Single and Double Photon Processes at High Energies

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Outline

- Real photons as probes of strong interaction dynamics
  - photons in initial or final states
  - similarities and differences

- Nomenclature and interaction mechanisms
  - theoretical background
  - input needed for calculations
  - parton distribution functions for photons
  - parton fragmentation functions for photons

- Examples
  - $\gamma p \rightarrow jets$
    - 3 jet production
    - dijet production
    - role of direct vs. resolved components
    - angular distributions for dijets
    - jet distributions
    - constraints on photon parton distributions
  - direct photon photoproduction
    - comparison to direct photon hadroproduction
    - photon p_T and η distributions
    - constraints on photon parton distributions
  - hadroproduction of photon pairs
    - novel NLO effect
    - fragmentation and isolation
    - role at LHC
  - $\gamma\gamma \rightarrow jets$
    - role of direct vs. resolved components

Why Photons?

- Known pointlike interaction
- Probe of hard scattering in both the initial (photo-production) and final states (direct photons)
- Provides information which complements that from jet production, without the complication of a jet definition
- important for signals of and backgrounds to new physics, *e.g.*, Higgs searches
- Photoproduction
  - additional subprocesses beyond those seen in hadroproduction
  - probe of photon structure complements photon structure function measurements
- Direct photon production
  - additional subprocesses beyond those seen in hadroproduction
  - importance for constraining the gluon distribution in hadrons

Will compare and contrast these two types of processes, exploiting common elements of each and emphasizing parallels between the two

Perturbative Mechanisms for Jet Photoproduction

- Lowest order: $\mathcal{O}(\alpha\alpha_s)$

  1. $\gamma q \to qg$ QCD Compton
  2. $\gamma g \to q\overline{q}$ annihilation (time reversed)

- Next order: $\mathcal{O}(\alpha\alpha_s^2)$

  1. $\gamma q \to qgg$ (added a gluon)
  2. $\gamma g \to q\overline{q}g$ (added a gluon)
  3. $\gamma q \to qq'\overline{q}'$ (replaced $g \to q'\overline{q}'$)
  4. also need 1-loop corrections to lowest order subprocesses

Leading Log Approximation

1. $\alpha_s \to \alpha_s(Q^2)$

2. Scale violations incorporated into parton distributions

$$E\frac{d^3\sigma}{dp^3}(\gamma p \to jet + X) = \sum_b \int dx_b\, G_{b/p}(x_b, Q^2)\, \frac{\hat{s}}{\pi}\, \frac{d\sigma}{d\hat{t}}(\gamma b \to jet + X)\, \delta(\hat{s} + \hat{t} + \hat{u})$$

- See the notes at the end of the lecture for some definitions and examples which may help you derive the following relations

- A $\hat{}$ indicates parton-level Mandelstam variables

$$\begin{aligned} \hat{s} &= x_b s \\ \hat{t} &= -p_T \sqrt{s}\, e^{-y} \\ \hat{u} &= -x_b p_T \sqrt{s}\, e^{y} \end{aligned}$$

- The single integration may be done using the δ function

$$\begin{aligned} \delta(\hat{s} + \hat{t} + \hat{u}) &= \delta(x_b s - p_T \sqrt{s}\, e^{-y} - x_b p_T \sqrt{s}\, e^{y}) \\ &= \frac{1}{s - p_T \sqrt{s}\, e^{y}} \delta(x_b - \frac{p_T\, e^{-y}}{\sqrt{s} - p_T\, e^{y}}) \end{aligned}$$

Single jet invariant cross section is given by the following simple expression:

$$E\frac{d^3\sigma}{d^3p} =$$
$$\sum_b x_b\, G_{b/p}(x_b, Q^2)\, \frac{1}{\pi}\, \frac{2}{2 - x_T e^y}\, \frac{d\sigma}{d\hat{t}}(\gamma b \to jet + X)$$

with $x_T = 2p_T/\sqrt{s}$ and $x_b = \frac{x_T e^{-y}}{2 - x_T e^y}$.

- No integrations remaining
- Rapidity of the recoil jet is fixed by kinematics

Conservation of energy: $1 + x_b = x_T(\cosh y_1 + \cosh y_2)$
Conservation of momentum: $1 - x_b = x_T(\sinh y_1 + \sinh y_2)$

$$==> y_2 = \ln(\tfrac{2}{x_T} - e^{y_1})$$

- Higher order contributions included in leading-log approximation via scaling violations and running coupling

Is this all there is?

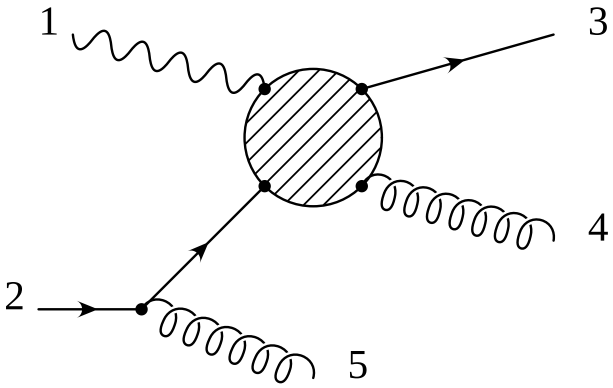
Look at $\mathcal{O}(\alpha\alpha_s^2)$ terms ...

$\mathcal{O}(\alpha\alpha_s^2)$ Contributions

- Parts of these contributions have already been included in the leading-log terms considered thus far

- Examine different regions of phase space to see how much...

- Consider $\gamma(1)q(2) \to q(3) + g(4) + g(5)$



- Look at the region where gluon 5 is nearly collinear with the incoming quark 2
  - Will have $t_{25} = (p_2 - p_5)^2 \approx 0$

- The squared matrix element may be approximated as

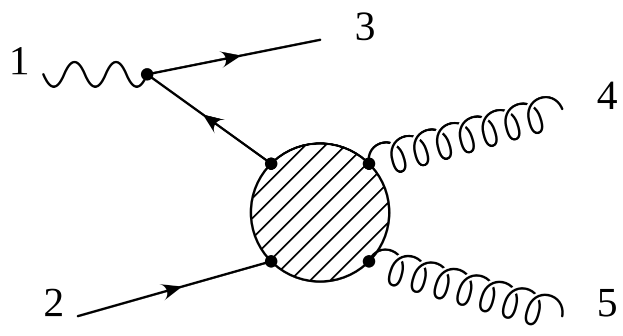
$$\overline{\sum}|M(\gamma q \to qgg)|^2 \approx$$
$$-\frac{\alpha_s}{2\pi}\, P_{qq}(z)\, \frac{1}{zt_{25}} \overline{\sum}|M(\gamma q \to qg)|^2$$

- Integration over t_{25} generates a logarithmic singularity
  - factorize singularity and absorb into the bare quark distribution
  - internal quark line going on-shell is indicative of long distance effects included in the nonperturbative parton distribution
- Use Altarelli-Parisi equations to sum the leading logs and generate the scale violating distributions

$$Q^2 \frac{dG_{q/p}(x,Q^2)}{dQ^2} = \frac{\alpha_s}{2\pi}\left[P_{qq}\otimes G_{q/p} + P_{qg}\otimes G_{g/p}\right]$$

$$Q^2 \frac{dG_{g/p}(x,Q^2)}{dQ^2} = \frac{\alpha_s}{2\pi}\left[P_{gq}\otimes \sum_q G_{q/p} + P_{gg}\otimes G_{g/p}\right]$$

- Next, consider another region of phase space where $t_{13}\to 0$



- The squared matrix element may be approximated as

$$\overline{\sum}|M(\gamma q\to qgg)|^2 \approx -\frac{\alpha_s}{2\pi}\, P_{q\gamma}(z)\,\frac{1}{zt_{13}}\overline{\sum}|M(\bar{q}q\to gg)|^2$$

- Again, an internal quark line goes on-shell which is indicative of long distance physics
- New configuration
  - two high-p_T partons (the gluons in this case)
  - a parton going in the beam direction
- Factorize the singularity and include in a parton distribution function for the *photon*
- Incoming photon can fluctuate into a hadronic state (vector meson)
  - include calculable pointlike part with the non-perturbative component
- Sum leading logs with modified Altarelli-Parisi equations

$$Q^2 \frac{dG_{q/\gamma}(x,Q^2)}{dQ^2} = \frac{\alpha}{2\pi} P_{q\gamma} + \frac{\alpha_s}{2\pi} \left[P_{qq} \otimes G_{q/\gamma} + P_{qg} \otimes G_{g/\gamma} \right]$$

$$Q^2 \frac{dG_{g/\gamma}(x,Q^2)}{dQ^2} = \frac{\alpha_s}{2\pi} \left[P_{gq} \otimes \sum_q G_{q/\gamma} + P_{gg} \otimes G_{g/\gamma} \right]$$

where $P_{q\gamma} = 6P_{qg}$.

- Parton distributions in a photon driven by the point-like splitting function $P_{q\gamma}$. Can add any solution of the homogeneous A-P equations (hadronic part).

- The situation has become more complex!
  - Expect to see two classes of events
  1. Direct (or pointlike) - no beam fragments
  2. Resolved - beam fragments present. Photon interacts like a hadron, but part of the parton distribution is perturbatively calculable.

- In (1) all of the photon energy goes into the hard-scattering subprocess.

- In (2) some of the energy is carried away by the beam fragments going approximately in the original photon direction.

