PH7363 Fall 2025

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Particle detection and detector physics

Resources

You are allowed to use one page of notes. You can write any equation that you want and you are allowed to use both sides of the paper. No additional resources are allowed for this midterm examination.

Question 1: Detector Design for Higgs Boson Physics

Points: 20

This question asks you to use your knowledge of particle detection by designing a general-purpose detector for Higgs boson physics. The main goals are to discover the Higgs at a proton-proton collider and its precision study at an e^+e^- collider.

Part 1: General-Purpose Detector Components

Points: 5

A. What are the major components of a general purpose particle detector?

Part 2: Higgs Discovery at a Proton-Proton Collider

Points: 5

The discovery of the Higgs boson ($m_H \approx 125$ GeV/c^2) at the LHC (a pp machine) was achieved in two "golden" channels: $H \to ZZ \to 4\ell$ and $H \to \gamma\gamma$.

B. Focus on the $H \to \gamma\gamma$ (di-photon) channel.

a) What is the enormous, dominant background that makes this search so difficult?

b) The intrinsic (natural) width of the Higgs is $\Gamma_H \approx 4.1$ MeV, which is small. The *observed* width of the mass peak (σ_{obs}) is therefore completely dominated by the detector's experimental resolution (σ_{det}). Which **single detector subsystem** from Part A is most critical for this measurement, and what **specific performance metrics** (e.g., stochastic term A, constant term B) must be optimized to create a narrow, statistically significant "bump"?

Part 3: Higgs Precision Study at an e^+e^- Collider

Points: 10

At a future e^+e^- "Higgs factory" (like the ILC or FCC-ee), the primary production mode is Higgs-strahlung, $e^+e^- \to ZH$. The collider's center-of-mass energy E_{cm} is known with exquisite precision. This allows for a "recoil mass" technique, where the Higgs boson can be identified without observing its decay.

C. The recoil mass m_{recoil}, which is equal to the Higgs mass ($m_{recoil} = m_H$), is calculated from the well-known initial state (E_{cm}, $p_{cm} = 0$) and the measured Z boson (E_Z, p_Z).

a) Write down the equation that relates the recoil mass to E_{cm}, E_Z and p_Z

b) If the $Z \to \ell^+\ell^-$ (leptonic) decay is used, what **single detector subsystem** primarily determines the precision of the Higgs mass measurement?

c) To get a statistically significant dataset, one must also use the $Z \to jj$ (hadronic) decay. What **overall detector capability** is required to get a precise enough measurement of E_Z and p_Z from the two jets, and what **key detector subsystems** enable this?

Question 2: Detector Design for Hadronic W/Z Boson Separation

Points: 40

Scenario: At a hadron collider, the separation of W and Z bosons is straightforward in their leptonic decay channels ($Z \to \ell^+\ell^-$ vs. $W \to \ell\nu$) but is a significant challenge in their hadronic channels ($W \to jj$ and $Z \to jj$). This is because their masses are relatively close, and the energy resolution for jets is typically much poorer than for leptons.

Your task is to perform an error analysis to determine the performance requirements for a detector system capable of distinguishing these two hadronic decay modes by reconstructing the dijet invariant mass (m_{jj}).

Given:

- W Boson Mass: m_W = 80.4 GeV
- W Boson Natural Width: Γ_W = 2.09 GeV
- Z Boson Mass: m_Z = 91.2 GeV
- Z Boson Natural Width: Γ_Z = 2.49 GeV

Part 1: The Separation Criterion

Points: 5

The observed mass peak for each particle is a convolution of its natural Breit-Wigner shape (width Γ) and the detector's Gaussian resolution (σ_{det}). The resulting *measured width* (σ_{meas}) can be approximated as:

$$\sigma_{meas} \approx \sqrt{\Gamma^2 + \sigma_{det}^2}$$

To "resolve" or "separate" two peaks, their mass difference ($\Delta m = m_Z - m_W$) must be larger than some function of their measured widths. A common criterion for separation is that the "dip" between the two peaks is statistically significant. For this problem, let's use the simpler criterion that the peak-to-peak separation must be at least the average of their measured widths:

$$\Delta m \geq \frac{\sigma_{meas,W} + \sigma_{meas,Z}}{2}$$

A. Using this criterion, and assuming for simplicity $\sigma_{meas,W} \approx \sigma_{meas,Z} \equiv \sigma_{meas}$ and $\Gamma_W \approx \Gamma_Z \approx 2.5$ GeV, calculate the **maximum permissible detector resolution (σ_{det})** for the dijet invariant mass that still allows for the separation of W and Z bosons.

Part 2: Error Propagation and Jet Energy Resolution

Points: 15

The dijet invariant mass resolution (σ_{det}) is driven almost entirely by the Jet Energy Resolution (JER). The invariant mass for two jets is:

$$m_{jj}^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$

For two high-energy jets ($E \gg m_{jet}$), this simplifies to $m_{jj}^2 \approx 2E_1E_2(1 - \cos\theta_{12})$.

The fractional JER (σ_E/E) is the dominant uncertainty; you can assume the uncertainty in the jet angle measurement ($\sigma_{\theta_{12}}$) is negligible.

