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’RODUCTION OF HEAVY)UARKONIUM IN HIGH-ENERGY :OLLIDERS

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ABSTRACT

Recent data from the Tevatron collider have revealed that the production rate of prompt charmonium at large transverse momentum is orders of magnitude larger than the best theories of a few years ago had predicted. These surprising results can be understood by taking into account two recent developments that have dramatically revised the theoretical description of heavy-quarkonium production. The first is the realization that fragmentation must dominate at large transverse momentum, which implies that most charmonium in this kinematic region is produced by the hadronization of individual high-p_T partons. The second is the development of a factorization formalism for quarkonium production based on nonrelativistic QCD that allows the formation of charmonium from color-octet $c\bar{c}$ pairs to be treated systematically. This review summarizes these theoretical developments and their implications for quarkonium production in high-energy colliders.

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1. INTRODUCTION

The goal of high-energy physics is to identify the elementary constituents of matter and to understand their fundamental interactions. Over the past 20 years, this endeavor has been extraordinarily successful. A gauge theory called the minimal Standard Model provides a satisfactory description of the strong, weak, and electromagnetic interactions of all the known elementary particles. There are few discrepancies between theory and experiment, and most of them are at the level of a few standard deviations or less. However, there is one process for which experimental results have differed from theoretical predictions by orders of magnitude: the production of charmonium at large transverse momentum at the Tevatron Collider. This dramatic conflict between experiment and theory presents a unique opportunity to make a significant step forward in our understanding of heavy-quarkonium physics.

Prior to 1993, conventional wisdom regarding production of charmonium in hadron collisions was based primarily on calculations in the color-singlet model. In this model, the production of a charmonium state is assumed to proceed through parton processes that produce a $c\bar{c}$ pair in a color-singlet state. It is the predictions of the color-singlet model at lowest order in α_s that disagree so dramatically with the Tevatron data. The data can, however, be explained by combining two recent theoretical developments in heavy-quarkonium physics. The first is the realization that heavy quarkonium at large transverse momentum is produced primarily by fragmentation, the hadronization of individual high-p_T partons. In the color-singlet model, fragmentation first contributes at a higher order in α_s; thus it was not taken into account in previous calculations. The second development is the realization that color-octet mechanisms, in which the $c\bar{c}$ pair is produced at short distances in a color-octet state, sometimes dominate the oduction of charmonium. These mechanisms can be analyzed systematically ing a new factorization formalism for heavy-quarkonium production that is ised on an effective field theory called nonrelativistic quantum chromodynam- s (NRQCD). The Tevatron data can be explained by including a color-octet rm in the fragmentation function for the formation of charmonium in a gluon t. Evidence in support of this explanation has been accumulating, but a great al of work, both experimental and theoretical, is required before it can be garded as conclusive.

This review summarizes the recent theoretical developments in heavy-quark- iium production. A brief review of these developments has been given pre- ously by Mangano (1). Our main focus is the production of quarkonium in gh-energy colliders, where both fragmentation and color-octet mechanisms e important. In Section 2, we review the color-singlet model, summarizing its edictions for charmonium production in $p\bar{p}$ colliders, which are in dramatic sagreement with Tevatron data. In Section 3, the production of quarkonium / fragmentation is reviewed. We describe the factorization theorems of per- rbative QCD that imply that fragmentation should dominate at large trans- rse momentum, illustrating them using the electromagnetic production of the $/\psi$ in Z^0 decay. We then discuss the predictions of the color-singlet model r fragmentation functions. Color-octet mechanisms for producing quarko- um are reviewed in Section 4. We first discuss the production of p-wave ates, where a color-octet mechanism is required for perturbative consistency, id then we describe the NRQCD factorization formalism, which implies that lor-octet mechanisms must also contribute to the production of s-wave states. 'e also discuss the implications of the factorization formalism for fragmen- tion functions. In Section 5, we summarize applications of the recent theo- tical developments to the production of prompt charmonium, bottomonium, id the B_c at the Tevatron and at the Large Electron-Positron collider (LEP). 'e conclude in Section 6 with an outlook on the work required to develop a mprehensive understanding of heavy-quarkonium production in high-energy ocesses.

. THE PROBLEM OF CHARMONIUM PRODUCTION

.1 *Color-Singlet Model*

efore 1993, most predictions for charmonium production were based on the lor-singlet model. Ascribing credit for this model is a perilous task as many hysicists were involved in its early development. Three groups—DeGrand & oussaint, Wise, and Kühn et al (2)—were first to treat the decays of B mesons ito charmonium states through the process $b \to c\bar{c} + s$ in the color-singlet model. The hadronic production of J/ψ through the parton process $gg \rightarrow c\bar{c} + g$ was calculated by Chang (3). A thorough treatment of charmonium production in hadron collisions through $2 \rightarrow 3$ parton processes was presented by Baier & Rückl (4). Guberina et al (5) applied the color-singlet model to charmonium production from the decay $Z^0 \rightarrow c\bar{c} + \gamma$. Berger & Jones (6) calculated the rate for photoproduction of charmonium and emphasized the kinematic restrictions on the applicability of the model. This model was also applied to the inclusive production of charmonium in e^+e^- annihilation by Keung and by Kühn & Schneider (7). Recently, a thorough review of the applications of the color-singlet model to heavy-quarkonium production was given by Schuler (8).

An alternative model for quarkonium production, called the color-evaporation model, was developed around the same time (9). In this model, it was assumed that all $c\bar{c}$ pairs with invariant mass between $2m_c$ and the $D\overline{D}$ threshold $2m_D$ produce charmonium states. The fraction f_H of the $c\bar{c}$ pairs that form a particular charmonium state H is assumed to be independent of the production process. This model, however, is incapable of describing the variation of the production ratios for charmonium states between processes and as functions of kinematical variables. Therefore, we do not consider it further.

To motivate the color-singlet model, we consider the production of charmonium as proceeding in two steps. The first step is the production of a $c\bar{c}$ pair, and the second step is the binding of the $c\bar{c}$ pair into a charmonium state. For significant probability of binding, the pair must be produced with relative momentum that, in the $c\bar{c}$ rest frame, is small compared with the mass, m_c, of the charm quark. Otherwise, the c and $\bar{c}$ will fly apart and ultimately form D and $\overline{D}$ mesons.

Assuming that the c and $\bar{c}$ are not present in the initial state, any Feynman diagram for the production of a $c\bar{c}$ pair must involve virtual particles that are off their mass shells by amounts on the order of m_c or larger. The part of the amplitude in which all internal lines are off-shell by such amounts is called the short-distance part, and it is calculable by using perturbation theory in $\alpha_s(m_c)$. The parts of the amplitude in which the c and $\bar{c}$ lines are off-shell by amounts much less than m_c can be considered part of the amplitude for the formation of the bound state. The short-distance part of the amplitude describes the production of a $c\bar{c}$ pair with a spatial separation that is on the order of $1/m_c$ or smaller. This follows from the fact that the short-distance part is insensitive to changes in the relative 3-momentum of the c and $\bar{c}$ that are much less than m_c. Since $1/m_c$ is much smaller than the length scale associated with the charmonium wavefunction, the $c\bar{c}$ pair is essentially pointlike on that scale. Thus, we need only consider the amplitude for a pointlike $c\bar{c}$ pair to bind form a charmonium state. This amplitude will necessarily depend on the armonium state H and on the quantum numbers of the $c\bar{c}$ pair.

For any given charmonium state, the dominant Fock state consists of a color-iglet $c\bar{c}$ pair in a definite angular-momentum state. We denote the two possible lor states of a $c\bar{c}$ pair by **1** for color-singlet and **8** for color-octet. We use e spectroscopic notation ${}^{2S+1}L_J$ for the angular momentum state, where S, and J are the quantum numbers associated with the total spin, the orbital gular momentum, and the total angular momentum, respectively. Thus, the minant Fock state for a charmonium state H is denoted $|c\bar{c}(\mathbf{1}, {}^{2S+1}L_J)\rangle$ for propriate values of S, L, and J. For example, for J/ψ, the dominant Fock ite is $|c\bar{c}(\mathbf{1}, {}^3S_1)\rangle$, whereas for the χ_{cJ}, it is $|c\bar{c}(\mathbf{1}, {}^3P_J)\rangle$.

The color-singlet model is a simple model for the amplitudes for a pointlike pair to form a charmonium state. If the dominant Fock state of the meson H $|c\bar{c}(\mathbf{1}, {}^{2S+1}L_J)\rangle$, the amplitude is assumed to be 0 unless the pointlike $c\bar{c}$ pair in a color-singlet ${}^{2S+1}L_J$ state. For this state, the amplitude can be expressed terms of the L-th derivative of the radial wavefunction at the origin for the eson H. For example, the amplitudes for producing the states ψ and χ_{cJ}, is some specific final state F, are assumed to have the forms

$$\mathcal{A}(\psi + F) = \widehat{\mathcal{A}}\left(c\bar{c}\left(\mathbf{1}, {}^3S_1\right) + F\right) \; R_\psi(0), \tag{1}$$

$$\mathcal{A}(\chi_{cJ} + F) = \widehat{\mathcal{A}}\left(c\bar{c}\left(\mathbf{1}, {}^3P_J\right) + F\right) \; R'_{\chi_c}(0). \tag{2}$$

ie $\widehat{\mathcal{A}}$'s are amplitudes for producing color-singlet $c\bar{c}$ pairs with vanishing rel-ve momentum in the angular-momentum states indicated. The factor $R_\psi(0)$ the radial wavefunction at the origin for the ψ, while $R'_{\chi_c}(0)$ is the derivative the radial wavefunction at the origin for the χ_c states. In the color-singlet odel, it is assumed that the amplitudes $\widehat{\mathcal{A}}$ can be calculated by using pertur-tive QCD and that all nonperturbative effects can be absorbed into the wave-iction factors. From Equations 1 and 2, we deduce that the corresponding lusive differential cross sections in the color-singlet model have the forms

$$d\sigma(\psi + X) = d\widehat{\sigma}\left(c\bar{c}\left(\mathbf{1}, {}^3S_1\right) + X\right) \; |R_\psi(0)|^2, \tag{3}$$

$$d\sigma(\chi_{cJ} + X) = d\widehat{\sigma}\left(c\bar{c}\left(\mathbf{1}, {}^3P_J\right) + X\right) \; |R'_{\chi_c}(0)|^2. \tag{4}$$

The color-singlet model has enormous predictive power. The cross section producing a quarkonium state in any high-energy process is predicted in ms of a single nonperturbative parameter for each orbital-angular-momentum iltiplet. For example, the nonperturbative factor is $R_\psi(0)$ for the s-wave states and η_c. It is $R'_{\chi_c}(0)$ for the p-wave states χ_{c0}, χ_{c1}, χ_{c2}, and h_c. Moreover, se parameters can be determined from decays of the charmonium states. For xample, $R_\psi(0)$ can be determined from the electronic width of the ψ:

$$\Gamma(\psi \rightarrow e^+e^-) \approx \frac{4\alpha^2}{9m_c^2}|R_\psi(0)|^2. \qquad 5.$$

'hus, the color-singlet model gives absolutely normalized predictions for the roduction rates of charmonium states in high-energy collisions.

In spite of its great predictive power, the color-singlet model is only a model. Io theorems guarantee that the amplitude factorizes in the simple way assumed n Equations 1 and 2. In particular, it was never proven that higher-order radiative orrections would respect the factorized form. In addition, the model is clearly ncomplete. For one thing, relativistic corrections, which take into account the elative velocity v of the quark and antiquark, are neglected. These corrections re probably not negligible for charmonium, because the average value of v^2 s only about one third. The color-singlet model also assumes that a $c\bar{c}$ pair roduced in a color-octet state will never bind to form charmonium. This ssumption must break down at some level, because a color-octet $c\bar{c}$ pair can nake a nonperturbative transition to a color-singlet state by radiating a soft luon. The most glaring evidence that the color-singlet model is incomplete omes from the presence of infrared divergences in the cross sections for p-vave states. This problem and its solution are discussed in Section 4. For the noment, we simply note that the infrared divergence violates the factorization ssumption implicit in Equation 4. It implies that the cross-section $d\hat{\sigma}$ is ensitive to small momentum scales, so it cannot be calculated reliably using erturbative QCD.