Questions to be examined in detail

1. To what extent does this simple decomposition make sense and can it be applied when higher order corrections are included?

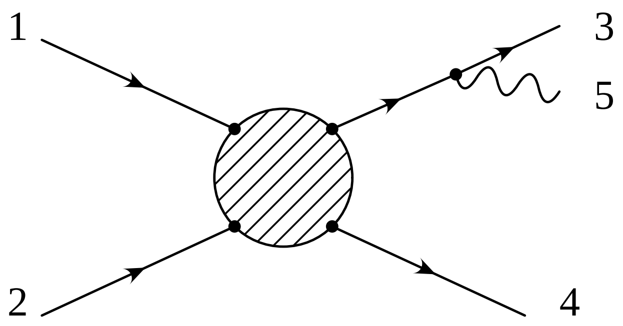
2. To what extent can one determine the parton distribution functions in a photon?

Direct Photon Production

- How do the phenomena discussed for photoproduction carry over to the production of direct photons?

- In lowest order, again have two subprocesses
  - $qg \to \gamma q$ and $q\bar{q} \to \gamma g$
  - kinematics corresponds to an isolated photon recoiling against a jet (analogue of the direct component)

- In next order one sees a new configuration
  - consider the subprocess $q(1)q(2) \to q(3)q(4)\gamma(5)$
  - examine the region where $s_{35} = (p_3 - p_5)^2 \approx 0$



$$\overline{\sum}|M(qq \to qq\gamma)|^2 \approx \frac{\alpha_s}{2\pi}\, P_{\gamma q}(z)\, \frac{1}{s_{35}} \overline{\sum}|M(qq \to qq)|^2$$

- Again, an internal quark line is going on-shell signalling long distance physics effects

- New configuration
  - photon is accompanied by jet fragments on the *same* side
  - high$-p_T$ jet recoils on the away side

- Factorize the singularity and include it in the bare *photon fragmentation function*

- Sum large logs with modified Altarelli-Parisi equations

$$
\begin{aligned}
Q^2 \frac{dD_{\gamma/q}(x,Q^2)}{dQ^2} &= \frac{\alpha}{2\pi} P_{\gamma q} + \frac{\alpha_s}{2\pi}\left[D_{\gamma/q} \otimes P_{qq} + D_{\gamma/g} \otimes P_{gq}\right] \\
Q^2 \frac{dD_{\gamma/g}(x,Q^2)}{dQ^2} &= \frac{\alpha_s}{2\pi}\left[\sum_q D_{\gamma/q} \otimes P_{qg} + D_{\gamma/g} \otimes P_{gg}\right]
\end{aligned}
$$

- Again, the situation has become more complex

- Expect to see two classes of events
  1. Isolated (or pointlike) - no hadrons accompanying the photon
  2. Accompanied - photon is a fragment of a high-p_T jet. Part of the fragmentation function is perturbatively calculable.

- Expect (1) to dominate at high-p_T since the energy is not shared with accompanying hadrons.

A nagging question

- The perturbative part of the photon parton distribution and fragmentation functions first showed up in our examples at the next to leading order in α_s, that is, one order beyond the Born term. Are they just higher order corrections then?

Not really. When one examines solutions of the relevant evolution equations, it becomes apparent that the leading behavior of the perturbative part of the solutions goes as $\ln Q^2$. This is quite different from the usual hadronic behavior where we are used to seeing the distributions decrease at large x and slowly increase at small x. (See additional notes at the end of the lecture)

The factor of $\ln Q^2$ effectively cancels out one factor of α_s, so that the resolved and direct contributions end up having similar dependences on Q^2. We should have included these pieces even at the lowest order!

Questions to be examined in detail

1. To what extent does this simple decomposition make sense and can it be applied when higher order corrections are included?

2. To what extent can one determine the photon fragmentation functions?

Summary

- Processes involving photons require the same type of nonperturbative functions as do hadronic processes, e.g., parton distribution and fragmentation functions.

- In addition, there are subprocesses where the photon interacts directly, in a pointlike manner.

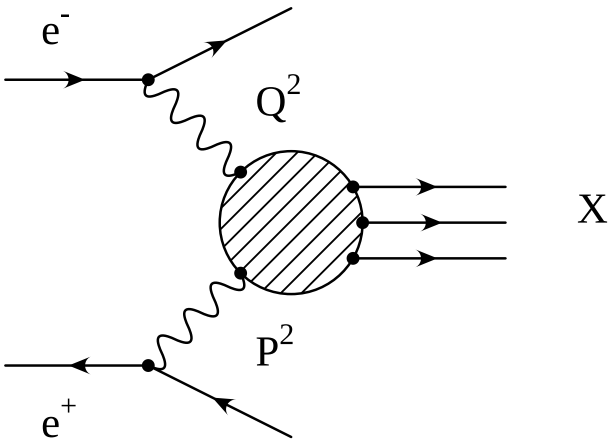
- Photon processes have all the complications of hadronic processes and more

- Offers new opportunities and predictions to to be tested

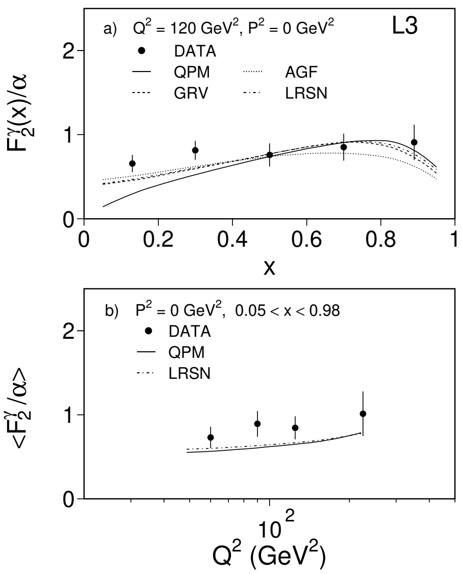
Parton distribution and fragmentation functions for photons

- Strategy is the same as for hadronic parton distributions
  - use evolution equations and fit data to determine the nonperturbative functions (boundary conditions)
  - provide convenient parametrizations or tables of (x, Q^2) values to be used with an interpolation routine)

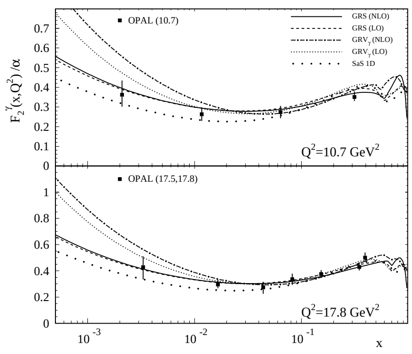
- For parton distributions in a photon, fits are usually made to data for the photon structure function F_2^γ measured with one photon far off-shell ($Q^2 >> \Lambda^2_{QCD}$) and one photon nearly on-shell ($P^2 \approx 0$).



- Comparison to data from L3: hep-ex/0004005



- Small x results from Glück, Reya, and Schienbein, hep-ph/0103216

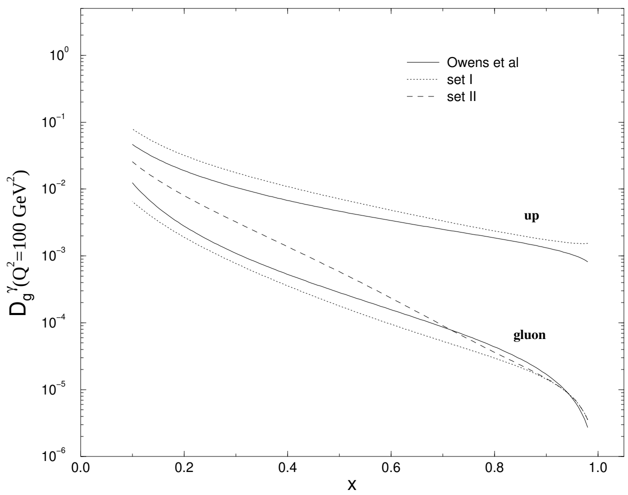


- Relatively few characteristic features to aid in fitting the parton distributions to the data

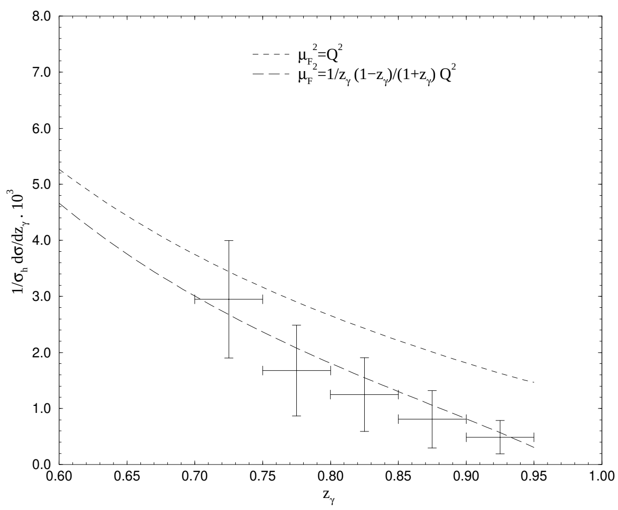
- Largest uncertainty is in the gluon distribution

- For fragmentation functions, evolution equations are solved with appropriate boundary conditions, usually taken from vector dominance estimates.

