Heavy Quark Production

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- Heavy quark decays
- Leading order
- Higher order corrections
- Bottom production
- Top production
- Single top production

Collaboration of B.R. Webber and W.J. Stirling in development of some of this material is gratefully acknowledged.

Approximate rates

- There are many measurements of heavy quark production.
- Note high rate of hadron machines for b-quarks.

Process	$\sqrt{s}$ [GeV]	Cross section	$\sigma_{Q\bar{Q}}/\sigma_{tot}$
$\pi N \to c\bar{c}$	25	20 μb	0.1%
$pN \to c\bar{c}$	38	40 μb	0.1%
$\gamma N \to c\bar{c}$	19	0.7 μb	0.4%
$pp \to b\bar{b}$	40	20 nb	10^{-6}
$p\bar{p} \to b\bar{b}$	1800	20 μb	0.03%
$pp \to b\bar{b}$	14000	300 μb	0.5%
$e^+e^- \to b\bar{b}$	10.48	1 nb	25%
$e^+e^- \to b\bar{b}$	91	7 nb	15%
$p\bar{p} \to t\bar{t}$	1800	5 pb	10^{-10}
$pp \to t\bar{t}$	14000	800 pb	10^{-8}
$e^+e^- \to t\bar{t}$	500	500 fb	5%

e^+e^- vs. pp

- A large sample of b quarks is needed to observe CP violation in the decays of hadrons containing b quarks and to measure $B_s - \overline{B}_s$ mixing.

- At the Fermilab Tevatron running at a luminosity of 10^{32} cm^{-2}s^{-1}, b quarks are produced at 2 kHz.

- At an e^+e^- machine such as PEP-II (SLAC) operating at a luminosity of 3×10^{33} cm^{-2}s^{-1}, the rate of b production is ~ 3 Hz

- hadronic production of b quarks gives access to a wide spectrum of states, including mesons containing both bottom and charm ($b\overline{c}$) and baryonic states containing b quarks.

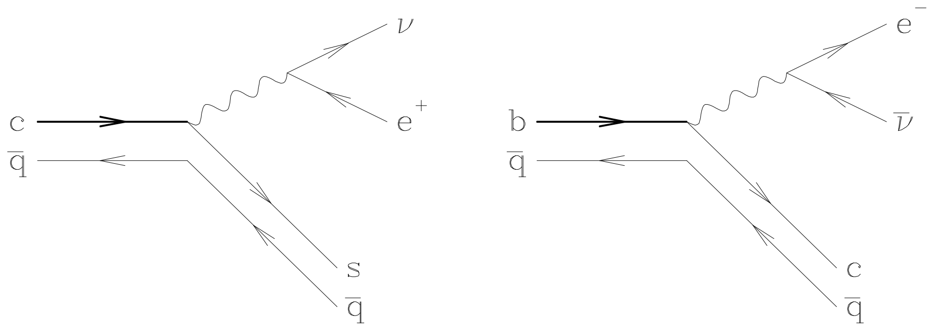
- At e^+e^- machines operating at the $\Upsilon(4S)$ resonance only B^0 and B^+ are produced.

Charm and bottom quark decays

- treat the semi-leptonic decays of hadrons containing c and b quarks in analogy with the decay of a free muon, (*spectator model*)

- Lagrangians for CKM-favoured decays are

$$\mathcal{L}^{(c)} = -\frac{G_F}{\sqrt{2}} V_{cs}\, \bar{s}\gamma^\mu(1-\gamma_5)c\, \bar{\nu}\gamma_\mu(1-\gamma_5)e\ ,$$
$$\mathcal{L}^{(b)} = -\frac{G_F}{\sqrt{2}} V_{cb}\, \bar{c}\gamma^\mu(1-\gamma_5)b\, \bar{e}\gamma_\mu(1-\gamma_5)\nu\ .$$



$$\overline{\sum}|\mathcal{M}^{(c)}|^2 = 64\, G_F^2\, |V_{cs}|^2\, c\cdot e^+\ s\cdot\nu\ ,$$
$$\overline{\sum}|\mathcal{M}^{(b)}|^2 = 64\, G_F^2\, |V_{cb}|^2\, b\cdot\bar{\nu}\ c\cdot e^-\ ,$$

where $b, c, s, \nu, \bar{\nu}, e^+$ and e^- now stand for the four-momenta of the particles in the decay.

- by angular momentum conservation s and ν momenta prefer to be anti-parallel in c quark decay. The endpoint configuration in which the e^+ recoils against the parallel s and ν is thus disfavoured. We expect a soft spectrum for the positron.

- Conversely we expect a hard spectrum for the neutrino (or for the electron coming from the decay of a b quark).

$$\Gamma^{(Q)}_{\mathrm{sl}} = \frac{m_Q}{2^8\pi^3}\int dxdy\,\theta(x+y-x_m)\,\theta(x_m-x-y+xy) \overline{\sum}|\mathcal{M}^{(Q)}|^2$$

- x and y are the rescaled energies of the charged and neutral leptons, $x = 2E_e/m_Q$, $y = 2E_\nu/m_Q$ in the frame in which the heavy quark Q is at rest. The kinematic endpoint of the spectrum is denoted by x_m and is given by $x_m = 1-\epsilon^2$ where $\epsilon = m_q/m_Q$. The result for the semi-leptonic widths of the c and the b is

$$\frac{d\Gamma^{(c)}_{\mathrm{sl}}}{dxdy} = |V_{cs}|^2\Gamma_0(m_c)[12x(x_m-x)]$$

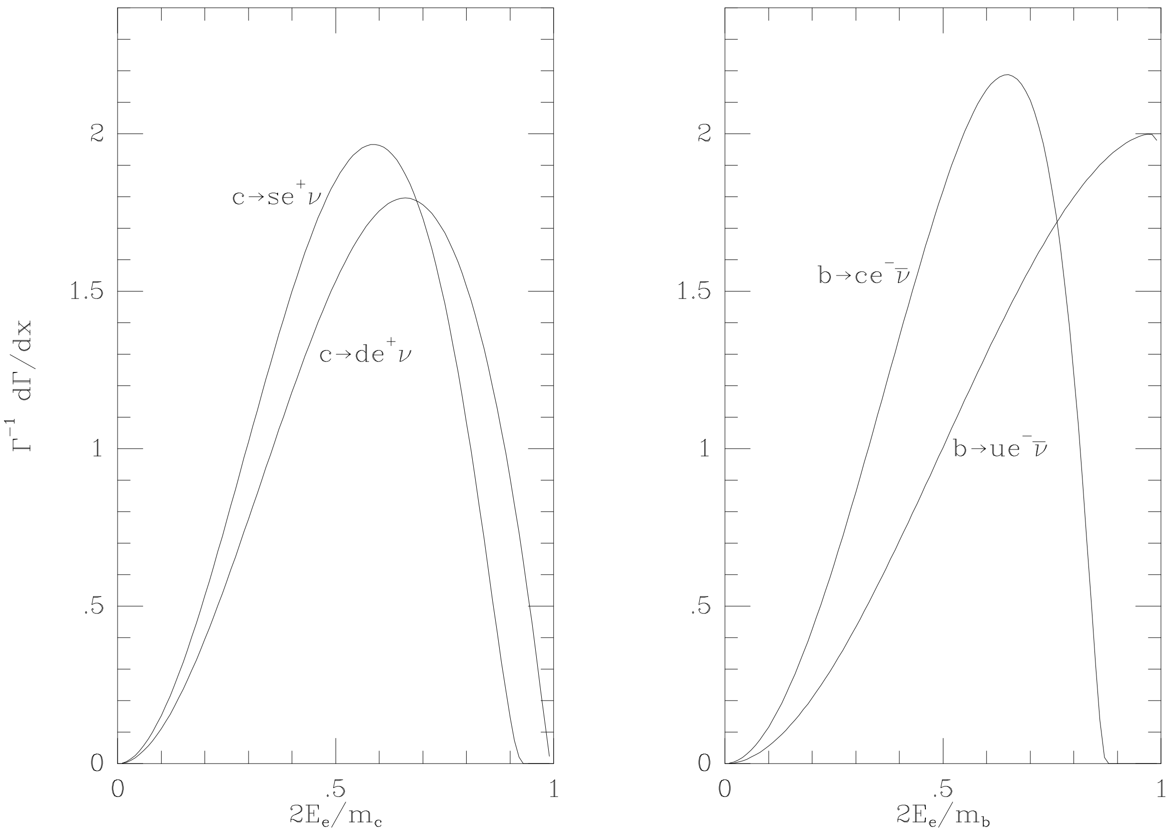
$$\frac{d\Gamma^{(b)}_{\mathrm{sl}}}{dxdy} = |V_{cb}|^2\Gamma_0(m_b)[12y(x_m-y)]$$

Γ_0 is the rescaled muon decay rate,

$$\Gamma_0(m_Q) = \frac{G_F^2 m_Q^5}{192\pi^3} \; .$$

The lepton spectra are

$$\begin{aligned} \frac{d\Gamma_{\mathsf{sl}}^{(c)}}{dx} &= |V_{cs}|^2 \Gamma_0(m_c) \left[\frac{12x^2(x_m - x)^2}{(1-x)}\right] \\ \frac{d\Gamma_{\mathsf{sl}}^{(b)}}{dx} &= |V_{cb}|^2 \Gamma_0(m_b) \left[\frac{2x^2(x_m - x)^2}{(1-x)^3}\right] \\ &\times \quad (6 - 6x + x x_m + 2x^2 - 3x_m). \end{aligned}$$



The e^+ from charm decay has a soft spectrum. The e^- from the CKM-disfavoured mode $b \to u$ has a hard spectrum.

