III. FACTORIZATION AND EVOLUTION

1. Factorization as a generalization of the Parton Model
2. Evolution
3. Hard hadron-hadron scattering
4. Prologue: pQCD beyond the basic theorems

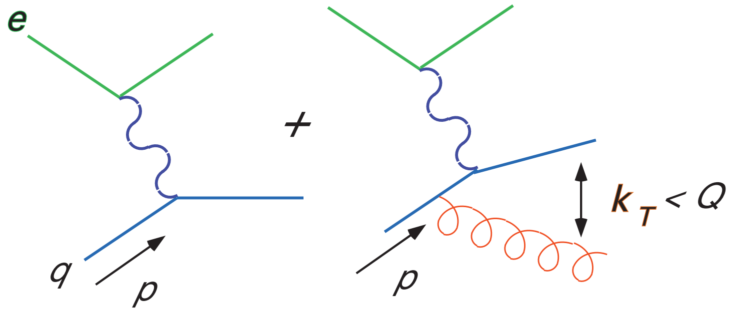
1. FACTORIZATION AS A GENERALZATION OF THE PARTON MODEL

- Challenge: use AF in observables (cross sections (σ) (also some amplitudes ...)) that are *not infrared safe*
- Possible *if:* σ has a short-distance subprocess. Separate *IR Safe* from IR: *this is factorization*
- *IR Safe part (short-distance) is calculable in pQCD*
- *Infrared part – example: parton distribution – measureable and universal*
- *Infrared safe – insensitive to soft gluon emission collinear rearrangements*
- Just like Parton Model except in Parton Model the infrared safe part is $\sigma_{\text{Born}} \Rightarrow \phi(x)$ normalized uniquely
- In pQCD must define parton distributions more carefully – yet another scheme:

  the factorization scheme

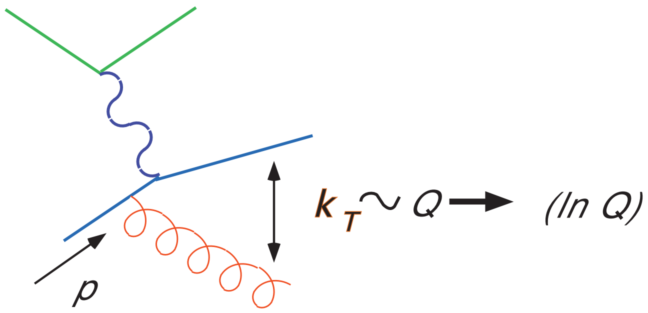
FIELD THEORY CORRECTIONS: DIS

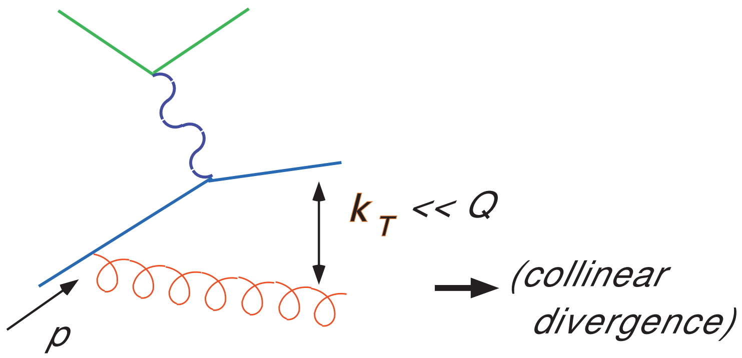
Basic observation: virtual states not truly frozen. Some states fluctuate on scale $1/Q$:



Short-lived states $\Rightarrow \ln(Q)$

Long-lived states $\Rightarrow$ *Collinear Logs (IR senstive)*





RESULT: FACTORIZED DIS

$$F_2^{\gamma q}(x, Q^2) \;=\; \int_x^1 d\xi \; C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right)$$
$$\times \; \phi_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

$$\equiv \; C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \otimes \phi_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

