SUPERSYMMETRY AND QCD

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June 8, 2002

- Why SUSY? What is SUSY? How is SUSY broken?
- The Feynman rules of SUSY-QCD
- QCD effects in SUSY and vice versa
- SUSY particle production at hadron, e^+e^-, and $\gamma\gamma$ colliders
- Virtual loop diagrams/calculations
- Real emission diagrams/calculations
- SUSY effects in α_s, PDFs, and FFs
- Summary

WHY SUPERSYMMETRY?

- The Standard Model is successful, but it has many deficiencies:
  - Gravity
  - Hierarchy of $m_h \ll m_{\mathrm{Pl.}}$
  - Electroweak symmetry breaking
  - Unification of the coupling constants
  - Cold dark matter in the universe
- Supersymmetry is a theoretically attractive extension:
  - SUSY is the only non-trivial extension of the Poincaré group
  - SUSY unifies fermions and bosons, matter and forces
  - SUSY as a local symmetry includes gravity [= supergravity]
  - SUSY appears naturally in string theories
  - SUSY stabilizes the mass of the Higgs boson
  - SUSY can break the electroweak symmetry radiatively
  - SUSY can explain the unification of couplings and $\sin^2\theta_W$
- Minimal Supersymmetric Standard Model (MSSM):
  - N=1 SUSY generators: One superpartner for each SM particle
  - Two Higgs doublets to give mass to up- and down-type quarks
  - Strongly interacting gluino: $\tilde{g}$, squarks: $\tilde{q}_{L,R}, \tilde{t}_{1,2}, \tilde{b}_{1,2}$
  - Weakly interacting gauginos: $\tilde{\chi}^0_{1-4}, \tilde{\chi}^\pm_{1,2}$, sleptons: $\tilde{l}_{L,R}, \tilde{\nu}_L$
  - Renormalizability, $B-L$ conservation $\longrightarrow$ R-parity conserved
  - SUSY particles must be produced in pairs, LSP is stable

WHAT IS SUPERSYMMETRY? [S.P. Martin, hep-ph/9709256]

- Only non-trivial extension of the Poincaré group
- Generated by an operator Q and $Q^\dagger$ [= anticommuting spinors]:

$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0 \quad ; \quad \{Q, Q^\dagger\} = P^\mu$$

Transforms as a Lorentz vector $\longrightarrow$ SUSY = space-time symmetry

- Chiral fermions: $Q|\phi\rangle = |\psi\rangle \; ; \; Q|\psi\rangle = |\phi\rangle$

  $m = 0$: ψ=fermion(2), ϕ=comp.scalar(2)

  $m \neq 0$: ψ=fermion(4), ϕ=comp.scalar(2), F=aux.comp.scalar(2)

- Gauge bosons: $Q|A\rangle = |\lambda\rangle \; ; \; Q|\lambda\rangle = |A\rangle$

  $m = 0$: A=vector boson(2), λ=fermion(2)

  $m \neq 0$: A=vector boson(3), λ=fermion(4), D=aux.real scalar(1)

- General SUSY Lagrangian:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\mathrm{chiral}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{SUSY-gauge}} \\
\mathcal{L}_{\mathrm{chiral}} &= -(D^\mu \phi_i^*)(D_\mu \phi_i) - \bar{\psi}_i i \not{D} \psi_i + F_i^* F_i \\
&\quad - \frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + (c.c.) \\
\mathcal{L}_{\mathrm{gauge}} &= -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \lambda^{\dagger a} i \not{D} \lambda_a + \frac{1}{2} D^a D_a \\
\mathcal{L}_{\mathrm{SUSY-gauge}} &= g_a (\phi^* T^a \phi) D_a - \sqrt{2} g_a [(\phi^* T^a \psi)\lambda_a + \lambda_a^\dagger (\psi^\dagger T^a \phi)]
\end{aligned}$$

- Superpotential: $W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k$

$$\begin{aligned}
M^{ij} &= \text{Fermion mass matrix} \\
y^{ijk} &= \text{Yukawa interactions} \\
W^{ij} &= \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \\
W^i &= \frac{\partial W}{\partial \phi_i} = -F^{*i} \quad \text{[eq. of motion]} \\
D^a &= -g_a (\phi^* T^a \phi) \quad \text{[eq. of motion]}
\end{aligned}$$

How is Supersymmetry broken?

- No SUSY particles observed $\longrightarrow$ SUSY masses, beyond exp. reach
- Soft SUSY breaking Lagrangian in the MSSM:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{soft}}^{\mathrm{MSSM}} = & -\frac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B}\right) \\
& -\tilde{Q}^\dagger\, \mathbf{m}_{\mathbf{Q}}^{\mathbf{2}}\, \tilde{Q} - \tilde{L}^\dagger\, \mathbf{m}_{\mathbf{L}}^{\mathbf{2}}\, \tilde{L} - \tilde{\bar{u}}\, \mathbf{m}_{\bar{\mathbf{u}}}^{\mathbf{2}}\, \tilde{\bar{u}}^\dagger - \tilde{\bar{d}}\, \mathbf{m}_{\bar{\mathbf{d}}}^{\mathbf{2}}\, \tilde{\bar{d}}^\dagger - \tilde{\bar{e}}\, \mathbf{m}_{\bar{\mathbf{e}}}^{\mathbf{2}}\, \tilde{\bar{e}}^\dagger \\
& -\left(\tilde{\bar{u}}\, \mathbf{a_u}\, \tilde{Q} H_u - \tilde{\bar{d}}\, \mathbf{a_d}\, \tilde{Q} H_d - \tilde{\bar{e}}\, \mathbf{a_e}\, \tilde{L} H_d\right) \\
& - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - b H_u H_d + (c.c.)
\end{aligned}
$$

  - $M_{3,2,1}$ = gluino, wino, bino masses [with complex phases]
  - $m^2_{Q,L,\ldots}$= squark and slepton masses [3×3 matrices]
  - $a_{u,d,e}$ = trilinear couplings [complex 3×3 matrices]
  - Only scalars and gauginos get mass, not their superpartners
  - These masses do not reintroduce quadratic divergences
  - MSSM has 124 (105 SUSY + 19 SM) free parameters!

- Low-energy (m_Z) constraints:
  - Conservation of L_e, L_μ, L_τ, and CP, no FCNC, EDM
  - Generation universality, diagonal mass matrices

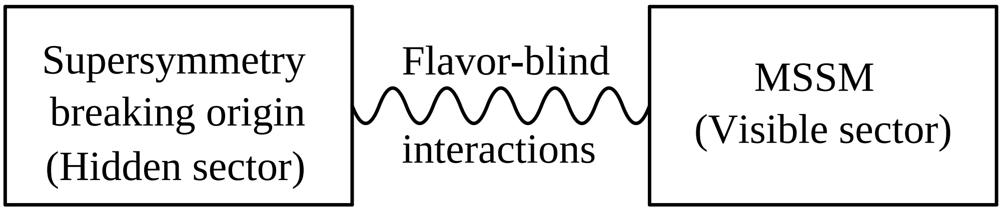
- High-energy ($m_{\mathrm{Pl.}}$) constraints:
  - Depend on different SUSY breaking models
  - Parameters must be evolved down to M_Z with RGE's
  - Gaugino mass relation:

$$
\frac{M_1(Q)}{\alpha_1(Q)} = \frac{M_2(Q)}{\alpha_2(Q)} = \frac{M_3(Q)}{\alpha_3(Q)} = \frac{m_{1/2}(M_X)}{\alpha_{\mathrm{GUT}}(M_X)}
$$

  - Radiative electroweak symmetry breaking

How is Supersymmetry broken?

- Spontaneous breaking: $Q|0\rangle \neq 0;\ Q^\dagger|0\rangle \neq 0;\ \langle 0|H|0\rangle \sim \langle 0|V|0\rangle \neq 0$
- Scalar potential: $V = F_i^* F^i + \frac{1}{2} D_a D^a = W_i^* W^i + \frac{1}{2} g_a^2 (\phi^* T^a \phi)^2$
- Fayet-Iliopoulos mechanism: $V = \frac{1}{2} D^2 - \kappa D + g D q_i \phi_i^* \phi^i$
- O'Raifeartaigh mechanism : $V = F_i^* F^i$



- Gravity-mediated models: $\mathcal{L} = \frac{-F_X}{m_{\mathrm{Pl.}}} \frac{f_a}{2} \lambda_a \lambda^a - \frac{F_X F_X^*}{m_{\mathrm{Pl.}}^2} k_j^i \phi_i \phi^{*j} + \ldots$

  $m_{1/2} = f \frac{\langle F_X \rangle}{m_{\mathrm{Pl.}}},\ m_0 = \sqrt{k} \frac{\langle F_X \rangle}{m_{\mathrm{Pl.}}},\ A_0 = \alpha \frac{\langle F_X \rangle}{m_{\mathrm{Pl.}}},\ B_0 = \beta \frac{\langle F_X \rangle}{m_{\mathrm{Pl.}}},\ \mathrm{sgn}(\mu)$
  - Auxiliary chiral field F_X from non-renormalizable SUGRA
- Gauge-mediated models: Ordinary gauge interactions

$$M_i = \frac{\alpha_i}{4\pi} \Lambda \quad , \quad m_\phi^2 = 2\Lambda^2 \left(\frac{\alpha_i}{4\pi} \right)^2 C_i$$

