PH7363 Fall 2025

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Particle detection and detector physics

Exercise 1: Light Attenuation in a Scintillator Bar

A particle interacts within a long, thin plastic scintillator bar of total length $\mathbf{L} = 200\,\mathrm{cm}$. Photodetectors (e.g., SiPMs or PMTs) are placed at each end: **Detector A** (at $x = 0$) and **Detector B** (at $x = L$).

The scintillator material has a characteristic **attenuation length of** $\lambda = 120\,\mathrm{cm}$.

An interaction at a position $\mathbf{x}$ (measured from Detector A) produces an initial isotropic flash of $\mathbf{N_0} = \mathbf{20,000}$ photons.

Assumptions:

- The initial light N_0 propagates equally in both directions (i.e., $N_0/2$ photons travel towards A and $N_0/2$ photons travel towards B).
- The photodetectors are 100% efficient.
- Ignore any reflections from the ends of the bar.

The number of photons $N(d)$ that survive after traveling a distance d from the interaction point is given by the light attenuation formula:

$$N(d) = N_{\text{initial}} \cdot e^{-d/\lambda}$$

where N_{initial} is the initial number of photons traveling in that direction.

Questions

Consider a single particle event that occurs at $\mathbf{x} = 40\,\mathrm{cm}$.

1. **Calculate** N_A: How many photons reach Detector A?
2. **Calculate** N_B: How many photons reach Detector B?
3. **Calculate Total Light**: What is the arithmetic sum of the photons detected ($N_A + N_B$)?
4. **Calculate Geometric Mean**: What is the geometric mean of the signals ($\sqrt{N_A \times N_B}$)?
5. **Calculate Asymmetry**: What is the signal asymmetry, defined as $A = \frac{N_A - N_B}{N_A + N_B}$?

Follow-up

1. **Position Independence:** Recalculate parts 1-4 for a new interaction at $\mathbf{x} = 100\,\mathrm{cm}$ (the center of the bar). What do you observe about the **geometric mean** ($\sqrt{N_A \times N_B}$)? Prove algebraically that the geometric mean is independent of the interaction position x and depends only on N_0, L, and λ.
2. **Position Reconstruction:** Show that the asymmetry A can be expressed as a hyperbolic tangent function of the position x. Based on your finding, if an event produced an asymmetry of $A = 0$, where would you conclude the interaction occurred?

Exercise 2: Statistical Limits of Time-of-Flight (TOF) Resolution

Calculate the fundamental position resolution of a scintillator-based Time-of-Flight (TOF) detector by combining:

1. Poisson statistics of photon production and detection.
2. Signal attenuation.
3. The intrinsic time jitter of the photodetectors.
4. Standard error propagation.

A minimum ionizing particle (MIP) passes transversely through a long plastic scintillator bar of length $\mathbf{L} = 2.0\,\mathrm{m}$. The interaction at position $\mathbf{x}$ (measured from one end) deposits $\mathbf{E_{dep}} = 2.0\,\mathrm{MeV}$ of energy.

The bar is read out at both ends ($x = 0$ and $x = L$) by two identical Silicon Photomultipliers (SiPMs), labeled **SiPM 1** and **SiPM 2**, respectively.

Given Parameters:

- **Scintillator Properties:**
  - Light Yield (Y): $8000\,\mathrm{photons/MeV}$
  - Attenuation Length (λ): $1.5\,\mathrm{m}$
  - Effective refractive index (n_{eff}): 1.58
  - Speed of light (c): $3.0 \times 10^8\,\mathrm{m/s}$
- **SiPM Properties:**
  - Photon Detection Efficiency (PDE, ϵ): 30% or 0.30
  - Single Photoelectron (p.e.) Time Jitter ($\sigma_{p.e.}$): $150\,\mathrm{ps}$

Consider a single event where the MIP interaction occurs at $\mathbf{x} = 0.5\,\mathrm{m}$.

Questions

The goal is to find the **position resolution (σ_x)** for this event.

Part 1: Mean Signal Calculation

1. **Initial Photons (N_0):** Calculate the total number of scintillation photons, N_0.
2. **Mean Detected Photons ($N_{p.e.}$):** Calculate the *mean* number of detected photoelectrons at **SiPM 1** ($\langle N_1 \rangle$) and **SiPM 2** ($\langle N_2 \rangle$).
3. **Mean Time Difference ($\langle \Delta T \rangle$):** Calculate the *true* (mean) time difference $\langle \Delta T \rangle = \langle T_1 - T_2 \rangle$.

Part 2: Resolution Calculation

The time resolution (σ_T) of a detector that detects $N_{p.e.}$ photoelectrons is modeled as:

$$\sigma_T = \frac{\sigma_{p.e.}}{\sqrt{N_{p.e.}}}$$

4. **Individual Time Resolutions (σ_{T1}, σ_{T2}):** Calculate the time resolution for SiPM 1 and SiPM 2.
5. **Time-Difference Resolution ($\sigma_{\Delta T}$):** Find the resolution of the time difference, $\sigma_{\Delta T}$.

Part 3: Position Resolution

The interaction position x is reconstructed from the time difference ΔT using:

$$x = \frac{L}{2} + \frac{v_{eff}}{2} \Delta T$$

6. **Position Resolution (σ_x):** Propagate the uncertainty $\sigma_{\Delta T}$ to determine the final position resolution, σ_x.

Part 4: Conceptual Analysis

7. **Impact of Position:** Without recalculating, explain how σ_x would change if the interaction occurred at the *center* of the bar ($x = 1.0\,\text{m}$). Better, worse, or the same? Justify your answer.