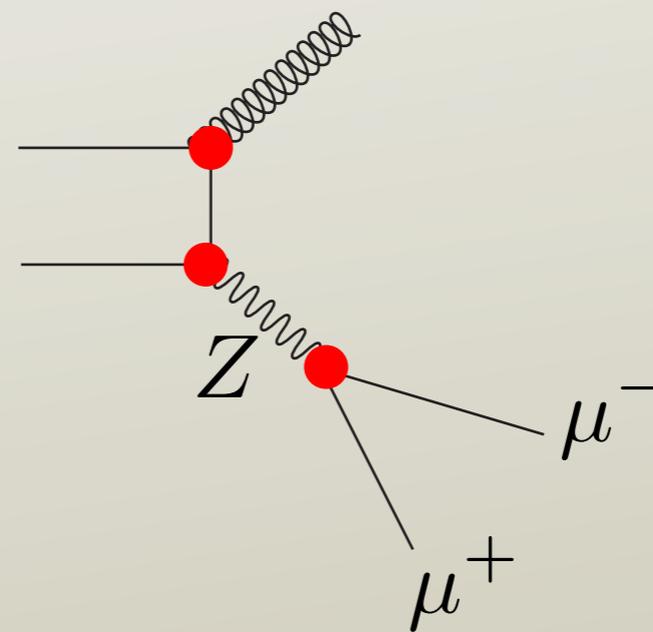


# Background events

- There are always background events that can look like the signal events.



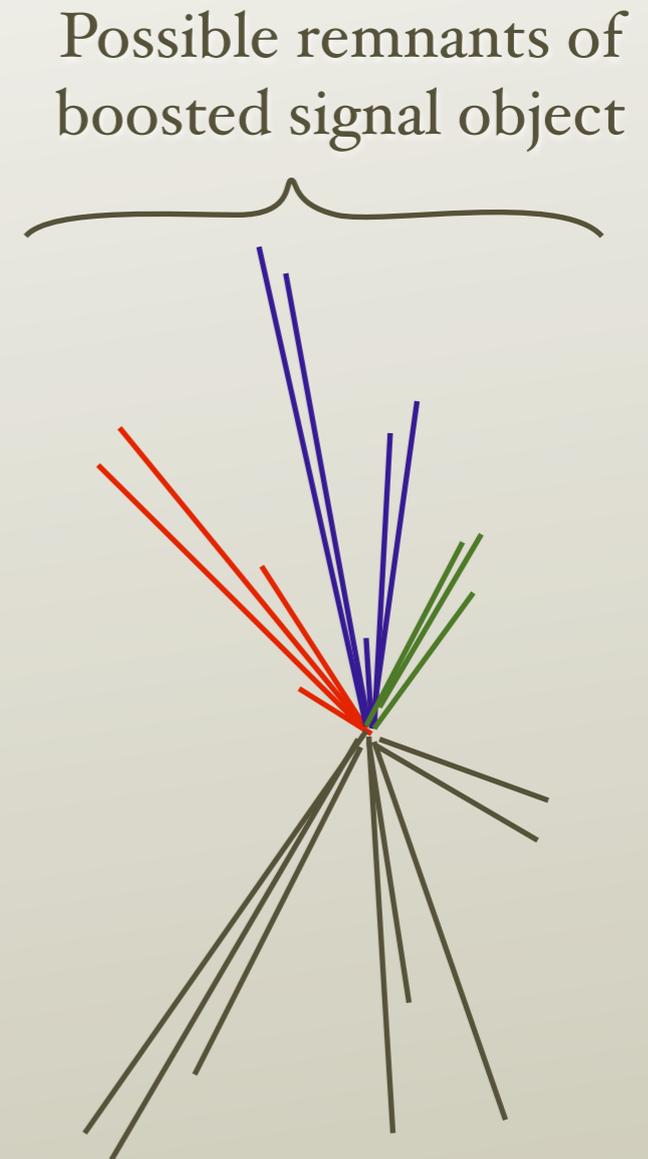
We want to find this.



In a background of this.

# Signal and background with jets

- Both signal and background events have jets.
- There are several techniques for separating signal from background using the structure of the jets.
- This is especially effective for highly boosted heavy objects.

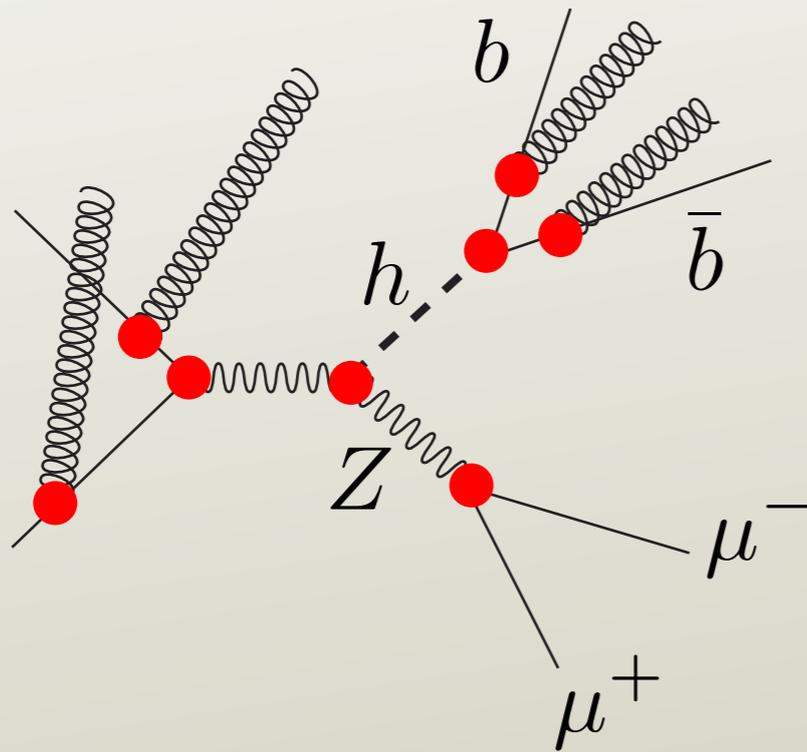


# What gauge field theory has to say

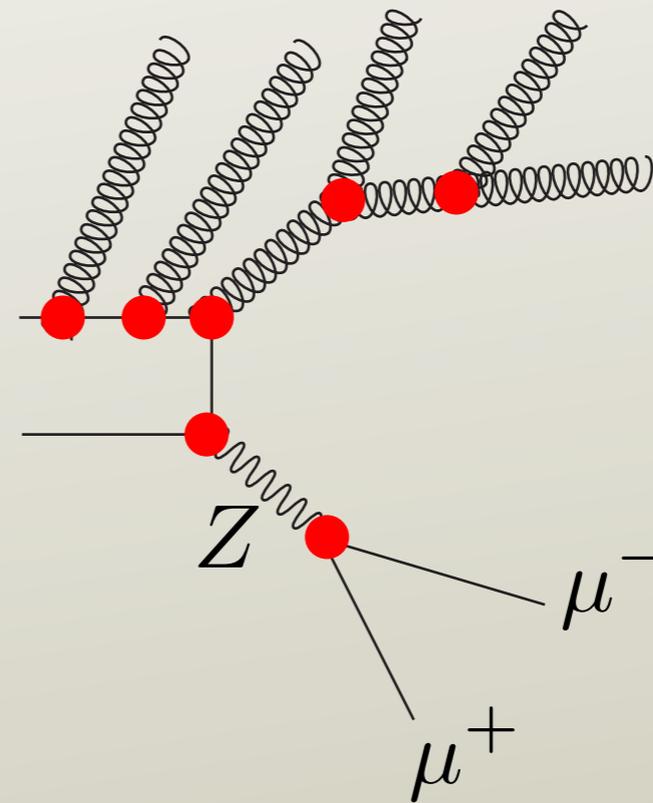
- One can use field theory to understand the characteristics of background events and the characteristics of signal events (with heavy particle decays).
- Schemes for selecting signal events depend on using the similarities and differences in signal and background events.
- I will try to outline the characteristics that can be used.

# Initial state radiation

- One immediate similarity is that both signal and background events have initial state radiation.



A Higgs signal event



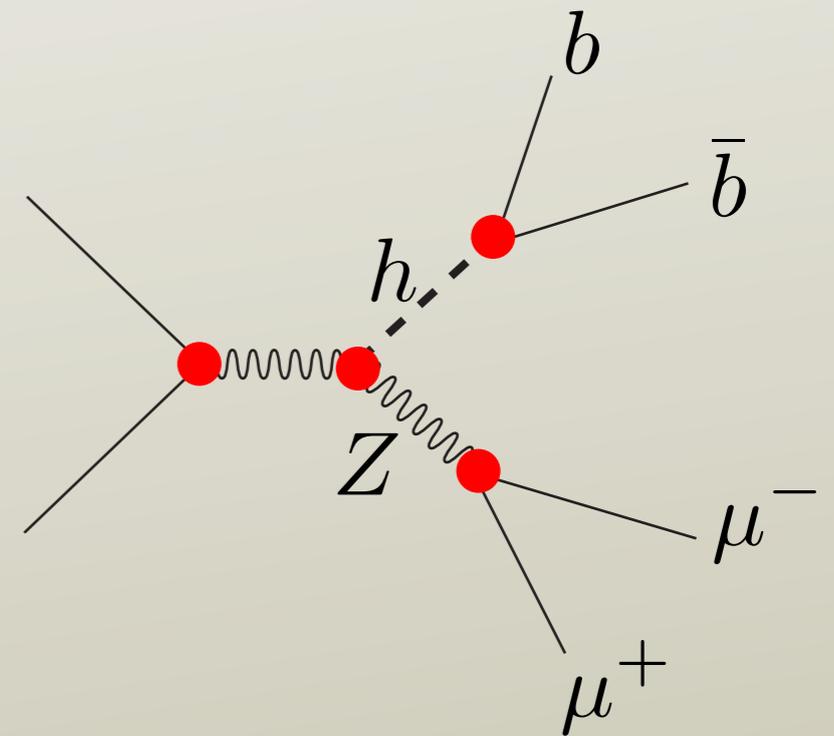
A similar background event

- This makes the signal/background separation difficult.
- More on this later.