:.2 *Expectations for Charmonium at Large p_T*

Iost calculations of charmonium production prior to 1993 were based on two rucial assumptions. The first was that the amplitude for the formation of a harmonium state from a $c\bar{c}$ pair was accurately described by the color-singlet nodel. The second was that the dominant production processes for color-inglet $c\bar{c}$ pairs were the Feynman diagrams that were lowest order in α_s. The rst thorough treatment of the problem of charmonium production at large ransverse momentum in hadron-hadron collisions was given by Baier & Rückl n 1983 (4). In subsequent calculations (10), the contributions from B-meson ecay into charmonium states were included. The results of these calculations re summarized below.

There is specific terminology for describing charmonium production in high-nergy colliders. Charmonium is *prompt* if the point at which the charmonium tate is produced and the collision point of the colliding beams cannot be re-olved using a vertex detector. Prompt charmonium is produced by QCD roduction mechanisms. The decay of b-hadrons produces charmonium that is ıot prompt. If a b-hadron is produced with large transverse momentum, it will ravel a significant distance before decaying weakly. For a b-hadron with p_T ıround 10 GeV, the displacement between the collision point and the secondary /ertex where the charmonium is produced is typically a fraction of a millimeter.

A charmonium state that is prompt but does not come from the decay of a ıigher charmonium state is called *direct*. For example, the ψ is produced in he decays of χ_{c0}, χ_{c1}, χ_{c2}, and ψ' with branching fractions of approximately).7, 27, 14, and 57%, respectively. Thus, the prompt ψ signal includes direct ψs as well as contributions from direct χ_{cJ} and direct ψ'.

A charmonium state with large momentum is *isolated* if there are no other ıadrons whose momentum is nearly collinear. Charmonium that is produced rom the decay of a b-hadron with large transverse momentum is never isoated, because the remnant hadrons from the decay of the b-hadron will have nomentum that is nearly collinear.

There are several mechanisms for the production of charmonium with large p_T in $p\bar{p}$ collisions. Charmonium produced from the weak decay of b-hadrons vith large p_T is neither prompt nor isolated. The dominant mechanism for ›roducing b-hadrons at large p_T is the gluon-fusion process $gg \to b\bar{b}$, folowed by the hadronization of the b or $\bar{b}$. The inclusive branching fractions 'or any b-hadron to decay into charmonium states are, presumably, close to hose for B mesons. The inclusive branching fractions for B-meson decays are ıpproximately 1.3% for ψ, 1% for χ_{c1}, and 0.5% for ψ'.

The color-singlet model also gives predictions for the production rate of charnonium from QCD mechanisms. If a $c\bar{c}$ pair is produced with large p_T through ›arton collisions, there must be a recoiling parton to balance the transverse momentum. Thus, the production mechanisms that are of leading order in α_s are $2 \to 3$ parton processes. Note that the charmonium states produced by these ›rocesses are prompt and isolated. In the color-singlet model, the only $2 \to 3$ ›rocess that can produce ψ or ψ' is $gg \to c\bar{c} + g$. The $2 \to 3$ processes that ›roduce χ_{cJ}s with large p_T are $gg \to c\bar{c} + g$, $gq \to c\bar{c} + q$, $g\bar{q} \to c\bar{c} + \bar{q}$, ınd $q\bar{q} \to c\bar{c} + g$. The cross sections for these $2 \to 3$ processes are all on the ›rder of α_s^3. At large p_T, the parton differential cross-sections $d\hat{\sigma}/dp_T^2$ scale ike $1/p_T^6$ for the χ_{cJ} states and like $1/p_T^8$ for ψ and ψ'. Thus, the parton cross ections for ψ and ψ' are suppressed relative to those for χ_{cJ} by a factor of m_c^2/p_T^2 at large p_T.

The color-singlet model at leading order in α_s gives the following predictions 'or charmonium production in $p\bar{p}$ collisions (10). The contribution from b-ıadron decay falls off the most slowly with p_T, and it was predicted to dominate ıt the Tevatron for $p_T > 7$ GeV. Of the prompt production mechanisms for ψ, he most important was found to be the decay of direct χ_{c1}. The contributions

:om direct χ_{c2} and direct ψ were down by factors of about 4 and 18 at $p_T =$ 0 GeV. Thus, the conventional wisdom before 1993 was that ψ production at ırge p_T at the Tevatron should be dominated by b-hadron decay, with decays of irect χ_{c1} and direct χ_{c2} being the only other important mechanisms. In the case f ψ', the decay of b-hadrons was predicted to be the only important production ıechanism at large p_T. It should be emphasized that these predictions were ased on the color-singlet model and on the additional assumption that the ominant parton processes were on the order of α_s^3.

..3 *Prompt Charmonium at the Tevatron*

'he first substantial data on the production of charmonium in $p\bar{p}$ collisions ame from the CERN S$p\bar{p}$S operating at a center-of-mass energy of 630 GeV. 'hat data, collected by the UA1 collaboration, already indicated deviations :om the predictions of the color-singlet model (11). The Tevatron, operating t the significantly higher energy of 1.8 TeV, provided an opportunity to investigate charmonium production in much greater detail. Working there, the CDF ollaboration accumulated large samples of data on the production of ψ, χ_{cJ}, nd ψ'. Although analysis of the data from the 1988–1989 collider run was ampered by the inability to separate prompt charmonium from charmonium roduced by the decays of b-hadrons, discrepancies between the data and the redictions of the color-singlet model were evident (12). However, assuming ıe conventional wisdom that ψ production at large p_T should be dominated by ıe decays of b-hadrons and direct χ_c's, CDF extracted a value for the b-quark ross section from their data on ψ and χ_c production (13). This value was ıown to be too large by about a factor of 2 in the subsequent collider run (14).

Before the 1992–1993 run of the Tevatron, CDF installed a silicon vertex etector that can resolve secondary vertices separated by distances greater than bout 10 μm from the $p\bar{p}$ collision point (15). Prompt charmonium is produced ssentially at the collision point, whereas charmonium from the decay of b-adrons is produced at a secondary vertex typically hundreds of microns away. 'hus, the silicon vertex detector can be used to separate prompt charmonium :om charmonium produced in b-hadron decays.

In the charmonium data sample collected during the 1992–1993 run, it was ›und that only 20% of ψs and only 23% of ψ's come from the decay of b-adrons (16). Furthermore, only 32% of ψs come from χ_c decay (17). Thus, ıe majority of ψs and ψ's must be produced by other mechanisms, in dramatic ontradiction to conventional wisdom. Furthermore, the fraction of ψ and ψ' :om b-hadron decay does not increase appreciably with increasing p_T, contrary › predictions of the color-singlet model.

An excellent review of the results on quarkonium production from the Teva-'on, and also from other experiments, has been presented by Sansoni (18).

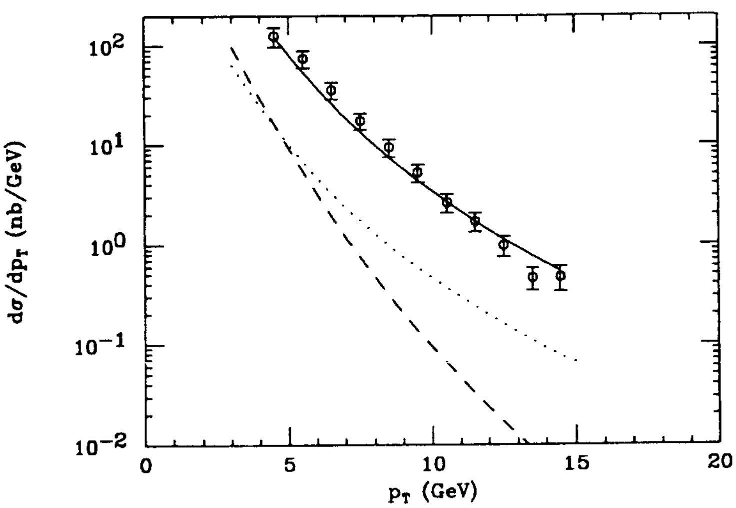
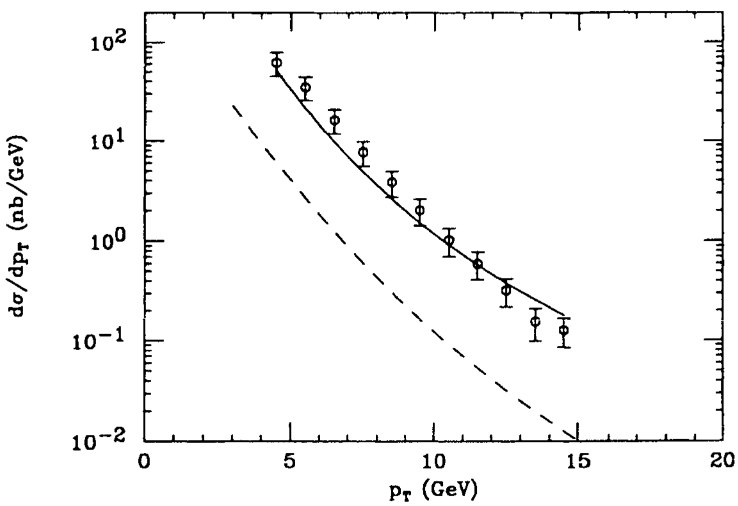


Figure 1 CDF data on the differential cross section for prompt ψs that do not come from χ_c decay as a function of p_T, the leading-order predictions of the color-singlet model (*dashed curve*), the predictions from fragmentation in the color-singlet model (*dotted curve*), and the contribution from gluon fragmentation via the color-octet mechanism (*solid curve*) with the normalization adjusted to fit the CDF data.

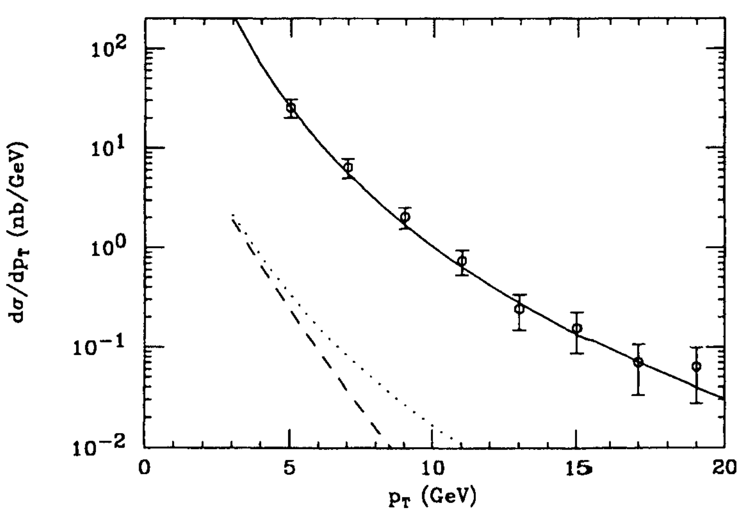
CDF has measured the cross sections for prompt ψ and prompt ψ' production as a function of p_T (16). The prompt ψ signal has been resolved into those ψs that come from χ_c decays and those that do not (17). In addition, the ratio of the production rates of χ_{c1} and χ_{c2} has been measured. The most recent CDF data on the production of prompt ψs that do not come from χ_c decay, prompt ψs from χ_c decay, and prompt ψs are shown in Figures 1, 2, and 3, respectively. The prompt ψs that do not come from χ_c decay include direct ψs and ψs from the decay of direct ψ's. The predictions of the color-singlet model at leading order in α_s are shown as dashed lines. These predictions fall several orders of magnitude below the data at large p_T. Thus, the predictions of the color-singlet model at lowest order in α_s fail dramatically when confronted with the data on charmonium production from the Tevatron.

3. FRAGMENTATION

The first major conceptual advance in the new picture of heavy quarkonium production was the realization that fragmentation dominates at sufficiently large transverse momentum. Fragmentation is the formation of a hadron within a



gure 2 CDF data on the differential cross section for prompt ψs from χ_c decay as a function p_T (*dashed, solid curves* as described in Figure 1).



gure 3 CDF data on the differential cross section for prompt ψ's as a function of p_T (*curves* described in Figure 1).

;t produced by a parton (quark, antiquark, or gluon) with large transverse ıomentum. As the parton emerges from the collision point, it radiates gluons ıd other partons, most of which are almost collinear. The partons ultimately ɔalesce into the hadrons that make up the jet. In the case of production of ıarmonium by fragmentation, the jet containing the charmonium state may ot qualify as a jet by conventional experimental definitions. For example, if ıost of the momentum of the jet is carried by the charmonium state, it may not ıtisfy a jet criterion that requires a specified number of tracks above a certain ıomentum threshold.

