# Polarized Scattering and the Spin Structure of the Nucleon

Werner Vogelsang

RIKEN-BNL Research Center
and Nuclear Theory, BNL

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# What this talk is about :

- What carries the nucleon spin ?

  Simple quark model picture:
  3 constituent quarks, $s_i = \frac{1}{2}$, $\mu_i = e_i \frac{e}{2m_i}$

  $$|P\rangle = \frac{1}{\sqrt{18}}\left[2u^{\uparrow}u^{\uparrow}d^{\downarrow} - u^{\uparrow}u^{\downarrow}d^{\uparrow} - u^{\downarrow}u^{\uparrow}d^{\uparrow} + \text{perm.}\right]$$

  $$\Rightarrow \begin{cases} \mu_n/\mu_p = -2/3 \quad \text{vs.} \quad (\mu_n/\mu_p)_{\text{exp}} = -0.685 & \checkmark \\ g_A = 5/3 \quad \text{vs.} \quad (g_A)_{\text{exp}} = 1.267 \pm 0.0035 & \times \\ \text{(relativistic corrections...)} \\ 2\,S_u = 4/3 \;\; , 2\,S_d = -1/3 \quad \rightsquigarrow & \boxed{S_q = S_u + S_d = \frac{1}{2}} \end{cases}$$

- nucleons have a rich internal structure:
  quarks, antiquarks (spin 1/2) and gluons (spin 1)
  What do we find in terms of partonic d.o.f. ?
  Relation between nucleon and parton spins ?

- How can we find out ?
  experiments of past, present, and future

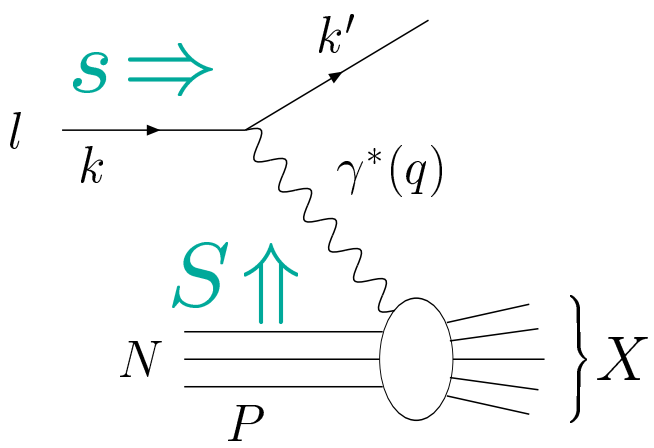
- What do we learn about QCD ?

# Outline :

- I. Spin-dependent deep-inelastic scattering
- II. Parton model for $g_1$, and beyond
- III. Lessons from polarized DIS
- IV. Ways of measuring $\Delta g$
- V. One example : Prompt Photon Production at RHIC
- VI. Further information on $\Delta q$, $\Delta \bar{q}$
- VII. Transversity
- VIII. $g_2$ – the other structure function
- IX. The proton's spin sum rule
- X. Summary & Outlook

# I. Spin-dependent DIS

Standard 'microscope' to explore nucleon structure




$$x = \frac{Q^2}{2\,P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot k}$$

$$s\cdot k = S\cdot P = 0$$

$$s^2 = S^2 = -1$$

Cross section : $\frac{d^2\sigma}{dxdy} \propto \underbrace{\mathcal{L}_{\mu\nu}(k,q,s)}_{\text{leptonic}} \cdot \underbrace{\mathcal{W}^{\mu\nu}(P,q,S)}_{\text{hadronic}}$

$\mathcal{L}_{\mu\nu}(k,q,s)$ = calculable in QED

$$\mathcal{W}^{\mu\nu}(P,q,S) = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\langle P,S|\,[\mathcal{J}_\mu(z),\mathcal{J}_\nu(0)]\,|P,S\rangle$$

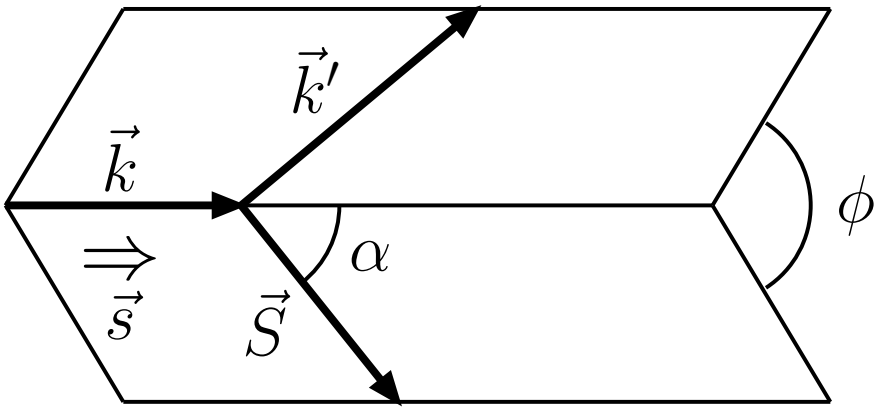
$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2) + \left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)F_2(x,Q^2)$$

$$+\, i\,M\,\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{P\cdot q}\,g_1(x,Q^2) + \frac{S_\sigma(P\cdot q) - P_\sigma(S\cdot q)}{(P\cdot q)^2}\,g_2(x,Q^2)\right]$$

$g_i(x,Q^2)$ : "spin-dependent" structure fcts.

Get the $g_i$ as differences of cross sections :

$$\mathcal{W}^{\mu\nu}(P,q,S) \;-\; \mathcal{W}^{\mu\nu}(P,q,-S)$$




specialize to lepton with helicity $\lambda$ and $\angle(\hat{k},\hat{S}) \equiv \alpha$:

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2} \times$$

$$\times \Bigg\{ \cos\alpha \left\{ \left[ 1 - \frac{y}{2} - \frac{y^2}{4}(\kappa-1) \right] g_1(x,Q^2) - \frac{y}{2}(\kappa-1) g_2(x,Q^2) \right\}$$

$$- \sin\alpha \cos\phi \sqrt{(\kappa-1)\left(1 - y - \frac{y^2}{4}(\kappa-1)\right)} \left( \frac{y}{2} g_1(x,Q^2) + g_2(x,Q^2) \right) \Bigg\}$$

$$\kappa - 1 = 4M^2x^2/Q^2$$

$\alpha = 0 \;:\; \Rightarrow \quad g_1$

$\alpha = \pi/2 \;:\; \Rightarrow \quad y\, g_1 + 2 g_2$ , suppressed $M/Q$

experimentally: spin-*asymmetries*,
e.g. case $\alpha = 0$ :

$$A_{\parallel} = \frac{d\sigma^{(\rightarrow\Leftarrow)} - d\sigma^{(\rightarrow\Rightarrow)}}{d\sigma^{(\rightarrow\Leftarrow)} + d\sigma^{(\rightarrow\Rightarrow)}}$$

$$A_{\parallel} \approx \frac{y(2-y)}{y^2 + 2(1-y)\left(1 + R(x,Q^2)\right)} A_1(x,Q^2)$$

$$= D(y)\,\frac{g_1(x,Q^2)}{F_1(x,Q^2)}$$

$$R = F_L/2xF_1 = (F_2 - 2xF_1)/2xF_1$$

so far only "fixed-target" expts.

SLAC E80,130 $(p)$; E142 $(n)$; E143 $(p,d)$; E154 $(n)$; E155 $(p,d)$

CERN spin muon collaboration CERN SMC EMC, SMC $(p,d)$

DESY hermes HERMES $(p,d,n)$

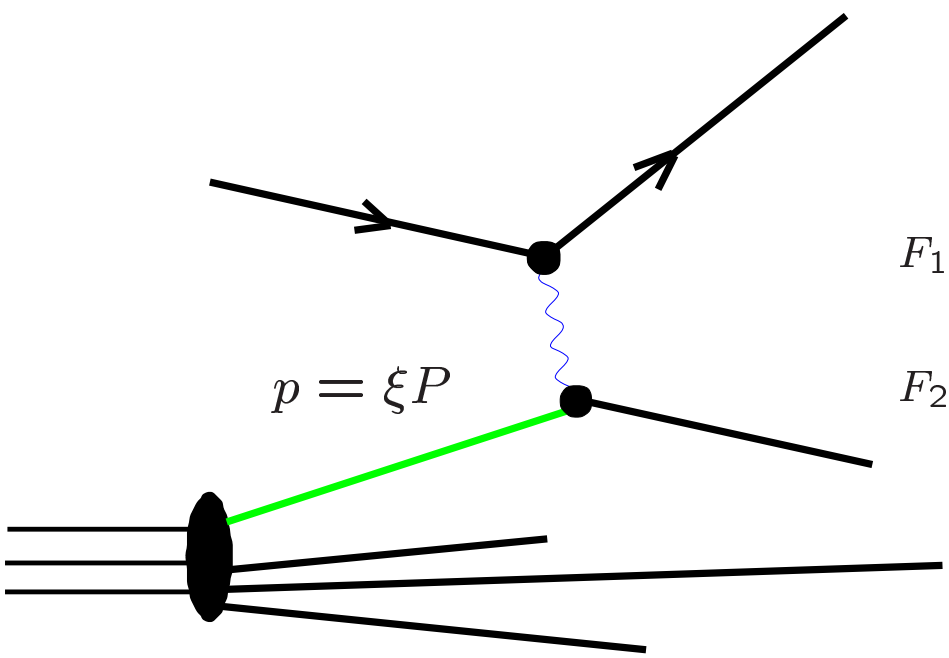
# II. Parton model for $g_1$, and beyond

Recall parton model for unpolarized DIS :

- life time of $\gamma^*$ $\ll$ time scale for "inter-partonic" interactions
  $\rightarrow$ lepton sees "snapshot" of nucleon in virtual parton state
- lepton scatters off "free" quarks, incoherently :

$$\frac{d\sigma^{ep}}{dxdy}(x) = \sum_f \int_x^1 \frac{d\xi}{\xi} \underbrace{f(\xi)}_{\uparrow} \frac{d\sigma^{ef}}{dzdy}\left(z = \frac{Q^2}{2p\cdot q} = \frac{x}{\xi}\right)$$

distr. fctn.
# of partons of type $f = q, \bar{q}, g$

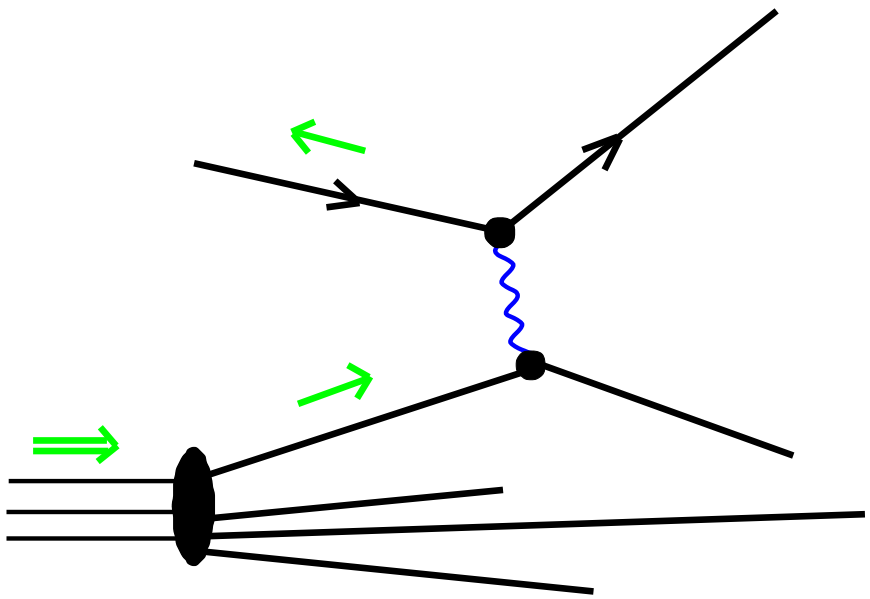



$$F_1(x) = \frac{1}{2}\sum_q e_q^2 \left[q(x) + \bar{q}(x)\right]$$

$$F_2(x) = 2\,x\,F_1(x)$$

"scaling"

Polarized scattering :
$g_1$, too, can be interpreted in parton model !



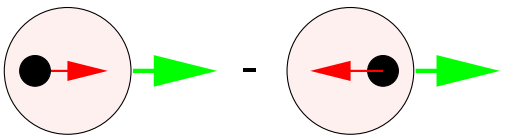
$\Rightarrow$ have to consider $e(\lambda_e)\, q(\lambda_q) \;\rightarrow\; eq$ etc., *and :*

$f^+(\xi)$ # of partons with *same* helicity as nucleon

$f^-(\xi)$ # of partons with *opposite* helicity

Define

$$\boxed{\Delta f(\xi) \equiv f^+(\xi) - f^-(\xi)}$$



$$\Rightarrow \qquad g_1(x) \;=\; \frac{1}{2}\sum_q e_q^2 \Big[\Delta q(x) + \Delta\bar{q}(x)\Big]$$

$\boxed{\Delta q,\ \Delta\bar{q}\ :\ \text{information on nucleon spin structure}}$

*qualitatively new* (compared to unpol. $q(\xi),\ \bar{q}(\xi)$)

(by the way, doesn't work for $g_2$) $\quad$ ($\rightarrow$ later)

Particularly interesting :

$$\int_0^1 \Delta f(\xi)\, d\xi \;=\; \int_0^1 \left[f^+(\xi) - f^-(\xi)\right] d\xi$$

$$\equiv \;\; \Delta F \qquad \text{“first moment”}$$

= net total parton polarization in nucleon

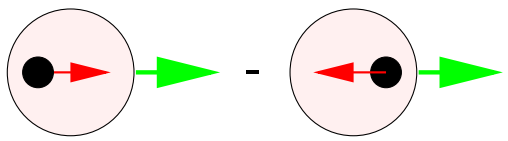
- quarks : $\frac{1}{2}\left[\Delta Q + \Delta \bar{Q}\right]$

  = contribution of flavor $q, \bar{q}$ to nucleon spin

  $$\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\Big[\Delta U + \Delta \bar{U} + \Delta D + \Delta \bar{D} + \Delta S + \Delta \bar{S}\Big]$$

  = total quark spin in nucleon

  recall, this was $\frac{1}{2}$ in simple quark model

- gluon as parton : 

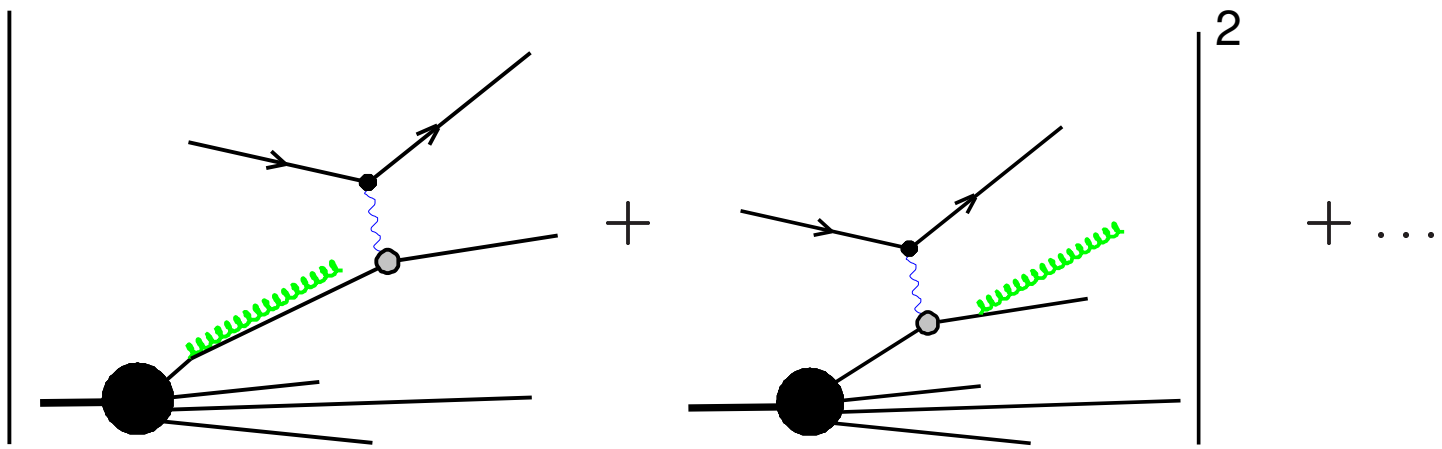
$$\Delta G \;\equiv\; \int_0^1 \Delta g(\xi)\, d\xi$$

$$= \int_0^1 \left[g^+(\xi) - g^-(\xi)\right] d\xi$$

In some sense : proton spin sum rule

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \underbrace{L_q + L_g}_{\text{parton OAM}}}$$

In QCD : partons *do* undergo interactions




- generates *scaling violations* in $g_1$ : nucleon structure depends on hardness ("resolving power") of probe

- scaling violations : *a quantitative prediction* of perturbative QCD !

