Tests of QCD

at e^+e^- colliders

Bill GARY

Department of Physics

U. California, Riverside

OPAL & BaBar Collaborations

email: bill.gary@ucr.edu

Outline

Part 1

(I) Introduction

- Elements of QCD
- QCD in e^+e^- annihilations
- Experiments and data samples

(II) Monte Carlo simulations

(III) Jet finding algorithms

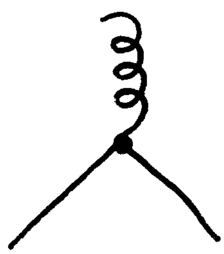
(IV) Tests of Matrix elements

(V) Measurements of α_S

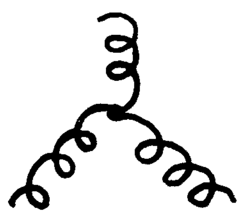
Fundamental Elements of QCD

- Non-Abelian SU(3) gauge theory describing the interactions of Spin $\frac{1}{2}$ quarks and Spin 1 gluons

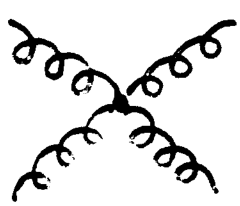
- Fundamental interactions:



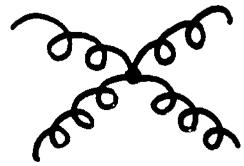
Quark-Gluon vertex



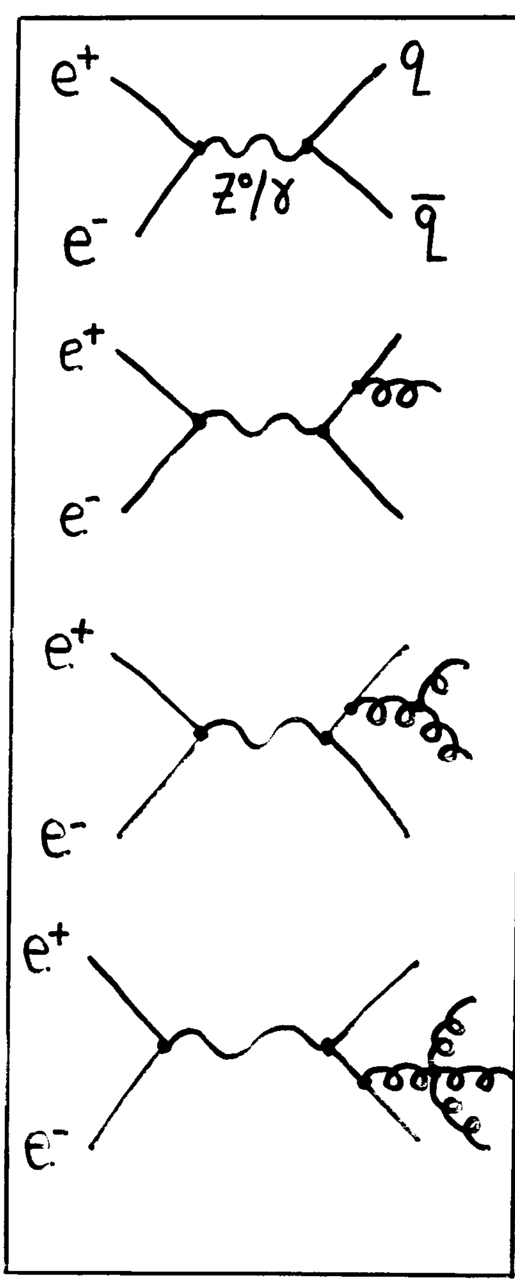
Triple Gluon vertex



Four Gluon vertex

Four gluon vertex  $\sim C_A^2$

$\rightarrow$ enters e^+e^- annihilations at $O(\alpha_s^3)$:



$O(\alpha_s^0)$ No QCD

2-jet tree level

<u>LO</u>: $O(\alpha_s)$

3-jet tree level

1-loop corrections to <u>2-jet σ</u>

<u>NLO</u>: $O(\alpha_s^2)$

4-jet tree level

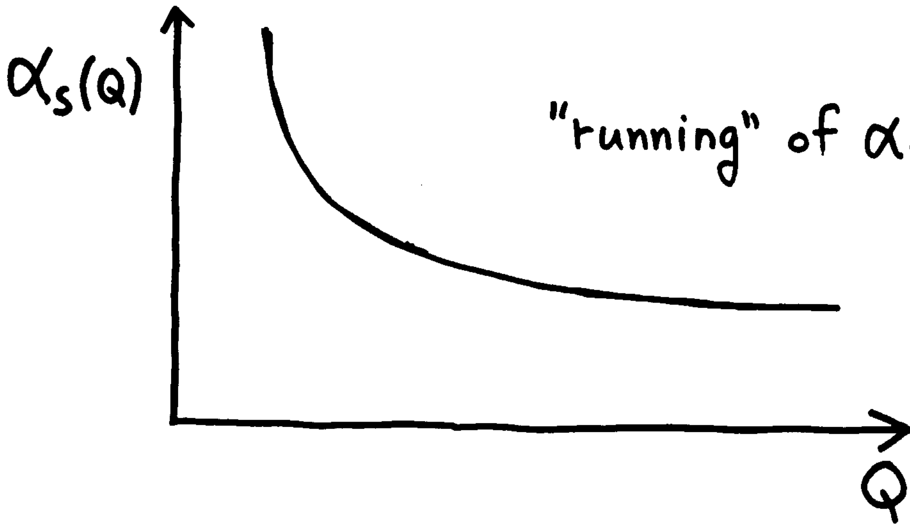
2-loop corrections to <u>2-jet σ</u>

<u>NNLO</u>: $O(\alpha_s^3)$

5-jet tree level

3-loop corrections to <u>2-jet σ</u>

- One free parameter: $\alpha_s(Q)$ or $\Lambda_{\overline{MS}}$



- α_s decreases with increasing energy scale Q:

$$\frac{\partial \alpha_s}{\partial Q} = -\left(\frac{\beta_0}{2\pi}\right)\frac{\alpha_s^2}{Q} - \left(\frac{\beta_1}{4\pi^2}\right)\frac{\alpha_s^3}{Q} - \left(\frac{\beta_2}{64\pi^3}\right)\frac{\alpha_s^4}{Q} - \dots$$

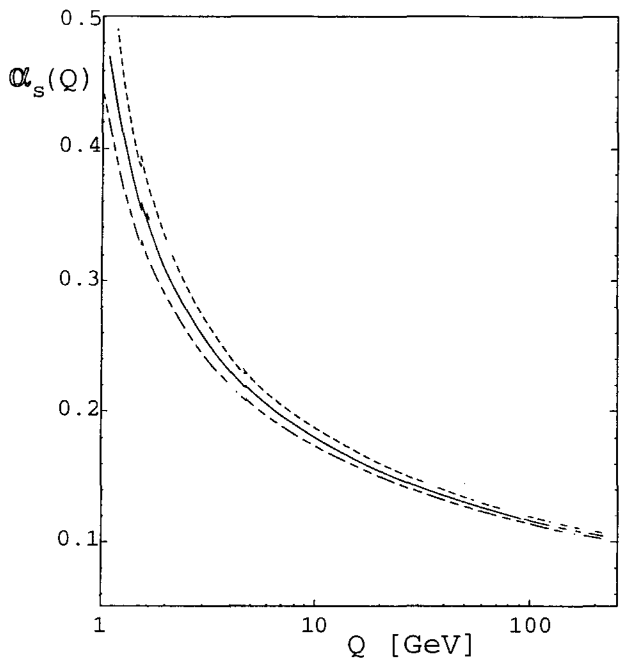
$\beta_0 = 11 - \frac{2}{3} n_f$; $\beta_1 = 51 - \frac{19}{3} n_f$; ...

n_f = effective nr. of active quark flavors

- use to compare the values of α_s measured at different scales Q

- <u>Standard scale</u>: $Q = M_Z$

The "shrinking error"



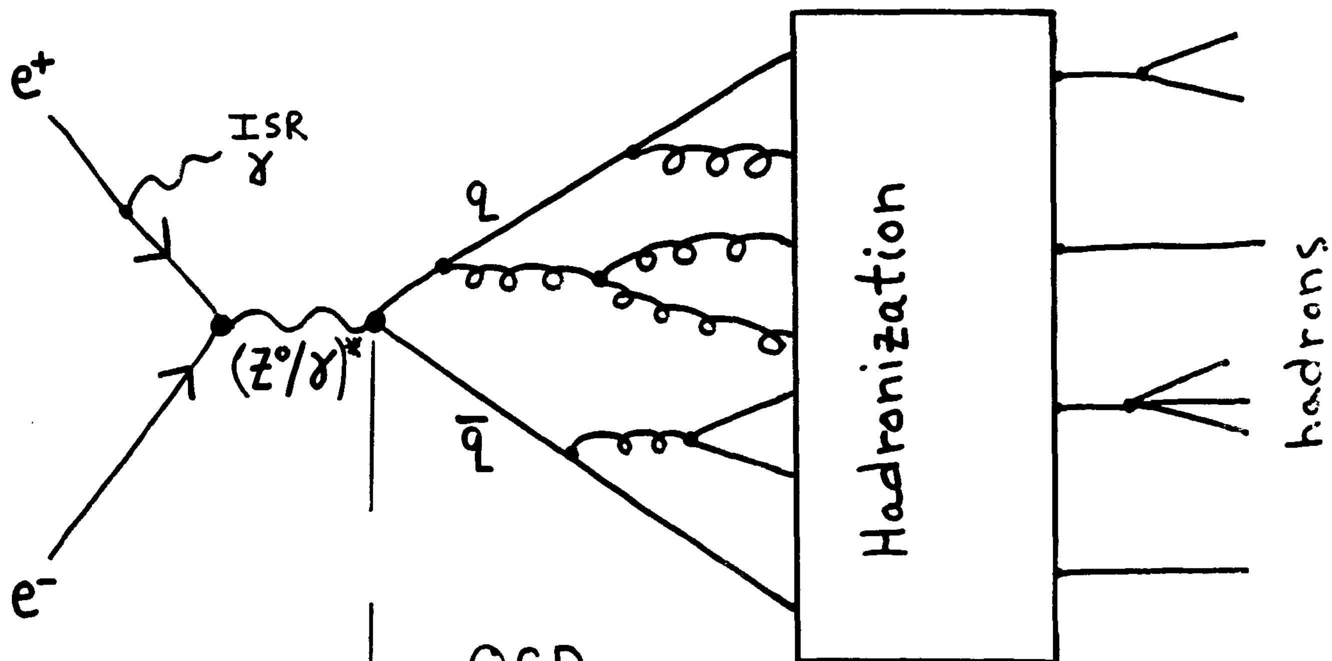
Evolve α_S from the physical scale Q to the standard scale M_Z

$$\frac{\delta\alpha_S(M_Z)}{\alpha_S(M_Z)} = \frac{\delta\alpha_S(Q)}{\alpha_S(Q)} \cdot \left(\frac{\alpha_S(M_Z)}{\alpha_S(Q)}\right)$$

The relative uncertainty of α_S decreases as the measurement is evolved from a low scale Q to M_Z

QCD in e^+e^- annihilations

QCD event: $e^+e^- \longrightarrow (Z^0/\gamma)^* \rightarrow$ hadrons



QCD

Electroweak "hard interaction"	Perturbative Phase $\alpha_s < 1$	Non-Perturbative Phase $\alpha_s \gtrsim 1$	Resonance Decay

Perturbative Phase $\alpha_s < 1$

Fixed order

- α_s^2 (two loop: NNLO)
- α_s^3 (some quantities)

or "parton shower"

- NLLA

Next-to-Leading Logarithm Summations to all orders

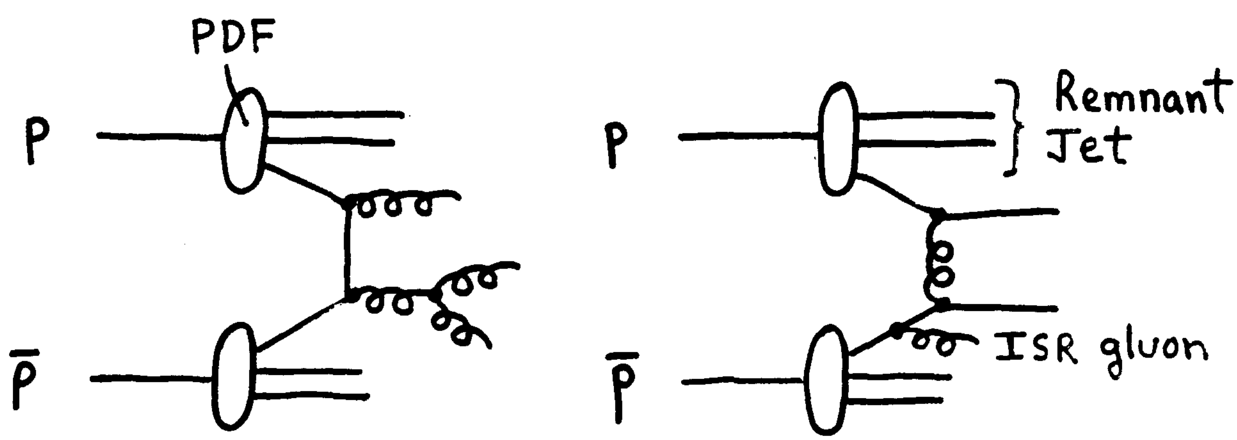
Non-Perturbative Phase $\alpha_s \gtrsim 1$

Resonance Decay

QCD at hadron colliders

→ Ultra-high energy jets { no "hadronization correction"

→ complementarity to e^+e^- colliders



with the complications of

- Parton Distribution Functions (PDFs)
- Strong interactions in initial state (ISR)
- Remnant Jets (underlying event)

and with perturbative expressions valid to α_s^3 (one loop : NLO) ONLY at fixed order (plus NLLA summations as in e^+e^-)

Principal e^+e^- accelerators which have contributed to QCD

SPEAR	(1972-1990)	8 GeV
CESR	(1979-present)	$\sim$ 10 GeV
PETRA	(1978-1986)	14-44 GeV
PEP	(1980-1990)	29 GeV
TRISTAN	(1987-1995)	52-64 GeV
SLC	(1989-2000)	91 GeV
	– SLD experiment	
LEP-1	(1989-1995)	91 GeV
	– ALEPH, DELPHI, L3 & OPAL experiments	
LEP-2	(1996-2000)	130-209 GeV
	– ALEPH, DELPHI, L3 & OPAL experiments	

LEP/SLC

Large collision energies compared to previous e^+e^- colliders

→ Perturbative calculations are more reliable (since α_S is smaller)

→ Hadronization uncertainties are less important (hadronization terms scale like $1/\sqrt{s}$)