The word fragmentation is sometimes used to describe the coalescence of artons into hadrons, whether or not these partons make up a jet. We prefer › use *hadronization* to describe this general process. The word fragmentation 'e reserve specifically for the formation of a hadron within the jet produced y a high-p_T parton. Thus, fragmentation involves the hadronization of the artons in the jet, but hadronization also occurs in low-energy processes that ave nothing to do with jets. Fragmentation is a useful concept because the robability for the formation of a hadron within a jet is universal, i.e. it is ıdependent of the process that produces the parton that initiates the jet.

In this section, we introduce the factorization theorems of perturbative QCD ıat guarantee that inclusive hadron production at large transverse momentum is ominated by fragmentation. We illustrate the factorization theorems using the mple example of the electromagnetic production of ψ in Z^0 decay. Finally, 'e discuss the color-singlet model predictions for the fragmentation functions f heavy quarkonium.

.1 *Factorization Theorems of Perturbative QCD*

'ne of the classic factorization theorems of perturbative QCD (19) guarantees ıat inclusive hadron production in e^+e^- annihilation at sufficiently large en- :gies is dominated by fragmentation. Consider the production of a hadron I with energy E at center-of-mass energy $\sqrt{s}$. We are interested in the cross :ction in the *scaling limit* in which $E, \sqrt{s} \to \infty$ with $E/\sqrt{s}$ held fixed. The roduction of the hadron also involves lower momentum scales, such as the adron mass and the scale $\Lambda_{\rm QCD}$ associated with nonperturbative effects in 'CD. The theorem states that the cross section in the scaling limit has the form [9)

$$d\sigma(e^+e^- \to H(E) + X) = \sum_i \int_0^1 dz\, d\hat{\sigma}(e^+e^- \to i(E/z) + X, \mu)\, D_{i\to H}(z, \mu), \quad 6.$$

'here the sum is over parton types i and the integral is over the longitudinal ıomentum fraction z of the hadron H relative to the parton i. In Equation 6, $d\hat{\sigma}$; the differential cross section for producing a parton i with total energy E/z. his cross section is only sensitive to momenta on the order of E, so it can be alculated by using QCD perturbation theory. The effects of lower momentum ;ales can be systematically factored into the functions $D_{i \to H}(z, \mu)$, which are alled fragmentation functions, or parton decay functions. The fragmentation ınction $D_{i \to H}(z, \mu)$ gives the probability that the jet initiated by parton i 'ill include a hadron H carrying a fraction z of the jet momentum. The ıctorization theorem holds to all orders in perturbation theory. For a light adron H whose mass is on the order of $\Lambda_{\rm QCD}$, all corrections to the factoriz- ıion formula (Equation 6) fall like powers of $\Lambda_{\rm QCD}/E$.

The essential ingredient in the proof of the factorization theorem was given y Collins & Sterman (20). They demonstrated that a diagram that contributes › the inclusive cross section in the scaling limit can be separated into a hard- ;attering subdiagram that produces hard partons, jet-like subdiagrams for each f the hard partons, and a soft part. The soft part includes soft gluon lines that ın couple to any of the jet-like subdiagrams. After summing all possible onnections of the soft gluons, one finds that the effects of the soft parts cancel ut, leaving a factorized form for this contribution to the inclusive cross section. ı the case of inclusive hadron production, these ideas were used by Curci : al and by Collins & Soper (21) to provide field theoretic definitions of the agmentation functions.

The factorization theorem (Equation 6) requires the introduction of an ·bitrary scale μ that separates the large momentum scale E from the lower ıomentum scales. The parton cross sections and the fragmentation functions ;pend on the arbitrary scale μ in such a way that the cross section is inde- ;ndent of μ. The μ dependence of the fragmentation functions is given by an /olution equation of the form

$$\mu^2 \frac{\partial}{\partial \mu^2} D_{i \to H}(z, \mu) = \sum_j \int_z^1 \frac{dy}{y} P_{i \to j}(z/y, \mu)\, D_{j \to H}(y, \mu). \qquad 7.$$

he kernel $P_{i \to j}(x, \mu)$ describes the splitting of a parton i into a parton j with ıomentum fraction x and is calculable as a perturbation series in $\alpha_s(\mu)$. At ading order in α_s, these kernels are identical to the Altarelli-Parisi functions ıat govern the evolution of parton distributions.

The factorization theorem for inclusive hadron production in e^+e^- annihila- on can be generalized to other high-energy processes. The real power of these ıctorization theorems lies in the fact that the fragmentation functions are uni- ;rsal, i.e. they are independent of the process that produces the fragmenting artons. Thus, if the fragmentation functions for a hadron H are determined 'om e^+e^- annihilation data, they can be used to predict the production rate of ıe hadron H in jets produced by other high-energy processes.

One of the important generalizations of the factorization theorem is to in-lusive hadron production at large transverse momentum p_T in hadron-hadron ɔllisions. The proof of the factorization theorem for this process is more dif-cult, because there are two hadrons in the initial state. In fact, such a proof as actually been carried out, but only for the simpler case of the Drell-Yan rocess for creating a muon pair (22). However, there are no apparent obstacles ı extending this proof to the case of inclusive hadron production at large p_T 19). The resulting factorization formula is

$$'\sigma(AB \to H(p_T) + X) = \sum_{ijk} \int_0^1 dx_1\, f_{j/A}(x_1) \int_0^1 dx_2\, f_{k/B}(x_2) \times \int_0^1 dz\, d\widehat{\sigma}(jk \to i(p_T/z) + X)\, D_{i\to H}(z). \quad 8.$$

ong-distance effects can be factored into the fragmentation functions and ıto the parton distributions $f_{j/A}(x_1)$ and $f_{k/B}(x_2)$ for hadrons A and B. The ɛstriction to large p_T is necessary so that the jets consisting of the remnants of adrons A and B after the hard scattering have large momentum relative to the ɛt containing hadron H. Equation 8 should hold to all orders in perturbation ıeory, with corrections falling like powers of Λ_{QCD}/p_T. Implicit in Equation are three arbitrary scales: the factorization scale μ_F, which cancels between ıe parton distributions and $d\widehat{\sigma}$; the fragmentation scale μ_{frag}, which cancels ɛtween $d\widehat{\sigma}$ and the fragmentation functions; and the renormalization scale $_R$ for the running coupling constant, which appears in $d\widehat{\sigma}$. In low-order alculations, μ_{frag}, μ_F, μ_R should all be chosen on the order of p_T/z, the ansverse momentum of the fragmenting parton.

In 1993, Braaten & Yuan (23) pointed out that the factorization theorems for ıclusive hadron production must apply to heavy quarkonium as well as to light adrons. The only difference is that the leading corrections fall as powers of $_Q/p_T$, where m_Q is the heavy quark mass, instead of as powers of Λ_{QCD}/p_T. herefore, the dominant production mechanism for charmonium at $p_T \gg m_c$ ıust be fragmentation. In retrospect, this statement may seem obvious, but 'agmentation contributions were not included in any of the previous calcula-ons of charmonium production in $p\bar{p}$ collisions summarized in Section 2.2. his can easily be seen from the p_T dependence of the parton cross sections. In ıe factorization formula (Equation 8), the only momentum scale that the par-ɔn cross-section $d\widehat{\sigma}$ can depend on is p_T. Therefore, by dimensional analysis, $\widehat{\sigma}/dp_T^2$ scales like $1/p_T^4$ at large p_T. The color-singlet model cross sections at leading order in α_s fall off much more rapidly with p_T: $d\widehat{\sigma}/dp_T^2$ scales like $1/p_T^8$ for ψ and ψ' and like $1/p_T^6$ for χ_{cJ}.

3.2 *Electromagnetic Production of ψ in Z^0 Decay*

Electromagnetic production of ψ is a particularly simple process. The QCD interaction can be treated nonperturbatively by expressing the production amplitude in terms of a matrix element of the electromagnetic current between the QCD vacuum and a ψ:

$$\langle\psi|\tfrac{2}{3}\bar{c}\gamma^{\mu}c|0\rangle \equiv g_{\psi}M_{\psi}^{2}\epsilon^{\mu}, \qquad 9.$$

where ϵ^{μ} is the polarization vector for the ψ, and g_{ψ} is dimensionless. The value of g_{ψ} can be determined from the decay rate for $\psi \to e^+e^-$:

$$\Gamma(\psi \to e^+e^-) = \frac{4\pi}{3}\alpha^2 M_{\psi} g_{\psi}^2. \qquad 10.$$

This gives the value $g_{\psi}^2 = 0.008$.

The various contributions to electromagnetic ψ production in Z^0 decay can be calculated by using perturbation theory in the electromagnetic coupling constant α. Since this coupling constant is small, we might naively expect the dominant production process to be the one that is lowest order in α. We must remember, however, that there is also another small, dimensionless parameter in this problem, namely M_{ψ}/M_Z. The relative importance of a production process will be determined both by its order in α and by how it scales with M_{ψ}/M_Z. The leading-order process is suppressed by M_{ψ}^2/M_Z^2, and the dominant production process is actually higher order in α.

The production process for ψ that is lowest order in α is $Z^0 \to \psi\gamma$, which proceeds at order α^2 through the process $Z^0 \to c\bar{c} + \gamma$. For the $c\bar{c}$ to form a ψ, the relative momentum of the c and $\bar{c}$ in the ψ rest frame must be small compared with m_c. This, in turn, is small compared to the momentum of the ψ, which is approximately $M_Z/2$. If the relative momentum of the c and $\bar{c}$ is neglected, the amplitude is proportional to the matrix element $\langle\psi|\bar{c}\gamma^{\mu}c|0\rangle$, and it, therefore, can be expressed in terms of g_{ψ}. The branching fraction for $Z^0 \to \psi\gamma$ is 5.2×10^{-8} (5). In the limit $M_{\psi} \ll M_Z$, the branching fraction is proportional to $\alpha g_{\psi}^2 M_{\psi}^2/M_Z^2$, so it vanishes in the scaling limit. The suppression factor M_{ψ}^2/M_Z^2 reflects the fact that the c or $\bar{c}$ must receive a momentum kick on the order of M_Z from the photon in order for the c and $\bar{c}$ to be produced with small enough relative momentum to form a bound state.

At next-to-leading order in α, the ψ can be produced electromagnetically via the decay $Z^0 \to \psi + \ell^+\ell^-$, which proceeds at order α^3 through the process $Z^0 \to c\bar{c} + \ell^+\ell^-$. The branching fraction for $Z^0 \to \psi + \ell^+\ell^-$ is 7.5×10^{-7} (24, 25). Thus, the decay rate for this order-α^3 process is an order of ıagnitude larger than the order-α^2 process $Z^0 \to \psi\gamma$. The reason it is larger that the factor of M_ψ^2/M_Z^2 in the decay rate for $Z^0 \to \psi\gamma$ provides larger ıppression than does the extra factor of α in the decay rate for $Z^0 \to \psi + \ell^+\ell^-$.

We introduce a scaling limit for the production of ψ with energy E in Z^0 ecay. This limit is $E, M_Z \to \infty$ with E/M_Z and M_ψ held fixed. In the :aling limit, the differential decay rate for the electromagnetic production of · satisfies a factorization theorem analogous to the one in Equation 6:

$$d\Gamma(Z^0 \to \psi(E) + X) = \sum_i \int_0^1 dz\, d\widehat{\Gamma}(Z^0 \to i(E/z) + X, \mu)\, D_{i\to\psi}(z, \mu), \qquad 11.$$

'here the sum over partons i includes photons and positive and negative leptons. he factor $d\widehat{\Gamma}$ is the inclusive rate for decay into a parton i with energy E/z. he factor $D_{i\to\psi}(z)$ is a fragmentation function that gives the probability for an ectromagnetic jet initiated by the parton i to include a ψ that carries a fraction of the jet momentum.

