

# 1 Bethe-Bloch: Question 1

Bethe equation - Ionization in Silicon and Argon

The mean energy loss per unit length via ionization is described by the Bethe equation. Neglecting the density corrections at very low and very high energy, the equation can be approximated as

$$-\left\langle \frac{dE}{dX} \right\rangle = 4\pi N_A r_e^2 m_e c^2 \frac{z^2 Z}{A\beta^2} \left[ \frac{1}{2} \ln \left( \frac{2m_e c^2 \beta^2 \gamma^2 \Delta T_{max}}{I_{eff}^2} \right) - \beta^2 \right] \quad (1.1)$$

where  $X = \rho x$  and the following parameters:

$N_A$ : Avogadro's constant

$r_e$ : Classical electron radius,  $r_e = 2.818$  fm

$m_e$ : Electron rest mass

$c$ : Speed of light

$z$ : Projectile particle charge number

$Z$ : Nuclear charge number

$A$ : Atomic mass in units of [g mol<sup>-1</sup>]

$\beta$ : Ratio speed of light  $\beta = v/c$

$\gamma$ : Lorentz factor

$\Delta T_{max}$ : Maximum energy transfer to electron

$I_{eff}$ : Mean excitation energy

$\rho$ : Material density

For protons in the energy range of several GeV, the maximum energy transfer can be approximated as

$$\Delta T_{max} \approx 2m_e c^2 (\beta\gamma)^2 \quad (1.2)$$

Consider a proton with a momentum of 5 GeV/c, traversing a silicon detector with a thickness of 300  $\mu\text{m}$  (Si,  $\rho=2.329$  g/cm<sup>3</sup>,  $A = 28.09$  g mol<sup>-1</sup>,  $I_{eff} = 173$  eV), or a gaseous detector filled with Argon and a thickness of 10 mm (Ar,  $\rho=0.00166$  g/cm<sup>3</sup>,  $A = 39.95$  g mol<sup>-1</sup>,  $I_{eff} = 188$  eV).

- Calculate the mean energy loss via ionization of the proton in the silicon detector.
- The energy required to produce an electron-hole pair in silicon is  $w_{\text{Si}} = 3.6$  eV. Determine the number of electron-hole pairs produced in the detector by the proton.
- Calculate the mean energy loss via ionization of the proton in the Argon filled detector.
- Determine the number of ions generated in the gaseous detector for a mean ionization energy of  $w_{\text{Ar}} = 26$  eV.