Charm and Bottom Quark Production

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OUTLINE

- Statement of the problem
- Status report:
  - Comparison of Data & Theory
- How do we make heavy quarks
- B-Hadroproduction: A case study
- Mass-Independent Evolution
  - Why is it valid?
- Conclusions
- Lunch

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A Thought Experiment:

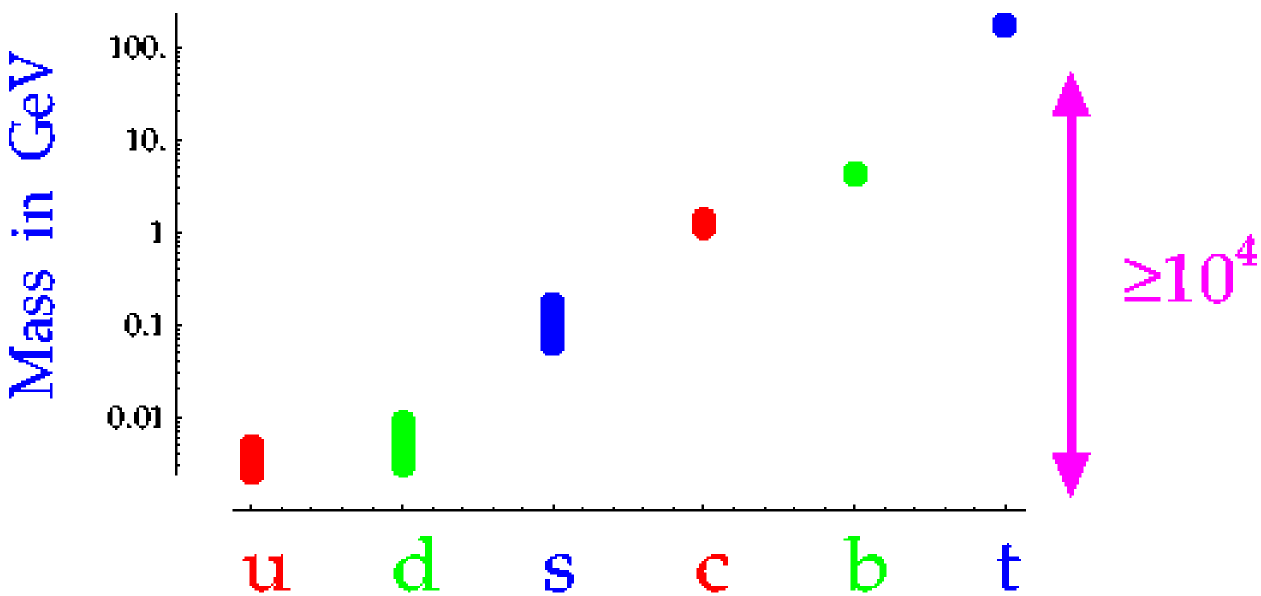
What is the ideal way to learn about quark masses and their effects on a physical process?

As a theorists, I simply run my calculation over the full range of mass values from 0 to ∞, and study the behavior.

Wouldn't it be great
if the experiments could do the same???

The UP Side

Quark Masses Span Wide Dynamical Range ~ 10^4



u	0.008
d	0.015
s	0.300
c	1.500
b	5.000
t	175.000

We can't vary the quark mass continuously, but these ``notches" on our control panel give us a lot of flexibility

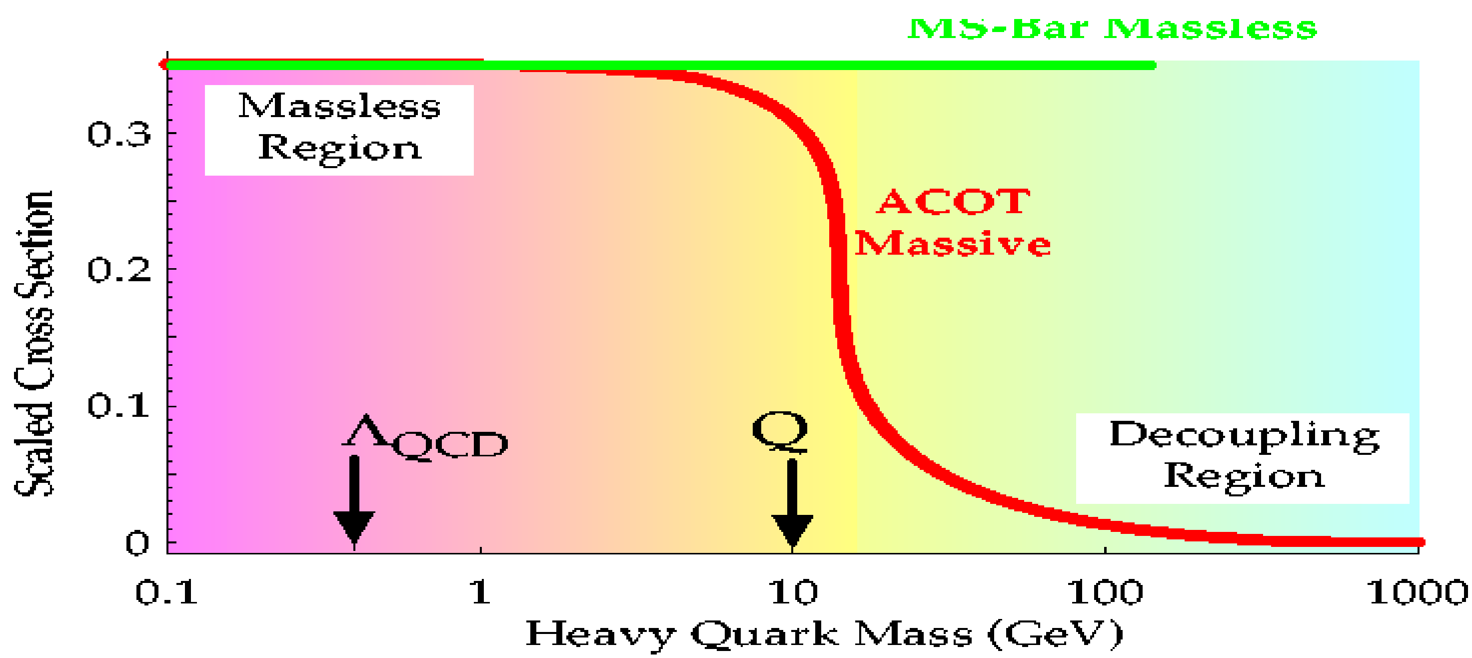
Theorists would much prefer that quark masses only come in 2 varieties:

$m = 0$: Massless case.

Mass plays no dynamic role
Well understood.

$m = \infty$: Infinite case.

Mass Decouples.
We can forget about this object



Welcome to the Multi-Scale Problem

Single-Scale Problem in Perturbation Theory:

$$\sigma = \sum_{N=1}^{\infty} (\alpha_s L)^N = \sum_{N=1}^{\infty} \left\{ \alpha_s(\mu) \log\left(\frac{E^2}{\mu^2}\right) \right\}^N$$

... where E is any relevant scale of the problem: Q, P_T, E_T,...

Use RGE to solve this. $\quad \dfrac{d\sigma}{d\log\mu^2} = \;...$

Multi-Scale Problem in Perturbation Theory:

What do we do if we have 2 scales???

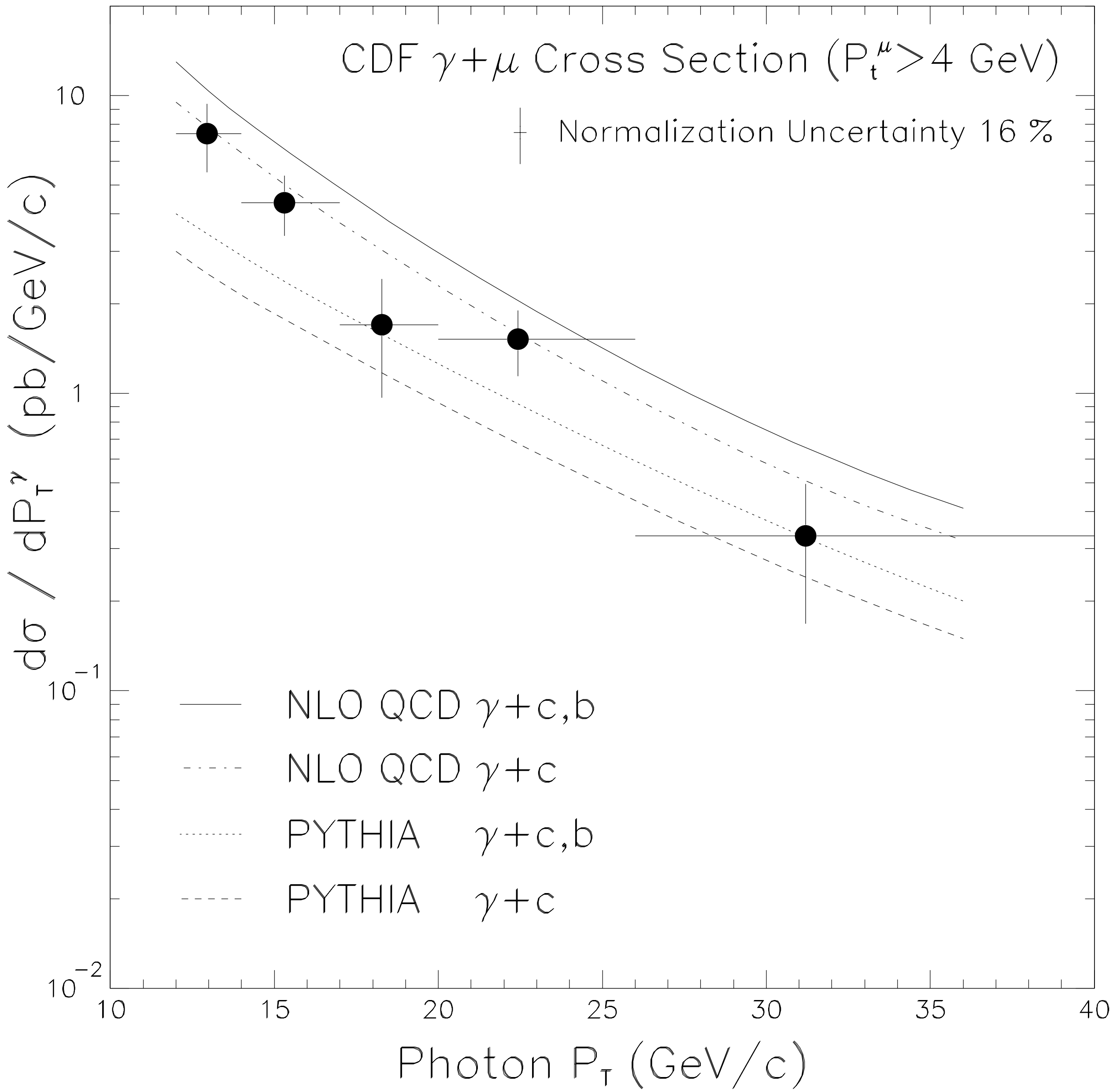
$$\log\left(\frac{E^2}{\mu^2}\right) \qquad \log\left(\frac{M_H^2}{\mu^2}\right)$$

... life gets interesting.

Status Report:

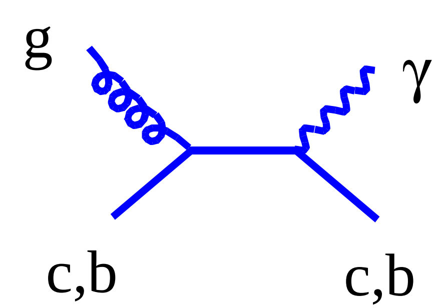
Comparison of Data & Theory

Hadroproduction of Charm (& Beauty) at Tevatron

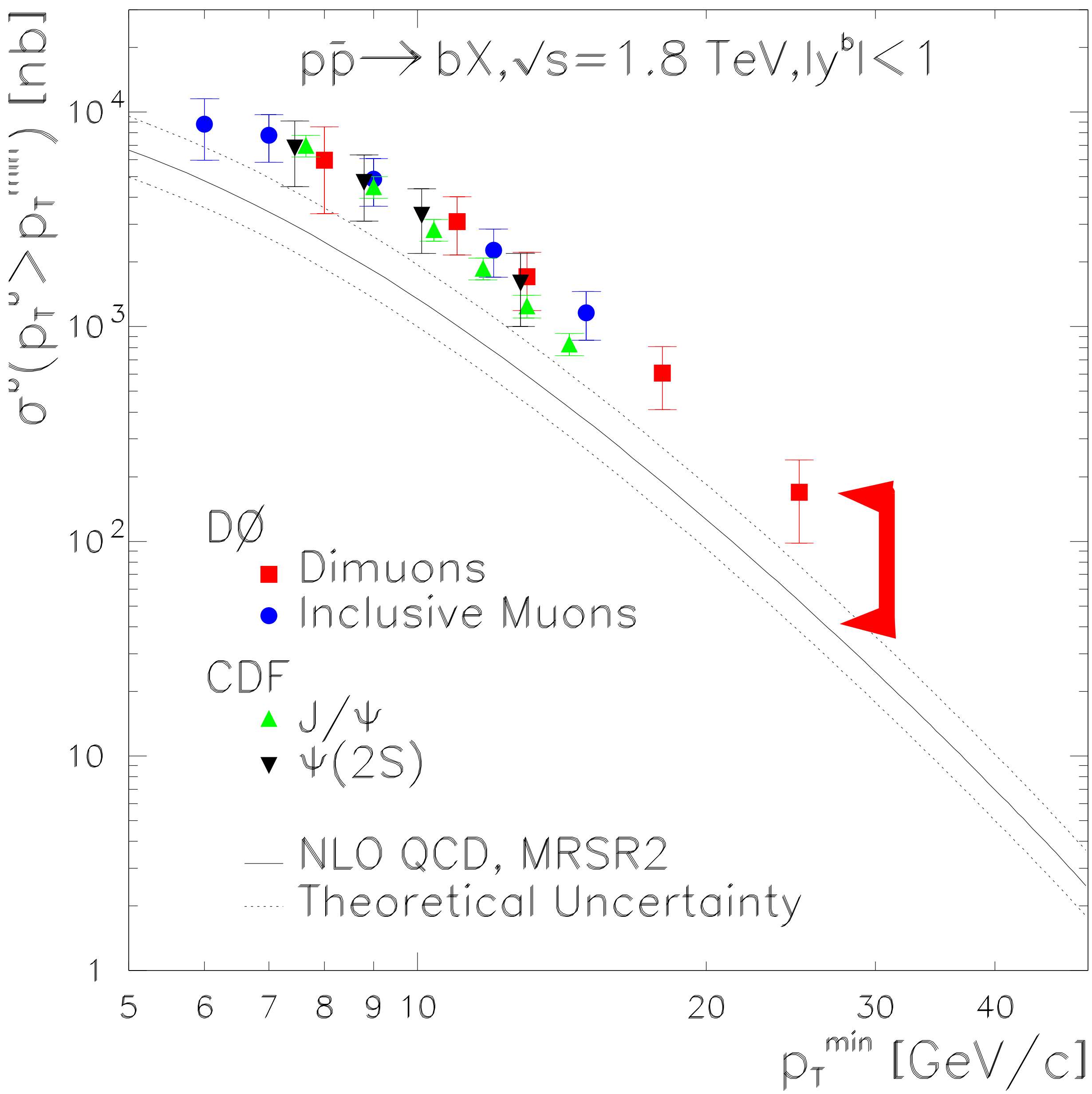


Comparison of Run I Data with NLO Theory

Reasonable agreement within theory uncertainty



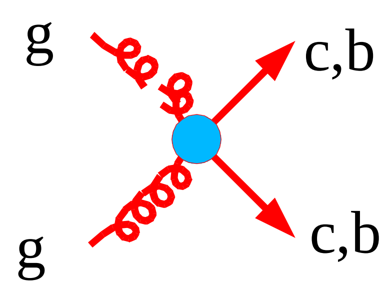
Hadroproduction of Beauty at Tevatron



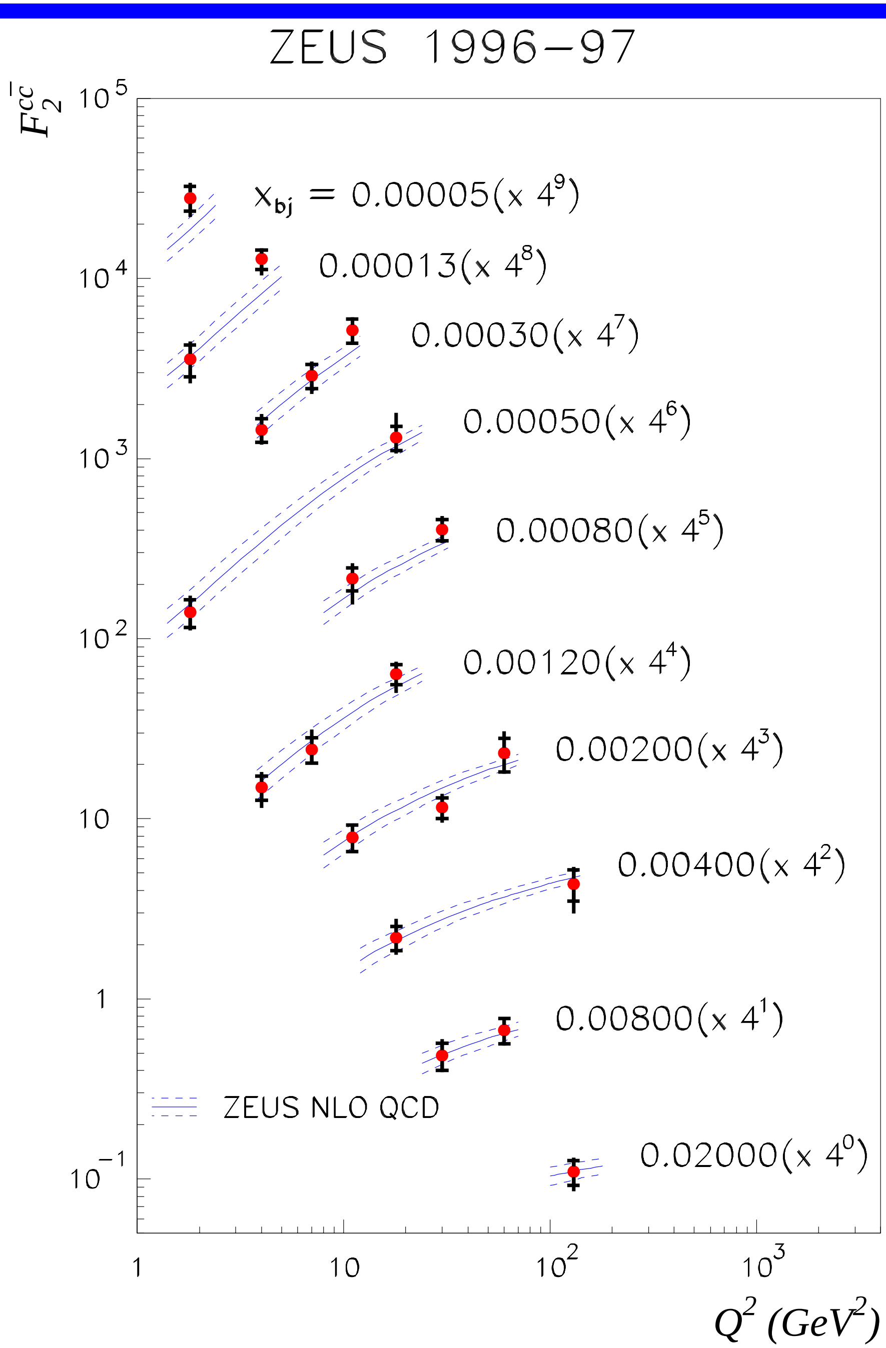
Comparison of Run I data with NLO Theory

Data is high by factor of 2x or 3x

... even given μ variation



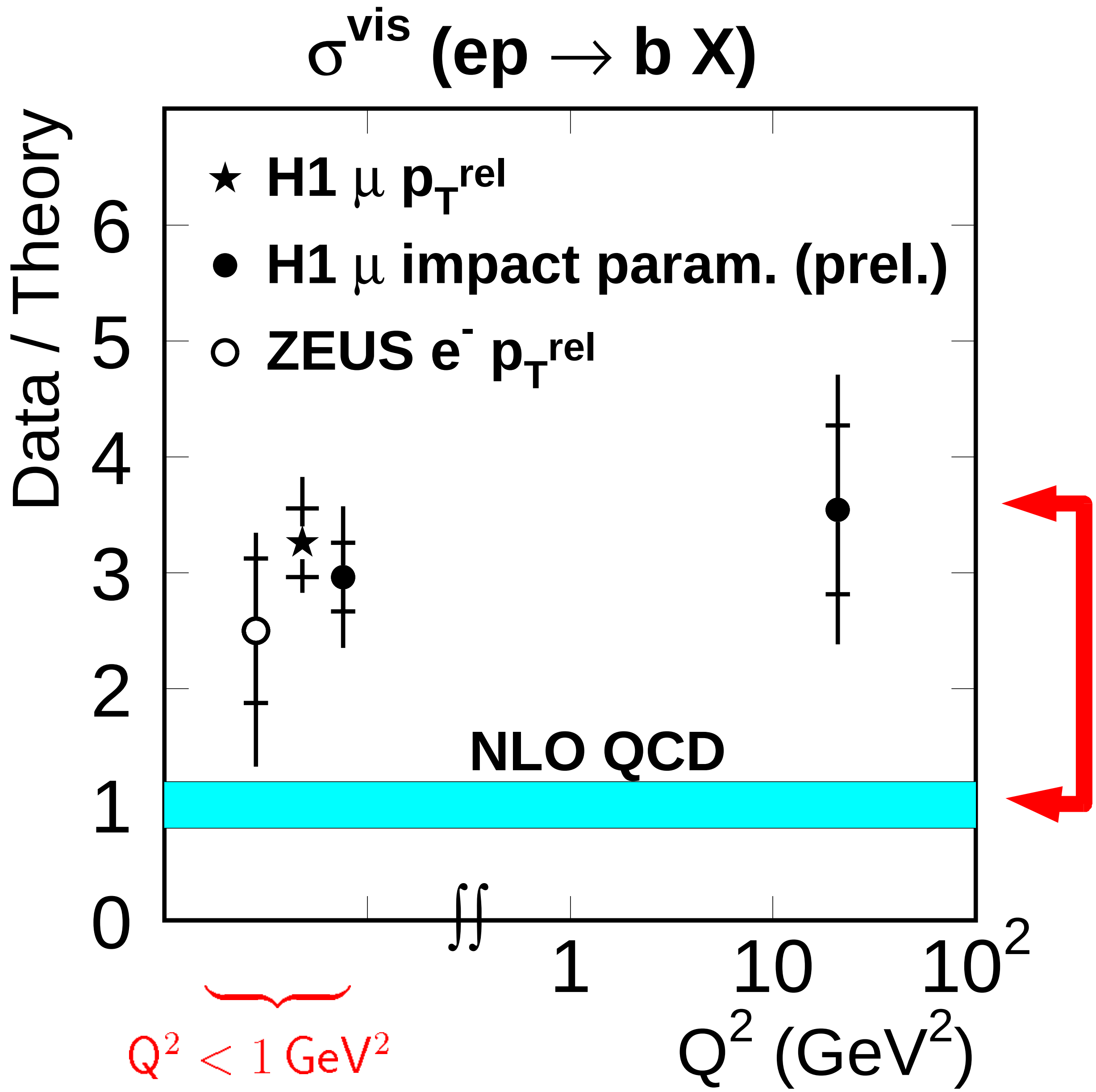
Charm Production at HERA



At HERA,
Charm production matches
well with NLO calculation

What about beauty???

Beauty Production at HERA

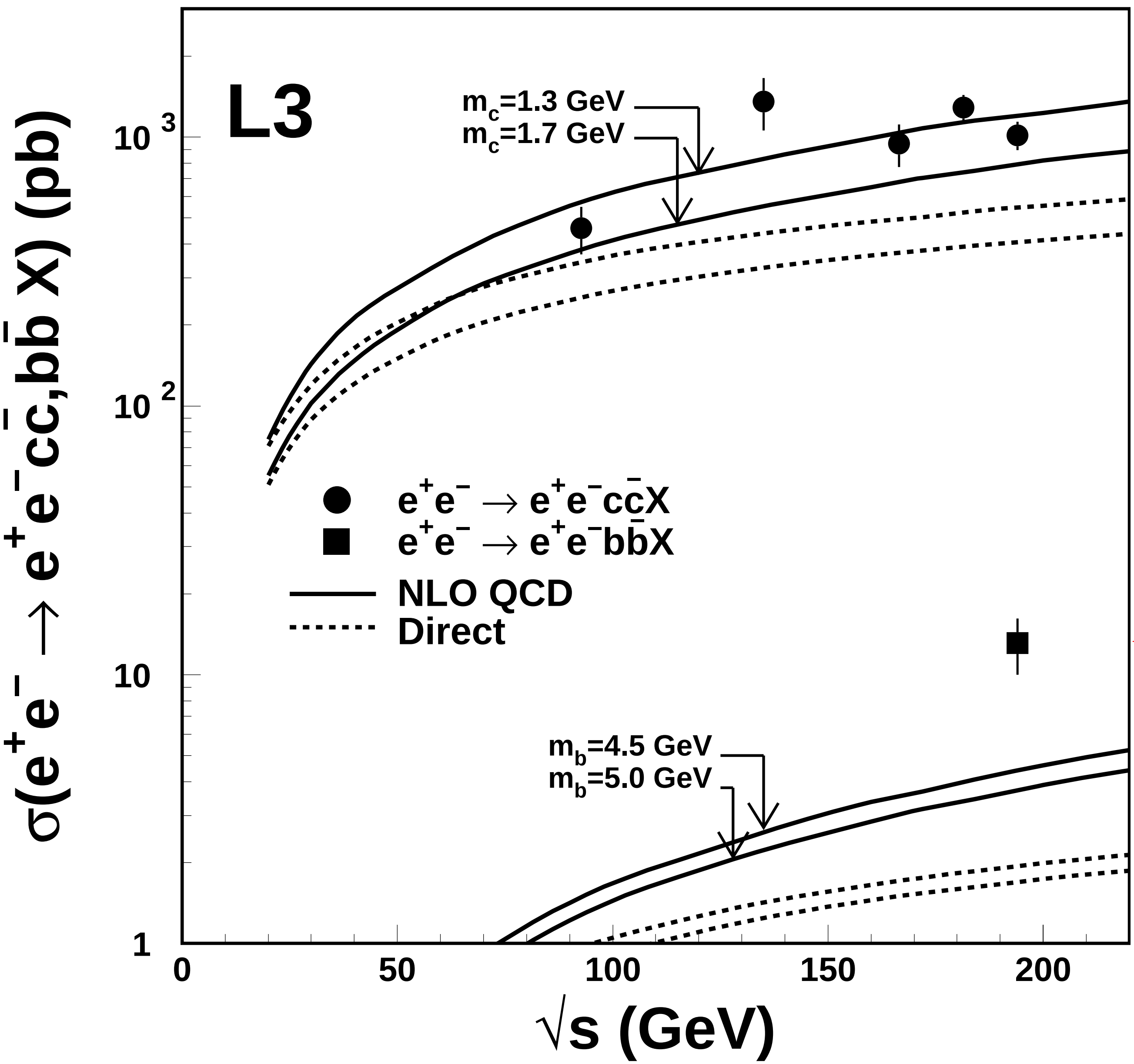


Comparison of HERA data with NLO Theory

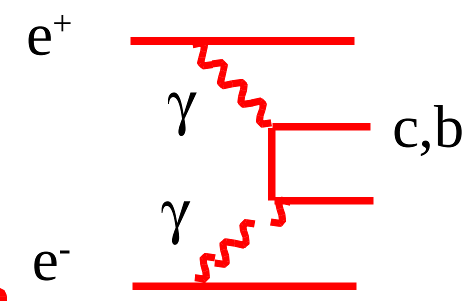
Data is high by factor of 2x or 3x

... even given μ variation

Charm and Bottom Production at LEP



Comparison of LEP data with NLO Theory



Charm is reasonable

Bottom data is high by factor of 2x or 3x

General Trend:

Charm: in good shape

Bottom: data lies above theory

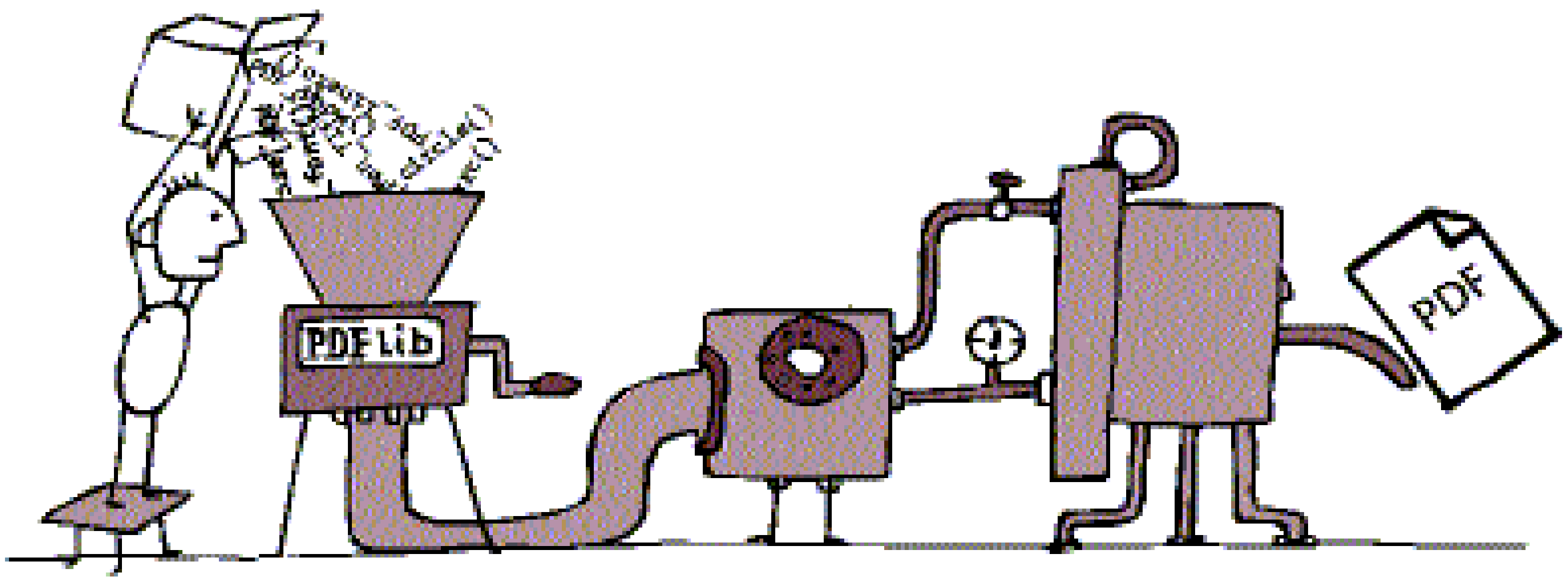
How can this make sense???

$m_b \approx 4.5$ GeV $\qquad m_c \approx 1.5$ GeV

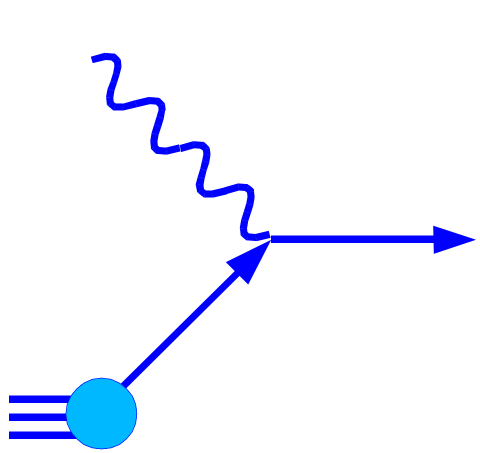
$\mathrm{Log}[m_b/m_c] \approx \mathrm{Log}[3] \approx$ Not a big number!!!

$\alpha_s(m_b) < \alpha_s(m_c)$

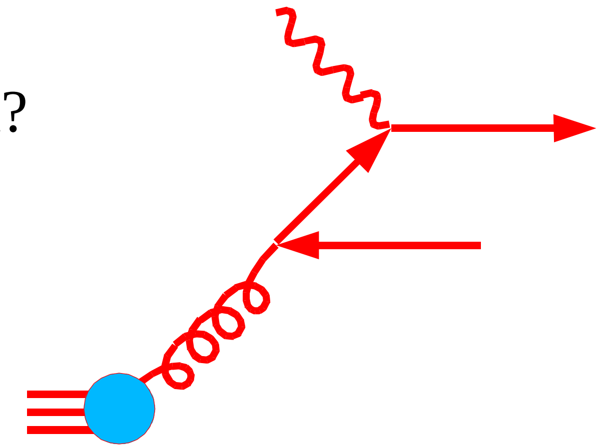
How do we make heavy quarks



Production of Heavy Quarks: The Problem



Which is the correct
production mechanism?



Heavy Excitation (HE)

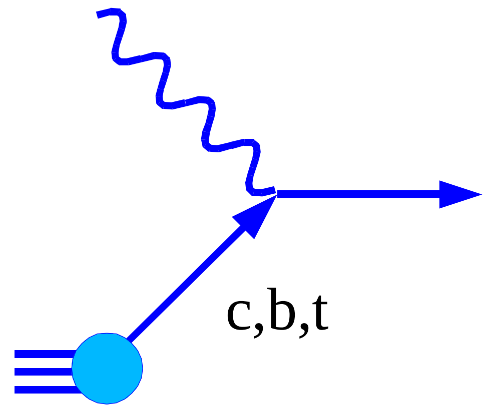
Quark	Channel
s	YES
t	NO
c	???
b	???

Heavy Creation (HC)

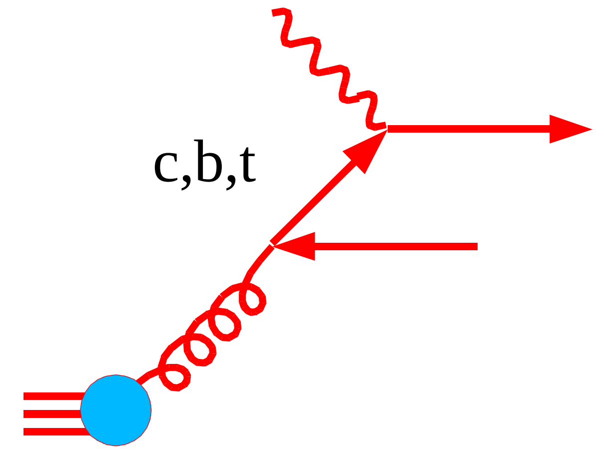
Quark	Channel
s	NO
t	YES
c	???
b	???

If you can't beat 'em, join 'em.

How to Join without ``Double Counting"???

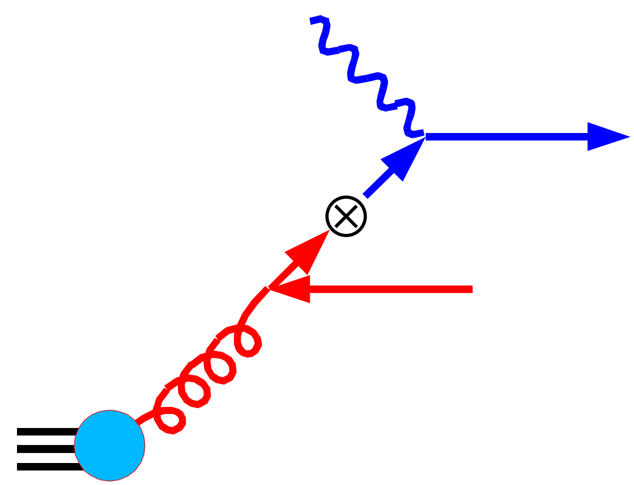


Heavy Excitation (HE)

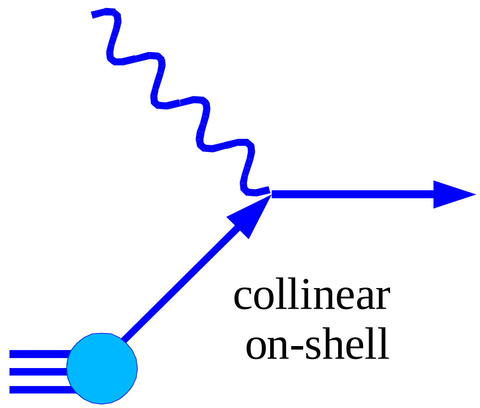


Heavy Creation (HC)

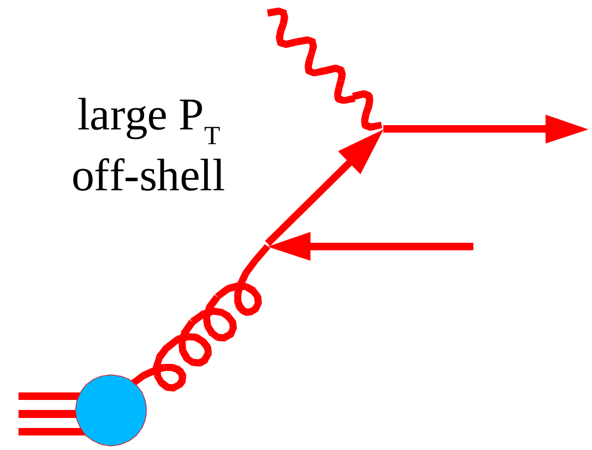
Wait a minute!
Since the heavy
quark originally came
from a gluon
splitting, these
diagrams are
Double Counting



How to Join without ``Double Counting"???

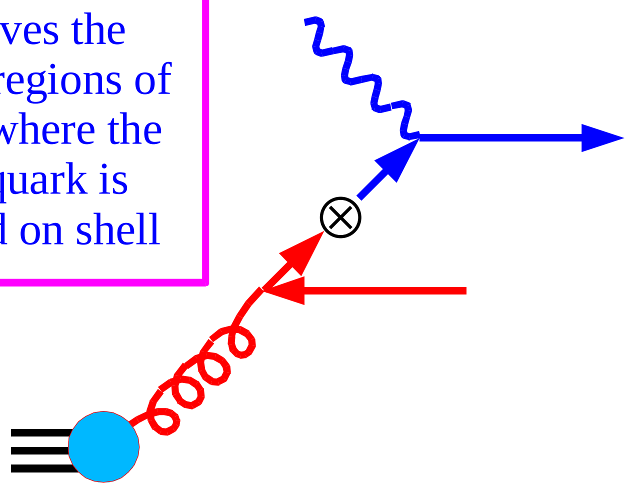


Heavy Excitation (HE)



Heavy Creation (HC)

SUB removes the overlapping regions of phase space where the t-channel quark is collinear and on shell



Subtraction (SUB)

TOT = HE + $\underbrace{\textbf{(HC – SUB)}}_{\textit{Formally, NLO}}$

There is a rigorous factorization proof ...

Ingredients of Factorization

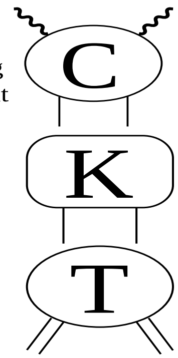
Decompose into (t-channel) 2PI amplitudes:

$$\sigma = \Sigma_{N=1}^{\infty} \; C \; (K)^N \; T \; + \textit{Non-leading}$$

Hard scattering coefficient

Kernel

Target



Collins, Soper, Sterman. Perturbative QCD, World Scientific (1989). Collins, in preparation

After reorganization of the infinite sum:

$$\sigma \approx \overbrace{\underbrace{C\,[1-(1-Z)\,K\,]^{-1}}_{\substack{\text{Wilson Coefficient} \\ (\text{Hard Scatt. } \hat{\sigma})}}\;Z\;\underbrace{[1-K\,]^{-1}\,T}_{\substack{\text{Parton} \\ \text{Distribution}}}}^{\text{Parton Model}} + \overbrace{C\,[1-(1-Z)\,K\,]^{-1}\,(1-Z)\,T}^{\text{Remainder}}$$

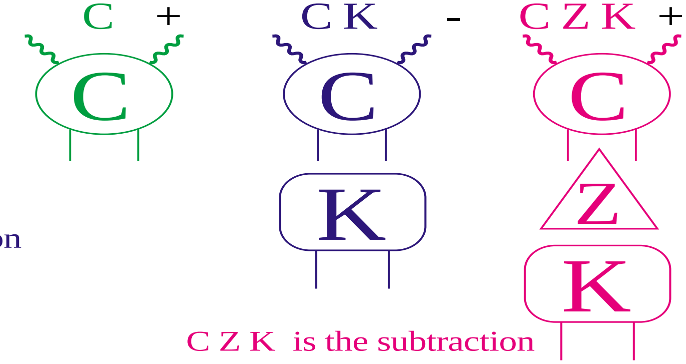
Z: collinear projection

Power Suppressed

Wilson Coefficient:

$$C\,[1-(1-Z)\,K\,]^{-1} \approx \underset{\text{Leading Order}}{C} + \overbrace{C\,K - C\,Z\,K}^{\text{Next to Leading Order}} + \ldots$$

All orders result



Wilson Coefficient:
IR safe "hard"
scattering cross section

C Z K is the subtraction

A formal proof was constructed by numerous groups.

This proof was explicitly extended to the case of massive quarks

(Collins,1998)

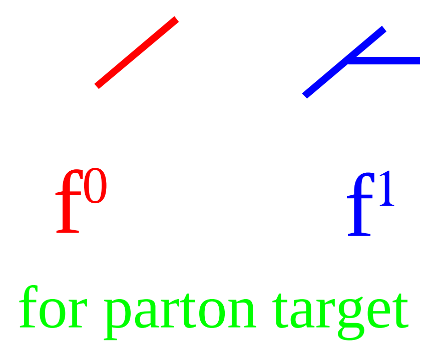
THOUGH EXPERIMENT
To keep things simple, let's consider scattering off a parton target.

Application of Factorization Formula at Leading Order (LO)

Basic Factorization Formula $$\sigma = f \otimes \omega \otimes d \ + O(\Lambda^2/Q^2)$$

At Zeroth Order:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 \ + O(\Lambda^2/Q^2)$$



Use: $f^0 = \delta$ and $d^0 = \delta$ for a parton target.

Therefore:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 = \delta \otimes \omega^0 \otimes \delta = \omega^0$$

$$\sigma^0 = \omega^0$$

Warning: This trivial result leads to many misconceptions at higher orders

Basic Factorization Formula

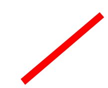
$$\sigma = f \otimes \omega \otimes d \; + O(\Lambda^2 / Q^2)$$

At First Order:

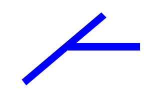
$$\sigma^1 = f^1 \otimes \omega^0 \otimes d^0 + f^0 \otimes \omega^1 \otimes d^0 + f^0 \otimes \omega^0 \otimes d^1$$

$$\sigma^1 = \quad f^1 \otimes \sigma^0 + \omega^1 + \sigma^0 \otimes d^1$$

We used: $f^0 = \delta$ and $d^0 = \delta$ for a parton target.