→ The perturbative structure of the jets is more developed: closer to the "asymptotic regime"

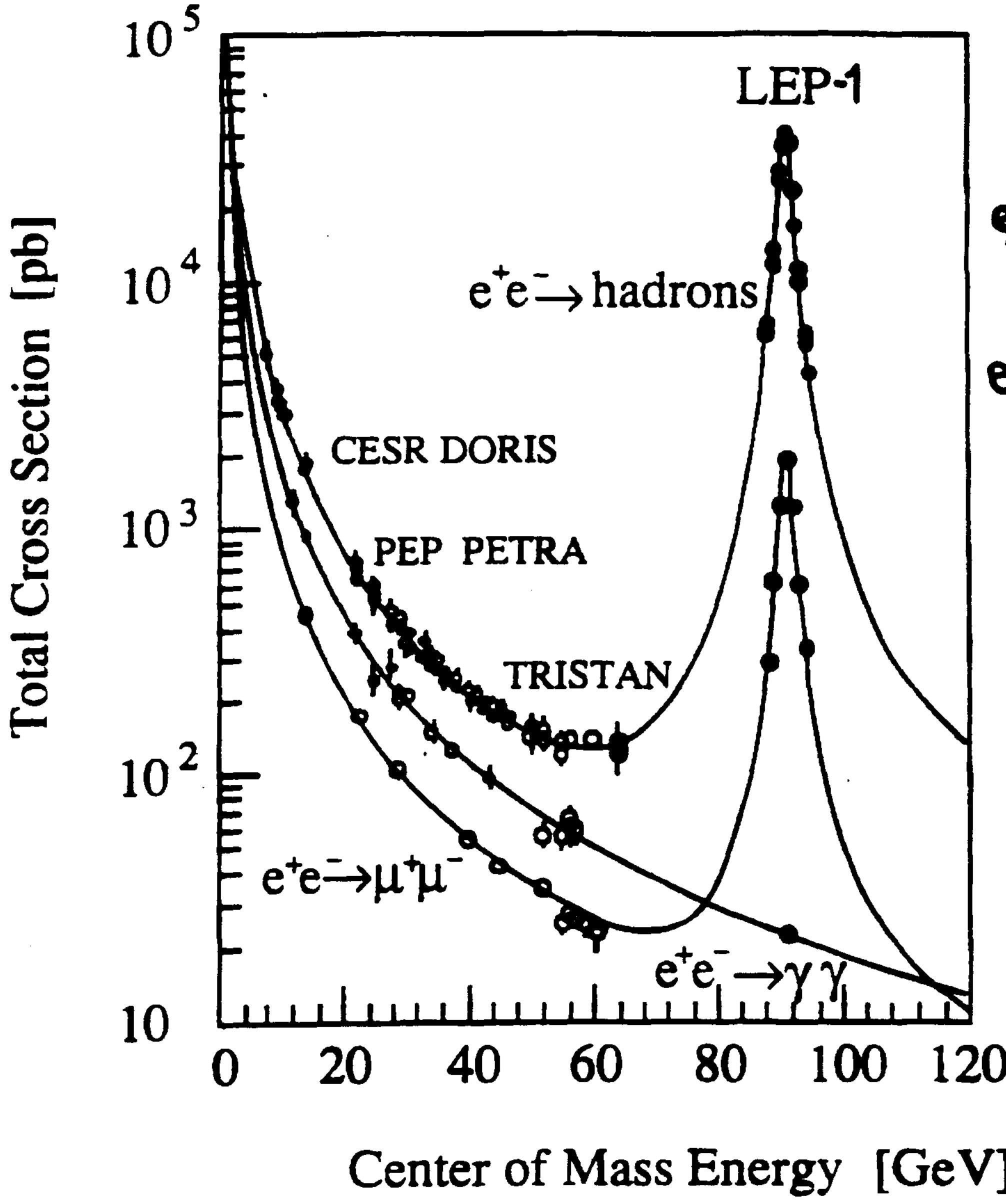
$$\Lambda_{\overline{MS}} \sim 200\ MeV << E_{particle} << \sqrt{s}$$

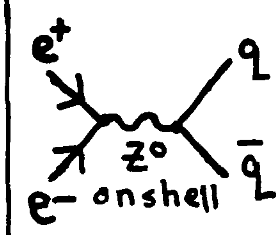
- Particle (parton) evolution is more likely to be governed by QCD dynamics than by kinematic constraints, compared to lower energy data
- Closer to the realm of validity of analytic calculations for multiparticle production, allowing quantitative tests of these QCD predictions

→ Almost all results shown here will be from LEP/SLC

QCD at LEP-1/SLC

$E_{c.m.} \approx m_Z = 91.2$ GeV





→ the largest e^+e^- data samples

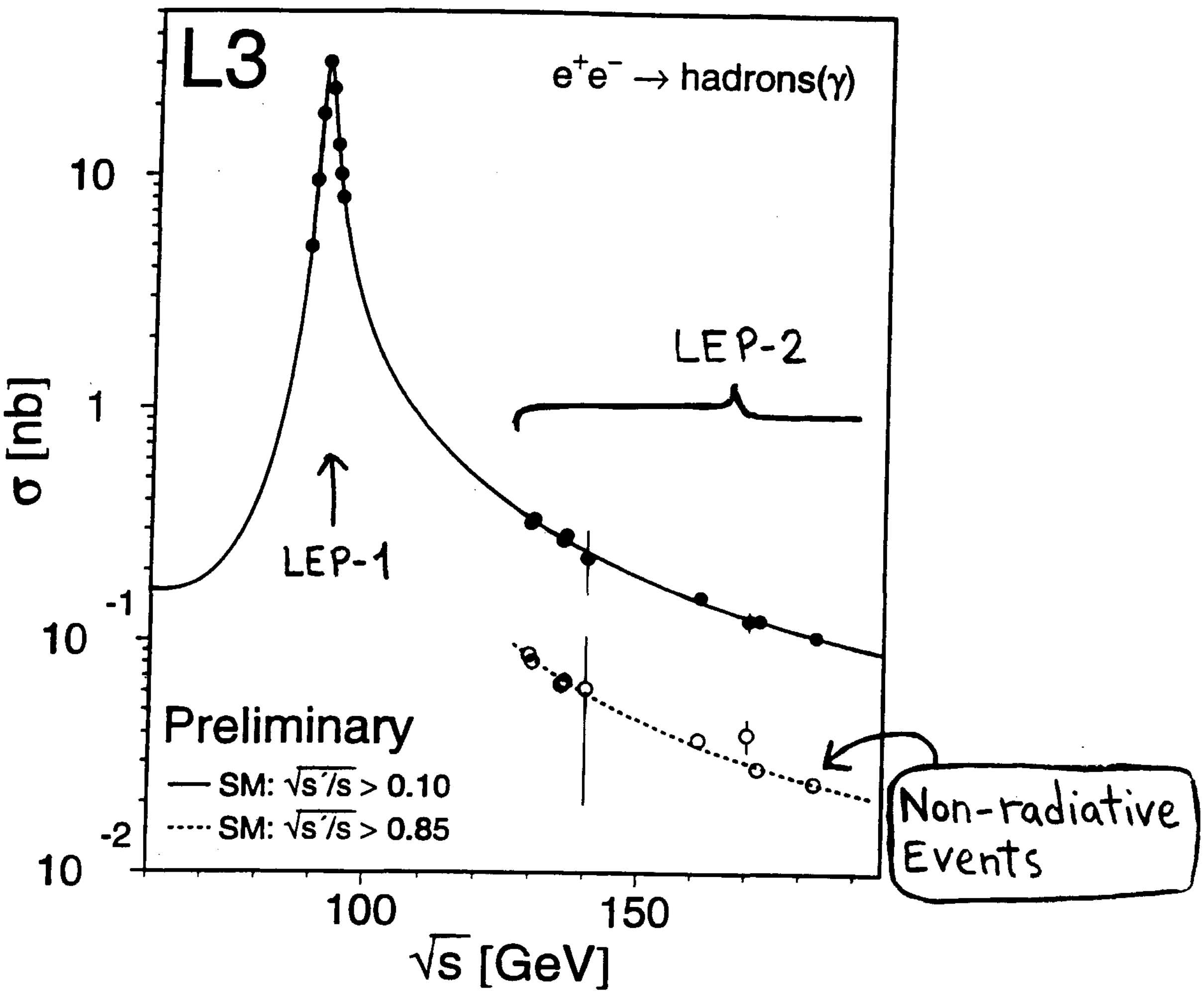
above the Υ region

LEP-1 event samples : $\underline{\sim 4 \times 10^6 \text{ events}}$ per experiment

Essentially no background and 100% efficiency!

QCD at LEP-2

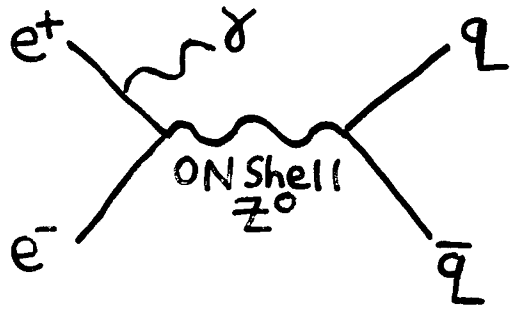
$130 \lesssim E_{c.m.} \lesssim 209$ GeV

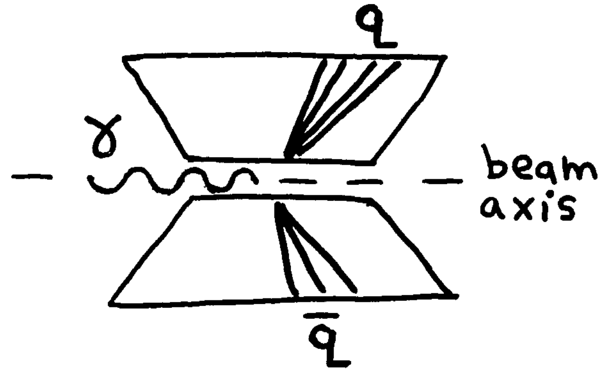


→ the highest e^+e^- energies

LEP-2: 1995-2000

- ~80% of QCD events are radiative returns to the Z^0





→ Reject by requiring $\sqrt{s}_{hadronic} \approx 2\,E_{beam}$

- "4-fermion events" $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$
  $\to Z^0 Z^0 \to q\bar{q}q\bar{q}$

  form a serious background to $e^+e^- \to Z^0/\gamma^* \to q\bar{q}$

→ Reject by cuts on matrix element probabilities (jet energies and angular distributions differ for 4-fermion and QCD processes) or an analogous technique

- ~10^4 QCD non-radiative events
  efficiency ~ 75% background ~ 5%

QCD Monte Carlo simulation programs

Essential

- ⟶ for the evaluation of detector response (also requires computer simulation of the detector)
- ⟶ to estimate hadronization effects (corrections)
- ⟶ to evaluate biases, e.g. introduced by jet-finders
- ⟶ to demonstrate sensitivity to a certain physics effect

The principal QCD programs

- Jetset/Pythia
- Herwig
- Ariadne
- Cojets

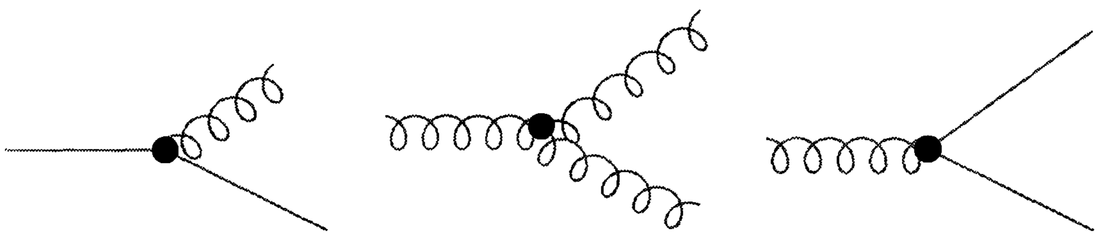
have been tuned to LEP-1 data using

- ⟶ the mean charged particle multiplicity, $\langle n_{ch} \rangle$
- ⟶ "event shape" variables, e.g. Thrust (the momentum structure of an event)
- ⟶ Inclusive identified particle production rates and distributions

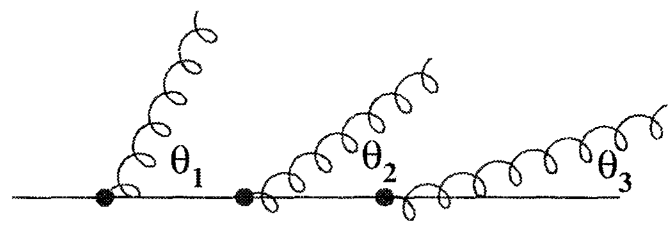
Jetset/Pythia

[T. Sjöstrand, Comp. Phys. Comm. 82 (1994) 74]

- LO parton shower based on Altarelli-Parisi splitting functions



with angular ordering:

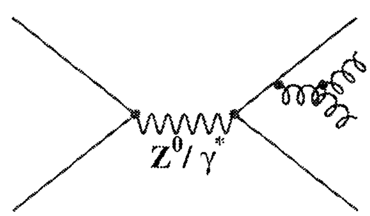


and azimuthal asymmetry in the soft gluon radiation pattern to simulate COHERENCE $\longrightarrow$ QCD interference effects

ALSO: matching of the first gluon branching to the e^+e^- 3-jet matrix element

OR

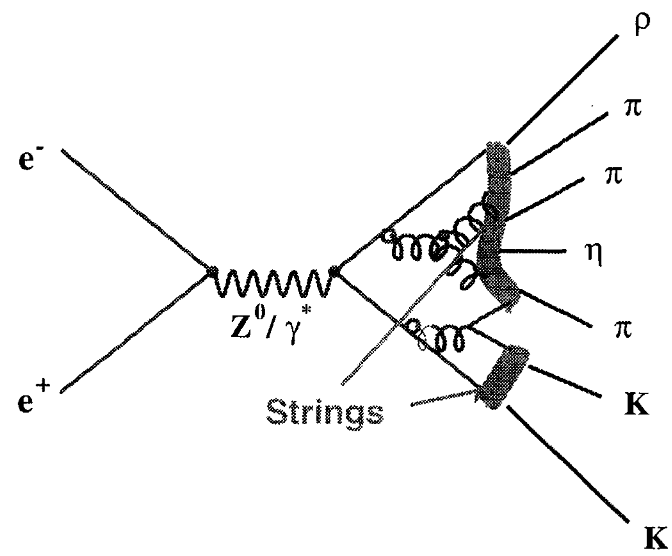
- fixed order $\mathcal{O}(\alpha_S^2)$ matrix elements