Fleming (26) pointed out that because the differential decay rate for the pro- ɛss $Z^0 \to \psi + \ell^+\ell^-$ does not vanish in the scaling limit, it must be expressible ı the form shown in Equation 11. He found that at leading order in α and in ıe scaling limit, it can be written as

$$\frac{d\Gamma}{dz}(Z^0 \to \psi(z) + \ell^+\ell^-) = 2\,\Gamma(Z^0 \to \ell^+\ell^-)\, D_{\ell\to\psi}(z, \mu) + \frac{d\widehat{\Gamma}}{dz}(Z^0 \to \gamma(z) + \ell^+\ell^-, \mu)\, P_{\gamma\to\psi}, \qquad 12.$$

'here $z = 2E/M_Z$. The photon fragmentation probability is $P_{\gamma\to\psi} = 4\pi\alpha g_\psi^2$: 7×10^{-4}. The lepton fragmentation function is

$$D_{\ell\to\psi}(z, \mu) = 2\alpha^2 g_\psi^2 \left[\frac{(z-1)^2 + 1}{z} \log\frac{z\mu^2}{M_\psi^2} - z\right], \qquad 13.$$

'here μ is an arbitrary factorization scale that separates the scales M_Z and I_ψ. The μ-dependence of the fragmentation function cancels against that of $\widehat{\Gamma}/dz$ in Equation 12. Note that all the effects of QCD, both perturbative and onperturbative, are absorbed into the factor g_ψ^2 in $P_{\gamma\to\psi}$ and into Equation 13. ı the decay rate for $Z^0 \to \psi + \ell^+\ell^-$, the absence of kinematic suppression ıctors like M_ψ^2/M_Z^2 can be attributed to the fact that the $c\bar{c}$ pair that forms the · is produced at a momentum scale on the order of m_c rather than M_Z. The ırge momentum scale M_Z only enters in the decay of the Z^0 into the partons $^+$, ℓ^-, and γ.

.3 *Fragmentation Functions in the Color-Singlet Model*

n 1993, Braaten & Yuan pointed out that the color-singlet model gives predic-ons for the fragmentation functions for the formation of heavy quarkonium n quark and gluon jets (23). They calculated the fragmentation functions ex-licitly for $g \to \eta_c, \psi$ to leading order in α_s. For example, the fragmentation unction for $g \to \eta_c$ is calculated from the parton process $g \to c\bar{c} + g$ and is n the order of α_s^2:

$$D_{g\to\eta_c}(z, 2m_c) = \frac{\alpha_s^2(2m_c)|R_{\eta_c}(0)|^2}{24\pi m_c^3}\left[3z - 2z^2 + 2(1-z)\log(1-z)\right]. \qquad 14.$$

his is the fragmentation function at an initial scale on the order of $2m_c$. It an be evolved up to a higher scale μ by using the evolution Equations 7. he fragmentation function for $g \to \psi$ is calculated from the parton process $\to c\bar{c} + gg$ and is on the order of α_s^3. The fragmentation functions for $\to \eta_c, \psi$ (27), and $b \to B_c, B_c^*$ (28) were subsequently calculated by Braaten t al. The fragmentation functions for $b \to B_c, B_c^*$ had actually been calculated arlier by Chang & Chen (29), although they did not note the universality of nese functions.

The fragmentation functions for p-wave quarkonium states have also been alculated in the color-singlet model to leading order in α_s. These fragmentation unctions include $g \to \chi_{cJ}$ (30, 31), $c \to \chi_{cJ}$, and $b \to p$-wave B_c (32, 33, 34). ome of the fragmentation functions for d-wave states have also been calculated 35, 36). The color-singlet model fragmentation functions for $g \to \chi_{cJ}$ are alculated from the parton process $g \to c\bar{c} + g$ and are logarithmically infrared-ivergent at leading order in α_s. The solution to this problem is discussed n Section 4. It should be noted that there is a discrepancy between the two alculations of the fragmentation functions for $g \to \chi_{cJ}$ that has not yet been esolved (30, 31).

The color-singlet model fragmentation functions can be resolved into the ontributions from each of the possible values of the helicity h of the char-nonium state. The fragmentation functions for $c \to \psi_L, \psi_T$, where ψ_L and ψ_T represent the longitudinal ($h = 0$) and transverse ($|h| = 1$) polarization omponents of the ψ, were first calculated by Chen (32) and by Falk et al (37). he fragmentation functions for $g \to \chi_{cJ}$ (31, 38), $b \to B_c^*$ (32, 39), and $b \to$ -wave $\bar{b}c$ states (32, 33) have also been resolved into the contributions from ne individual helicities.

In the method developed by Braaten & Yuan, the fragmentation functions vere extracted by taking the scaling limit of color-singlet model cross sections nd expressing them in the factorized form demanded by the QCD factorization ıeorems. This method would be rather cumbersome beyond leading order in $_s$. The fragmentation functions can also be calculated directly from the field ıeoretic definitions (31, 34, 40). This method provides a significant advantage ɔr calculating fragmentation functions beyond leading order in α_s. Higher-rder corrections may be particularly important if the leading-order calculation ives a soft fragmentation function that is small for z near 1 (41).

. COLOR-OCTET MECHANISMS

he second major conceptual advance in the recent revolution in heavy-quark-nium production is the realization that color-octet mechanisms can be impor-ınt. Contrary to the basic assumption of the color-singlet model, a $c\bar{c}$ pair that . produced in a color-octet state can bind to form charmonium. Although a $\bar{c}$ pair that forms charmonium must ultimately be in a color-singlet state, the ıort-distance part of the process can involve the production of a color-octet $c\bar{c}$ air.

.1 *Color-Octet Mechanism for p-Waves*

he most glaring evidence that the color-singlet model is incomplete comes om the presence of infrared divergences in the production cross sections for -wave states. There are analogous divergences in the annihilation decay rates ɔr p-wave states, which were discovered by Barbieri and collaborators as early ; 1976 (42). They found that the decay rate for $\chi_{cJ} \rightarrow q\bar{q}g$ depends logarith-ıically on the minimum energy of the final-state gluon. In phenomenological eatments of the annihilation decay rates of p-wave states, the infrared singu-rity was avoided by imposing an ad hoc infrared cutoff on the gluon energy. his cutoff was sometimes identified with the binding energy of the charmo-ium state, but without any good justification.

The presence of infrared divergences in the color-singlet model is not a roblem if the divergences can be absorbed into the nonperturbative factor in ıe cross section. For example, there are linear infrared divergences associated ith the Coulomb singularity in radiative corrections to s-wave cross sections, ıt they can be factored into $|R(0)|^2$. Similarly, the radiative corrections for -waves include a linear infrared divergence that can be factored into $|R'(0)|^2$. owever, the structure of the logarithmic infrared divergences for p-waves is ıch that they cannot be factored into $|R'(0)|^2$.

The solution to this problem was given by Bodwin et al in 1992 (43). They ɔted that the infrared divergence in the annihilation rate for $\chi_{cJ} \rightarrow q\bar{q}g$ arises hen the final-state gluon becomes soft. After radiating the soft gluon from ther the c or the $\bar{c}$ in the color-singlet ^3P_J bound state, the $c\bar{c}$ pair is in a color-octet $^3S_1$ state. It then annihilates through the process $c\bar{c} \to q\bar{q}$. In this region of phase space, the short-distance part of the decay amplitude is, therefore, the annihilation of a $c\bar{c}$ pair in a color-octet $^3S_1$ state. Thus, in addition to the conventional term in the color-singlet model, the factorization formula for the decay rate of the χ_{cJ} into light hadrons must include a second term. In the color-singlet model term, the short-distance factor is the annihilation rate of a $c\bar{c}$ pair in a color-singlet ^3P_J state, and the long-distance nonperturbative factor is proportional to $|R'_{\chi_c}(0)|^2$. In the second term, the short-distance factor is the annihilation rate of a $c\bar{c}$ pair in a color-octet $^3S_1$ state, and the long-distance factor is the probability for the χ_{cJ} to contain a pointlike $c\bar{c}$ pair in a color-octet $^3S_1$ state. The color-octet term can be interpreted as a contribution to the decay rate from the $c\bar{c}g$ component of the χ_{cJ} wavefunction.

The solution to the problem of infrared divergences in the production cross sections for the χ_{cJ} states is similar (44). The infrared divergence arises from the radiation of a soft gluon from the c or $\bar{c}$ that form the color-singlet ^3P_J bound state. Before the radiation, the $c\bar{c}$ pair is in a color-octet $^3S_1$ state. The short-distance part of the process in this region of phase space is the production of a $c\bar{c}$ pair with small relative momentum in a color-octet $^3S_1$ state. The factorization formula for p-wave cross sections must, therefore, include two terms:

$$d\sigma(\chi_{cJ}+X) = d\widehat{\sigma}\left(c\bar{c}\left(\mathbf{1},{}^3P_J\right)+X\right)\ \left|R'_{\chi_c}(0)\right|^2 + (2J+1)\, d\widehat{\sigma}\left(c\bar{c}\left(\mathbf{8},{}^3S_1\right)+X\right)\ \left\langle \mathcal{O}_8^{\chi_c}\right\rangle . \qquad 15.$$

The first term is the conventional term of the color-singlet model. In this term, the short-distance factor is the cross section for producing a $c\bar{c}$ pair with small relative momentum in a color-singlet ^3P_J state. The second term represents a contribution to the cross section from a color-octet mechanism. The short-distance factor in this term is the cross section for producing a $c\bar{c}$ pair with small relative momentum in a color-octet $^3S_1$ state. The long-distance nonperturbative factor $\langle \mathcal{O}_8^{\chi_c}\rangle$ is proportional to the probability for a pointlike $c\bar{c}$ pair in a color-octet $^3S_1$ state to bind and form a χ_c. Equation 15 has been applied to χ_{cJ} production in B-meson decays (44), in Υ decays (45), and in photoproduction (46).

Both factorization Equations 3, for s-waves, and 15, for p-waves, hold to all orders in perturbation theory in the nonrelativistic limit. This is the limit in which the typical velocity v of the heavy quark in the bound state goes to 0. Unlike the case of s-waves, a color-octet production mechanism is required by perturbative consistency in the case of p-wave states.

This can be understood from the NRQCD factorization approach discussed in Section 4.2. This approach indicates that the color-singlet term for p-waves is suppressed by a factor of v^2 compared with s-waves. It is this suppression of the color-singlet term that makes it necessary to include the color-octet term in the case of p-waves.

4.2 *NRQCD Factorization*

The simple Equations 3, for s-waves, and 15, for p-waves, are correct in the nonrelativistic limit, i.e. in the limit $v \to 0$, where v is the typical relative velocity of the c and $\bar{c}$ in charmonium. Potential-model calculations indicate that v^2 is only about one third for charmonium and about one tenth for bottomonium. Thus, the neglect of relativistic corrections suppressed by powers of v^2 introduces a systematic theoretical error that may not be negligible. For specific processes, the problem may be worse than the magnitude of v^2 suggests. If there are other small parameters in the problem, such as $\alpha_s(m_c)$ or kinematical parameters such as m_c^2/p_T^2, the terms that are of leading order in v^2 may be suppressed by these other parameters. If this suppression is large enough, terms that are subleading in v^2 may actually dominate.

The key to understanding the structure of the relativistic corrections to quarkonium production is to unravel the momentum scales in the problem. If there is a very large momentum scale that is set by the kinematics of the production process, such as M_Z in the case of Z^0 decay or p_T in the case of production at large transverse momentum in $p\bar{p}$ collisions, this scale can be removed from the problem by using the factorization theorems of perturbative QCD discussed in Section 3. The next largest momentum scale is the heavy-quark mass m_c, which is certainly important in the production of a $c\bar{c}$ pair. The actual formation of the bound state from the $c\bar{c}$ pair involves smaller momentum scales, including the typical momentum $m_c v$ of the heavy quark in quarkonium, its typical kinetic energy $m_c v^2$, and the scale Λ_{QCD} of generic nonperturbative effects in QCD. In charmonium and bottomonium, the mass m_Q is large enough that effects associated with this scale should be calculable by using perturbation theory in the running coupling constant $\alpha_s(m_Q)$. Nonperturbative effects become important at smaller momentum scales.

A powerful tool for isolating the effects of the scale m_Q is NRQCD, an effective field theory developed by Caswell & Lepage (47). NRQCD is a formulation of QCD in which the heavy quark and antiquark are treated nonrelativistically. They are described by a Schroedinger field theory with separate two-component spinor fields, rather than with a single four-component Dirac field. The gluons and light quarks are described by the relativistic Lagrangian for ordinary QCD.

NRQCD can accurately describe the physics of heavy quarkonium at energies that are much less than m_Q above the rest mass. This follows from the fact that virtual states with energies on the order of m_Q or larger have lifetimes on the order of $1/m_Q$ or smaller. In this time, quanta can propagate over distances that are at most of order $1/m_Q$. Since this distance is much smaller than the ze of the quarkonium bound state, the effects of these virtual states can be ken into account through local interaction terms in the NRQCD Lagrangian.