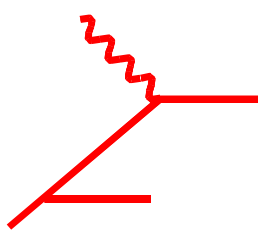


f^0

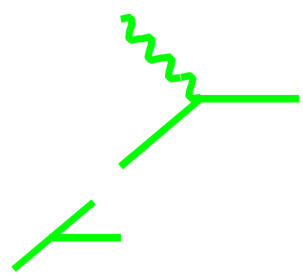


f^1

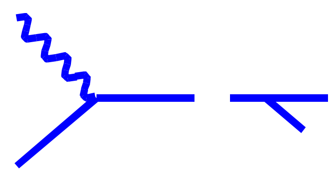
Therefore: $$\omega^1 = \sigma^1 - f^1 \otimes \sigma^0 - \sigma^0 \otimes d^1$$



σ^1



$f^1 \otimes \sigma^0$



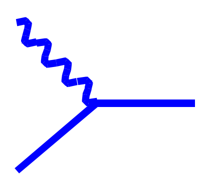
$\sigma^0 \otimes d^1$

Combined Result:

$$\underbrace{\omega^0+\omega^1}_{\text{TOT}}=\underbrace{\sigma^0}_{\text{HE}}+\underbrace{\sigma^1}_{\text{HC}}-\underbrace{\left\{f^1\otimes\sigma^0+\sigma^0\otimes d^1\right\}}_{\text{SUB}}$$

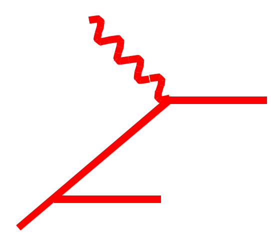
TOT

HE



Heavy Excitation

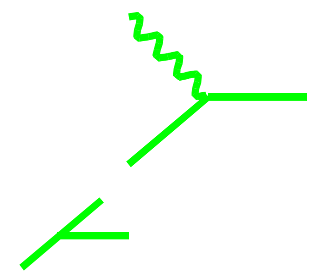
HC



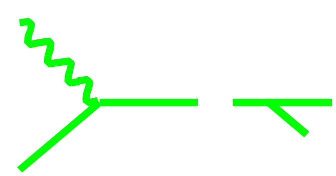
Heavy Excitation

SUB

Subtraction



$f^1\otimes\sigma^0$

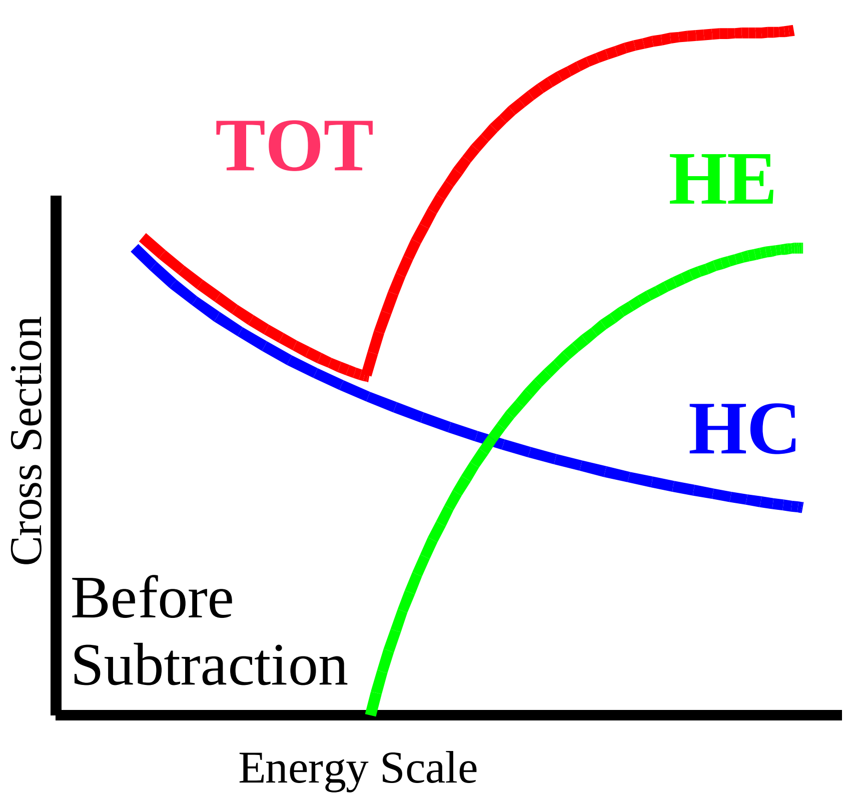


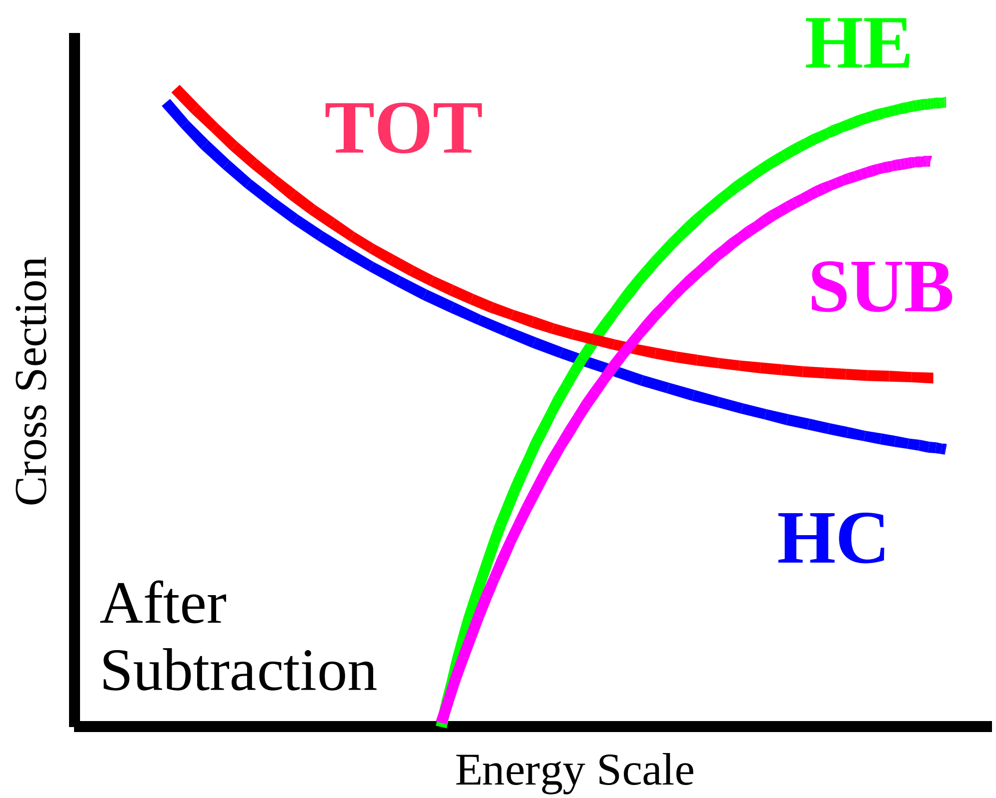
$\sigma^0\otimes d^1$

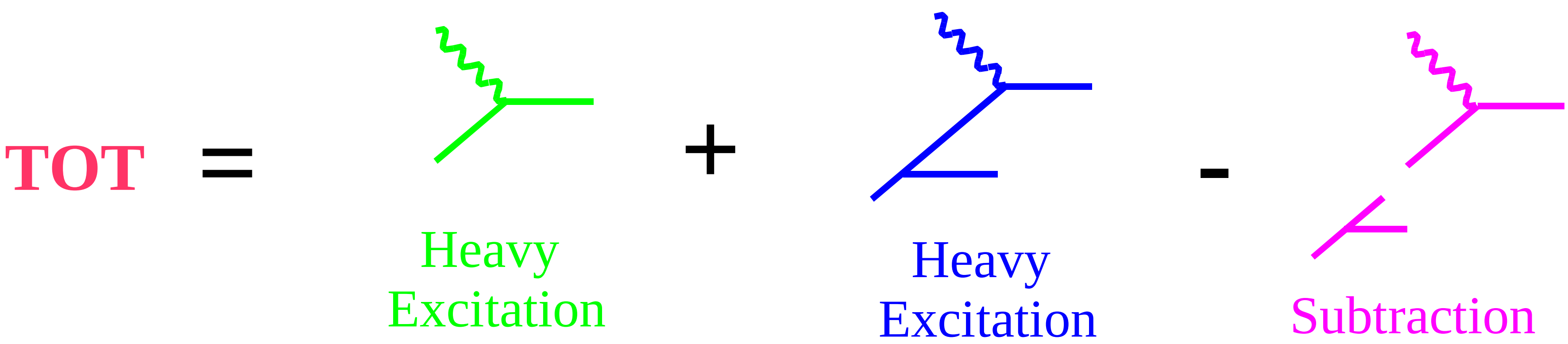
$$\boxed{\textbf{TOT} = \textbf{HE} + \textbf{HC} - \textbf{SUB}}$$

Interaction of the separate contributions vs. energy scale

Note, we'll see this again ...







An Example: How the separate pieces can conspire

Expand $f(x)=x$ in Taylor Series about x_0.

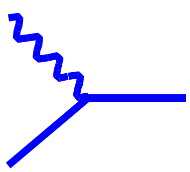
For $x_0=0$: $f(\varepsilon) = 0 + (\varepsilon - 0) + \ldots = \varepsilon$

For $x_0=1$: $f(\varepsilon) = 1 + (\varepsilon - 1) + \ldots = \varepsilon$

TOT

HE

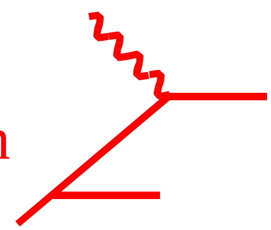
Heavy Excitation



σ^0

HC

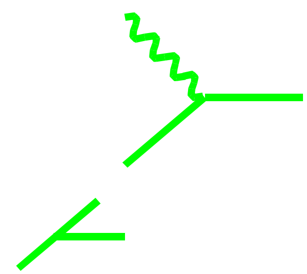
Heavy Excitation



σ^1

SUB

Subtraction



$f^1 \otimes \sigma^0$

$$\boxed{\mathbf{TOT = HE + HC - SUB}}$$

The Moral

**It doesn't matter which expansion point you use;
QCD will compensate (if you go to high enough order).**

In practice ...

**we are often limited to low-order calculations,
so it is wise to choose your expansion point carefully.**

HOMEWORK PROBLEM: WILSON COEFFICIENTS

Use the Basic Factorization Formula $\sigma = f \otimes \omega \otimes d \; + O(\Lambda^2/Q^2)$

At Second Order:

$$\sigma^2 = f^2 \otimes \omega^0 \otimes d^0 + \ldots$$
$$f^1 \otimes \omega^1 \otimes d^0 + \ldots.$$

Therefore:

$$\omega^2 = ???$$

- Compute ω^2 at second order.
- Make a diagrammatic representation of each term.

HOMEWORK PROBLEM: CONVOLUTIONS

Part 1) Show these 3 definitions are equivalent; work out the limits of integration.

$$f \otimes g = \int f(x)\, g(y)\, \delta(z - x * y)\, dx\, dy$$

$$f \otimes g = \int f(x)\, g\left(\frac{z}{x}\right) \frac{dx}{x}$$

$$f \otimes g = \int f\left(\frac{z}{y}\right) g(y) \frac{dy}{y}$$

Part 2) Show convolutions are the ``natural" way to multiply probabilities.

If f represents the heads/tails probability distribution for a single coin flip, show that the distribution of 2 coins is $f \oplus f$ and 3 coins is $f \oplus f \oplus f$.

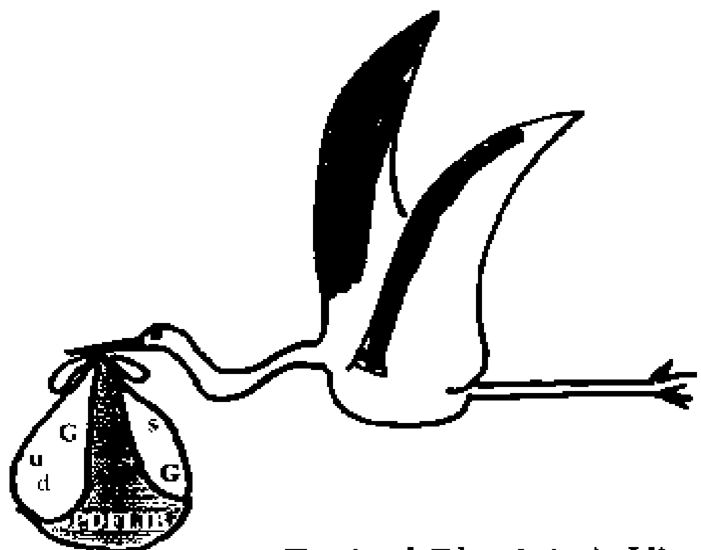
$$f \oplus g = \int f(x)\, g(y)\, \delta(z - (x + y))\, dx\, dy$$

$$f(x) = \frac{1}{2}(\delta(1 - x) + \delta(1 + x))$$

BONUS: How many processes can you think of that don't factorize?