- The measurement of leptons with energies beyond the kinematic limit for $b \to c$ gives information about V_{ub}.

- Allowing for theoretical uncertainty in endpoint region, the measured value is $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$

After integration the semi-leptonic width including mass effects is found to be

$$\Gamma_{\mathsf{sl}}^{(Q)} = |V_{Qq}|^2 \Gamma_0(m_Q) f(\frac{m_q}{m_Q})$$

where the function f is given by

$$f(\epsilon) = (1 - \epsilon^4)(1 - 8\epsilon^2 + \epsilon^4) - 24\epsilon^4 \ln \epsilon \ .$$

Including the CKM-disfavoured mode $c \to d$ the result for the semi-leptonic decay of the c quark is

$$\Gamma_{\mathsf{sl}}^{(c)} = \Gamma_0(m_c)[f(m_s/m_c)|V_{cs}|^2 + f(m_d/m_c)|V_{cd}|^2].$$

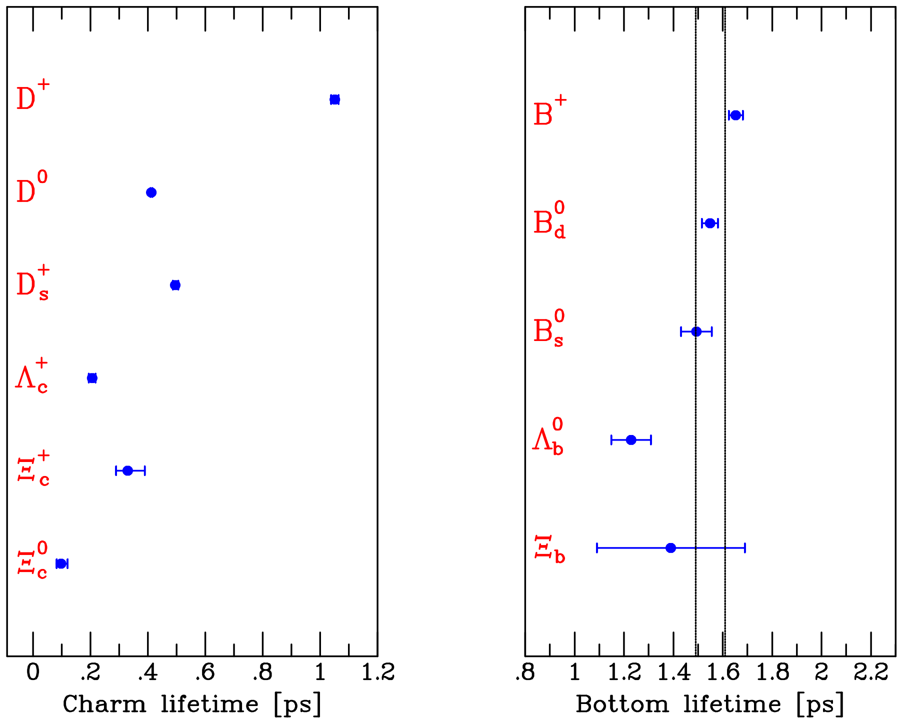
- For a rough estimate ignore strong interaction corrections and choose $m_c = 1.4$ GeV. The theoretical estimate for the semi-leptonic width is

$$\Gamma_{\mathsf{sl}} = 1.1 \times 10^{-10}\ \text{MeV}.$$

From the measured semi-leptonic branching ratios of the $D^+, (17.2 \pm 1.9\%)$ and $D^0, (7.7 \pm 1.2\%)$ and the inverse of known lifetimes, we can calculate the semi-leptonic widths.

$$\begin{aligned} \Gamma_{\mathsf{sl}}(D^0) &= (1.22 \pm 0.20) \times 10^{-10}\ \text{MeV} \\ \Gamma_{\mathsf{sl}}(D^+) &= (1.07 \pm 0.13) \times 10^{-10}\ \text{MeV}. \end{aligned}$$

- The spectator model gives a fair description of the semi-leptonic decays of D mesons.



- For the semi-leptonic decays of B mesons, the theoretical decay width is

$$\Gamma_{\text{sl}}^{(b)} = \Gamma_0(m_b) f(\frac{m_c}{m_b}) |V_{cb}|^2 \eta_0,$$

- The CKM-disfavoured mode makes a negligible contribution to the total rate. $\eta_0 \approx 0.87$ due to strong interaction corrections.

  Using the measured semi-leptonic branching ratios of the $B^{\pm}$, $(10.1 \pm 1.8 \pm 1.5\%)$ and B^0, $(10.9 \pm 0.7 \pm 1.1\%)$, the semi-leptonic widths of the B mesons are

$$\Gamma_{\text{sl}}(B^0) \quad = \quad (0.48 \pm 0.12) \times 10^{-10} \text{ MeV}$$

$$\Gamma_{\,\mathsf{sl}}(B^+) \quad = \quad (0.43 \pm 0.18) \times 10^{-10}\ \mathsf{MeV}.$$

We can estimate V_{cb}. Choosing the values $m_b = 4.8$ GeV, $m_c = 1.4$ GeV and $V_{cb} = 0.04$ one obtains

$$\Gamma_{\,\mathsf{sl}} = 2.7 \times 10^{-8} |V_{cb}|^2\ \mathsf{MeV} = 0.44 \times 10^{-10}\ \mathsf{MeV}\ .$$

Hadronic decays

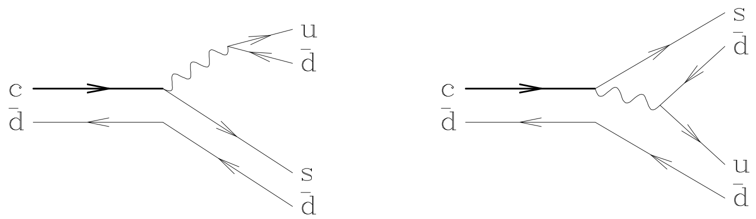
- Estimate for the total width using the spectator model. The width in the spectator model is given simply by the weak decay of the heavy quark followed by the subsequent decay of the resulting virtual W boson.

- Diagrams involving spectators are suppressed by powers of the heavy quark mass. For example, in the decay of a D_s^+ meson, a non-spectator diagram would result from the annihilation of the charm quark with the anti-strange quark.

$$BR(c \to eX) = \frac{1}{1+1+3},$$

  ignoring strong interaction effects. Therefore the prediction of the simplest spectator model for the total width of a charmed hadron is given by multiplying the semi-leptonic width by five.

- This leads to an expected common lifetime for all charmed hadrons of the order of 1.2 ps. However the lifetimes of the D^+ and the D^0 mesons are very different.



- Since the semi-leptonic widths are approximately equal, we can conclude that the failure of the spectator model is due to differences in the hadronic widths of the charmed hadrons.

- Reasons for the failure of the spectator model for D decays include non-spectator diagrams and strong radiative corrections.

- Interference effects should be suppressed by powers of m_c because of the small overlap of the $\bar{d}$ coming from the charm decay with the spectator $\bar{d}$. The former is initially localized in a volume of order $1/m_c^3$, whereas the latter is distributed throughout the D meson state.

- Since these interference effects appear to be important, we conclude that the charm quark is too light to be treated as a heavy quark in this context.

B total width

- Apply the spectator model to hadronic B decays.

- *A priori*, one might expect the B lifetime to be a factor of $(m_c/m_b)^5$ shorter than the estimate for the charm quark lifetime given above. However this mass effect is almost entirely cancelled by the factor of $|V_{cb}|^2$ which occurs in the expression for the width.

- The calculation of the semi-leptonic branching ratio for B decays is complicated even in the spectator model, because of the many channels which are kinematically allowed for the decay. A detailed calculation gives $BR(b \to eX) > 12.5\%$.

- The ground-state hadrons containing b quarks have roughly equal lifetimes.

- Calculating the total width from the theoretical semi-leptonic width and the measured semi-leptonic branching ratio, we obtain a lifetime of about 1.6 ps. The measured average b lifetime is 1.55 ± 0.06 ps

- This corresponds to a proper lifetime expressed in units of length of $c\tau = 463 \pm 18$ μm. A b quark with momentum

20 GeV has a relativistic γ factor of about 4. A B meson of this momentum decaying after one lifetime will travel about 1.9 mm. This decay distance large enough to be measured with a detector, typically a silicon vertex detector.

Top quark decays

- Standard Model. Since $m_t > M_W + m_b$ a top quark decays predominantly into a b quark and an on-shell W boson

$$\begin{aligned} t &\rightarrow W^+ + b \\ &\qquad \hookrightarrow l^+ + \nu \\ t &\rightarrow W^+ + b \\ &\qquad \hookrightarrow q + \bar{q} \end{aligned}$$

- the branching ratio to leptons is given by counting the decay modes of the W, ($e\bar{\nu}_e$, $\mu\bar{\nu}_\mu$, $\tau\bar{\nu}_\tau$ and three colours of $u\bar{d}$ and $c\bar{s}$,

$$\mathrm{BR}(W^+ \rightarrow e^+\bar{\nu}) = \frac{1}{3+3+3} \approx 11\%.$$

With a perfect detector the numbers of events expected at $\sqrt{s} = 1.8$ TeV per 100 pb^{-1} are

$$\begin{aligned} \mathrm{N}(e\mu\ +\text{jets}) = &\quad 2 \times .11 \times .11 \times 500 \approx 12 \\ \mathrm{N}(e\ +\ \text{jets}) = &\quad 2 \times .11 \times .66 \times 500 \approx 72. \end{aligned}$$

The existence of both of these decay modes with the correct ratio is a first test of the decay modes of the top.