- Results from Bourhis, Fontannaz, and Guillet, hep-ph/ 9704447



- Comparison of BFG predictions to ALEPH data for γ production associated with a jet (not fully inclusive). $z-$dependent scale corrects for cuts associated with the jet. See hep-ph/9704447



Some basic references

- Parton distribution functions

1. H. Abramowicz, K. Charchula, and A. Levy, Phys. Lett. B269, 458 (1991)

2. H. Abramowicz, K. Krawczyk, K. Charchula, A. Levy, and U. Maor, Int. J. Mod. Phys. A8, 1005 (1993)

3. L.E. Gordon and J.K. Storrow, Z.Phys.C56, 307 (1992); Nucl.Phys.B489, 405 (1997)

4. M. Glück, E. Reya, and A. Vogt, Phys. Rev. D45, 3986 (1992), Phys. Rev. D46, 1973 (1992).

5. P. Aurenche, J. P. Guillet, and M. Fontannaz, Z. Phys. C64, 621 (1994)

- Photon fragmentation functions

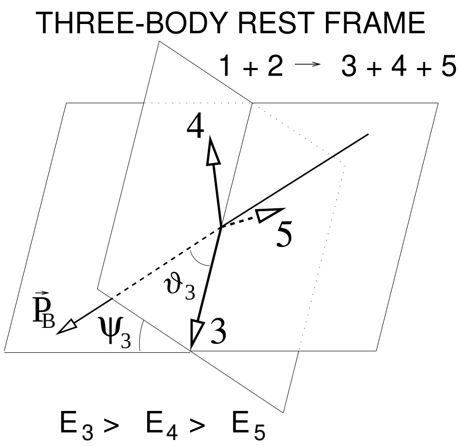
1. M. Glück, E. Reya, and A. Vogt, Phys.Rev.D48,116 (1993), Erratum-ibid.D51, 1427 (1995)

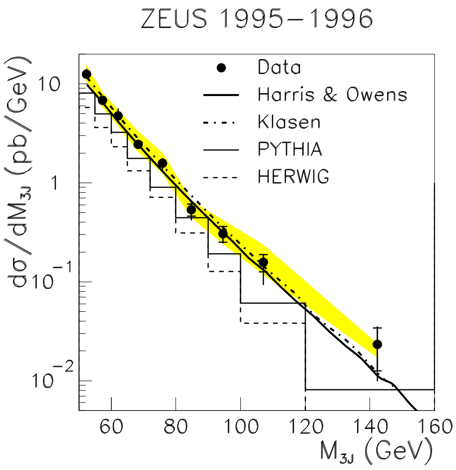
2. L. Bourhis, M. Fontannaz, and J.P. Guillet, Eur.Phys.J.C2, 529 (1998)

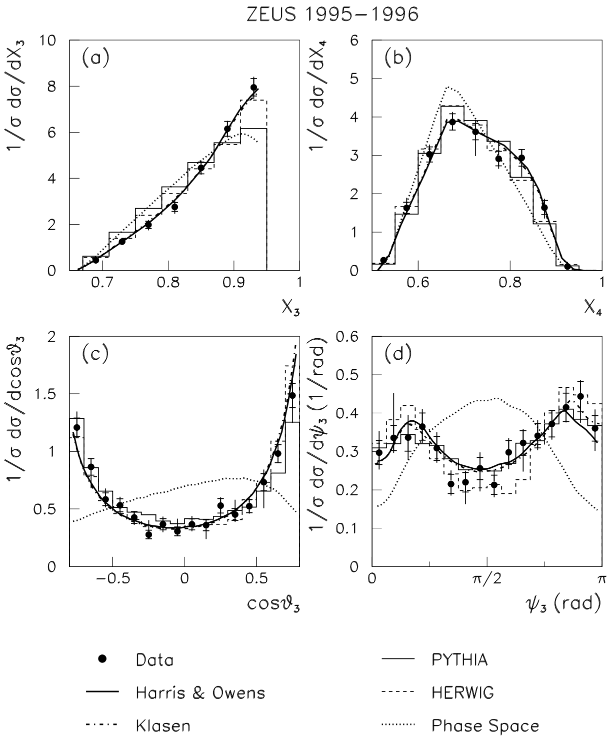
Examples

- Consider several different processes in order of increasing complexity concerning the interactions of the photons

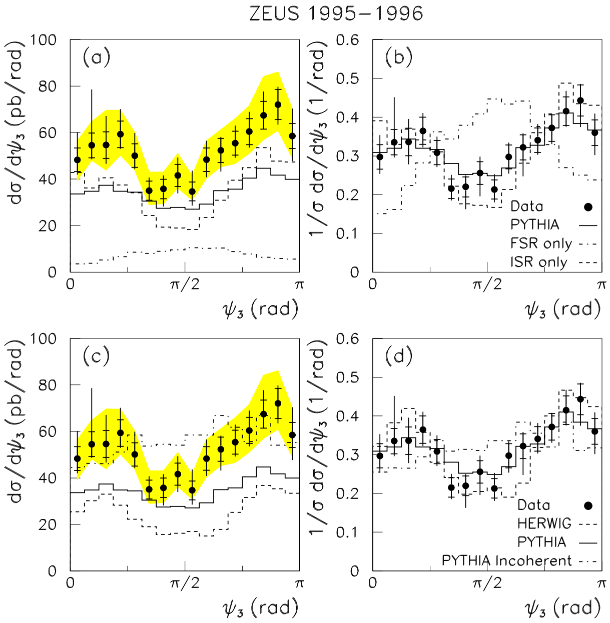
- Photoproduction of three-jet final states
  - Calculation based on $\mathcal{O}(\alpha\alpha_s^2)$ subprocesses
    1. $\gamma q \to qgg$
    2. $\gamma g \to q\overline{q}g$
    3. $\gamma q \to qq'\overline{q}'$
  - Consider three identified high-p_T jets with high three-jet mass
  - Only looking at the pointlike interaction of the photon - no resolved contribution
  - Calculation at this level is lowest order (LO) and also leading-log (LL)
  - Compare to data from the ZEUS Collaboration (hep-ex/9810046)
    * three-jet mass M_{3j}
    * energy fractions: jets ordered in energy with $E_3 > E_4 > E_5$ and energy fractions defined as $x_i = 2E_i/M_{3j}$
    * jet angular distributions







Observe good agreement between matrix element calculations and the data and also between shower Monte Carlo calculations and the data



- Data show an asymmetry about $\psi_3 = \pi/2$

- Effect is well described by matrix element and shower calculations

- Effect is predominantly coming from initial state radiation

- For small ψ the soft jet tends to go in the same general direction as the incoming photon while for large ψ it is more aligned with the incoming proton direction

- Expect more initial state radiation from the proton than from the photon as is shown by the data

Single jet and Dijet Photoproduction

- As noted in the first part of the lecture, there will be two components
  - direct or pointlike - photon contributes directly to the hard scattering
  - resolved - both photon and proton parton distributions convoluted with parton-parton scattering subprocesses
  - at $\mathcal{O}(\alpha\alpha_s^2)$ and beyond, specific diagrams can contribute to both. Need an experimental definition to distinguish between the two classes. For example the ZEUS Collaboration has used the following for dijet final states:

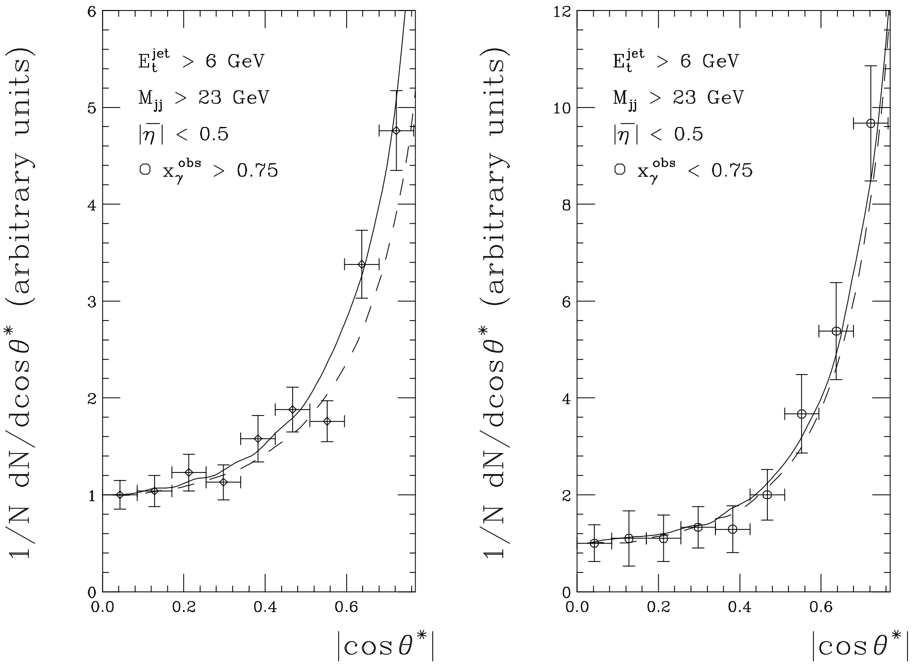
$$x_\gamma^{obs} = (E_T^{jet_1} e^{-\eta_{jet_1}} + E_T^{jet_2} e^{-\eta_{jet_2}})/2E_\gamma$$