- ϕ has $\ln(\mu_F/\Lambda_{\rm QCD})$ ...
- C has $\ln(Q/\mu)$, $\ln(\mu_F/\mu)$
- Often pick $\mu = \mu_F$ and often pick $\mu_F = Q$. So often see:

$$F_2^{\gamma q}(x, Q^2) = C_2^{\gamma q}\left(\frac{x}{\xi}, \alpha_s(Q)\right) \;\otimes\; \phi_{q/q}(\xi, Q^2)$$

- But we still need to specify what we *really* mean by factorization: *scheme* as well as *scale*
- For this, compute $F_2^{\gamma q}(x, Q)$
- Keep $\mu = \mu_F$ for simplicity

"Compute quark-photon scattering" – *What does this mean?*

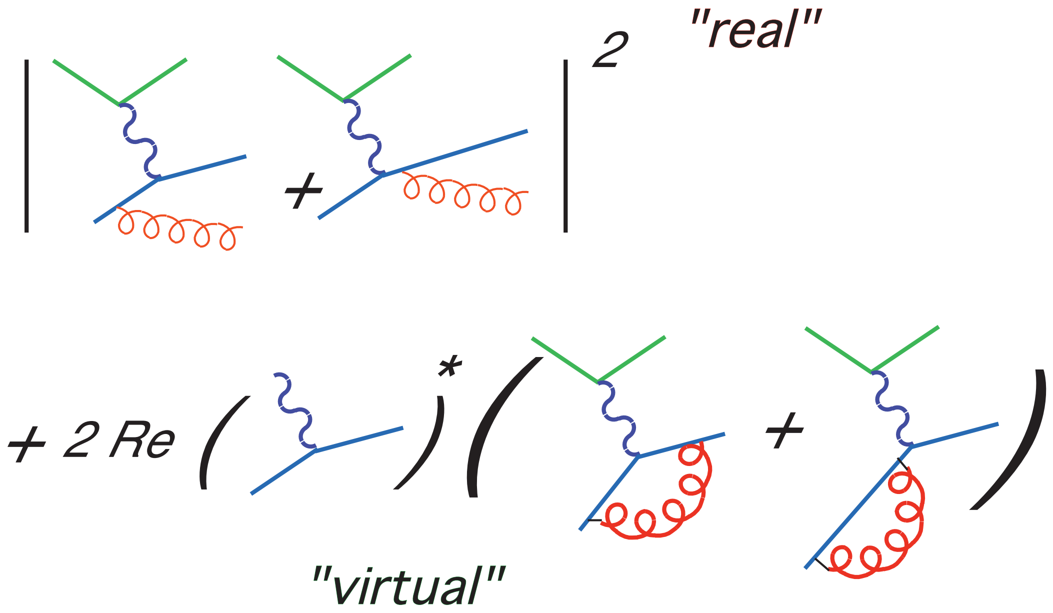
- Must use an *IR-regulated* theory
- Extract the *IR Safe part then* take away the regularization
- *Let's see how it works . . .*
- At *zeroth order – no interactions:*
- $C^{\gamma q_f\,(0)} = Q_f^2\ \delta(1 - x/\xi)$
  (Born cross section; parton model)
- $\phi^{(0)}_{q_f/q_{f'}}(\xi) = \delta_{f\,f'}\ \delta(1-\xi)$
  (at zeroth order, momentum fraction conserved)

$$
\begin{aligned}
F_2^{\gamma q_f\,(0)}(x,Q^2) &= \int_x^1 d\xi\ C_2^{\gamma q_f\,(0)}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right) \\
&\qquad \times\ \phi^{(0)}_{q_f/q_f}(\xi,\mu_F,\alpha_s(\mu)) \\
&= Q_f^2\ \int_x^1 d\xi\ \delta(1-x/\xi)\ \delta(1-\xi) \\
&= Q_f^2\ x\,\delta(1-x)
\end{aligned}
$$

- *On to one loop . . .*

$F^{\gamma q}$ @ AT ONE LOOP: FACTORIZATION SCHEMES

Start with F_2 for a *quark*:



Have to combine final states with different phase space

$$\Downarrow$$

"Plus Distributions":

$$\int_0^1 dx\, \frac{f(x)}{(1-x)_+} \quad \equiv \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)}$$

$$\int_0^1 dx\ f(x)\left(\frac{\ln(1-x)}{1-x}\right)_+ \quad \equiv \int_0^1 dx\ \left(\, f(x) - f(1)\,\right)\ \frac{\ln(1-x)}{(1-x)}$$

and so on ... where

- $f(x)$ will be parton distributions
- $f(x)$ term: real gluon, with momentum fraction $1-x$
- $f(1)$ term: virtual, with elastic kinematics

A Special Distribution
DGLAP "evolution kernel" = "splitting function"

$$P_{qq}^{(1)}(x) = C_F\, \frac{\alpha_s}{\pi}\ \left[\frac{1+x^2}{1-x}\right]_+$$

- Will see: P_{qq} a probability per unit log k_T

Expansion and Result:

$$F_2^{\gamma q}(x,Q^2) \;=\; \int_x^1 d\xi\; C_2^{\gamma q}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right)$$
$$\times\; \phi_{q/q}(\xi,\mu_F,\alpha_s(\mu))$$

$$F_2^{\gamma q_f}(x,Q^2) \;=\; C_2^{(0)}\;\phi^{(0)}$$
$$+\frac{\alpha_s}{2\pi}\;C^{(1)}\;\phi^{(0)}$$
$$+\frac{\alpha_s}{2\pi}\;C^{(0)}\;\phi^{(1)}+\dots$$

$$F_2^{\gamma q_f}(x,Q^2) \;=\; Q_f^2\,\{\;x\;\delta(1-x)$$
$$+\;\frac{\alpha_s}{2\pi}\;C_F\left[\frac{1+x^2}{1-x}\left(\frac{\ln(1-x)}{x}\right)+\frac{1}{4}\,(9-5x)\right]_+$$
$$+\;\frac{\alpha_s}{2\pi}\;C_F\int_0^{Q^2}\frac{dk_T^2}{k_T^2}\;\left[\frac{1+x^2}{1-x}\right]_+\;\}\;+\dots$$

$$F_1^{\gamma q_f}(x,Q^2) \;=\; \frac{1}{2x}\;\left\{F_2^{\gamma q_f}(x,Q^2)-C_F\,\alpha\,\frac{\alpha_s}{\pi^2}\,2x\;\right\}$$

Factorization Schemes

$\overline{\mathrm{MS}}$

$$\phi^{(1)}_{q/q}(x,\mu^2) = \frac{\alpha_s}{\pi^2}\ P_{qq}(x)\ \int_0^{\mu^2} \frac{dk_T^2}{k_T^2}$$

With k_T-integral "IR regulated".

Advantage: technical simplicity; not tied to process.

$C^{(1)}(x)_{\overline{\mathrm{MS}}} = (\alpha_s/2\pi)\ P_{qq}(x) \ln(Q^2/\mu^2) +$ μ-independent

DIS:

$$\phi_{q/q}(x,\mu^2) = \frac{\alpha_s}{\pi^2}\ F^{\gamma q_f}(x,\mu^2)$$

Absorbs all uncertainties in DIS into a PDF.

Closer to experiment for DIS.

$C^{(1)}(x)_{\overline{DIS}} = (\alpha_s/2\pi)\ P_{qq}(x) \ln(Q^2/\mu^2) + 0$

Using the Regulated Theory
and
Getting Parton Distributions for Real Hadrons

IR-regulated QCD is not *REAL* QCD

BUT it only differs at low momenta

THUS we can use it for IR Safe functions: $C_2^{\gamma q}$, etc.