The W boson coming from top decay can be either left-handed (L) or longitudinally (0) polarized.

$$\begin{aligned}\overline{\sum}|\mathcal{M}_L|^2 &= \frac{2G_F m_t^4}{\sqrt{2}}|V_{tb}|^2[2x^2(1-x^2+y^2)] \\ \overline{\sum}|\mathcal{M}_0|^2 &= \frac{2G_F m_t^4}{\sqrt{2}}|V_{tb}|^2[1-x^2-y^2(2+x^2-y^2)],\end{aligned}$$

where $x = M_W/m_t, y = m_b/m_t$.

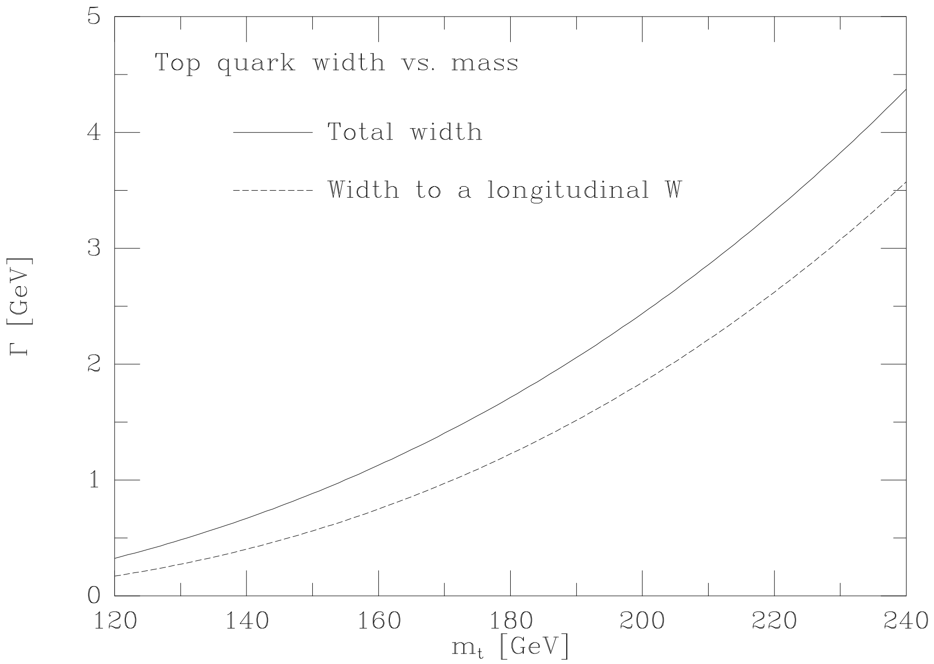
In the limit $m_t \gg M_W$ the result for the total width is

$$\Gamma(t \to bW) = \frac{G_F m_t^3}{8\pi\sqrt{2}}|V_{tb}|^2 \approx 1.76 \text{ GeV}(\frac{m_t}{175 \text{ GeV}})^3.$$

$V_{tb} \approx 1$ as suggested by the unitarity relation

$$|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2 = 1.$$

- The top quark decays before it has time to hadronize.



The top quark has a 'semi-weak' decay rate The lifetime of the top quark is only of order 10^{-25} seconds and it therefore decays before it has time to hadronize.

- The polarization state of the W controls the angular distribution of the leptons into which it decays. We may define the lepton helicity angle θ_e^*, which is the angle of the charged lepton in the rest frame of the W, with respect to the original direction of travel of the W (*i.e.* anti-parallel to the recoiling b quark). If the b quark jet is identified, this angle can be defined experimentally as

$$\cos\theta_e^* \approx \frac{b\cdot(e^+-\nu)}{b\cdot(e^++\nu)} \approx \frac{4b\cdot e^+}{m_t^2-M_W^2}-1,$$

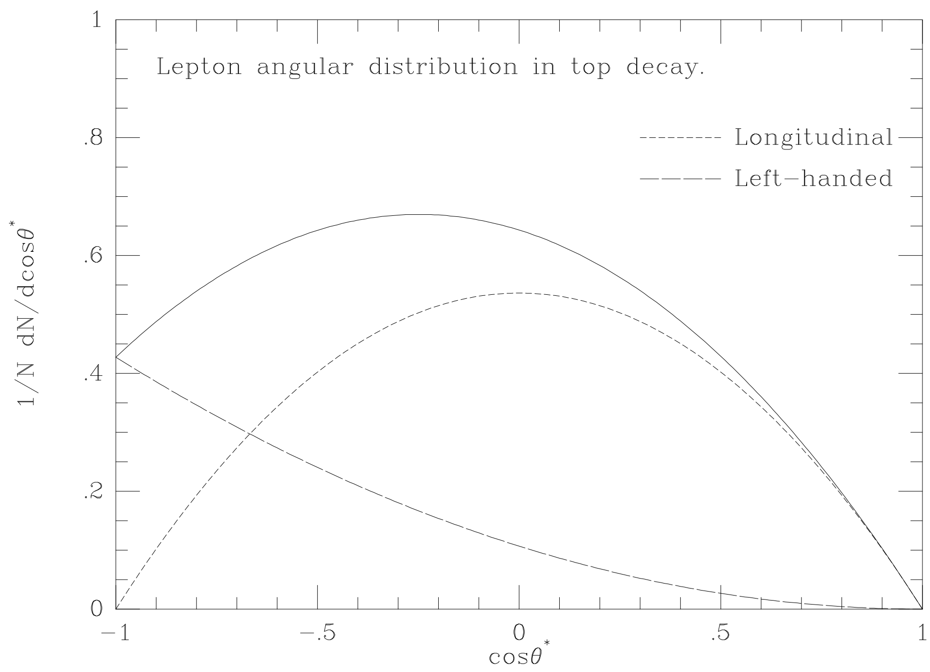
in an obvious notation where t, b, e^+ and ν represent four-momenta.

$$\overline{\sum}|\mathcal{M}^{(t)}|^2 = [\overline{\sum}|\mathcal{M}_0|^2 \times |D_0|^2 + \overline{\sum}|\mathcal{M}_L|^2 \times |D_L|^2] \times \frac{\pi}{M_W\Gamma_W}\,\delta(w^2 - M_W^2).$$

Here M_0, M_L are given in Eq. (1) with $y = 0$ and D_0, D_L are the helicity amplitudes for the decay of a longitudinal and left-handed W boson respectively:

$$|D_0|^2 = \frac{G_F M_W^4}{\sqrt{2}}\tfrac{1}{2}\sin^2\theta_e^*$$

$$|D_L|^2 = \frac{G_F M_W^4}{\sqrt{2}}\tfrac{1}{4}(1-\cos\theta_e^*)^2.$$

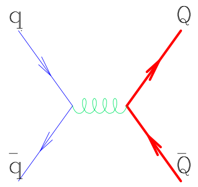


Leading order

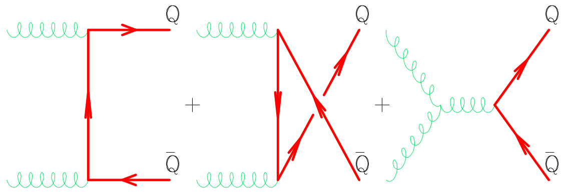
The leading-order processes for the production of a heavy quark Q of mass m in hadron-hadron collisions

$$
\begin{array}{ll}
(a) & q(p_1) + \overline{q}(p_2) \to Q(p_3) + \overline{Q}(p_4) \\
(b) & g(p_1) + g(p_2) \to Q(p_3) + \overline{Q}(p_4)
\end{array}
$$

where the four-momenta of the partons are given in brackets.



(a)



(b)

Process	$\overline{\sum}\|\mathcal{M}\|^2/g^4$
$q\ \overline{q} \to Q\ \overline{Q}$	$\frac{4}{9}(\tau_1^2 + \tau_2^2 + \frac{\rho}{2})$
$g\ g \to Q\ \overline{Q}$	$(\frac{1}{6\tau_1\tau_2} - \frac{3}{8})(\tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2})$

The matrix elements squared have been averaged (summed) over initial (final) colours and spins, as indicated by $\overline{\sum}$.

We have introduced the following notation for the ratios of scalar products:

$$\tau_1 = \frac{2p_1.p_3}{\hat{s}}, \quad \tau_2 = \frac{2p_2.p_3}{\hat{s}}, \quad \rho = \frac{4m^2}{\hat{s}}, \quad \hat{s} = (p_1+p_2)^2.$$

- the short-distance cross section is obtained from the invariant matrix element in the usual way:

$$d\hat{\sigma}_{ij} = \frac{1}{2\hat{s}} \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

  The first factor is the flux factor for massless incoming particles. The other terms come from the phase space for $2 \to 2$ scattering.