# Start with event selection

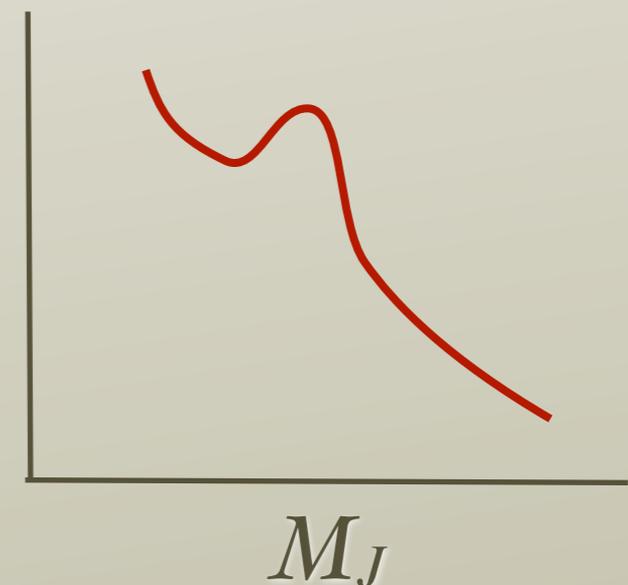
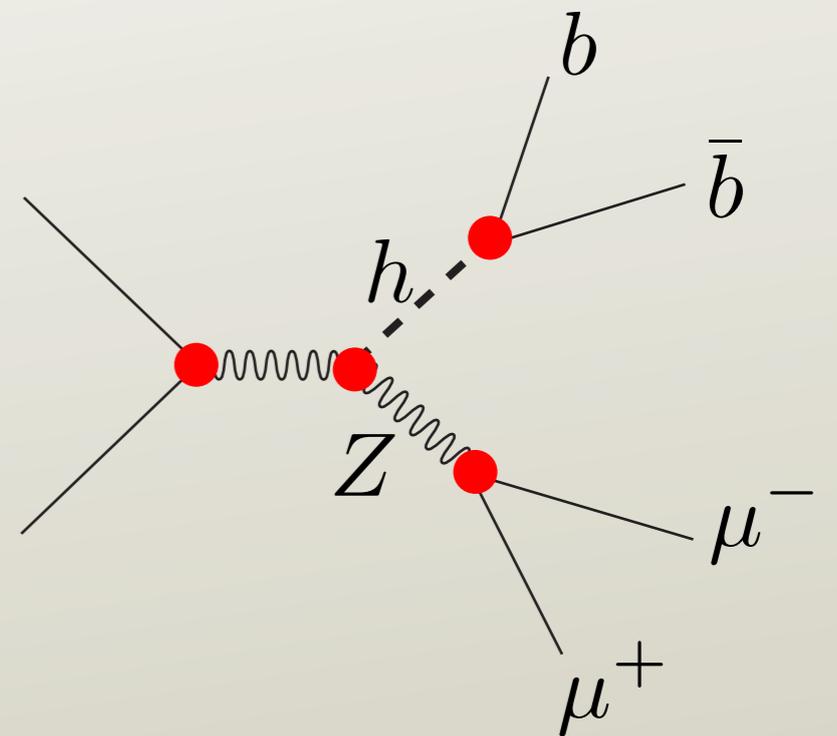
- Electron or muon pair near the  $Z$  mass.
- Large  $P_T$  of the lepton pair and of recoiling jet ( $>200$  GeV).
- Large  $P_T$  implies that the possible Higgs decay products are easier to isolate: they are part of a (rather fat) jet.

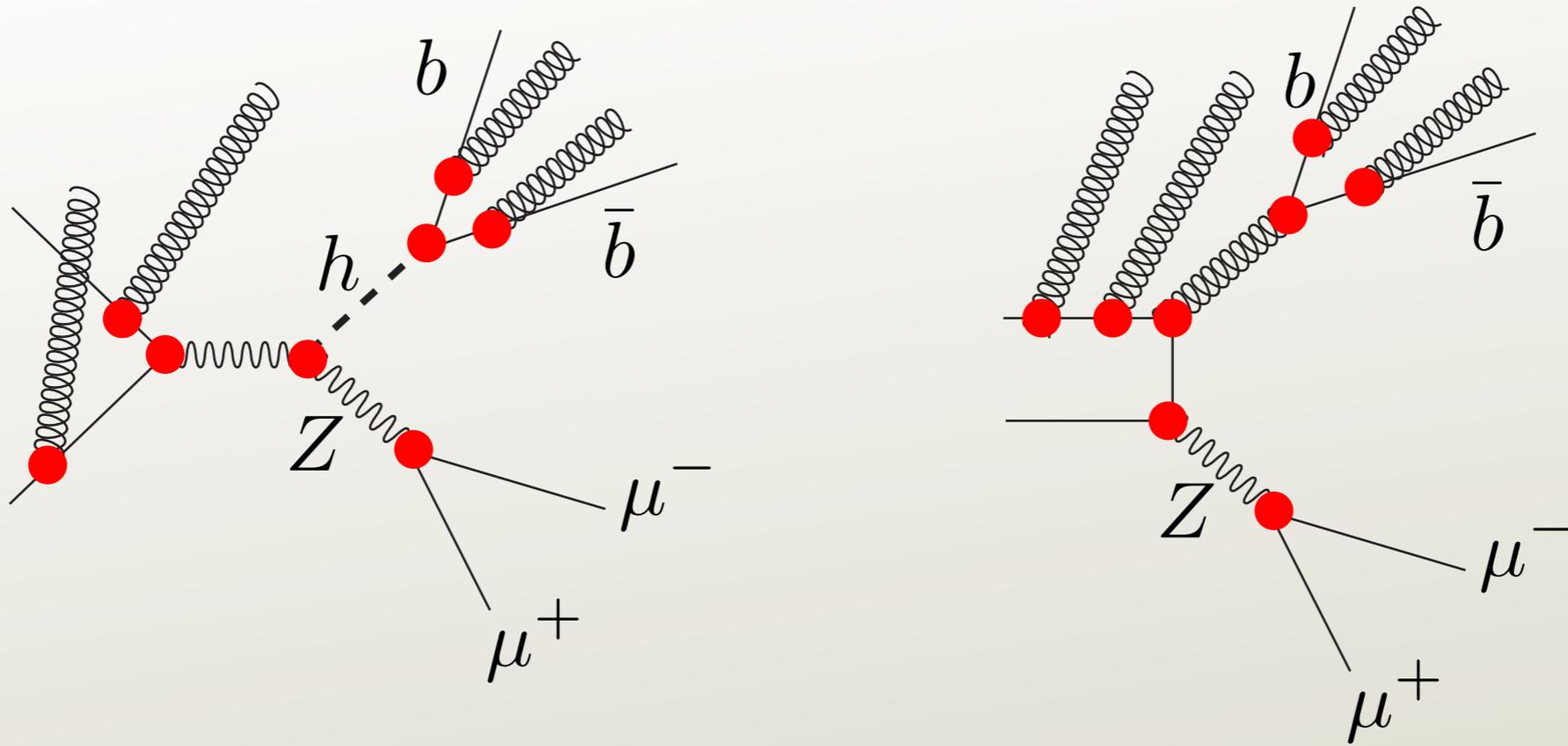
Butterworth, Davison,  
Rubin, and Salam (2008)



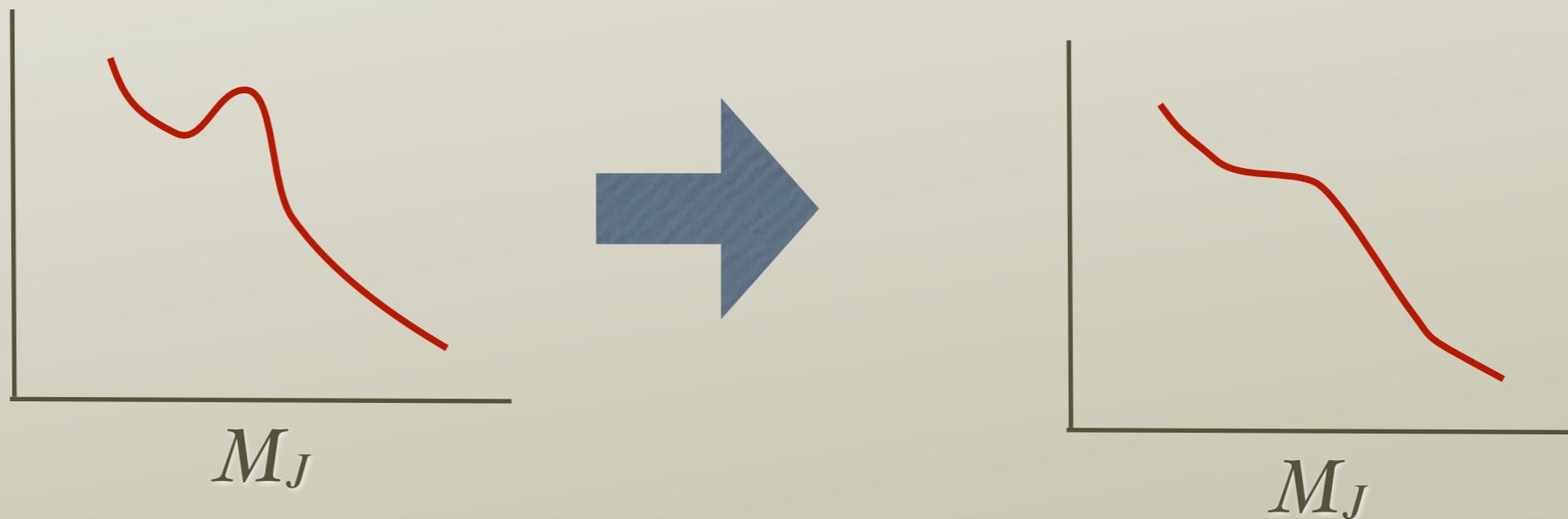
# Define the fat jet

- Look for a high  $P_T$  jet using the Cambridge-Aachen (angle) algorithm with  $R=1.2$ .
- We might hope that the distribution of the mass of the fat jet shows a bump at the Higgs mass.



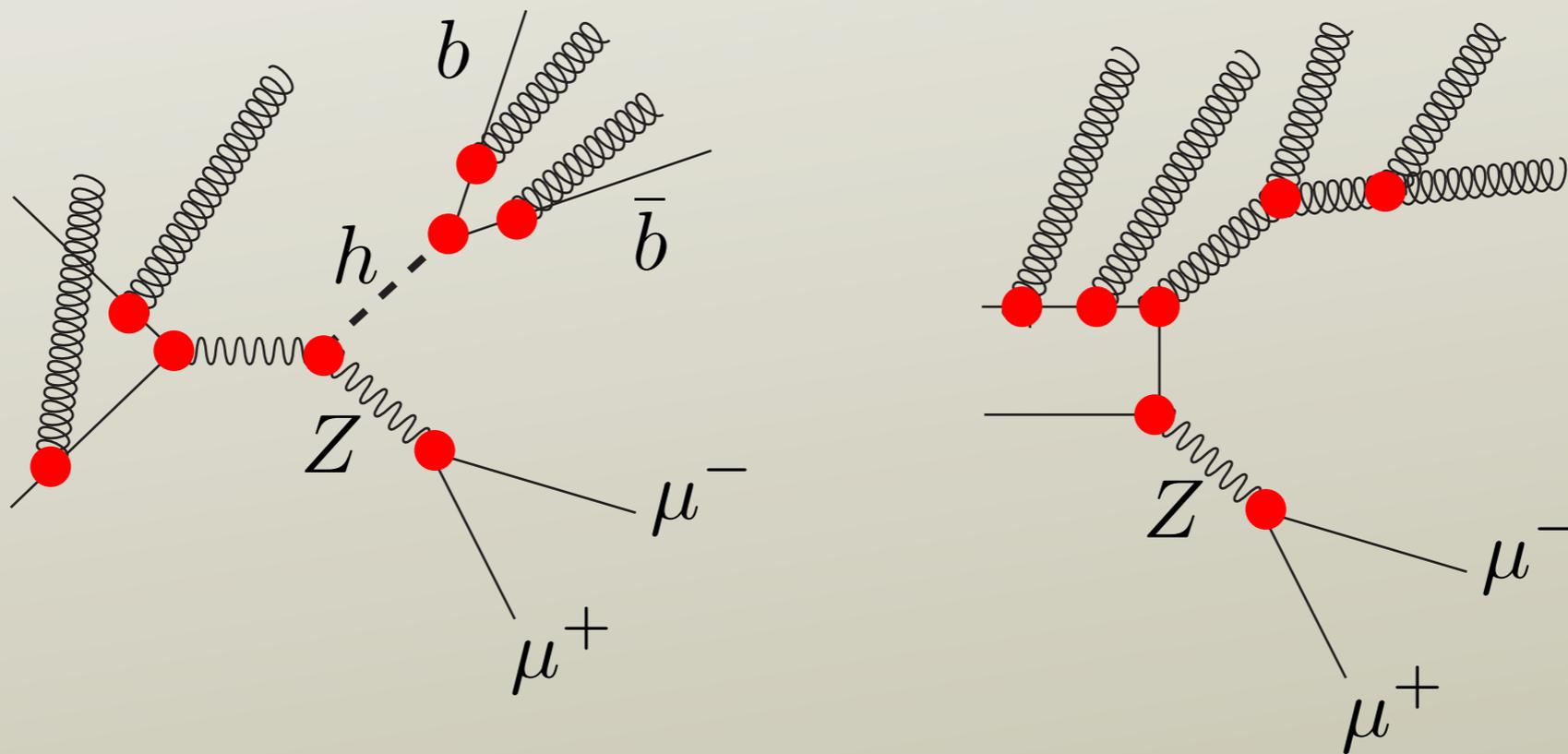


- Since QCD is operating, the mass bump gets smeared out.