This enables us to get PDFs for real hadrons:

- Compute $F_2^{\gamma q}$, $F_2^{\gamma G}$ ...
- Define factorization scheme; find IR Safe C's
- Use factorization in the full theory

$$F_2^{\gamma N} = \sum_{a=q_f,\bar{q}_f,G} C^{\gamma a} \otimes \phi_{a/N}$$

- Measure F_2; then use the known C's to derive $\phi_{a/N}$
- Multiple flavors and cross sections complicate technicalities; not logic (Global Fits)

NOW HAVE $\phi_{a/N}(\xi, \mu^2)$

USE IT IN ANY OTHER PROCESS THAT FACTORIZES

2. EVOLUTION

- Q^2-dependence
  - In general, Q^2/μ^2 dependence still in $C_a\left(x/\xi, Q^2/\mu^2, \alpha_s(\mu)\right)$

    Choose $\mu = Q$

    $$F_2^{\gamma A}(x,Q^2) = \sum_a \int_x^1 d\xi\; C_2^{\gamma a}\left(\frac{x}{\xi}, 1, \alpha_s(Q)\right) \phi_{a/A}(\xi,\mu^2)$$

    $Q \gg \Lambda_{\rm QCD} \rightarrow$ compute C's in PT.

    $$C_2^{\gamma a}\left(\frac{x}{\xi}, 1, \alpha_s(Q)\right) = \sum_n \left(\frac{\alpha_s}{\pi}\right)^n C_2^{\gamma a(n)}\left(\frac{x}{\xi}\right)$$

    But still need PDFs at $\mu = Q$: $\phi_{a/A}(\xi, Q^2)$

  - Remarkable result: EVOLUTION

    Can use $\phi_{a/A}(x, Q_0^2)$
    to determine $\phi_{a/A}(x, Q^2)$ and hence $F_{1,2,3}(x, Q^2)$
    for any Q !
    So long at $\alpha_s(Q)$ is still small

- *Illustrate by a 'nonsinglet' distribution*

$$F_a^{\gamma \mathrm{NS}} = F_a^{\gamma p} - F_a^{\gamma n}$$

$$F_2^{\gamma \mathrm{NS}}(x,Q^2) = \sum_a \int_x^1 d\xi \; C_2^{\gamma \mathrm{NS}}\left(\frac{x}{\xi},\frac{Q}{\mu},\alpha_s(\mu)\right) \; \phi_{\mathrm{NS}}(\xi,\mu^2)$$

Gluons, antiquarks cancel

At one loop: $C_2^{\mathrm{NS}} = C_2^{\gamma N}$

- *'Mellin' Moments and Anomalous Dimensions*

$$\bar{f}(N) = \int_0^1 dx \; x^{N-1} \; f(x)$$

- *Reduces convolution to a product*

$$f(x) = \int_x^1 dy \; g\left(\frac{x}{y}\right) \; h(y) \to \bar{f}(N) = \bar{g}(N) \; \bar{h}(N+1)$$

- *Moments applied to NS structure function:*

$$\bar{F}_2^{\gamma\mathrm{NS}}(N,Q^2) = \bar{C}_2^{\gamma\mathrm{NS}}\left(N,\frac{Q}{\mu},\alpha_s(\mu)\right)\,\bar{\phi}_{\mathrm{NS}}(N,\mu^2)$$

(Note $\phi_{\mathrm{NS}}(N,\mu^2) \equiv \int_0^1 d\xi \xi^N \phi(\xi,\mu^2)$ here.)

- $\bar{F}_2^{\gamma\mathrm{NS}}(N,Q^2)$ is *PHYSICAL*

$$\Rightarrow \quad \mu\frac{d}{d\mu}\,\bar{F}_2^{\gamma\mathrm{NS}}(N,Q^2) = 0$$

- *'Separation of variables'*

$$\begin{aligned}\mu\frac{d}{d\mu}\ln\bar{\phi}_{\mathrm{NS}}(N,\mu^2) &= -\gamma_{\mathrm{NS}}(N,\alpha_s(\mu))\\ \gamma_{\mathrm{NS}}(N,\alpha_s(\mu)) &= \mu\frac{d}{d\mu}\ln\bar{C}_2^{\gamma\mathrm{NS}}(N,\alpha_s(\mu))\end{aligned}$$