B-Hadroproduction: A Case Study

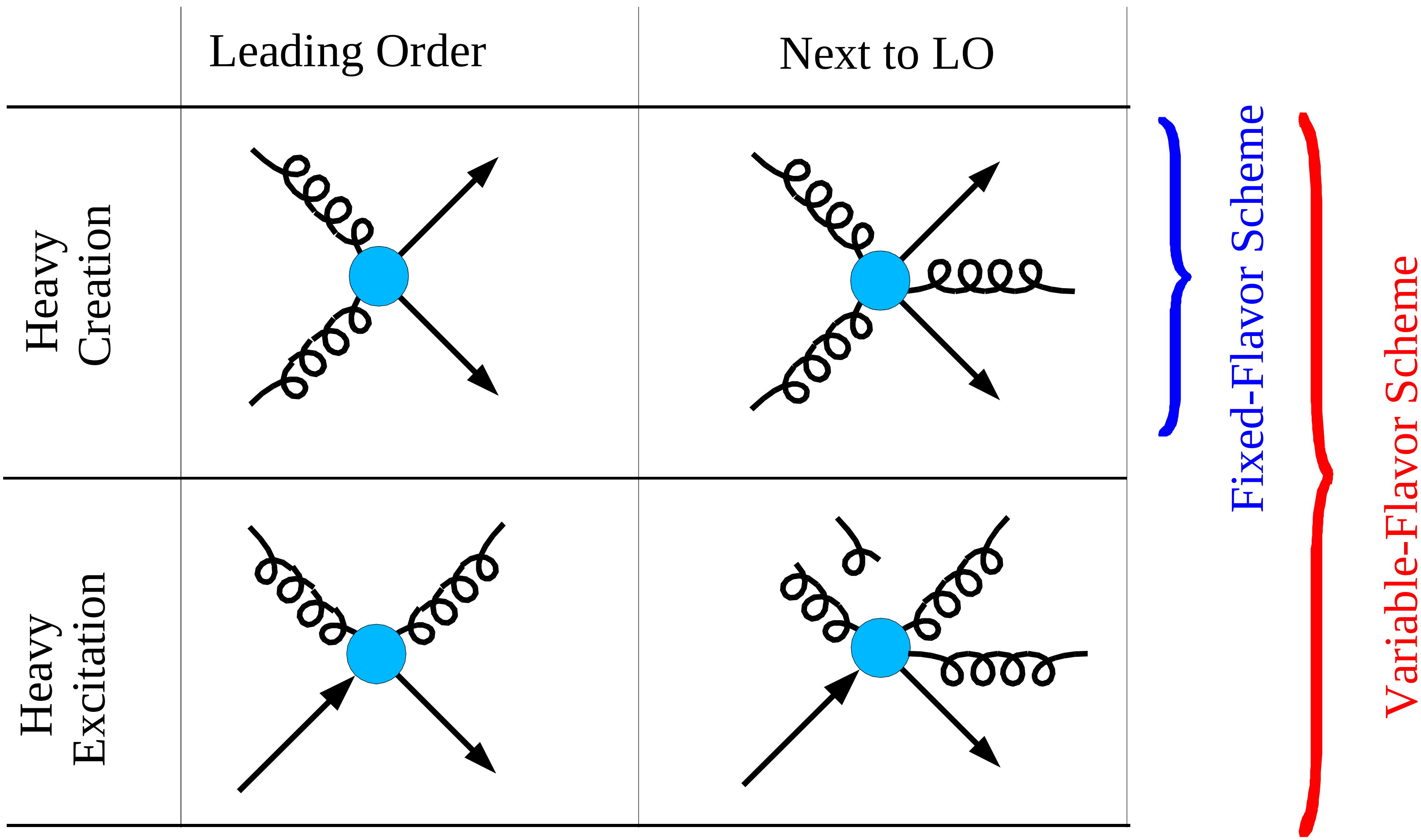
Where do PDF's come from???



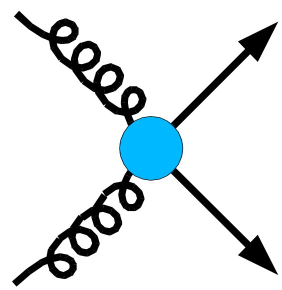
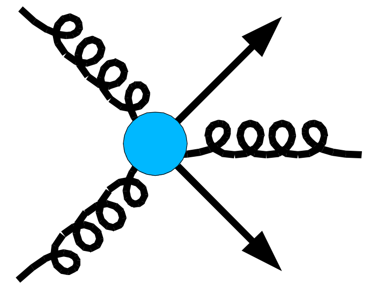
Typical Physicist's View

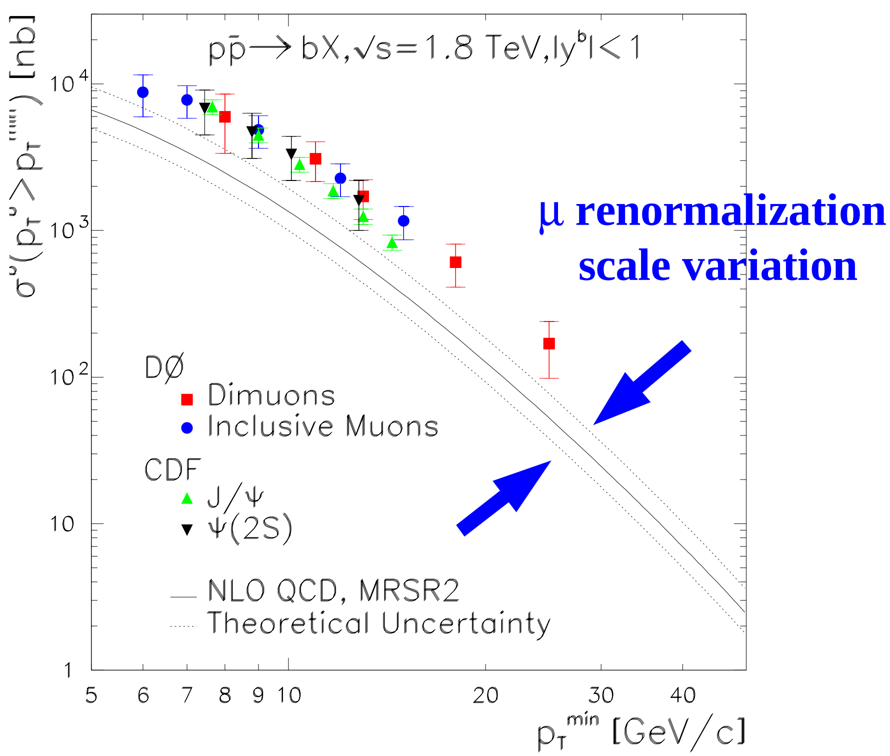
Drawing by Heidi Schellman

The Basic Contributions to Heavy Flavor Production



NLO Fixed-Flavor Scheme

	Leading Order		Next to LO
Heavy Creation		Surprise <<	

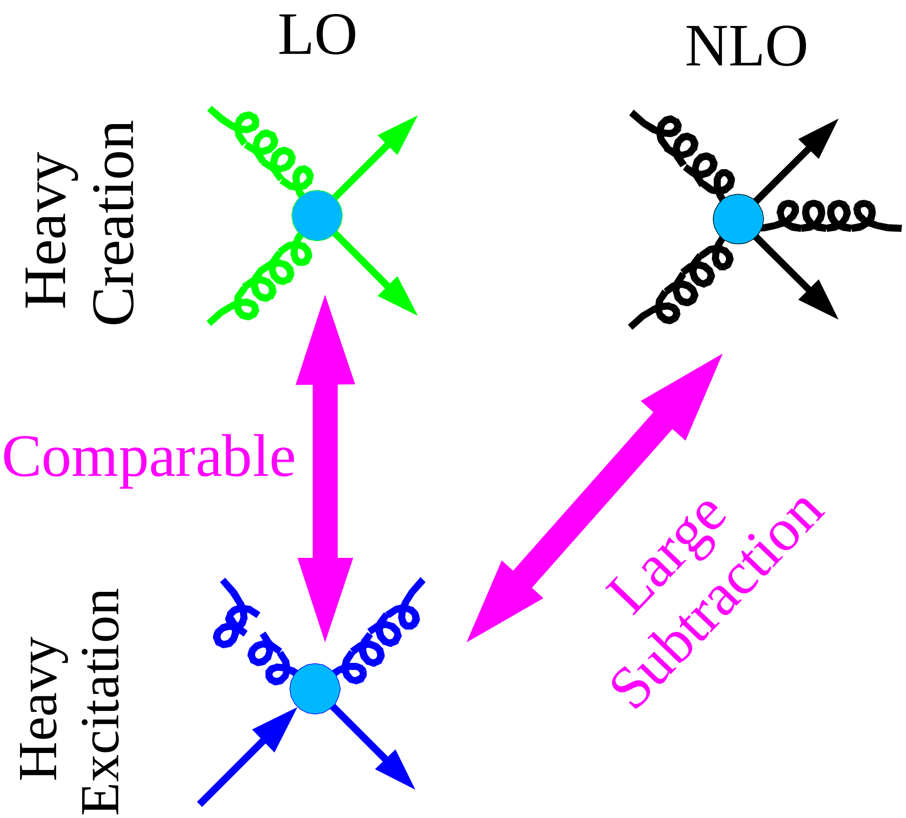


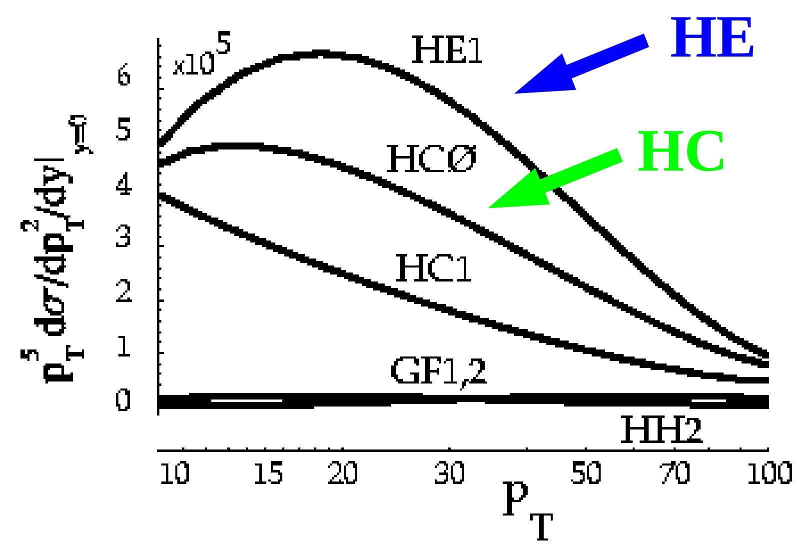
Surprise: NLO / LO ~ 2

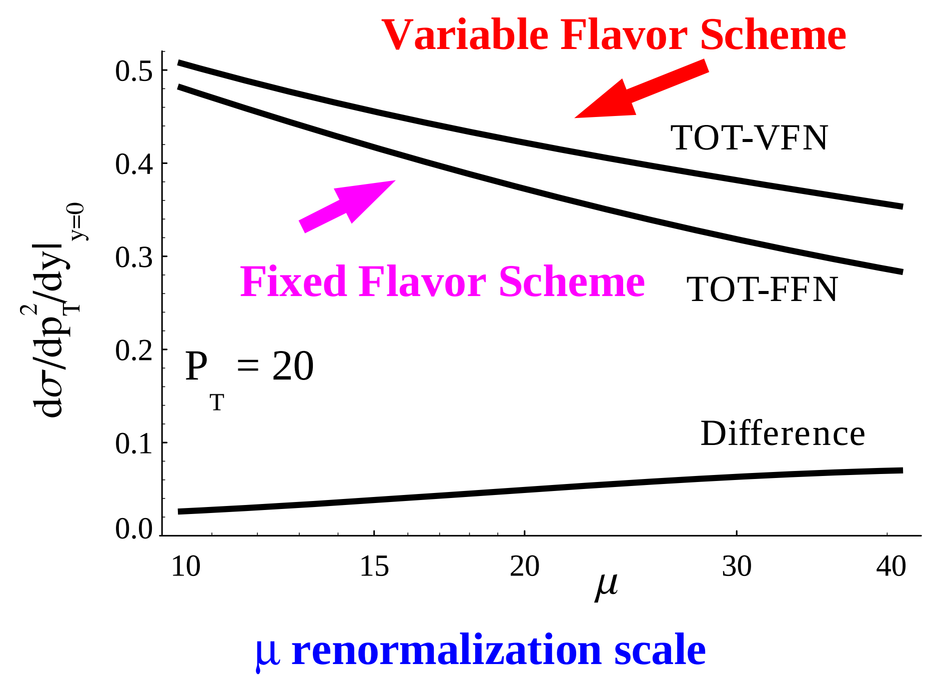
But, theory still below data

Nason, Dawson, Ellis
Beenakker, Kuijf, Van Neerven, Smith

NLO Variable Flavor Scheme





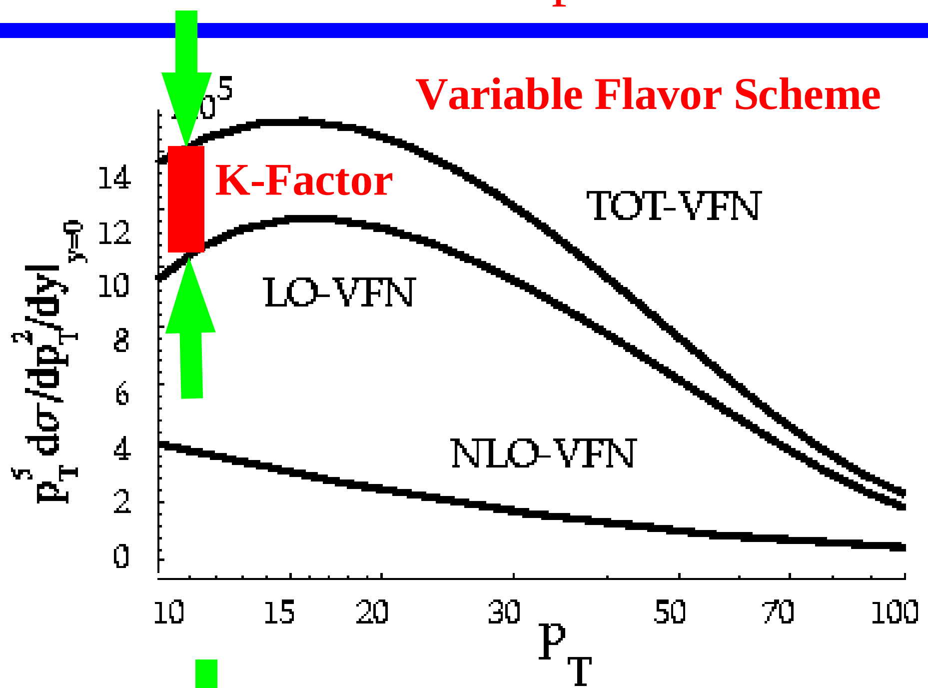


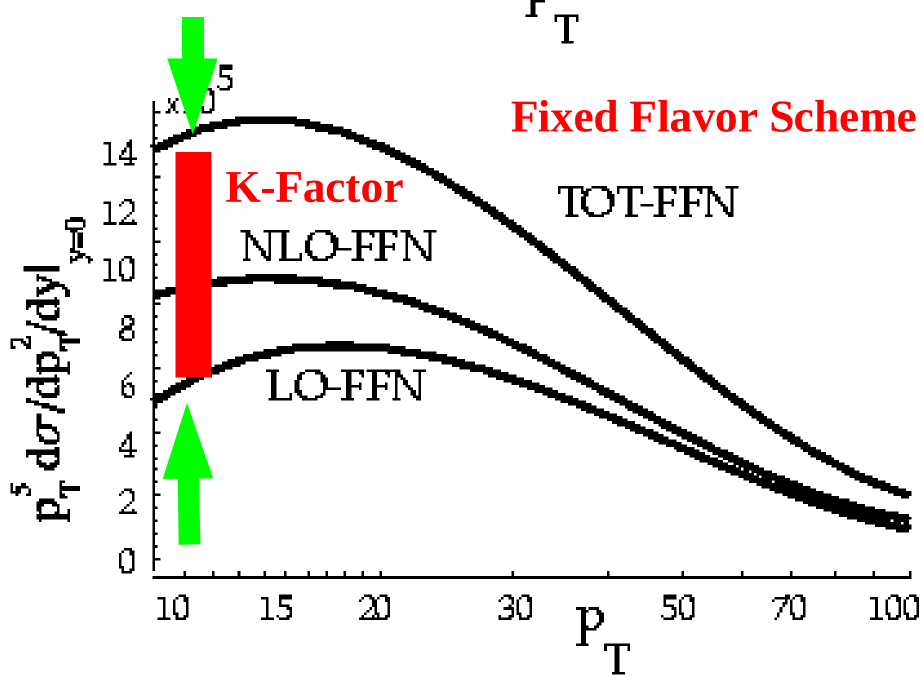
Net result:

- Reduction of μ-dependence
- 30% to 50% increase in σ

Aivazis, Collins, Olness, Tung
Olness, Scalise, Tung

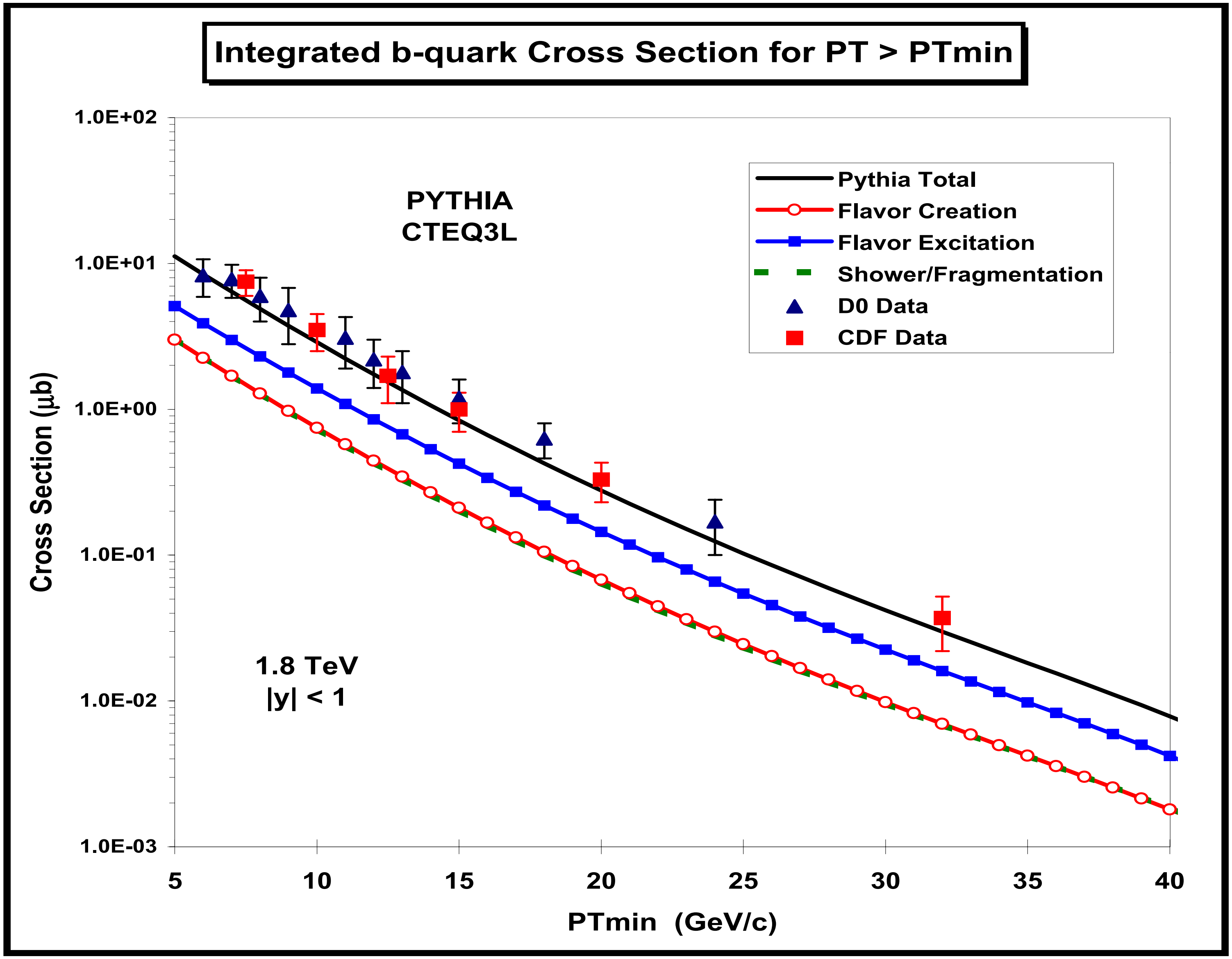
Compare Fixed & Variable Flavor Scheme



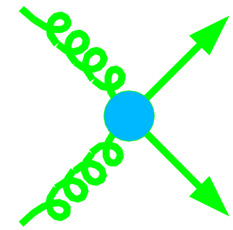


- To NLO, different schemes are comparable.
- K-Factor very different.
- Suggestion: VFS may provide more efficient organization of perturbation series than FFS.
- Recall: Choice of expansion point x_0 in Taylor series.

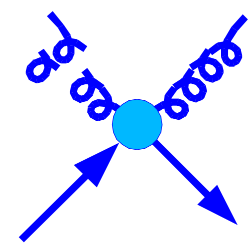
b-Quark Hadroproduction via Monte Carlo



Flavor Creation



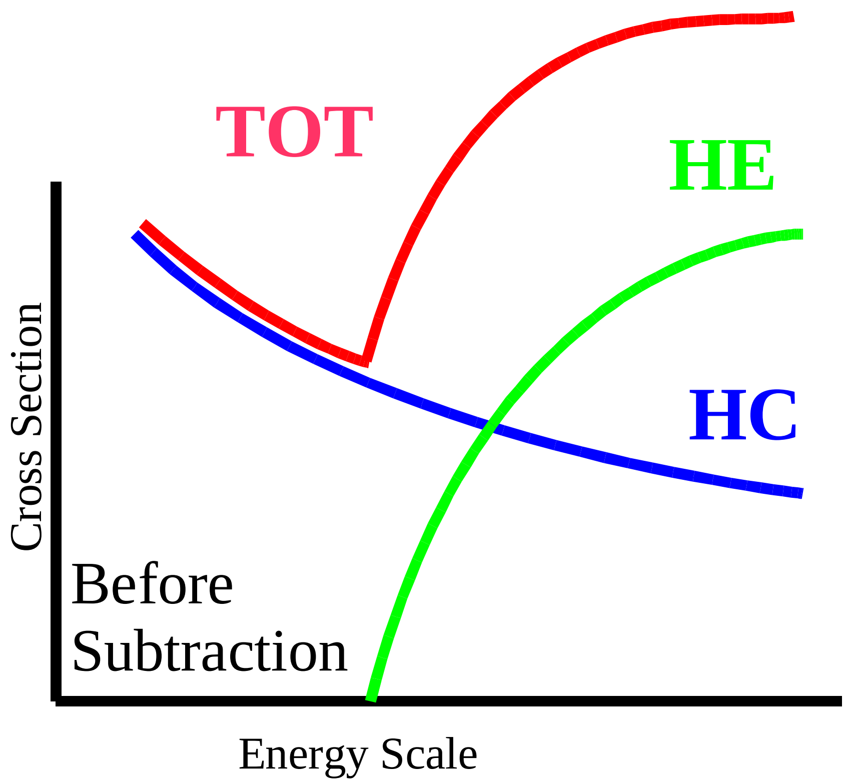
Flavor Excitation

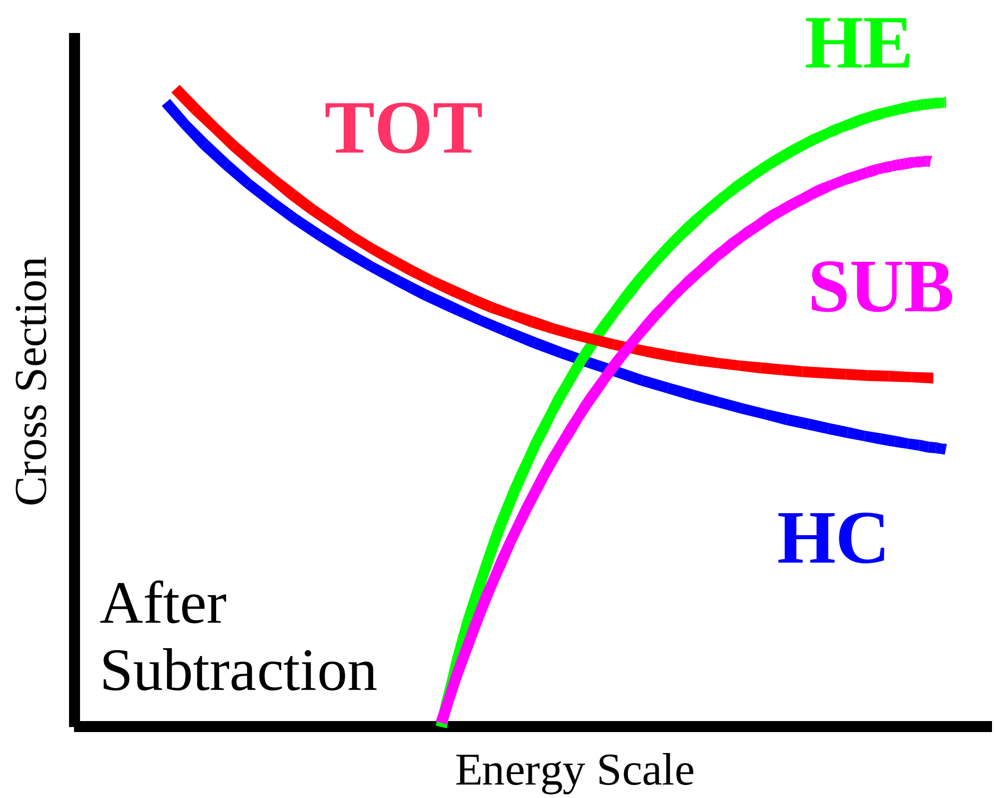


R. Field, hep-ph/0201112

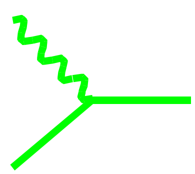
Recall

Note, we'll see this again ...





TOT =



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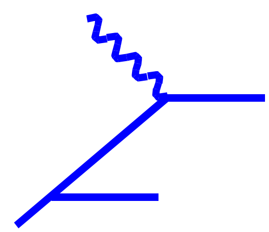

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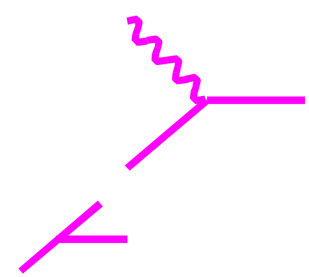
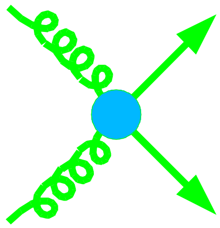
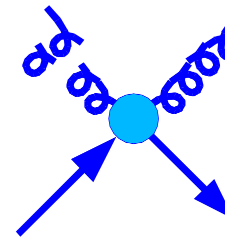