- main effect : $\Delta f(x) \longrightarrow \Delta f(x, \mu)$ ("DGLAP")

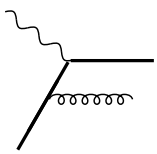
- $\mu$ dependence similar to unpolarized case, but *different quantitatively*

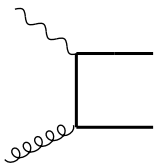
Full QCD picture for $g_1(x,Q^2)$ :

$$g_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \Bigg\{ \left[\Delta q + \Delta\bar{q}\right](x,Q)$$

$$+ \left| \quad + \ldots \right|^2 \otimes \left[\Delta q + \Delta\bar{q}\right](x,Q) + \ldots$$

$$+ \left| \quad + \ldots \right|^2 \otimes \Delta g(x,Q) \; + \ldots \Bigg\} \; + \; \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$





where pdf's obey DGLAP evolution equations :

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\begin{pmatrix}\Delta q(x,\mu)\\ \Delta g(x,\mu)\end{pmatrix} = \int_x^1 \frac{dz}{z}\begin{pmatrix}\Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg}\\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg}\end{pmatrix}_{(z,\alpha_s(\mu))} \cdot \begin{pmatrix}\Delta q\\ \Delta g\end{pmatrix}\left(\frac{x}{z},\mu\right)$$

$\Delta\mathcal{P}_{ij}(z,\alpha_s(\mu))$ : spin-dep. 'splitting'-functions calculable in QCD perturbation theory,

$$\Delta\mathcal{P}_{ij}(z,\alpha_s) \; = \; \frac{\alpha_s}{2\pi}\Delta P_{ij}^{(0)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{(1)}(z) + \ldots$$

(Ahmed,Ross; Altarelli,Parisi; Mertig,van Neerven; WV)

With first-order corrections to $g_1$, and $\Delta P_{ij}^{(1)}$ :

"Next-to-leading order" (NLO)

# III. Lessons from polarized DIS

20 years of beautiful data
– matched by theoretical efforts :

What have we learned ?

- NLO QCD fits describe data well
- Bjorken sum rule confirmed to $\sim 10\%$
- Ellis-Jaffe sum rule is violated
- quarks do *not* carry the proton spin !

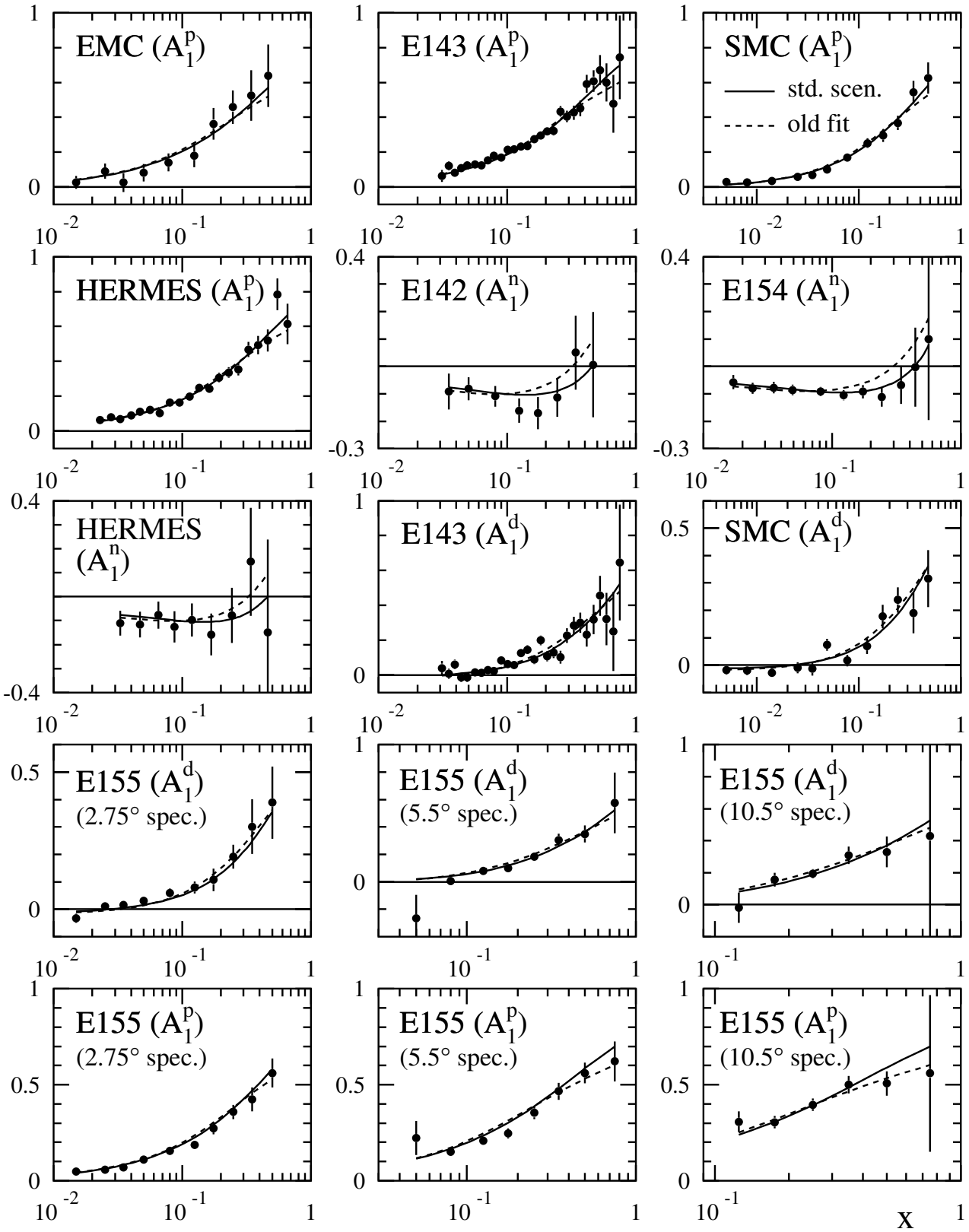
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g \qquad \frac{1}{2}\Delta\Sigma \approx 0$$

- still considerable lack of knowledge regarding
  - polarized gluon density $\Delta g(x,\mu)$
  - orbital angular momenta of quarks and gluons in nucleon
  - separation of quark and antiquark distrib.; flavor separation $u, \bar{u}, d, \bar{d}, s, \bar{s}, \ldots$
  - spin effects with *transverse* polarization

Table 1:

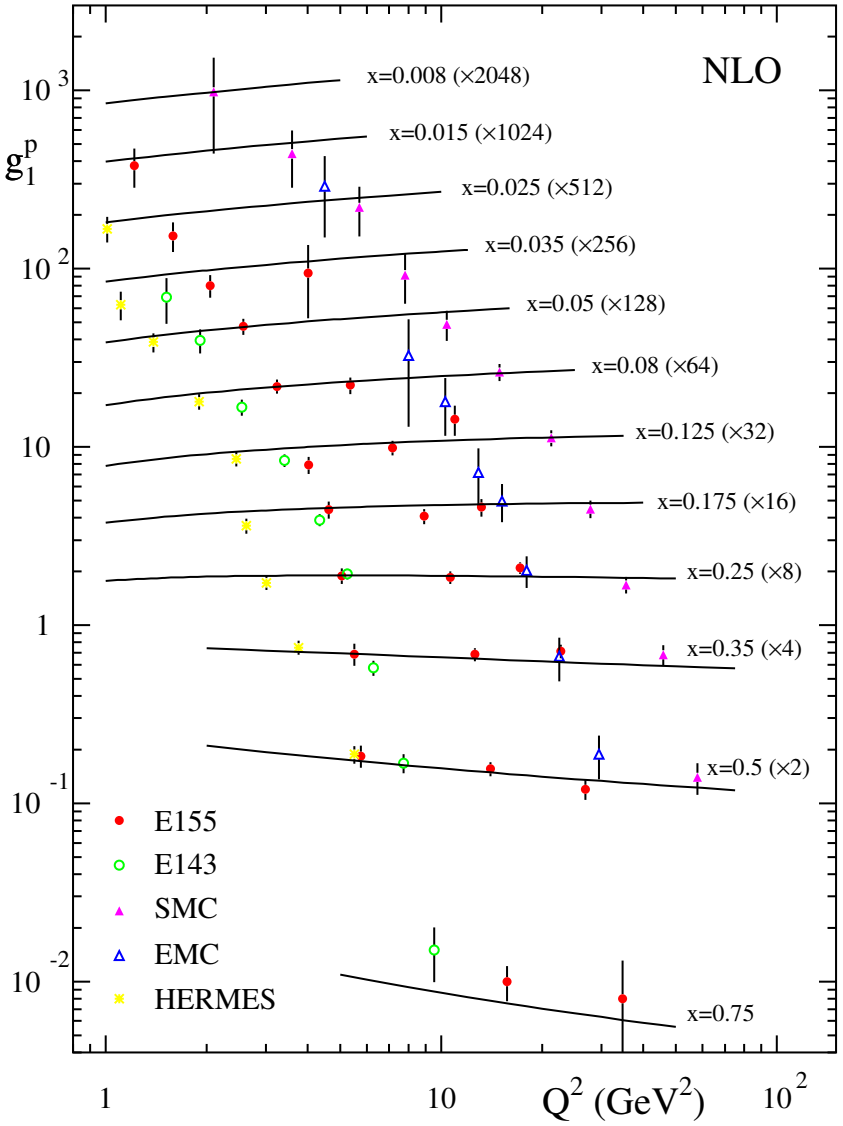
| Lab | Exp. | Year | Beam | $\langle Q^2 \rangle$ GeV$^2$ | $x$ | $P_B$ | Target | $P_T$ | $f$ | $\mathcal{L} \times 10^{-32}$ cm$^{-2}$-s |
|---|---|---|---|---|---|---|---|---|---|---|
| SLAC | E80 | 75 | 10-16 GeV $e^-$ | 2 | 0.1 – 0.5 | 85% | H-butanol | 50% | 0.13 | 400 |
| | E130 | 80 | 16-23 GeV $e^-$ | 5 | 0.1 – 0.6 | 81% | H-butanol | 58% | 0.15 | 400 |
| | E142 | 92 | 19-26 GeV $e^-$ | 2 | 0.03 – 0.6 | 39% | $^3$He | 35% | 0.35 | 2000 |
| | E143 | 93 | 10-29 GeV $e^-$ | 3 | 0.03 – 0.8 | 85% | $NH_3$ | 70% | 0.15 | 1000 |
| | | | | | | | $ND_3$ | 25% | 0.24 | 1000 |
| | E154 | 95 | 48 GeV $e^-$ | 5 | 0.01 – 0.7 | 82% | $^3$He | 38% | 0.55 | 3000 |
| | E155 | 97 | 48 GeV $e^-$ | 5 | 0.01 – 0.9 | 81% | $NH_3$ | 90% | 0.15 | 1000 |
| | | | | | | | LiD | 22% | 0.36 | 1000 |
| | E155' | 99 | 30 GeV $e^-$ | 3 | 0.02 – 0.9 | 83% | $NH_3$ | 75% | 0.16 | 1000 |
| | | | | | | | LiD | 22% | 0.36 | 1000 |
| CERN | EMC | 85 | 100-200 GeV $\mu^+$ | 11 | 0.01 – 0.7 | 79% | $NH_3$ | 78% | 0.16 | 0.3 |
| | SMC | 92 | 100 GeV $\mu^+$ | 4.6 | 0.006 – 0.6 | 82% | D-butanol | 35% | 0.19 | 0.3 |
| | | 93 | 190 GeV $\mu^+$ | 10 | 0.003 – 0.7 | 80% | H-butanol | 86% | 0.12 | 0.6 |
| | | 94-95 | | | | 81% | D-butanol | 50% | 0.20 | 0.6 |
| | | 96 | | | | 77% | $NH_3$ | 89% | 0.16 | 0.6 |
| DESY | HERMES | 95 | 28 GeV $e^+$ | 2.5 | 0.02 – 0.6 | 55% | $^3$He | 46% | 1.0 | 1 |
| | | 96-97 | | | | 55% | H | 88% | 1.0 | 0.1 |
| | | 98 | 28 GeV $e^-$ | | | 55% | D | 85% | 1.0 | 0.2 |
| | | 99-00 | 28 GeV $e^+$ | | | 55% | D | 85% | 1.0 | 0.2 |
| CERN | COMPASS | 01 | 190 GeV $\mu^+$ | 10 | 0.005 – 0.6 | 80% | $NH_3$ | 90% | 0.16 | 3 |
| | | | | | | | LiD | 40% | 0.50 | 3 |
| BNL | RHIC | 02 | 200 GeV p - p | $\sim 100$ | 0.05 – 0.6 | 70% | Collider | 70% | 1.0 | 2 |
| DESY | ZEUS/H1 | ?? | $28 \times 800$ GeV e - p | 22 | 0.00006 – 0.6 | 70% | Collider | 70% | 1.0 | 0.2 |