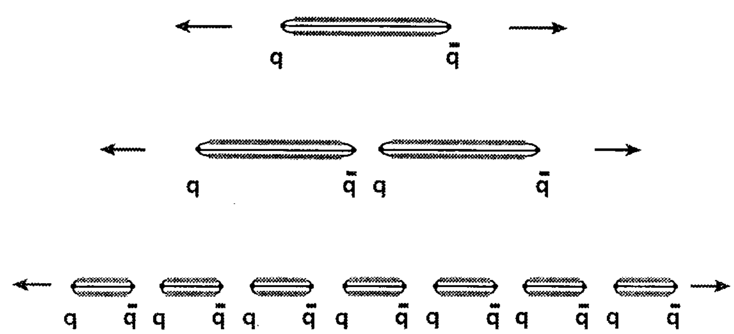
$\longrightarrow$ up to four partons in the final state: $q\bar{q}gg$, $q\bar{q}q\bar{q}$

Hadronization ⟶ the Lund String model

[Bo Andersson et al., Phys.Rep. 97 (1983) 31]



Each string segment hadronizes in its rest frame according to a longitudinal phase space model



<u>Principal parameters tuned to data</u>

Λ, Q_0: Perturbative phase

a, b: Longitudinal momentum distribution

σ_q: Transverse momentum distribution

⟶ Very successful description of data

⟶ The most widely used Monte Carlo for QCD studies at e^+e^- colliders

Herwig

[B.R.Webber, G. Marchesini et al., JHEP 0101 (2001) 010]

- Parton shower based on AP splitting functions:

$$q \rightarrow qg \qquad g \rightarrow gg \qquad g \rightarrow q\bar{q}$$

with NLO accuracy at high x

$\longrightarrow$ Herwig at the parton level is $\sim$ equivalent to an NLO calculation with exact $E - \vec{p}$ conservation

- Like Jetset/Pythia, the Herwig parton shower includes

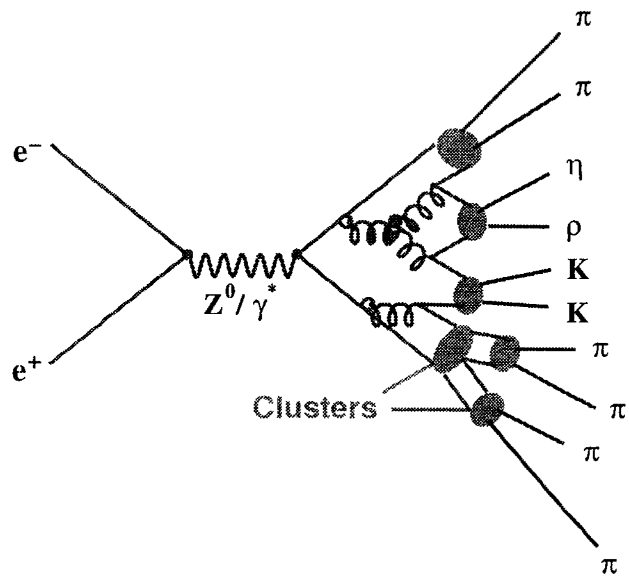
$\longrightarrow$ Angular ordering and azimuthal asymmetries to simulate the effects of coherence

$\longrightarrow$ Matching of the first gluon branching to the 3-jet matrix element

- As an alternative to the parton shower, Herwig also includes the fixed order $\mathcal{O}(\alpha_S^2)$ matrix elements

Hadronization $\longrightarrow$ the Cluster model

[G.C. Fox and S. Wolfram, Nucl. Phys. B168 (1980) 285]



- Gluons from the parton shower are forcibly split to $q\overline{q}$ pairs
- Color singlet clusters are formed from neighboring q and $\overline{q}$
- Clusters with a mass above a threshold $\sim$3 GeV evolve through string model-like splitting before decaying
- The low mass clusters decay into hadrons according to 2-body phase space

Principal parameters tuned to data

Λ, M_g:	Perturbative phase
CLMAX, CLPOW	Cluster mass threshold
PSPLT:	Cluster mass spectrum from string decay
CLSMR:	Angular correlation between $q\overline{q}$ and a hadron

$\longrightarrow$ Overall description of data very good, but generally worse than Jetset except for gluon jets

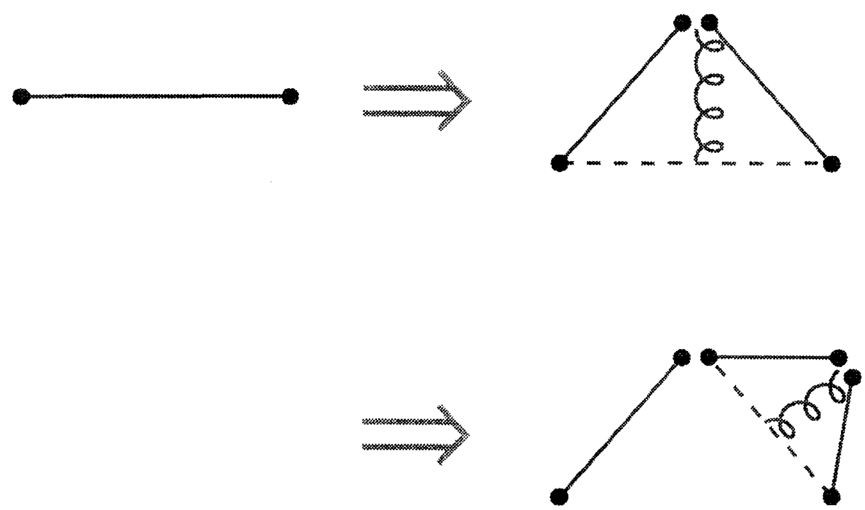
$\longrightarrow$ The hadronization model provides an important alternative to the Lund model

Ariadne

[L. Lönnblad, Comp. Phys. Comm. 71 (1992) 15]

- LO Parton shower based on a dipole cascade:

  [G. Gustafson, Phys. Lett. B75 (1986) 453]



$\longrightarrow$ An alternative to Altarelli-Parisi-type splittings

- The soft gluon radiation is inherently coherent (the gluon radiation is not from independently evolving partons)
- Hadronization $\longrightarrow$ The Lund string model
- Principal parameters: same as Jetset except $Q_0 \rightarrow p_\perp$ to terminate the parton shower
- Description of data similar to Jetset, sometimes better

Cojets

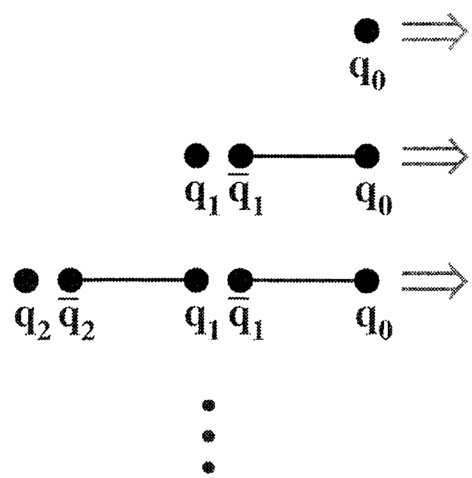
[R. Odorico, Comp. Phys. Comm. 72 (1992) 238]

- LO Altarelli-Parisi-type Parton shower without simulation of coherence effects: (no angular ordering)
- Hadronization ⟶ the Independent Fragmentation model

[R.D. Field & R.P. Feynman, Nucl. Phys. B136 (1978) 1]

⟶ Split gluons into $q\overline{q}$ pairs

⟶ Each q and $\overline{q}$ hadronizes independently according to a longitudinal phase space model



- Describes the main features of LEP-1 data . . . but

⟶ it provides a poor description of the $\sqrt{s}$ evolution of basic measurements: $n_{ch.}$, Thrust, etc.

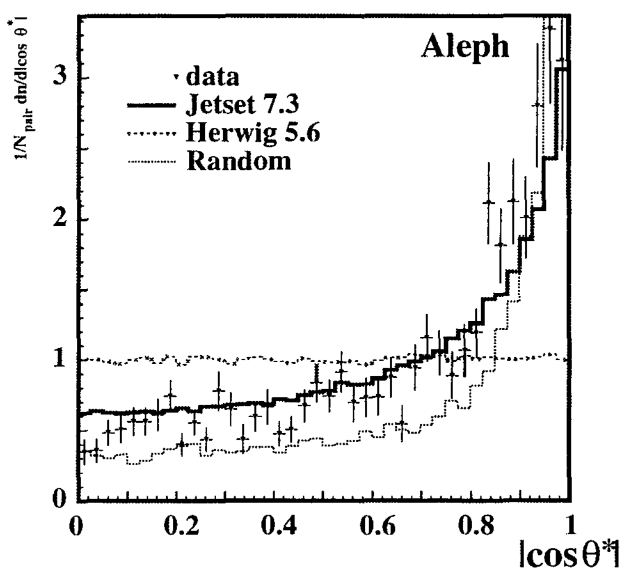
⟶ it fails to describe experimental distributions sensitive to color flow ⟶ The String Effect (see lecture 2)

- Useful as a "toy model" to establish the sensitivity of a measurement to coherence effects and/or color flow

Experimental discrimination between the string & and cluster models

⟶ $\cos\theta^*$ distribution of identified proton-antiproton ($p\bar{p}$) pairs

$\cos\theta^*$ = angle between the proton and event axis (sphericity axis) in the $p\bar{p}$ rest frame



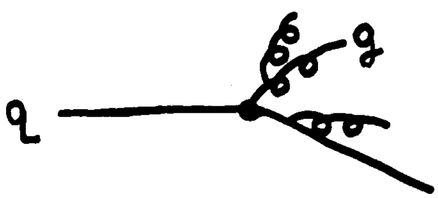
⟶ "Naive" isotropic cluster decay is disfavored

⟶ Tune-able angular correlations implemented in later versions of Herwig partially correct this problem: CLSMR

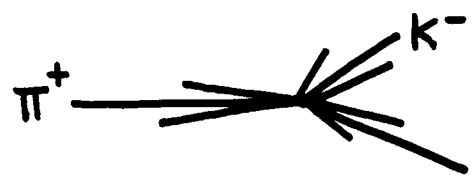
⟶ An example of how the e^+e^- data has been used to test and improve the hadronization models

Jet Definition

Many tests of QCD are based on the grouping of particles into **JETS**



Theory → partons



Experiment → hadrons

Group particles close together in phase space into a single jet

Want a good correspondence between the parton and hadron levels

The results of the jet algorithm should correspond to intuition
→ all particles in a jet move in about the same direction

Good theoretical properties:

- infrared and colinear "safe" (insensitive to soft and colinear radiation)

- small hadronization corrections

- allows "resummation:" the summation of leading and next-to-leading terms in $\ln(y)$; y = resolution parameter to all orders (NLLA calculation)

  → improves reliability in the two-jet region

Jet finders

① Standard recombination algorithms:

$$y_{ij} = M_{ij}^2 / E_{vis}^2$$

→ combine the particle pair (i, j) with smallest y_{ij}

$$(i, j) \rightarrow k$$

E scheme: $p_k = p_i + p_j$ → massive jets
(Lorentz invariant)

EΦ scheme: $E_k = E_i + E_j$ → massless jets

$$\vec{p}_k = E_k \frac{\vec{p}_i + \vec{p}_j}{|\vec{p}_i + \vec{p}_j|}$$

→ iterate until all particle pairs satisfy

$$y_{ij} > y_{CUT}$$

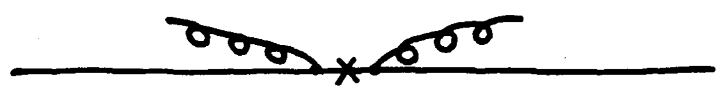
y_{cut} → the (single) resolution parameter

JADE jet finder

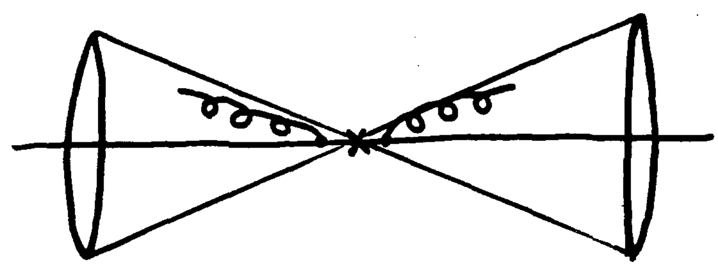
$$M_{ij}^2 = 2E_iE_j(1-\cos\theta_{ij}) \approx (\text{invariant mass})^2$$

$\longrightarrow$ does not allow resummation

i.e. consider two soft, colinear gluons 1 and 2:



the event should "resum" in the 2-jet class



The invariant mass of the two gluons can be less than that of any quark-gluon pair

3-jet class

Unnatural factorization of n-jet phase space spoils resummation

[Brown, Stirling, PLB 252 (1990) 657]

$k_\perp$ or "Durham" jet finder

$$M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$$

~ same experimental characteristics as JADE algorithm

Small θ_{ij} →

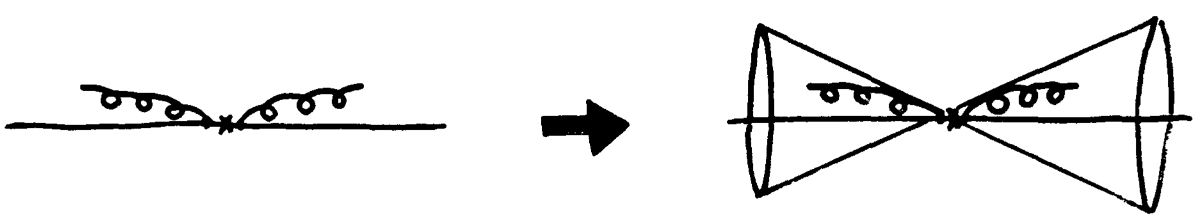
$$M_{ij}^2 \approx 2\min(E_i^2, E_j^2)\left[1-\left(1-\frac{\theta_{ij}^2}{2}+\cdots\right)\right]$$

$$\approx \min(E_i^2, E_j^2)\,\theta_{ij}^2 \approx \min(E_i^2, E_j^2)\sin^2\theta_{ij}$$

$\approx$ ("$k_\perp$")^2 minimum relative transverse energy

soft colinear radiation →

→ attached to the quark jet

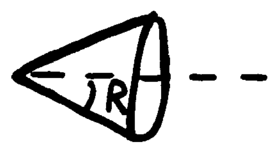


→ allows resummation

③ Cone algorithm:

(standard jet finder used at hadron colliders)