The inverse of the mean radius of a quarkonium state is a dynamically gen- ated momentum scale that is much smaller than its rest mass. The ratio of iese momentum scales defines a small parameter v, which can be identified ith the typical velocity of the heavy quark in the quarkonium state. This small arameter v, which is defined nonperturbatively, can be exploited to organize alculations of heavy-quarkonium observables into expansions in powers of ^{2}. NRQCD is equivalent to full QCD in the sense that the parameters in the RQCD Lagrangian can be tuned so that its predictions agree with those of ill QCD to any desired order in v^2 (48).

Bodwin et al (49) have recently developed a theoretical framework for inclu- ve quarkonium production that allows relativistic corrections to be included any desired order in v^2. This framework provides a factorization formula for iclusive cross sections in which all effects of the scale m_c are separated from ie effects of lower momentum scales, including $m_c v$. The derivation of this ictorization formula involves two steps. The first step is the topological fac- ization of diagrams that contribute to the cross section. This step is similar the derivations of the factorization theorems of perturbative QCD discussed Section 3.1. For simplicity, we consider the case of charmonium production e^+e^- annihilation at a center-of-mass energy $s^{1/2}$ that is significantly larger ian $2m_c$. We define a scaling limit by $s^{1/2}, m_c \to \infty$ with $s^{1/2}/m_c$ fixed. diagram that contributes to the cross section in this limit can be separated ito a hard-scattering subdiagram that produces hard partons and a $c\bar{c}$ pair with mall relative momentum, a jet-like subdiagram for each of the hard partons, a ubdiagram that involves the c and $\bar{c}$, and a soft part. The soft part includes soft luons that can couple to the jet-like subdiagrams and to the $c\bar{c}$ subdiagram. fter summing over all possible connections of the soft gluons, the effects of ie soft gluons that are connected to the jet-like subdiagrams cancel out, leav- ig a factored form for this contribution to the cross section. The derivation of pological factorization requires all the hard partons to have large momentum elative to the $c\bar{c}$ pair. Presumably, this can be generalized to processes in which iere are hadrons in the initial state, provided that the charmonium is produced ith transverse momentum that is much larger than Λ_{QCD}.

After the topological factorization of the diagram, the hard-scattering am- litude contains all effects of the scale m_c, but it also depends on the relative iomentum $\mathbf{p}$ of the $c\bar{c}$ pair. To complete the derivation of the factorization ormula, the dependence on $\mathbf{p}$ must be removed from the hard-scattering ampli- ude, so that all effects of the scale $m_c v$ reside in the $c\bar{c}$ subdiagram. This could e accomplished simply by Taylor-expanding the hard-scattering amplitude in

›owers of **p** were it not for the fact that this generates ultraviolet divergences n the $c\bar{c}$ subdiagram. This is where the power of NRQCD becomes evident. This effective field theory can be used to systematically unravel the scales m_c ınd $m_c v$ so that all effects of the scale $m_c v$ are contained in the $c\bar{c}$ subdiagram. Essentially, the $c\bar{c}$ subdiagram is defined to include all the parts of the diagram hat are reproduced by NRQCD. Those parts involving the $c\bar{c}$ pair that are not eproduced by NRQCD must involve the scale m_c and, therefore, can be in- :luded in the hard-scattering subdiagram. The result is a factorization formula or the inclusive cross section for producing a quarkonium state H that holds o all orders in α_s. It has the form

$$d\sigma(H+X) = \sum_n d\widehat{\sigma}(c\bar{c}(n)+X)\ \langle \mathcal{O}_n^H \rangle, \qquad 16.$$

vhere $d\widehat{\sigma}$ is the inclusive cross section for producing a $c\bar{c}$ pair in a color and ıngular-momentum state labeled by n, and having vanishing relative momen- um. The parton cross sections $d\widehat{\sigma}$ involve only momenta on the order of m_c ›r larger, and therefore they can be calculated as perturbation expansions in $\alpha_s(m_c)$. Because $d\widehat{\sigma}$ is insensitive to relative momenta that are much smaller han m_c, it describes the production of a $c\bar{c}$ pair with separation less than or ›n the order of $1/m_c$. This separation is essentially pointlike on the scale of a :harmonium wavefunction, which is on the order of $1/(m_c v)$. Thus, the non- ›erturbative long-distance factor $\langle \mathcal{O}_n^H \rangle$ is proportional to the probability for a ›ointlike $c\bar{c}$ pair in the state n to form the bound state H. Note that the only de- ›endence on the quarkonium state H in Equation 16 resides in the factor $\langle \mathcal{O}_n^H \rangle$.

Equation 16 holds only for inclusive cross sections. The matrix elements $\mathcal{O}_n^H \rangle$ are proportional to the probabilities for the formation of the charmonium tate H, plus light hadrons whose total energy in the H rest frame is on the ›rder of $m_c v^2$. Summation over these light hadronic states is essential for the eparation of short-distance and long-distance effects. The methods required to lerive the factorization formula break down for exclusive cross sections. The lerivation also implies that factorization does not hold at the amplitude level, ontrary to the basic assumption of the color-singlet model.

The factors $\langle \mathcal{O}_n^H \rangle$ in Equation 16 can be expressed as vacuum matrix elements ›f four-quark operators in NRQCD (49). The operator $\mathcal{O}_n^H$ creates a pointlike $\bar{c}$ pair in the state n, projects onto states that in the asymptotic future include he quarkonium state H, and finally annihilates the $c\bar{c}$ pair at the creation point, gain in the state n. The matrix elements that play the most important roles n quarkonium production were denoted $\langle \mathcal{O}_1^H(^{2S+1}\mathrm{L}_J) \rangle$ and $\langle \mathcal{O}_8^H(^{2S+1}\mathrm{L}_J) \rangle$ n Reference 49. The operator $\mathcal{O}_n(^{2S+1}\mathrm{L}_J)$ creates and annihilates a $c\bar{c}$ pair n the state $c\bar{c}(\mathbf{n}, {}^{2S+1}\mathrm{L}_J)$. If the dominant Fock state of the meson H is $c\bar{c}(\mathbf{1}, {}^{2S+1}\mathrm{L}_J)\rangle$, the matrix element $\langle \mathcal{O}_1^H(^{2S+1}\mathrm{L}_J) \rangle$ can be related to the radial ɾavefunction. For example, up to corrections on the relative order of v^4, we ave

$$\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle = \frac{9}{2\pi}|R_\psi(0)|^2, \qquad 17.$$

$$\langle \mathcal{O}_1^{\chi_{cJ}}(^3P_J)\rangle = (2J+1)\frac{9}{2\pi}|R'_{\chi_c}(0)|^2. \qquad 18.$$

f we keep only these terms in Equation 16, we recover Equations 3 and 4 of the olor-singlet model. NRQCD has approximate heavy-quark spin symmetry, nd this implies relations between color-octet matrix elements. For example, ıp to corrections on the relative order of v^2, we have

$$\langle \mathcal{O}_8^{\chi_{cJ}}(^3P_J)\rangle = (2J+1)\langle \mathcal{O}_8^{\chi_{c0}}(^3P_0)\rangle. \qquad 19.$$

dentifying $\langle \mathcal{O}_8^{\chi_{c0}}(^3P_J)\rangle = \langle \mathcal{O}_8^{\chi_c}\rangle$, we find that the corresponding term in Equaion 16 reproduces the color-octet term in the nonrelativistic Equation 15 for ›-wave production.

Equation 16 is not particularly useful in its most general form, because t involves infinitely many nonperturbative factors $\langle \mathcal{O}_n^H\rangle$. However, one can leduce from NRQCD how the various matrix elements scale with v for any ›articular quarkonium state H. For example, the scaling of $\langle \mathcal{O}_n^H(^{2S+1}L_J)\rangle$ vith v is determined by the number of electric dipole and magnetic dipole ransitions that are required to go from the dominant Fock state of the meson H to a state of the form $|c\bar{c}(\mathbf{n}, {}^{2S+1}L_J) + \text{gluons}\rangle$. The matrix element scales ıs v^{3+2L}, multiplied by v^2 for each electric dipole transition and by v^4 for each nagnetic dipole transition.

The relative importance of the various terms in Equation 16 is determined by he order in v of the matrix element $\langle \mathcal{O}_n^H\rangle$ and by the order in $\alpha_s(m_c)$ of the parton ross sections $d\widehat{\sigma}(c\bar{c}(n)+X)$. The NRQCD factorization framework acquires ›redictive power when the expansion (Equation 16) is truncated to some low ›rder in v^2. If we truncate Equation 16 to lowest nontrivial order in v, regardless ›f the order in α_s, we recover Equation 3 for s-waves and Equation 15 for p-vaves. However, if the cross sections $d\widehat{\sigma}$ for these terms are suppressed, other erms in the factorization formula may be important. This observation is the ›asis for a suggestion by Braaten & Fleming (50) that color-octet mechanisms nay also play an important role in s-wave production. The leading color-octet natrix elements for the ψ are $\langle \mathcal{O}_8^{\psi}(^1S_0)\rangle$, $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$, and $\langle \mathcal{O}_8^{\psi}(^3P_J)\rangle$, all of vhich are suppressed by v^4 relative to the leading color-singlet matrix element $\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle$.

If the parton cross sections $d\widehat{\sigma}$ multiplying the color-octet matrix elements ıave the same magnitudes as the parton cross section multiplying $\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle$, hen the corrections from the color-octet terms should be small. However, if the arton cross section multiplying the color-singlet matrix element is suppressed y powers of $\alpha_s(m_c)$ or by small kinematic parameters such as m_c^2/p_T^2, the color-ctet terms could dominate. An example is the gluon fragmentation function or producing a ψ, discussed in Section 4.3.

The implications of color-octet mechanisms for the production of quarko-ium in high-energy colliders are discussed in Section 5. There have also been iany recent investigations of color-octet production mechanisms in lower-nergy experiments, including fixed-target πN and pN collisions (51), B-ieson decays (52), e^+e^- annihilation (53), and photoproduction (54, 55). 'hese applications lie outside the scope of this review.

.3 *Fragmentation Functions for Heavy Quarkonium*

'he NRQCD factorization formula Equation 16 implies that fragmentation inctions for charmonium have the general form

$$D_{i\to H}(z,\mu) = \sum_n d_{i\to n}(z,\mu)\,\langle \mathcal{O}_n^H \rangle, \qquad 20.$$

'here $d_{i\to n}(z,\mu)$ gives the probability for the parton i to form a jet that in-ludes a $c\bar{c}$ pair in the state labeled by n, and where $\langle \mathcal{O}_n^H \rangle$ is proportional to the robability for a pointlike $c\bar{c}$ pair in the state n to bind to form a charmonium tate H. The coefficient $d_{i\to n}(z, 2m_c)$ at the initial scale $\mu = 2m_c$ involves only iomenta on the order of m_c and, therefore, can be calculated as a perturbation xpansion in the running coupling constant $\alpha_s(2m_c)$. The fragmentation func-on can be evolved up to higher scales of μ by using the evolution equations Equation 7). Note that the only dependence on the quarkonium state H on the ght side of Equation 20 resides in the matrix elements.

If the dominant Fock state for the meson H is $|c\bar{c}(\mathbf{1}, {}^{2S+1}\mathrm{L}_J)\rangle$, the fragmen-ition function in the color-singlet model is obtained by keeping only the term i Equation 20 that involves $\langle \mathcal{O}_1^H({}^{2S+1}\mathrm{L}_J)\rangle$. In the case of gluon fragmentation ito χ_{cJ}, the coefficient of $\langle \mathcal{O}_1^{\chi_{cJ}}({}^3\mathrm{P}_J)\rangle$ is logarithmically infrared divergent at ading order in α_s. The corresponding parton process is $g \to c\bar{c}(\mathbf{1}, {}^3\mathrm{P}_J) + g$, nd the divergence arises when the final-state gluon becomes soft. In this re-ion of phase space, the short-distance part of the fragmentation process is $\to c\bar{c}(\mathbf{8}, {}^3\mathrm{S}_1)$, and the divergence from the radiation of the soft gluon should e absorbed into the matrix element $\langle \mathcal{O}_8^{\chi_{cJ}}({}^3\mathrm{S}_1)\rangle$. The resulting expression for ie fragmentation function at leading order in α_s is (30)

$$D_{g\to\chi_{cJ}}(z, 2m_c) = \frac{\alpha_s^2(2m_c)}{m_c^5} d_J(z)\,\langle \mathcal{O}_1^{\chi_{cJ}}({}^3\mathrm{P}_J)\rangle + \frac{\pi\alpha_s(2m_c)}{24m_c^3}\delta(1-z)\,\langle \mathcal{O}_8^{\chi_{cJ}}({}^3\mathrm{S}_1)\rangle, \qquad 21.$$

vhere $d_J(z)$ is a dimensionless function of z. Since the matrix elements are the ame order in v and the color-octet term is lower order in α_s, it can be expected o dominate.