- In terms of the rapidity $y = \frac{1}{2}\ln((E+p_z)/(E-p_z))$ and transverse momentum, p_T, the relativistically invariant phase space volume element of the final-state heavy quarks is

$$\frac{d^3p}{E} = dy\, d^2p_T\ .$$

  The result for the invariant cross section may be written as

$$\frac{d\sigma}{dy_3 dy_4 d^2p_T} = \frac{1}{16\pi^2 \hat{s}^2} \sum_{ij} x_1 f_i(x_1,\mu^2)\, x_2 f_j(x_2,\mu^2) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

x_1 and x_2 are fixed if we know the transverse momenta and rapidity of the outgoing heavy quarks. In the centre-of-mass system of the incoming hadrons we may write

$$\begin{aligned}
p_1 &= \tfrac{1}{2}\sqrt{s}(x_1, 0, 0, x_1) \\
p_2 &= \tfrac{1}{2}\sqrt{s}(x_2, 0, 0, -x_2) \\
p_3 &= (m_T \cosh y_3, p_T, 0, m_T \sinh y_3) \\
p_4 &= (m_T \cosh y_4, -p_T, 0, m_T \sinh y_4).
\end{aligned}$$

Applying energy and momentum conservation, we obtain

$$\begin{aligned}
x_1 &= \frac{m_T}{\sqrt{s}}(e^{y_3} + e^{y_4}) \\
x_2 &= \frac{m_T}{\sqrt{s}}(e^{-y_3} + e^{-y_4}) \\
\hat{s} &= 2m_T^2(1 + \cosh \Delta y).
\end{aligned}$$

The quantity $m_T = \sqrt{(m^2 + p_T^2)}$ is the transverse mass of the heavy quarks and $\Delta y = y_3 - y_4$ is the rapidity difference between them.

In these variables the leading order cross section is

$$\frac{d\sigma}{dy_3 dy_4 d^2p_T} = \frac{1}{64\pi^2 m_T^4 (1+\cosh(\Delta y))^2} \times \sum_{ij} x_1 f_i(x_1,\mu^2)\, x_2 f_j(x_2,\mu^2) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

Expressed in terms of m, m_T and Δy, the matrix elements for the two processes are

$$\overline{\sum}|\mathcal{M}_{q\bar{q}}|^2 = \frac{4g^4}{9}\left(\frac{1}{1+\cosh(\Delta y)}\right)\left(\cosh(\Delta y) + \frac{m^2}{m_T^2}\right),$$

$$\overline{\sum}|\mathcal{M}_{gg}|^2 = \frac{g^4}{24}\left(\frac{8\cosh(\Delta y)-1}{1+\cosh(\Delta y)}\right)\left(\cosh(\Delta y) + 2\frac{m^2}{m_T^2} - 2\frac{m^4}{m_T^4}\right).$$

- As the rapidity separation Δy between the two heavy quarks becomes large

$$\overline{\sum}|\mathcal{M}_{q\bar{q}}|^2 \sim \text{ constant}, \quad \overline{\sum}|\mathcal{M}_{gg}|^2 \sim \exp\Delta y\ .$$

- The cross section is damped at large Δy and heavy quarks produced by $q\bar{q}$ annihilation are more closely correlated in rapidity those produced by gg fusion.

Applicability of perturbation theory?

- Consider the propagators in the diagrams.

$$\begin{aligned} (p_1+p_2)^2 = &\quad 2p_1.p_2 &= 2m_T^2(1+\cosh\Delta y)\ , \\ (p_1-p_3)^2-m^2 = &\ -2p_1.p_3 &= -m_T^2(1+e^{-\Delta y})\ , \\ (p_2-p_3)^2-m^2 = &\ -2p_2.p_3 &= -m_T^2(1+e^{\Delta y})\ . \end{aligned}$$

  Note that the propagators are all off-shell by a quantity of least of order m^2.

- Thus for a sufficiently heavy quark we expect the methods of perturbation theory to be applicable. It is the mass m (which by supposition is very much larger than the scale of the strong interactions Λ) which provides the large scale in heavy quark production. We expect corrections of order Λ/m

- This does not address the issue of whether the charm or bottom mass is large enough to be adequately described by perturbation theory.

Heavy quark production in $O(\alpha_s^3)$

In NLO heavy quark production m is the heavy quark mass.

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2\; \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) F_i(x_1, \mu^2) F_j(x_2, \mu^2)$$

$$\hat{\sigma}_{i,j}(\hat{s}, m^2, \mu^2) = \sigma_0 c_{ij}(\hat{\rho}, \mu^2)$$

where $\hat{\rho} = 4m^2/\hat{s}, \bar{\mu}^2 = \mu^2/m^2, \sigma_0 = \alpha_s^2(\mu^2)/m^2$ and $\hat{s}$ in the parton total c-of-m energy squared. The coupling satisfies

$$\frac{d\alpha_s}{d\ln\mu^2} = -b_0 \frac{\alpha_s^2}{2\pi} + O(\alpha_s^3),\; b_0 = \frac{11N - 2n_f}{6}$$

$$c_{ij}(\rho, \frac{\mu^2}{m^2}) = c_{ij}^{(0)}(\rho) + 4\pi\alpha_s(\mu^2)[c_{ij}^{(1)}(\rho) + \bar{c}_{ij}^{(1)}(\rho)\ln(\frac{\mu^2}{m^2})] + O(\alpha_s^2)$$

The lowest-order functions $c_{ij}^{(0)}$ are obtained by integrating the lowest order matrix elements

$$c_{q\bar{q}}^{(0)}(\rho) = \frac{\pi\beta\rho}{27}\left[(2+\rho)\right] ,$$

$$c^{(0)}_{gg}(\rho) = \frac{\pi\beta\rho}{192}[\frac{1}{\beta}[\rho^2 + 16\rho + 16]\ln(\frac{1+\beta}{1-\beta}) - 28 - 31\rho] \ ,$$

$$c^{(0)}_{gq}(\rho) = c^{(0)}_{g\overline{q}}(\rho) = 0 \ ,$$

and $\beta = \sqrt{1-\rho}$.

- The functions $c^{(0)}_{ij}$ vanish both at threshold $(\beta \to 0)$ and at high energy $(\rho \to 0)$.
- Note that the quark-gluon process is zero in lowest order, but is present in higher orders.