## Glück,Reya,Stratmann,WV :

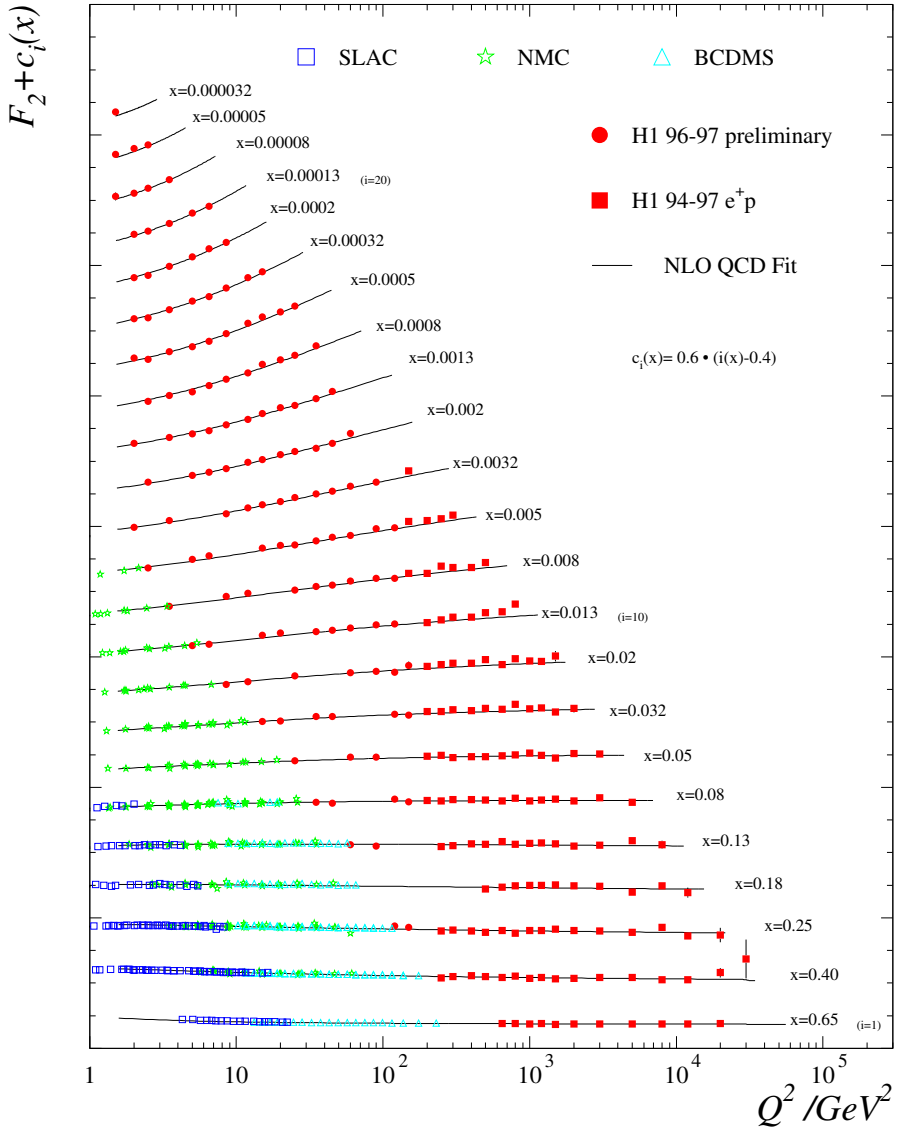


# World data on pol. and unpol. deep-inelastic scattering

polarized

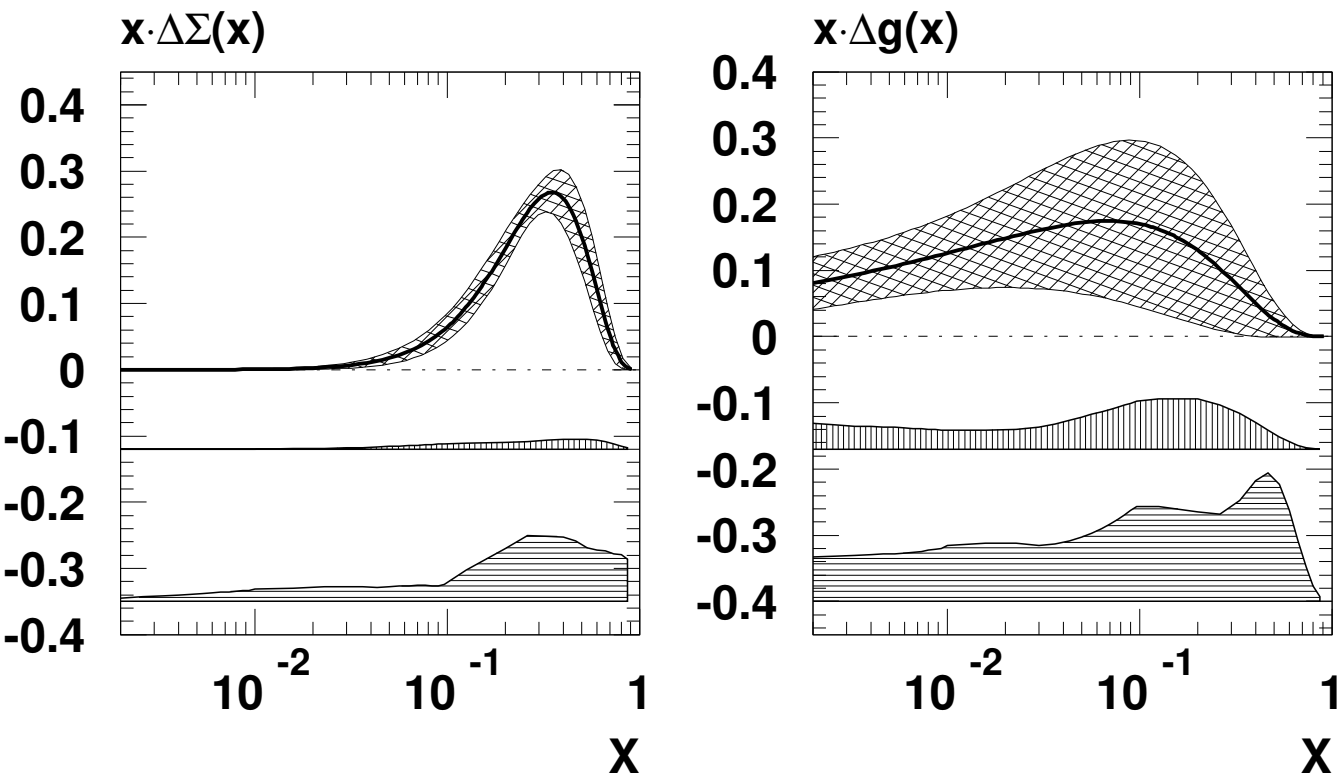



unpolarized



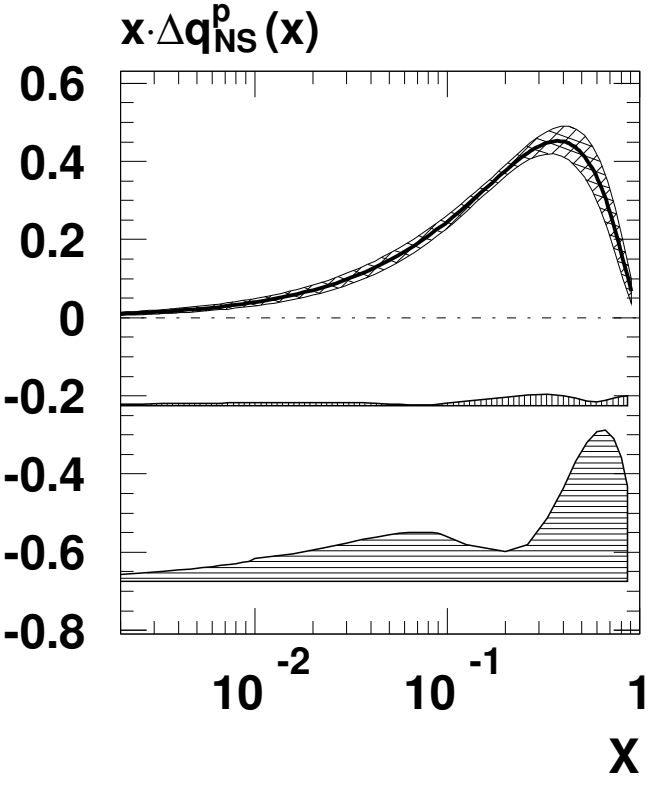

NLO QCD fit: Glück, Reya, Vogelsang, MS (2000 update)

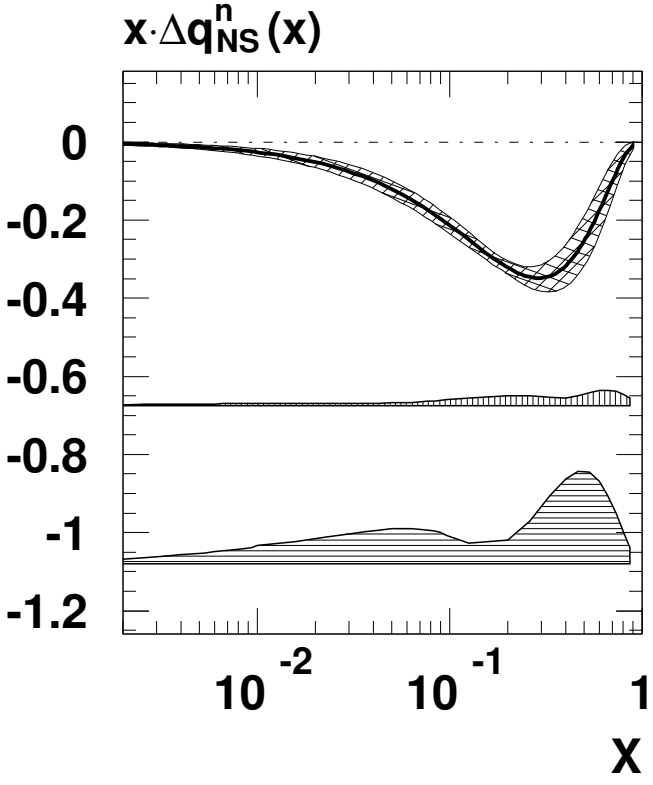
# SMC analysis :









# Bjorken's sum rule

Originally, a main motivation for measuring $g_1$ !

$$\int_0^1 dx \left[ g_1^p(x,Q^2) - g_1^n(x,Q^2) \right] = \frac{1}{6} \underbrace{g_A}_{\Uparrow \; n \to p\, e^-\, \bar{\nu}_e}$$

when expressed in terms of parton densities :

$$\begin{aligned} \mathcal{G}_1^{p-n}(Q^2) &= \frac{1}{6}\left[ \Delta U(Q) + \Delta \bar{U}(Q) - \Delta D(Q) - \Delta \bar{D}(Q) \right] \\ &\equiv \frac{1}{6}\, \Delta \mathcal{A}_3(Q) \end{aligned}$$

one can show :

$$S^\mu \; \Delta \mathcal{A}_3(Q) \;=\; \langle P,S \mid \underbrace{\bar{q}\, \gamma^\mu\, \gamma^5\, \frac{\lambda_3}{2}\, q}_{\text{non-sing. axial current } j_5^{\mu,3}} \mid P,S \rangle$$

isospin rotation yields :

$$\langle p \mid j_5^{\mu,3} \mid p \rangle = \langle p \mid j_5^{\mu,+} \mid n \rangle \;\propto\; g_A$$

non-singlet axial current is conserved,

$$\partial_\mu \, j_5^{\mu,3} \;=\; 0 \quad \Rightarrow \quad \Delta \mathcal{A}_3(Q) \;\equiv\; \Delta \mathcal{A}_3(\cancel{Q})$$

consistent with DGLAP evolution :

$$\mu \frac{\mathrm{d}\, \Delta \mathcal{A}_3(\mu)}{\mathrm{d}\mu} \;=\; 0 \cdot \Delta \mathcal{A}_3(\mu)$$

Recall :

'QCD formula' for $g_1$ contains $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha_s^2), \ldots$ corrections to partonic cross sections

$$\Rightarrow \quad \mathcal{G}_1^{p-n}(Q^2) \;=\; \frac{1}{6}\,\mathcal{C}_{\rm Bj}\Big(\alpha_s(Q)\Big)\,g_A$$

where

$$\mathcal{C}_{\rm Bj}(\alpha_s) \;=\; 1 - \frac{\alpha_s}{\pi} - \left[\frac{55}{12} - \frac{n_f}{3}\right]\left(\frac{\alpha_s}{\pi}\right)^2$$

$$- \quad \left[41.4 - 7.6\,n_f + \frac{115}{648}\,n_f^2\right]\left(\frac{\alpha_s}{\pi}\right)^3 \pm \ldots$$

(Kodaira et al.; Gorishny,Larin; Larin,Vermaseren)

(can even be used to determine $\alpha_s$ !
(Ellis,Karliner; Altarelli,Ball,Forte,Ridolfi))

In addition, target-mass and 'higher-twist' corrections
(Mankiewicz,Schäfer,Stein;
Shuryak,Vainshtein; Uematsu et al.; ...)

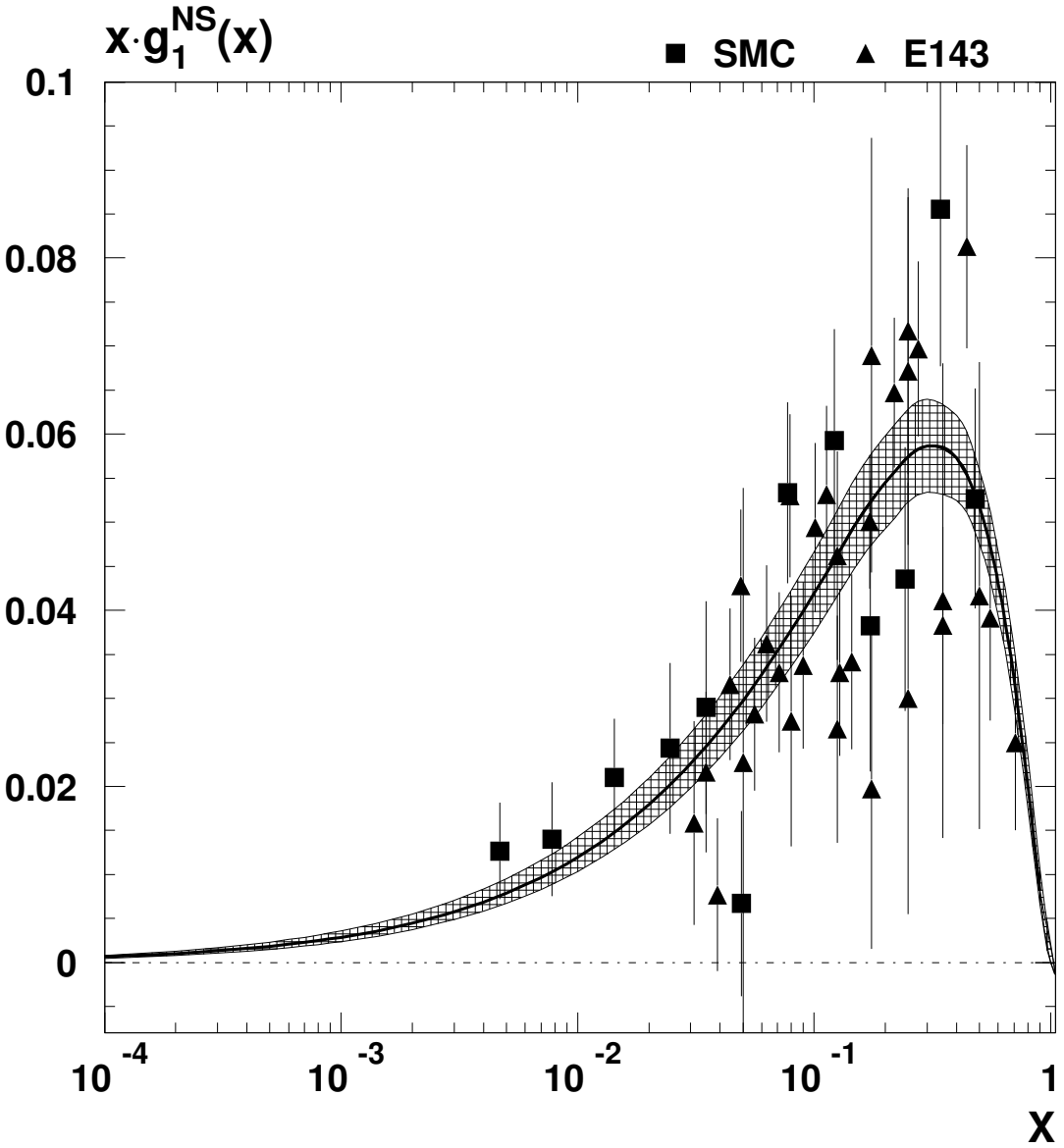
$Q^2 = 5\ \mathrm{GeV}^2$ :

Theory :

$$\Gamma_1^{p-n}(Q^2) = 0.181 \pm 0.003$$

SMC : (E155 similar)

$$\Gamma_1^{p-n}(Q^2) = 0.174 \pm 0.005\ {}^{+0.011}_{-0.009}\ {}^{+0.021}_{-0.006}$$



# Ellis-Jaffe sum rule

We had (to lowest order)

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 \Big[\Delta q(x) + \Delta\bar{q}(x)\Big]$$

Reshuffle a little, and integrate over $x$,

$$\mathcal{G}_1^{p,n}(Q^2) \;=\; \int_0^1 dx\, g_1^{p,n}(x,Q^2) = \pm\frac{1}{12}\Delta\mathcal{A}_3 + \frac{1}{36}\Delta\mathcal{A}_8 + \frac{1}{9}\Delta\Sigma$$

where

$$\Delta\mathcal{A}_3 \;\equiv\; \Delta U + \Delta\bar{U} - \Delta D - \Delta\bar{D} \qquad \text{Bj. sum rule . . .}$$

$$\Delta\mathcal{A}_8 \;\equiv\; \Delta U + \Delta\bar{U} + \Delta D + \Delta\bar{D} - 2\left(\Delta S + \Delta\bar{S}\right)$$

$$\Delta\Sigma \;\equiv\; \Delta U + \Delta\bar{U} + \Delta D + \Delta\bar{D} + \Delta S + \Delta\bar{S}$$

One finds :

$$S^\mu\,\Delta\mathcal{A}_3 \;=\; \langle P,S \mid \bar{q}\,\gamma^\mu\,\gamma^5\,\frac{\lambda_3}{2}\,q \mid P,S\rangle$$

$$S^\mu\,\Delta\mathcal{A}_8 \;=\; \langle P,S \mid \bar{q}\,\gamma^\mu\,\gamma^5\,\frac{\lambda_8}{2}\,q \mid P,S\rangle$$

$$S^\mu\,\Delta\Sigma \;=\; \langle P,S \mid \bar{q}\,\gamma^\mu\,\gamma^5\,\frac{\mathbb{1}}{2}\,q \mid P,S\rangle$$