B. By propagating the errors, show that the fractional invariant mass resolution (σ_{det}/m_{jj}) is related to the fractional jet energy resolution (σ_E/E) by:

$$\frac{\sigma_{det}}{m_{jj}} \approx \frac{1}{\sqrt{2}} \frac{\sigma_E}{E}$$

(You may assume two symmetric jets, $E_1 = E_2$, for this derivation.)

C. The JER for a calorimeter is often parameterized as $\frac{\sigma_E}{E} = \frac{A}{\sqrt{E}} \oplus B$, where A is the stochastic term (from sampling fluctuations) and B is the constant term (from calibration, uniformity, etc.).

Using your result from **Part A** (the required σ_{det}) and **Part B** (the propagation), determine the **maximum allowable constant term (B)**. Assume you are measuring jets with a typical energy of $E = 50$ GeV and that the stochastic term A is 0.5 GeV$^{1/2}$ (i.e., 50% GeV$^{1/2}$).

Part 3: Detector Design Implications

Points: 10

D. Your calculation in Part C defines the most critical performance benchmark for your calorimeter.

a) Which detector subsystem is responsible for this measurement?

b) What specific design choices (e.g., technology, materials, granularity, depth) are necessary to achieve a constant term (B) this small?

c) Briefly explain *why* these design choices directly impact (i.e., reduce) the constant term of the energy resolution.

Part 4: Comparison with e^+e^- Colliders

Points: 10

The analysis in Parts 1-3 was for a **hadron collider**, where the $W \to jj$ and $Z \to jj$ signals are overwhelmed by an enormous background from QCD dijet production ($gg \to jj$, $qg \to qj$, etc.). In that scenario, the known W and Z masses are often used as a "standard candle" to *calibrate* the Jet Energy Resolution (JER), rather than as a discovery channel.

Now, consider an e^+e^- **collider** (like LEP or a future Higgs factory) operating at a center-of-mass energy well above $2m_Z$. Here, the environment is exceptionally clean. The dominant processes are $e^+e^- \to q\bar{q}$ (which forms two jets) and, at high enough energy, diboson production ($e^+e^- \to W^+W^-$ and $e^+e^- \to ZZ$).

In this clean environment, the goal is to fully reconstruct events, for example, to separate $e^+e^- \to W^+W^- \to (jj)(jj)$ (a 4-jet final state) from $e^+e^- \to ZZ \to (jj)(jj)$ (also a 4-jet final state).

E. How does the **primary challenge** of separating hadronic W and Z bosons in an e^+e^- environment differ fundamentally from the challenge at a hadron collider? (Hint: Think about "signal-to-background" vs. "signal-to-signal".)

F. At a hadron collider, the main requirement for the HCAL was a low constant term (B) to manage backgrounds over a wide energy range. At an e^+e^- collider, the key to separating $W \to jj$ from $Z \to jj$ in 4-jet events is achieving the best possible jet energy resolution (e.g., $\sigma_E/E \sim 3-4\%$). This performance is not achieved by the HCAL alone, but by a technique called **Particle Flow**.

Briefly explain the concept of a Particle Flow Algorithm (PFA) and why it allows for a much better jet energy resolution than just using the calorimeter system. Which detector subsystems are most critical for a PFA?

G. Given the separation criterion from Part 1 ($\Delta m \geq \sigma_{meas}$), what would the required **detector-only mass resolution (σ_{det})** be to separate a W^+W^- pair from a ZZ pair, assuming you could perfectly pair the jets? Why is this a much more stringent requirement than the one you calculated in Part A for a hadron collider?

Question 3: Particle Identification with dE/dx

Points: 10

Scenario:

A basic detector setup in high-energy physics measures two key properties of a charged particle:

1. Its **momentum (p)** from the curvature of its track in the magnetic field.
2. Its **mean energy loss per unit length ($-dE/dx$)** from the amount of ionization it deposits in the detector gas.

The goal is to use these two measurements to identify the particle's type (e.g., distinguish a pion $\pi^{\pm}$, a kaon $K^{\pm}$, and a proton p).

Your task is to use the Bethe-Bloch formula for particle identification in different momentum regimes.

Part 1: The $1/\beta^2$ Region (Low Momentum)

Points: 5

At low momentum (e.g., $p \ll m_p c$), the $1/\beta^2$ term is dominant, and the logarithmic term changes very slowly.

A. Consider a pion ($m_\pi \approx 140$ MeV/c^2), a kaon ($m_K \approx 494$ MeV/c^2), and a proton ($m_p \approx 938$ MeV/c^2) all measured to have the **exact same momentum**, $p = 500$ MeV/c.

Which particle will have the highest $-dE/dx$ signal? Which will have the lowest? Justify your answer by relating momentum to the dominant term in the Bethe-Bloch equation.

Part 2: The Relativistic Rise Region (High Momentum)

Points: 5

At high momentum, $\beta \approx 1$ for all three particles. The $1/\beta^2$ term is now nearly constant (≈ 1), and the dominant velocity-dependent term becomes the logarithmic one: $\ln(\beta^2\gamma^2)$.

B. Now consider the same three particles (π, K, p) all measured to have the **exact same high momentum**, $p = 10$ GeV/c.

In this "relativistic rise" region, which particle will have the highest $-dE/dx$ **signal?** Does the ordering change with respect to your answer in Part 1?