- *Because α_s is the only variable held in common!*

$$
\begin{aligned}
\mu \frac{d}{d\mu} \ln \bar{\phi}_{\mathrm{NS}}(N, \mu^2) &= -\gamma_{\mathrm{NS}}(N, \alpha_s(\mu)) \\
\gamma_{\mathrm{NS}}(N, \alpha_s(\mu)) &= \mu \frac{d}{d\mu} \ln \bar{C}_2^{\gamma \mathrm{NS}}(N, \alpha_s(\mu))
\end{aligned}
$$

- *Only need to know C's* $\Rightarrow \gamma_n$ *from IR regulated theory!*

$$\Downarrow$$

Q-DEPENDENCE DETERMINED BY PT

EVOLUTION

THIS WAS HOW WE FOUND OUT QCD IS 'RIGHT'

THIS IS HOW QCD PREDICTS PHYSICS
AT NEW SCALES

$\gamma_{\rm NS}$ AT ONE LOOP

$$
\begin{aligned}
\gamma_{\rm NS}(N,\alpha_s) &= \mu\frac{d}{d\mu}\ \ln \bar{C}_2^{\gamma\rm NS}\,(N,\alpha_s(Q)) \\
&= \mu\frac{d}{d\mu}\ \left\{\ (\alpha_s/2\pi)\ \bar{P}_{qq}(N)\ln(Q^2/\mu^2) + \mu \text{ indep. }\right\} \\
&= -\frac{\alpha_s}{\pi}\ \int_0^1 dx\ x^{N-1}\ P_{qq}(x) \\
&= -\frac{\alpha_s}{\pi}C_F\ \int_0^1 dx\ \left[\left(x^{N-1}-1\right)\ \frac{1+x^2}{1-x}\right] \\
&= -\frac{\alpha_s}{\pi}C_F\ \left[\ 4\sum_{m=2}^{N}\frac{1}{m} - 2\frac{2}{N(N+1)} + 1\ \right] \\
&\equiv -\frac{\alpha_s}{\pi}\ \gamma_{\rm NS}^{(1)}
\end{aligned}
$$

Hint:

$$
(1-x^2)/(1-x) = 1+x\ldots(1-x^k)/(1-x) = \sum_{i=0}^{k-1} x^k
$$

SOLUTION: SCALE BREAKING

$$\mu \frac{d}{d\mu}\, \bar{\phi}_{\mathrm{NS}}(N,\mu^2) = -\gamma_{\mathrm{NS}}(N,\alpha_s(\mu))\ \bar{\phi}_{\mathrm{NS}}(N,\mu^2)$$

$$\begin{aligned} \bar{\phi}_{\mathrm{NS}}(N,\mu^2) \;&=\; \bar{\phi}_{\mathrm{NS}}(N,\mu_0^2) \\ &\quad \times \exp\left[-\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\ \gamma_{\mathrm{NS}}(N,\alpha_s(\mu)) \right] \end{aligned}$$

$$\Downarrow$$

$$\bar{\phi}_{\mathrm{NS}}(N,Q_0^2) \;=\; \bar{\phi}_{\mathrm{NS}}(N,Q_0^2)\ \left(\frac{\ln(Q^2/\Lambda^2_{\mathrm{QCD}})}{\ln(Q_0^2/\Lambda^2_{\mathrm{QCD}})} \right)^{-2\gamma_N^{(1)}/\beta_0}$$

Hint:

$$\alpha_s(Q) = \frac{4\pi}{\beta_0\ \ln(Q^2/\Lambda^2_{\mathrm{QCD}})}$$

So also:

$$\bar{\phi}_{\mathrm{NS}}(N,Q_0^2) \;=\; \bar{\phi}_{\mathrm{NS}}(N,Q_0^2)\ \left(\frac{\alpha_s(Q_0^2}{\alpha_s(Q^2)} \right)^{-2\gamma_N^{(1)}/\beta^{(1)}}$$

$$\bar{\phi}_{\rm NS}(N,Q_0^2) \;=\; \bar{\phi}_{\rm NS}(N,Q_0^2)\,\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{-2\gamma_N^{(1)}/\beta^{(1)}}$$

