3306 physics lectures, Spring 2026

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https://www.physics.smu.edu/saptaparnab/PH3306_Spring_2026/
Based on Simon Dalley's lectures delivered in spring 2025

WARM UP 11: Entropy (Halliday: 20.1 & 20.4)

Write your answers in the space following the warm-up question if you can. Write as if you are explaining to a fellow student. If you need more space, you are probably over-thinking things.

What is entropy?

What is the 2nd law of thermodynamics?

Is it possible that all the air molecules in a room are found in one corner? Explain.

Explain the relationship between the number of microstates and the entropy. **The correct answer is 2. b $>$ a.**

Here is a step-by-step breakdown of why the entropy change is greater for path b:

1. Identify the Processes

First, let's look at what is happening to the ideal gas on each path:

- **Path a:** The line goes straight up. This means the pressure (P) is increasing while the volume (V) stays exactly the same. This is a **constant volume** (isochoric) process.
- **Path b:** The line goes straight to the right. This means the volume (V) is increasing while the pressure (P) stays exactly the same. This is a **constant pressure** (isobaric) process.

2. The Relationship Between Heat, Temperature, and Entropy

Both paths start at temperature T_1 and end at a higher temperature T_2 (we know T_2 is higher because the Ideal Gas Law, $PV = nRT$, tells us that increasing pressure at a constant volume, or increasing volume at a constant pressure, requires an increase in temperature).

The change in entropy (ΔS) is directly related to the amount of heat (Q) added to the system to cause that temperature change.

To raise the temperature of a gas by a certain amount:

- **At constant volume (Path a):** All the heat you add goes directly into raising the internal energy (temperature) of the gas.
- **At constant pressure (Path b):** The gas expands as it heats up. Because it expands, it does work on its surroundings. This means you have to add **extra heat** to the system—some to do the work of expanding, and the rest to raise the temperature to T_2.

Because you have to pump more heat into the system along path **b** to reach the same final temperature T_2, path **b** experiences a greater increase in disorder, or entropy.

3. The Mathematical Proof

If you prefer to see this using thermodynamics equations, we can look at the formulas for entropy change where n is the number of moles:

- **For Path a (constant volume):**

$$\Delta S_a = nC_v \ln\left(\frac{T_2}{T_1}\right)$$

  (Where C_v is the molar heat capacity at constant volume)

- **For Path b (constant pressure):**

$$\Delta S_b = nC_p \ln\left(\frac{T_2}{T_1}\right)$$

  (Where C_p is the molar heat capacity at constant pressure)

For any ideal gas, it is a fundamental rule that $C_p > C_v$ (specifically, $C_p = C_v + R$).

Since both equations share the exact same $n \ln\left(\frac{T_2}{T_1}\right)$ term, the equation multiplied by the larger heat capacity (C_p) will result in the larger entropy change. Therefore, $\Delta S_b > \Delta S_a$.

The correct answer is e) 3 ¿ 2 = 1.

Here is a step-by-step breakdown of how to find the multiplicity for each scenario.

In statistical mechanics and probability, **multiplicity** (W) is the number of different ways (microstates) a specific outcome (macrostate) can happen.

1) Getting 10 heads total when I throw 10 coins

Think about how many different arrangements result in exactly 10 heads. There is only **one** possible way this can happen: every single coin must land on heads (HHHHHHHHHH).

- $W_1 = 1$

2) Getting 6 dots total when I throw 6 dice

A standard die has a minimum value of 1. If you roll 6 dice, the absolute lowest sum you can possibly get is 6 $(1 + 1 + 1 + 1 + 1 + 1)$. Just like the coins, there is only **one** specific combination of dice rolls that yields a total of 6 dots: every single die must show a 1.

- $W_2 = 1$

3) Getting 2 tails total when I throw 5 coins

Unlike the previous two scenarios, there are multiple ways to get exactly 2 tails out of 5 coins. For example, the tails could be the first two coins (TTHHH), the last two coins (HHHTT), or scattered (THTHH).

To find the exact number of ways, we use the binomial combination formula (often read as "n choose k"):

$$W = \frac{N!}{n!(N-n)!}$$

Where N is the total number of coins (5) and n is the number of tails we want (2):

$$W_3 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \times 4}{2 \times 1} = 10$$

- $W_3 = 10$

The Final Ranking

Comparing our multiplicities:

- $W_1 = 1$
- $W_2 = 1$
- $W_3 = 10$

Therefore, scenario 3 has the highest multiplicity, while 1 and 2 are equal (**3** $>$ **2 = 1**).

The correct answer is a. $P = P_A \cdot P_B$

Here is a breakdown of why this is the case:

In statistical thermodynamics, **thermodynamic probability** (often denoted as W, Ω, or in this case, P) is not a standard probability between 0 and 1. Instead, it represents the **total number of possible microstates** (specific microscopic arrangements) that correspond to a specific macrostate (the overall observable condition of the system).

The Multiplicative Rule

If you have two independent parts of a system, A and B, the total number of ways the combined system can be arranged is the **product** of their individual arrangements.

This is based on the fundamental counting principle in combinatorics: if part A can exist in P_A different states, and part B can exist in P_B different states, then for every single state of part A, part B can be in any of its P_B states. Therefore, the total number of combined states is $P_A \times P_B$.

Connecting it to Entropy

You might remember this from the earlier question regarding the equation $S = S_1 + S_2$. The reason entropies add together is precisely because their thermodynamic probabilities multiply!

Using Boltzmann's equation, where k is the Boltzmann constant:

$$S = k \ln(P)$$

If we look at the combined system:

$$S_{total} = k \ln(P_{total})$$

We know that entropy is additive ($S_{total} = S_A + S_B$):

$$k \ln(P_{total}) = k \ln(P_A) + k \ln(P_B)$$

According to the rules of logarithms, $\ln(A) + \ln(B) = \ln(A \cdot B)$. Therefore:

$$k \ln(P_{total}) = k \ln(P_A \cdot P_B)$$

Which leaves us with:

$$P_{total} = P_A \cdot P_B$$