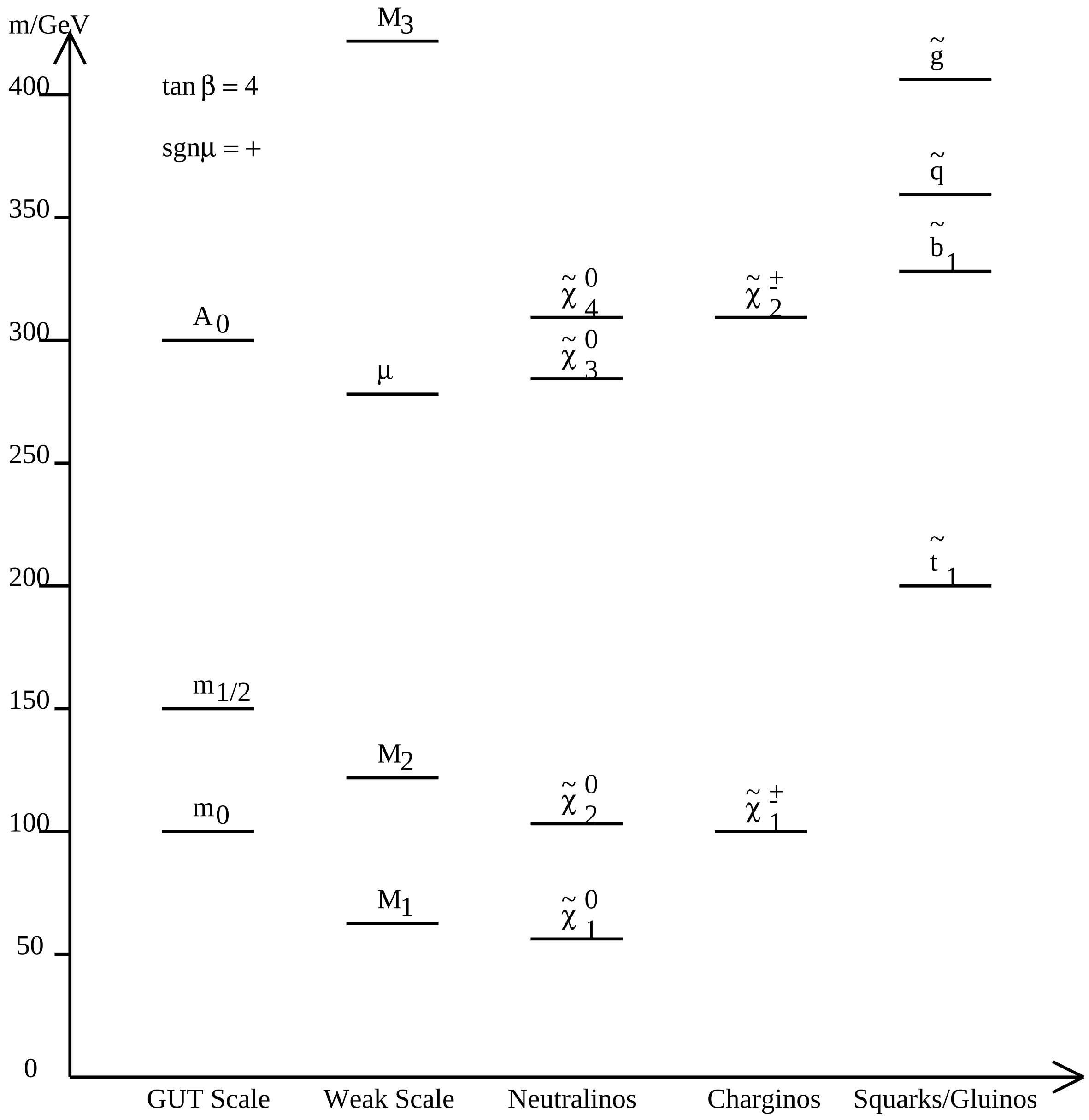
  - Auxiliary chiral field S and chiral messenger fields
  - Typically one messenger generation in SU(5)
  - Messenger scale: $\Lambda \in [40; 150]$ TeV
- Anomaly-mediated models: [Giudice, Rattazzi; Randall, Sundrum]

$$M_i = \frac{b_i \alpha_i}{4\pi} m_{3/2}$$

  - Gravity supermultiplet $\longrightarrow$ Super-Weyl-Anomaly
  - Gravitino mass $m_{3/2} \in [30; 60]$ TeV

Low energy SUSY particle masses

- Universal boundary conditions at high energies ($m_{\mathrm{Pl.}}$)
- Renormalization group equations predict physical masses (m_Z)
- Loop corrections to masses, couplings [BPMZ, NPB 491 (1997) 3]
- Programs: SUSPECT, SOFTSUSY, SUSYGEN; ISAJET, SPYTHIA
- Snowmass (2001) benchmarks: hep-ph/0202233
- Mass Spectrum in a Typical SUGRA Scenario:



The Feynman rules of SUSY-QCD

- Standard references:
  - H.E. Haber, G.L. Kane, Phys. Rep. 117 (1985) 75
  - J.F. Gunion, H.E. Haber, Nucl. Phys. B 272 (1986) 1
- More recent compilations:
  - J. Rosiek, Phys. Rev. D 41 (1990) 3464 and hep-ph/9511250(E)
  - W. Hollik, D. Stöckinger, Eur. Phys. Journ. C 20 (2001) 105
- Treamtent of Majorana fermions (such as gluinos):
  - A. Denner *et al.*, Nucl. Phys. B 387 (1992) 467
  - Avoid explicit charge conjugation matrices
  - Fix reference order for spinors, fermion flow for fermion chains
  - Multiply with permutation parity of the spinors

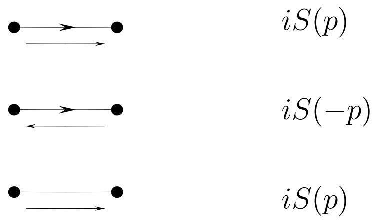
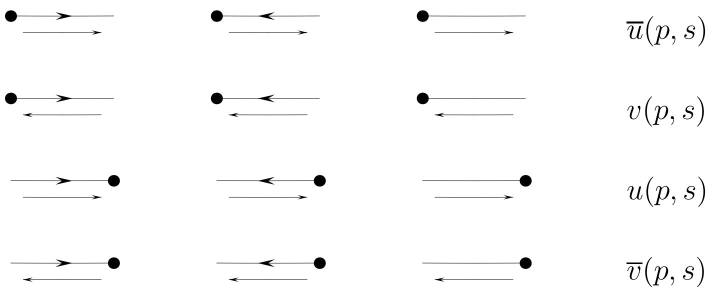
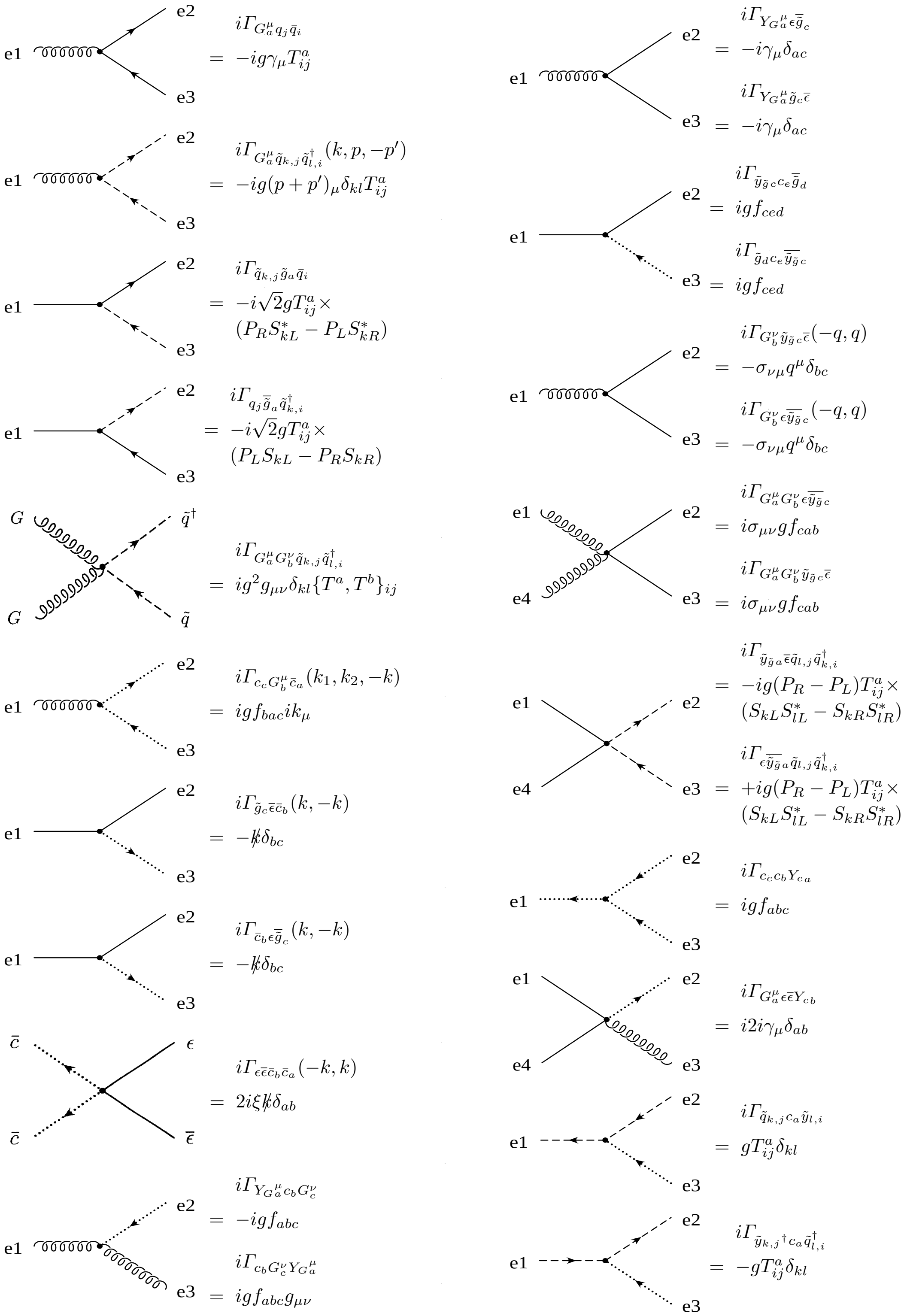


Figure 2.2: The Feynman rules for fermion propagators with orientation (thin arrows) The momentum p flows from left to right.