- *'Mild' scale breaking*
- For $\alpha_s \to \alpha_0 \neq 0$, get a power Q-dependence:

$$\left(Q^2\right)^{\gamma^{(1)}\frac{\alpha_S}{2\pi}}$$

- *QCD's consistency with the Parton Model (73-74)*

$$\mu\frac{d}{d\mu}\,\bar{\phi}_{\rm NS}(N,\mu^2) = -\gamma_N(\alpha_s(\mu))\,\bar{\phi}_{\rm NS}(N,\mu^2)$$

$$\Downarrow$$

$$\mu\frac{d}{d\mu}\,\bar{\phi}_{\rm NS}(N,\mu^2) = \int_x^1 \frac{d\xi}{\xi}\,P_{\rm NS}(\xi,\alpha_s(\mu))\,\bar{\phi}_{\rm NS}(\xi,\mu^2)$$

Splitting function $\leftrightarrow$ Moments

$$\int_0^1 dx\; x^{N-1}\; P_{qq}(x,\alpha_s) = \gamma_{qq}(N,\alpha_s)$$

BEYOND NONSINGLET

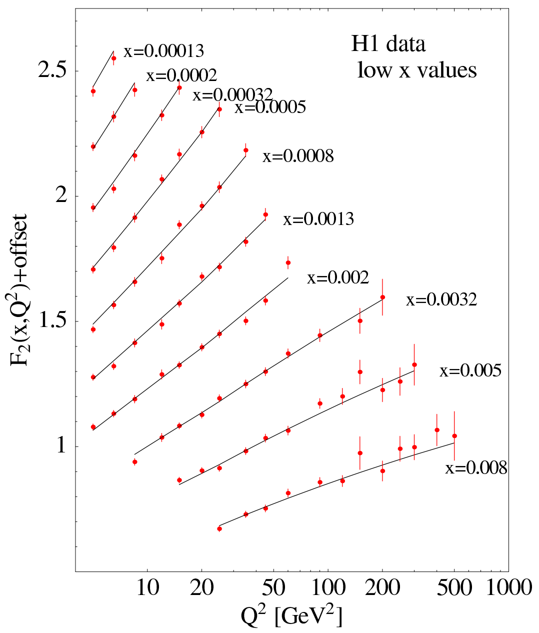
COUPLED EVOLUTION

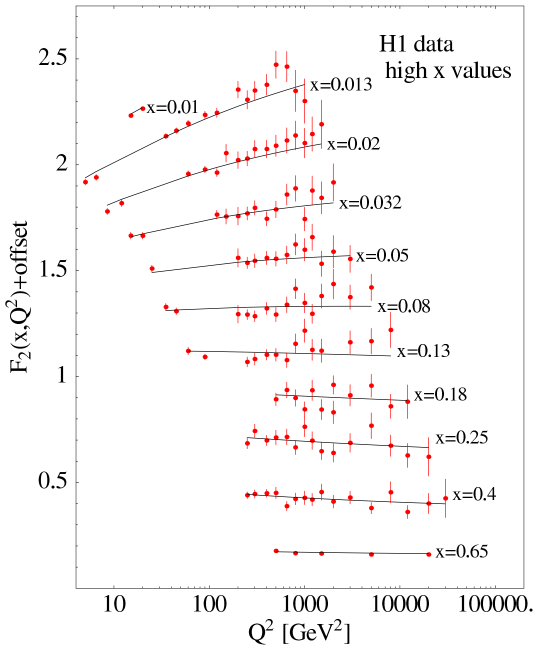
$$\mu\frac{d}{d\mu}\,\bar{\phi}_{q/A}(N,\mu^2) = \sum_{b=q,\bar{q},G}\int_x^1 \frac{d\xi}{\xi}\, P_{\rm ab}(\xi,\alpha_s(\mu))\;\bar{\phi}_{b/A}(\xi,\mu^2)$$