Isospin : $\Delta\mathcal{A}_3 = g_A$ Bj. sum rule

SU(3) : $\Delta\mathcal{A}_8$ from baryonic $\beta$ decays

| Zerfall | SU(3) | exp. |
|---|---|---|
| $n \to pe^-\bar{\nu}_e$ | $F + D$ | $1.2670 \pm 0.0035$ |
| $\Lambda \to pe^-\bar{\nu}_e$ | $F + D/3$ | $0.718 \pm 0.015$ |
| $\Sigma^- \to ne^-\bar{\nu}_e$ | $F - D$ | $-0.340 \pm 0.017$ |
| $\Xi^- \to \Lambda e^-\bar{\nu}_e$ | $F - D/3$ | $0.25 \pm 0.05$ |

$$\Delta\mathcal{A}_3 = F + D = 1.267 \pm 0.0035$$

$$\Delta\mathcal{A}_8 = 3F - D = 0.58 \pm 0.03$$

Ellis,Jaffe : assume $\Delta S + \Delta\bar{S} \approx 0$

$$\mathcal{G}_1^p(Q^2) \approx \frac{1}{2}F - \frac{1}{18}D = 0.186 \pm 0.004$$

E155 $0.118 \pm 0.004 \pm 0.007$

$$\mathcal{G}_1^n(Q^2) \approx \frac{1}{3}F - \frac{2}{9}D = -0.025 \pm 0.004$$

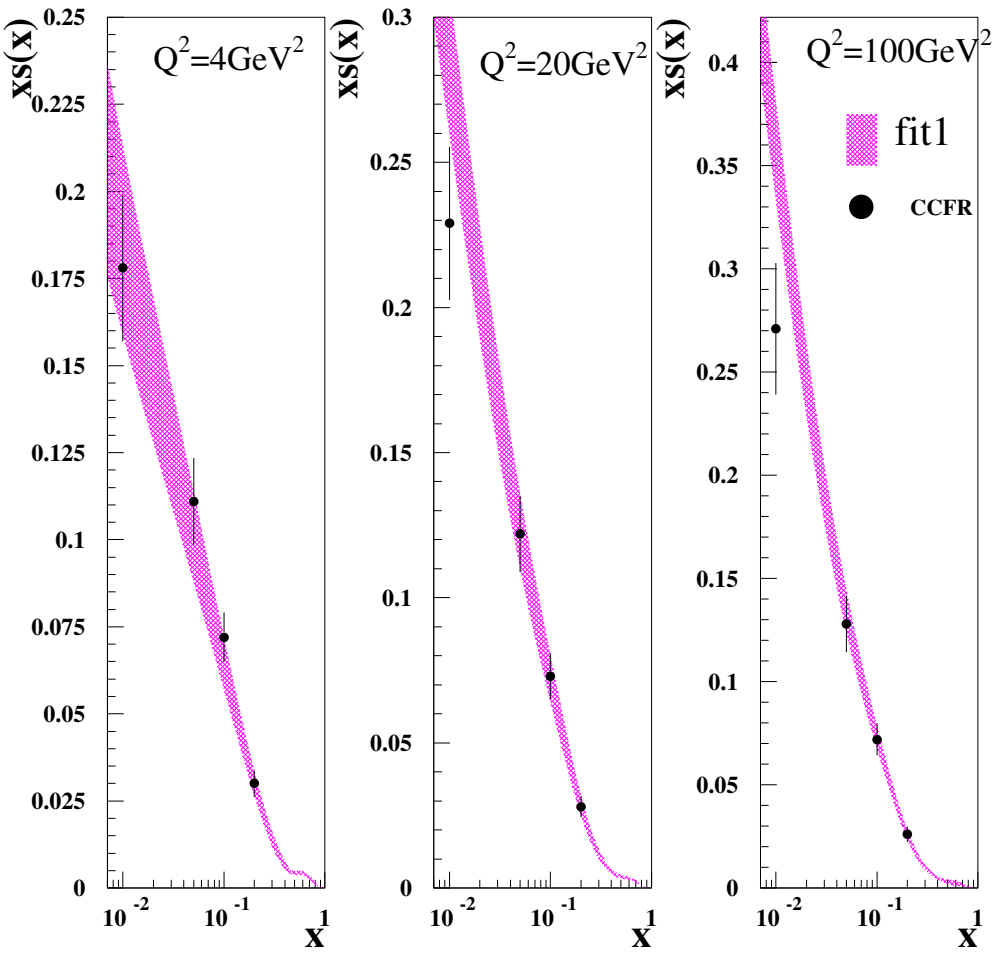
E155 $-0.058 \pm 0.005 \pm 0.008$

Hence, we find that $\Delta S + \Delta\bar{S} \approx -0.14$, or

$$\frac{1}{2}\Delta\Sigma \approx 0.08 \pm 0.04 \ll \frac{1}{2}$$ "spin surprise"

# Strangeness in the Nucleon ?

- unpol. : CCFR/NuTeV measure $F_3^{\nu_\mu,\bar{\nu}_\mu}$




(Barone et al.)

$$\frac{\int x\, s(x)}{\int x\, [\bar{u}(x)+\bar{d}(x)]/2} \sim 0.4$$

- matrix element

$$\langle P \mid \bar{s}\, \gamma^\mu\, \gamma_5 \;\; s \mid P \rangle$$

also appears in elastic $\nu p$ scattering, and is sizable and negative

(Kaplan,Manohar; Ellis,Karliner)

# Radiative QCD Corrections ?

Recall :

$$g_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \left\{ \left[\Delta q + \Delta\bar{q}\right](x,Q) \right.$$

$$+ \left| \quad + \ldots \right|^2 \otimes \left[\Delta q + \Delta\bar{q}\right](x,Q) + \ldots$$

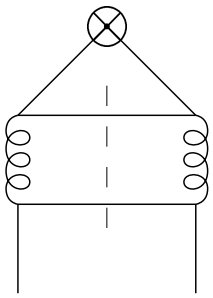
$$\left. + \left| \quad + \ldots \right|^2 \otimes \Delta g(x,Q) \; + \ldots \right\} \; + \; \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

- for integral $\int_0^1 dx$, in $\overline{\text{MS}}$ scheme :

$$\mathcal{G}_1^{p,n}(Q^2) = \left[1 - \frac{\alpha_s(Q)}{\pi}\right]\left\{\pm\frac{1}{12}\Delta\mathcal{A}_3 + \frac{1}{36}\Delta\mathcal{A}_8 + \frac{1}{9}\Delta\Sigma(Q)\right\}$$

- no correction $\propto \Delta g$ here
- $\mathcal{A}_{3,8}(\not{Q})$, but $\Delta\Sigma = \Delta\Sigma(Q)$ !
- $[1 - \frac{\alpha_s(Q)}{\pi}]$ as in Bjorken sum rule; moderate correction

What's behind $\Delta\Sigma = \Delta\Sigma(Q)$ ?



$$S^\mu \, \Delta\Sigma \; = \; \langle P,S \,| \quad \underbrace{\bar{q} \, \gamma^\mu \, \gamma^5 \, \frac{\mathbb{1}}{2} \, q}_{\text{singlett axial current } j_5^{\mu,0}} \quad | \, P,S \rangle$$

*not conserved* due to axial anomaly :

$$\partial_\mu \, j_5^{\mu,0} \equiv \partial_\mu \left[ \bar{q} \, \gamma^\mu \, \gamma^5 \, q \right] = n_f \, \frac{\alpha_s}{2\pi} \, \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} \left[ G_{\mu\nu} \, \tilde{G}^{\mu\nu} \right]$$

$$\Rightarrow \Delta\Sigma \propto \langle P,S| \, j_5^{\mu,0} \, |P,S\rangle \to \Delta\Sigma(Q) \quad \text{(Kodaira)}$$

Has led to alternative definitions of quark spin :
(Altarelli, Ross; Carlitz, Collins, Mueller)

$$\Delta\Sigma(Q) \; = \; \Delta\Sigma'(\cancel{Q}) - n_f \, \frac{\alpha_s(Q)}{2\pi} \underbrace{\int_0^1 dx \Delta g(x,Q)}_{\equiv \Delta G(Q)}$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix} = \frac{\alpha_s}{2\pi} \begin{pmatrix} 0 & 0 \\ \frac{3}{2} C_F & \frac{\beta_0}{2} \end{pmatrix} \otimes \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix}$$

- $\alpha_s(Q) \, \Delta G(Q) \; \to$ const. as $Q \to \infty$ (!)
- $\Delta\Sigma' =$ "true quark spin" ? Large ?
- however, *a matter of definition*
- separation not gauge invariant
- nonetheless, boost for interest in $\Delta g$ : only relevant if $\Delta G$ is large

# IV. Ways of measuring $\Delta g$

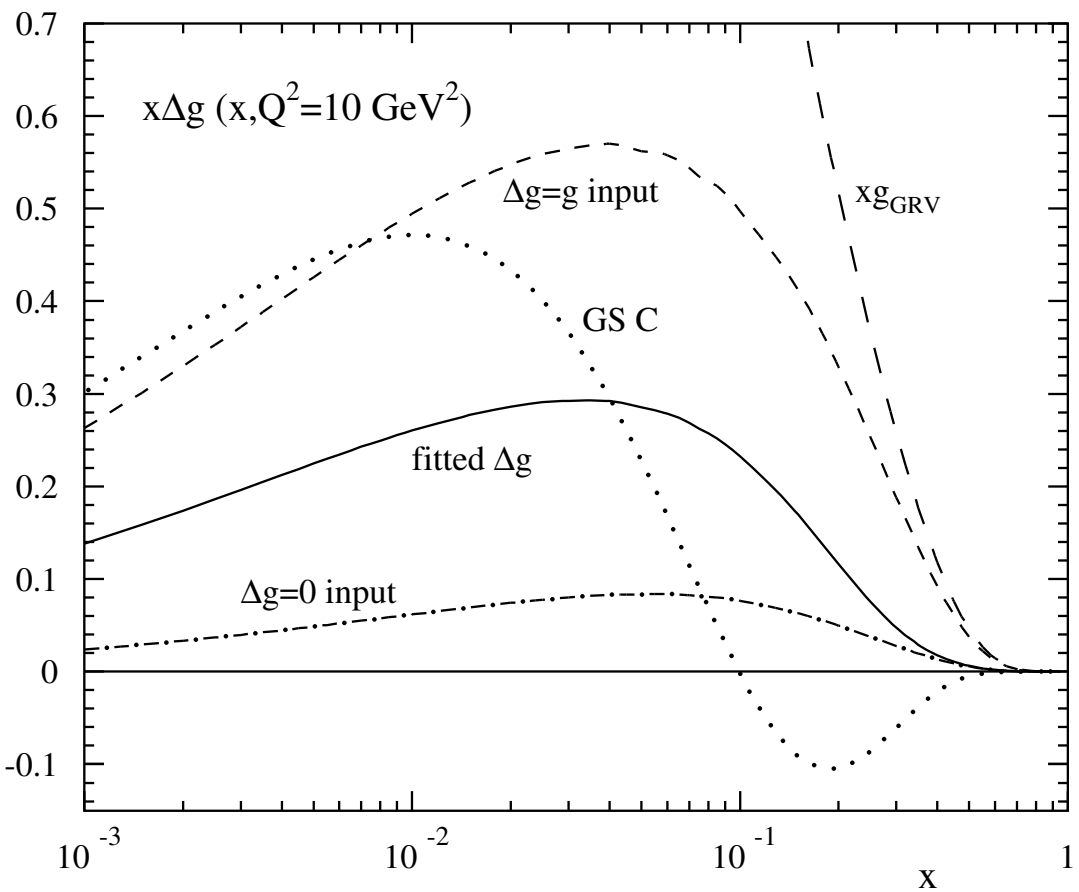
- DIS rather indirect, scaling violations

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}=\frac{\alpha_s}{2\pi}\begin{pmatrix}\Delta P_{qq} & \Delta P_{qg}\\ \Delta P_{gq} & \Delta P_{gg}\end{pmatrix}\otimes\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}$$

- to be viable, need large lever arm in $Q^2$ and/or high precision
  unpol. case: H1, ZEUS, NMC, ...
  pol. case: more difficult
  (so far, only fixed-target; pol. HERA?)

- more promising: study reactions which have gluon participation already at lowest order

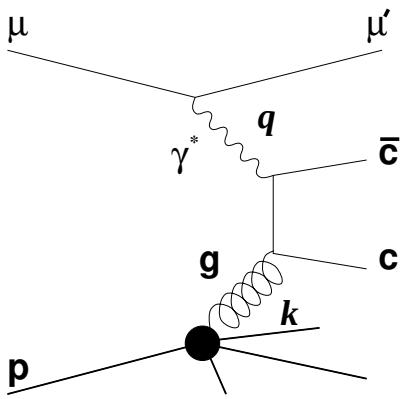
## Polarized gluon densities :
## (Glück,Reya,Stratmann,WV; Gehrmann,Stirling)



- lepton-nucleon scattering :

  look at *less inclusive* reactions; examples :

  * $\vec{l}\vec{N}\,(\vec{\gamma}\vec{N}) \rightarrow Q\bar{Q}\,X$
  * $\vec{l}\vec{N}\,(\vec{\gamma}\vec{N}) \rightarrow$ jet $X$
  * $\vec{l}\vec{N}\,(\vec{\gamma}\vec{N}) \rightarrow h^{+}h^{-}\,X$

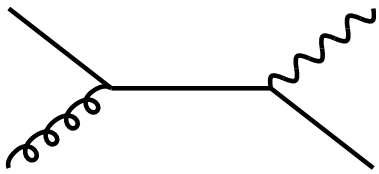



(COMPASS, HERMES, pol. HERA?)

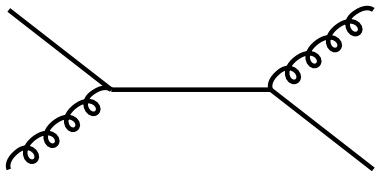
- hadron-hadron scattering :

  a rich new program commencing at RHIC !

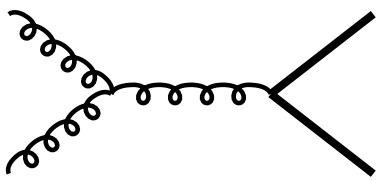
  * $\vec{p}\vec{p} \rightarrow \gamma\,X$
    "prompt photon"



  * $\vec{p}\vec{p} \rightarrow$ jet $X$

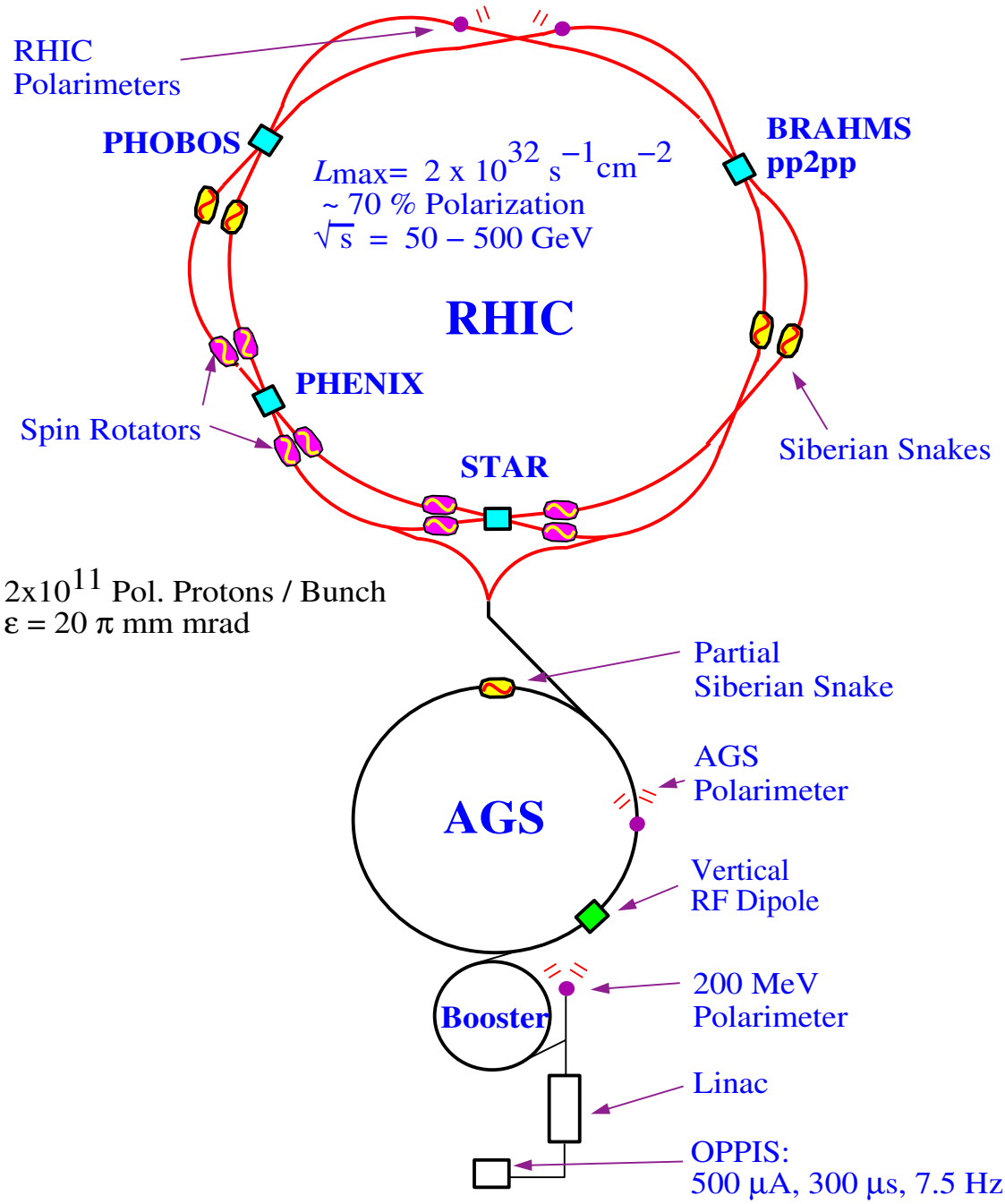


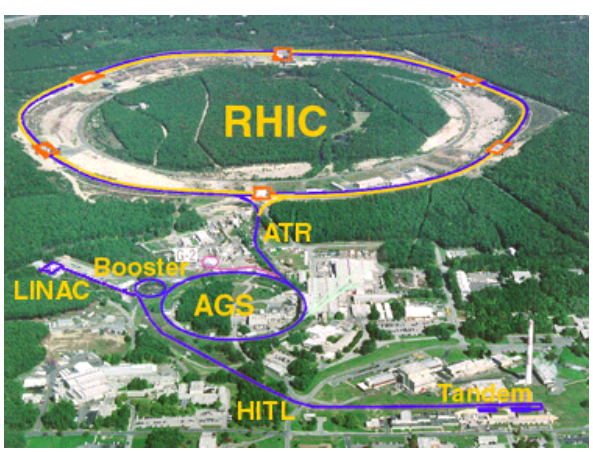
  * $\vec{p}\vec{p} \rightarrow Q\bar{Q}\,X$