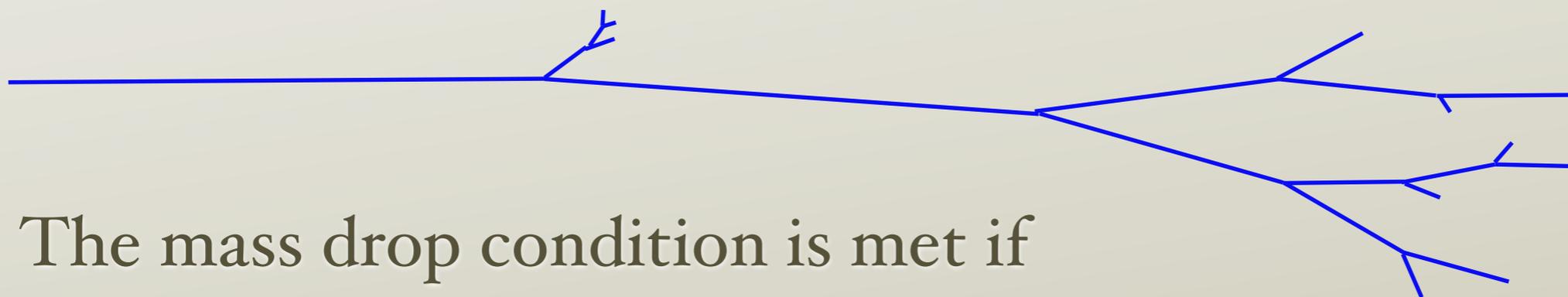
# Subjet analysis

- We would like to take apart the fat jet in order to get rid of the contaminating initial state radiation.



# Jet mass drop and filtering

- Step I: mass drop.
  - Examine the C-A splitting tree, starting at the trunk.



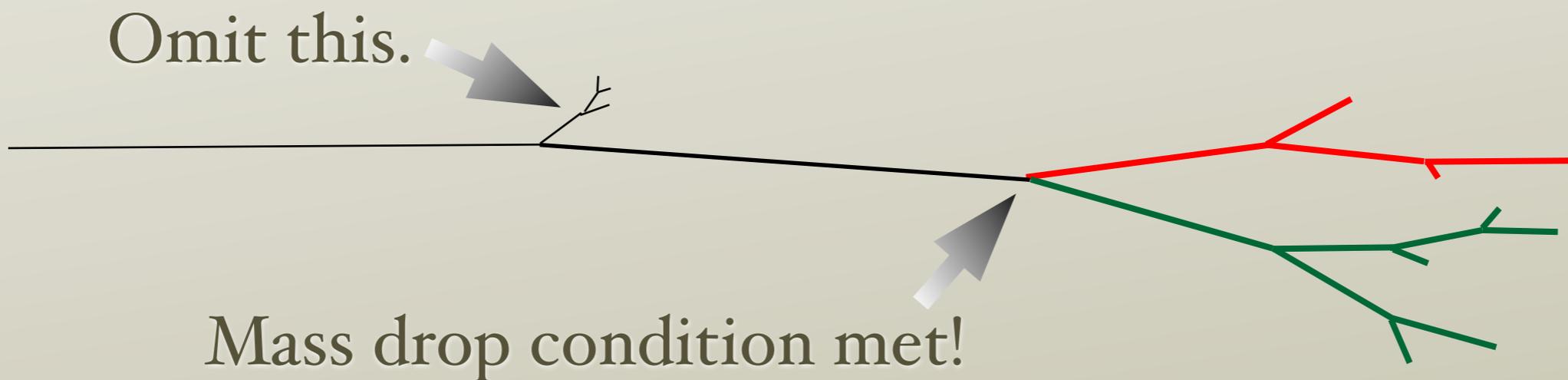
- The mass drop condition is met if

$$\max(M_i, M_j) < 0.67 M_{\{i,j\}}$$

$$\min(p_{T,i}^2, p_{T,j}^2) [(y_i - y_j)^2 + (\phi_i - \phi_j)^2] > 0.09 M_{\{i,j\}}^2$$

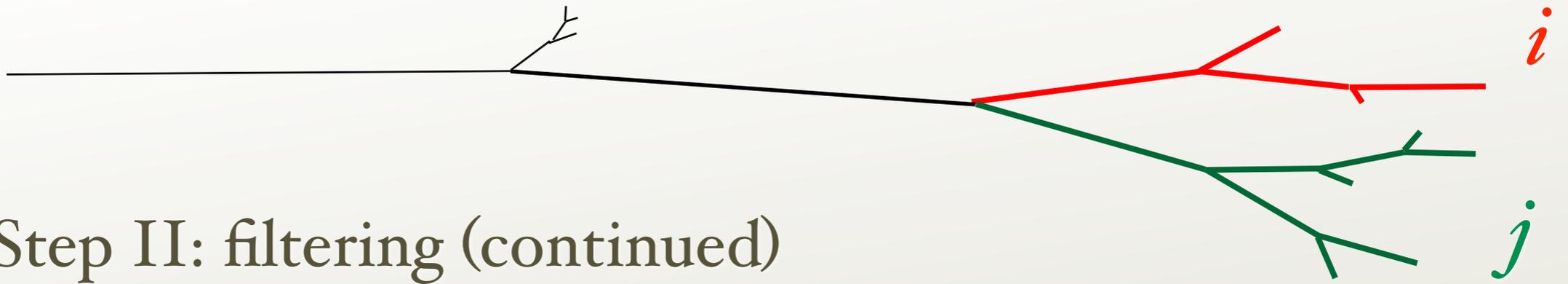


- If mass drop condition isn't met, omit smaller  $p_T$  daughter and keep looking.
- If it is never met, remove the event from your sample.





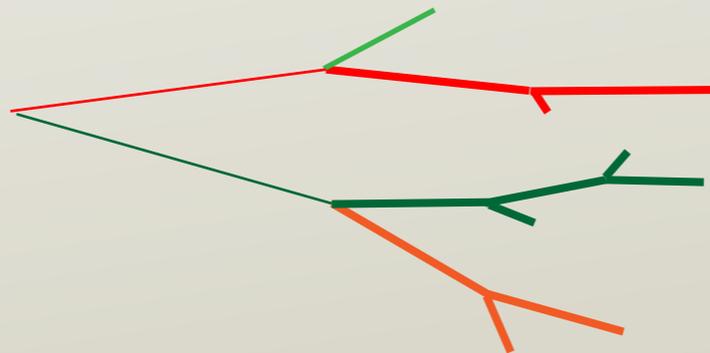
- Step II: filtering the prospective  $b$ -jets,  $i$  and  $j$ .
  - Are both prospective  $b$ -jets tagged as containing  $b$ -quarks?
  - If  $i$  and  $j$  are not  $b$ -tagged, reject the event.



- Step II: filtering (continued)

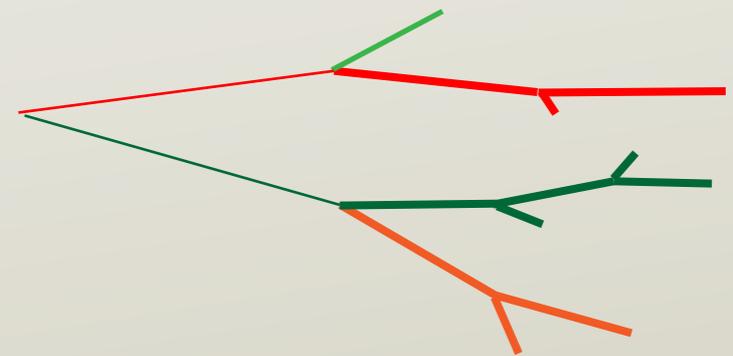
- Apply the C-A algorithm with to protojets  $i$  and  $j$  with

$$R = \min \left( \frac{1}{2} [(y_i - y_j)^2 + (\phi_i - \phi_j)^2]^{1/2}, 0.3 \right)$$

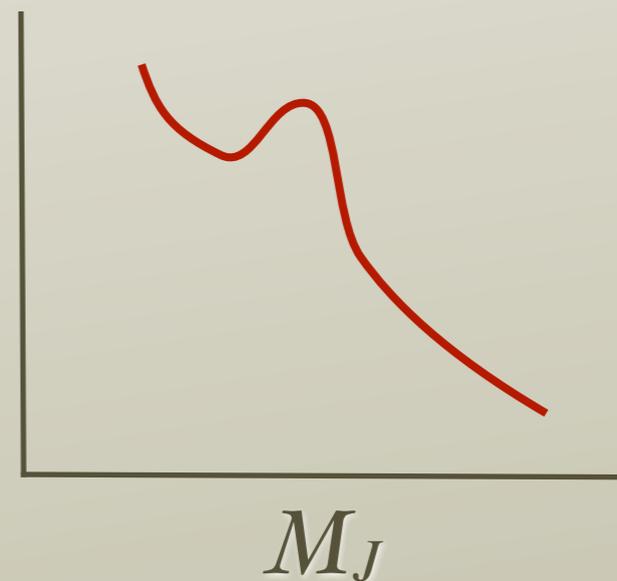
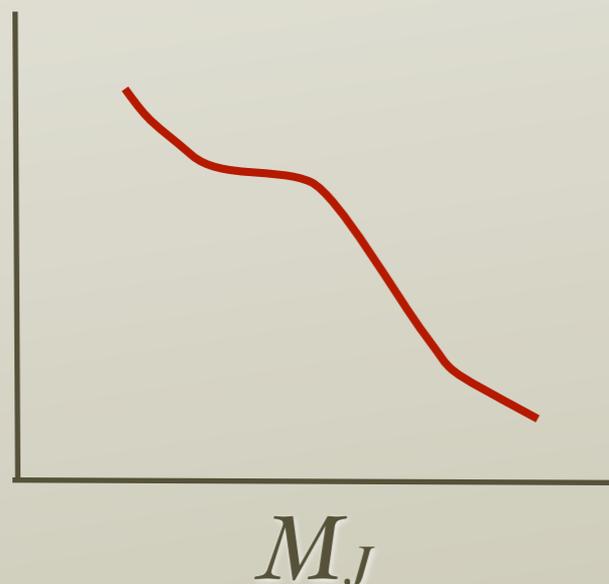


- Are the two highest  $p_T$  subjects thus found tagged as containing b-quarks?
- If not, throw out the event.