- The functions $c_{ij}^{(1)}$ are also known

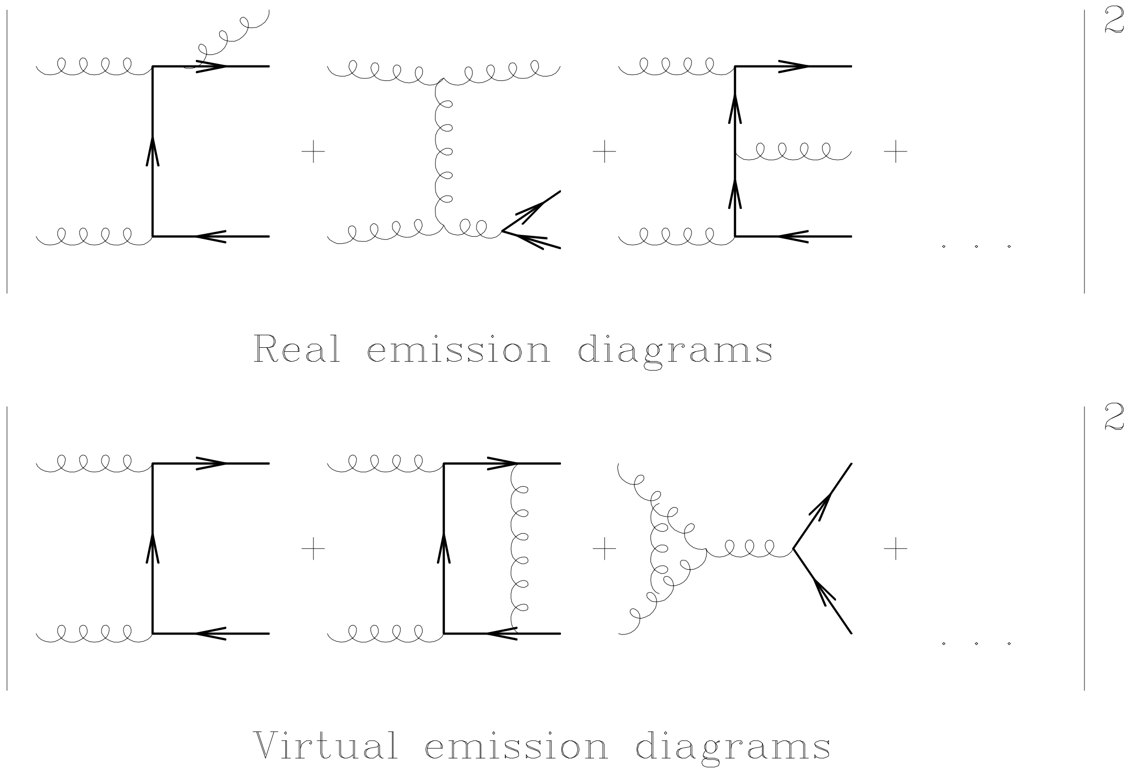
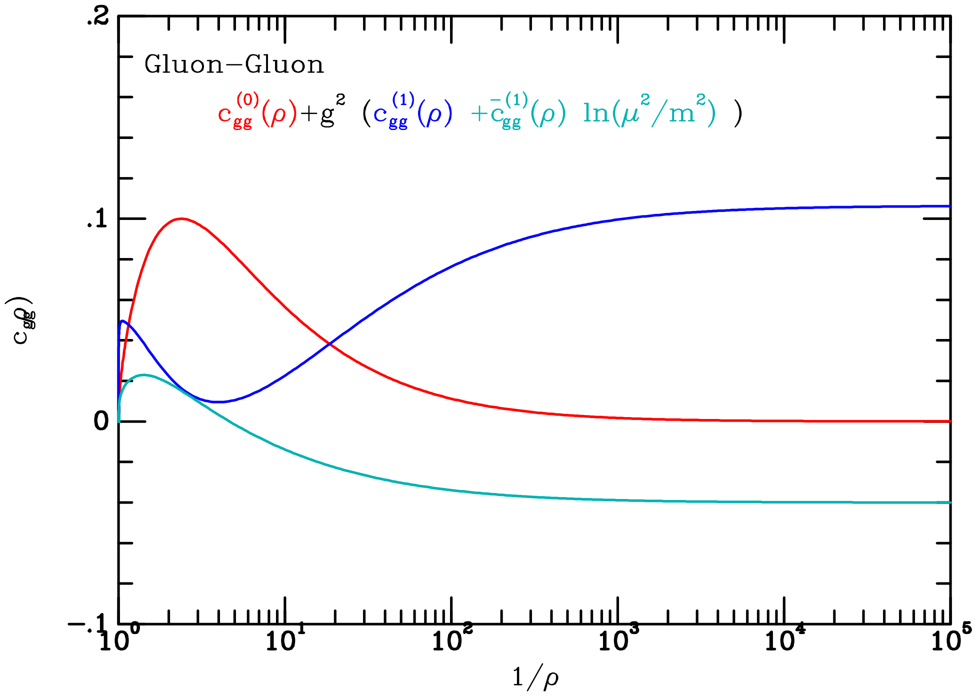
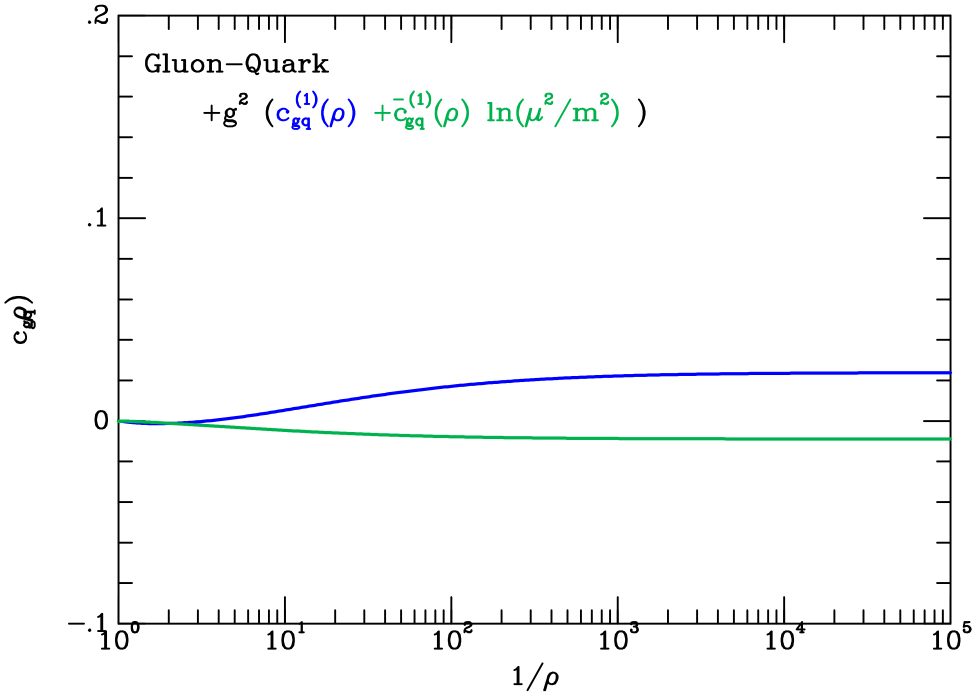
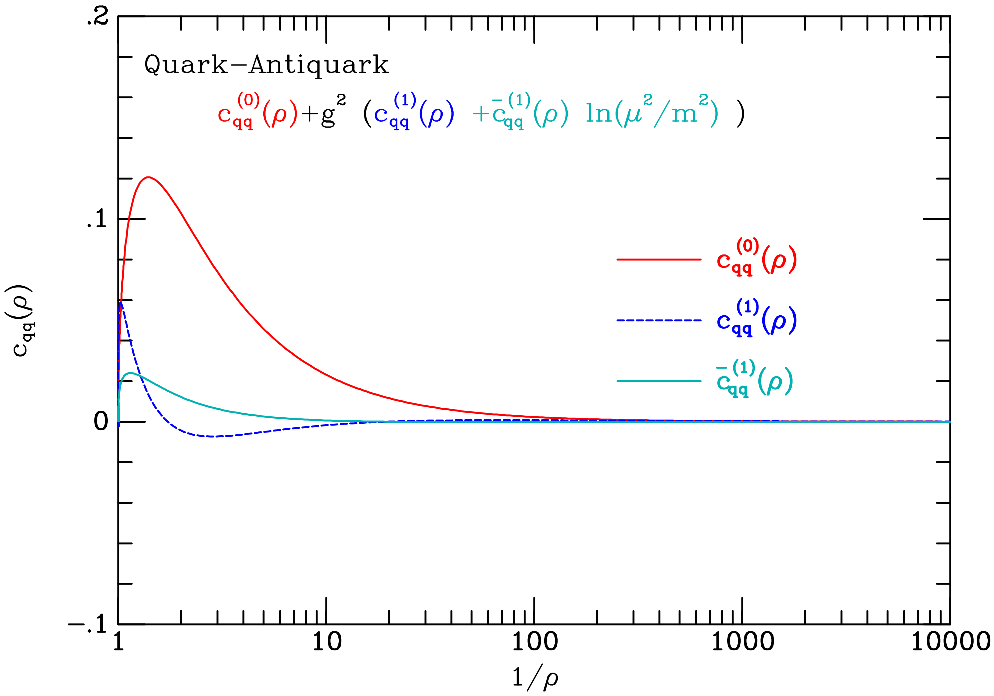


Figure 1: Examples of higher-order corrections to heavy quark production.

- In order to calculate the c_{ij} in perturbation theory we must perform both renormalization and factorization of mass singularities. The subtractions required for renormalization and factorization are done at mass scale μ.


Gluon−Gluon
$c^{(0)}_{gg}(\rho)+g^2\ (c^{(1)}_{gg}(\rho)\ +\bar{c}^{(1)}_{gg}(\rho)\ \ln(\mu^2/m^2)\ )$
.2
.1
0
−.1
10^0
10^1
10^2
10^3
10^4
10^5
$1/\rho$
$c_{gg}(\rho)$


Gluon−Quark
$+g^2\ (c^{(1)}_{gq}(\rho)\ +\bar{c}^{(1)}_{gq}(\rho)\ \ln(\mu^2/m^2)\ )$
.2
.1
0
−.1
10^0
10^1
10^2
10^3
10^4
10^5
$1/\rho$
$c_{gq}(\rho)$



Quark−Antiquark
$c^{(0)}_{q\bar{q}}(\rho)+g^2\ (c^{(1)}_{q\bar{q}}(\rho)\ +\bar{c}^{(1)}_{q\bar{q}}(\rho)\ \ln(\mu^2/m^2)\ )$
$c^{(0)}_{q\bar{q}}(\rho)$
$c^{(1)}_{q\bar{q}}(\rho)$
$\bar{c}^{(1)}_{q\bar{q}}(\rho)$
$c_{q\bar{q}}(\rho)$
$1/\rho$
.2
.1
0
−.1
1
10
100
1000
10000

μ dependence

μ is an unphysical parameter. The physical predictions should be invariant under changes of μ at the appropriate order in perturbation theory. If we have performed a calculation to $O(\alpha_S^3)$, variations of the scale μ will lead to corrections of $O(\alpha_S^4)$,

$$\mu^2 \frac{d}{d\mu^2} \sigma = O(\alpha_S^4).$$

- The term $\overline{c}^{(1)}$, which controls the μ dependence of the higher-order perturbative contributions, is fixed in terms of the lower-order result $c^{(0)}$:

$$\begin{aligned} \overline{c}_{ij}^{(1)}(\rho) &= \frac{1}{8\pi^2} \Big[4\pi b c_{ij}^{(0)}(\rho) - \int_\rho^1 dz_1 \sum_k c_{kj}^{(0)}(\frac{\rho}{z_1}) P_{ki}^{(0)}(z_1) \\ &\quad - \int_\rho^1 dz_2 \sum_k c_{ik}^{(0)}(\frac{\rho}{z_2}) P_{kj}^{(0)}(z_2) \Big]. \end{aligned}$$

In obtaining this result we have used the renormalization group equation for the running coupling