  - positive η corresponds to the direction of the proton
  - consider two samples with $x_\gamma^{obs} > .75$ (mostly direct) and $x_\gamma^{obs} < .75$ (mostly resolved)

- Dijet angular distribution
  - $\gamma q \to qg$ and $\gamma g \to q\overline{q}$ are fermion exchange processes which have an angular distribution going as $1/(1-\cos\theta)$ in the dijet center of mass system
  - typical boson exchange processes in the resolved part $qq \to qq, qg \to qg, gg \to gg$ etc., behave as $1/(1-\cos\theta)^2$ which is much steeper

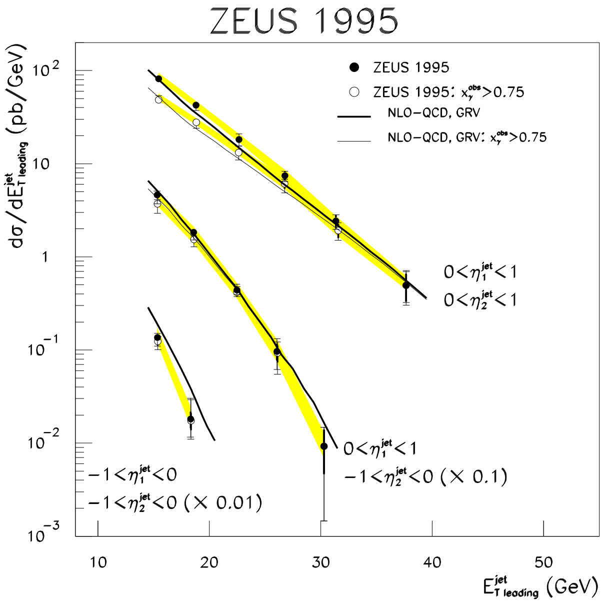
Compare to data from the ZEUS Collaboration (M. Derrick *et al.*, Phys./ Lett. B**384**, 401 (1996).)

Curves from Harris and Owens, Phys. Rev. D**56**, 4007 (1997). Dashed lines are LO and solid are NLO.

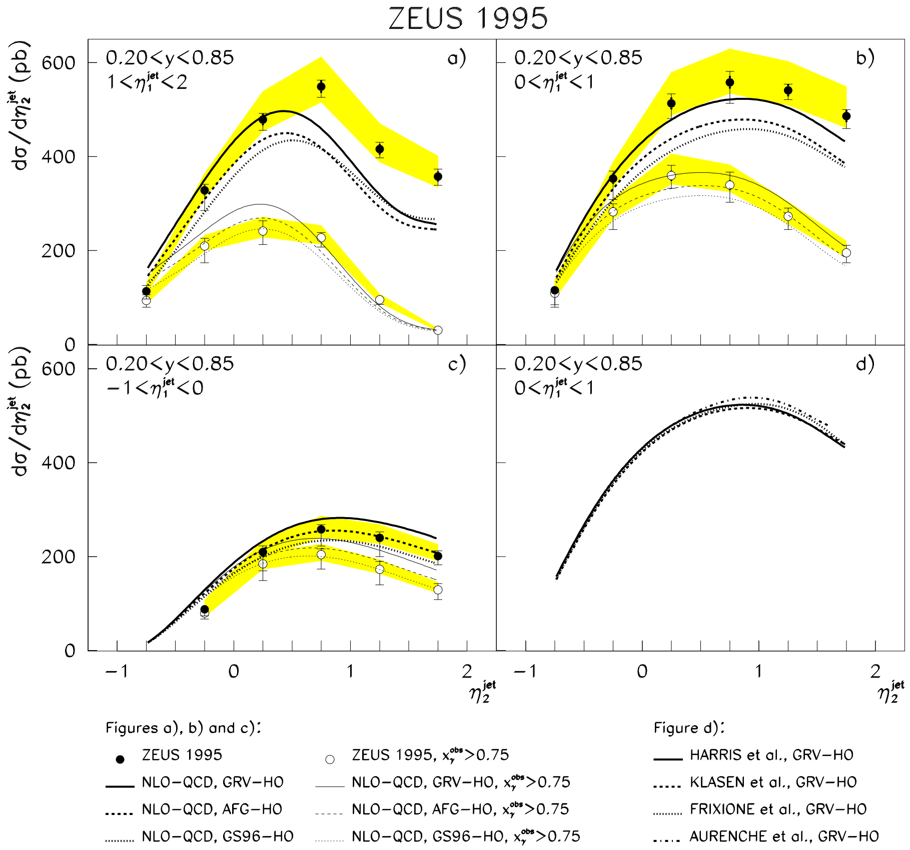


Note the different scales on these normalized distributions - the dominantly resolved sample has a much steeper distribution

- Next, compare NLO results with dijet data on E_T and η distributions
  - by measuring the E_T and η for both jets, one can study regions dominated by the direct or resolved contributions
  - the coordinate system used in the following has positive η in the direction of the proton
  - moving the jets to more forward rapidity values *decreases* x_γ^{obs} thereby emphasizing the resolved component
  - moving the jets to backward rapidities emphasizes the direct component
- Data from the ZEUS Collaboration hep-ex/9905046

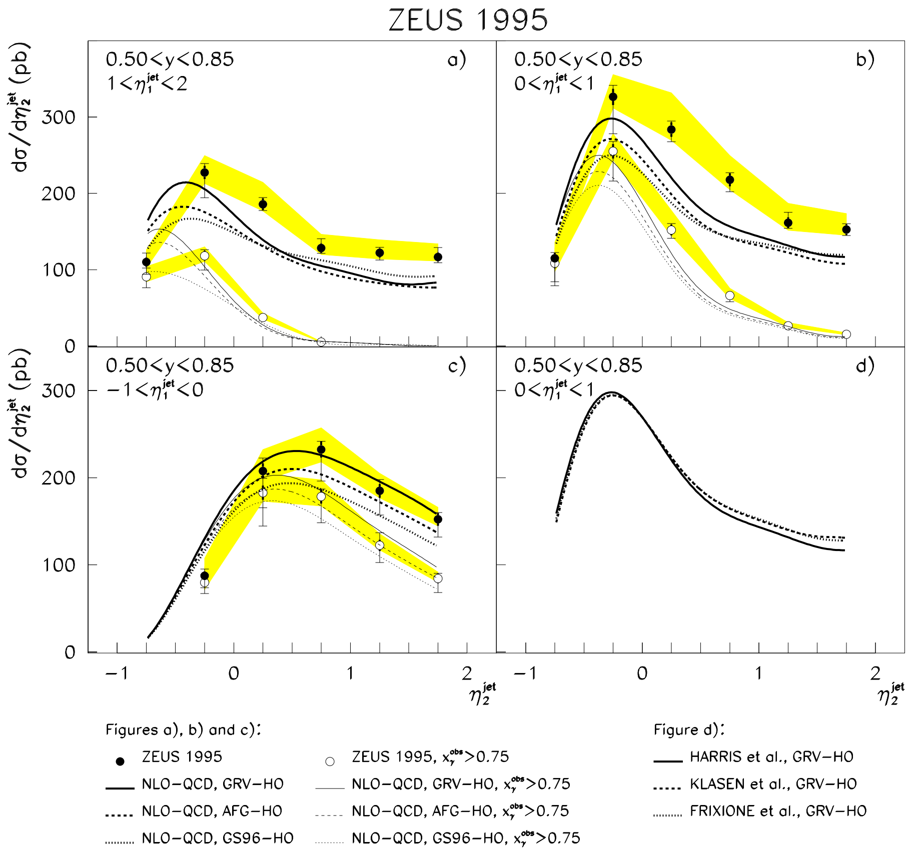


- NLO curves agree well except when both jets are at negative rapidities - more on this shortly



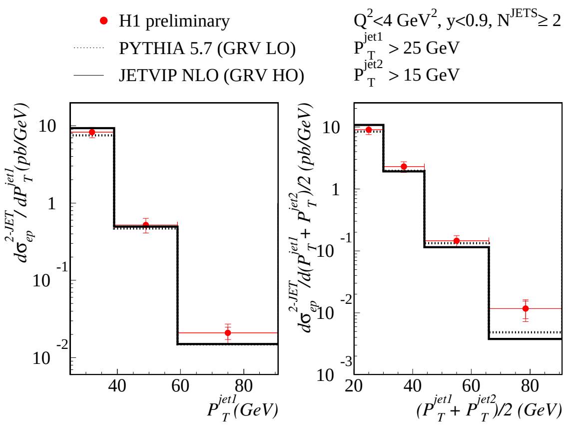
- Theory describes the direct part well, but the total rate is underestimated.