- Step II: filtering (continued some more)
  - Throw out all but the three highest  $p_T$  subjets thus found.
  - What remains is the filtered jet.



- Measure the mass of this filtered jet.



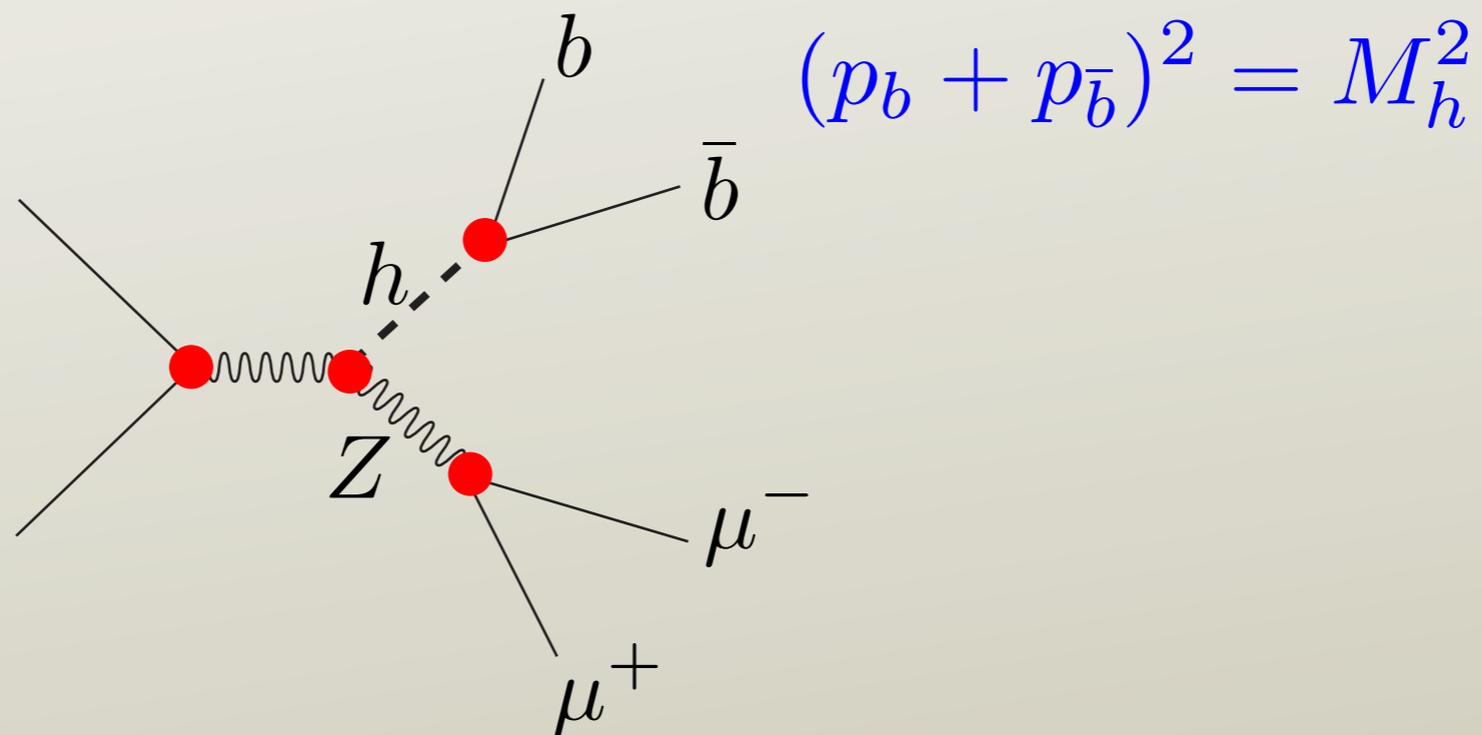
# Other methods

- There are other algorithms for subject analysis.
- Some are looking for something specific, as in the BDRS method above and the Johns Hopkins and HEP top quark taggers.
- Others are more generic, as in pruning and trimming.

Features of gauge field theory  
that we are looking for

# Mass bumps

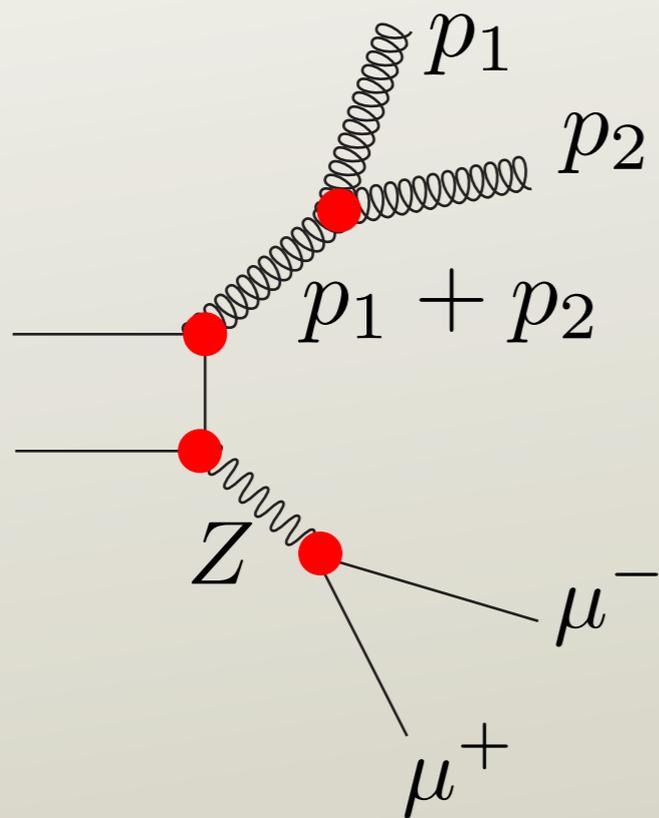
- One key feature of signal events is that the decay products of a heavy particle have a fixed mass.



- The theoretical uncertainty of this is  $M_h \Gamma_h$ .
- The practical uncertainty is much larger.

# Mass in QCD splittings

- QCD splittings have a very different distribution as a function mass.



- I will use a scaled virtuality as a “hardness variable”:

$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$

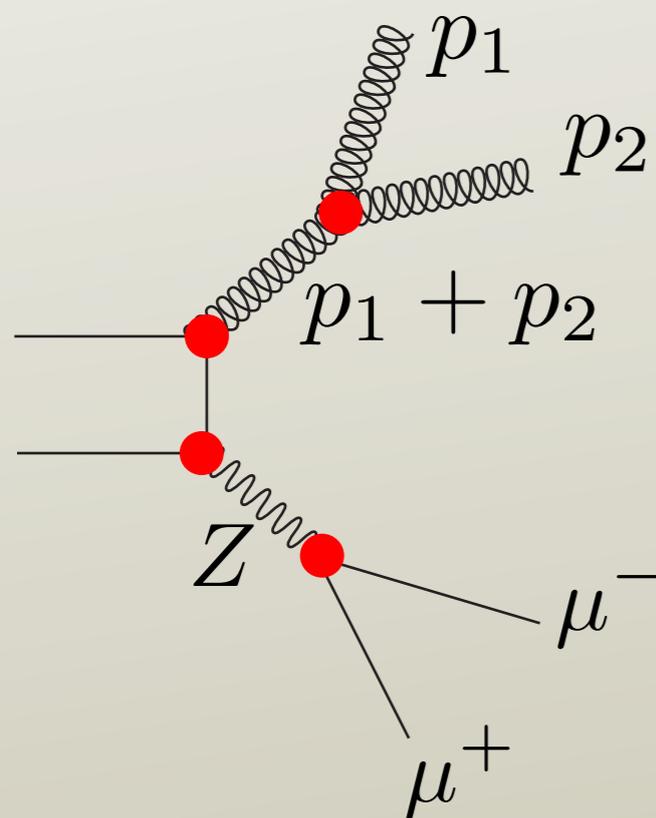
- $Q_0$  is the momentum of the outgoing partons in the hard process.

# QCD tree graph results

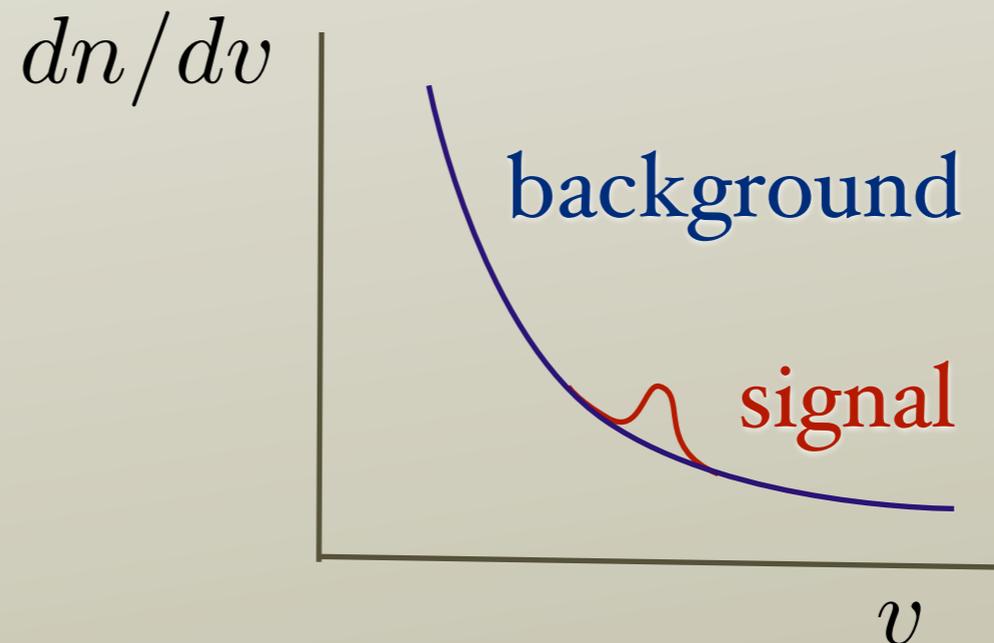
$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$

- Differential probability is

$$dn \approx \frac{dv}{v} C \alpha_s \log(\tilde{v}/v)$$

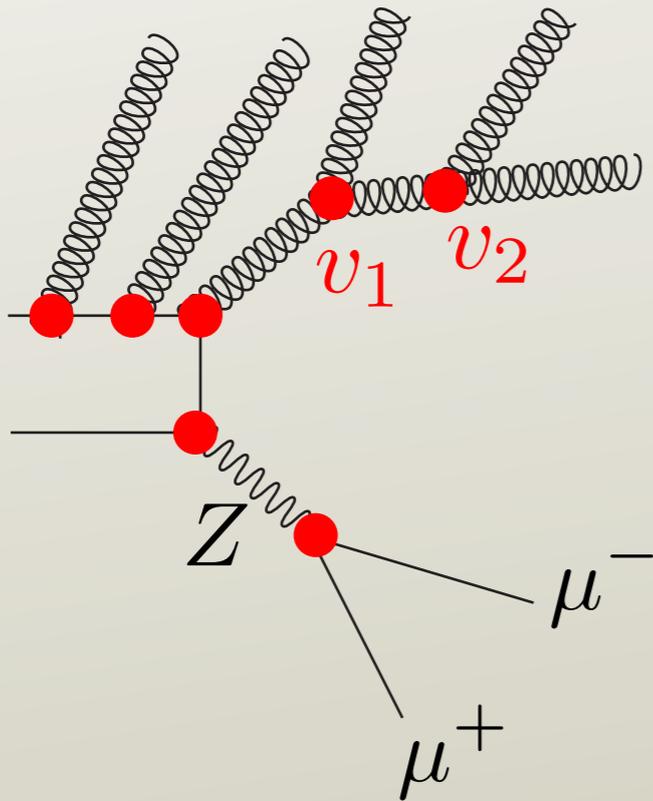


- The log (and  $\tilde{v}$ ) comes from  $\int dz$ .