- all these reactions studied in unpolarized case

# Polarized Proton Collisions at BNL





# V. One example :

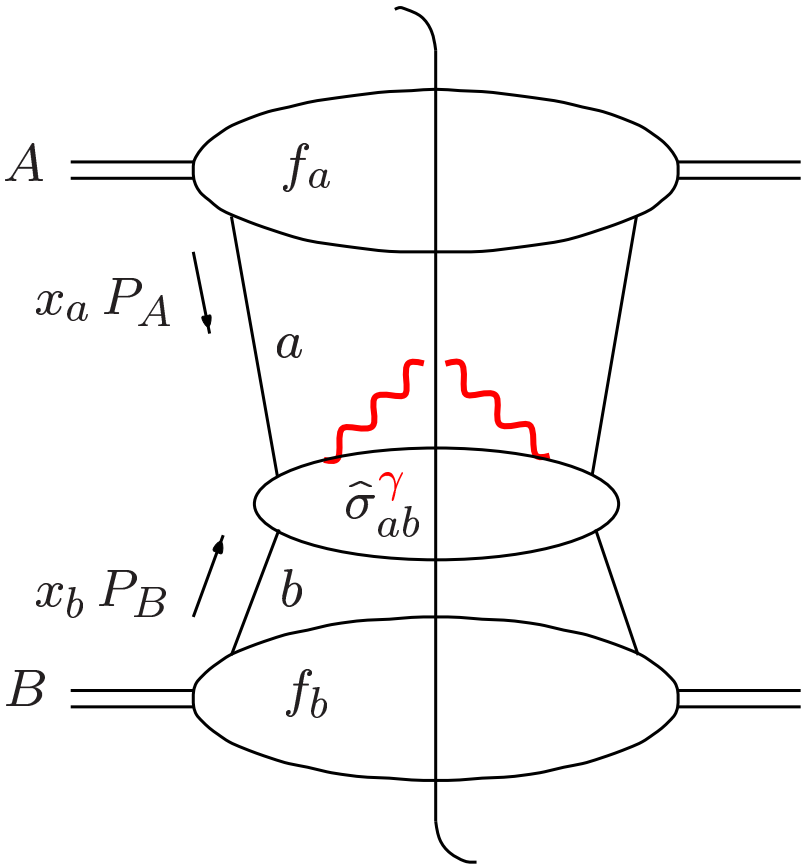
## Prompt Photon Production at RHIC

$p\,p \;\to\; \gamma(p_T)\; X$ :

- clean electromagnetic probe of QCD hard scattering, and of gluon density $g(x,\mu)$

Perturbative QCD / factorization theorem :

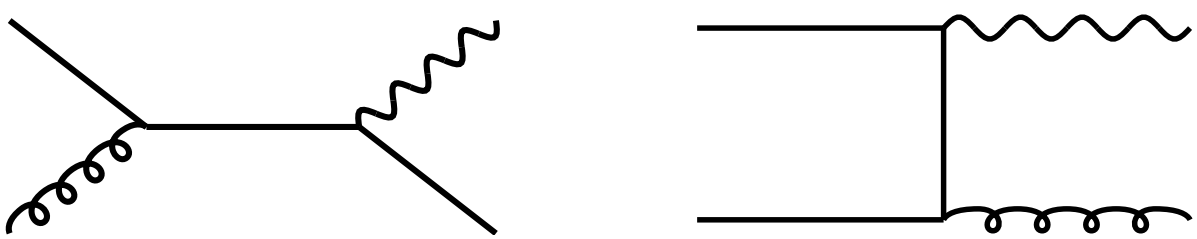
$$d\sigma^{\gamma}_{AB} \;=\; \sum_{a,b} \int dx_a\, dx_b\; f_a(x_a,\mu)\, f_b(x_b,\mu)$$
$$\times\; d\hat{\sigma}^{\gamma}_{ab}(x_a\,P_A\,, x_b\,P_B\,, p_T^{\gamma}, \eta^{\gamma}, \underbrace{\mu}_{\text{ren/fact scale}}\;)$$



Partonic cross sections perturbative :

$$d\hat{\sigma}_{ab}^{\gamma} = \underbrace{d\hat{\sigma}_{ab}^{\gamma,(0)}}_{\text{LO}} + \alpha_s \underbrace{d\hat{\sigma}_{ab}^{\gamma,(1)}}_{\text{NLO}} + \ldots$$

Lowest order $\mathcal{O}(\alpha\alpha_s)$ :



Compton annihilation

$d\hat{\sigma}_{ab}^{\gamma,(1)}$ : Aurenche et al.
Baer,Ohnemus,Owens
Gordon,WV
Frixione

At RHIC, measure spin asymmetry

$$A_{LL}^{\gamma} \equiv \frac{d\sigma_{++} - d\sigma_{+-}}{d\sigma_{++} + d\sigma_{+-}} \equiv \frac{d\,\Delta\sigma^{\gamma}}{d\,\sigma^{\gamma}}$$

One finds : (Collins)

$$d\Delta\sigma_{AB}^{\gamma} = \sum_{a,b} \int dx_a\, dx_b\; \Delta f_a(x_a,\mu)\, \Delta f_b(x_b,\mu) \times d\Delta\hat{\sigma}_{ab}^{\gamma}(x_a P_A, x_b P_B, p_T^{\gamma}, \eta^{\gamma}, \mu)$$

where

$$\Delta f(\xi,\mu) \equiv f^{+}(\xi,\mu) - f^{-}(\xi,\mu)$$

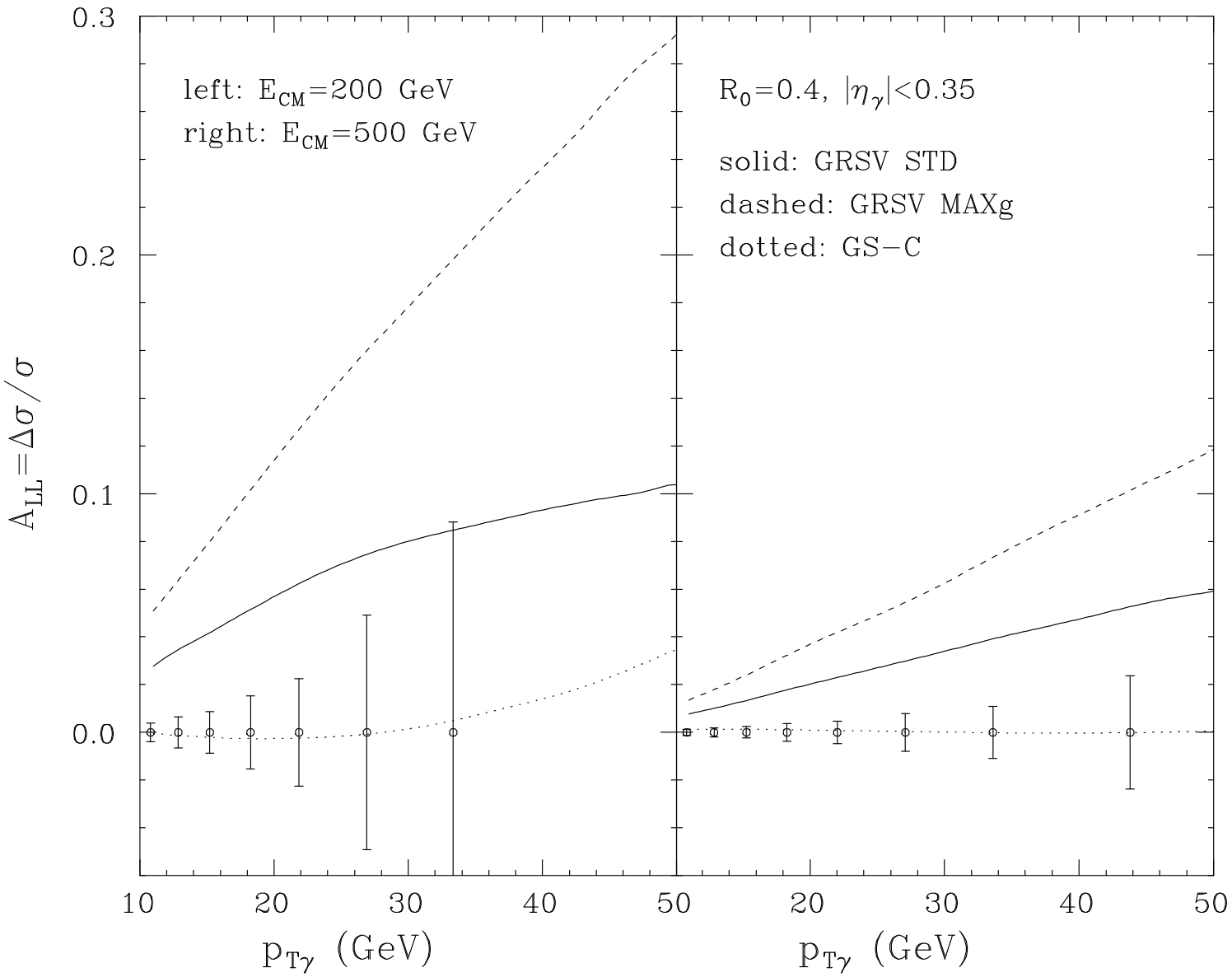
$$d\Delta\hat{\sigma}_{ab}^{\gamma} = \frac{1}{2}\left[ d\hat{\sigma}_{a(+)b(+)}^{\gamma} - d\hat{\sigma}_{a(+)b(-)}^{\gamma} \right]$$

$$d\Delta\hat{\sigma}_{ab}^{\gamma} = d\Delta\hat{\sigma}_{ab}^{\gamma,(0)} + \alpha_s\, d\Delta\hat{\sigma}_{ab}^{\gamma,(1)} + \ldots$$

$d\Delta\hat{\sigma}_{ab}^{\gamma,(1)}$ : Contogouris et al.;
Gordon,WV; Frixione,WV

$|\eta^\gamma| \leq 0.35$ PHENIX-exp.

isolation with cone $R_0 = 0.4$



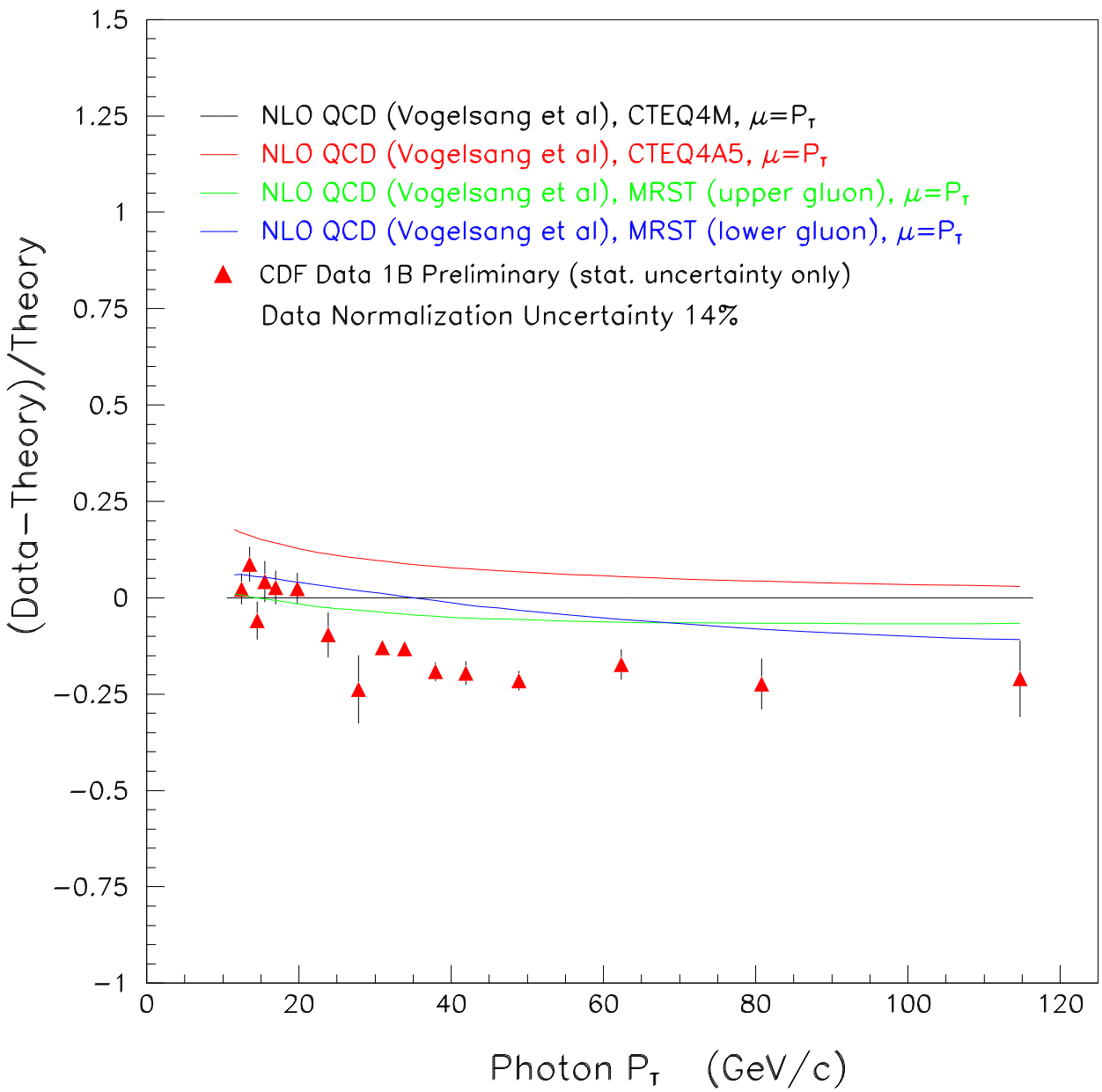

(Frixione,W.V.)

## Trouble with prompt photons...?

Unpolarized case :

comparison data $\longleftrightarrow$ NLO theory not satisfactory

one example : CDF, Run-1B

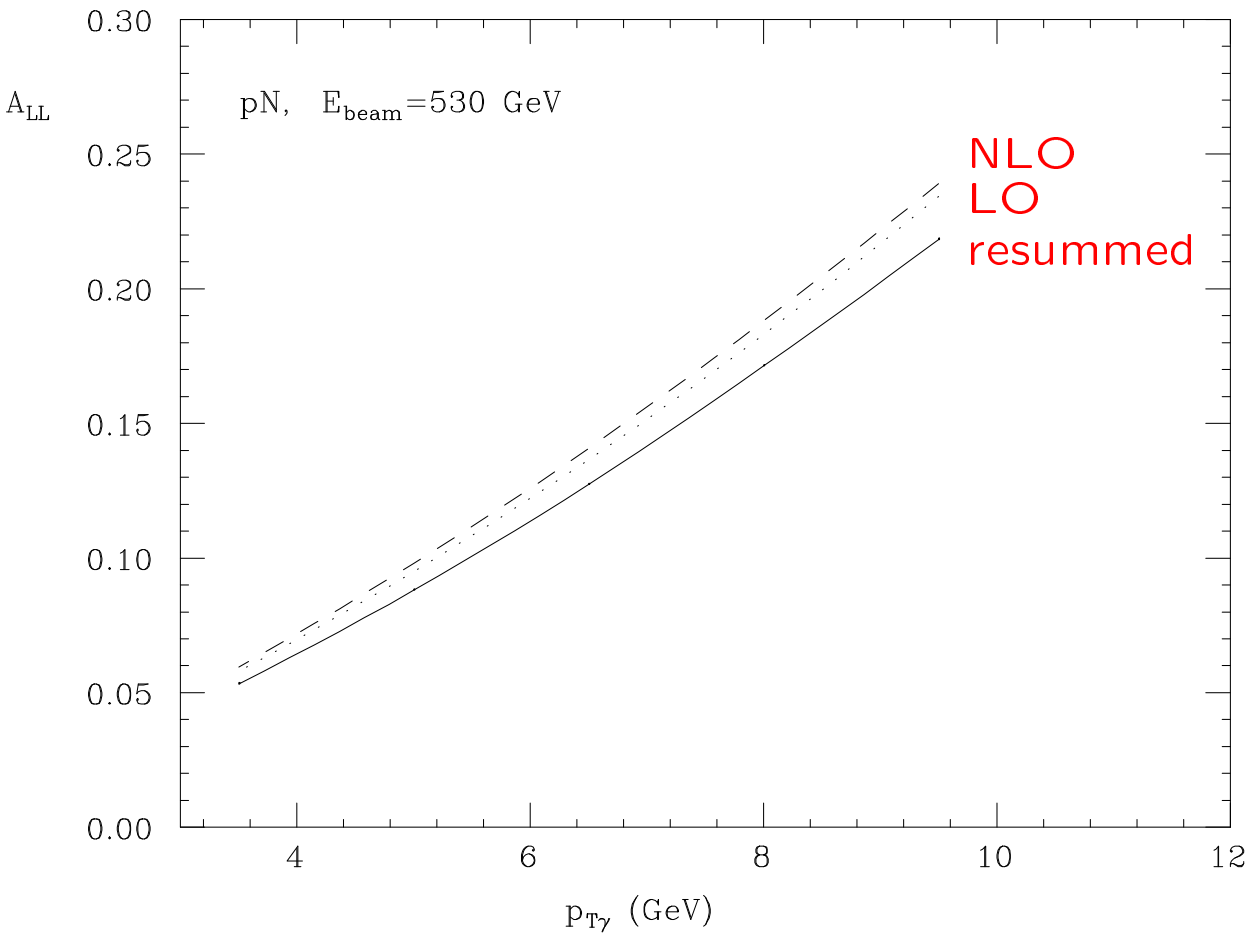


Theory :