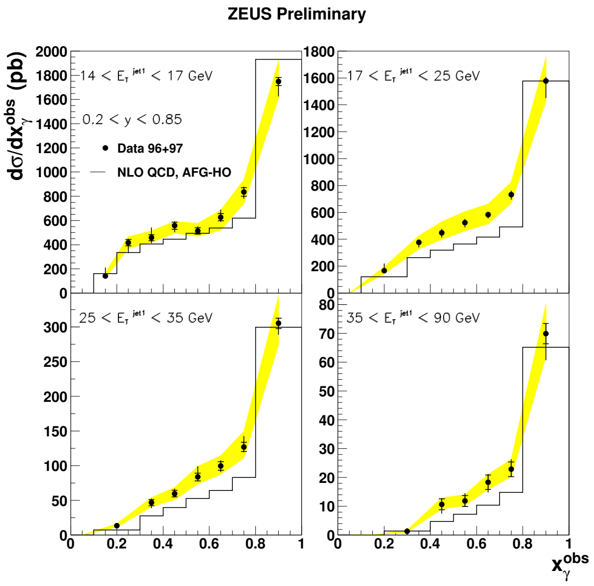
- Examine the high y end of the data sample where $E_\gamma = yE_e$
  - Increasing E_γ increases s and decreases x_γ^{obs} thereby increasing the emphasis on the resolved contribution to the forward jet cross section



- Discrepancy grows for the higher energy sample
  - suggests that the parametrizations of the photon parton distribution functions used for the NLO predictions are too low

- Previous conclusion supported by new data from H1 and ZEUS - see M. Wing, hep-ex/0007011

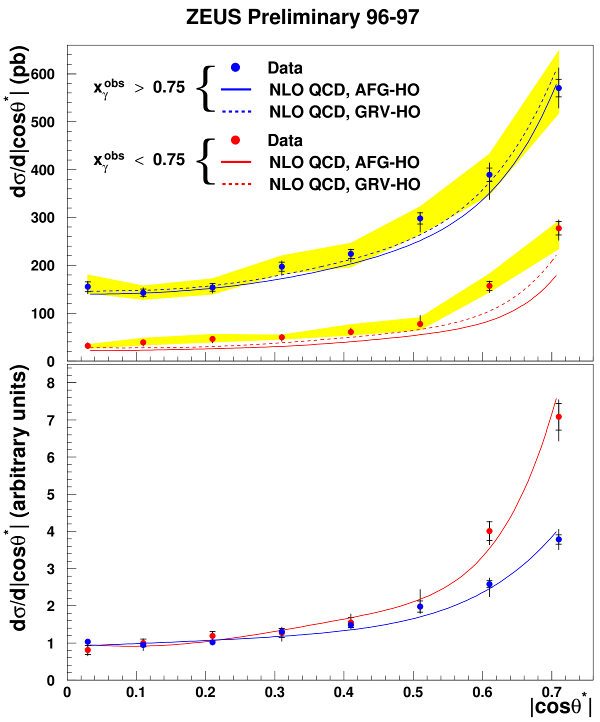




- Overall p_T distribution well described

- Data underestimated by theory in the low x_γ^{obs} region

- New data for the dijet $\cos\theta^*$ distribution also support the previous conclusion

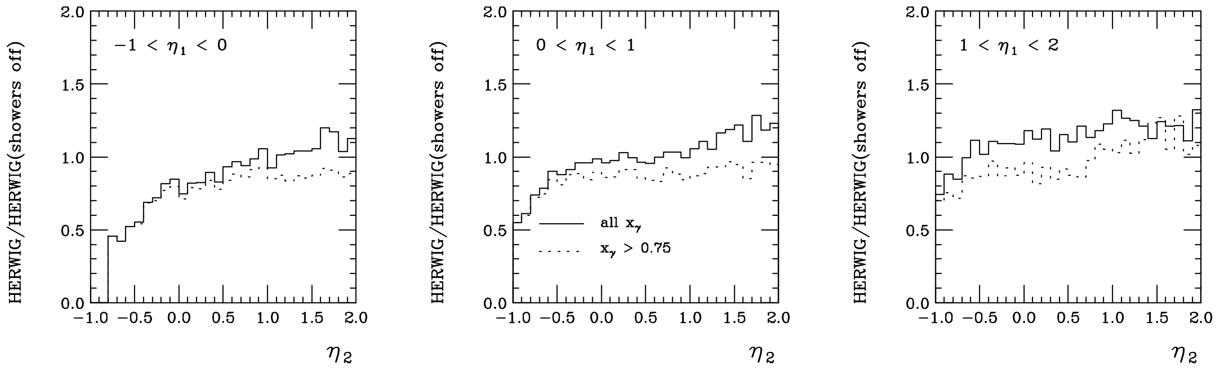


- Shapes are correct for both the resolved and direct contributions

- Normalization is again too low in the resolved component

- More work to be done on photon pdf's

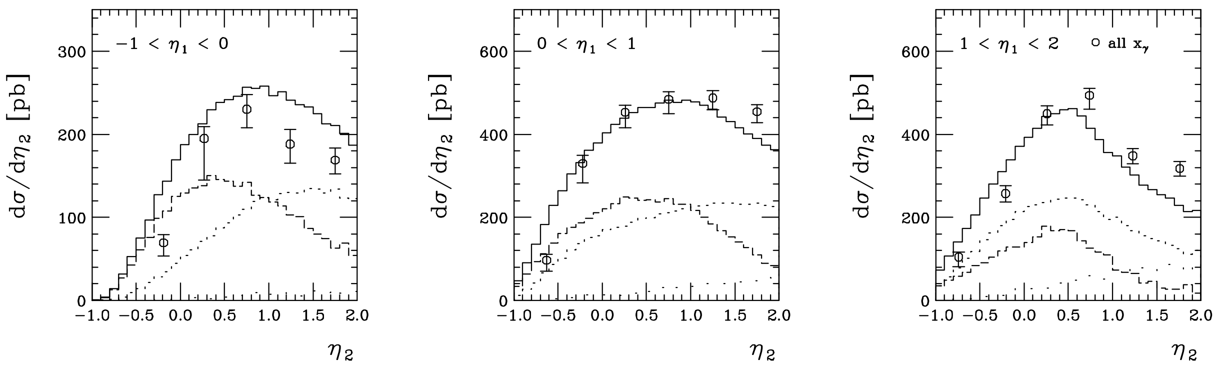
- Problem in the very backward direction is due to the neglect of parton showering effects in the NLO calculations
  - Following figure shows the ratio of HERWIG to HERWIG(showers off) (from Harris and Owens, hep-ph/9712299)



- Showering corrections are near unity everywhere except the region where both jets have negative rapidity - there it drops to about 1/2

- Explains why all the NLO calculations were too high in this region

- Constraints on photon pdf's
  - relative contribution of the resolved component increases as the jet rapidities are made more forward - x_γ^{obs} decreases
  - gluon content in the photon is concentrated at small values of x_γ^{obs}



Total (solid line), Direct (dashed),

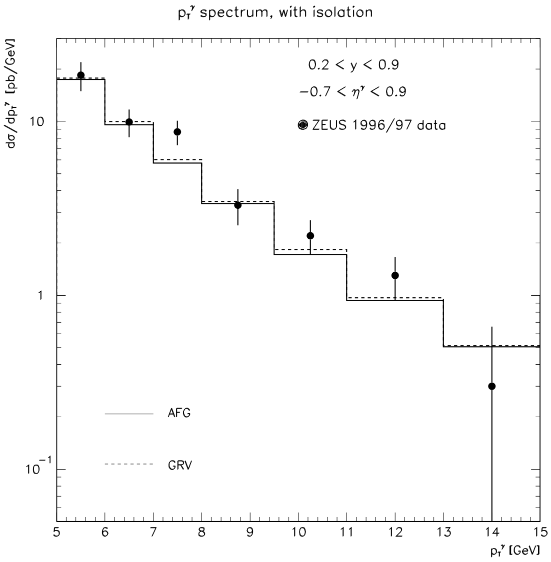
Resolved quark (.....), Resolved gluon (. . . .)

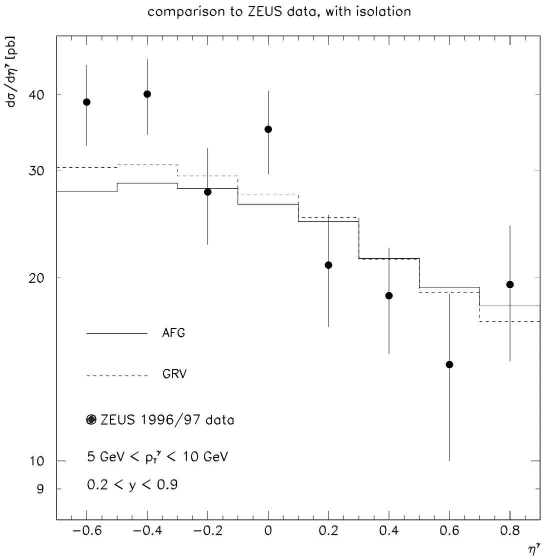
- To enhance sensitivity to the gluon distribution, need smaller x which means more forward jet rapidities and/or higher energy (or a subset of the data with higher y)

Photoproduction of Photons

- Direct photon production has a long history of being used as a probe of hard scattering dynamics in hadron-hadron interactions (See lecture by Steve Kuhlmann)
- Similar role can be played in photoproduction processes
- Now have a photon in the initial state and one in the final state
- Leads to a more complex set of subprocesses
- Taking into account the possibility of photon parton distributions in the initial state and photon fragmentation functions in the final state we have the following structure
  - Direct Direct, e.g., $\gamma q \to \gamma q$
  - Resolved Direct, e.g., $G_{a/\gamma} \otimes \sigma(ab \to \gamma c)$
  - Direct Fragmentation, e.g., $\sigma(\gamma b \to cd) \otimes D_{\gamma/c}$
  - Resolved Fragmentation, e.g. $G_{a/\gamma} \otimes \sigma(ab \to cd) \otimes D_{\gamma/c}$
- Recent calculation includes all these contributions to NLO - Fontannaz, Guillet, and Heinrich, hep-ph/0105121