# Nested splittings

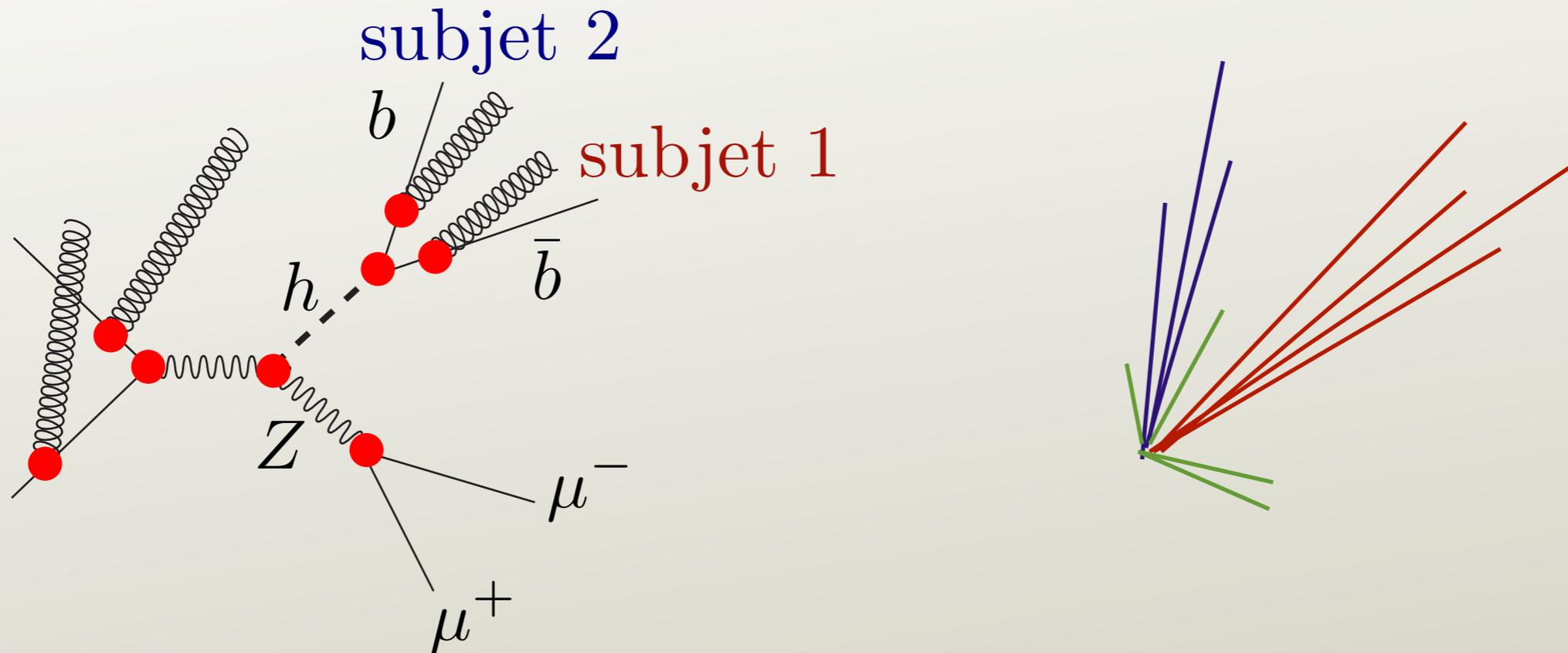
- The same structure can be (approximately) iterated.



- $v_2 < v_1$ .
- Set  $v_2 = 0$  in splitting 1.

$$\begin{aligned}
 dn &\approx \frac{dv_1}{v_1} C_1 \alpha_s \log(\tilde{v}_1/v_1) \\
 &\times \frac{dv_2}{v_2} C_2 \alpha_s \log(\tilde{v}_2/v_2) \\
 &\times \theta(v_2 < v_1)
 \end{aligned}$$

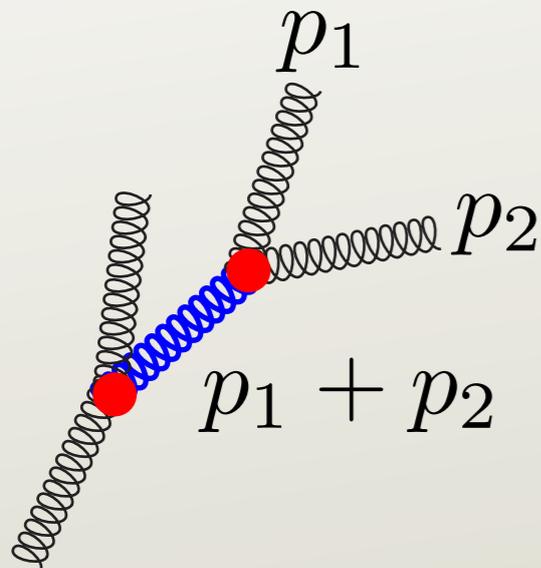
# An application of this



- A Higgs boson will likely decay to two low mass subjets, plus soft gluons.
- Try to find the low mass subjets.
- A “mass drop” condition can be part of this.  
(Butterworth, Davison, Rubin & Salam.)
- Measure the mass of subjet pair.

# Sudakov factor

- A better approximation for  $dn$  includes a Sudakov factor.



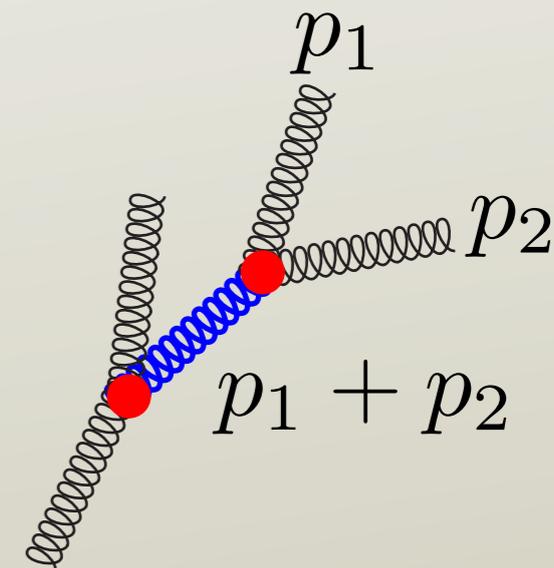
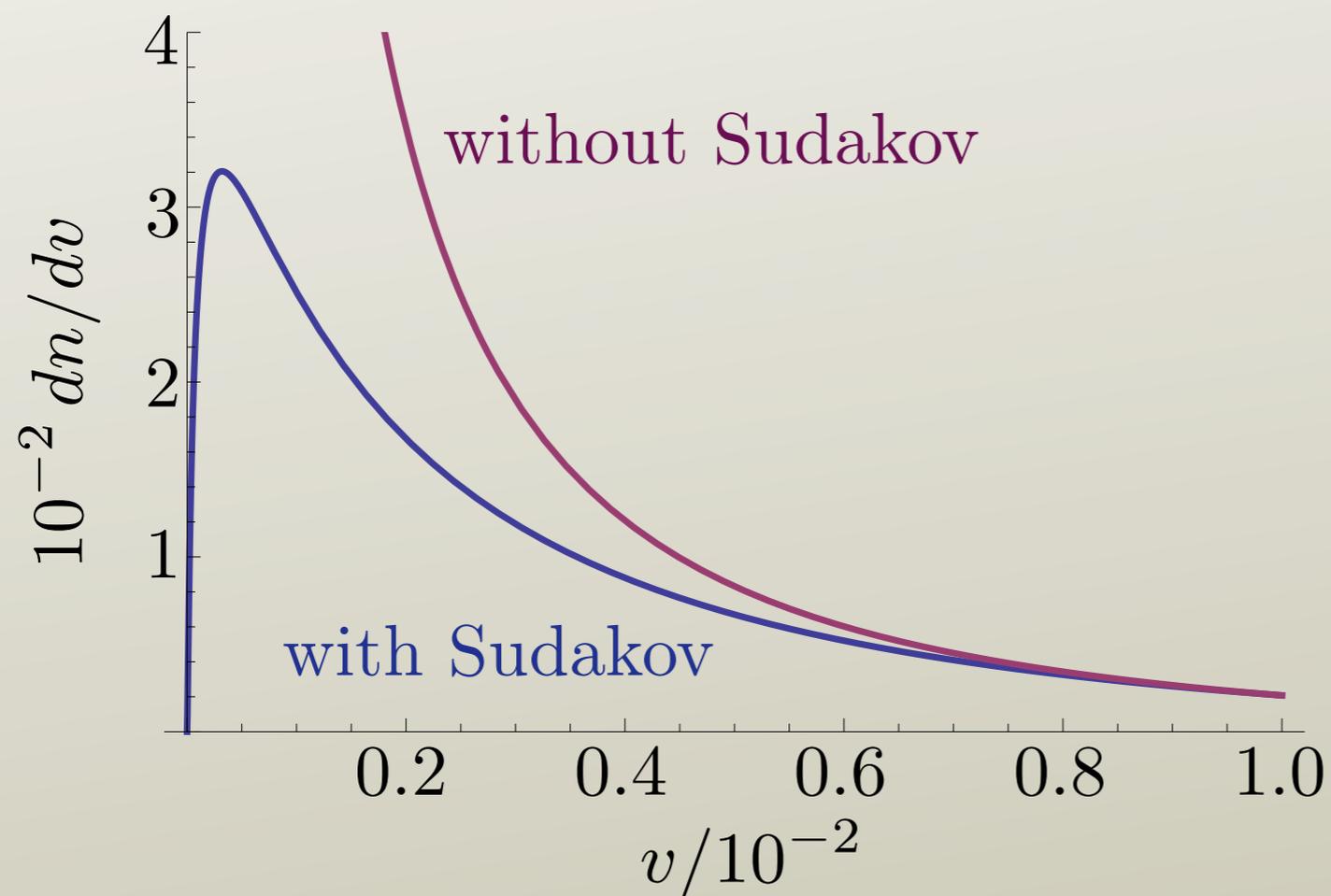
$$dn \approx \frac{dv}{v} C \alpha_s \log(\tilde{v}/v)$$

$$\exp \left( - \int_v^{v_1} \frac{d\bar{v}}{\bar{v}} C \alpha_s \log(\tilde{v}/\bar{v}) \right)$$

- We think of a renormalization group approach with a running resolution scale.
- The Sudakov factor accounts for virtual graphs and unresolved real parton emissions.