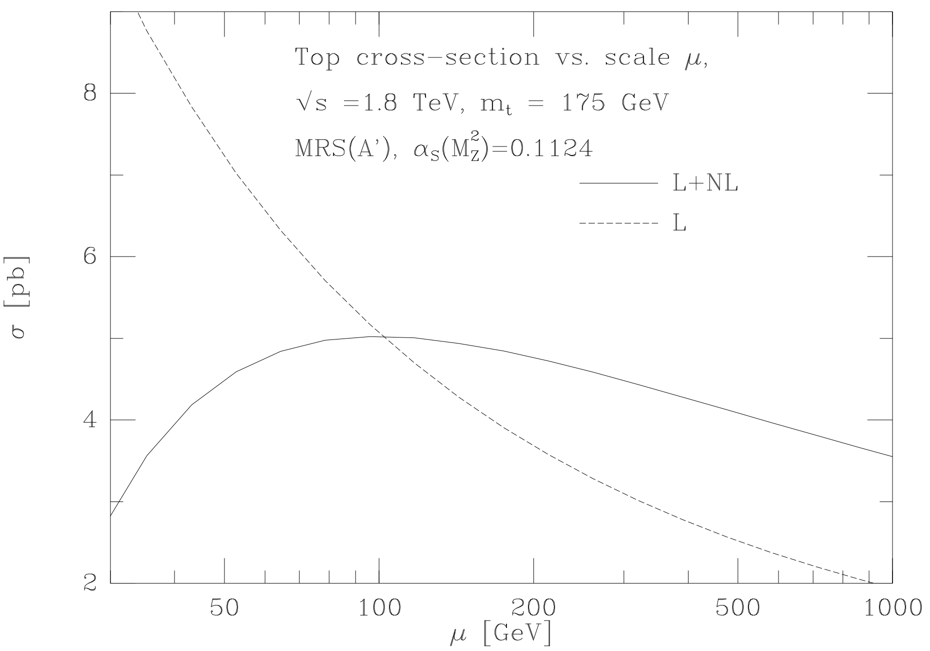
$$\mu^2 \frac{d}{d\mu^2} \alpha_S(\mu^2) = -b\alpha_S^2 + \ldots$$

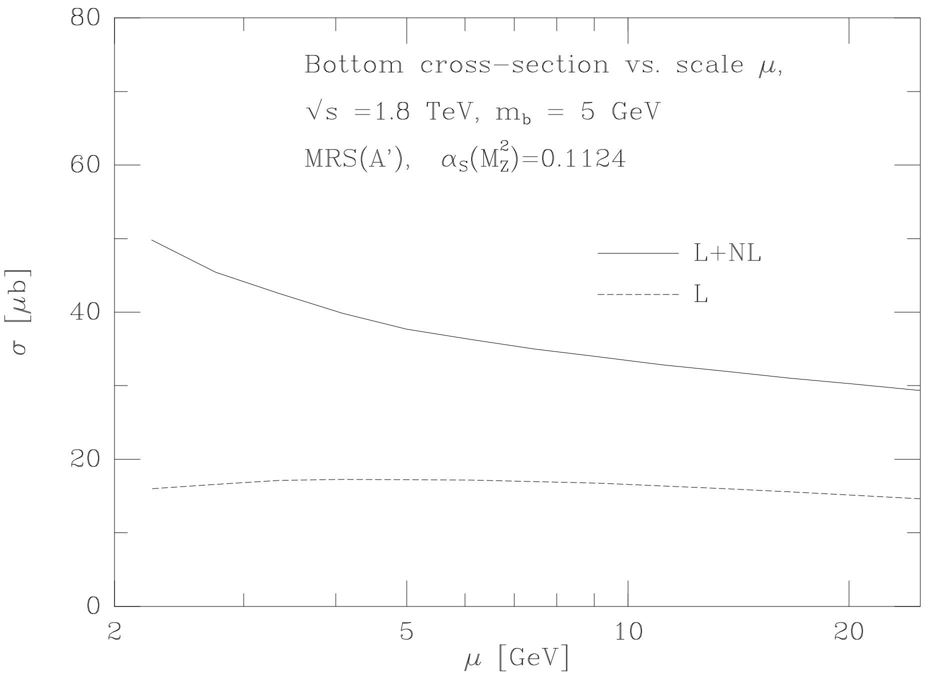
and the lowest-order form of the GLAP equation

$$\mu^2 \frac{d}{d\mu^2} f_i(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \sum_k \int_x^1 \frac{dz}{z} P_{ik}^{(0)}(z) f_k(\frac{x}{z},\mu^2) + \ldots.$$

This illustrates an important point which is a general feature of renormalization group improved perturbation series in QCD. The coefficient of the perturbative correction depends on the choice made for the scale μ, but the scale dependence changes the result in such a way that the physical result is independent of that choice. Thus the scale dependence is formally small because it is of higher order in α_S. This does not assure us that the scale dependence is actually *numerically* small for all series. A pronounced dependence on the scale μ is a signal of an untrustworthy perturbation series.

Scale dependence





Features of HO corrections

Two features of $c_{ij}^{(1)}$ are worthy of note.

- The quark-antiquark and gluon-gluon initiated processes have a very rapid rise at threshold.

$$c_{q\bar{q}}^{(1)} \to \frac{c_{q\bar{q}}^{(0)}(\rho)}{8\pi^2}\left[-\frac{\pi^2}{6\beta} + \frac{16}{3}\ln^2(8\beta^2) - \frac{82}{3}\ln(8\beta^2)\right]$$

$$c_{gg}^{(1)} \to \frac{c_{gg}^{(0)}(\rho)}{8\pi^2}\left[\frac{11\pi^2}{42\beta} + 12\ln^2(8\beta^2) - \frac{366}{7}\ln(8\beta^2)\right]$$

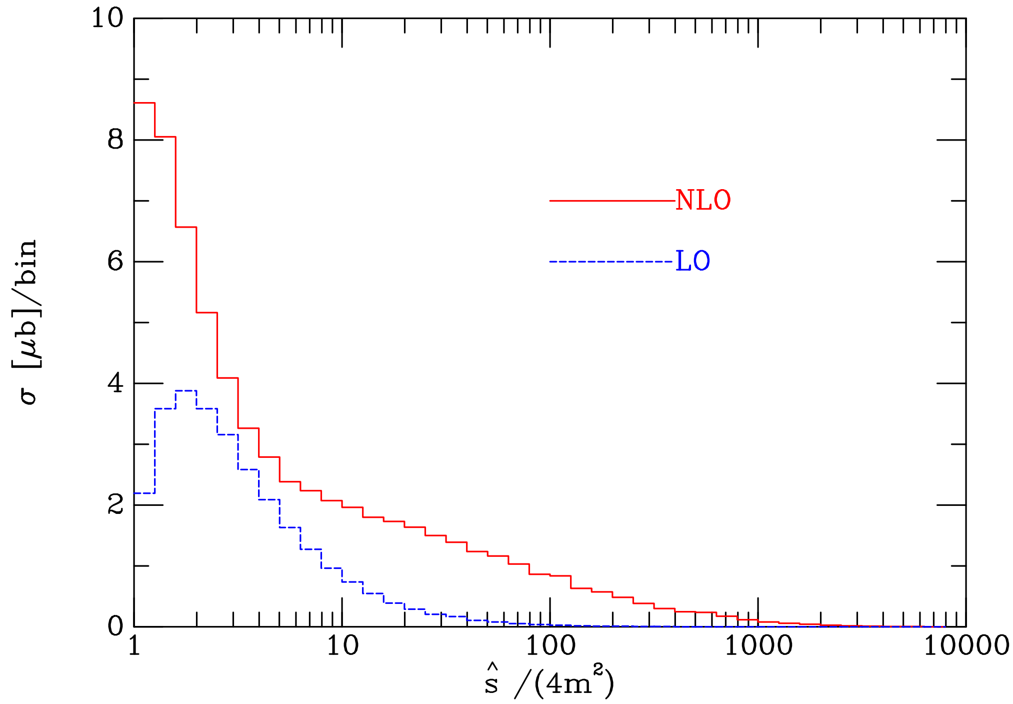
due to Coulomb $1/\beta$ singularities and to Sudakov double logarithms. Near threshold, these terms give large contributions and require special treatment, such as resummation to all orders.

- The gg and gq contributions to $c_{ij}^{(1)}$, which involve spin-one gluon exchange in the t-channel, tend to a constant value at large $\hat{s}$.

$$\begin{aligned} c_{gg} &\to 6k \sim 0.1074 \\ c_{gq} &\to \frac{4}{3}k \sim 0.02388 \end{aligned}$$

$$k = \frac{7291}{16200}\frac{1}{8\pi} \approx 0.01791 \; .$$

- At the energy of the Tevatron, both the threshold region and the large $\hat{s}$ region are important at NLO.



The resummation of the threshold soft gluon corrections is best carried out in ω-space, where ω is the moment variable after Mellin transform with respect to $\hat{s}$. In ω-space the threshold region corresponds to the limit $\omega \to \infty$ and the structure of the threshold corrections is as follows

$$\hat{\sigma} = \sigma^{LO}\{1 + \sum_{j=1}^{\infty}\sum_{k=1}^{2j} b_{jk}\alpha_s^j \ln^k \omega\} \ .$$

- Threshold resummation is of limited importance at the Tevatron where b-quarks are normally produced far from threshold.

- Threshold Resummation effects lead to minor changes in the predicted cross section and a slight reduction in the scale dependence.