The process $g \rightarrow c\bar{c}(\mathbf{8}, {}^3S_1)$ gives a term on the order of α_s in the gluon ragmentation function for any quarkonium state H:

$$D_{g\rightarrow H}(z, 2m_c) \approx \frac{\pi\alpha_s(2m_c)}{24m_c^3}\delta(1-z)\,\langle \mathcal{O}_8^H({}^3S_1)\rangle. \qquad 22.$$

The delta function in Equation 22 should be interpreted as a distribution in z hat is peaked near $z = 1$, with a width on the order of v^2. All other terms n the fragmentation function either have coefficients $d_{g\rightarrow n}(z)$ on the order of α_s^2 or higher or else have matrix elements that are higher order in v^2 than $\mathcal{O}_8^H({}^3S_1)\rangle$. The importance of the term (Equation 22) in the fragmentation unction depends on the charmonium state H. Surprisingly, it may be the nost important term in the fragmentation functions for the s-wave states ψ ind ψ'. Although the matrix element $\langle \mathcal{O}_8^\psi({}^3S_1)\rangle$ is suppressed by v^4 relative o $\langle \mathcal{O}_1^\psi({}^3S_1)\rangle$, the leading term in the coefficient of the color-singlet matrix element comes from the parton process $g \rightarrow c\bar{c}(\mathbf{1}, {}^3S_1) + gg$ and is therefore on the order of α_s^3. The suppression of the color-singlet term by α_s^2 may be more effective than the suppression of the color-octet term by v^4. The elative importance of the two terms can only be assessed after determining the value of the color-octet matrix element (see Section 5.1 for Tevatron data and in estimate).

5. COLLIDER APPLICATIONS

Our understanding of heavy-quarkonium production in high-energy colliders ias been completely revolutionized by the theoretical developments described n Sections 3 and 4. It is clear that fragmentation contributions can dominate at arge transverse momentum, even if they are higher order in α_s. In addition, the ormation of quarkonium from color-octet $Q\bar{Q}$ pairs can, in some cases, dominate over the color-singlet contributions. In this section, we discuss several applications to collider physics in which these developments play an important ole. These applications are the production of prompt charmonium at the Tevatron, the production of bottomonium and prompt charmonium at LEP, and the production of the B_c in high-energy colliders.

5.1 *Prompt Charmonium at Large p_T in $p\bar{p}$ Collisions*

Much of the impetus for the recent theoretical developments in heavy quarkonium production has come from Tevatron data on prompt charmonium production at large p_T. As discussed in Section 2.3, the experimentally measured rate ıt large p_T is orders of magnitude greater than the predictions of the color-inglet model at leading order in α_s. The Tevatron results, however, can be xplained by a combination of the fragmentation and color-octet mechanisms liscussed in Sections 3 and 4.

In 1993, Braaten & Yuan (23) pointed out that fragmentation should be he most important charmonium production mechanism at sufficiently large p_T. The first explicit calculation of the fragmentation contribution to prompt harmonium production was carried out by Doncheski et al (56). They used the luon and charm quark fragmentation functions from the color-singlet model to alculate the fragmentation contributions to direct ψ and direct ψ' production at arge p_T at the Tevatron. They found that although fragmentation does indeed lominate over the leading-order color-singlet contribution for p_T greater than ıbout 7 GeV, the predictions still fell more than an order of magnitude below he CDF data for inclusive ψ and ψ' production.

In subsequent calculations of prompt ψ production (57), contributions from ragmentation into direct χ_c, followed by the radiative decay of χ_{cJ} into ψ, vere included. These contributions are roughly an order of magnitude larger han the direct ψ contributions and are dominated by the color-octet term in he gluon fragmentation function for χ_{cJ}. The value of the NRQCD matrix lement $\langle \mathcal{O}_8^{\chi_{c0}}({}^3S_1)\rangle$ that was used in this calculation was estimated from data n B-meson decays (44). The inclusion of the contribution from gluon fragnentation into χ_{cJ} brings the theoretical prediction for inclusive ψ production o within a factor of 3 of the CDF data (16). Because of the many theoretical ıncertainties that enter into the calculation, this factor of 3 discrepancy was onsidered acceptable at the time. However, in the case of prompt ψ' producion, the theoretical prediction remained about a factor of 30 below the data. This dramatic discrepancy became known as the "CDF ψ' anomaly."

The main difference between the prompt production of ψ and ψ' is that the ψ signal is fed by direct χ_{cJ}'s, whereas the ψ' signal is not fed by any known harmonium states. The ψ' anomaly could be explained if there existed undisovered charmonium states with sufficiently large branching fractions into ψ'. Among the possibilities that have been considered are d-wave states, higher p-vave states, and "hybrid" states whose dominant Fock state is $c\bar{c}g$ (38, 58). The nain difficulty with these proposals is explaining why these states should have he large branching fractions into ψ' that would be required to explain the data.

An alternative solution to the ψ' anomaly, based on the NRQCD factorization ormalism, was proposed by Braaten & Fleming (50). They suggested that the lominant contribution to the production of ψ' with large p_T comes from the olor-octet term (22) in the gluon fragmentation function for ψ'. This term epresents the fragmentation of a gluon into a $c\bar{c}$ pair in a color-octet ${}^3S_1$

tate at short distances, followed by the formation of the ψ' from the $c\bar{c}$ pair ırough nonperturbative QCD interactions. The probability for the formation f the ψ' is proportional to the NRQCD matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$. Although ıe normalization of this term in the cross section for ψ' production depends n the undetermined matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$, the p_T dependence of this ontribution is predicted and found to be in good agreement with the CDF ata (16). By fitting to CDF data, one finds that the value of the matrix element ; $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle \approx 0.0042$ GeV^3. This value is consistent with the NRQCD rediction that it should be suppressed by v^4 with respect to the corresponding olor-singlet matrix element $\langle \mathcal{O}_1^{\psi'}(^3S_1)\rangle \approx 0.573$ GeV^3. Thus, this solution to ıe ψ' anomaly is at least plausible.

This proposal was given further support by subsequent CDF data in which ıe prompt ψs were separated into those that come from χ_c decay and those ıat do not (17). The observed rate for prompt ψs not from χ_c decay was about factor of 30 larger than was predicted for direct ψ from fragmentation in the olor-singlet model. Cacciari et al showed that this data could be explained y color-octet gluon fragmentation (59, 60). As in the case of ψ', the p_T ependence predicted by color-octet fragmentation is in good agreement with ıe data. The value of the color-octet matrix element that is required to fit the ata is $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle \approx 0.014$ GeV^3. This is small compared to the corresponding olor-singlet matrix element $\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle \approx 1.13$ GeV^3, consistent with the ıppression by v^4 predicted by NRQCD.

The simple picture that seems to emerge from the CDF data is that the ominant mechanism for the production of ψ, ψ', or χ_{cJ} at large p_T is the erturbative fragmentation of a gluon into a $c\bar{c}$ pair in a color-octet $^3S_1$ state, ıllowed by the nonperturbative formation of charmonium from the $c\bar{c}$ pair. he differential cross section, therefore, reduces to that for producing a gluon ɔnvoluted with a fragmentation function:

$$
\begin{aligned}
& d\sigma(p\bar{p} \to H(p_T) + X) \\
& \quad = \sum_{jk} \int_0^1 dx_1\ f_{j/p}(x_1) \int_0^1 dx_2\ f_{k/\bar{p}}(x_2) \\
& \quad \int_0^1 dz\ d\hat{\sigma}(jk \to g(p_T/z) + X, \mu_{\rm frag}) D_{g\to H}(z, \mu_{\rm frag}). \qquad 23.
\end{aligned}
$$

he dominant term in the gluon fragmentation function is assumed to be the term roportional to $\langle \mathcal{O}_8^{H}(^3S_1)\rangle$. It is given by Equation 22 at the scale $\mu_{\rm frag} = 2m_c$, ut it should be evolved up to the scale $\mu_{\rm frag} = p_T/z$ using the evolution equaons (Equation 7). Of course, a thorough analysis of charmonium production at arge p_T must include additional terms in the gluon fragmentation function that re on the order of α_s^2 and higher. It must also include the contributions from he fragmentation of other partons, such as the charm quark and light quarks.

In Figures 1, 2, and 3, the predictions of this simple picture of charmonium roduction are compared with the most recent CDF data on prompt ψ not from χ_c decay (17), prompt ψ from χ_{c1} and χ_{c2} decay (17), and prompt ψ' (16), espectively. The predictions of the color-singlet model at leading order in α_s re shown by dashed lines; they fall orders of magnitude below the data. In igures 1 and 3, the dotted lines show the fragmentation contributions calculated n the color-singlet model. Although they increase the theoretical predictions by nore than an order of magnitude at the largest values of p_T, they still fall about factor of 30 below the experimental measurements. Contributions from color-ctet fragmentation with the color-octet matrix elements $\langle \mathcal{O}_8^H({}^3\mathrm{S}_J)\rangle$ adjusted o fit the data are shown as solid curves in Figures 1, 2, and 3; they differ only n their normalizations. The normalizations are proportional to $\langle \mathcal{O}_8^{\psi}({}^3\mathrm{S}_1)\rangle +$ $).57\langle \mathcal{O}_8^{\psi'}({}^3\mathrm{S}_1)\rangle$, $0.27\langle \mathcal{O}_8^{\chi_{c1}}({}^3\mathrm{S}_1)\rangle + 0.14\langle \mathcal{O}_8^{\chi_{c2}}({}^3\mathrm{S}_1)\rangle = 1.51\langle \mathcal{O}_8^{\chi_{c0}}({}^3\mathrm{S}_1)\rangle$, and $\langle \mathcal{O}_8^{\psi'}({}^3\mathrm{S}_1)\rangle$, in Figures 1, 2, and 3, respectively. Their shapes agree reasonably vell with the data. The values of the matrix elements for ψ and ψ' are given ibove. The value $\langle \mathcal{O}_8^{\chi_{c0}}({}^3\mathrm{S}_1)\rangle = 0.0076$ GeV^3 obtained for the χ_c matrix element is significantly larger than the most recent value obtained from B-neson decay (50). In calculating these curves, we imposed a pseudorapidity cut of $|\eta| < 0.6$ on the charmonium state and used the MRSDO parton distribution set. The scales μ_R, μ_F, and μ_{frag} were all chosen to be equal to the transverse nomentum p_T/z of the fragmenting gluon.

Although the fragmentation contribution must dominate at sufficiently large p_T, the contributions to the cross section that are suppressed by factors of m_c^2/p_T^2 nay be important at the values of p_T that are experimentally accessible. These contributions have been included in a recent calculation by Cho & Leibovich (61). They calculated the cross sections for all $2 \to 3$ parton processes of the form $ij \to c\bar{c} + k$ that produce $c\bar{c}$ pairs in color-octet states with angular mo-nentum quantum numbers ${}^3\mathrm{S}_J$, ${}^1\mathrm{S}_0$, and ${}^3\mathrm{P}_J$. They used these to calculate the cross section for direct ψ and ψ' production, including all color-octet contribu-ions that are suppressed only by v^4. By fitting the p_T distributions for all p_T greater than 5 GeV, they extracted values for the matrix elements $\langle \mathcal{O}_8^{\psi}({}^1\mathrm{S}_0)\rangle$ and $\langle \mathcal{O}_8^{\psi}({}^3\mathrm{P}_J)\rangle$, as well as for $\langle \mathcal{O}_8^{\psi}({}^3\mathrm{S}_1)\rangle$. The best fit requires a value for $\langle \mathcal{O}_8^{\psi}({}^3\mathrm{S}_1)\rangle$ significantly smaller than that obtained above. The terms involving $\langle \mathcal{O}_8^{\psi}({}^1\mathrm{S}_0)\rangle$ and $\langle \mathcal{O}_8^{\psi}({}^3\mathrm{P}_J)\rangle$ dominate in the range 5 GeV $< p_T <$ 10 GeV. At $p_T >$ 10 GeV, the term involving $\langle \mathcal{O}_8^{\psi}({}^3\mathrm{S}_1)\rangle$ dominates, and it asymp-totically approaches the contribution from the leading color-octet term in the gluon fragmentation function. The difference between the full calculation of the fferential cross section and the fragmentation approximation is less than 20% $p_T = 10$ GeV.