THE FEYNMAN RULES OF SUSY-QCD



$i\Gamma_{G^\mu_a q_j \bar{q}_i} = -ig\gamma_\mu T^a_{ij}$

$i\Gamma_{G^\mu_a \tilde{q}_{k,j} \tilde{q}^\dagger_{l,i}}(k,p,-p') = -ig(p+p')_\mu \delta_{kl} T^a_{ij}$

$i\Gamma_{\tilde{q}_{k,j}\bar{\tilde{g}}_a \bar{q}_i} = -i\sqrt{2} g T^a_{ij} \times (P_R S^*_{kL} - P_L S^*_{kR})$

$i\Gamma_{q_j \bar{\tilde{g}}_a \tilde{q}^\dagger_{k,i}} = -i\sqrt{2} g T^a_{ij} \times (P_L S_{kL} - P_R S_{kR})$

$i\Gamma_{G^\mu_a G^\nu_b \tilde{q}_{k,j} \tilde{q}^\dagger_{l,i}} = ig^2 g_{\mu\nu} \delta_{kl} \{T^a, T^b\}_{ij}$

$i\Gamma_{c_c G^\mu_b \bar{c}_a}(k_1, k_2, -k) = ig f_{bac} i k_\mu$

$i\Gamma_{\tilde{g}_c \bar{\epsilon} \bar{c}_b}(k,-k) = -\not{k}\delta_{bc}$

$i\Gamma_{\bar{c}_b \epsilon \bar{\tilde{g}}_c}(k,-k) = -\not{k}\delta_{bc}$

$i\Gamma_{\epsilon\bar{\epsilon}\bar{c}_b\bar{c}_a}(-k,k) = 2i\xi\not{k}\delta_{ab}$

$i\Gamma_{Y_{G^\mu_a} c_b G^\nu_c} = -ig f_{abc}$

$i\Gamma_{c_b G^\nu_c Y_{G^\mu_a}} = ig f_{abc} g_{\mu\nu}$

$i\Gamma_{Y_{G^\mu_a}\epsilon\bar{\tilde{g}}_c} = -i\gamma_\mu\delta_{ac}$

$i\Gamma_{Y_{G^\mu_a}\tilde{g}_c\bar{\epsilon}} = -i\gamma_\mu\delta_{ac}$

$i\Gamma_{\tilde{y}_{\tilde{g}_c} c_e \bar{\tilde{g}}_d} = ig f_{ced}$

$i\Gamma_{\tilde{g}_d c_e \overline{\tilde{y}_{\tilde{g}}}_c} = ig f_{ced}$

$i\Gamma_{G^\nu_b \tilde{y}_{\tilde{g}_c}\bar{\epsilon}}(-q,q) = -\sigma_{\nu\mu} q^\mu \delta_{bc}$

$i\Gamma_{G^\nu_b \epsilon \overline{\tilde{y}_{\tilde{g}}}_c}(-q,q) = -\sigma_{\nu\mu} q^\mu \delta_{bc}$

$i\Gamma_{G^\mu_a G^\nu_b \epsilon \overline{\tilde{y}_{\tilde{g}}}_c} = i\sigma_{\mu\nu} g f_{cab}$

$i\Gamma_{G^\mu_a G^\nu_b \tilde{y}_{\tilde{g}_c}\bar{\epsilon}} = i\sigma_{\mu\nu} g f_{cab}$

$i\Gamma_{\tilde{y}_{\tilde{g}_a}\bar{\epsilon}\tilde{q}_{l,j}\tilde{q}^\dagger_{k,i}} = -ig(P_R - P_L) T^a_{ij} \times (S_{kL} S^*_{lL} - S_{kR} S^*_{lR})$

$i\Gamma_{\epsilon\overline{\tilde{y}_{\tilde{g}}}_a \tilde{q}_{l,j}\tilde{q}^\dagger_{k,i}} = +ig(P_R - P_L) T^a_{ij} \times (S_{kL} S^*_{lL} - S_{kR} S^*_{lR})$

$i\Gamma_{c_c c_b Y_{c_a}} = ig f_{abc}$

$i\Gamma_{G^\mu_a \epsilon\bar{\epsilon} Y_{c_b}} = i2i\gamma_\mu\delta_{ab}$

$i\Gamma_{\tilde{q}_{k,j} c_a \tilde{y}_{l,i}} = g T^a_{ij} \delta_{kl}$

$i\Gamma_{\tilde{y}_{k,j}{}^\dagger c_a \tilde{q}^\dagger_{l,i}} = -g T^a_{ij}\delta_{kl}$

QCD EFFECTS IN SUPERSYMMETRY

- Standard Model particle decays:
  - $b \longrightarrow s\gamma$: $\tilde{\chi}, \tilde{g}$ loops [Barger *et al.*, PRL 70 (1993) 1368; PRD 51 (1995) 2438; Arnowitt, Nath, PRL 74 (1995) 4592; Carena *et al.*, PLB 499 (2001) 141; Becher, Braig, Kagan, Neubert, hep-ph/0205274]
  - $t \longrightarrow \tilde{t}\tilde{\chi}$ [Mrenna, Yuan, PLB 367 (1996) 188]
- Standard Model particle production:
  - $p\bar{p} \longrightarrow b\bar{b}$: Light $\tilde{b}$, $\tilde{g}$ [Berger *et al.*, PRL 86 (2001) 4231]
  - $p\bar{p} \longrightarrow t\bar{t}$ [Alam *et al.*, PRD 55 (1997) 1307; Sullivan, PRD 56 (1997) 451]
- SUSY particle decays (LEP, TESLA, Tevatron, LHC searches):
  - $\tilde{g} \longrightarrow g\tilde{\chi}$ (1-loop) [Baer, Tata, Woodside, PRD 42 (1990) 1568]
  - $\tilde{\chi} \longrightarrow q\tilde{q}$ [Berge, Klasen, to be published]
  - $\tilde{q} \longrightarrow q\tilde{\chi}, \tilde{q}W/Z/H$ [Bartl *et al.*, PLB 386 (1996) 175; 419 (1998) 243; PRD 59 (1999) 115007]
  - $\tilde{q} \longrightarrow q\tilde{g}, \tilde{g} \longrightarrow q\tilde{q}$ [Beenakker *et al.*, PLB 378 (1996) 159; ZPC 75 (1997) 349]
  - $H \longrightarrow q\bar{q}', \tilde{q}\tilde{q}'$ [Bartl *et al.*, PLB 373 (1996) 117; 378 (1996) 167; 402 (1997) 303]
- SUSY particle production (LEP, TESLA, Tevatron, LHC searches):
  - $e^+e^- \longrightarrow \tilde{q}\tilde{q}$ [Bartl, Eberl, Majerotto, NPB 472 (1996) 481]
  - $e^+e^- \longrightarrow \tilde{g}\tilde{g}$ (1-loop) [Kileng, Osland, ZPC 66 (1995) 503; Berge, Klasen, to be published]
  - $p\bar{p} \longrightarrow \tilde{\chi}\tilde{\chi}, \tilde{l}\tilde{l}$ [Baer, Harris, Reno, PRD 57 (1998) 5871; Beenakker *et al.*, PRL 83 (1999) 3780]
  - $p\bar{p} \longrightarrow \tilde{q}\tilde{q}, \tilde{g}\tilde{g}, \tilde{\chi}\tilde{q}, \tilde{\chi}\tilde{g}$ [Berger, Klasen, Tait, PLB 459 (1999)165; PRD 62 (2000) 095014; Beenakker *et al.*, NPB 492 (1997) 51; 515 (1998) 3]
- SUSY particle scattering ($\longrightarrow$ dark matter searches, cosmic rays):
  - $\tilde{\chi}N \longrightarrow \tilde{\chi}X$ [Djouadi, Drees, PLB 484 (2000) 183]
  - $\tilde{q}, \tilde{g}$ parton densities [Kounnas *et al.*, NPB 211 (1983) 216; 214(1983)317; Corianò,627(2002)66]
  - $\tilde{q}, \tilde{g}$ fragmentation functions [Corianò, Faraggi, PRD 65 (2002) 075001]

IMPLICATIONS OF SUPERSYMMETRY FOR QCD

[Dixon, hep-ph/9507214]

- QCD without fermions (pure Yang-Mills theory):
  - Assume fermions are gluinos, so QCD $\longrightarrow$ SUSY-QCD
  - Useful for multi-gluon scattering amplitudes
- Supersymmetric Ward identities:
  - In exact SUSY $Q|0\rangle = 0 \rightarrow [Q, \phi_i] = 0$ in helicity amplitudes
  - Useful relations for helicity amplitudes:

$$
\begin{aligned}
A_n^{\mathrm{SUSY}}(1^{\pm}, 2^+, 3^+, \ldots, n^+) &= 0 \\
A_n^{\mathrm{SUSY}}(1^-, 2_P^-, 3_P^+, 4^+, \ldots, n^+) &= \left(\frac{\langle 12\rangle}{\langle 13\rangle}\right)^{2|h_P|} \\
&\quad \times A_n^{\mathrm{SUSY}}(1^-, 2_\phi^-, 3_\phi^+, 4^+, \ldots, n^+)
\end{aligned}
$$

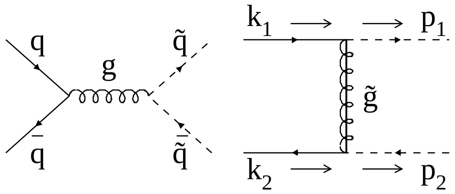
  - h_P =helicity $(0,\ \frac{1}{2},\ 1)$, $\langle jl\rangle = \bar{u}_-(k_j)u_+(k_l)$
- One-loop amplitudes via unitarity
  - Absorptive parts of loop amplitudes: integrate lower amplitudes
  - Simplify tree amplitudes before integration
  - Tree amplitudes possess “effective” supersymmetry
  - On-shell conditions for intermediate particles
  - Polynomial ambiguities only for masses, not for massless QCD
- Application: $gg \longrightarrow gg$ at two loops