# Example of effect of Sudakov

$$dn \approx \frac{dv}{v} C \alpha_s \log(v/\tilde{v}) \exp\left(-\int_v^{v_1} \frac{d\bar{v}}{\bar{v}} C \alpha_s \log(\bar{v}/\tilde{v})\right)$$



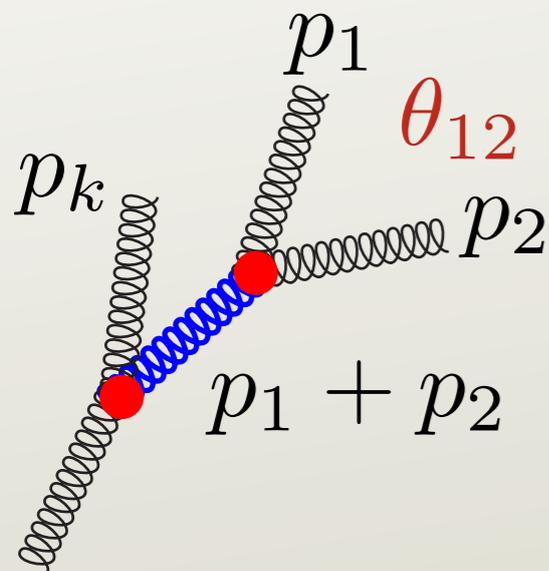
Example with

$$C\alpha_s = 0.3$$

$$\tilde{v} = 0.02$$

$$v_1 = 0.01$$

# Distribution in angles



- For a given  $v$ , what is the distribution in the angle  $\theta_{12}$  between  $\vec{p}_1$  and  $\vec{p}_2$ ?

- Kinematically,

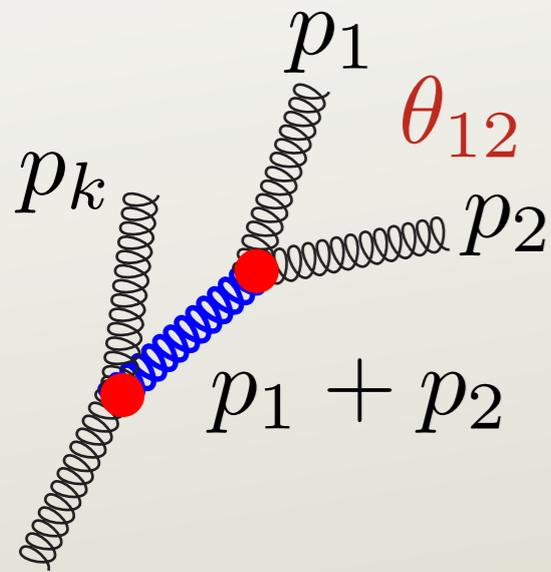
$$\theta_{12}^2 > v \frac{4Q_0^2}{(p_1 + p_2) \cdot Q_0}$$

- For larger, but not too large,  $\theta_{12}$ ,

$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$

$$dn \propto \frac{d\theta_{12}^2}{\theta_{12}^2}$$

# Large angles



- Large  $\theta_{12}$  corresponds to  $|\vec{p}_1| \ll |\vec{p}_2|$ .
- (Or gluon 2 could be the soft one.)
- For emission of soft gluon 1, interference with emission from parton  $k$  is important.
- Parton  $k$  is the color connected partner.

- With interference,

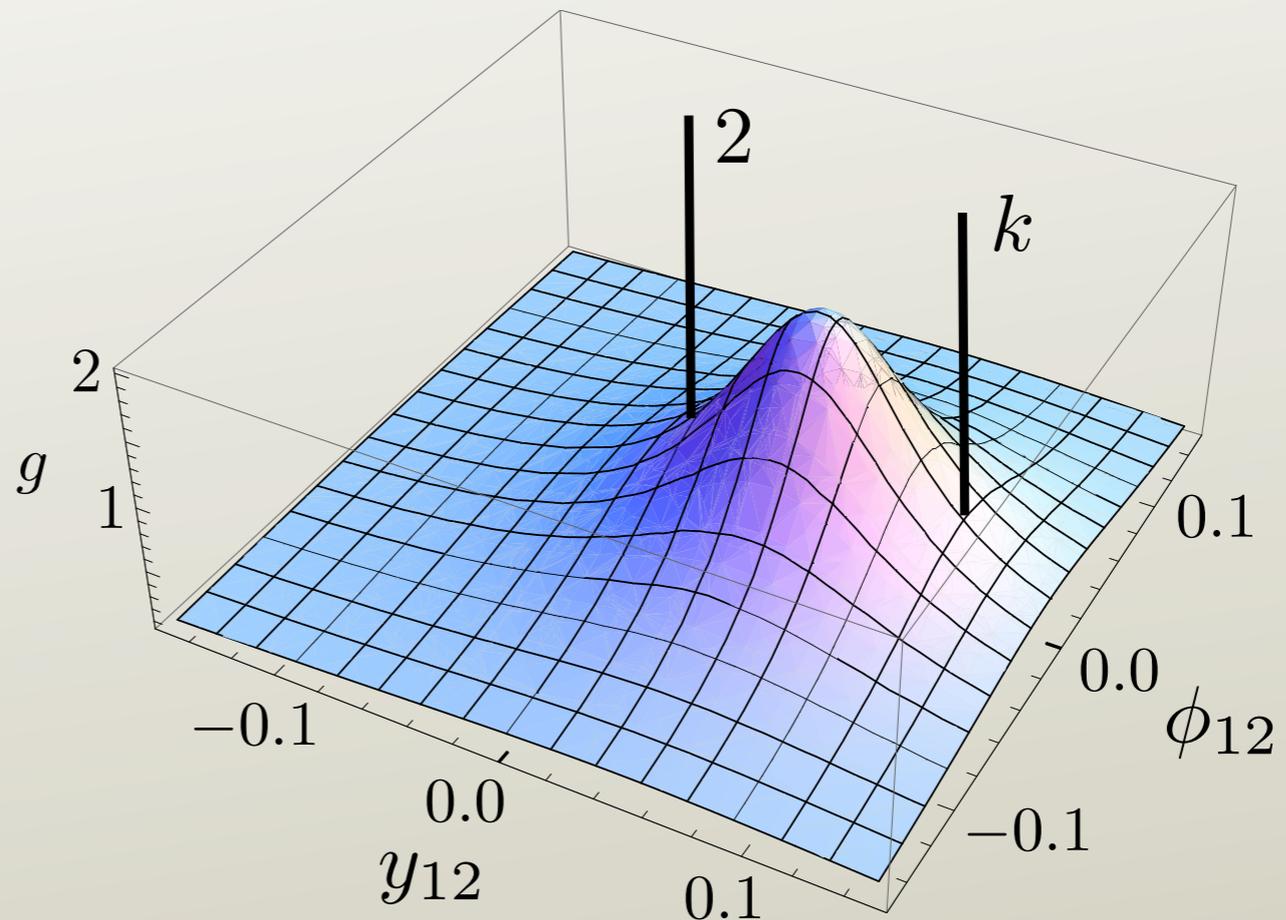
$$dn \propto \frac{d\theta_{12}^2}{\theta_{12}^2} g(\theta_{12}^2)$$

$$g(\theta_{12}^2) = \frac{\theta_{2k}^2}{\theta_{12}^2 + \theta_{1k}^2}$$

# Effect of the angle factor

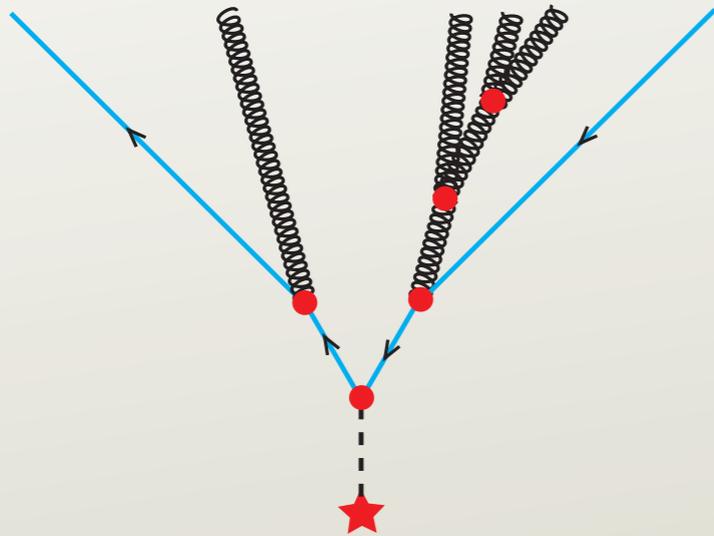
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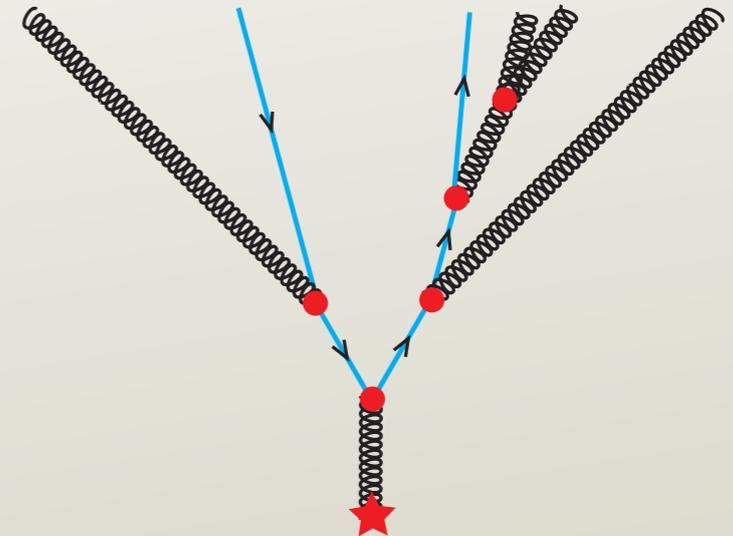
- Emission with  $\theta_{12} \gg \theta_{2k}$  is suppressed.
- Emission between partons 2 and  $k$  is enhanced.

# An application of this



After  $H \rightarrow b + \bar{b}$

Soft gluons between  $b$  and  $\bar{b}$



After  $g \rightarrow b + \bar{b}$

Soft gluons away from  $b$  and  $\bar{b}$

- Cf. “pull” (Gallicchio & Schwartz).

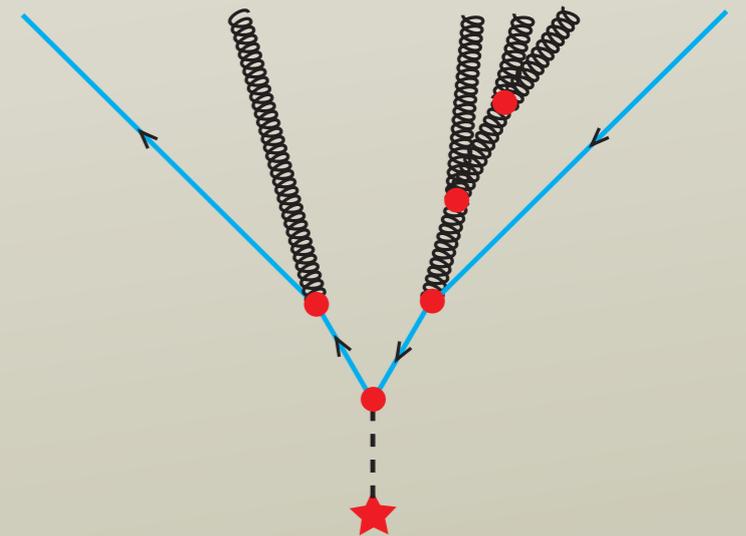
# Gluons want to be soft

- Large angles  $\implies$  small  $z$ :