- Compare to recent ZEUS data, Phys. Lett B **472**, 175 (2000), hep-ex/9910045

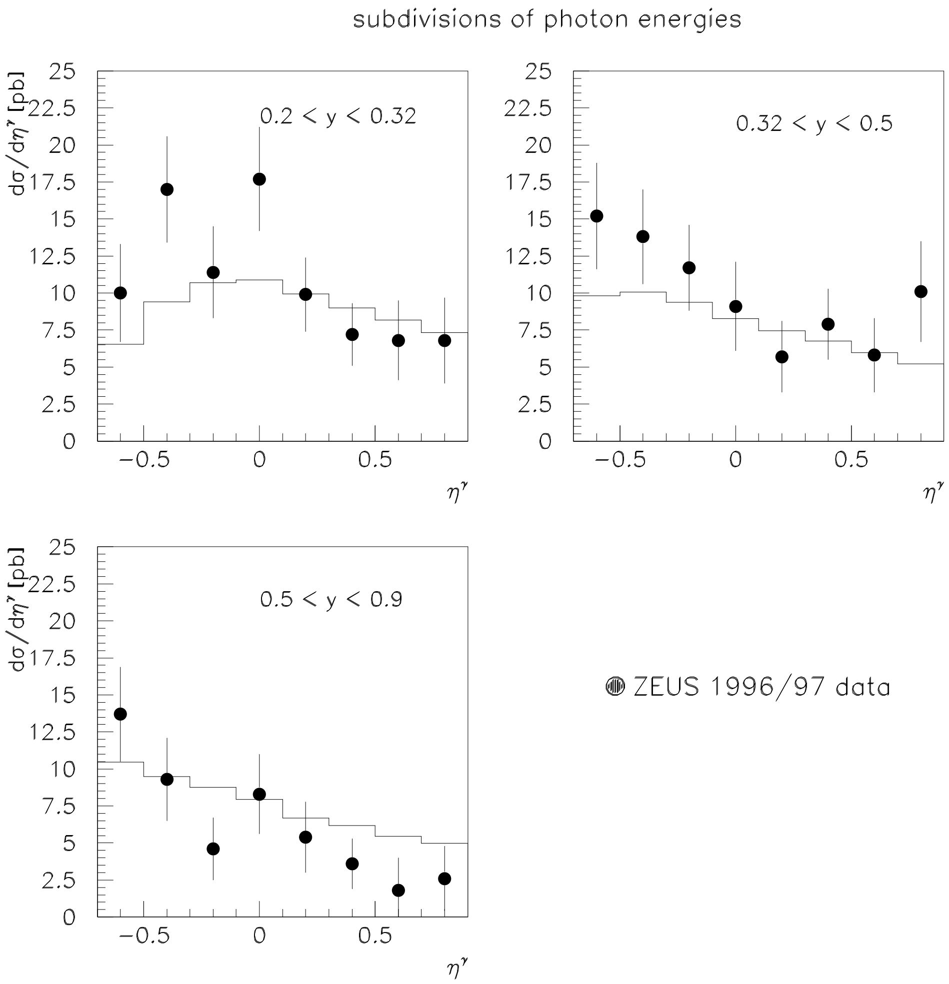




- Photon p_T distribution well described

- Photon η distribution too low at negative η^γ

- Split data set into three ranges of incident photon energy



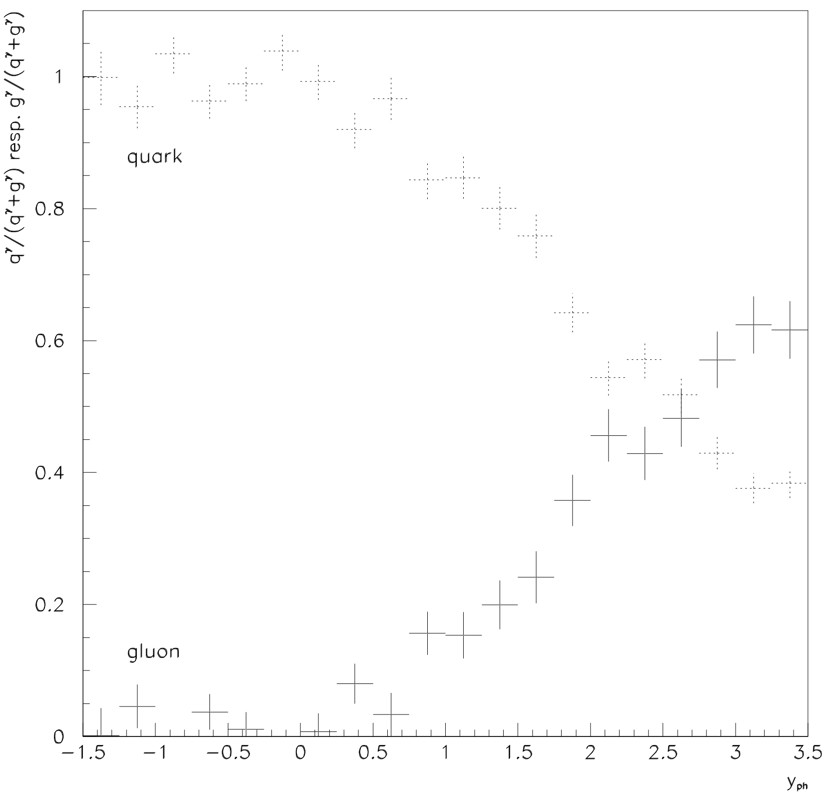
- Problem at low η^γ is at the lower energies

- See overestimate at high η^γ at the highest energy

- Need better statistics to study this in detail

- Note: similar conclusions reached in a recent paper by Krawczyk and Zembrzuski, hep-ph/0105166

- Direct photon photoproduction may help constrain the gluon distribution in the photon
  - Photon parton distributions primarily fit to data for F_2^γ which is sensitive mostly to the quark content of the photon
  - As shown previously, dijet production may help
  - Direct photon photoproduction may help through the resolved direct subprocess $gq \to \gamma q$



- Need extended range in photon rapidity

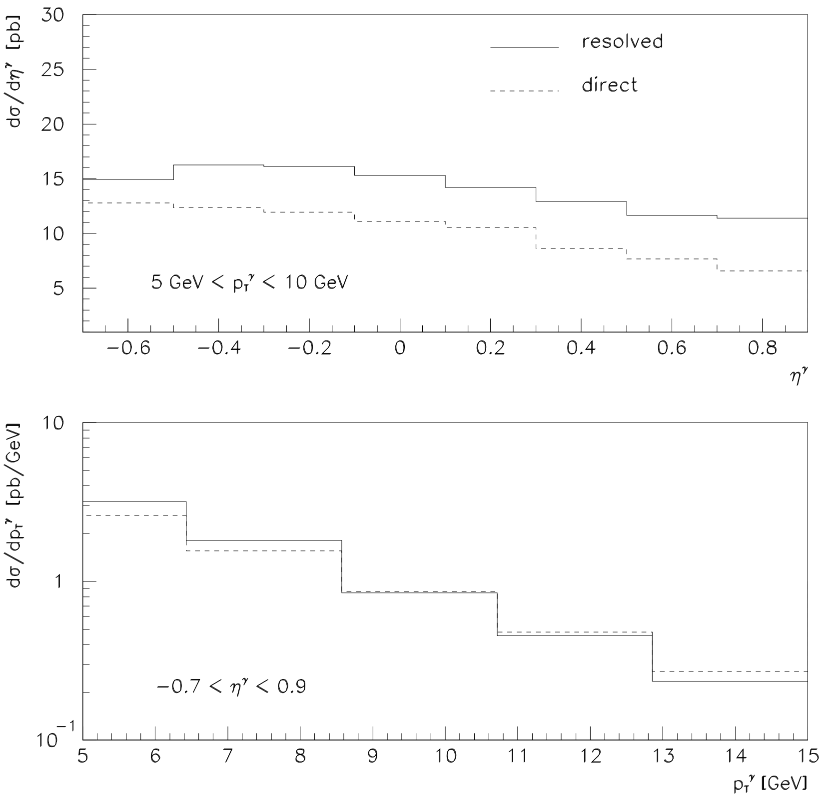
- γ + jet final state may help through better control of the subprocess kinematics

k_T Effects?

- For hadroproduction of direct photons, NLO QCD calculations are known to underestimate the measured cross sections for a number of experiments, especially at fixed target energies

- Both phenomenological analyses and calculations based on QCD resummation techniques suggest that the effects of initial state radiation smear the photon p_T distributions, thereby enhancing the single photon inclusive cross section at fixed p_T

- No such deficit appears to exist in the p_T spectrum of direct photon photoproduction. Why?