- look for additional corrections
  beyond the NLO framework

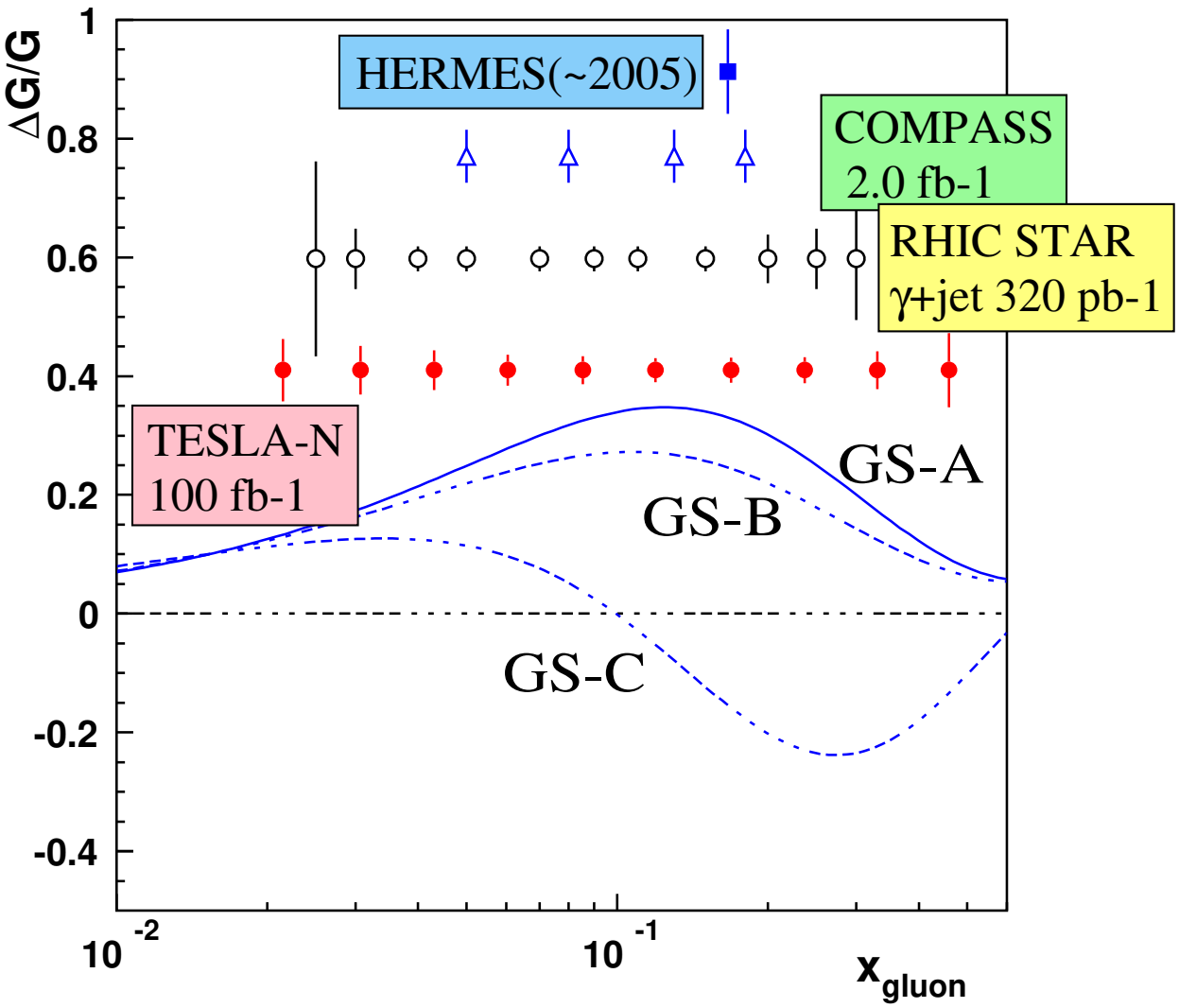
- in particular, soft-gluon resummations
  (Laenen,Sterman,Oderda;
  Catani,Mangano,Nason,Oleari,WV; Li;
  Kidonakis,Owens; Laenen,Sterman,WV; Fink)

- spin asymmetry and resummation :
  (Sterman,WV)



"World comparison" for $\Delta g$ measurements

(TESLA-N Study Group)



# VI. Further information on $\Delta q$, $\Delta\bar{q}$

- inclusive DIS via $\gamma^*$ :

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 \Big[\Delta q(x) + \Delta\bar{q}(x)\Big]$$

  $\Rightarrow$ only information on $\Delta q + \Delta\bar{q}$

- (how) can one determine $\Delta u$, $\Delta\bar{u}$, $\Delta d$, $\Delta\bar{d}$, $\Delta s$, $\Delta\bar{s}$ separately ?

- learn more about nucleon structure
  $\mathrm{SU}(2)_f$, $\mathrm{SU}(3)_f$ breaking in sea

  * $\Delta\bar{u} - \Delta\bar{d} \quad > 0$ ? Or $< 0$ ? Or $= 0$ ?

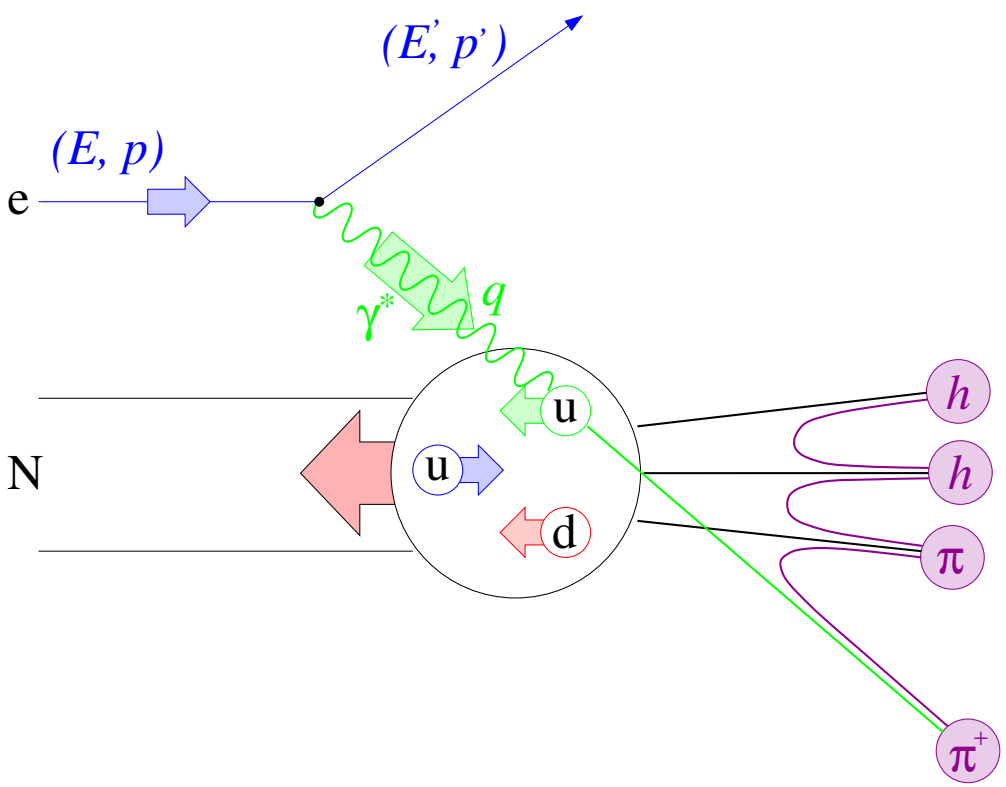
    $|\Delta\bar{u} - \Delta\bar{d}| \; > |\bar{d} - \bar{u}|$ ? Or $<$ ? Or $=$ ?

  * confront with models
    (pion cloud, chiral soliton, Pauli exclusion)
    (Thomas,Holt,Speth,Fäßler;
    Diakonov,Petrov,Pobylitsa,Polyakov,Weiss;
    Gamberg,Ruiz Arriola,Weigel; Wakamatsu,Watabe,
    Glück,Reya)

- tests of $\Delta\mathcal{A}_i \leftrightarrow F, D$

# (a) Semi-inclusive DIS

**To do**: detect a hadron $h = \pi^+, \pi^-, K^\pm, \ldots$ in coincidence with the scattered electron

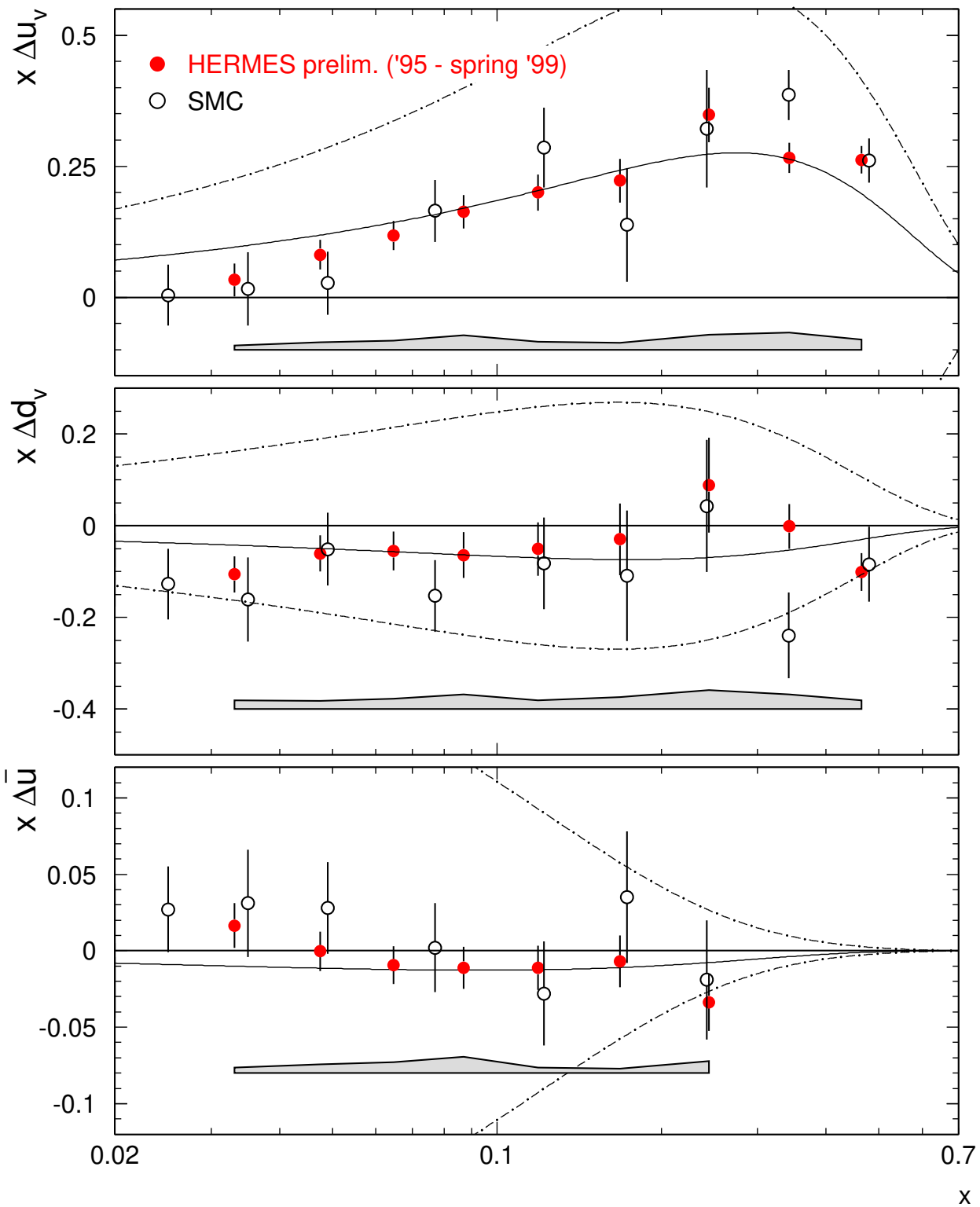



**Idea**: flavor content of the observed hadron is related to the flavor of the struck quark through *fragmentation functions* $D_q^h$

$D_q^h(z, Q)$: prob. to find $h$ in $q$ with mom. fraction $z$
*weigh* differently the flavors of struck quark

$$A_1^h(x,z) = \frac{g_1^h(x,z)}{F_1^h(x,z)} = \frac{\sum_q e_q^2 \left[\Delta q(x)\ D_q^h(z) + \Delta\bar{q}(x)\ D_{\bar{q}}^h(z)\right]}{\sum_q e_q^2 \left[q(x)\ D_q^h(z) + \bar{q}(x)\ D_{\bar{q}}^h(z)\right]}$$

(HERMES)

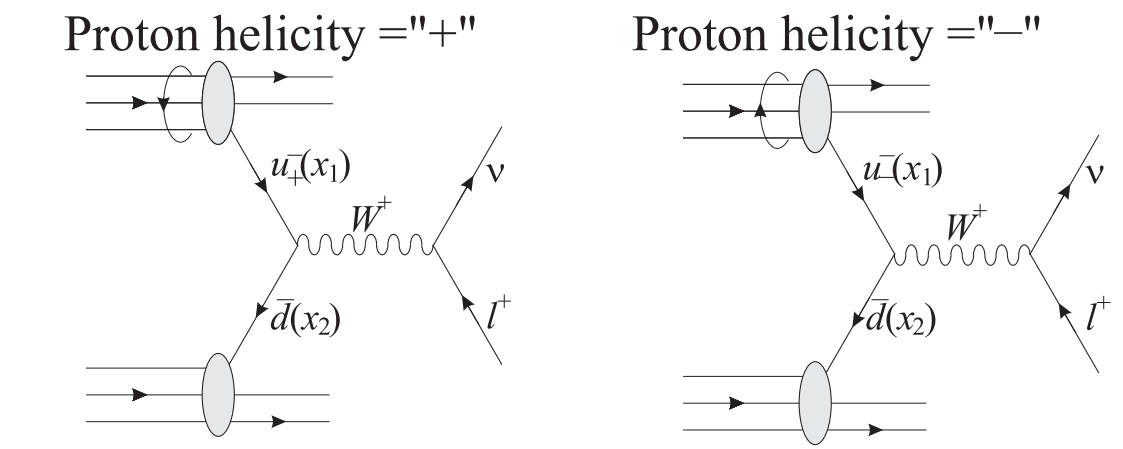

x Δu_v
0.5
0.25
0
HERMES prelim. ('95 - spring '99)
SMC
x Δd_v
0.2
0
-0.2
-0.4
x Δū
0.1
0.05
0
-0.05
-0.1
0.02
0.1
0.7
x

# (b) $W^{\pm}$ production at RHIC

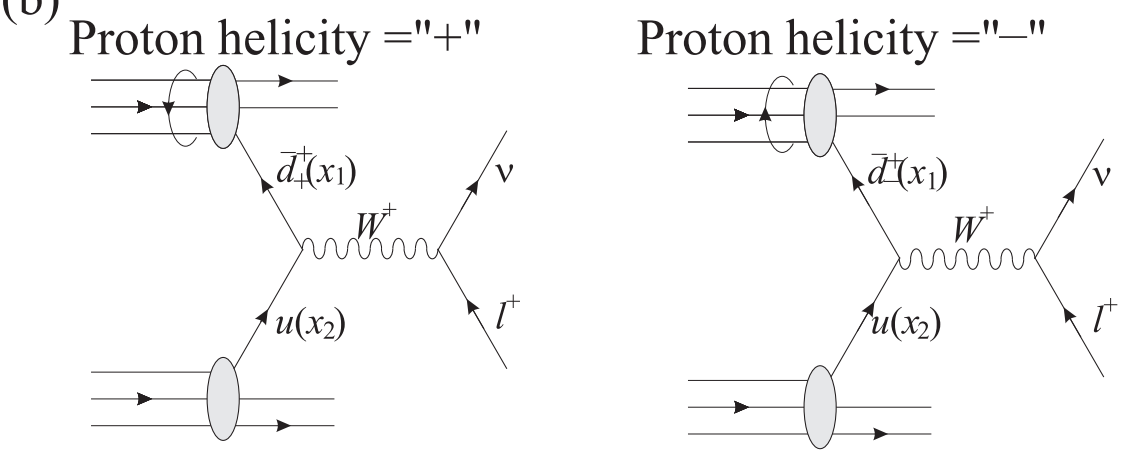
(Bourrely,Soffer; Kamal; Gehrmann; Dressler,Goeke et al.)