→ Cluster particles which lie within a cone of half angle R into a jet



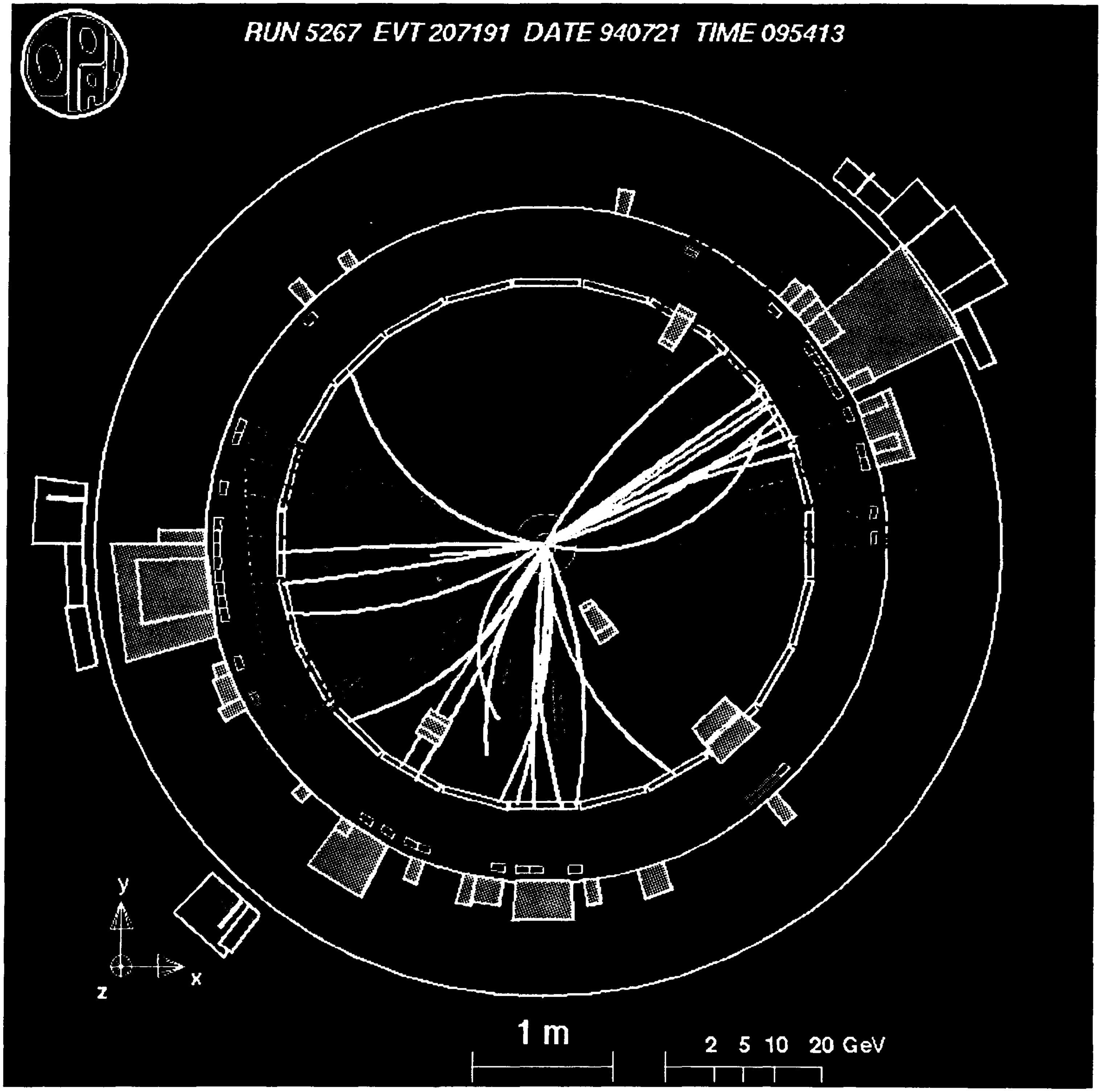
→ Require minimum jet energy

$$E^{jet} > \underline{\varepsilon}$$

→ Eliminate/merge overlapping jets

($\underline{R, \varepsilon}$ → two resolution parameters)

- a more intuitive representation of a jet than that given by recombination jet finders
- not all particles are necessarily assigned to a jet (unlike the case with recombination algorithms)


RUN 5267 EVT 207191 DATE 940721 TIME 095413
y
z
x
1 m
2 5 10 20 GeV

Test of "matrix elements"

→ energy and angular distributions of jets

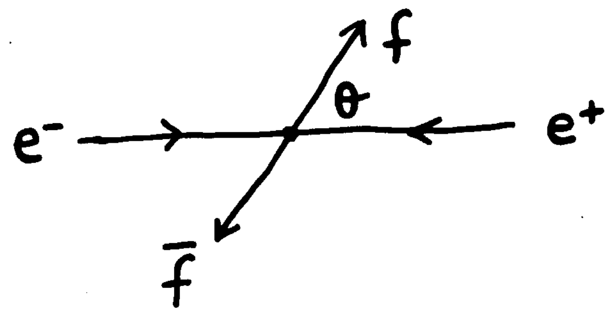
Sensitive to:

- Spin of the partons
- group structure of the gauge theory

Two jet "matrix element"

(spin of the quark)

$$e^+e^- \rightarrow Z^0/\gamma \rightarrow f\bar{f}$$



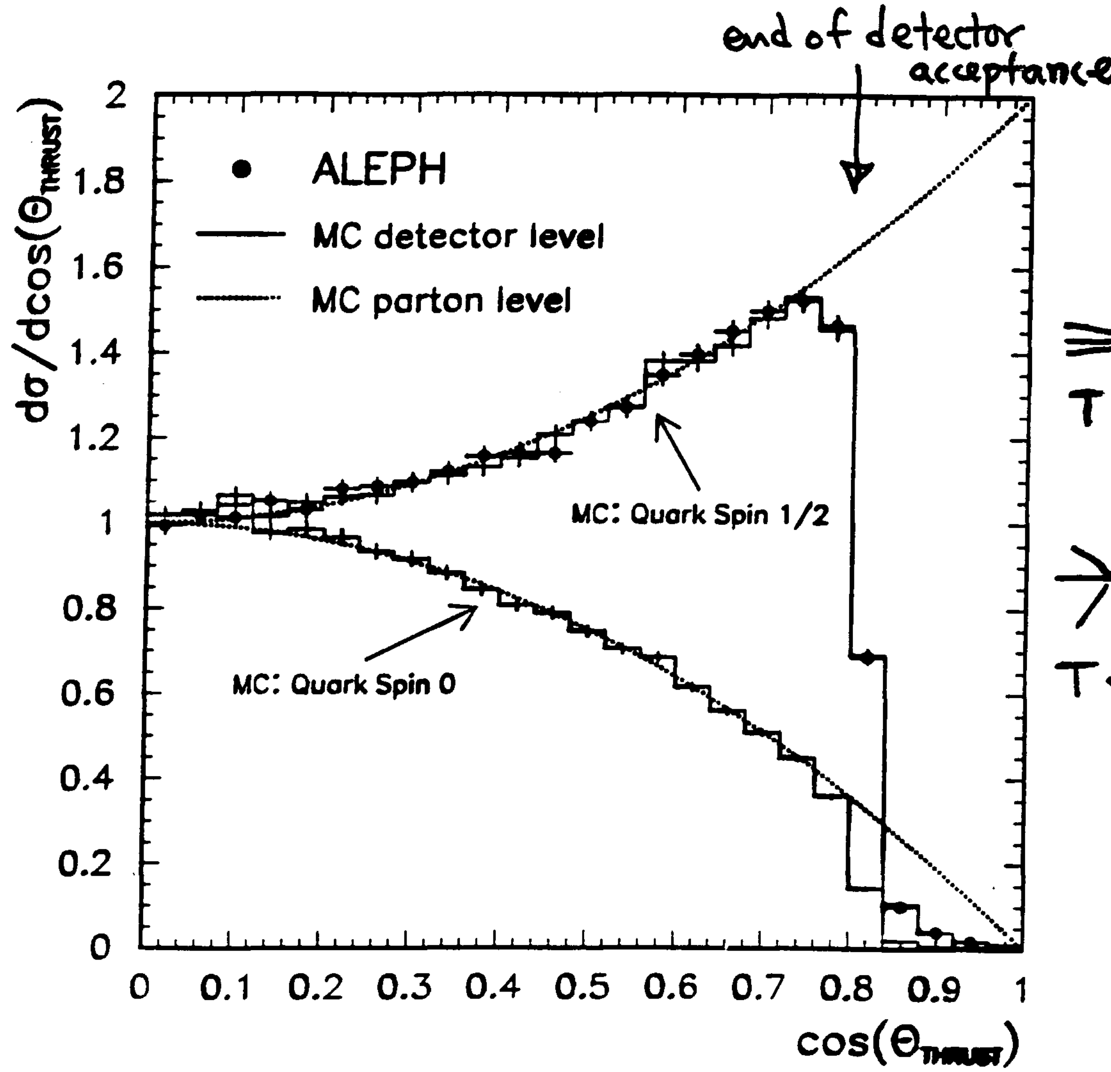
$$\frac{d\sigma}{d\cos\theta} \propto 1 + \cos^2\theta + \frac{8}{3}A_{FB}\cos\theta$$

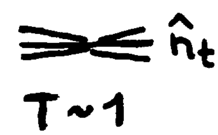
for spin 1/2, massless fermions

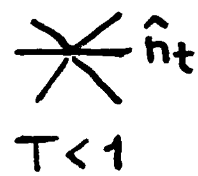
Forward-backward asymmetry cancels between f and $\bar{f}$ (i.e. if charge information is not used)

$$\frac{d\sigma}{d\cos\theta} \propto 1 + \cos^2\theta$$

Angle θ_{thrust} between the Thrust and beam axes







Thrust axis: the direction $\hat{n}_t$ which minimizes $P_\perp$ of the particles (maximizes $P_\parallel$):

$$T = \max\left(\frac{\sum \vec{P}_i \cdot \hat{n}_t}{\sum |\vec{P}_i|}\right)$$

= Thrust value

$\vec{P}_i$ = particle momentum

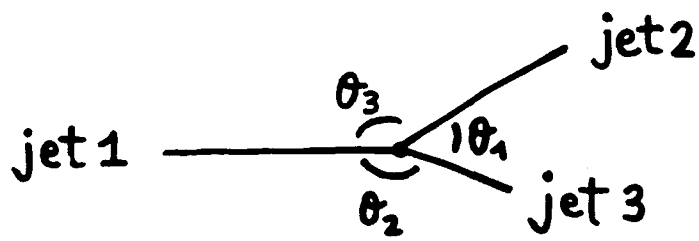
Test of the 3-jet matrix element

(spin of the gluon)

- Select 3-jet events ⟶

  L3: JADE jet finder, $y_{cut} = 0.02$

  ⟶ 25% of events classified as having 3-jet structure



- Calculated jet energies ⟶

$$E_i = E_{c.m.} \frac{\sin\theta_i}{\sum_{i=1,3} \sin\theta_i} \quad \begin{pmatrix}\text{massless} \\ \text{jets}\end{pmatrix}$$

$$E_1 > E_2 > E_3$$

⟶ jet 3 is the gluon jet in ~75% of the events (energy tagging of g jets)

- Scaled energies: $X_i = \frac{2E_i}{E_{c.m.}}$; $X_1 + X_2 + X_3 = 2$

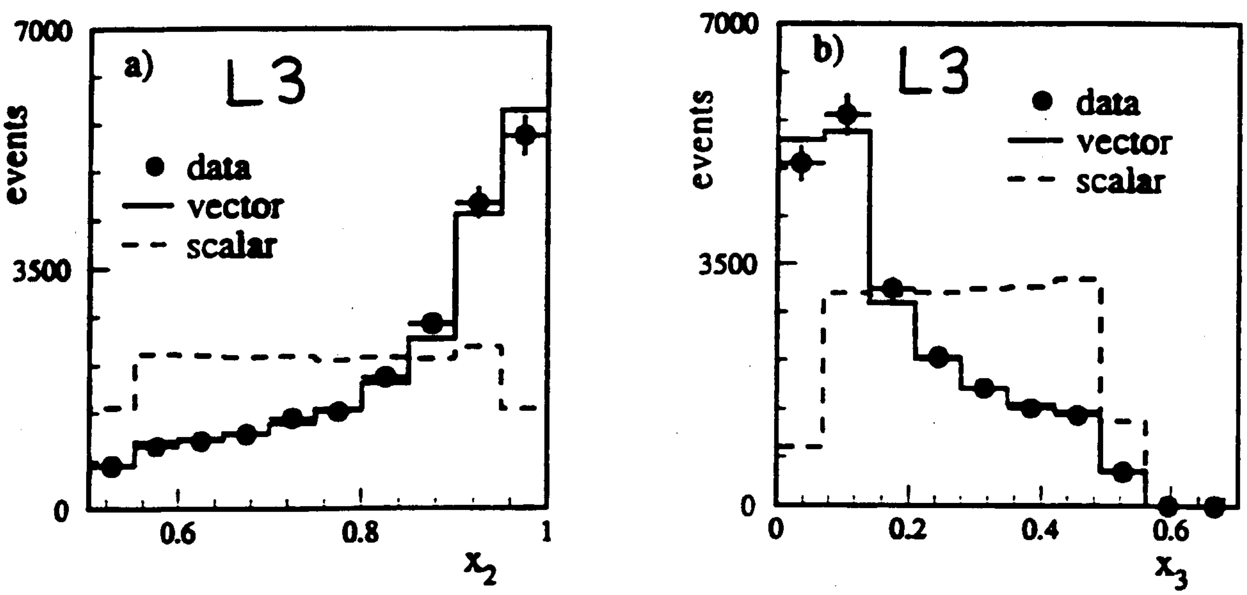
$O(\alpha_s)$ differential cross section:

$e^+e^- \to 3\,\text{jets}$: $$\frac{d^2\sigma}{dx_1 dx_2} \propto \frac{x_1^2 + x_2^2 + x_3^2}{(1-x_1)(1-x_2)(1-x_3)}$$

(2nd order corrections and hadronization don't make much difference)