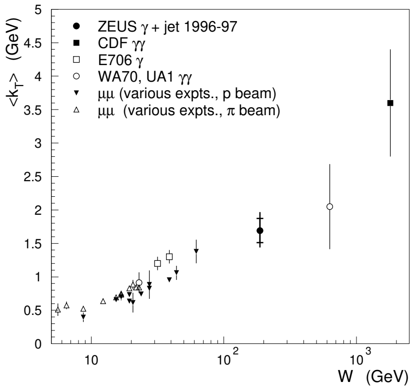


- Significant fraction of the cross section is from direct processes, and there is no shower (and hence k_T) associated with the photon in that component

- Recent ZEUS analysis (hep-ex/0104001) looks at γ + jet events where $E_T^{\gamma} > 5$ GeV, $E_T^{jet} > 5$ GeV, and $x_{\gamma} > .9$ to isolate the direct component

  – found $< k_T > = 1.69 \pm 0.186^{+.18}_{-.20}$ GeV

  – includes effects of showering in PYTHIA plus the input PYTHIA parameter k_0



  – consistent with other determinations of $< k_T >$

- Conclude that showering effects are present in direct photon photoproduction, but they are less important, at least for the direct component, since only one initial leg is showering

- Of course, this is not true for the resolved component

- Conclude that more study is needed, but the relationship between photoproduction and hadroproduction is interesting

Hadroproduction of Photon Pairs

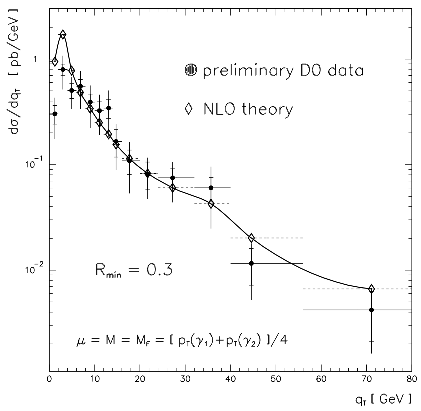
- Process complements single photon production as a probe of the hard scattering dynamics

- Important background for LHC Higgs searches

- Recent full NLO calculation from Binoth, Guillet, Pilon, and Werlen, hep-ph/9911340

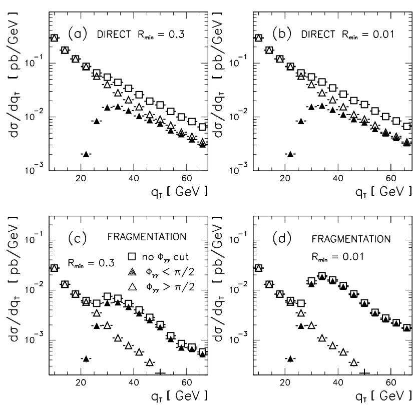
- As in the previous examples, there are a variety of production mechanisms
  - Annihilation: $q\bar{q} \to \gamma\gamma$
  - Single fragmentation, e.g., $\sigma(ab \to \gamma c) \otimes D_{\gamma/c}$
  - Double fragmentation, e.g., $\sigma(ab \to cd) \otimes D_{\gamma/c} \otimes D_{\gamma/d}$
  - Gluon box: $gg \to \gamma\gamma$

- Compare to recent D0 data
  - $E_T > 14.9(13.85)$ GeV for the higher (lower) E_T photon
  - Photon rapidities between -1 and 1
  - Acollinearity cut for the two photons requires $\Delta y^2 + \Delta\phi^2 \geq R^2_{min}$



- Both the data and the theory show a small shoulder in the photon pair q_T plot. See Binoth, Guillet, Pilon, and Werlen, hep-ph/0012191

- True NLO effect in the theory - unusual to see such a shape change due to NLO effects
  - shoulder is due to the opening up of a contribution where both photons are on the same side of the event
  - dominant contribution is where a quark radiates a photon and the quark remnant fragments into a photon
  - configuration is enhanced by the near collinearity of the parent quark and the brems photon
  - nearly collinear configuration requires $q_T > 28.75$ GeV



Some comments on isolation cuts

- Previously referenced calculation shows some problems with the usual photon isolation cuts

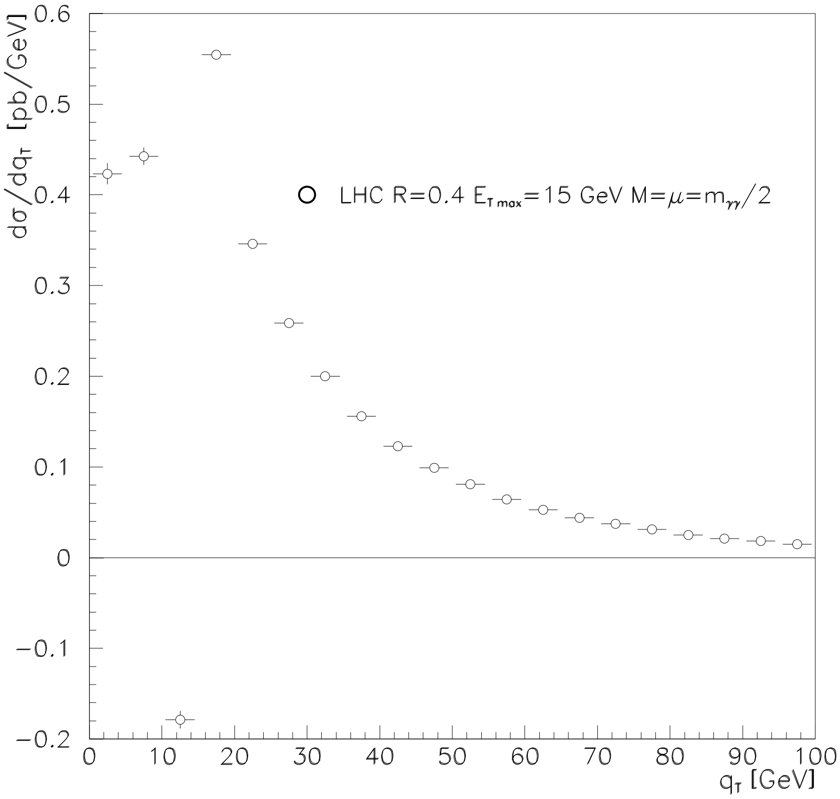
- Problem discussed in Berger, Guo, and Qiu, Phys. Rev. **D54** 5470 (1996)

- Basic isolation cut vetoes events in which more than a certain amount of hadronic energy (or transverse energy) accompanies the photon within a cone of some specified radius

- consider a subprocess like $\sigma(qg \to \gamma qg) \otimes D_{\gamma/q}$
  - the isolation cut on the photon from the fragmentation function places a limit on the allowed range of z for the fragmentation
  - it is possible for the fragmentation remnants to saturate the isolation cut, thus curtailing the gluon contribution within the isolation cone
  - this results in a lack of cancellation between the real and infrared gluon singularities

- the singularities are integrable and the act of binning smoothes over them

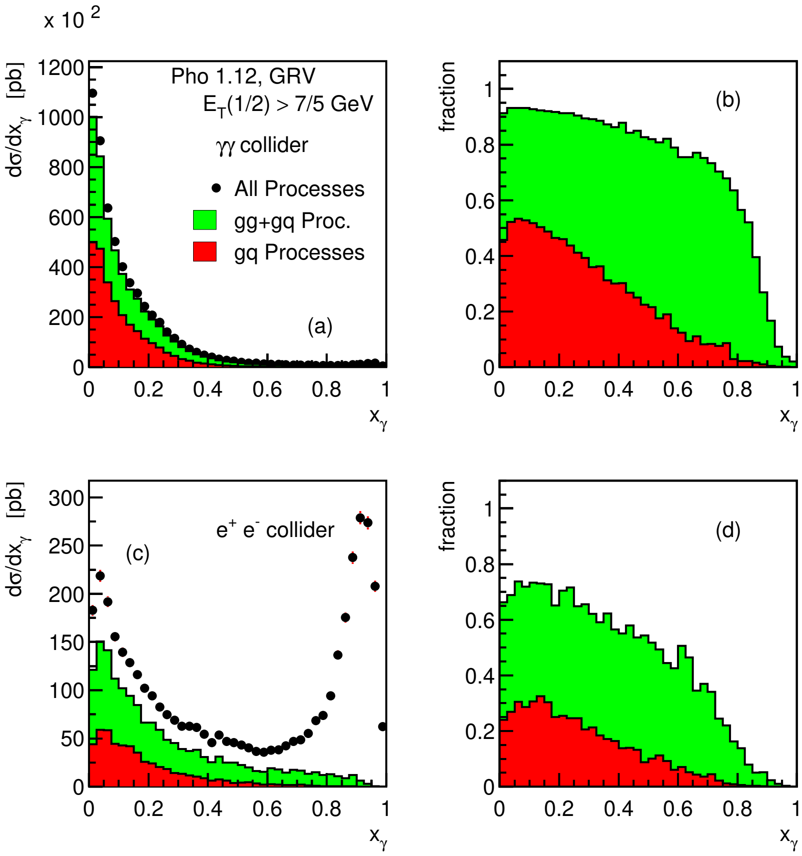
- if very tight isolation cuts are used and the binning is not large enough, then artificially induced dips/bumps appear



- The use of very tight isolation cuts may require the application of resummation techniques

Final Example: e^+e^- Linear Collider

- Two photon processes for either an e^+e^- or $\gamma\gamma$ collider will have three types of contributions
  1. Direct-Direct: no photon beam fragments
  2. Direct-Resolved: beam fragments from one photon
  3. Resolved-Resolved: beam fragments from both photons
- For classes (2) and (3) above, one needs photon parton distributions
- Can expect significant gluon contributions for kinematic regions which reach low values of x_γ
- Predictions for dijet production from Wengler and De Roeck, hep-ph/0010293