$W$ selects parton helicity !

(a)



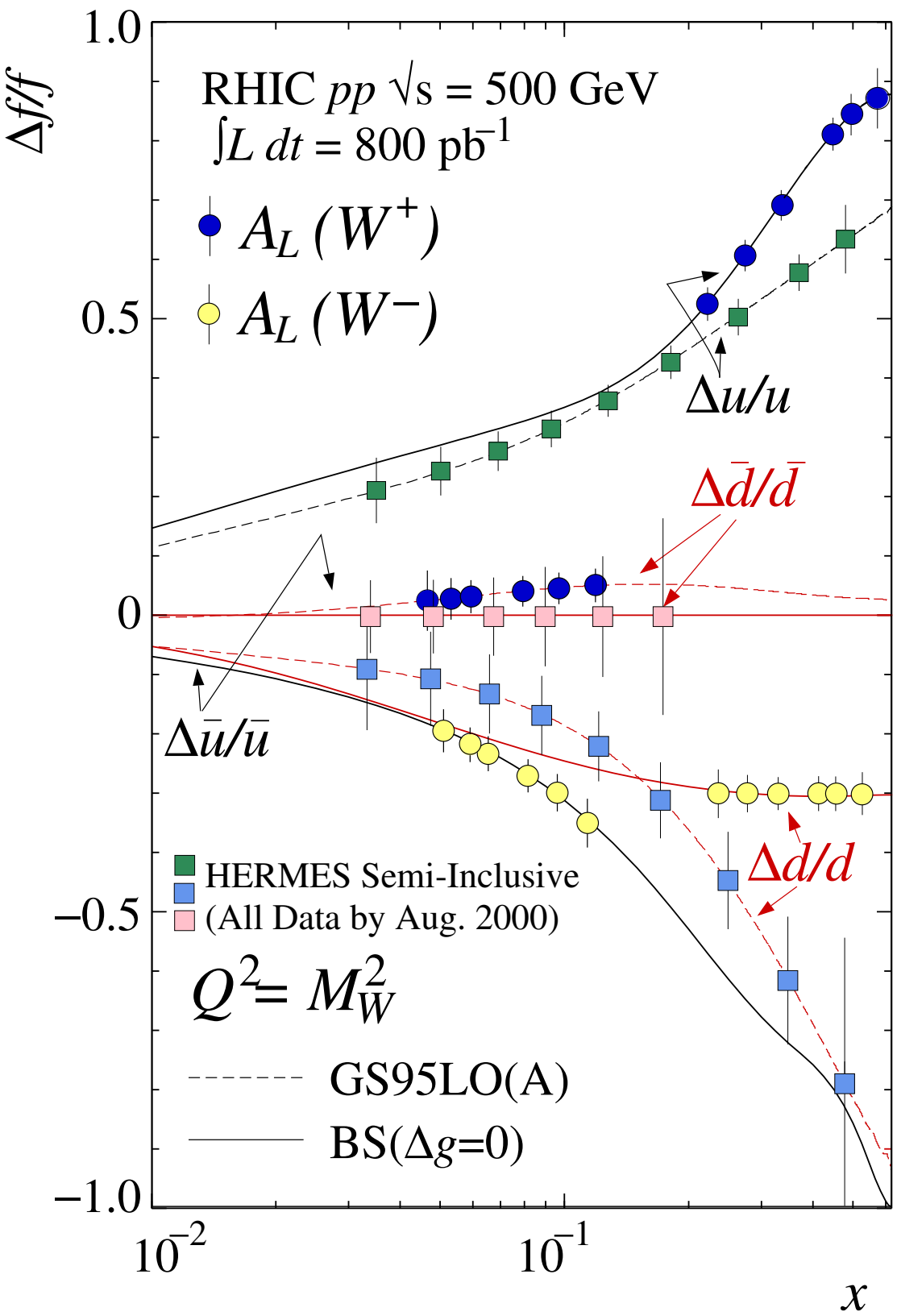

(b)




$$A_L^W \equiv \frac{\sigma_-(W) - \sigma_+(W)}{\sigma_-(W) + \sigma_+(W)}$$

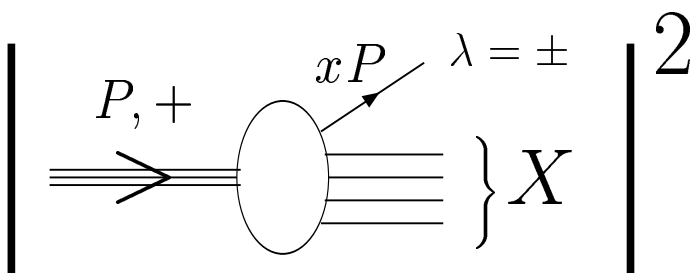
for ex. $$A_L^{W^+} \approx \frac{\Delta u(x_1)\,\bar{d}(x_2) - \Delta\bar{d}(x_1)\,u(x_2)}{u(x_1)\,\bar{d}(x_2) + \bar{d}(x_1)\,u(x_2)}$$

$$x_1 = M_W\, e^{y_W}/\sqrt{s}, \qquad x_2 = M_W\, e^{-y_W}/\sqrt{s}$$

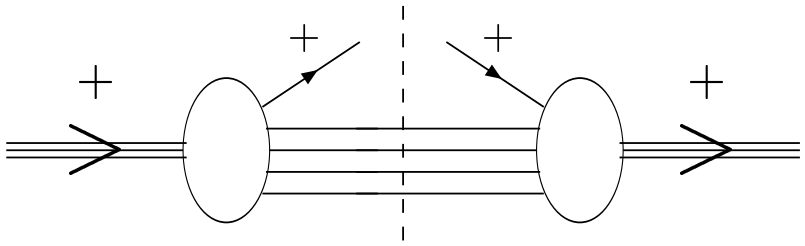

1.0
0.5
0
−0.5
−1.0
Δf/f
RHIC pp √s = 500 GeV
∫L dt = 800 pb−1
AL (W+)
AL (W−)
Δu/u
Δd̄/d̄
Δū/ū
Δd/d
HERMES Semi-Inclusive
(All Data by Aug. 2000)
Q2 = MW2
GS95LO(A)
BS(Δg=0)
10−2
10−1
x

# VII. Transversity

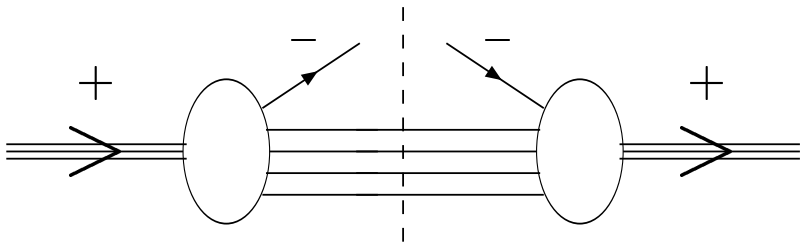
A parton distribution :




$f^{+}(x) \propto$



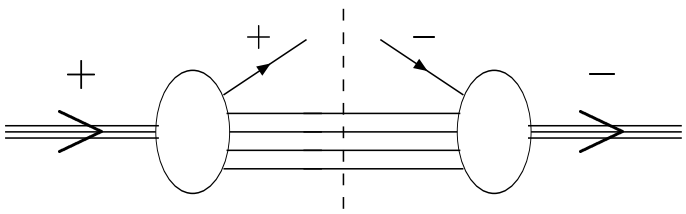

$f^{-}(x) \propto$




$$q(x) = \frac{1}{2}\sum_X \delta(P_X^+ - (1-x)P^+)$$
$$\times \left[ \left| \langle X | \mathcal{P}^+ \psi_+(0) | P, \Lambda = \frac{1}{2} \rangle \right|^2 + \left| \langle X | \mathcal{P}^- \psi_+(0) | P, \Lambda = \frac{1}{2} \rangle \right|^2 \right]$$

$$\Delta q(x) = \frac{1}{2}\sum_X \delta(P_X^+ - (1-x)P^+)$$
$$\times \left[ \left| \langle X | \mathcal{P}^+ \psi_+(0) | P, \Lambda = \frac{1}{2} \rangle \right|^2 - \left| \langle X | \mathcal{P}^- \psi_+(0) | P, \Lambda = \frac{1}{2} \rangle \right|^2 \right]$$

$$\mathcal{P}^{\pm} \equiv \frac{\mathbf{1} \pm \gamma_5}{2}$$

How about 

?

Allowed by angular momentum conservation; would involve helicity flip !

- helicity-flip amplitudes occur for transverse polarization:

$$|\perp\rangle \;=\; \frac{1}{\sqrt{2}}\Big(|+\rangle \,+\, |-\rangle\Big)$$

$$|\top\rangle \;=\; \frac{1}{\sqrt{2}}\Big(|+\rangle \,-\, |-\rangle\Big)$$

In cross section *difference* with transverse polarization one finds

$$\delta\sigma_\perp - \delta\sigma_\top \;\equiv\; \frac{1}{2}\Big[\,\langle\perp|\cdots|\perp\rangle \,-\, \langle\top|\cdots|\top\rangle\,\Big]$$

$$\propto \quad \langle+|\cdots|-\rangle \,+\, \langle-|\cdots|+\rangle$$

$$\delta q\,(x) \;=\; \frac{1}{2}\sum_X \delta(P_X^+ - (1-x)P^+)$$

$$\times\;\left[\left|\langle X|\,\mathcal{P}^\uparrow\psi_+(0)\,|P,S_\perp=\tfrac{1}{2}\rangle\right|^2 - \left|\langle X|\,\mathcal{P}^\downarrow\psi_+(0)\,|P,S_\perp=\tfrac{1}{2}\rangle\right|^2\right]$$

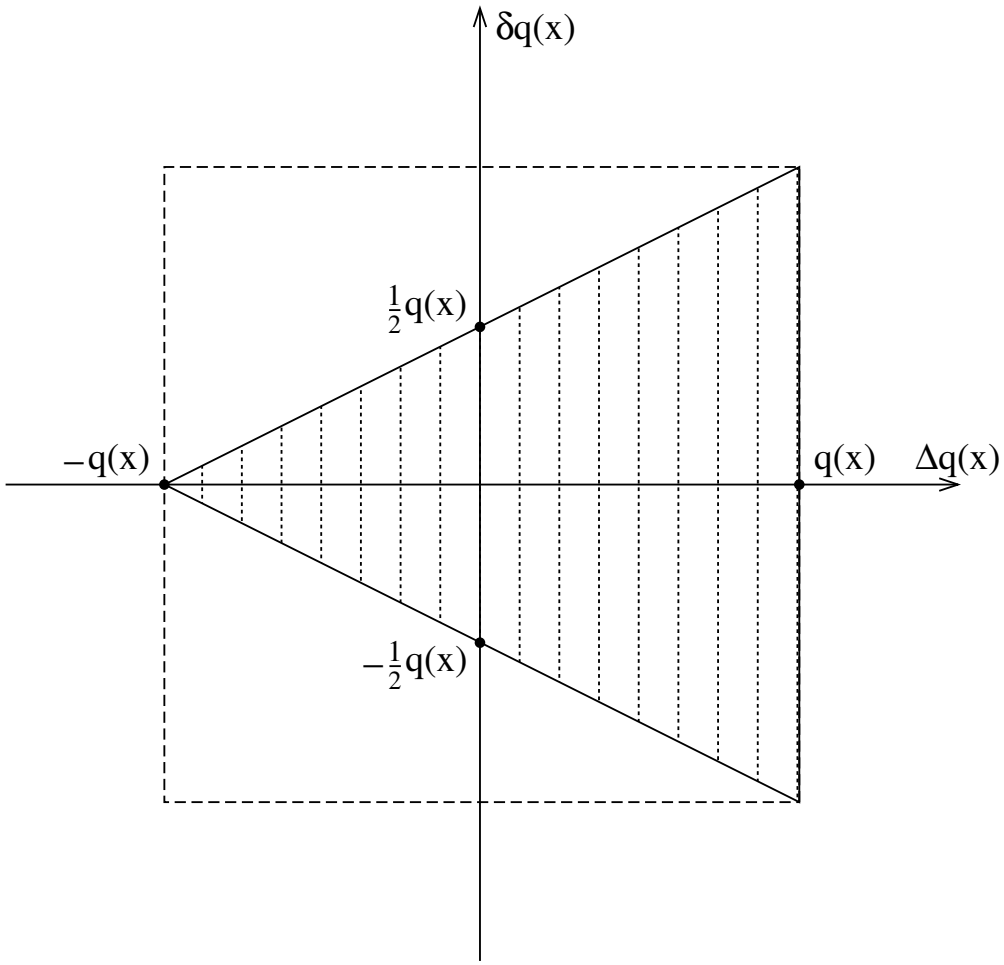
$$\mathcal{P}^{\uparrow\downarrow} \;\equiv\; \frac{\mathbb{1}\pm\gamma_\perp\gamma_5}{2}$$

"as partonic as" $q, \Delta q$ ! (Ralston,Soper)

- note, $\gamma_\perp\gamma_5 \neq \Sigma_\perp \quad\rightsquigarrow\quad$ "transversity"
- spin-1/2 target : *no* helicity flip for gluons possible

From helicity structure :
Soffer's inequality

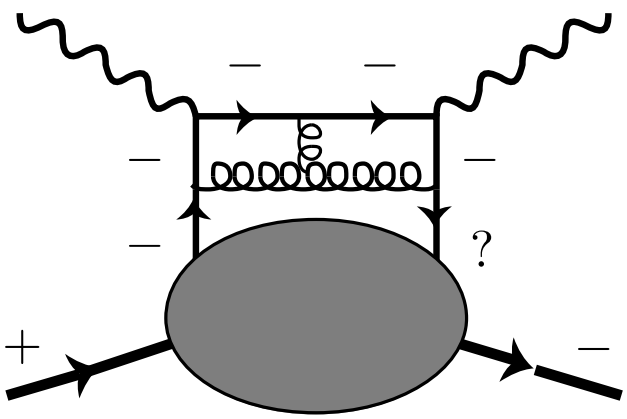
$$2\,|\delta q(x,\mu)| \;\leq\; q(x,\mu) + \Delta q(x,\mu)$$




- for all flavors
- constraint for models for transversity
- preserved under QCD evolution
  (Barone; Bourrely,Soffer,Teryaev;
  Martin,Schäfer,Stratmann,WV)

# Measurement of $\delta q$ ? Difficult !

Helicity flip required $\Rightarrow$ not in incl. DIS :




Possibilities (among others) :

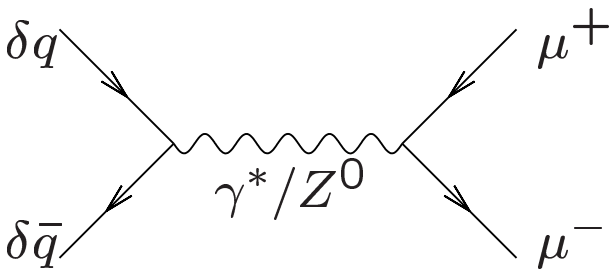
- collisions of transversely pol. protons at RHIC

$$A_{TT} = \frac{d\sigma^{p^\uparrow p^\uparrow} - d\sigma^{p^\uparrow p^\downarrow}}{d\sigma^{p^\uparrow p^\uparrow} + d\sigma^{p^\uparrow p^\downarrow}}$$

Drell-Yan dimuon production, $pp \to \mu^+\mu^- X$

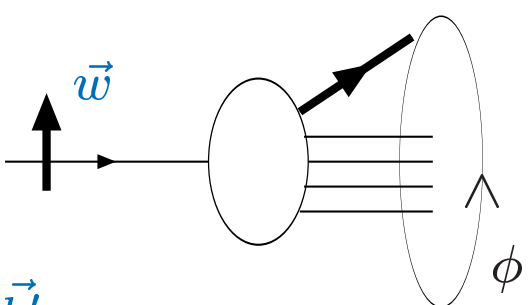
$+$ : LO $q\bar{q} \to \gamma^* \to \mu^+\mu^-$

$-$ : presumably $\delta\bar{q}$ small




- Collins effect :