$\longrightarrow$ poles for soft gluon radiation

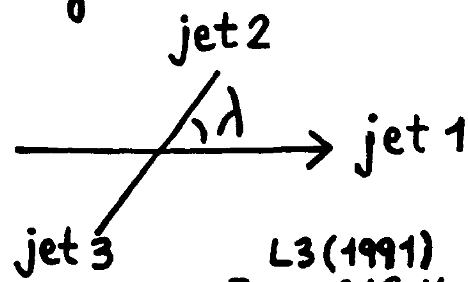
$x_3 \to 0 \qquad x_1, x_2 \to 1$



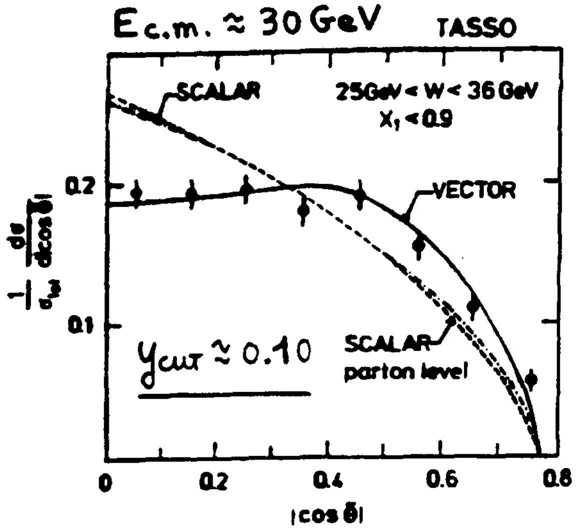
Comparison to $O(\alpha_s^2)$ (ERT) QCD matrix element and to $O(\alpha_s)$ scalar gluon matrix element, including hadronization

Ellis-Karliner angle

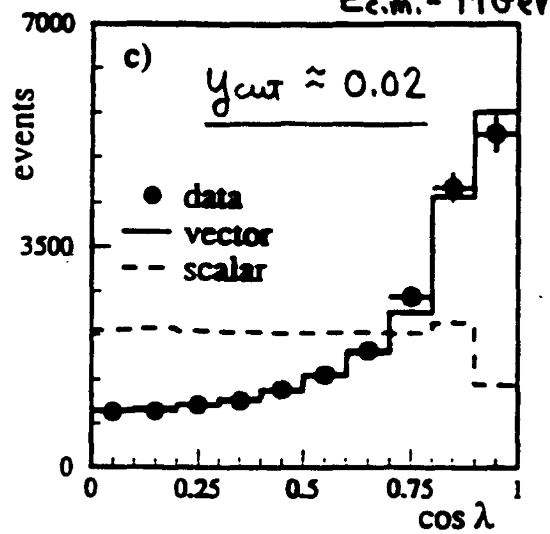
$$\cos\lambda \equiv \frac{X_2 - X_3}{X_1}$$



TASSO (1980)
$E_{c.m.} \approx 30$ GeV



L3 (1991)
$E_{c.m.} = 91$ GeV



The larger jet energies at LEP/SLC permit the use of smaller y_{cut} values

$$E_{jet} \simeq \sqrt{y_{cut}}\, E_{c.m.}$$

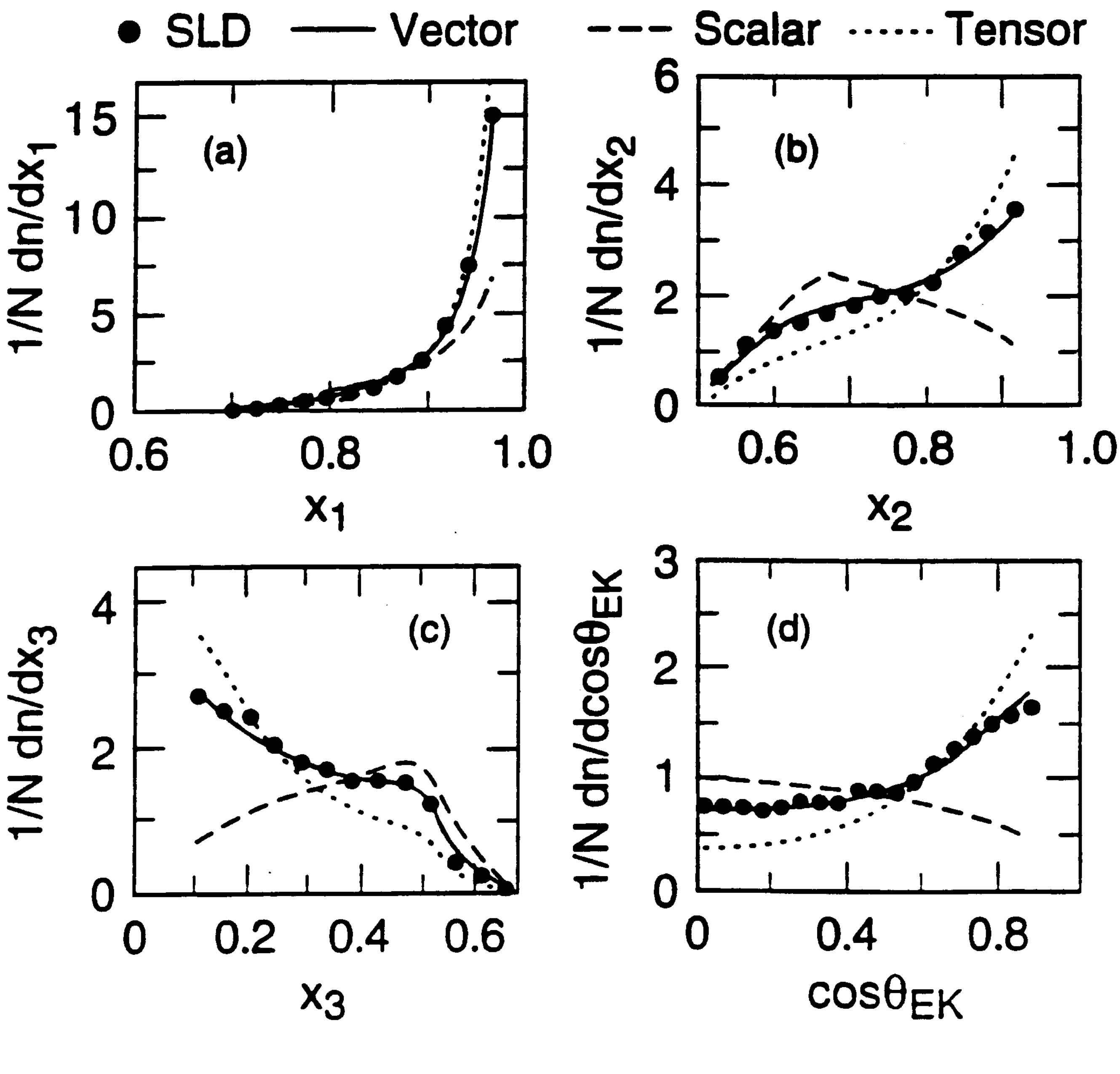
Factor of 3 in $E_{c.m.}$
→ factor of 9 in y_{cut}

→ the pole structure of the matrix element becomes visible

$X_3 \approx 0$, $X_2 \approx 1$ → the region most sensitive to the gluon spin

SLD (1996)

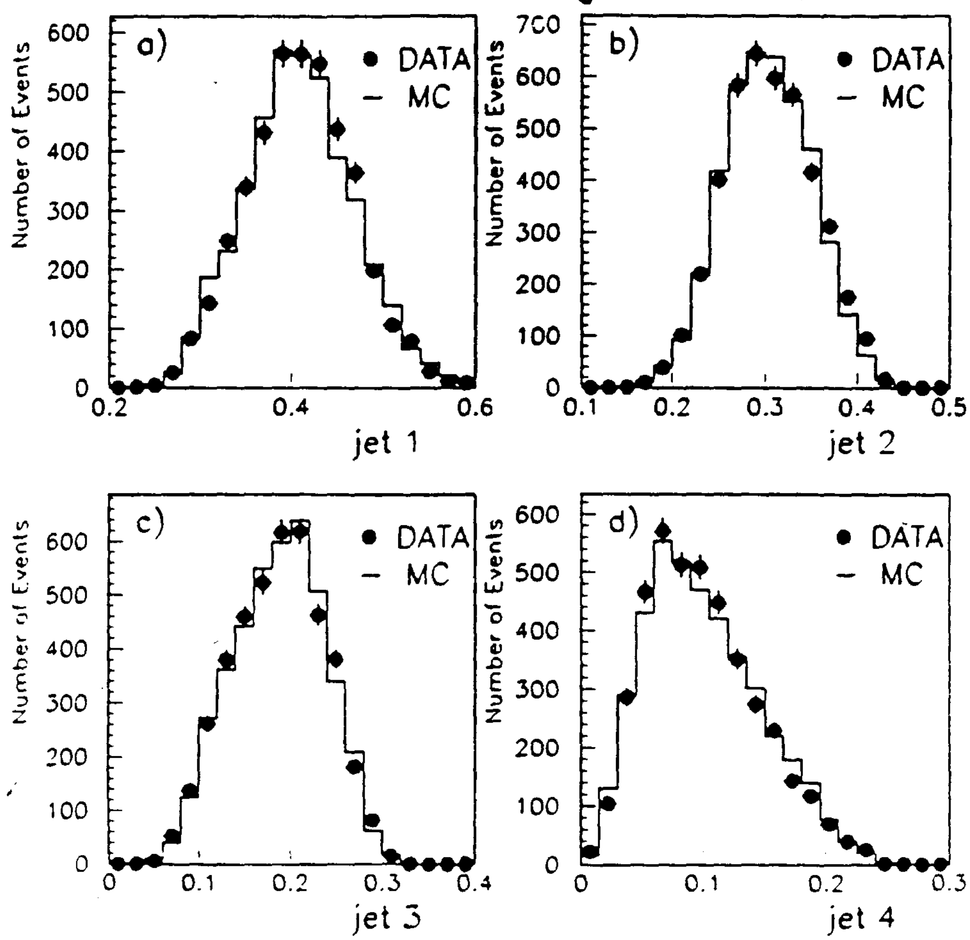
→ Update and include <u>tensor gluon</u> predictions



Test of the 4-jet matrix element

(ALEPH/DELPHI/L3/OPAL)

L3 (JADE-E ; $y_{cut} = 0.02$)



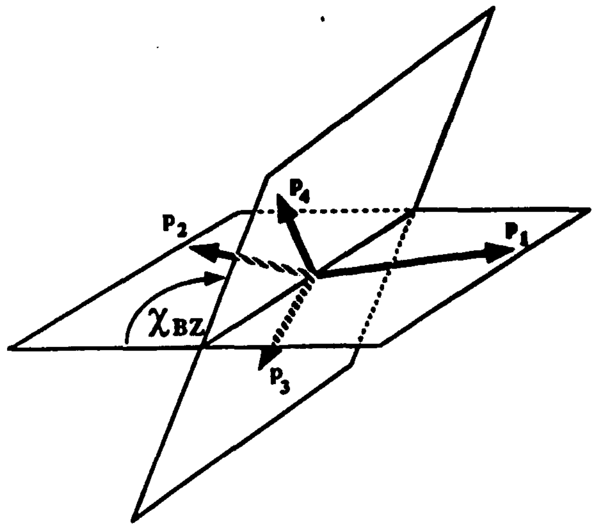
$E_{jet} / E_{c.m.}$

~9% of events classified as 4-jet events

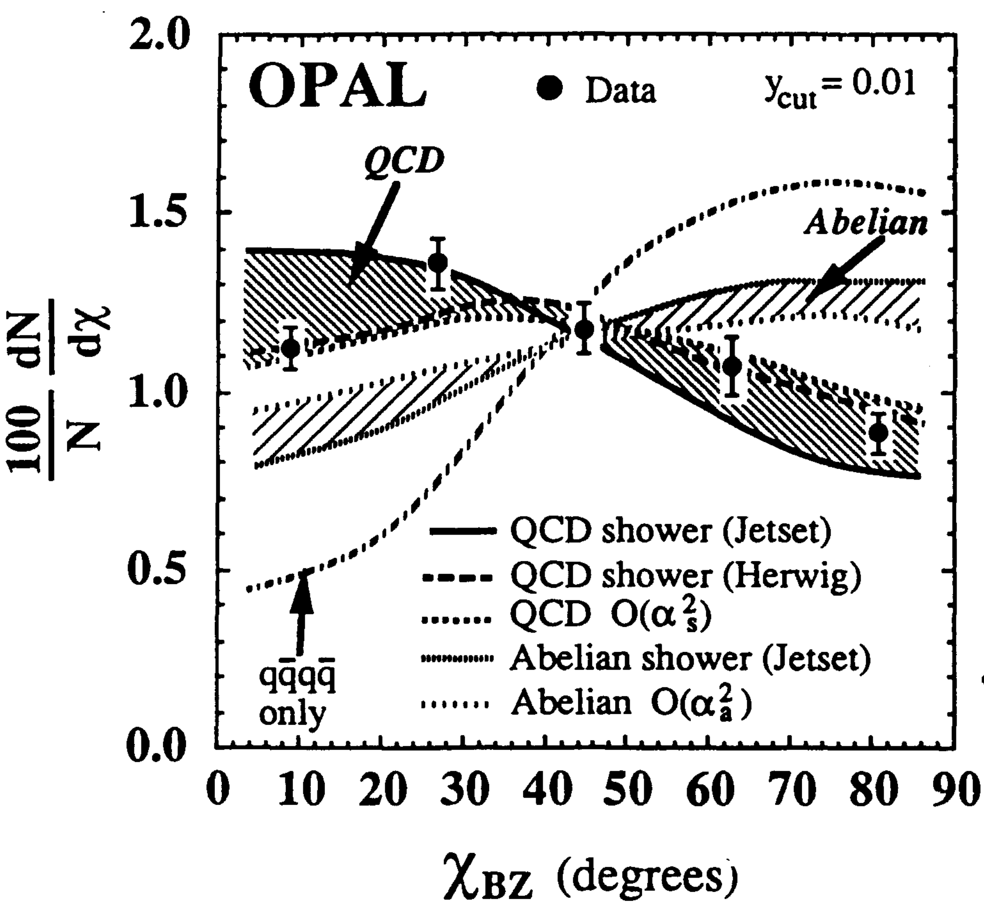
(comparison with Jetset parton shower MC)