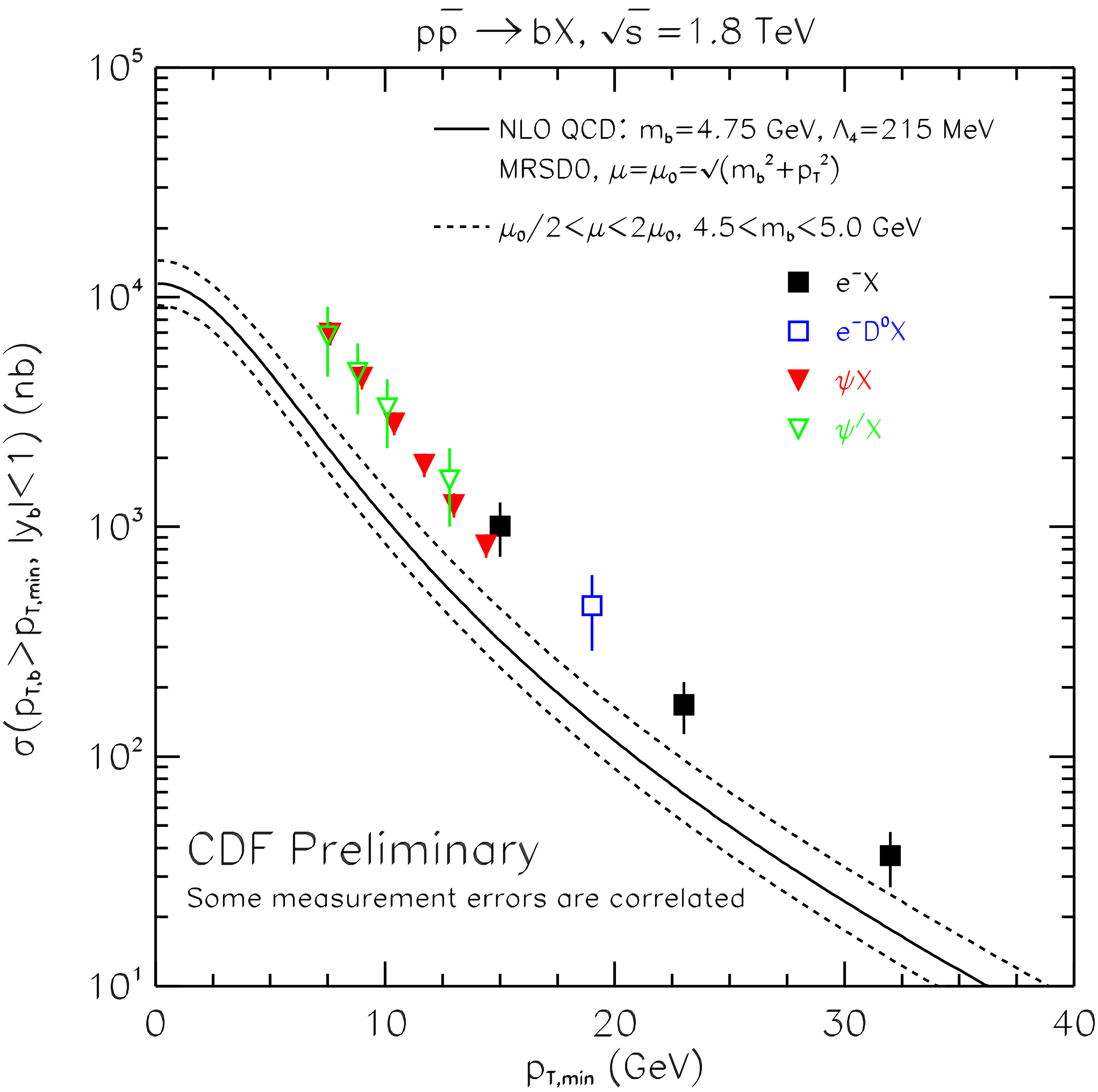
- At high energy, heavy quark production becomes a two scale problem, since $S \gg m^2 \gg \Lambda^2$ and the short distance cross section contains logarithms of $\hat{s}/m^2$. At small ω the dominant terms in the heavy quark production short distance cross section are of the form

$$\hat{\sigma} \sim \frac{\alpha_s^2}{m^2} \sum_{j=0}^{\infty} \alpha_{\mathrm{S}}^j \sum_{k=0}^{\infty} c_{jk} (\frac{\alpha_{\mathrm{S}}}{\omega})^k \ .$$

For $j = 0$ we obtain the leading logarithm series (LLx). Detailed numerical estimates of the importance of small x resummation are not yet available.

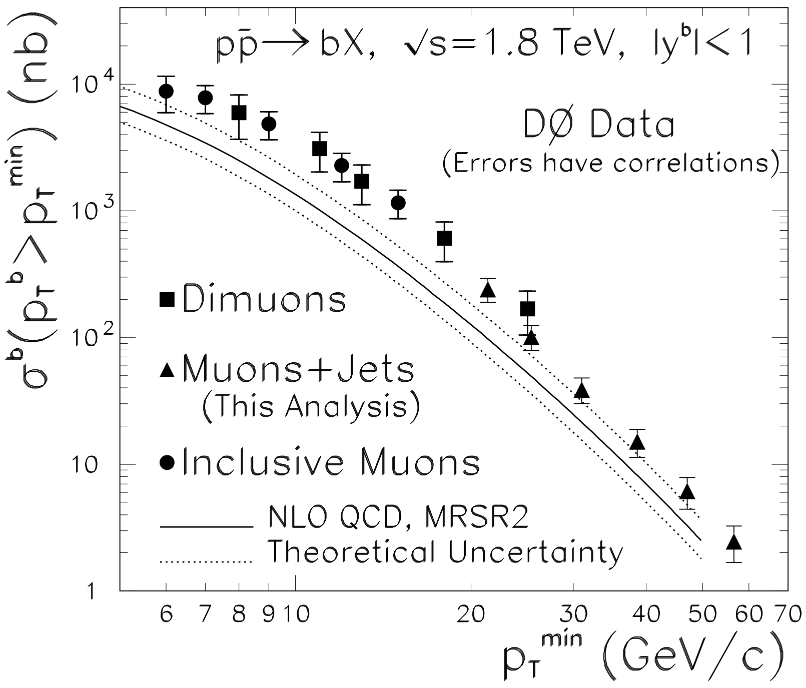
Beauty production at CDF

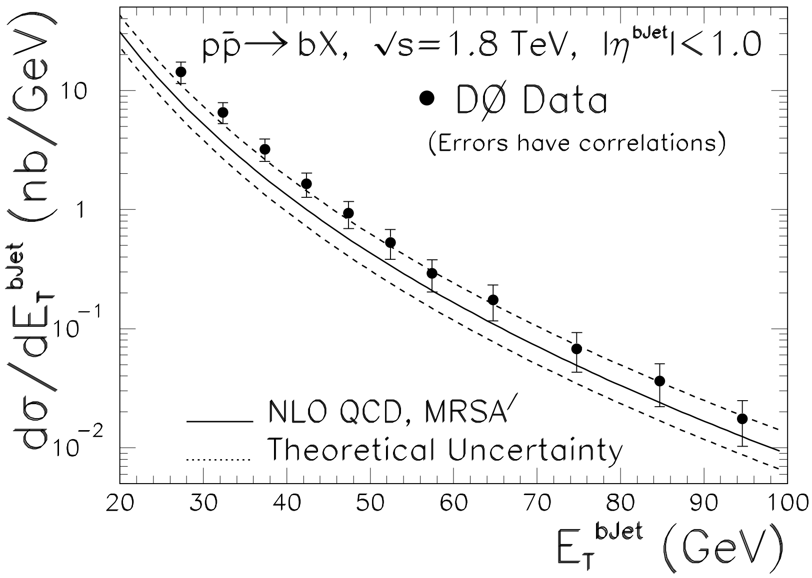
- Data lies above the theory by a factor of ~ 2



Beauty production at D0

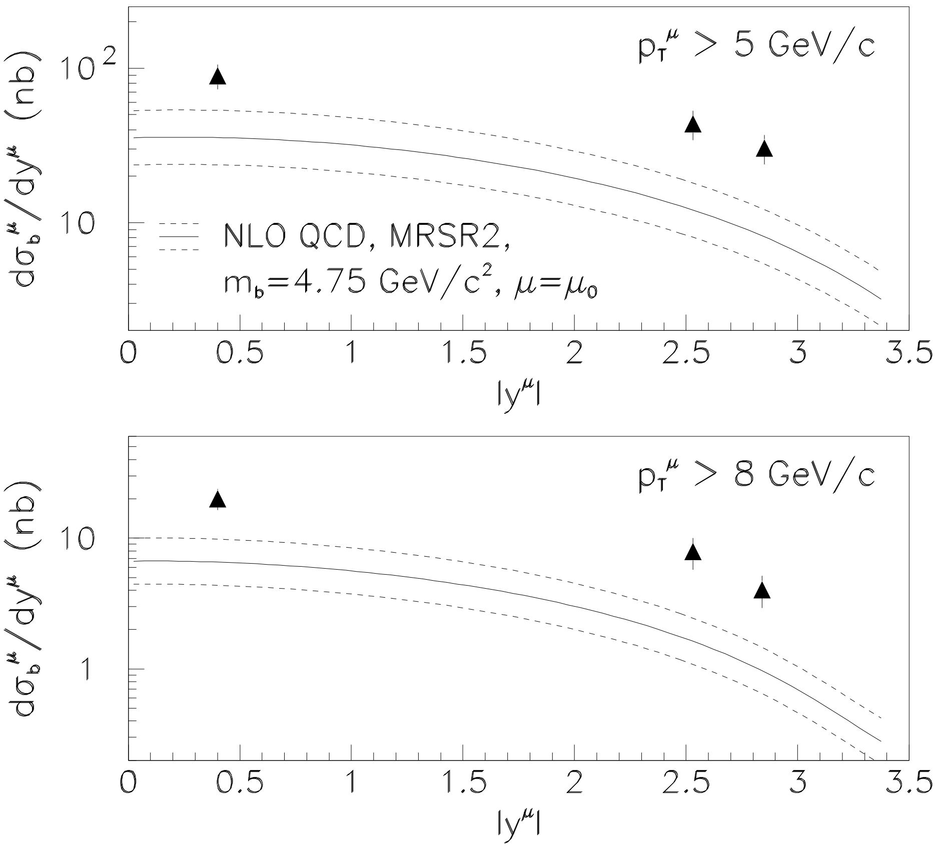
b-quark data lies above NLO theory by a factor of ~ 2