[Bern, de Freitas, Dixon, JHEP 0203 (2002) 018]

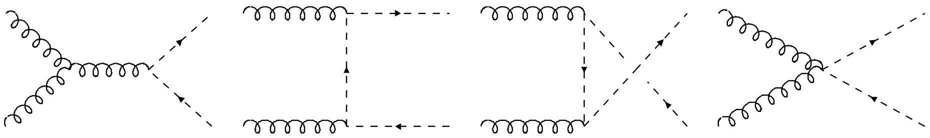
SUSY PARTICLE PRODUCTION AT HADRON COLLIDERS

- Squarks and gluinos:

(a)

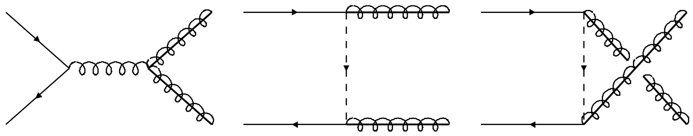


(b)



(c)

(d)



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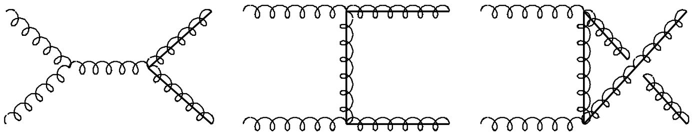

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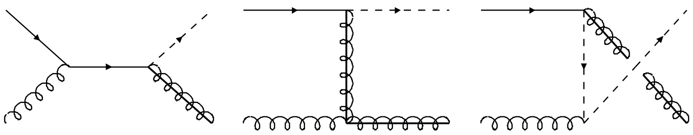
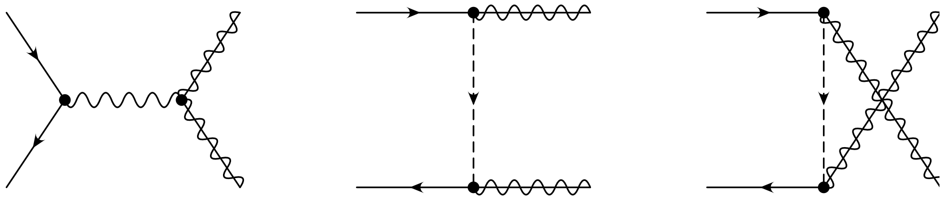


  - Cross sections depend only on physical squark and gluino masses
  - Mixing is important for $\tilde{t}$ – (a2) and (c) do not occur $[f_{t/p} = 0]$
  - Off-diagonal squark pairs can be produced from quark pairs (c)
  - Squarks can be produced in association with gluinos (f)
  - LO: Dawson, Eichten, Quigg, PRD 31 (1985) 1581
  - NLO: Beenakker *et al.*, NPB 492 (1997) 51; 515 (1998) 3

SUSY PARTICLE PRODUCTION AT HADRON COLLIDERS

- Gauginos and sleptons:



  - Neutralino production: Z-exchange in s-channel
  - Chargino production: also γ-exchange in s-channel
  - Slepton production: only s-channel, γ- and Z-exchange, like Drell-Yan
  - Associated production of $\tilde{g}\tilde{\chi}$: only t- and u-channel, $\tilde{q}_{\mathrm{L,R}}$-exchange
  - LO: Dawson, Eichten, Quigg, PRD 31 (1985) 1581
    Baer, Karatas, Tata, PRD 42 (1990) 2259

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(q\overline{q}' \to \tilde{\chi}_i\tilde{\chi}_j) = \frac{\pi}{s^2}\frac{N_C}{4N_C^2}$$

$$\left[\frac{A_s[(t-m_i^2)(t-m_j^2)+(u-m_i^2)(u-m_j^2)]+2A_s' m_i m_j s}{s^2}\right.$$

$$+A_t\frac{(t-m_i^2)(t-m_j^2)}{(t-m_{\tilde{q}}^2)^2}+A_u\frac{(u-m_i^2)(u-m_j^2)}{(u-m_{\tilde{q}}^2)^2}$$

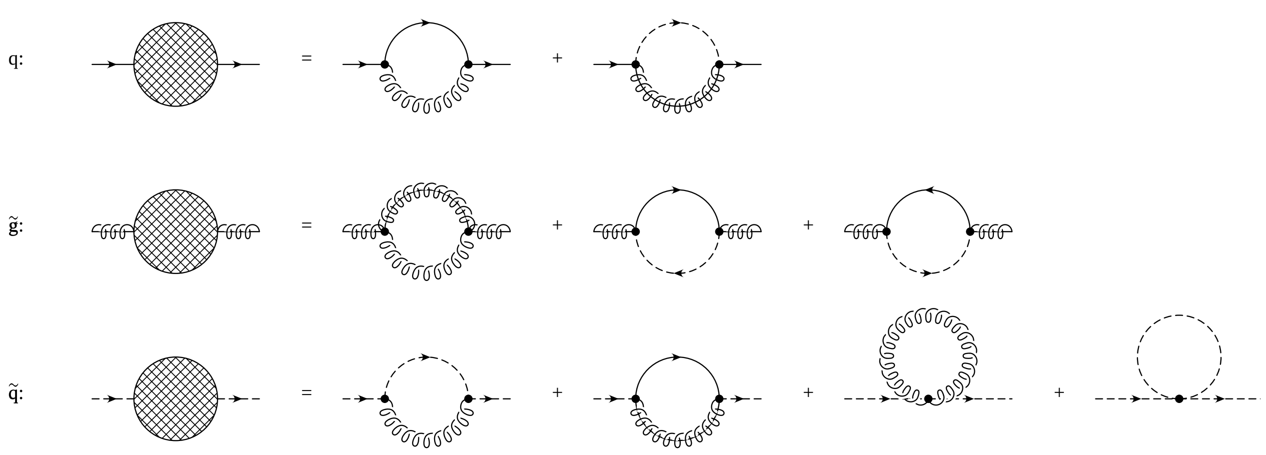
$$+\frac{A_{st}(t-m_i^2)(t-m_j^2)+A_{st}' m_i m_j s}{s(t-m_{\tilde{q}}^2)}$$

$$\left.+\frac{A_{su}(u-m_i^2)(u-m_j^2)+A_{su}' m_i m_j s}{s(u-m_{\tilde{q}}^2)}+A_{tu}\frac{m_i m_j s}{(t-m_{\tilde{q}}^2)(u-m_{\tilde{q}}^2)}\right]$$

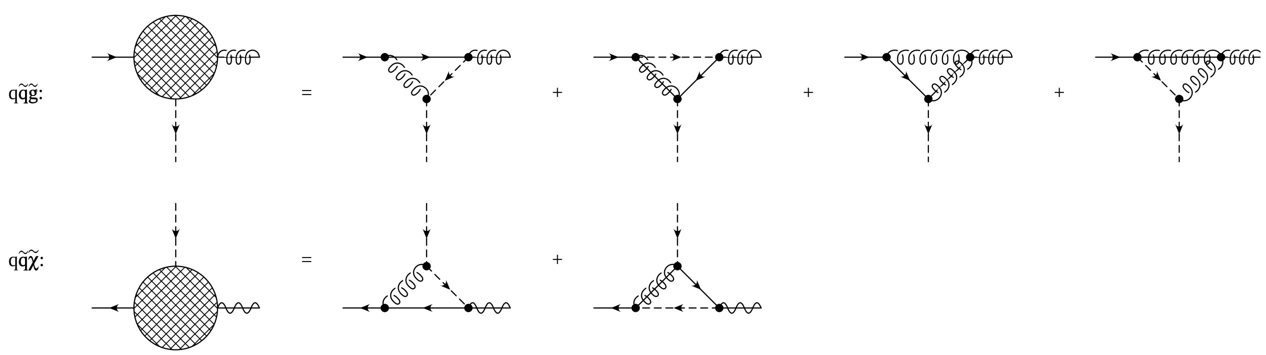
  - s, t, u are partonic Mandelstam variables, m_i are physical masses, $A_s, A_t, A_u, A_{st}, A_{su}, A_{tu}$ contain electroweak/strong couplings
  - NLO: Baer, Harris, Reno, PRD 57 (1998) 5871;
    Beenakker *et al.*, PRL 83 (1999) 3780;
    Berger, Klasen, Tait, PLB 459 (1999)165; PRD 62 (2000) 095014

VIRTUAL LOOP DIAGRAMS

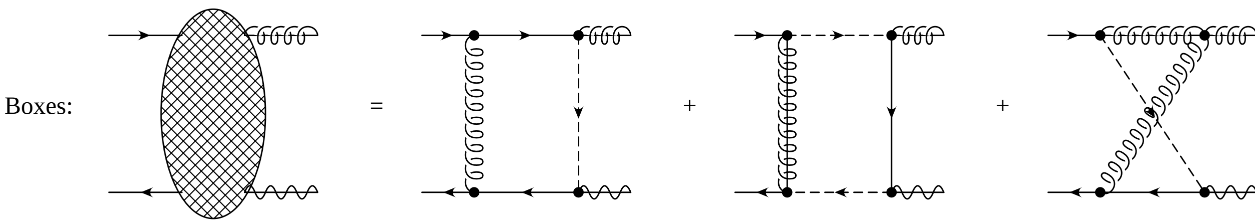
- Self-energy corrections (factorize the LO cross section):



- Vertex corrections (factorize the LO amplitude):



- Box diagrams (factorize the LO amplitude):