$$z(1 - z) = \frac{v}{\theta^2} \frac{Q_0^2}{(p_1 + p_2) \cdot Q_0}$$

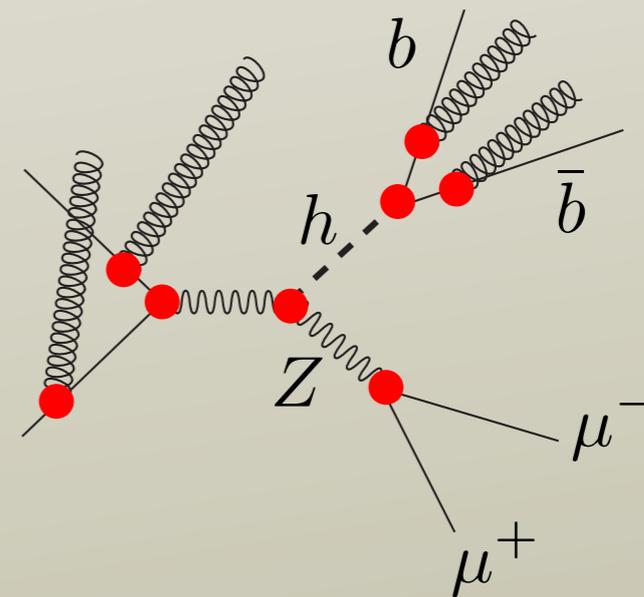
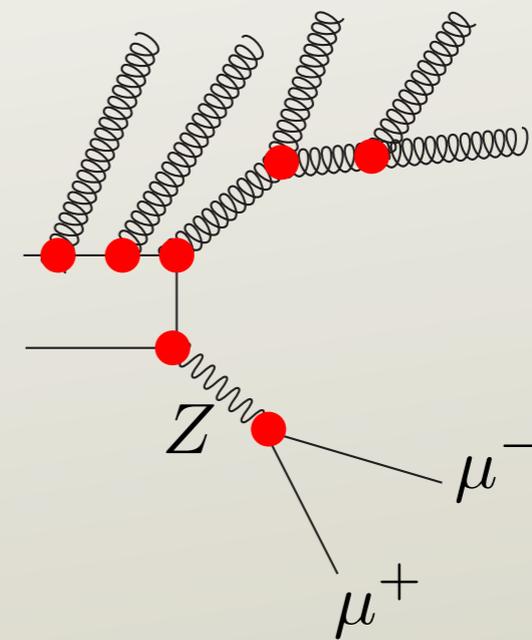
- So we expect lots of soft gluon radiation.

- However, soft gluon radiation is limited by “angular ordering” from  $g(\theta)$ .



# More soft gluons come from initial state radiation

- There is initial state radiation in both signal and background events.
- It can come at central rapidities.
- It is largely rather low transverse momentum.



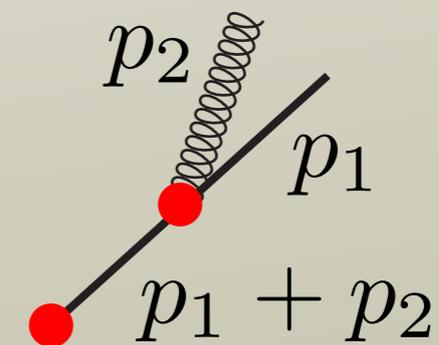
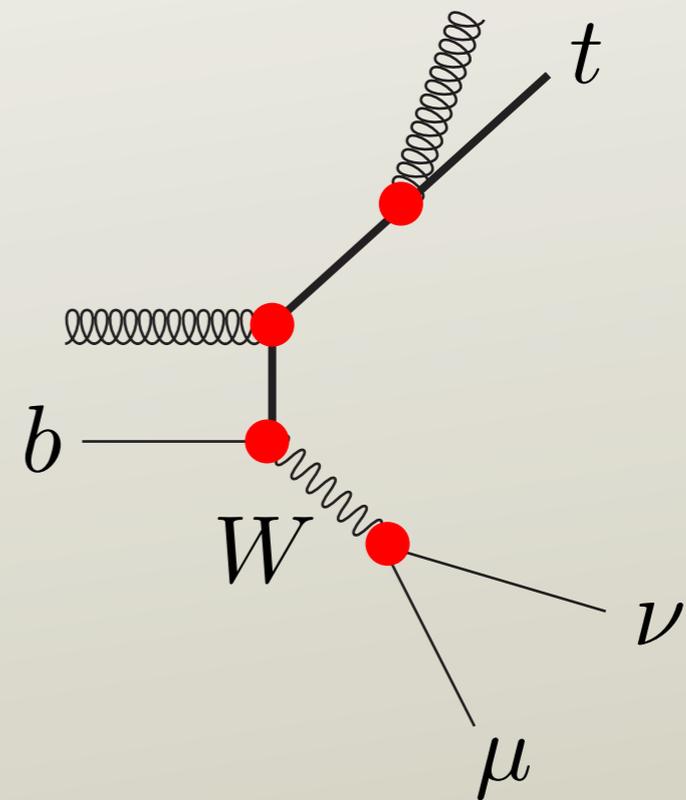
# Heavy particles with color

- A heavy particle with color will radiate gluons.
- There is singularity in  $|\mathcal{M}|^2$ .

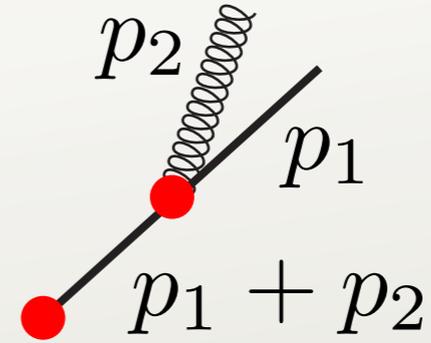
$$dn \propto \frac{dv}{v} \alpha_s \log(\theta_{\min}/\theta_{\max})$$

where the hardness  $v$  is now

$$v = \frac{(p_1 + p_2)^2 - M^2}{(p_1 + p_2) \cdot Q_0}$$



- The collinear singularity is cut off:

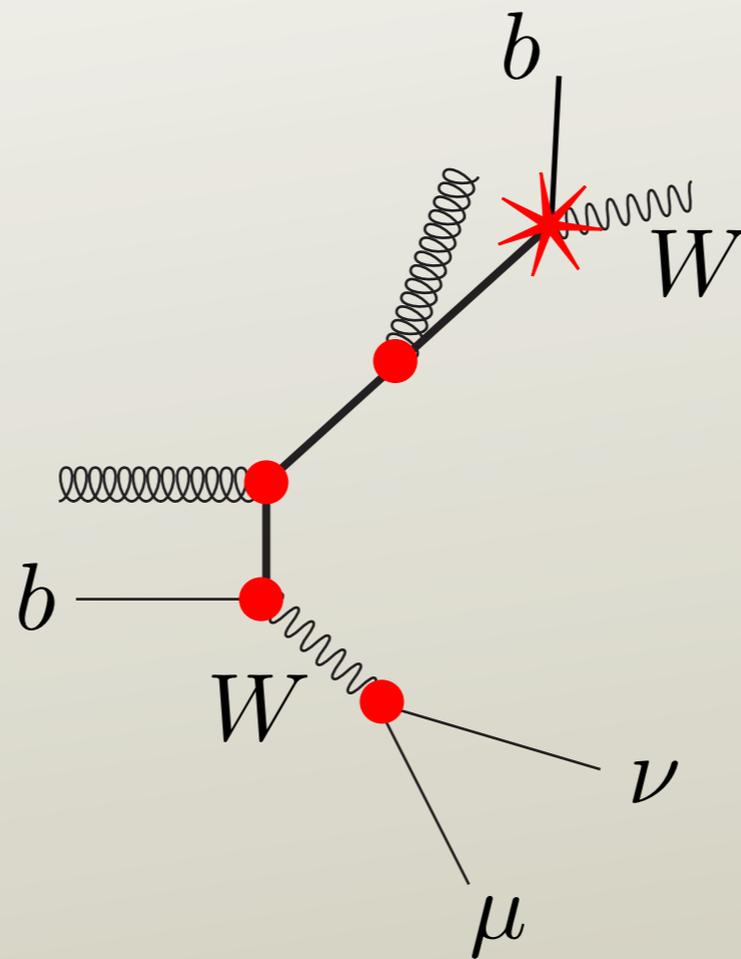


$$\theta_{\min}^2 \sim \min \left( \frac{M^2 Q_0^2}{((p_1 + p_2) \cdot Q_0)^2}, v \frac{4Q_0^2}{(p_1 + p_2) \cdot Q_0} \right)$$

- There are no emissions that take longer than the particle lifetime:

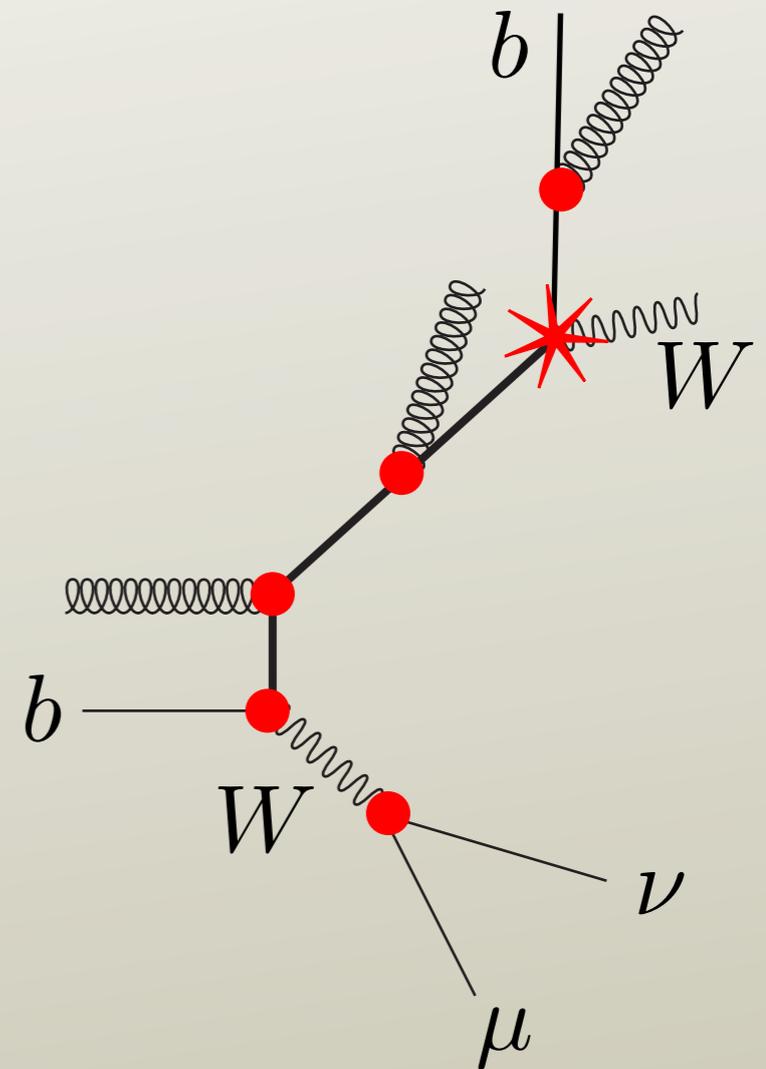
$$v > \frac{M\Gamma}{(p_1 + p_2) \cdot Q_0}$$