The greatest weakness of the proposal to explain the CDF data by color-:tet production mechanisms is that the color-octet matrix elements are free rameters that can be adjusted to fit the data. The predictive power in this oposal lies in the fact that the NRQCD matrix elements are universal and will so enter into other charmonium production processes. The most convincing oof that the color-octet proposal is correct would be to show that the values the matrix elements determined from the CDF data are required to explain armonium production in other high-energy processes. One possibility is Z^0 cay, discussed in Section 5.2.

The color-octet fragmentation mechanism also gives other predictions that n be tested experimentally. The most dramatic prediction is that direct ψs d ψ's at large p_T will have a large spin alignment. Cho & Wise (62) pointed .t that ψs and ψ's produced by color-octet gluon fragmentation tend to inherit transverse polarization of the nearly on-shell gluon. The process $g \rightarrow$ $(\mathbf{8}, {}^3S_1)$ produces a $c\bar{c}$ pair whose total spin is transversely polarized. The proximate heavy-quark spin symmetry of NRQCD implies that the spin state the $c\bar{c}$ pair will be affected very little by the nonperturbative QCD effects volved in binding the $c\bar{c}$ pair into a ψ. At leading order in α_s, the predicted larization is 100% transverse. Beneke & Rothstein analyzed the radiative rrections and concluded that they can decrease the polarization by only about % (63). Cho & Leibovich (61) have shown that the dominant corrections to spin alignment at Tevatron energies come from terms in the cross section that volve $\langle \mathcal{O}_8^{\psi}({}^1S_0)\rangle$ and $\langle \mathcal{O}_8^{\psi}({}^3P_J)\rangle$ and that fall as m_c^2/p_T^2. These corrections nain small at the largest values of p_T measured at the Tevatron. A large nsverse spin alignment, therefore, is a robust signature for the dominance by lor-octet fragmentation of the production of direct ψs and ψ's at high p_T.

Other predictions of the color-octet fragmentation mechanism can be tested at Tevatron. As pointed out by Barger et al, the production of two quarkonium ites with large p_T will be dominated by the production of two high-p_T gluons it both fragment into quarkonium (64). Taking into account the color-octet chanism, they found that $\psi\psi$ events at large p_T should be detectable at the vatron. The color-octet fragmentation mechanism also gives predictions for rrelations between charmonium at large p_T and the other jets produced by $p\bar{p}$ collision, but these have not yet been explored.

The differential cross sections for the production of bottomonium states at Tevatron have also been calculated by Cho & Leibovich (60, 61). The edictions of the color-singlet model fall more than an order of magnitude low the CDF data for $p_T > 5$ GeV (65). The production rates can, however,

›e explained by including color-octet terms in the cross section for the states $\Upsilon(nS)$ and $\chi_{bJ}(n)$, $n = 1, 2, 3$. The values of the color-octet matrix elements equired to fit the CDF data are consistent with expectations from NRQCD. 'he calculations of Cho & Leibovich show that fragmentation is not important n bottomonium production at the Tevatron. The error from neglecting contri-›utions that fall as m_c^2/p_T^2 is greater than 100% even if p_T is as large as 20 GeV.

Color-singlet and color-octet fragmentation processes in inelastic ψ pho-›oproduction at the Hadron-Elektron Ring Anlage (HERA) *ep* collider have lso been investigated (55, 66). The dominant contributions for values of p_T ccessible to experimental studies appear to be color-singlet charm quark frag-nentation and color-octet processes that are suppressed by m_c^2/p_T^2. In the olor-octet contributions, the $c\bar{c}$ pair is created at short distances in either a S_0 or ^3P_J state. Because the numerical values of the color-octet matrix el-ments $\langle \mathcal{O}_8^{\psi}(^1S_0)\rangle$ and $\langle \mathcal{O}_8^{\psi}(^3P_J)\rangle$ are not measured very precisely, it is not ›ossible to determine which process is the dominant one at HERA. However, t asymptotic values of p_T, the fragmentation contribution will be the most nportant contribution. Thus, for large p_T, data from HERA can be used to tudy a fragmentation mechanism not so easily probed at the Tevatron and at ›e Large Hadron Collider (LHC).

׀.2 *Charmonium and Bottomonium in Z^0 Decay*

'he decay of the Z^0 provides a laboratory for the study of heavy quarkonium ›ith large transverse momentum that is complementary to $p\bar{p}$ collisions. The olor-singlet model predicts that the production rates for prompt charmonium nd for bottomonium are so small that they are unlikely to be observed at .EP. However, the dramatic failure of the color-singlet model in $p\bar{p}$ collisions uggests that its predictions for Z^0 decay should also be reexamined.

The predictions of the color-singlet model for charmonium production in Z^0 decay have been studied thoroughly (5, 7, 67, 68). The largest production rocess for ψ in the color-singlet model is $Z^0 \to \psi c\bar{c}$. The rate for this process ›as first calculated in 1990 by Barger et al (67), who found it to be surprisingly ırge. It is two orders of magnitude larger than the rate for $Z^0 \to \psi gg$ (7), espite the fact that both processes are of the same order in α_s. This was xplained in 1993 by Braaten et al, who pointed out that the reason $Z^0 \to \psi c\bar{c}$; so large is that this process has a fragmentation contribution not suppressed y m_c^2/M_Z^2 (27). They showed that the dominant terms can be factored into ›e decay rate for $Z^0 \to c\bar{c}$ and into fragmentation functions for the processes $\to \psi c$ and $\bar{c} \to \psi \bar{c}$:

$$\frac{d\Gamma}{dz_\psi}(Z^0 \to \psi(z_\psi) + X) \approx 2\,\Gamma(Z^0 \to c\bar{c})\, D_{c\to\psi}(z_\psi), \qquad 24.$$

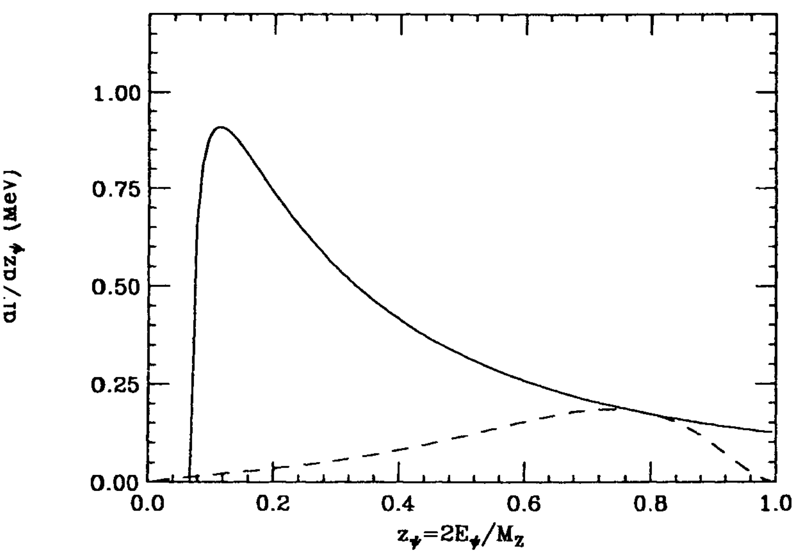
here $z_\psi = 2E_\psi/M_Z$. Because the momentum scale for the fragmentation pro-ss is on the order of m_c, the rate is proportional to $\alpha_s^2(m_c)$ instead of $\alpha_s^2(M_Z)$. ıe resulting prediction for the branching fraction for direct ψ production in) decay is 2.9×10^{-5}. The experimental study of prompt charmonium in Z^0 cay is complicated by the large background from the decay of b-hadrons, hich must be removed using vertex detectors. There are preliminary results om LEP (69) that indicate that the branching fraction for prompt ψ production around 10^{-4}, well above the predictions of the color-singlet model.

Another fragmentation process in the color-singlet model was considered by agiwara et al (68). They calculated the rates for the process $Z^0 \to q\bar{q}g^*$, with armonium being produced by the virtual gluon through the processes $g^* \to gg$ and $g^* \to \chi_{cJ}g$. The rate is at least an order of magnitude smaller than e charm fragmentation process described above, but the experimental back-ounds are less severe. The DELPHI collaboration (69) has recently reported 1×10^{-4} as a limit on the branching fraction for $Z^0 \to \psi X$ from this process.

The color-octet mechanism that was introduced to explain prompt charmo-um production at the Tevatron also offers new possibilities for charmonium oduction at the Z^0 resonance. Cheung et al (70) and Cho (71) have studied ompt charmonium production at LEP via the color-octet mechanism. The lor-octet process that is leading order in α_s involves the short-distance decay) $\to c\bar{c}(\mathbf{8}, {}^3S_J) + g$, but it has a negligible branching fraction because it is sup-essed by a short-distance factor of m_c^2/M_Z^2. The dominant color-octet process on the order of α_s^2 and involves the short-distance decay $Z^0 \to c\bar{c}(\mathbf{8}, {}^3S_1) +$ $\bar{q}$, where q can be one of the light quarks u, d, or s, or one of the heavy quarks or b. Apart from different coupling constants, color factors, and long-distance atrix elements, this process is identical to the electromagnetic production of studied by Fleming (26) and discussed in Section 3.2. At leading order in , the perturbative QCD factorization formula for this process has the form

$$\frac{d\Gamma}{dz_\psi}(Z^0 \to \psi(z_\psi) + X) \approx 2\sum_q \Gamma(Z^0 \to q\bar{q}) D_{q\to\psi}(z_\psi, \mu)$$
$$+ \sum_q \int_{z_\psi}^1 dy \frac{d\widehat{\Gamma}}{dz_g}(Z^0 \to g(z_\psi/y)q\bar{q}, \mu) D_{g\to\psi}(y), \qquad 25.$$

ıere $z_\psi = 2E_\psi/M_Z$ and $z_g = z_\psi/y$. The fragmentation function $D_{g\to\psi}(y)$ r the formation of a ψ in a gluon jet through the color-octet mechanism is ven in Equation 22, except that it must be evolved up to the scale E_ψ/y set the gluon energy. The fragmentation function $D_{q\to\psi}(z)$ for the formation a ψ in a light quark jet can be obtained from Equation 13 by replacing $\alpha^2 g_\psi^2$ th $\alpha_s^2\langle\mathcal{O}_8^\psi({}^3S_1)\rangle/72m_c^3$.



;ure 4 Predictions for the energy spectrum $d\Gamma/dz$ for prompt ψs from Z^0 decay in the color- glet model (*dashed curve*) and from the leading color-octet process (*solid curve*).

In Figure 4, we compare the ψ energy distribution predicted by the color- iglet model with the distribution from the most important color-octet process $^{)} \to c\bar{c}(\mathbf{8}, {}^3S_1) + q\bar{q}$. The most striking feature in Figure 4 is that the energy stribution of the color-octet process dominates over that of the color-singlet odel for all values of $z_\psi = 2E_\psi/M_Z$. There is also a dramatic difference tween the shapes of the two distributions. The energy distribution from e color-singlet process is rather hard because of the nature of heavy-quark igmentation. The distribution for the color-octet process is very soft and has pronounced peak near $z_\psi = 0.1$. The prediction for the branching ratio for $^{)} \to \psi + X$ from the color-octet process is 1.4×10^{-4}. This is consistent with e DELPHI bound for the color-singlet process $Z^0 \to \psi g g q\bar{q}$, which produces s with a similar event topology.

The color-octet fragmentation process is also important in the production of and χ_c. The energy distributions are predicted to be similar to that for the ψ own in Figure 4. The production rates are predicted to be smaller by about a ctor of 3 for the ψ' and larger by about a factor of 5 for the three spin states the χ_{cJ} combined.

As in the case of $p\bar{p}$ annihilation, the color-octet fragmentation mechanism ikes definite predictions for the spin alignment of the ψ in Z^0 decay. The

:olor-octet process $Z^0 \to c\bar{c}(\mathbf{8}, {}^3S_1) + q\bar{q}$ involves contributions from both ;luon fragmentation and light-quark fragmentation. At leading order in α_s, ;luon fragmentation only produces ψs that are transversely polarized. How- :ver, a light quark can fragment into a longitudinally polarized ψ even at lead- ng order in α_s. This leads to significant degradation of the spin alignment of he ψ.