4-jet angular correlations

→ Bengtsson-Zerwas angle



$E_1 > E_2 > E_3 > E_4$



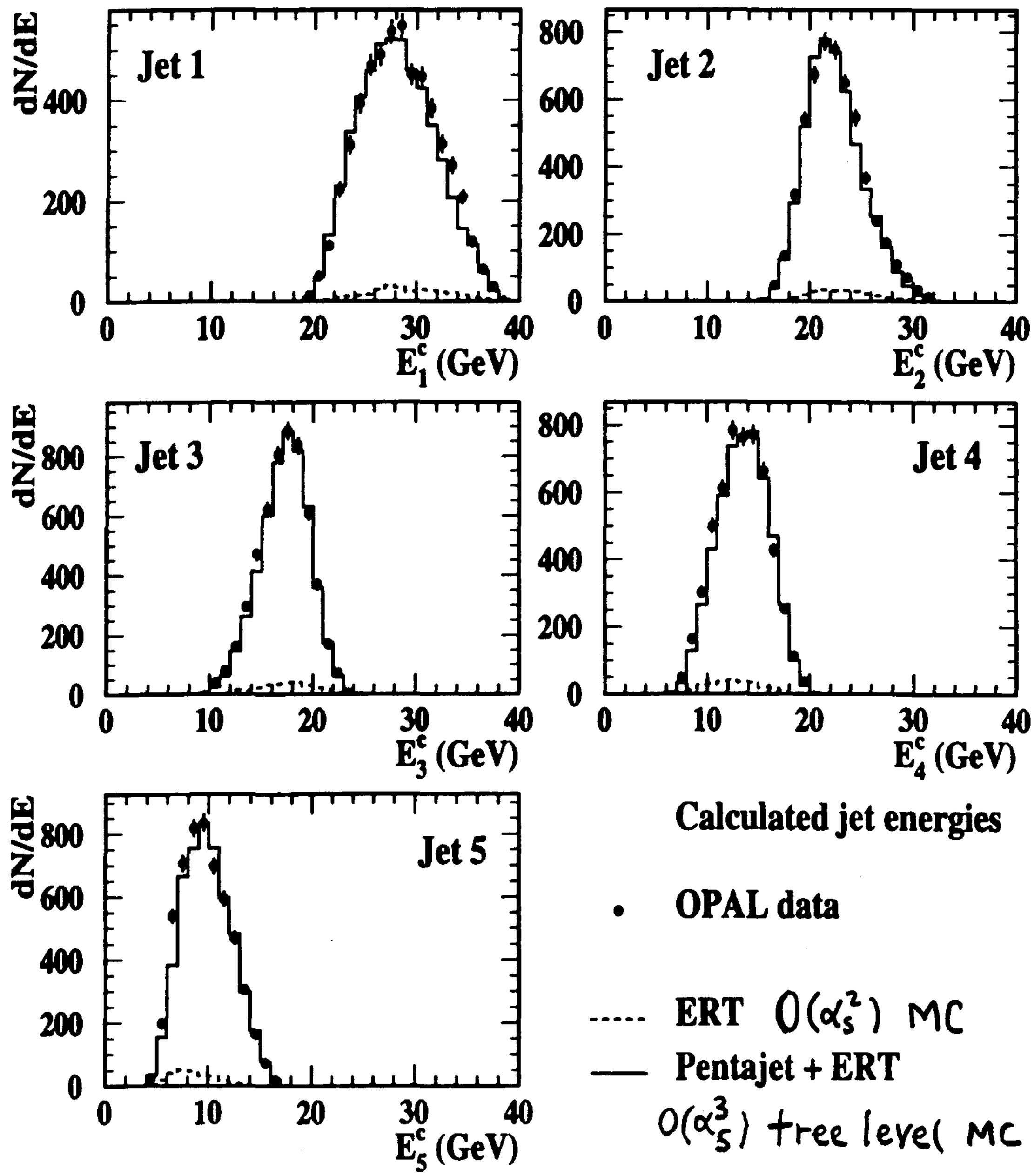
Abelian: no coupling

→ Sensitive to the group structure of QCD

Test of the 5-jet matrix element

$\sqrt{s} = 91$ GeV

OPAL preliminary

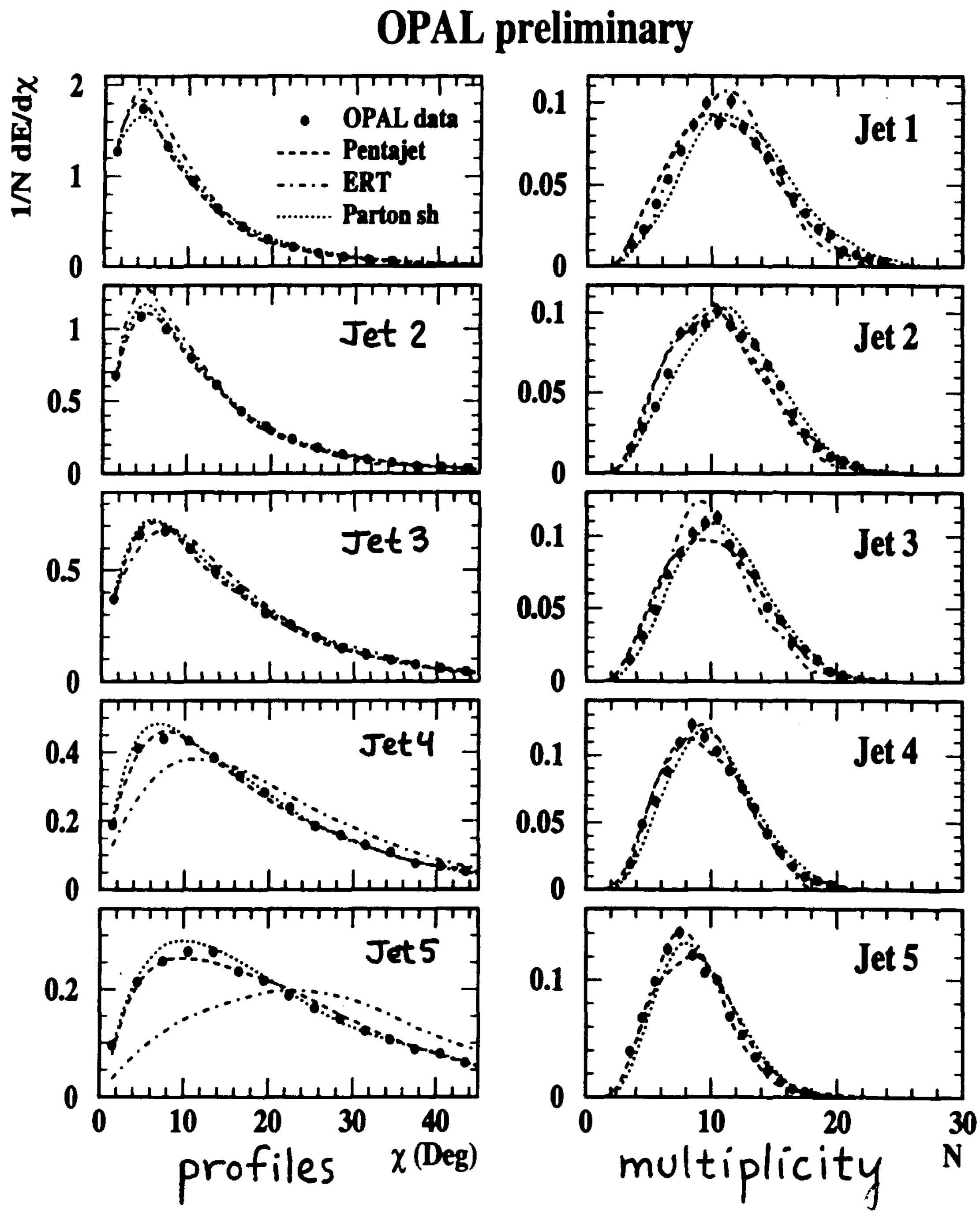


<u>JADE algorithm</u>

gluon jet content ~ 35, 50, 60, 70, 80 %
for the five jets $E_1 > E_2 > E_3 > E_4 > E_5$

Data show a need for 5-jet structure

jet profiles → distribution of jet energy around the jet axis



Cannot describe the 5-jet jet profiles using $O(\alpha_s^2)$ matrix element (ERT)

Measurements of α_S

→ Inclusive:

- $R_\ell = \dfrac{\Gamma(Z^0 \to hadrons)}{\Gamma(Z^0 \to \ell^+\ell^-)}$ $\ell = e, \mu$ or τ
- $R_\tau = \dfrac{\Gamma(\tau \to hadrons)}{\Gamma(\tau \to \ell^+\ell^-)}$ $\ell = e$ or μ
- Standard Model (SM) fit result for σ_ℓ , the leptonic pole cross section at the Z^0

→ 3-jet dominated:

- Event shapes: Thrust, Jet broadening, $\cdots$
- N-jet rates
- Energy-energy correlations

→ Scaling violations:

- "Q^2" evolution of fragmentation functions

α_S from R_ℓ

$R_\ell =$ ratio of the total hadronic to the single species (massless) leptonic branching ratios of the Z^0

$$= \frac{\Gamma(Z^0 \to hadrons)}{\Gamma(Z^0 \to \ell^+ \ell^-)} \qquad \ell = e, \mu \text{ or } \tau$$

Experiment

⟶ based exclusively on <u>event counting</u>

⟶ no "analysis" except to understand the acceptance for the hadronic and charged leptonic events

⟶ no hadronization correction (there is some hadronization uncertainty in the acceptance corrections)

Theory

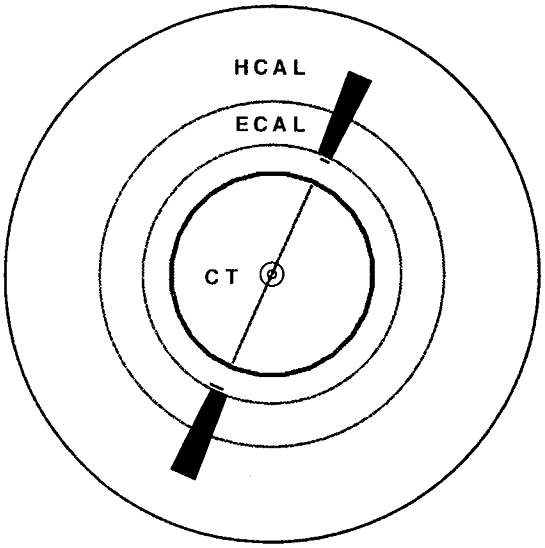
⟶ Complete $\mathcal{O}(\alpha_S^3)$ (3 loop) calculation available

⟶ The only observable, along with R_τ, for which this is true

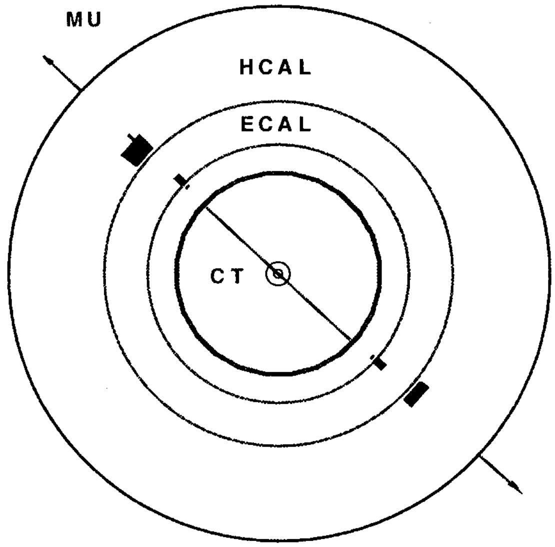
α_S from R_ℓ has intrinsically small experimental and theoretical uncertainties !

$Z^0 \rightarrow \ell^+\ell^-$ and $Z^0 \rightarrow hadrons$

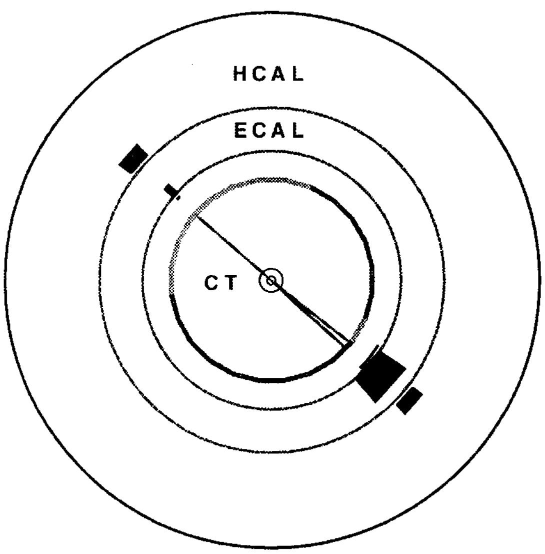
e+e- --> e+e-



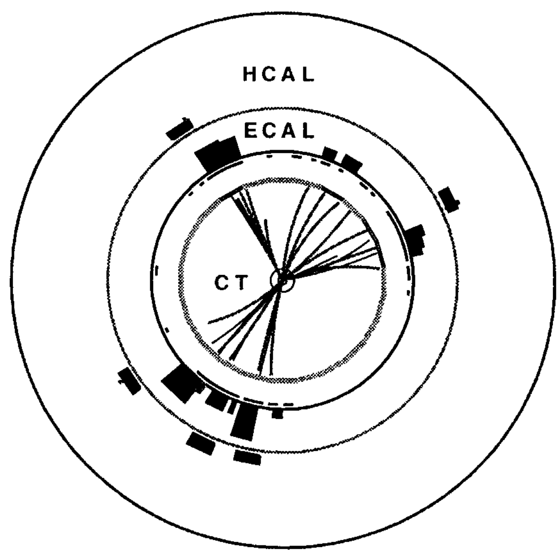
e+e- --> μ+μ−



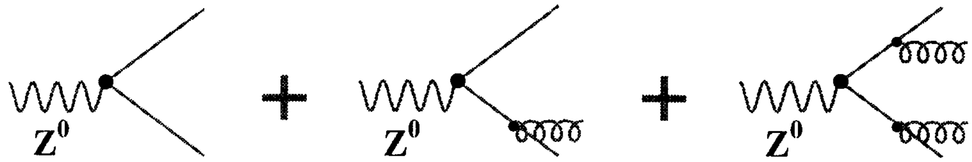
e+e- --> τ+τ−



e+e- --> $q\bar{q}$(g)



However, the dependence of R_ℓ on α_S is non-leading and therefore weak:

$\mathbf{R_l} \sim$ 

or $R_\ell = R_\ell^0 \left(1 + \delta_{QCD}\right)$

with

$$R_\ell^0 = R_\ell(\alpha_S = 0) = 19.934$$

and

$$\delta_{QCD} = 1.045 \left(\frac{\alpha_S}{\pi}\right) + 0.94 \left(\frac{\alpha_S}{\pi}\right)^2 - 15 \left(\frac{\alpha_S}{\pi}\right)^3 \approx 0.042$$

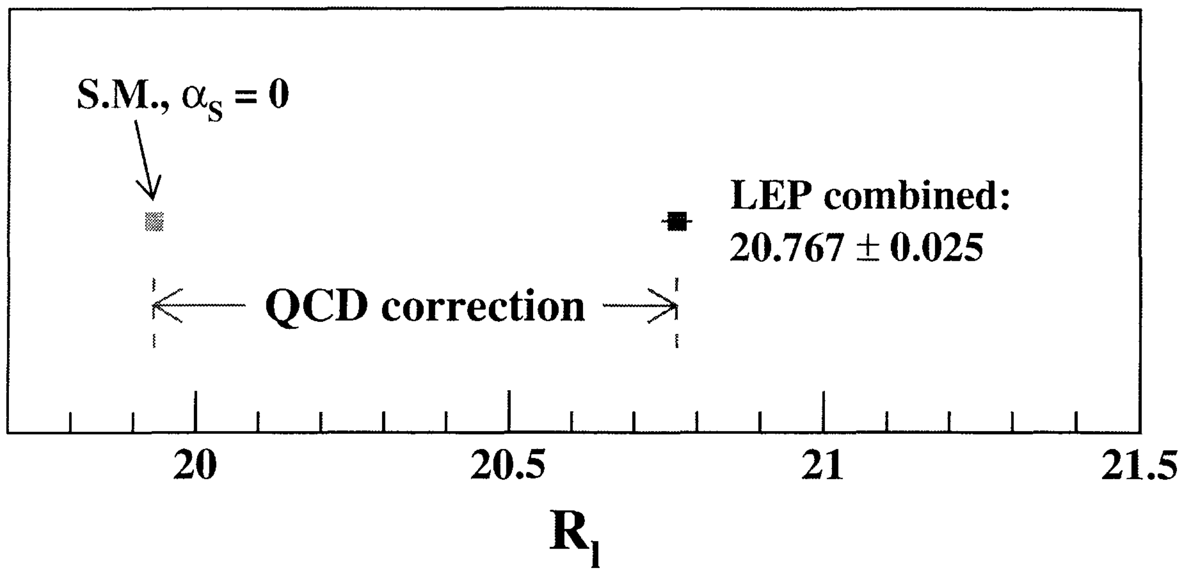
where $\alpha_S = \alpha_S(m_Z)$ [see E. Tournefier, hep-ex/9810042]

$\longrightarrow$ An accurate determination of α_S from R_ℓ requires the total LEP-1 event statistics from the four LEP experiments combined !