# Heavy particles decay



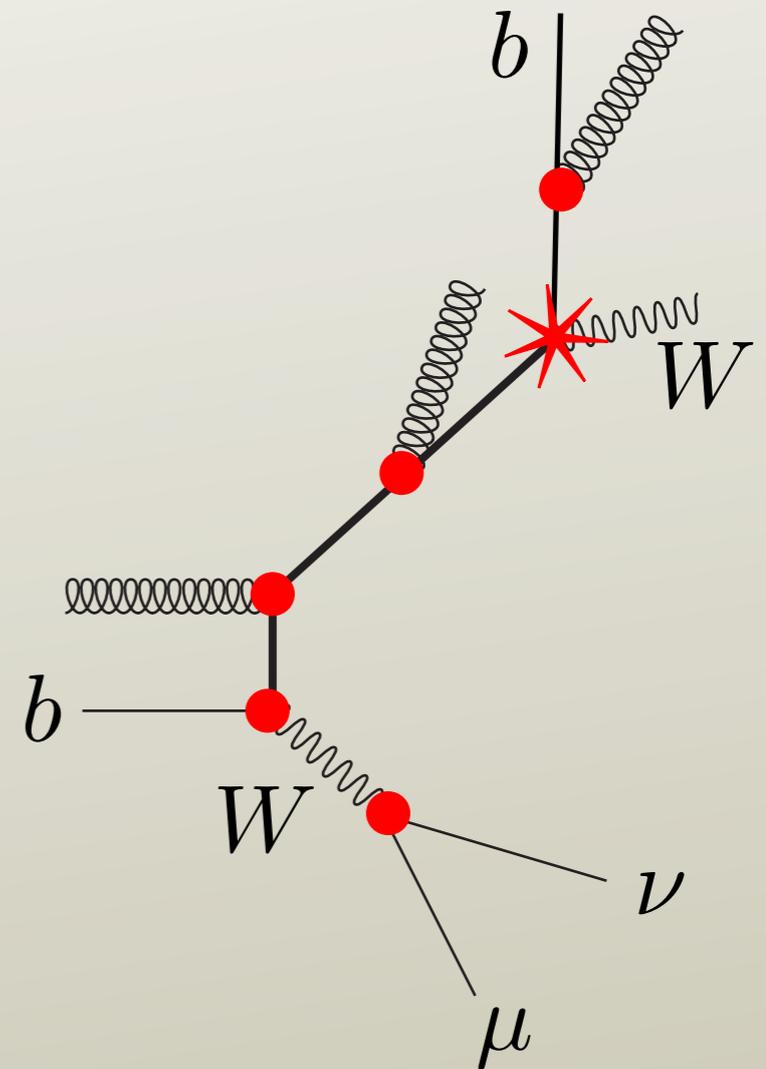
# Daughters radiate

- In  $t \rightarrow b + W$ , a lot of energy is released.
- Color charge is accelerated.
- Gluons are radiated.
- The first radiation is from a color dipole of
  - The final state  $b$ -quark;
  - The “initial state”  $t$ -quark;



# Where does the radiation go?

- The first radiation is from a color dipole of
  - The final state  $b$ -quark;
  - The “initial state”  $t$ -quark;
- Direction of  $g$  is likely collinear with the  $b$ .
- Direction of  $g$  is likely within angle  $M/|\vec{P}_t|$  of the top.
- If the top is highly boosted, this is a narrow cone.





# Daughters radiate

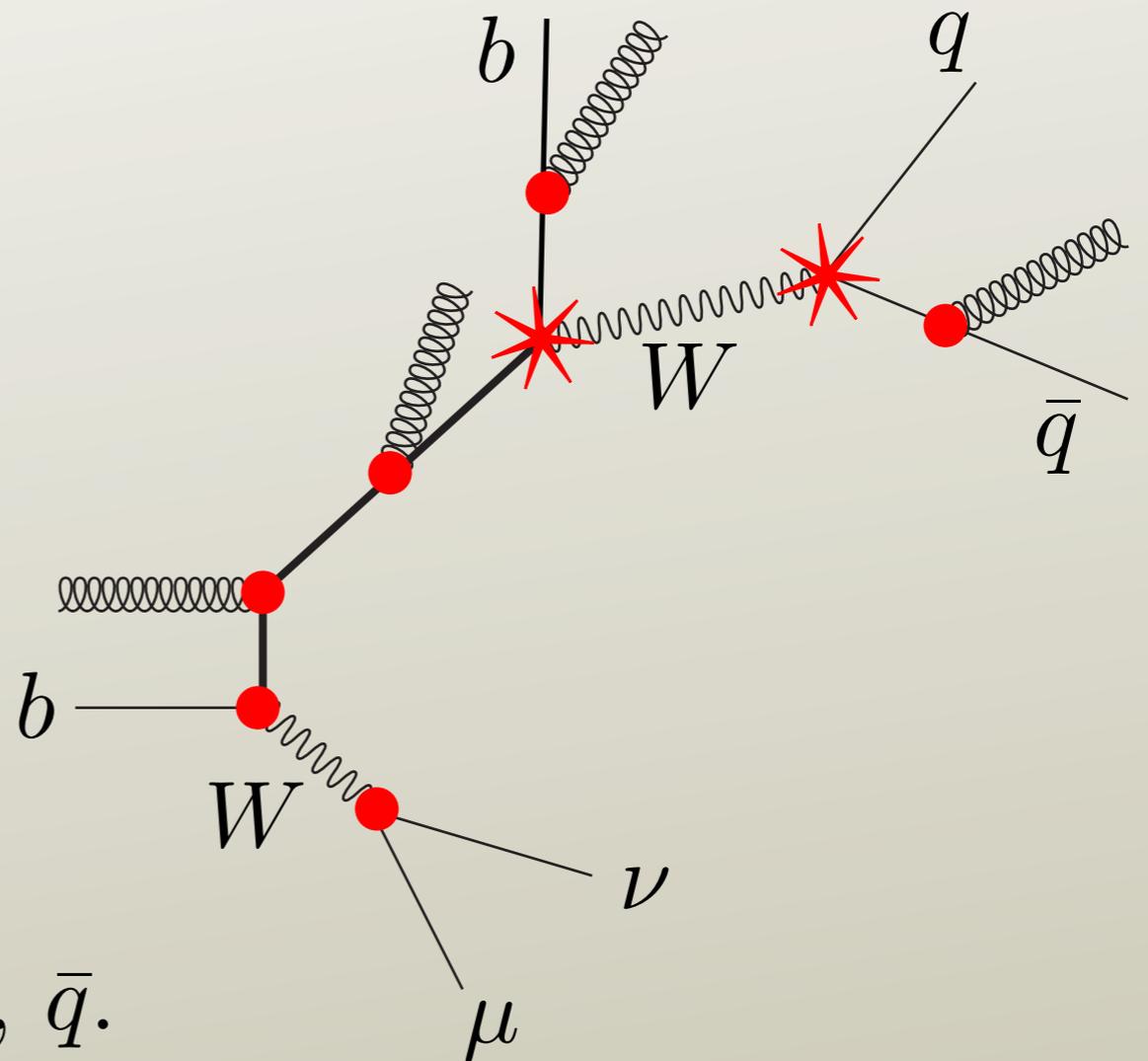
- In  $W \rightarrow q + \bar{q}$ , a lot of energy is released.

- Color charge is accelerated.

- Gluons are radiated.

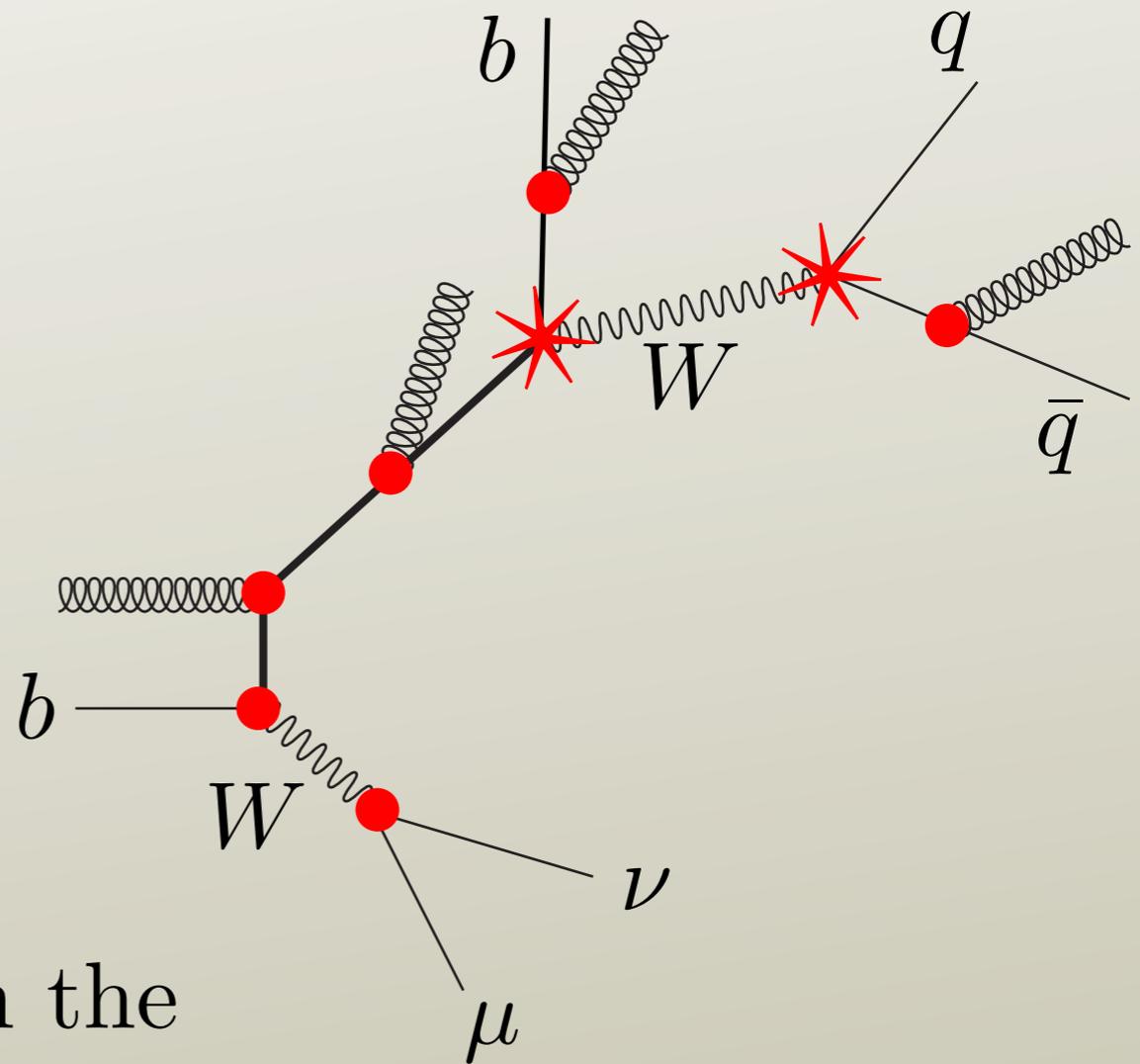
- The first radiation is from a color dipole of

- The final state quark,  $q$ ;
- The final state anti-quark,  $\bar{q}$ .



# Where does the radiation go?

- The first radiation is from a color dipole of
  - The final state quark,  $q$ ;
  - The final state anti-quark,  $\bar{q}$ .
- Direction of  $g$  is likely collinear with the  $q$  or  $\bar{q}$ .
- In any case, it is likely between the  $q$  and  $\bar{q}$ .



# Comments

- Since we know something about where the collinear or soft radiation goes in a heavy particle decay, sequential heavy particle decays should have good signatures.
- We do have the problem of contamination from initial state radiation.
- Having highly boosted heavy objects helps with this.