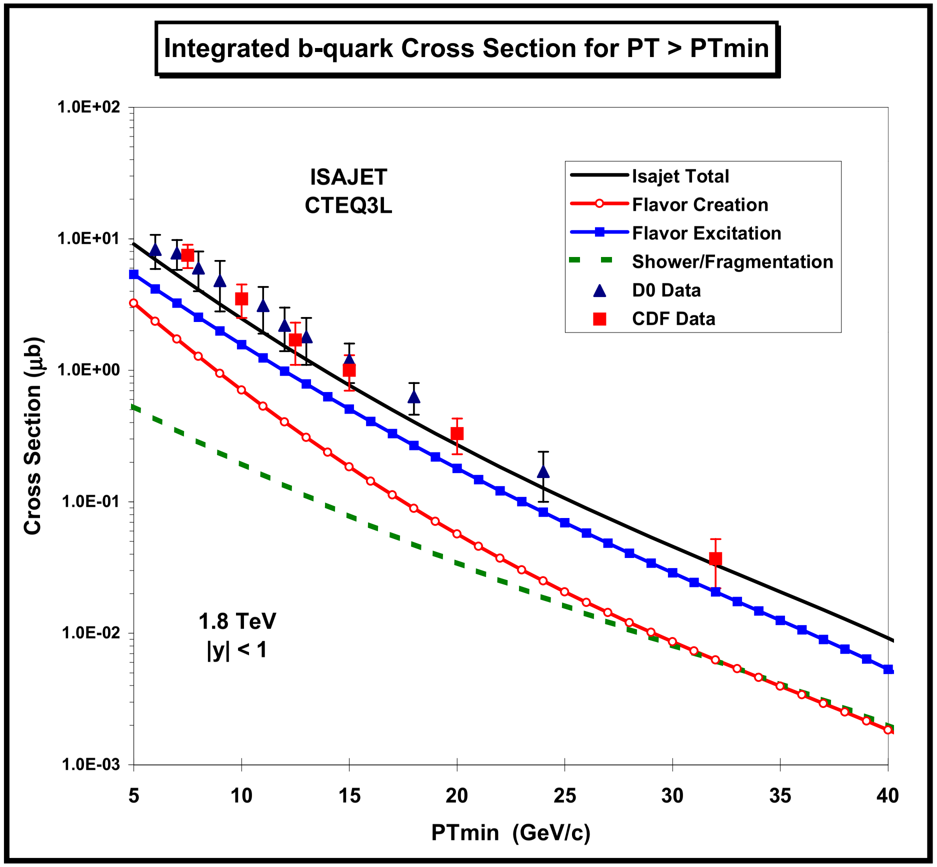
b-Quark Hadroproduction via Monte Carlo

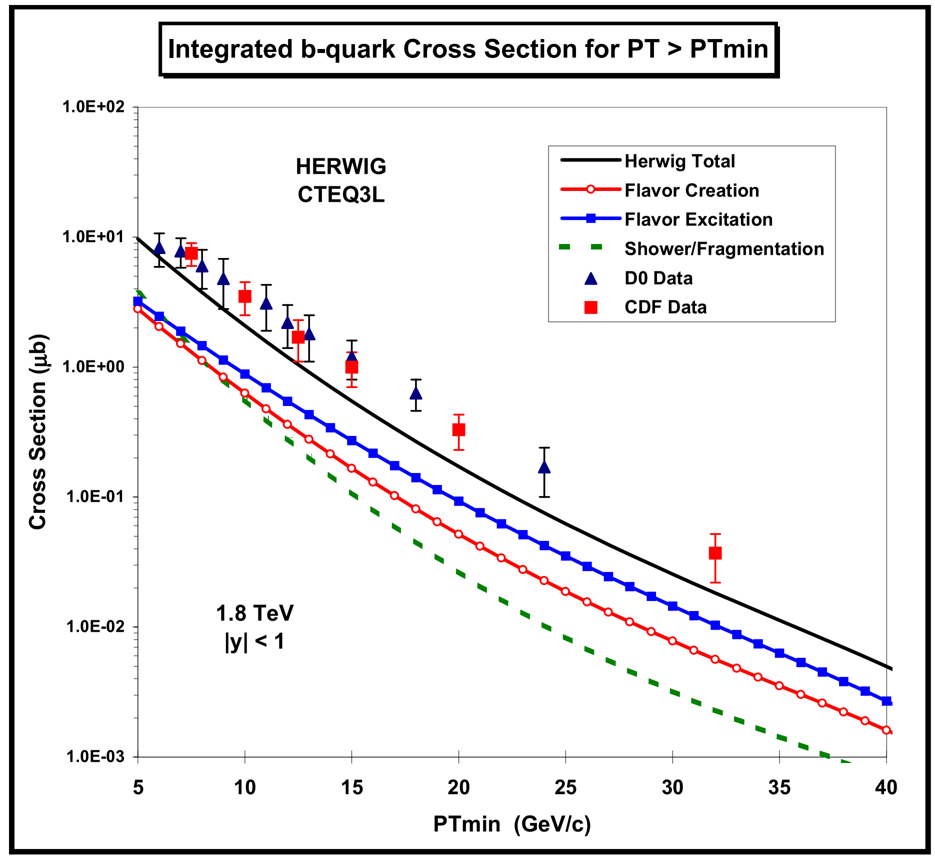
Flavor Creation



Flavor Excitation

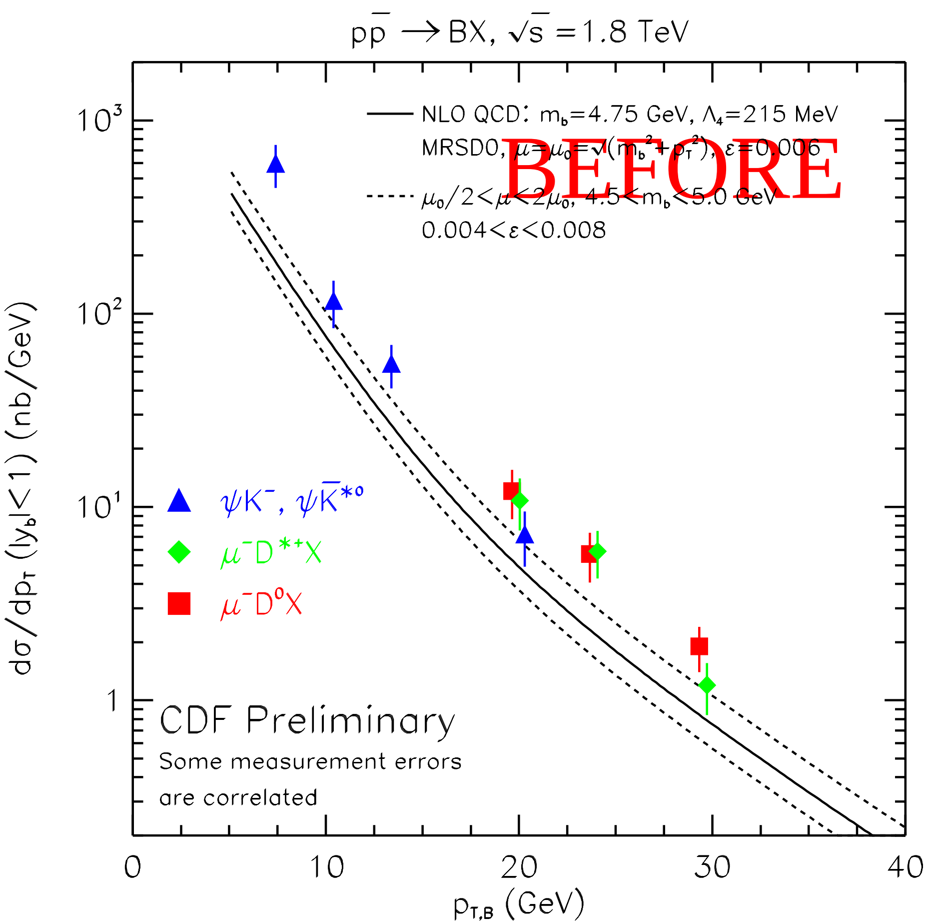


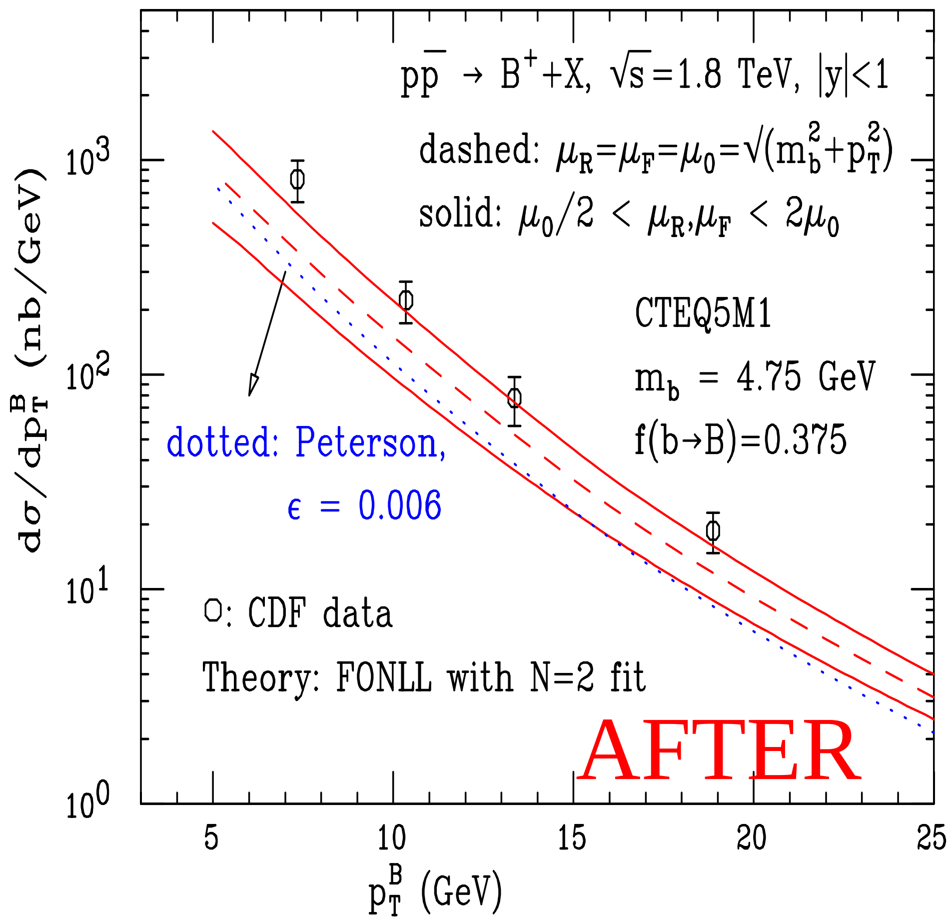




R. Field, hep-ph/0201112

B production with modified fragmentation function

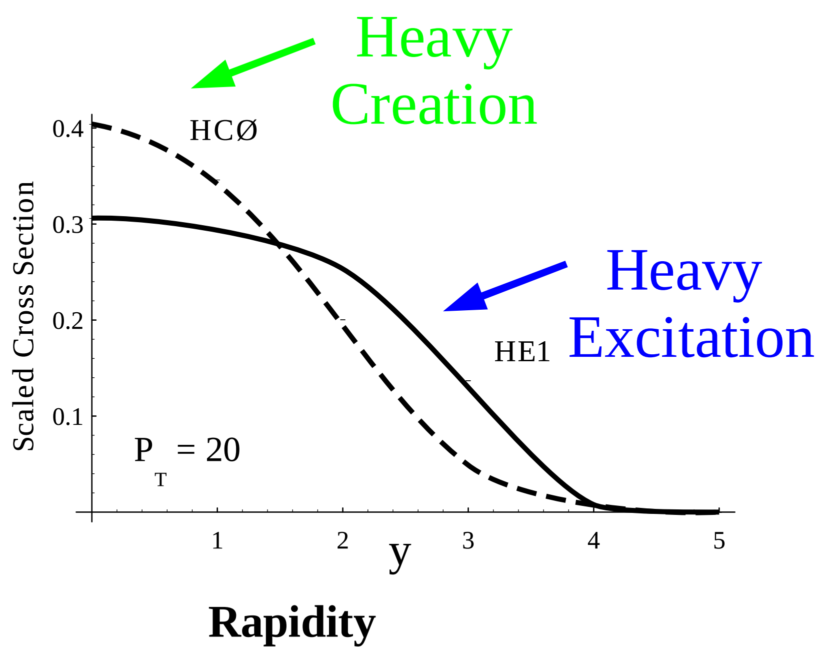


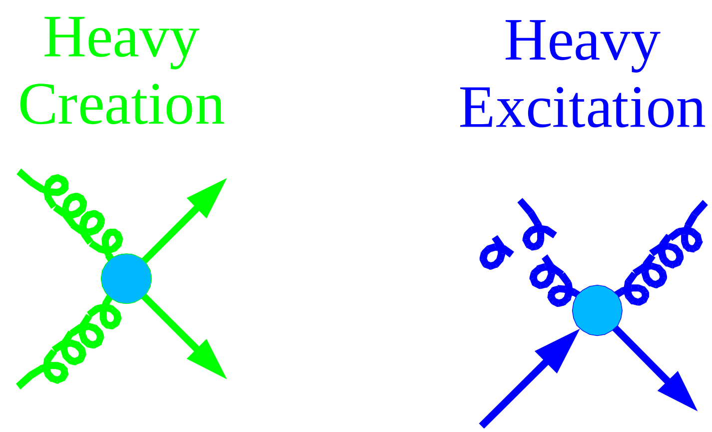


Cacciari & Nason hep/ph-0204025
Cacciari, Greco, Nason, JHEP 9805:007, 1998
Cacciar & Greco, Nuc.Ph.B421:530, 1994

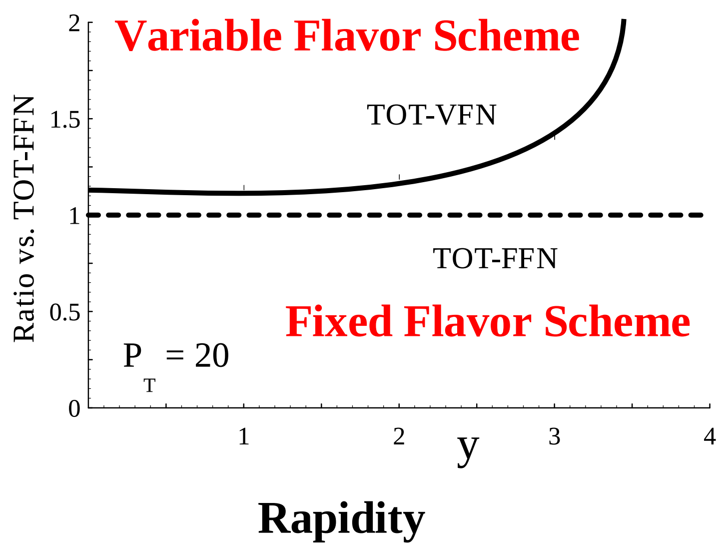
The effect of modifying the bottom fragmentation function
raises theory predictions in the range of data

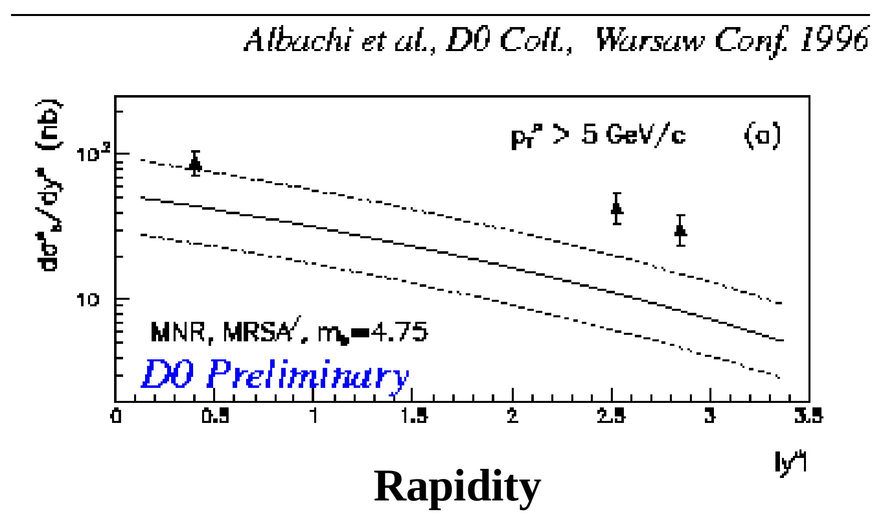
Rapidity Distribution





Different processes have different kinematic distributions

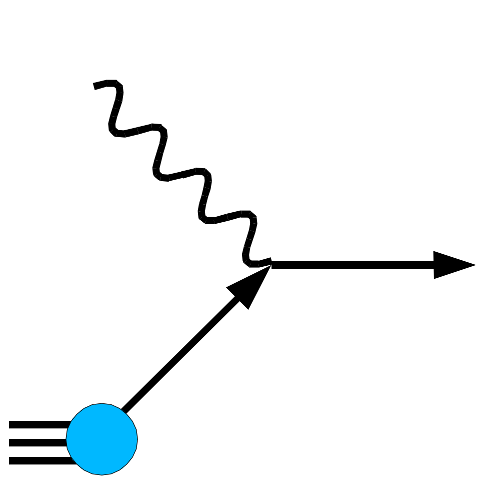


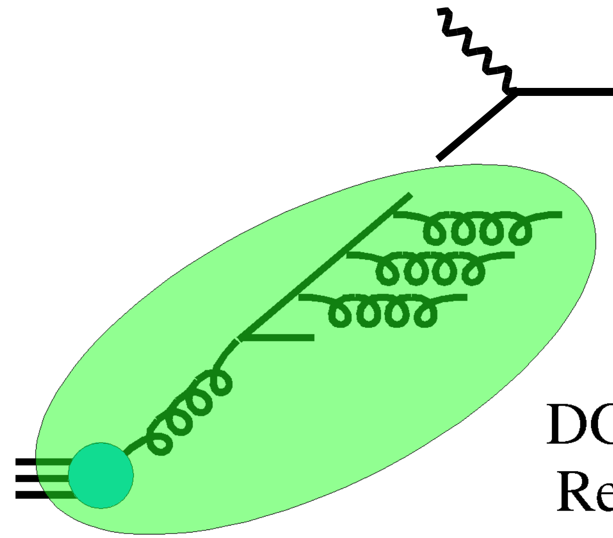


Mass-Independent Evolution.