- Additional Feynman rules:
  - Colored parts of LO diagrams: SM/SUSY particle exchanges
  - Factor (-1) for loop diagrams with a closed fermion line
  - Factor 1/2 for loop diagrams with identical particles
  - Need interference of loop and LO diagrams $\longrightarrow$ only real part

VIRTUAL LOOP CALCULATIONS

- 1- to 4-point tensor loop integrals (loop four-momentum l):

$$
\begin{aligned}
A_0 &= (2\pi\mu)^{4-n} \int \frac{\mathrm{d}^n l}{i\pi^2} \frac{1}{D_1}, \\
B_{0,\mu,\mu\nu} &= (2\pi\mu)^{4-n} \int \frac{\mathrm{d}^n l}{i\pi^2} \frac{\{1, l_\mu, l_\mu l_\nu\}}{D_1 D_2}, \\
C_{0,\mu,\mu\nu,\mu\nu\rho} &= (2\pi\mu)^{4-n} \int \frac{\mathrm{d}^n l}{i\pi^2} \frac{\{1, l_\mu, l_\mu l_\nu, l_\mu l_\nu l_\rho\}}{D_1 D_2 D_3}, \\
D_{0,\mu,\mu\nu,\mu\nu\rho} &= (2\pi\mu)^{4-n} \int \frac{\mathrm{d}^n l}{i\pi^2} \frac{\{1, l_\mu, l_\mu l_\nu, l_\mu l_\nu l_\rho\}}{D_1 D_2 D_3 D_4}.
\end{aligned}
$$

- Denominators:

$$
\begin{aligned}
D_1 &= l^2 - m_1^2 + i\eta, \\
D_2 &= (l + p_1)^2 - m_2^2 + i\eta, \\
D_3 &= (l + p_1 + p_2)^2 - m_3^2 + i\eta, \\
D_4 &= (l + p_1 + p_2 + p_3)^2 - m_4^2 + i\eta
\end{aligned}
$$

- Variables:

$$
\begin{aligned}
p_1, \ldots, p_3 &= \text{external particle momenta} \\
m_1, \ldots, m_4 &= \text{internal particle masses}
\end{aligned}
$$

- Reduction to scalar integrals [Passarino, Veltman, NPB 160 (1979) 151]
  - Based on Lorentz invariance
  - UV divergences: $|l| \longrightarrow \infty$ in A_0, B_0
  - I R divergences: $|l| \longrightarrow 0$ and coll. splittings in B_0, C_0, D_0
- Numerical evaluation of tensor integrals [Oldenborgh, Vermaseren, ZPC46 (1990) 425]

VIRTUAL LOOP CALCULATIONS

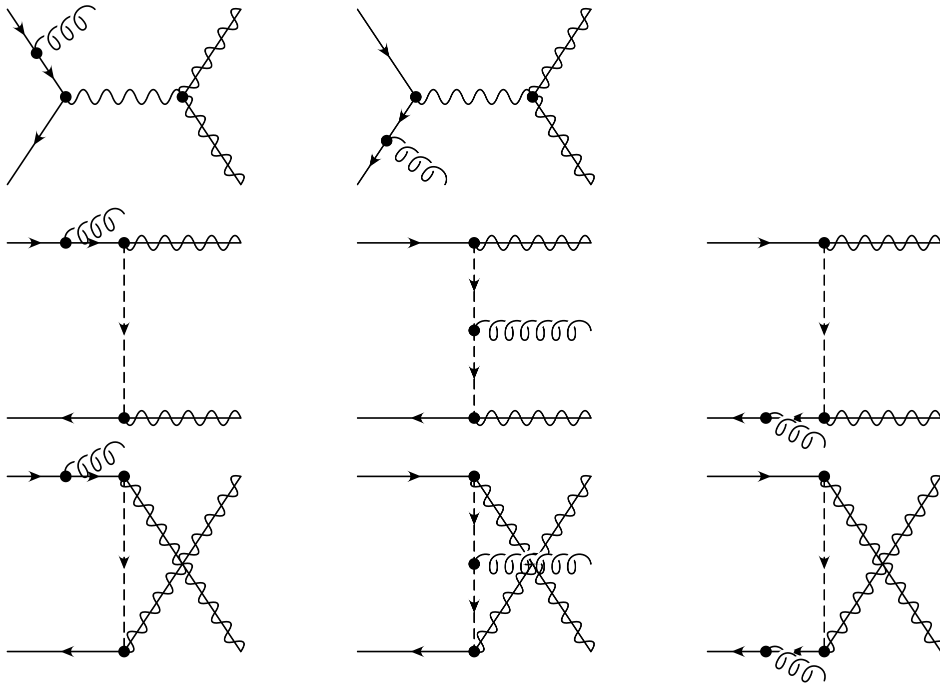
- Dimensional regularization: ['t Hooft, Veltman, NPB 44 (1972) 189]
  - Dirac traces and loop integrals in n dimensions
  - γ_5 anti-commutes in 4 dimensions, commutes in $n-4$
  - Breaks SUSY: g has n-2 degrees of freedom, but $\tilde{g}$ has 2
- Dimensional reduction: [Siegel, PLB 84 (1979) 193; Capper *et al.*, NPB 167 (1980) 479]
  - Dirac traces in 4 dimensions, loop integrals in n dimensions
  - γ_5 anti-commutes in all (4) dimensions
  - Manifestly supersymmetric: g and $\tilde{g}$ have 2 degrees of freedom
- Evaluation of scalar integrals:
  - Feynman parameters ['t Hooft, Veltman, NPB 153 (1979) 365]
  - Cutkosky cutting, dispersion integral ['t Hooft, Veltman, New York, NY, 1973]
  - Analytical continuation of logarithms $\longrightarrow$ large π^2 terms
- Renormalization:
  - Heavy particle masses: on-shell scheme
  - Couplings: $\overline{\mathrm{MS}}$ scheme
  - Finite renormalization to restore supersymmetry in $\overline{\mathrm{MS}}$

$$\hat{g} = g\left[1 + \frac{g^2}{32\pi^2}\left(\frac{4}{3}N_C - C_F\right)\right]$$

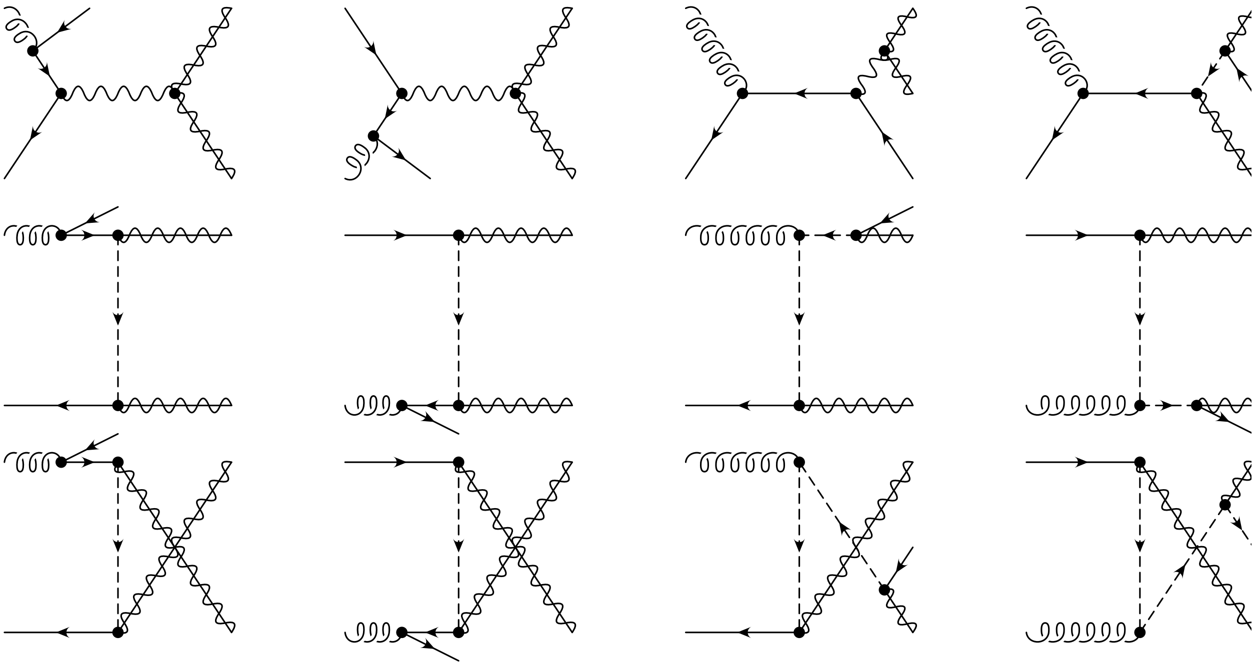
[Martin, Vaughn, PLB 318 (1993) 331]

Real emission diagrams

- Gluons:

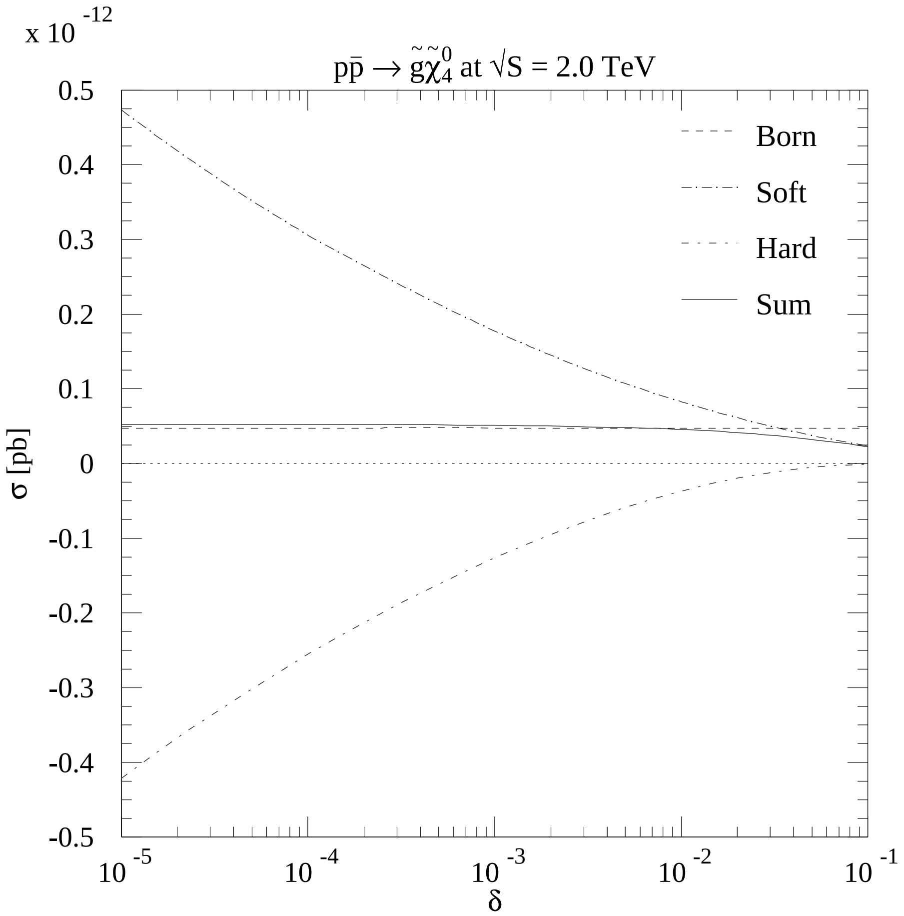


- Quarks / Antiquarks:



Real emission calculations

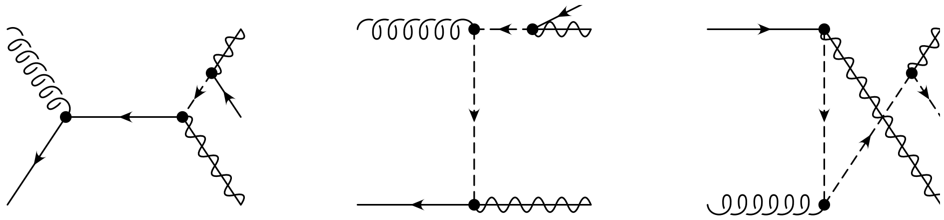
- Phase space slicing method (heavy quarks): [Beenakker *et al.*, PRD 40 (1989) 54]
  - Simplification of matrix elements in soft/collinear limit
  - Analytical integration up to cut-off Δ
  - Numerical integration above cut-off Δ
  - Numerical cancellation of cut-off dependence



- Subtraction method (massless QCD): [Catani *et al.*, NPB 485 (1997) 291; 627 (2002) 189]
  - Construct counter terms form dipole form of parton splittings
  - Add integrated counter terms to virtual corrections
  - Subtract unintegrated counter terms from real corrections
  - Point-by-point cancellation of singularities

REAL EMISSION CALCULATIONS

- Treatment of IR singularities:
  - KLN-cancellation between real and virtual corrections
  - Factorization of collinear divergences into $\overline{\mathrm{MS}}$ parton densities
  - On-shell particle decays (intermediate squarks):



    - $*$ Assoc. production $p\bar{p} \longrightarrow \tilde{q}\tilde{\chi}$, subsequent decay $\tilde{q} \longrightarrow q\tilde{\chi}$
    - $*$ To avoid double counting, one must subtract

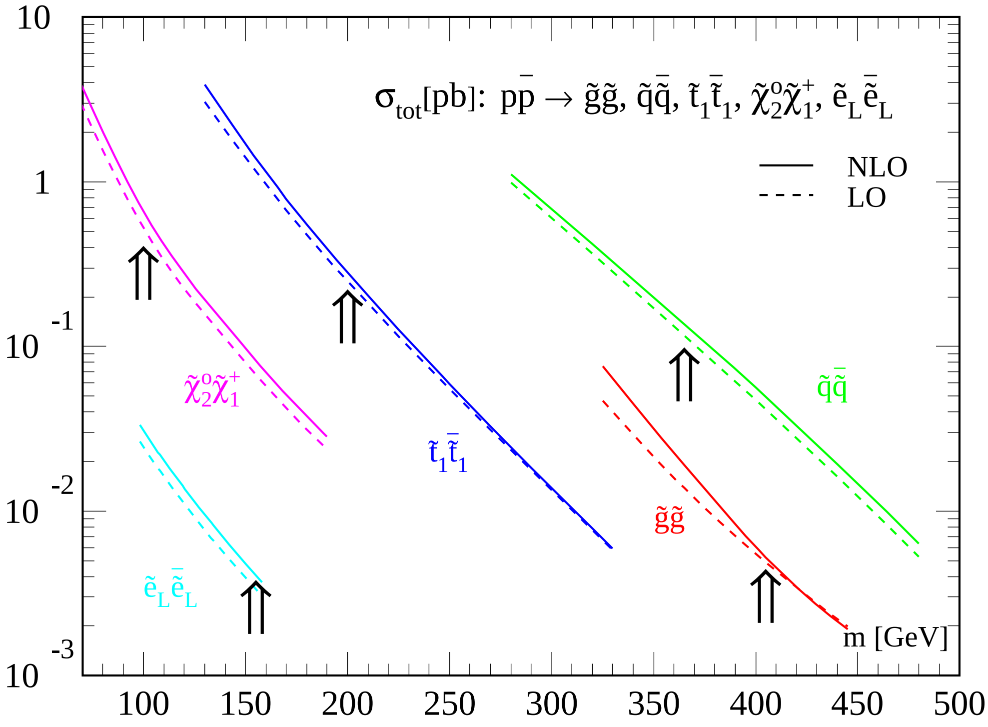
$$\frac{\mathrm{d}\sigma}{\mathrm{d}M^2} = \sigma(gq \to \tilde{q}\tilde{\chi}_i)\mathrm{BR}(\tilde{q} \to q\tilde{\chi}_j)\frac{m_{\tilde{q}}\Gamma_{\tilde{q}}/\pi}{(M^2 - m_{\tilde{q}}^2)^2 + m_{\tilde{q}}^2\Gamma_{\tilde{q}}^2}$$

$$\to \sigma(gq \to \tilde{q}\tilde{\chi}_i)\mathrm{BR}(\tilde{q} \to q\tilde{\chi}_j)\delta(M^2 - m_{\tilde{q}}^2)$$

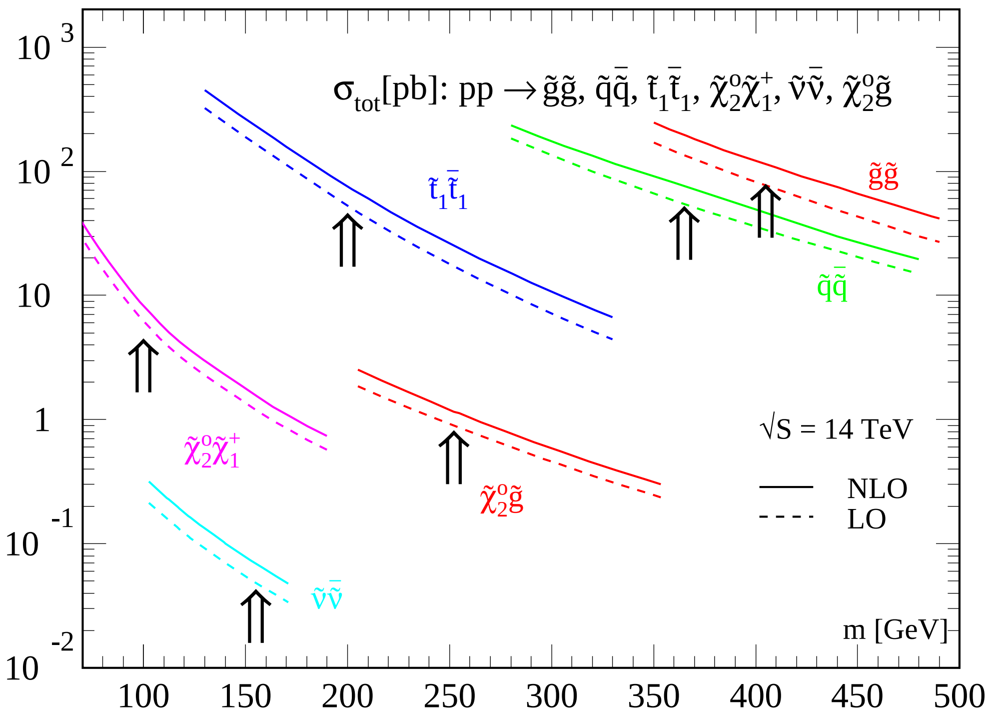
- Implementation in flexible Monte Carlo (FORTRAN,C++) programs:
  - Partonic scaling functions: $\hat{\sigma}_{ij}(\hat{s})$
  - Total hadronic cross sections: $\sigma(m^2)$
  - K-factors: $\sigma^{\mathrm{NLO}}/\sigma^{\mathrm{LO}}$
  - Distributions: $\mathrm{d}\sigma/\mathrm{d}E_T, \mathrm{d}\sigma/\mathrm{d}\eta$
- Implementation in event generators (ISAJET, SPYTHIA, HERWIG):
  - SUSY mass spectra, LO scattering matrix elements, K-factors
  - Parton showering, (s)particle decays, hadronization
  - Detector simulation

SUSY PARTICLE PRODUCTION AT HADRON COLLIDERS

- Tevatron:

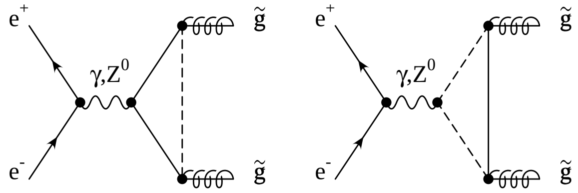


- LHC:



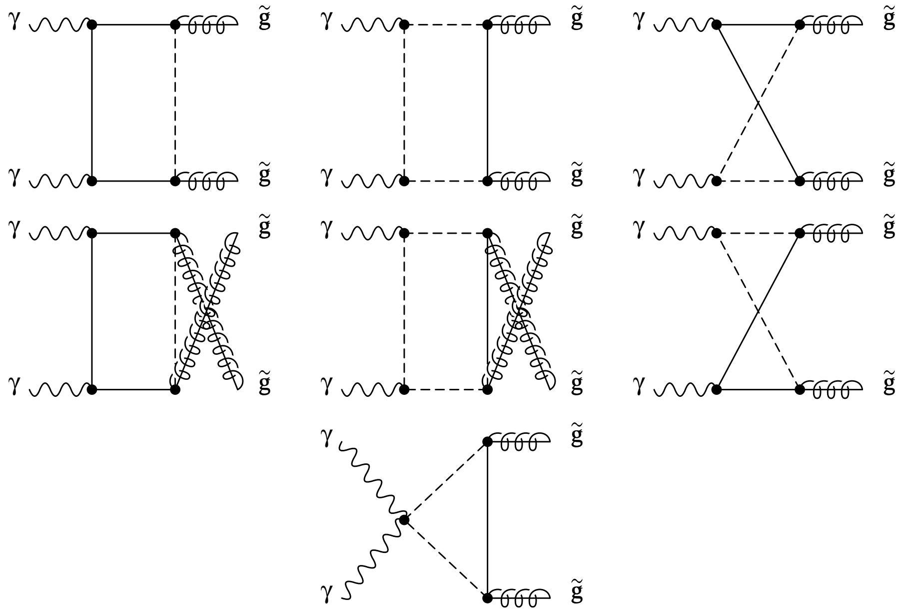
SUSY PARTICLE PRODUCTION AT e^+e^- AND $\gamma\gamma$ COLLIDERS

- $e^+e^- \longrightarrow \tilde{g}\tilde{g}$: [Kileng, Osland, ZPC 66 (1995) 503; Berge, Klasen, to be published]



  - $g_{\gamma/Z^0\tilde{g}\tilde{g}} = 0 \longrightarrow$ 1-loop, UV-finite (C-functions), IR-finite ($m_{\tilde{q}} \neq 0$)
  - $\tilde{g}$ =Majorana fermions $\longrightarrow$ $\not{P}$ axial vector coupling
  - Photon exchange cancels for $m_{\tilde{u}_L} = m_{\tilde{u}_R}; m_{\tilde{d}_L} = m_{\tilde{d}_R}$
  - Z^0 exchange cancels for $m_u = m_d; m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_{\tilde{d}_L} = m_{\tilde{d}_R}$
  - $m_t \gg m_b,\ m_{\tilde{t}_2} \gg m_{\tilde{t}_1} \longrightarrow$ largest contribution

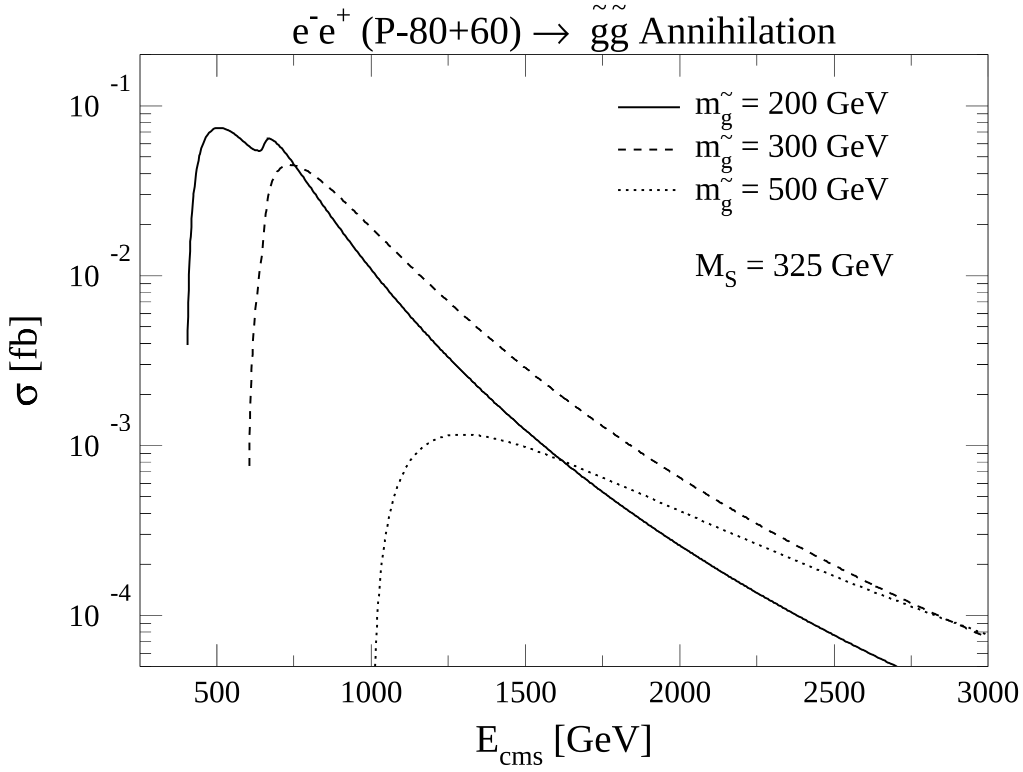
- $\gamma\gamma \longrightarrow \tilde{g}\tilde{g}$: [Berge, Klasen, to be published]



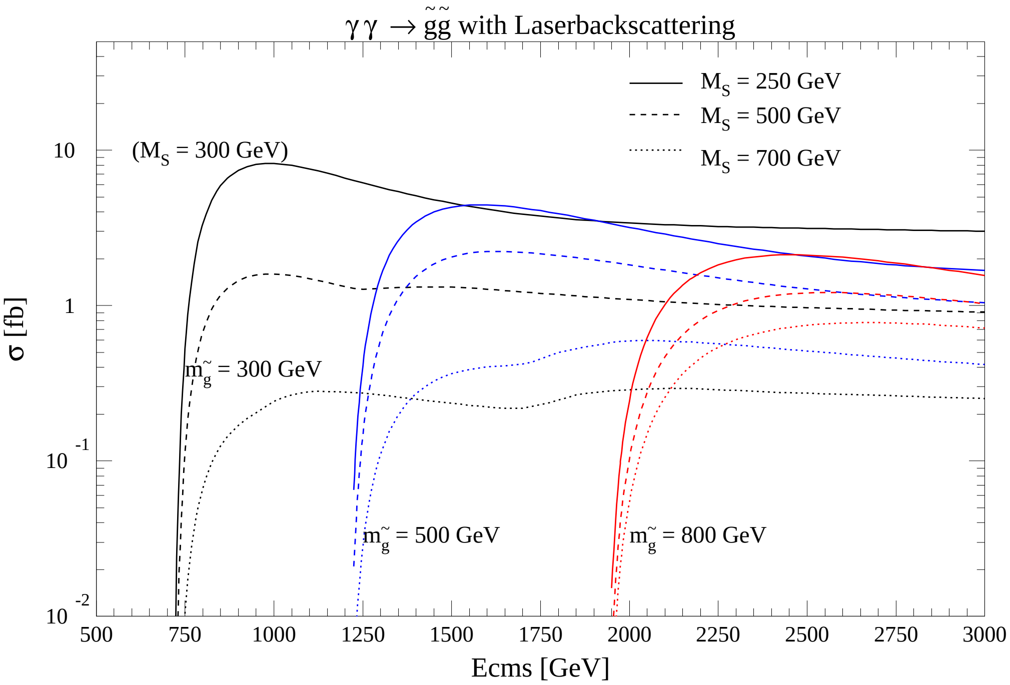
  - $g_{\gamma\tilde{g}\tilde{g}} = 0 \longrightarrow$ 1-loop, UV-finite (C, D-functions), IR-finite ($m_{\tilde{q}} \neq 0$)
  - Depends on physical $(m_q,\ m_{\tilde{q}},\ m_{\tilde{g}}),\ (e_q,\ e_{\tilde{q}})$
  - No cancellations, single squark exchange dominates

SUSY PARTICLE PRODUCTION AT e^+e^- AND $\gamma\gamma$ COLLIDERS

- $e^+e^- \longrightarrow \tilde{g}\tilde{g}$: [Berge, Klasen, to be published]



- $\gamma\gamma \longrightarrow \tilde{g}\tilde{g}$: [Berge, Klasen, to be published]



SUSY STRONG COUPLING CONSTANT AND PARTON DENSITIES

- Usually heavy particles ($t,\ \tilde{q},\ \tilde{g}$) are decoupled, since $m^2 \gg Q^2$
- Strong coupling constant: [Antoniadis *et al.*, PLB 262 (1991) 109; Jezabek, Kühn, 301 (1993) 121] [Machacek, Vaughn, NPB 222 (1983) 83]

$$\frac{\alpha(Q^2)}{2\pi} = \frac{2}{\beta_0}\frac{1}{\ln(Q^2/\Lambda^2)}\left(1 - \frac{\beta_1}{\beta_0}\frac{\ln\ln(Q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)} + O(\frac{1}{\ln^2(Q^2/\Lambda^2)})\right)$$

$$\beta_0^{\mathrm{SM}} = \frac{11}{3}C_A - \frac{4}{3}T_R n_f,\ \beta_1^{\mathrm{SM}} = \frac{34}{3}C_A^2 - \frac{20}{3}C_A T_R n_f - 4C_F T_R n_f$$

$$\beta_0^{\mathrm{SUSY}} = \beta_0^{\mathrm{SM}} - \frac{2}{3}C_A n_{\tilde{g}} - \frac{2}{3}T_R n_{\tilde{q}},\ \beta_1^{\mathrm{SUSY}} = \beta_1^{\mathrm{SM}} - 16C_A n_{\tilde{g}} - \frac{4}{3}C_A T_R n_{\tilde{q}} - 8C_F T_R n_{\tilde{q}}$$

