Studies of the combination of $\nu$WT histogram and fit methods.

Peter Renkel, Robert Kehoe
2 Southern Methodist University, Dallas, Texas, USA

June 29, 2009

We have two general questions: "Can we combine two very correlated methods, $\nu$WT$_h$ and $\nu$WT$_f$? Do we gain in statistical error by combining them?" To answer these questions requires a careful evaluation of the source of the decorrelation observed and a verification that a combination does indeed improve the actual measured mass resolution. It also requires that we can properly estimate our final statistical uncertainty.

1 Decorrelation between $\nu$WT$_h$ and $\nu$WT$_f$

The source of the decorrelation can arise from several sources, such as the different forms of likelihood calculation in Eqs. 12 and 19. More important sources concern the shape differences between probability density histograms and functions due to binning, interpolation of empty bins, or the structure of the chosen fit functions. We discuss each of these in the subsections below.

1.1 Gaussian and Poisson Constraints

To test the impact of the Gaussian and Poisson constraints on the decorrelation between $\nu$WT$_h$ and $\nu$WT$_f$ methods, we have performed the latter using a fixed $n_s$ and $n_b$. We obtain the results given in Table 1. The results for the $\nu$WT$_h$ method are also given for comparison. The observed shifts are very small and have no impact on the final measured mass from all channels.

1.2 Binning

To address the effect of coarse binning in the $h_s$ and $h_b$, let us take the idealized case when we know our probability density functions ($f_s$ and $f_b$) precisely, and we coarsely bin them to provide probability density histograms ($h_s$ and $h_b$). We can then measure $m_t$ in each of many pseudoexperiments using the histogram-based method and the function-based method. Because the histograms have
channel  | $\nu_{WT_f}$ | $\nu_{WT_f}$ (no $n_s$ and $n_b$ constraints) | $\nu_{WT_h}$
---|---|---|---
$e\mu$  | 166.5 | 166.5 | 172.8
$ee$    | 181.1 | 181.2 | 181.6
$\mu\mu$ | 185.9 | 185.7 | 186.6
$e+\text{track}$ | 161.2 | 161.3 | 166.1
$\mu+\text{track}$ | 181.2 | 181.2 | 178.8

Table 1: Measured mass in binned and unbinned $\nu_{WT_f}$ methods. The $\nu_{WT_h}$ result is provided for comparison.

introduced a binning to the fit functions, there will be some evident decorrelation between the two measurements. However, combining the two methods would not be correct because all of the information is contained in the perfect $f_s$ and $f_b$. The metric by which to judge the sensibility of the combination is the $rms$ of the distribution of measured masses in many pseudoexperiments. These are the actual measurements, and we have gained sensitivity if a combination of the two methods gives an $m_t$ that correlates better with the ‘true’ top quark mass than the original histogram and fit methods. In the scenario considered in this subsection, we will not observe any reduction in $rms$ because the histogram adds no information.

This situation is approximated by the following test. We take the $f_s$ and $f_b$ for the $\nu_{WT_f}$. We bin these functions in the $\mu_w$ vs. $\sigma_w$ plane by taking the values of the probability density functions at the centers of the bins. The binning was taken to be exactly the same as for the $h_s$ and $h_b$ of the $\nu_{WT_h}$ method. We do not bin the $m_t$ so that, for each choice of $\mu_w^i, \sigma_w^i$, we still have an analytic function of $m_t$ to fit and no parabolic fits are involved. The results are given in Table 2. We see that combining $\nu_{WT_f}$ and binned $\nu_{WT_f}$ does not help in this case.

<table>
<thead>
<tr>
<th>$\nu_{WT_f}$</th>
<th>$\nu_{WT_f}$ binned</th>
<th>combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t$ $rms$</td>
<td>5.3</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Table 2: Mass $rms$ values for binned and unbinned $\nu_{WT_f}$ methods, plus their combination.

1.3 Interpolation in Histograms

The $\nu_{WT_h}$ method introduces an interpolation for bins which have zero entries. This interpolation introduces a potential difference in shape to the probability density histograms. In pseudoexperiments, we employed the linear interpolation used in the default analysis and compared this with a constant interpolation scheme. The observed difference in measured mass was < 100 MeV in all channels.
Table 3: Observed width of $m_t$ as measured in 300 pseudoexperiments for the $\nu WT_h$, $\nu WT_f$, and combined analyses.

<table>
<thead>
<tr>
<th></th>
<th>$\nu WT_h$ [GeV]</th>
<th>$\nu WT_f$ [GeV]</th>
<th>combination [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass rms</td>
<td>5.27</td>
<td>5.31</td>
<td>4.60</td>
</tr>
<tr>
<td>Gaussian fit $\sigma$</td>
<td>5.24±0.25</td>
<td>5.21±0.25</td>
<td>4.66±0.21</td>
</tr>
</tbody>
</table>

Table 4: Calibration metrics for combined $\nu$WT measurement.

<table>
<thead>
<tr>
<th></th>
<th>slope offset pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>combination</td>
<td>1.007±0.018 −0.03 ± 0.15 1.001±0.020</td>
</tr>
</tbody>
</table>

### 1.4 Assumed Fit Function Shape

Now let us consider the actual case we have in this analysis where the fit functions are chosen *ad hoc* and are fitted to unmodified $h_s$ and $h_b$. We know that the fit functions have a large $\chi^2/dof$ relative to the histograms, and therefore there is a shift in shape due to the chosen function. To determine if the fit functions are supplying unique information to measure $m_t$, we use the BLUE weight equations. Using our estimated correlation (85%) and the measured statistical and estimated systematic uncertainties, we obtain weights of 0.76 and 0.24 for $\nu WT_h$ and $\nu WT_f$, respectively, for the data events. We combined the two sets of measurements (each with 300 pseudoexperiments) and the result is shown in Fig. 1. We see three mass distributions: for $\nu WT_h$, $\nu WT_f$, and the combination. The $rms$ for the combination is smaller than the $rms$ for $\nu WT_h$ and the $rms$ for $\nu WT_f$. The estimated statistical uncertainty for the combination is obtained from the 15% correlation coefficient, and the estimated statistical uncertainties from each pseudoexperiment corrected for the pull width. The agreement of this uncertainty with the mass $rms$ demonstrates that our statistical errors are estimated correctly.

In Fig. 1 we fitted these three plots to gaussians, and the results are also shown. The numerical $rms$ and fit values are given in Table 3. The conclusion of this study is that we are allowed to combine the measurements in our case.

### 2 Performance for combining $\nu WT_f$ and $\nu WT_h$

The average expected statistical uncertainty for the combination is 4.8 GeV. To confirm that combining the $\nu WT_f$ and $\nu WT_h$ can correctly provide this estimate, we also tested the calibration of the combination. In 300 pseudoexperiments, we measured the slopes, offsets and pulls of the combined $m_t$ measurement with the true $m_t$. The results are shown in Fig. 2 and Table 4.
Figure 1: (left) Mass distribution for $\nu_{WT_f}$, $\nu_{WT_h}$ and combination and fits for this mass distributions.

Figure 2: (left) Calibration and slope for the combination of $\nu_{WT_f}$ and $\nu_{WT_h}$ and (right) pulls for the combination.
3 Acknowledgments

We would like to thank Aurelio Juste, Scott Snyder and Elizaveta Shabalina for helpful comments toward these studies.