b-jet data lies at the upper end of predicted range

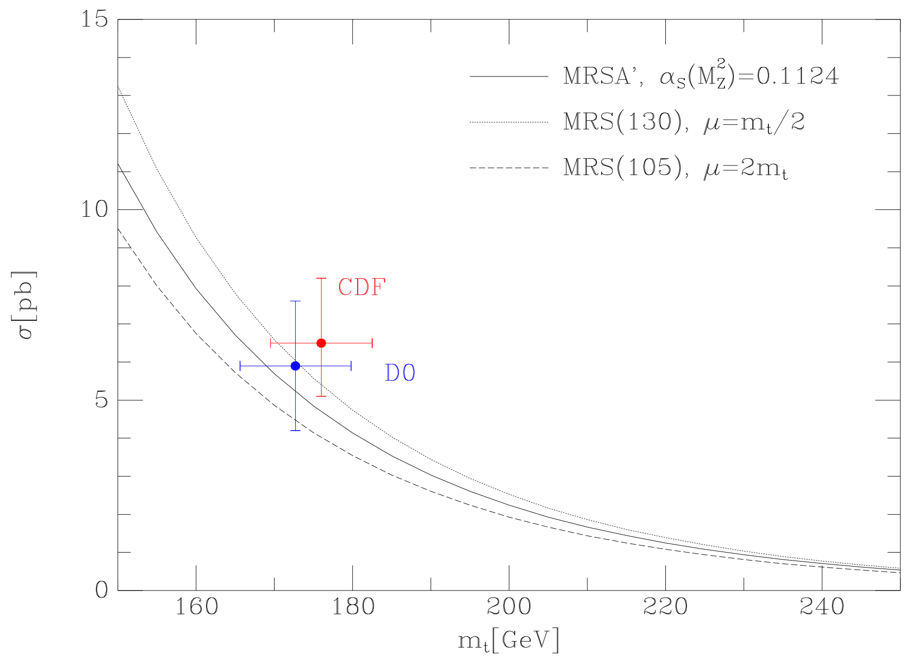
From D0 excess over data is even larger at large rapidity



Top production

- All the information on the top quark is still rather limited and crude
- Within errors agreement between three generation theory and experiment

	Experiment	Theory
m_t	174.3 ± 5.1 GeV	$168.2^{+9.6}_{-7.4}$ GeV
$\sigma(t\bar{t})$	6.2 ± 1.7 pb	$4.75 - 5.5$ pb
$BR(t \to Wb)/BR(t \to Wq)$	$0.94^{+0.31}_{-0.24}$	≈ 1
$BR(t \to W_0 b)$	0.91 ± 0.39	≈ 0.7
$BR(t \to W_+ b)$	0.11 ± 0.15	≈ 0



The CKM matrix

The primed down-type quarks q' differ from the mass eigenstate combinations q by a unitary transformation, V:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} .$$

V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

The constraint of unitarity is $V^\dagger V = 1$. Written out explicitly, this imposes nine conditions

$$\sum_{j=1}^{3} |V_{ij}|^2 = \sum_{i=1}^{3} |V_{ij}|^2 = 1 ,$$

$$\sum_{k=1}^{3} V_{ki}^* V_{kj} = 0 \quad (i \neq j) .$$

The experimental limits on the magnitudes of individual matrix elements, taking into account constraints derived from unitarity, are

$$V = \begin{pmatrix} 0.9742-0.9757 & 0.219-0.226 & 0.002-0.005 \\ 0.219-0.225 & 0.9734-0.9749 & 0.037-0.043 \\ 0.004-0.014 & 0.035-0.043 & 0.9990-0.9993 \end{pmatrix}$$

If we do not assume three generation unitarity,

$$V = \begin{pmatrix} 0.9722-0.9748 & 0.216-0.223 & 0.002-0.005 & \ldots \\ 0.209-0.228 & 0.959-0.976 & 0.037-0.043 & \ldots \\ 0-0.09 & 0-0.16 & 0.07-0.993 & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

Single Top production

- CDF has measured

$$\frac{BR(t \to Wb)}{BR(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.94^{+0.31}_{-0.24}$$

If we assume just three generations of quarks, unitarity of the CKM matrix implies that the denominator is equal to one, so that we can extract

$$|V_{tb}| = 0.97^{+0.16}_{-0.12}$$

- But assuming unitarity we know already that $V_{tb} = 0.9990 - -0.9993$.

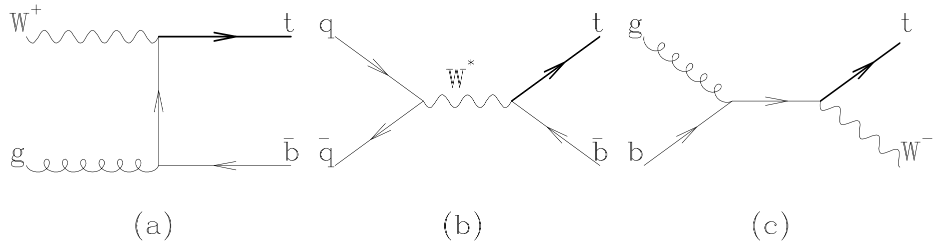
- The current CDF measurement shows that $|V_{tb}| \gg |V_{td}|, |V_{ts}|$.

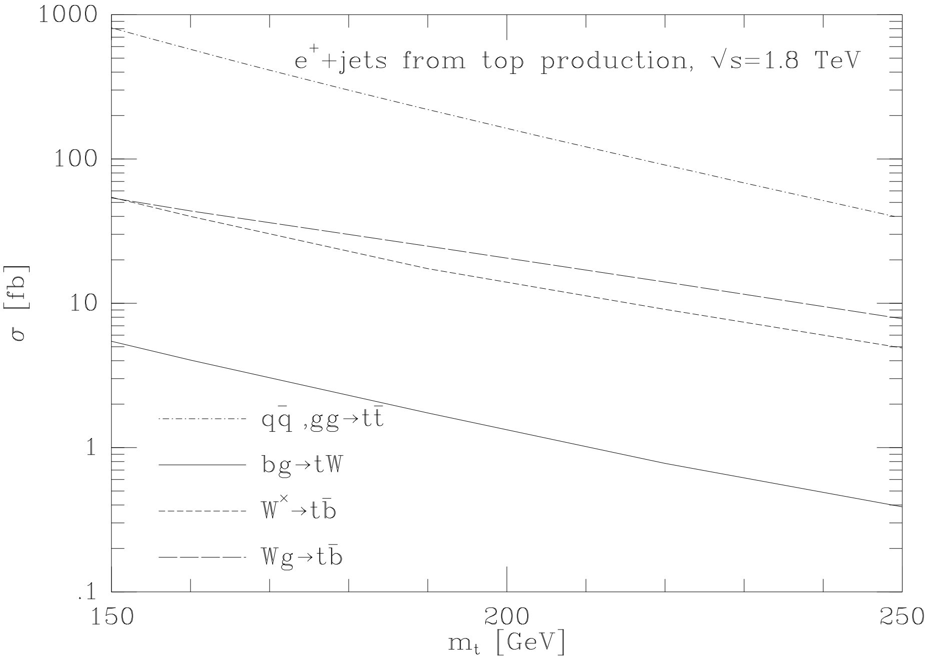
- For a real measurement of V_{tb} we must look at the electroweak production of a top quark.

Three possible weak-interaction single-top quark production mechanisms are

$$\begin{aligned} (a) \quad & Wg && \to t\bar{b}\,, \\ (b) \quad & q\bar{q} && \to t\bar{b}\,, \\ (c) \quad & gb && \to tW^-\,. \end{aligned}$$

The raw rate for these processes as observed in the lepton + jets channel is about a factor of ten below the standard production mechanism at $\sqrt{s} = 1.8$ TeV,





	$\sqrt{s} = 1.8$TeV	LHC	Run I
σ_t^{NLO}	1.45 ± 0.08 pb	238 ± 12 pb	< 15.8 pb
σ_s^{NLO}	0.73 ± 0.04 pb	10.2 ± 0.6 pb	< 15.4 pb
σ_{LL}		56 ± 8 pb	

- Better statistics needed for significant measurements in Run II.

- V_{tb} will be measured at the Tevatron to an accuracy of about 12% with 2 fb^{-1}.

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