- Non-singlet evolution equations: [Kounnas, Ross, NPB 214 (1983) 317]

$$\begin{aligned} Q^2\frac{d}{dQ^2}q_V(x,Q^2) &= \frac{\alpha(Q^2)}{2\pi}\left(P_{qq}\otimes q_V + P_{q\tilde{q}}\otimes \tilde{q_V}\right) \\ Q^2\frac{d}{dQ^2}\tilde{q}_V(x,Q^2) &= \frac{\alpha(Q^2)}{2\pi}\left(P_{\tilde{q}q}\otimes q_V + P_{\tilde{q}\tilde{q}}\otimes \tilde{q_V}\right), \end{aligned}$$

- Singlet evolution equations: [Kounnas, Ross, NPB 214 (1983) 317]

$$Q^2\frac{d}{dQ^2}\begin{bmatrix} G(x,Q^2) \\ \lambda(x,Q^2) \\ q^+(x,Q^2) \\ \tilde{q}^+(x,Q^2)\end{bmatrix} = \begin{bmatrix} P_{GG} & P_{G\lambda} & P_{Gq} & P_{G\tilde{q}} \\ P_{\lambda G} & P_{\lambda\lambda} & P_{\lambda q} & P_{\lambda\tilde{q}} \\ P_{qG} & P_{q\lambda} & P_{qq} & P_{qs} \\ P_{sG} & P_{s\lambda} & P_{\tilde{q}q} & P_{\tilde{q}\tilde{q}}\end{bmatrix}\otimes\begin{bmatrix} G(x,Q^2) \\ \lambda(x,Q^2) \\ q^+(x,Q^2) \\ \tilde{q}^+(x,Q^2)\end{bmatrix}.$$

- SUSY relations among splitting functions:

$$\begin{aligned} P_{gg} + P_{\lambda g} &= P_{g\lambda} + P_{\lambda\lambda} \quad , \quad P_{qg} + P_{sg} = P_{q\lambda} + P_{s\lambda} \\ P_{gq} + P_{\lambda q} &= P_{gs} + P_{\lambda s} \quad , \quad P_{qq} + P_{sq} = P_{qs} + P_{ss} \end{aligned}$$

- SUSY sum rules (momentum and baryon number conservation):

$$\begin{aligned} 1 &= \int_0^1 x\,dx\left(xG(x) + x\lambda(x) + xq^{(+)}(x) + x\tilde{q}^{(+)}(x)\right) \\ 3 &= \int_0^1 dx\left(q^{(-)}(x) + \tilde{q}^{(-)}(x)\right) \end{aligned}$$

SUSY SPLITTING FUNCTIONS

[Kounnas, Ross, NPB 214 (1983) 317]

$$P_{GG} = 2C_A\left[\frac{1+x^2}{(1-x)_+} + \frac{1+(1-x)^2}{x} - \left(x^2+(1-x)^2\right)\right] + [3C_A - T_R]\,\delta(1-x)$$

$$P_{\lambda G} = 2C_A\left[x^2+(1-x)^2\right]$$

$$P_{qG} = 2T_R\left[x^2+(1-x)^2\right]$$

$$P_{sG} = 2T_R\left[1-\left[x^2+(1-x)^2\right]\right]$$

$$P_{G\lambda} = 2C_A\left[\frac{1+(1-x)^2}{x}\right]$$

$$P_{\lambda\lambda} = 2C_A\left[\frac{1+x^2}{(1-x)_+}\right] + (3C_A - T_R)\,\delta(1-x)$$

$$P_{q\lambda} = 2T_R[1-x]$$

$$P_{s\lambda} = 2T_R[x]$$

$$P_{Gq} = 2C_F\left[\frac{1+(1-x)^2}{x}\right]$$

$$P_{\lambda q} = 2C_F(1-x)$$

$$P_{qq} = 2C_F\left[\frac{1+x^2}{(1-x)_+}\right] + 2C_F\delta(1-x)$$

$$P_{sq} = 2C_F[x]$$

$$P_{Gs} = 2C_F\left[\frac{1+(1-x)^2}{x} - x\right]$$

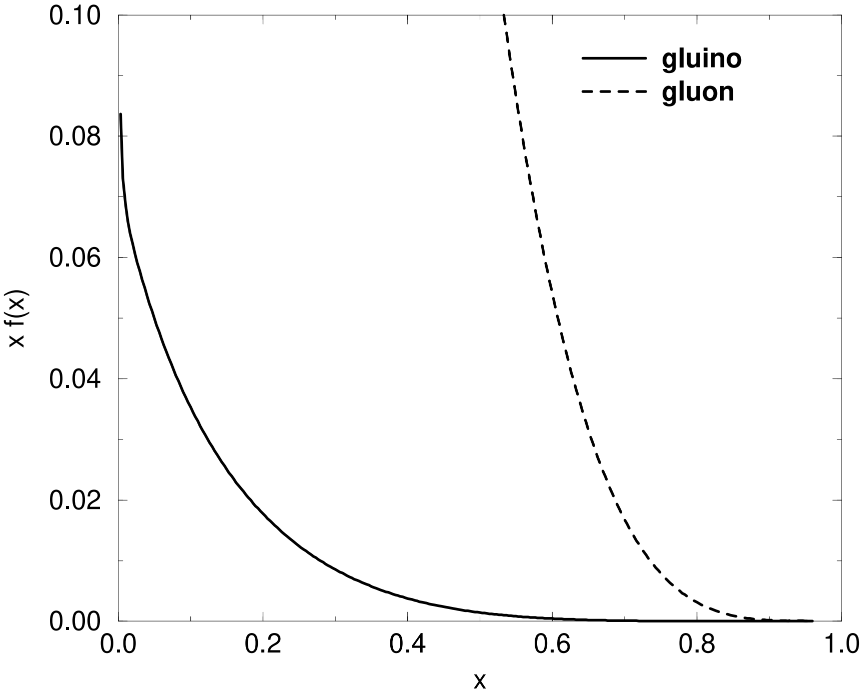
$$P_{\lambda s} = 2C_F[1]$$

$$P_{qs} = 2C_F[1]$$

$$P_{ss} = 2C_F\left[\frac{1+x^2}{(1-x)_+} - (1-x)\right] + 2C_F\delta(1-x)$$

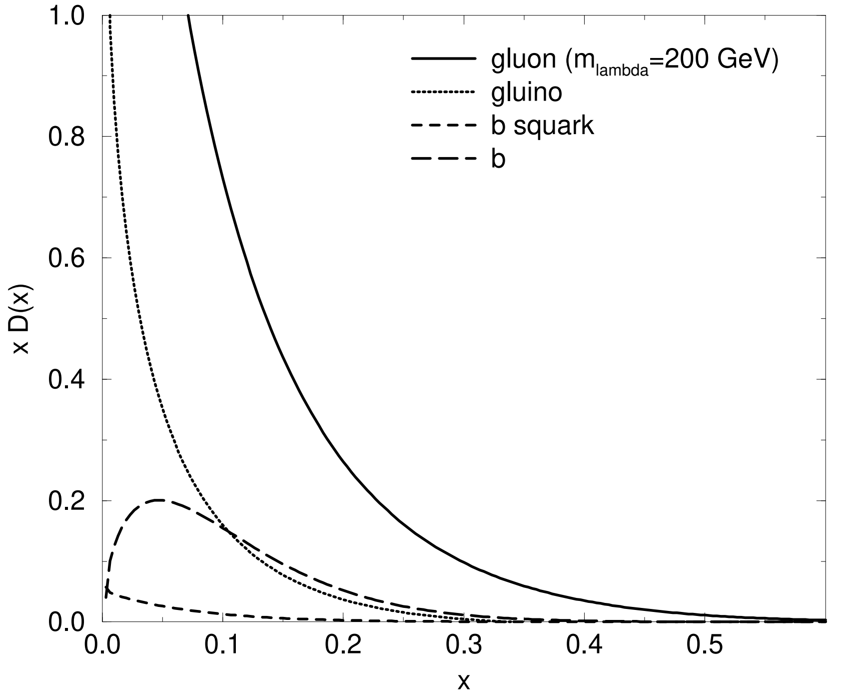
SUSY PARTON DENSITIES AND FRAGMENTATION FUNCTIONS

- Gluon/gluino densities in protons: [Corianò, NPB 627 (2002) 66]



  – $m_{\tilde{g}} = 30$ GeV, $Q = 100$ GeV

- (S)bottom/gluino fragmentation functions: [Corianò, Faraggi, PRD 65 (2002) 075001]



  – $m_{\tilde{g}} = 100$ GeV, $Q = 10^5$ GeV

SUMMARY

- What is SUSY and why is it interesting?
  - Unifies fermions and bosons, matter and forces, couplings
  - Can include gravity, appears in string theories
  - Stabilizes Higgs mass, can break electroweak symmetry
  - MSSM has one superpartner for each SM particle, 2HDM
  - SUSY-breaking introduces soft masses (and phases)
- The Feynman rules of SUSY-QCD
  - Fermion direction for Majorana fermions
  - Yukawa couplings contain γ_5
  - Dimensional regularization vs. dimensional reduction
  - Treatment of intermediate unstable particles
- QCD effects in SUSY and vice versa
  - SUSY Ward identities for QCD helicity amplitudes
  - SUSY effects in SM (bottom) production and decay
  - SUSY particle production at hadron, e^+e^-, and $\gamma\gamma$ colliders
  - Higher order SUSY-QCD corrections
  - SUSY effects in α_s, PDFs, and FFs