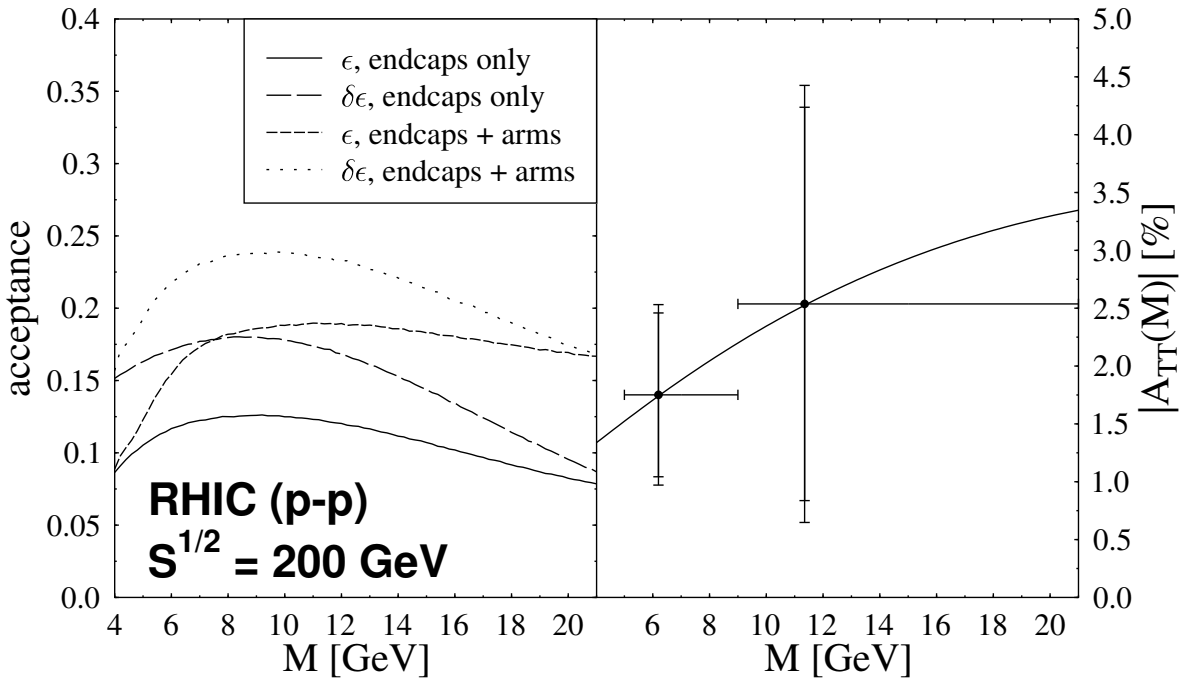
$$\tilde{D}^h(z) = \int d^2k_\perp \, D^h(z, \vec{k}_\perp) \cos(\phi)$$



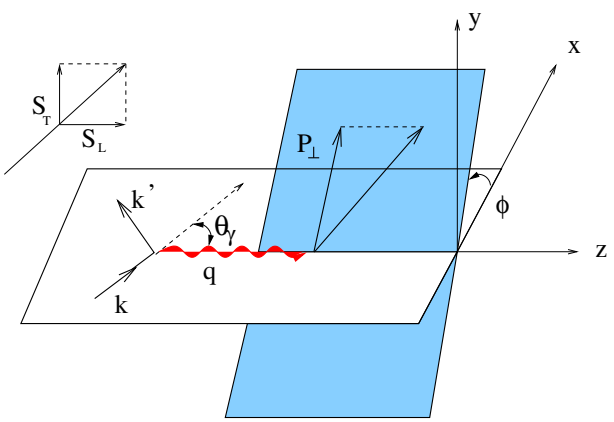

$$\cos(\phi) = \frac{\vec{k}_\perp \cdot \vec{w}}{|\vec{k}_\perp||\vec{w}|} \qquad \vec{w} = \vec{S}_T \times \vec{k}'$$

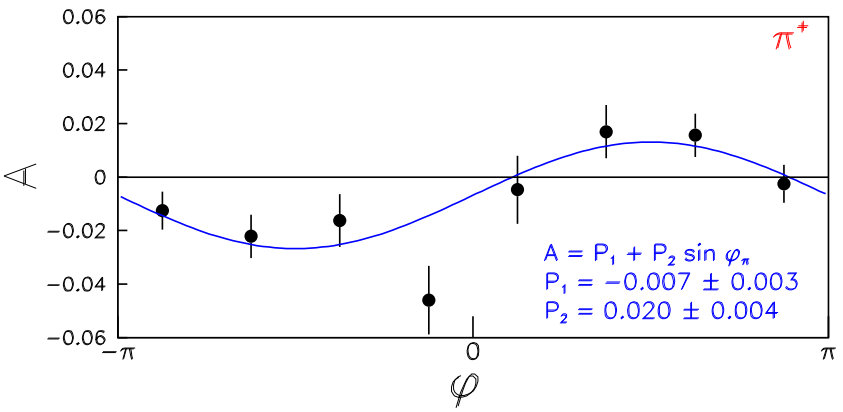
## $A_{TT}$ in Drell-Yan dimuon production :

(Martin,Schäfer,Stratmann,WV)




## Collins effect at HERMES ?



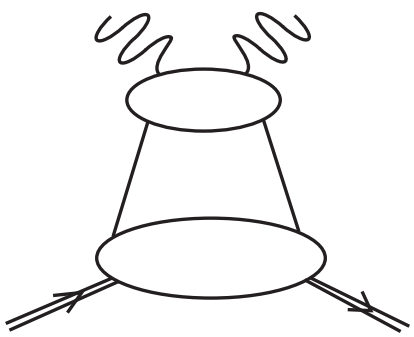
# VIII. $g_2$ – the other structure fctn.

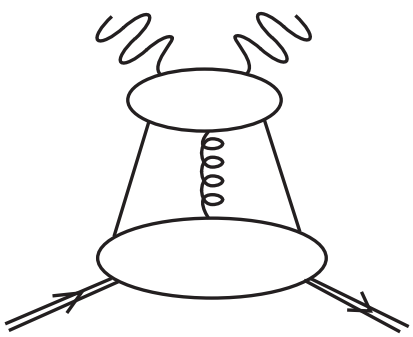
- recall, accessible with transversely pol. target
- *no* partonic interpretation : $g_2 \equiv 0$ for free quark
  very sensitive to $m_q$, $k_q^2$, $k_T^2$ etc.
  (Jaffe,Ji; Anselmino,Efremov,Leader)
- operator product exp. : (Hey,Mandula; Kodaira)

$$\int_0^1 dx\, x^{n-1}\, g_1(x,Q^2) \;=\; \sum_i C_{i,1}^n\, a_i^n$$

$$\int_0^1 dx\, x^{n-1}\, g_2(x,Q^2) \;=\; -\frac{n-1}{n}\left[\sum_i C_{i,1}^n\, a_i^n - \sum_i C_{i,2}^n\, d_i^n\right]$$

$a_i^n \sim \langle\langle \mathcal{O}^n_{\text{TW-2}} \rangle\rangle$ $\qquad$ $d_i^n \sim \langle\langle \mathcal{O}^n_{\text{TW-3}} \rangle\rangle$

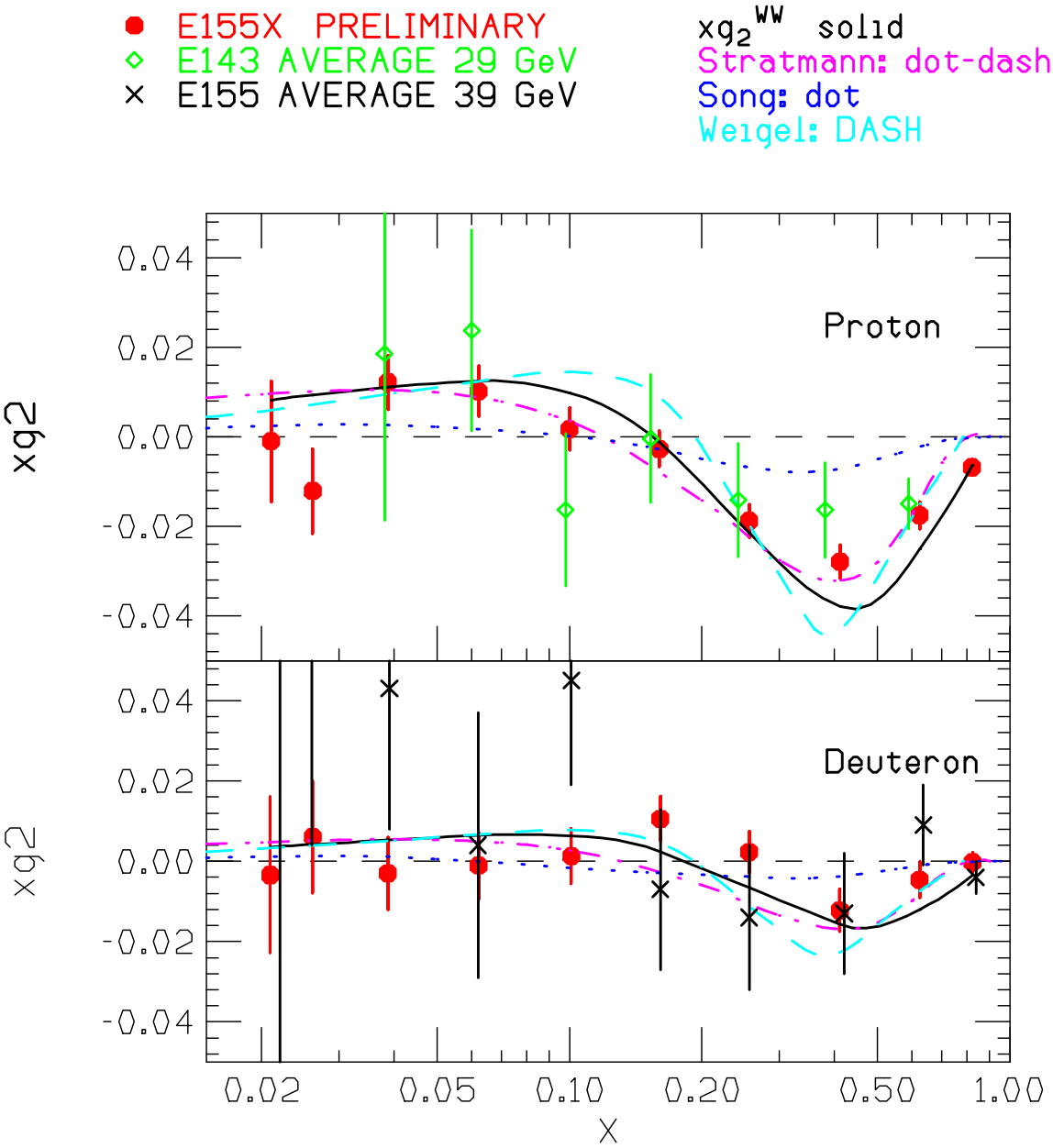




- probes "quark-gluon correlations" (Shuryak,Vainshtein)
- twist 2 & 3 appear at same level !
- assumption $d_i^n \approx 0$ gives (Wandzura; Wilczek)

$$g_2^{\text{WW}}(x,Q^2) \;=\; -g_1(x,Q^2) + \int_x^1 \frac{dy}{y}\, g_1(y,Q^2)$$

# Recent measurement by E155x



# IX. The proton's spin sum rule

- The proton spin :

$$\frac{1}{2} = \langle P, \frac{1}{2} | J_3 | P, \frac{1}{2} \rangle = \langle P, \frac{1}{2} | \int d^3x \, M_{012}(x) | P, \frac{1}{2} \rangle$$

where

$$M^{\mu\nu\lambda}(x) = x^\nu T^{\mu\lambda}(x) - x^\lambda T^{\mu\nu}(x)$$

One finds (Jaffe,Manohar; Ji)

$$M_{012} = \underbrace{\bar{\psi}\gamma^3\gamma_5\psi}_{\sim \text{ quark spin}} + \underbrace{\psi^\dagger \left(\vec{x} \times \left(-i\vec{\mathcal{D}}\right)\right)_3 \psi}_{\sim \text{ quark OAM}} + \underbrace{\left[\vec{x} \times \left(\vec{E}(\vec{x}) \times \vec{B}(\vec{x})\right)\right]_3}_{\sim \textit{total} \text{ gluon ang. mom.}}$$

- Problem :

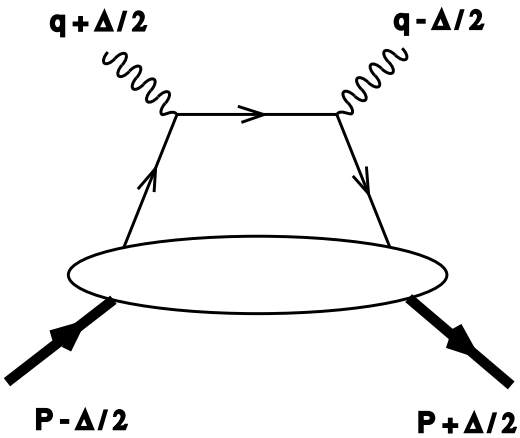
$$\vec{x} \times \left(\vec{E} \times \vec{B}\right) = \underbrace{E_j \left(\vec{x} \times i\vec{\mathcal{D}}\right) A_j}_{\text{gluon OAM ?}} + \underbrace{\vec{E} \times \vec{A}}_{\Delta G \text{ ?}}$$

*not* a gauge invariant separation

- Asset of this definition : terms in

$$\frac{1}{2} = J_q + J_g$$

*in principle* measurable in
"deeply-virtual Compton scattering" (Ji)




lower part :
"off-forward scattering ampl."

form factors $H_q(x,\xi,\Delta^2)$, $E_q(x,\xi,\Delta^2)$
$\Delta^2 = (P_i - P_f)^2\,, \quad \xi = -n\cdot\Delta/2$

$$J_q \;=\; \frac{1}{2}\lim_{\Delta^2\to 0}\int dx\, x\,\left[H_q(x,\xi,\Delta^2) + E_q(x,\xi,\Delta^2)\right]$$

however, *very* challenging experimentally :
$J_{q=u}$ dominates, $J_g$ only from evolution, backgrounds . . .
(DESY, JLAB, . . . )

- evolution of Ji's angular momentum :

$$\mu\frac{d}{d\mu}\begin{pmatrix} J_q(\mu) \\ J_g(\mu)\end{pmatrix} = \frac{\alpha_s(\mu)}{\pi}\frac{1}{9}\begin{pmatrix} -16 & 3n_f \\ 16 & -3n_f\end{pmatrix}\begin{pmatrix} J_q(\mu) \\ J_g(\mu)\end{pmatrix}$$

$$J_q(\infty) \;=\; \frac{1}{2}\,\frac{3n_f}{16+3n_f}\,, \qquad J_g(\infty) \;=\; \frac{1}{2}\,\frac{16}{16+3n_f}$$

- "parton model" analysis in terms of $M_{+12}$
(Jaffe,Manohar; Jaffe,Bashinsky)

$$\left[\vec{E}\times\vec{A}\right]_3 = F^{01}A^2 - F^{02}A^1 \;\longrightarrow\; F^{+1}A^2 - F^{+2}A^1$$

- in light-cone gauge $A^+ = 0$ :

$$\langle P,\frac{1}{2}|\int d^3x\,\left[F^{+1}A^2 - F^{+2}A^1\right]|\,P,\frac{1}{2}\rangle$$

indeed coincides with the operator definition of the quantity $\Delta G$ *measurable* at RHIC

- improvement in gauge invariance (Jaffe,Bashinsky)

# X. Summary & Outlook

- Studies of spin dependence of deeply inelastic high-energy reactions yield fundamental new information on nucleon structure
  * spin-dependent parton distributions

- Tests of QCD

- Considerable experimental and theoretical progress over the past few years
  * huge success of polarized-DIS program

- Spin structure of nucleon still far from fully understood

- Further dedicated experiments are running *or are about to start*
  * HERMES, COMPASS, RHIC, . . .

## If you'd like to see more . . .

- R.L. Jaffe, SPIN, TWIST AND HADRON STRUCTURE IN DEEP INELASTIC PROCESSES, hep-ph/9602236;
  THE THEORY OF THE NUCLEON SPIN, hep-ph/0008038

- A. Manohar, AN INTRODUCTION TO SPIN DEPENDENT DEEP INELASTIC SCATTERING, hep-ph/9204208

- B.W. Filippone, Xiangdong Ji, THE SPIN STRUCTURE OF THE NUCLEON, hep-ph/0101224

- Hai-Yang Cheng, STATUS OF THE PROTON SPIN PROBLEM, Int.J.Mod.Phys.A11 (1996) 5109 (hep-ph/9607254)

- M. Anselmino, A. Efremov, E. Leader, THE THEORY AND PHENOMENOLOGY OF POLARIZED DEEP INELASTIC SCATTERING, Phys.Rept.261 (1995) 1 (hep-ph/9501369)

- B. Lampe, E. Reya, SPIN PHYSICS AND POLARIZED STRUCTURE FUNCTIONS, Phys.Rept.332 (2000) 1 (hep-ph/9810270)

- G. Bunce, N. Saito, J. Soffer, W. Vogelsang, PROSPECTS FOR SPIN PHYSICS AT RHIC, Ann.Rev.Nucl.Part. Sci.50 (2000) 525 (hep-ph/0007218)

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