Total LEP-1 event statistics (combined):

$Z^0 \rightarrow hadrons: \sim 15\,000\,000$

$Z^0 \rightarrow leptons: \sim 1\,724\,000$
(all species)



[see LEP Electroweak Working Group Note LEPEWWG/2002-01]

$$\longrightarrow \boxed{\alpha_S = 0.1224 \pm 0.0038}$$

3% precision

$\longrightarrow$ Uncertainty dominated by experimental systematics (e.g. acceptance for narrow 2-jet-like events near the beam axis) and event statistics.

α_S from σ_ℓ

$\sigma_\ell =$ cross section for $e^+e^- \rightarrow \ell^+\ell^-$ at the Z^0 pole

$$= \frac{12\pi}{M_Z^2} \frac{\Gamma^2(Z^0 \rightarrow \ell^+\ell^-)}{\Gamma^2(Z_{total})}$$

with

- $\Gamma(Z_{total}) = \Gamma_{had.} + 3\Gamma_{\ell^+\ell^-} + 3\Gamma_{\nu\bar{\nu}}$
- M_Z

$\longrightarrow$ Determined in a Standard Model fit of the Z^0 resonance parameters

[LEP Electroweak Working Group Note LEPEWWG/2002-01]

$$\longrightarrow \boxed{\alpha_S = 0.1180 \pm 0.0030}$$

2.5% precision

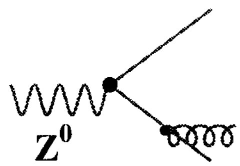
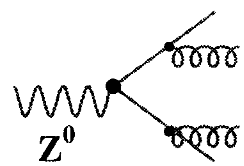
$\longrightarrow$ Uncertainty dominated by experimental measurement of the absolute luminosity.

α_S from Event Shapes

⟶ Measures of the momentum structure of an event

⟶ 3-jet dominated quantities with one entry "y" per event

⟶ Leading terms are $\sim \ \alpha_S$

y ~  **+** 

- Thrust T:

$$T = \max \left(\frac{\sum_i \vec{p}_i \cdot \hat{n}}{\sum |\vec{p}_i|} \right) \qquad i = particles$$

$$\text{resulting } \hat{n} = \hat{n}_T \longrightarrow \text{the thrust axis}$$

- Jet broadening variables B_T and B_W:

⟶ Divide event into hemispheres using plane $\perp$ to $\hat{n}_T$

⟶ Calculate the transverse momentum components

$$B_k = \frac{\sum_{k \ni i} |\vec{p}_i \times \hat{n}_T|}{\sum |\vec{p}_i|} \qquad k = 1, 2 \text{ (hemispheres)}$$

$$B_T = B_1 + B_2 \quad \underline{\textit{Total}} \text{ jet broadening}$$

$$B_W = \max(B_1, B_2) \quad \underline{\textit{Wide}} \text{ jet broadening}$$

- Jet rate "flip" value y_{23}: (also known as y_3 or D_2)
  - ⟶ Define jets using the Durham jet finder
  - ⟶ Find the y_{cut} value at which an event flips from the 2-jet to the 3-jet class
- plus many others : C parameter, major, minor, oblateness ...

⟶ Many of these variables are equivalent to each other at LO but have different higher order corrections

⟶ Unlike σ_ℓ, the distributions in "y" require a hadronization correction before being fitted by theoretical expressions

⟶ The hadronization correction of $\sim 10\%$ is determined using the ratio of the Monte Carlo predictions at the parton & hadron levels and is one of the main sources of systematic uncertainty

QCD predictions for event shape variables

- Exact $\mathcal{O}(\alpha_S^2)$ expressions:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dy} = A(y)\frac{\alpha_S(\mu)}{2\pi} + [B(y) + A(y)2\pi b_0 \log f]\left(\frac{\alpha_S(\mu)}{2\pi}\right)^2$$

$$f = \frac{\mu^2}{s} \quad ; \mu = \text{renormalization scale}$$

$$b_0 = (33 - 2n_f)/12\pi$$

→ The renormalization scale μ is an unphysical parameter

→ If the calculation were available to all orders in perturbation theory, there would be no dependence on μ

→ For finite orders, a residual dependence $\sim \left(\log \frac{\mu^2}{s}\right)^n$ is present

→ Need $\mu \approx \sqrt{s}$ for the effects of higher order terms to be negligible

→ Two parameter fits of $\alpha_S(M_Z)$ and μ to the hadronization corrected data typically yield $\mu \approx \sqrt{s}/20$, indicating the importance of the missing higher order terms

→ We need $\mathcal{O}(\alpha_S^3)$ expressions for event shapes !
(not yet available, several groups are working on them)

→ Theory uncertainties due to the missing higher orders (renormalization scale dependence) dominate the total uncertainty of α_S measurements from event shapes

- $\mathcal{O}(\alpha_S^2)$ + NLLA expressions:

→ A perturbative expansion can be performed for the cumulative event shape cross section $R(y)$

$$R(y) = \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy'}dy'$$

→ Expressed as a series in $L = \ln(1/y)$

→ Most singular (largest) terms are in the 2-jet region, $y \to 0$

→ Leading and next-to-leading logarithmic terms have been summed to all orders of α_S for a number of event shape variables: NLLA

Perturbative structure of $R(y)$:

NLLA (brace over "Leading logs" and "Next-to-leading logs")

	Leading logs	Next-to-leading logs	Sub-leading terms
$\mathcal{O}(\alpha_S)$	$\alpha_S L^2$	$\alpha_S L$	$\alpha_S, \alpha_S \frac{1}{L}$
$\mathcal{O}(\alpha_S^2)$	$\alpha_S^2 L^3$	$\alpha_S^2 L^2$	$\alpha_S^2 L, \alpha_S^2, \alpha_S^2 \frac{1}{L}$
	$\alpha_S^3 L^4$	$\alpha_S^3 L^3$	$\cdots$
	$\cdots$	$\cdots$	

$\}\ \mathcal{O}(\alpha_S^2)$ (brace over the $\mathcal{O}(\alpha_S)$ and $\mathcal{O}(\alpha_S^2)$ rows)

Common to $\mathcal{O}(\alpha_S^2)$ and NLLA (dashed box around the leading and next-to-leading log terms of the $\mathcal{O}(\alpha_S)$ and $\mathcal{O}(\alpha_S^2)$ rows)

- Terms up to $\mathcal{O}(\alpha_S^2)$ in the NLLA expression are replaced by the exact $\mathcal{O}(\alpha_S^2)$ results

$$\longrightarrow \boxed{\mathcal{O}(\alpha_S^2) + \text{NLLA}}$$

- This provides the most complete analytic descriptions currently available for event shapes

- Fits of the $\mathcal{O}(\alpha_S^2)$ + NLLA expressions to data yield results for μ much closer to the physical scale $\sqrt{s}$ than the pure $\mathcal{O}(\alpha_S^2)$ expressions

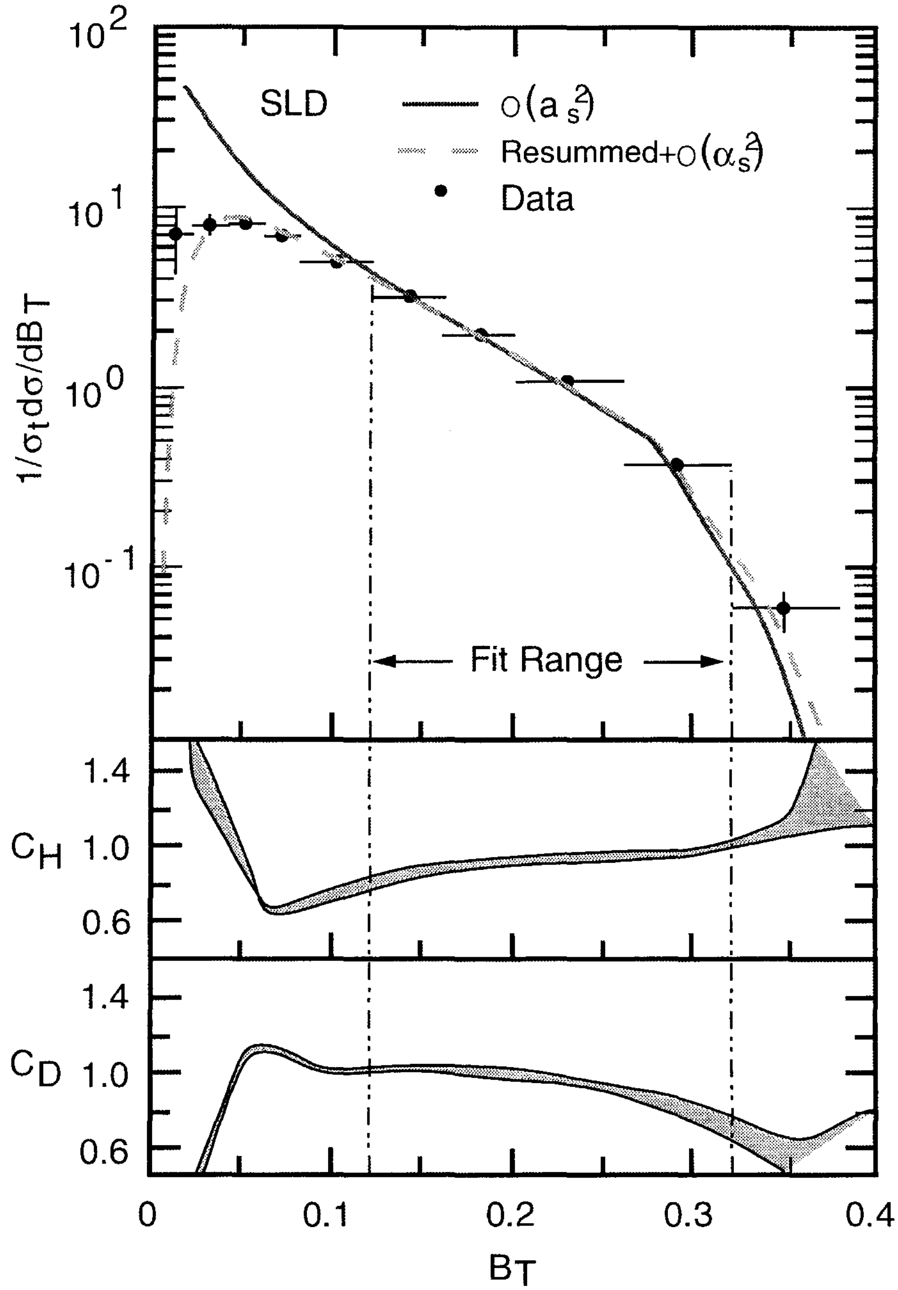
- The perturbative description of the data is more sensible

- The description of the 2-jet region is improved

- The NLLA terms reduce the sensitivity of the α_S result to the choice of μ

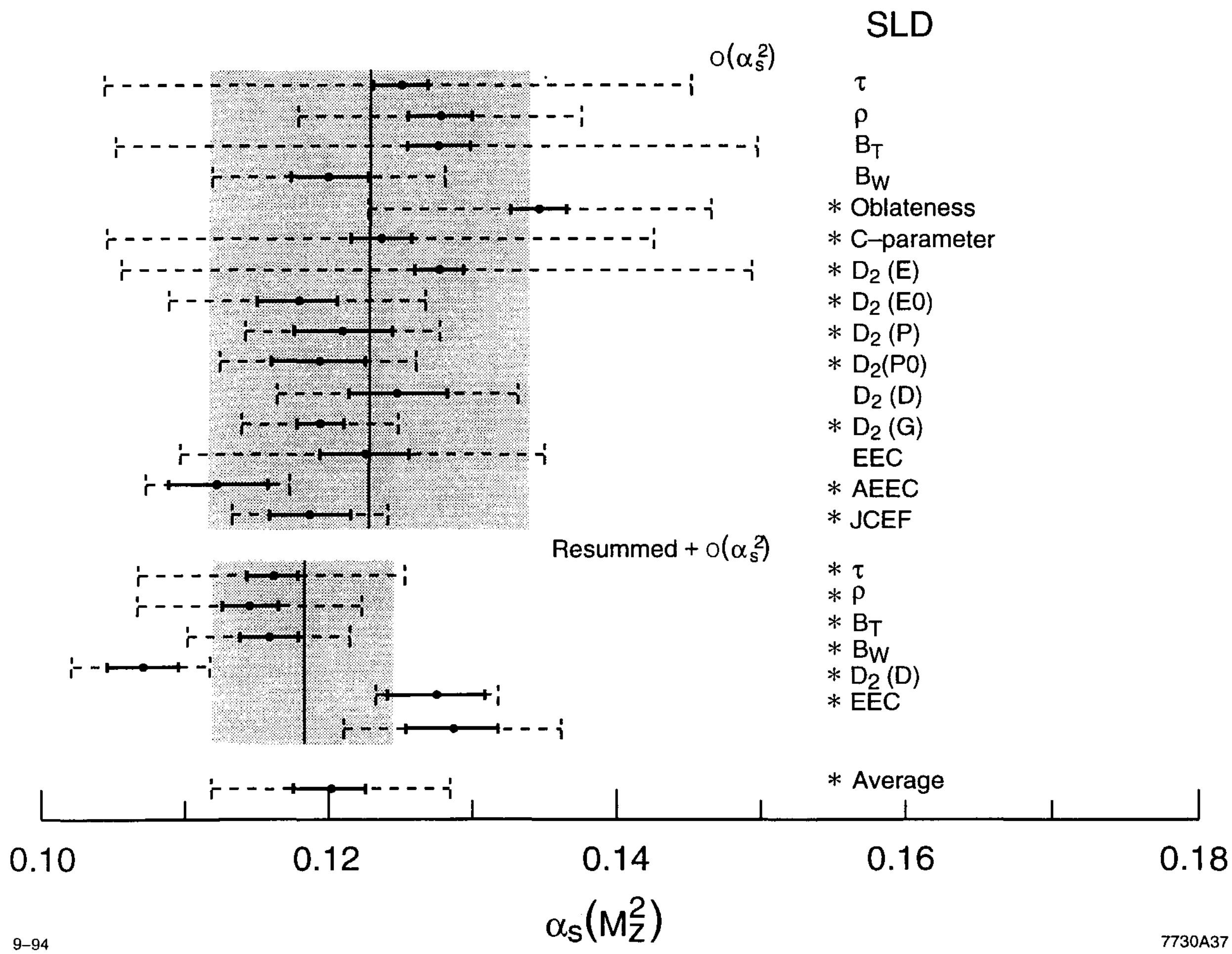
- The theory uncertainty due to missing higher order terms remains the dominant uncertainty for α_S, however