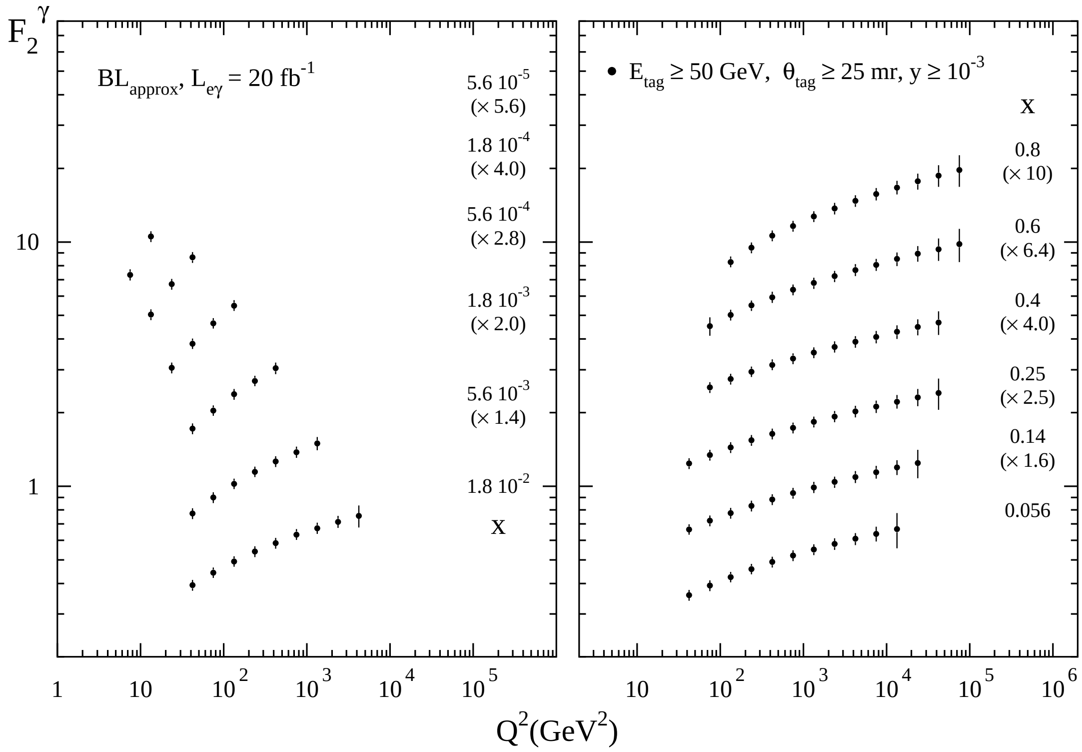


- 7/5 means $E_{Tmin} = 7$ GeV for the highest E_T jet and $E_{Tmin} = 5$ GeV for the second jet

- Note dominance of gluon subprocesses especially for the $\gamma\gamma$ mode

- Can also measure F_2^γ at such a facility

- Kinematic coverage for $e\gamma$ mode from De Roeck, hep-ph/0101075



- Note characteristic linear rise of F_2^γ as discussed earlier

- Plentiful opportunities for the study of QCD dynamics exist at such machines

- Necessary to understand the sizeable QCD contributions in order to be able to search for new physics signals

Concluding Remarks

- Photons are important participants in many processes studied at hadron-hadron, hadron-lepton, and lepton-lepton machines

- When the photons participate in hadronic processes one encounters nonperturbative parton distribution and fragmentation functions for photons

- The additional subprocesses which are present when considering photon interactions provide new probes of strong dynamics

- Potentially useful for constraining parton distributions

- Photons are important components of searches for new physics

- As such, they also contribute to backgrounds for such searches

- Important to understand the production of and interactions of photons

- Topics worth investigation

  - new constraints on photon pdf's provided by jet photoproduction data

  - relationship between hadroproduction and photoproduction of photons

  - treatment of photon isolation cuts

  - new opportunities for photon physics at e^+e^-, $e\gamma$, and $\gamma\gamma$ machines

Some miscellaneous and hopefully useful stuff

1. Check out the CTEQ web page at www.cteq.org
   - information on past summer schools, including transparencies of many of the lectures
   - information and links for parton distributions
   - CTEQ List of Challenges in Perturbative QCD
   - CTEQ Pedagogical Page
   - CTEQ Handbook of Perturbative QCD

2. Four-vectors and rapidity
   - rapidity is defined as $y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}$. For massless particles this reduces to the pseudorapidity which is defined as $\eta = \ln\cot\theta/2$.
   - the four-vector for a massless particle with transverse momentum p_T and rapidity y may be conveniently expressed as

$$p^\mu = (p_T \cosh y, p_T, 0, p_T \sinh y)$$

3. Mandelstam variables
   - For a two-body process $p_1 + p_2 \to p_3 + p_4$ the three Mandelstam variables are defined as

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - p_3)^2 \\ u &= (p_1 - p_4)^2 \end{aligned}$$

- For processes with more particles one sometimes encounters variables such as $s_{ij} = (p_i + p_j)^2$ which is just the squared invariant mass of particles i and j and $t_{ij} = (p_i - p_j)^2$ which is the squared four-momentum transfer between particles i and j.

4. Another example: jet photoproduction $\gamma q \to jet + X$

   - four-vectors

$$\begin{aligned} p_\gamma &= \frac{\sqrt{s}}{2}(1,0,0,1) \\ p_q &= \frac{\sqrt{s}}{2} x_b (1,0,0-1) \\ p_{jet} &= p_T(\cosh y, 1, 0, \sinh y) \end{aligned}$$

   - Substituting these four-vectors into the expressions for the Mandelstam variables above yields

$$\begin{aligned} \hat{s} &= x_b\ s \\ \hat{t} &= -p_T\sqrt{s}\ e^{-y} \\ \hat{u} &= -x_b\ p_T\sqrt{s}\ e^{y} \end{aligned}$$

5. Convolutions

   - The symbol $\otimes$ has been used throughout this lecture to denote a convolution:

$$\begin{aligned} f \otimes g &= \int_0^1 dy \int_0^1 dz\ f(y)\ g(z)\ \delta(x - yz) \\ &= \int_x^1 \frac{dz}{z}\ f(x/z)\ g(z) \end{aligned}$$

6. Some comments on the asymptotic solution of the evolution equations for parton distributions in a photon

- Rewrite the evolution equations by taking moments of both sides using the following definitions:

$$
\begin{aligned}
M_q^n &= \int_0^1 dx\ x^{n-1}\ G_{q/\gamma}(x) \\
M_g^n &= \int_0^1 dx\ x^{n-1}\ G_{g/\gamma}(x) \\
A_{ij}^n &= \frac{1}{2\pi b} \int_0^1 dx\ x^{n-1}\ P_{ij}(x) \\
a^n &= \frac{\alpha}{2\pi} \int_0^1 dx\ x^{n-1}\ P_{q\gamma} \\
\alpha_s(t) &= \frac{1}{bt}
\end{aligned}
$$

  where $t = \ln(Q^2/\Lambda^2)$.

- The evolution equations can now be written as

$$
\begin{aligned}
\frac{dM_q^n}{dt} &= e_q^2\ a^n\ + \frac{1}{t}\ [A_{qq}^n\ M_q^n\ + A_{qg}^n\ M_g^n] \\
\frac{dM_g^n}{dt} &= \frac{1}{t}\ \left[\sum_q A_{gq}^n M_q^n + A_{gg}^n M_g^n\right]
\end{aligned}
$$

- If each of the moments is proportional to t, the t dependence drops out of the equations and they may be solved algebraically

- The asymptotic solution is

$$
\begin{aligned}
M_q^n &= a^n \left(\frac{e_q^2 - 5/18}{1 - A_{qq}^n} + \frac{5}{18} \frac{1 - A_{gg}^n}{F^n} \right) t \\
M_g^n &= \frac{5f}{9} a^n \frac{A_{gg}^n}{F^n} t \\
F^n &= 1 - A_{qq}^N - A_{gg}^n + A_{qq}^n A_{gg}^n - 2f A_{qg}^n A_{gq}^n
\end{aligned}
$$

  where f is the number of flavors

- Note how the moments are each proportional to t

- Compare to the case where $P_{q\gamma} = 0$ where the moments are of the form

$$M^n(t_0) \left(\frac{t}{t_0} \right)^{A^n}$$

- Note that one can add any solution of the homogeneous evolution equations to this asymptotic solution

7. HERA Kinematics

   - It is conventional for the HERA experiments to present results in the lab frame with positive rapidity in the direction of the proton beam. The kinematics I used earlier for jet photoproduction used the photon-hadron center-of-momentum frame with positive rapidity in the direction of the photon.

   - Suppose that the proton and photon lab four-vectors at HERA are given as $p = p_p(1, 0, 0, 1)$ and $p_\gamma = yE_e(1, 0, 0, -1)$.

- To go from the lab to the photon-proton cms, the boost parameter is $\beta = \frac{p_p - yE_e}{p_p + yE_e}$.

- Use

$$E^{lab} = \gamma(E + \beta p_z) \text{ and } p_z^{lab} = \gamma(p_z + \beta E)$$

to derive

$$\eta^{lab} = \eta + \frac{1}{2} \ln \frac{1+\beta}{1-\beta} = \eta + \frac{1}{2} \ln \frac{p_p}{yE_e}$$

- Taking into account the sign reversal on the convention for rapidity one arrives at the final relation between the two cases

$$\eta^{lab} = -\eta + \frac{1}{2} \ln \frac{p_p}{yE_e}$$