Why is it valid?

DGLAP Equation and the Heavy Quark PDF



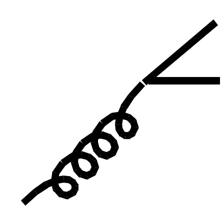


$$\mathrm{HE} = \int f(P \to a) \otimes \sigma(a \to c)$$

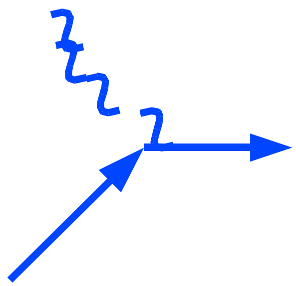
DGLAP equation
Resums iterative
splittings inside
the proton

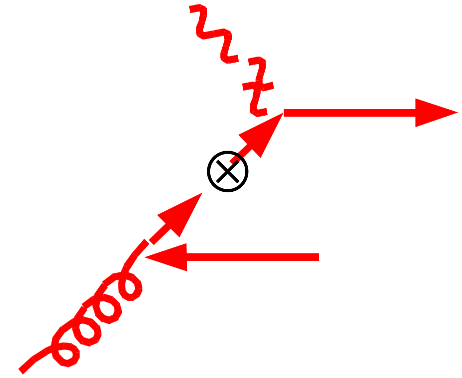
DGLAP Equation
$$\frac{df_i}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\, {}^1P_{j\to i} \otimes f_j + \ldots$$

Splitting Function
$${}^1P_{g\to q} = \frac{1}{2}\left[x^2 + (1-x)^2\right] + \left(\frac{M_H^2}{\mu^2}\right)\left[x(1-x)\right]$$



Effect of Heavy Quark Mass in the Calculation





$$\mathrm{HE} = \underbrace{\int f(P \to a)}\otimes \sigma(a \to c) \qquad \mathrm{SUB} = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$

$$\approx \; f(P \to g) \otimes {}^{1}P(g \to a)$$

${}^{1}P$ splittings must match

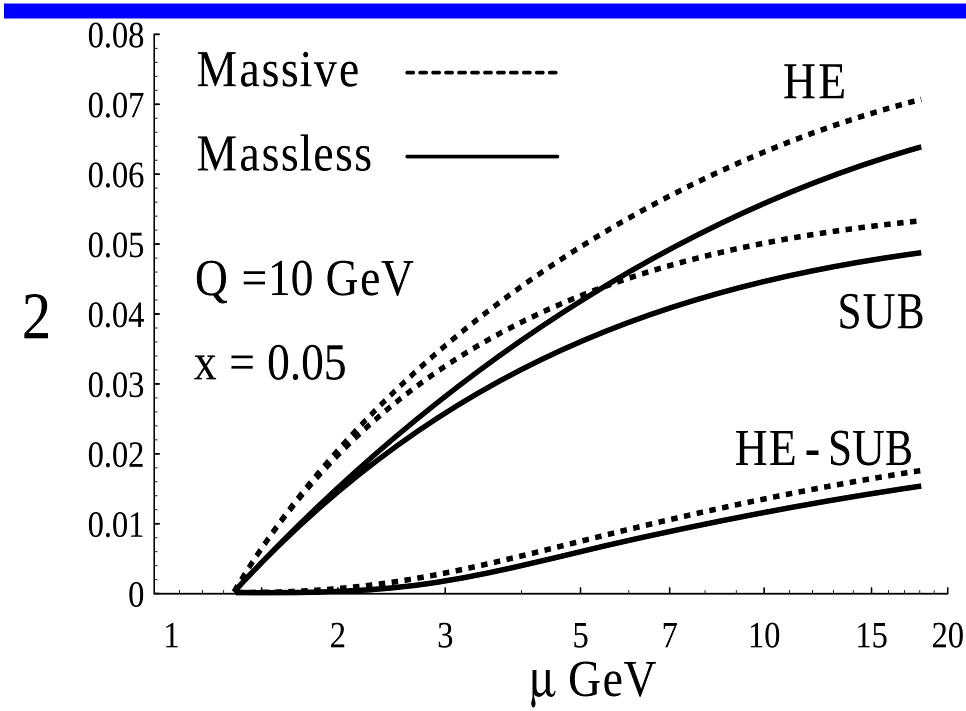
valid near threshold (M_H~Q)

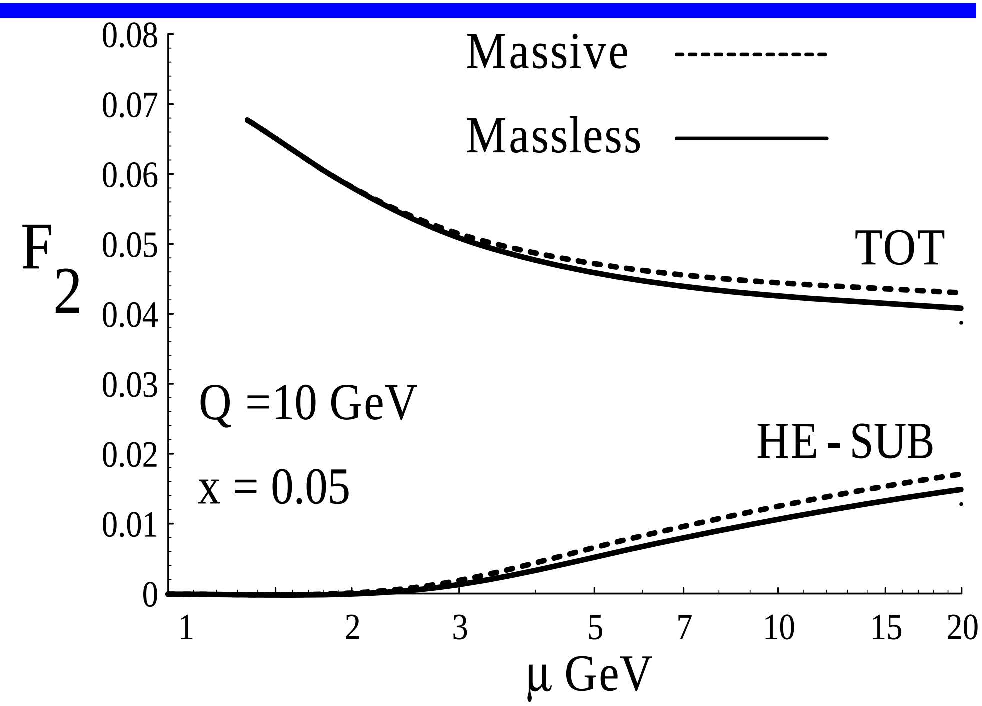
In Summary:

Near threshold(M_H~Q), mass effects cancel between HE and SUB

Above threshold(M_H<<Q), mass effects can be ignored

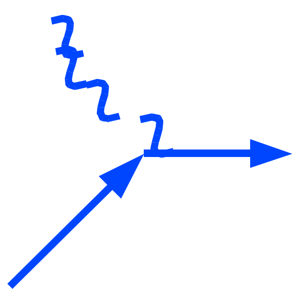
Effect of Heavy Quark Mass in the Calculation is Trivial

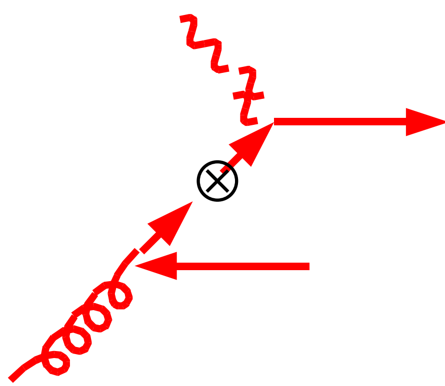


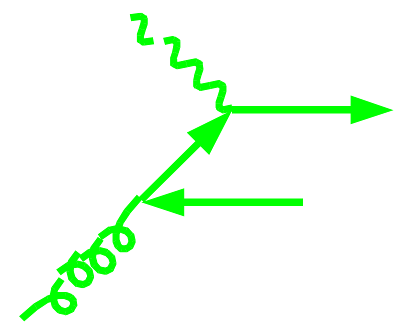


$$HE = \int f(P \to a) \otimes \sigma(a \to c)$$

$$HC = \int f(P \to g) \otimes \sigma(g \to c)$$

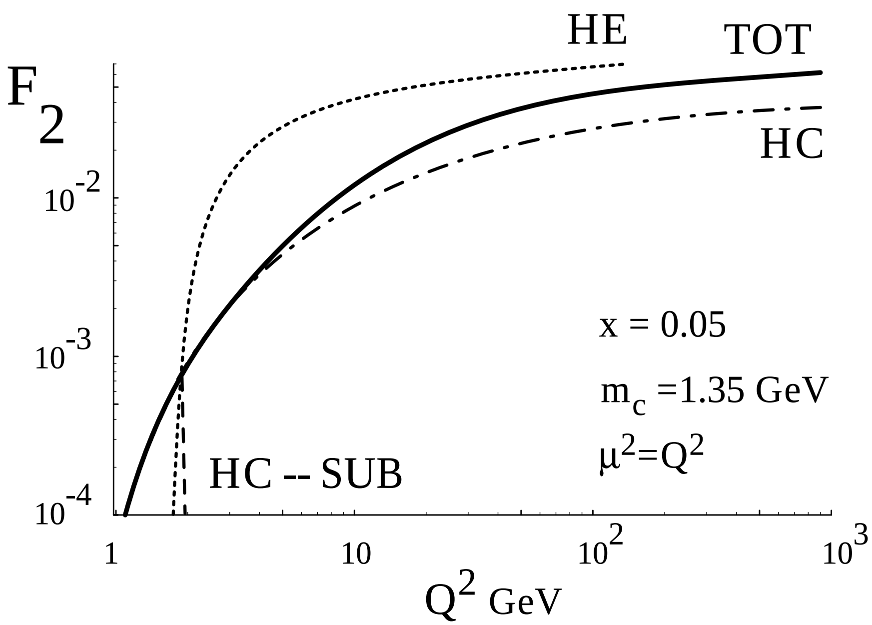


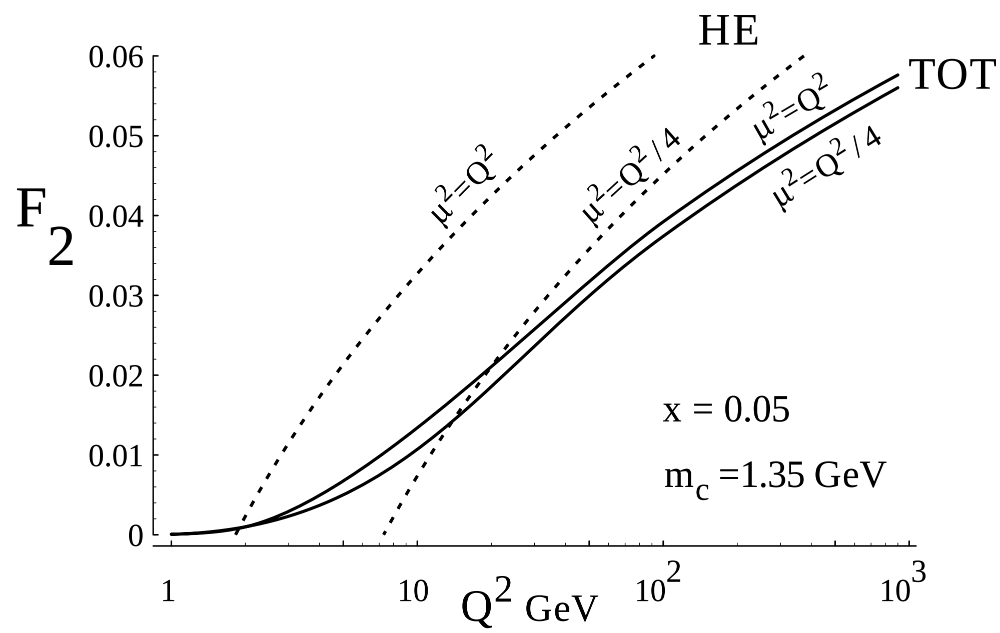




$$SUB = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$

Variation of σ vs. renormalization scale μ





LO = HE result is very sensitive to the choice of scale (i.e., $\mu^2=Q^2$ or $Q^2/4$)
TOT result (higher order) is stable w.r.t. the choice of scale

An accurate calculation must be stable as the renormalization scale varies

Conclusions

... or, lunch is just around the corner

Charm & Bottom Quark Production

An interesting subject because:

- Lots of data at present; more in near future
- Theoretical issues of multi-scale problem

A fascinating subject because:

- Theory & data not fully consistent
- This should be a region we can compute

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