[SLD Collab., Phys. Rev. D51 (1995) 962]



C_H = Hadronization correction

C_D = Correction for detector acceptance & resolution



Solid bars	⟶	Experimental uncertainties
Dashed bars	⟶	Experimental & theory uncertainties
Top section	⟶	$\mathcal{O}(\alpha_S^2)$ results
Bottom section	⟶	$\mathcal{O}(\alpha_S^2)$ + NLLA results
Shaded regions	⟶	Average α_S value and total uncertainty

α_S from event shapes [S. Bethke, hep-ex/0004021]

$\longrightarrow$ $\boxed{\alpha_S = 0.121 \pm 0.006}$ 5% precision

α_S from scaling violations

Inclusive cross section for the production of a hadron with scaled energy $x = 2E/\sqrt{s}$

$$\frac{d\sigma}{dx}\left(e^+e^- \to h + X\right) = \int_x^1 \frac{dz}{z} \sum_{f=u,d,s,c,b,g} C_f\left(z, \alpha_S, \sqrt{s}\right) D_f^h\left(\frac{x}{z}, \mu\right)$$

$C_f =$ "coefficient functions" describing the probability to create a parton f with energy fraction $z = 2\,E/\sqrt{s}$ (Known to NLO, so far used only to LO by expts.)

$D_f =$ "fragmentation functions," i.e. the probability that parton f yields a hadron with energy fraction x

- D_f is not predicted by theory

  $\longrightarrow$ Measure the charged particle fragmentation functions

$$D\left(x, s\right) = \sum_f D_f\left(x, s\right) \equiv \frac{1}{N}\frac{dn_{ch}}{dx}$$

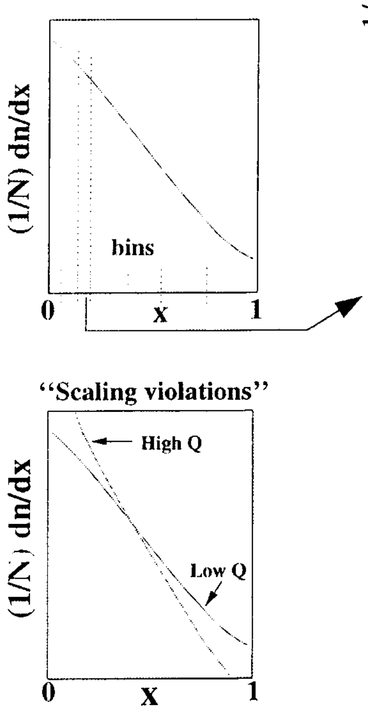
  using e$^+$e$^-$ data from $\sqrt{s} \sim 20$ to 91 GeV

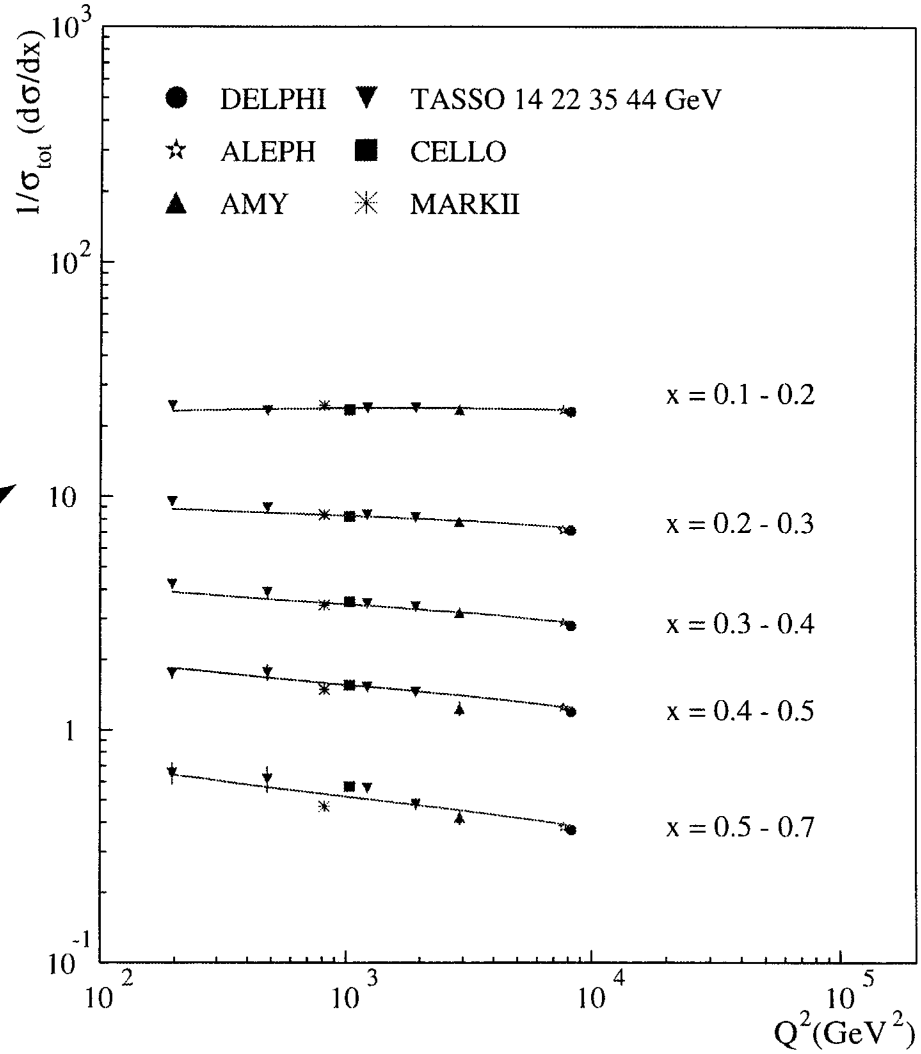
- $\sqrt{s}$ dependence of D_f is predicted by DGLAP evolution equations
- A priori unknowns are α_S and analytic expressions for D_f

$$D_f \longrightarrow \text{parametrized}$$

- Define D_f at an arbitrary scale μ, evolve the D_f over the relevant range of scales using DGLAP, fit the expression for $d\sigma/dx$ to the measurements of $(1/N)dn_{ch}/dx$ versus $\sqrt{s}$

[DELPHI Collab., Phys. Lett. B398 (1997) 194]



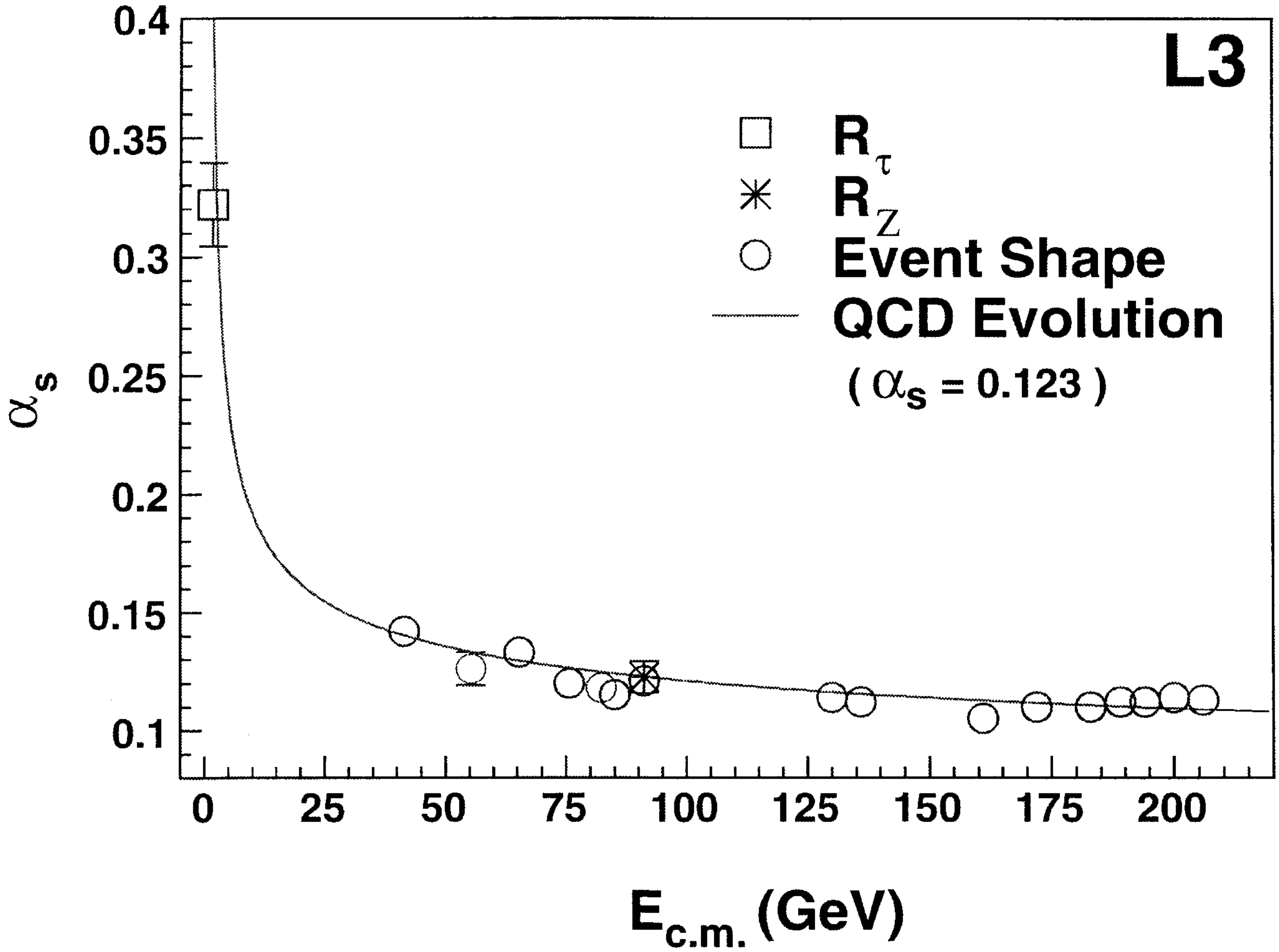


α_S from scaling violations

$\longrightarrow$ $\boxed{\alpha_S = 0.124 \pm 0.011}$ 9% precision

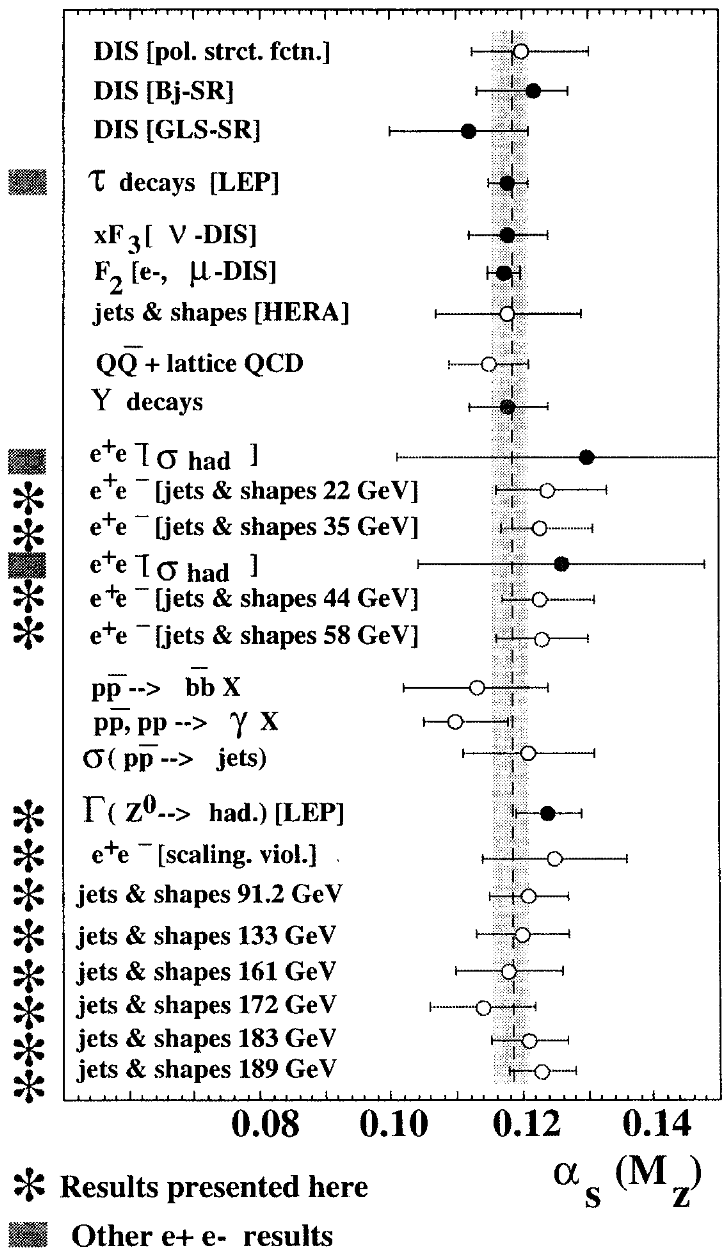
α_S: LEP summary → the running of α_S

[L3 Collab., CERN-EP-2002-015]



α_S: Overall summary

[S. Bethke, hep-ex/0004021: $\alpha_S(M_Z) = 0.1884 \pm 0.0031$ (2.6% precision)]



Note the smallness of the α_S result from τ decays

$\rightarrow$ The "shrinking error" of QCD !