Bottomonium states can also be produced in Z^0 decay. These states are impler to study experimentally because there is no background to bottomonium roduction from non-prompt production mechanisms analogous to b-hadrons' lecay into charmonium. A signal from the $J^{PC} = 1^{--}$ states $\Upsilon(1S)$, $\Upsilon(2S)$, ınd $\Upsilon(3S)$ has in fact been observed at LEP. The preliminary result from the)PAL detector is that the branching fraction summed over the three states s (73)

$$\sum_{n=1}^{3} \mathrm{Br}(Z^0 \to \Upsilon(nS) + X) = (1.2^{+0.9}_{-0.6} \pm 0.2) \times 10^{-4}. \qquad 26.$$

The direct production mechanisms for $\Upsilon(nS)$ in Z^0 decay are essentially iden- ical to those for prompt ψ. The dominant color-singlet process is the decay $Z^0 \to \Upsilon(nS) + b\bar{b}$. This process has a fragmentation contribution that corre- ponds to the decay $Z^0 \to b\bar{b}$ followed by the formation of $\Upsilon(nS)$ through the ragmentation of the b or $\bar{b}$. The prediction for the sum of branching fraction ;iven in Equation 26 is 1.6×10^{-5}, which is about an order of magnitude below he OPAL result. Color-octet mechanisms for the production of $\Upsilon(nS)$ and $\chi_{bJ}(n)$ have been studied by Cho (71). The dominant mechanism is the forma- ion of bottomonium through the short-distance process $Z^0 \to b\bar{b}(\mathbf{8}, {}^3S_1) + q\bar{q}$, $q = u, d, s$, which involves both gluon fragmentation and light-quark frag- nentation. Using values for the color-octet matrix elements $\langle \mathcal{O}_8^H({}^3S_1) \rangle$ for the ottomonium states that were extracted from CDF data (65), Cho predicted that he branching fraction in Equation 26 should be 4.1×10^{-5}. Given the large ıncertainties in the color-octet matrix elements, this is in reasonable agreement vith the OPAL result. Showing that the same values of the color-octet matrix :lements explain Υ production in $p\bar{p}$ colliders and in Z^0 decay would provide trong support for the proposal that the production is dominated by color-octet nechanisms.

;.3 *Production of the B_c*

n the standard model, the only bound states consisting of two heavy quarks vith different flavors are the $\bar{b}c$ mesons. The ground state of the $\bar{b}c$ system s the pseudoscalar state B_c. The mass spectrum of the $\bar{b}c$ mesons can be redicted reliably from quark potential models that are tuned to reproduce the spectra of charmonium and bottomonium (74). According to the potential-model calculations, the first two sets of s-wave states, the first and probably the entire second set of p-wave states, and the first set of d-wave states all lie below the BD flavor threshold. Because QCD interactions are diagonal in flavors, the annihilation of $\bar{b}c$ mesons can occur only through a virtual W^+ and, therefore, is suppressed relative to the electromagnetic and hadronic transitions to lower-lying $\bar{b}c$ states. Thus, all the excited states below the BD threshold will cascade down to the ground state B_c via emission of photons and/or pions. A calculation of the inclusive production of the B_c meson must, therefore, include the contributions from production of all the s-wave, p-wave, and d-wave states below the BD threshold.

The production of the B_c and the lowest $^3S_1$-state B_c^* in e^+e^- annihilation was first computed to leading order in α_s in the color-singlet model by Clavelli and by Amiri & Ji (75). The lowest-order process for B_c production is $e^+e^- \to B_c + b\bar{c}$. Chang & Chen (29) showed that the energy distributions for direct B_c and B_c^* production could be expressed in terms of the fragmentation functions $D_{\bar{b}\to B_c}(z)$ and $D_{\bar{b}\to B_c^*}(z)$. For example, the production rate for B_c at the Z^0 resonance in the scaling limit $M_Z \to \infty$ with $z = 2E_{B_c}/M_Z$ fixed has the form

$$\frac{d\Gamma}{dz}(Z^0 \to B_c(z) + X) \approx \Gamma(Z^0 \to b\bar{b})\; D_{\bar{b}\to B_c}(z). \qquad 27.$$

The fragmentation functions $D_{\bar{b}\to B_c}(z)$ and $D_{\bar{b}\to B_c^*}(z)$ were also calculated later by Braaten et al (28), who pointed out that they were universal and could also be used to calculate direct B_c production in other high-energy processes, such as $p\bar{p}$ collisions. The first calculations of B_c production in hadron colliders using fragmentation functions were carried out by Cheung (76). The fragmentation functions were subsequently calculated for the p-wave states of the $\bar{b}c$ system (32, 33) and even for the d-waves (36), allowing a complete calculation of the inclusive B_c production rate. The advantage of the fragmentation approach over carrying out a full calculation of the production rate to leading order in α_s is that the calculation of the fragmentation function is much simpler and can be extended to p-wave and d-wave states relatively easily. The limitation of the fragmentation approach is that it is accurate only at sufficiently large p_T.

Color-octet production mechanisms are expected to be much less important for $\bar{b}c$ mesons than for charmonium or bottomonium. All terms in the gluon fragmentation functions are on the order of α_s^3 or higher. For the b-quark fragmentation functions, the leading color-singlet terms and the leading color-octet terms are both on the order of α_s^2. For s-wave states like B_c and B_c^*, the color-octet terms are therefore suppressed relative to the color-singlet terms by a factor of v^4 from the ratio of the NRQCD matrix elements. For p-wave states, a color-octet term in the fragmentation function is necessary to avoid an ıfrared divergence in the color-singlet fragmentation function. However, this vergence occurs first at next-to-leading order in α_s. Thus, the color-octet term not as important as it is for the case of gluon fragmentation into χ_c, where e divergence occurs at leading order.

The production of the B_c and B_c^* at hadronic colliders like the Tevatron and e LHC has recently been calculated by several groups, both through complete (α_s^4) calculations (77, 78) and by using the simpler fragmentation approx- ıation (76). Discrepancies among the earlier calculations raised questions out the accuracy of the fragmentation approximation in the $\bar{b}c$ case. Recent lculations (78) have shown that the fragmentation approximation is very ac- ırate for the B_c for p_T as low as 10 GeV. However, for the B_c^*, the accuracy is uch lower, decreasing to 20% only for p_T around 30 GeV. For B_c production $\gamma\gamma$ collisions, even larger values of p_T are required before the fragmentation proximation becomes accurate (80). The fact that such large values of p_T are quired in order for fragmentation to dominate can be attributed to the presence an additional small parameter m_c/m_b in the problem. Contributions that fall f asymptotically as m_b^2/p_T^2 may dominate at subasymptotic values of p_T if ey are enhanced by powers of m_b/m_c.

The only calculations presently available for the production of the p-wave mesons have been carried out using the fragmentation approximation. This proach has been used by Cheung & Yuan to calculate the inclusive production the B_c in $p\bar{p}$ collisions, including the contributions from all the s-wave and wave states below the BD threshold. With acceptance cuts of $p_T > 6$ GeV ıd $|y| < 1$, the inclusive production cross section for the B_c meson at the Teva- ɔn is about 5 nb. The contributions from the excited s-wave states and from e p-wave states are 58 and 23%, respectively, and, hence, are significant. The wave states were not included in this analysis because they are expected to ntribute only about 2% to the inclusive production of B_c (36). The search for e B_c is now underway at the Tevatron, and preliminary results have already en presented by the CDF collaboration (81). If the B_c is not found in the esent run of the Tevatron, it should certainly be discovered after the instal- ion of the main injector, which will boost the luminosity by about a factor 10.

The B_c may also be discovered at LEP. Candidate events for the B_c have eady been reported by the ALEPH collaboration (82). The branching fraction edicted from the color-singlet processes $Z^0 \rightarrow B_c + b\bar{c}$ and $Z^0 \rightarrow B_c^* + b\bar{c}$ e 5.9×10^{-5} and 8.3×10^{-5}, respectively (28). If the fragmentation functions e used to estimate the contributions of higher s-wave, p-wave, and d-wave ıtes, the inclusive branching fraction for B_c is predicted to be larger by about actor of 5.

OUTLOOK

ıe NRQCD factorization formalism provides a very general framework for ıalyzing the production of heavy quarkonium. It implies that the produc- ɔn process is much more complex than has been assumed in the color-singlet odel. There are infinitely many nonperturbative matrix elements that con- bute to the cross sections, although only a finite number of them contribute any given order in v^2. The most dramatic consequence of this formalism that color-octet mechanisms sometimes give the largest contributions to the oss section. In the case of p-waves, there is a color-octet term at leading der in v^2 that must be included for perturbative consistency. In the case of waves, color-octet terms are suppressed by v^4, but they may be important if e color-singlet term is suppressed by other small parameters such as α_s.

The NRQCD factorization formalism suggests an explanation for the large oduction rates for bottomonium and prompt charmonium that have been ob- rved at the Tevatron. They can be attributed to color-octet terms in the cross ction that are important because the color-singlet terms are suppressed by ıwers of α_s and m_Q^2/p_T^2. The magnitudes of the color-octet matrix elements quired to explain the Tevatron data are in accord with expectations from RQCD. This explanation leads to many predictions that can be tested exper- ıentally. The ultimate test is that the same matrix elements must be able to plain heavy-quarkonium production in other high-energy processes, such as) decay.

In the NRQCD factorization formalism, there are many nonperturbative ma- x elements that must be determined phenomenologically to make theoretical edictions for quarkonium production. Fortunately, there is a wealth of exper- ıental data that can be used to determine these matrix elements. Quarkonium produced as a byproduct of almost every high-energy experiment. In this view, we only discussed data from the highest-energy colliders, the Tevatron d LEP. However, there is also data on quarkonium production from lower- ergy e^+e^- and ep colliders. There is also an abundance of data on pN, πN, d γN collisions from fixed-target experiments. By carrying out a compre- nsive analysis of all the data on quarkonium production, it should be possible determine most of the NRQCD matrix elements of phenomenological im- ırtance.

An enormous amount of theoretical work will be required to carry this pro- am to completion. For every process, the production rate should be calculated next-to-leading order in all the small parameters in the problem, including , α_s, and kinematic parameters such as m_Q^2/p_T^2. Most calculations until very ɔently were carried out within the color-singlet model and, therefore, include ly contributions that are of leading order in v^2. Even in the color-singlet nodel, the only process for which the corrections at next-to-leading order in α_s ıave been calculated is photoproduction of J/ψ (83). The effort to calculate :ontributions that are beyond leading order in v^2, such as color-octet production nechanisms, has just begun.

Although the NRQCD factorization approach provides a very general framevork for analyzing quarkonium production, it should not be regarded as a com›lete theory. The derivation of the factorization formula breaks down when here are hadrons in the initial state if the quarkonium is produced with small ransverse momentum. Therefore, processes involving diffractive scattering, uch as the elastic scattering process $\gamma p \rightarrow \psi p$, are not described by this ormalism. The NRQCD formalism also is not appropriate for describing the ormation of charmonium from intrinsic $c\bar{c}$ pairs that come from the parton listribution of a colliding hadron (84).

The complications of diffractive scattering and intrinsic $c\bar{c}$ pairs do not arise in he production of heavy quarkonium at large p_T, and therefore, the theoretical nalysis of this process is particularly clean. The factorization theorems of erturbative QCD guarantee that the production is dominated by fragmentation, he formation of heavy quarkonium within jets that are initiated by single highnergy partons. The large cross sections for prompt charmonium production bserved at the Tevatron can be explained by including a color-octet term in he gluon fragmentation function. Although the normalization of the cross ection is not predicted, this mechanism does give other predictions that can e tested experimentally. The most dramatic prediction is a large transverse pin alignment for direct ψ and ψ' at large p_T, but the spin alignment has not et been measured. This mechanism also gives predictions for the correlations etween high-p_T charmonium states and other jets produced by the collision ıat can be tested.

An enormous amount of theoretical work remains to be done in order to obtain recise predictions for fragmentation that can be compared with experiment. Quantitative estimates of all the relevant NRQCD matrix elements are required ı order to determine which terms in the fragmentation functions are numerically nportant. The important terms should all be calculated to next-to-leading rder in α_s. It is also important to calculate the power corrections to the cross ections that fall as m_Q^2/p_T^2, because they may give the largest corrections to ome observables.

New experimental data from high-energy colliders is providing stringent ests of our understanding of the production of heavy quarkonium. The NRQCD ıctorization approach suggests that cross sections should be calculable in terms f the heavy-quark mass m_Q, the running coupling constant α_s, and a few matrix lements that can be determined phenomenologically. This approach provides n explanation for the large production rates that have been observed at the 'evatron and at LEP, but further effort, both theoretical and experimental, will e required to show conclusively that this explanation is correct. If this effort ; successful, it will mark a major milestone on the road to a comprehensive escription of heavy-quarkonium production in all high-energy processes.

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