Physical Contxt of Evolution

- Parton Model: $\phi_{a/A}(x)$ density of parton a with momentum fraction x, assumed independent of Q
- PQCD: $\phi_{a/A}(x,\mu)$: same density, but with transverse momentum $\leq \mu$
- If there *were* a maximum transverse momentum Q_0, $\phi(x,Q_0)$ would freeze for $\mu \geq Q_0$
- *Not so* in renormalized PT
- Scale breaking measures the change in the density as maximum transverse momentum increases
- Cross sections we compute still depend on our choice of μ through uncomputed “higher orders” in C and evolution

- Evolution in DIS (with CTEQ6 fits)





3. HADRON HADRON SCATTERING

- Same generalization from Parton Model as DIS
- For final state system F, mass Q

$$\begin{aligned}\sigma_{AB\to F(Q)} &= \sum_{a,b=q,\bar{q},G} \int_0^1 d\xi\, d\eta\, \phi_{a/A}(x,\mu)\ \phi_{b/B}(\eta,\mu)\\ &\qquad \times \hat{\sigma}_{ab\to F(Q)}(xp_A, yp_B, Q, \mu, \alpha_s(\mu))\\ &\qquad \times \theta(\xi\eta p_A\cdot p_B - Q^2)\end{aligned}$$

Convolution form:

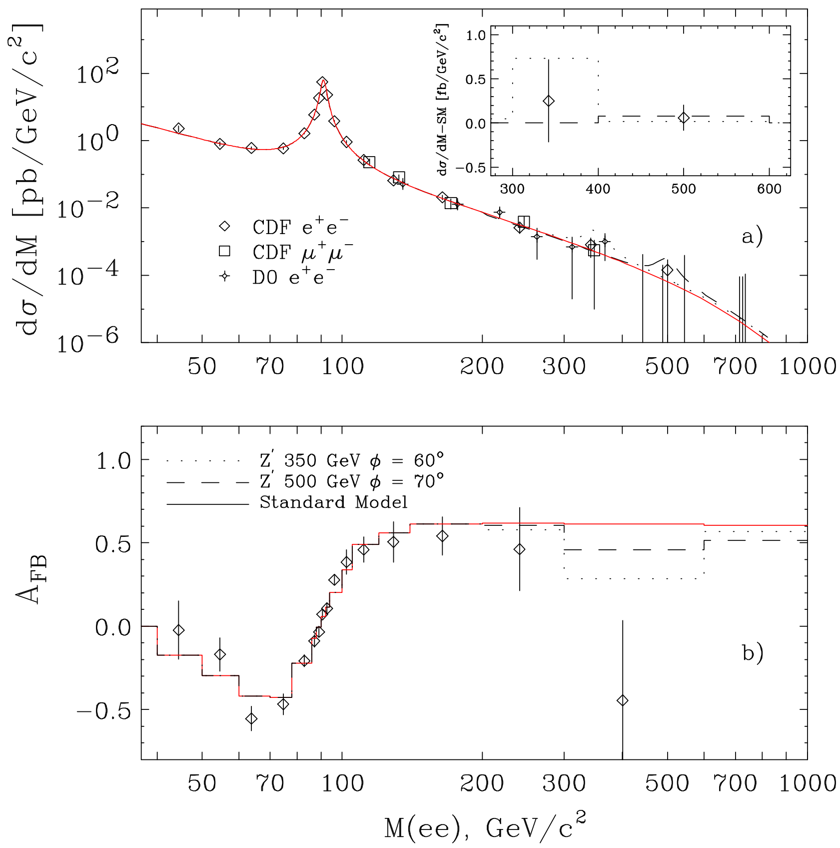
$$\sigma_{AB\to F(Q)} = \hat{\phi}_{a/A} \otimes \hat{\sigma}_{ab\to F} \otimes \hat{\phi}_{b/B}$$

- Depends on choice of μ through uncomputed "higher orders" in $\hat{\sigma}$ and evolution

- A typical $\mathcal{O}(\alpha_s)$ (NLO) hard-scattering cross section at one loop: $\hat{\sigma})_{\bar{q}q}$ for Drell-Yan

$$\hat{\sigma}_{\bar{q}q} \;=\; \sigma_{\rm Born}(Q^2)\,\left(\frac{\alpha_s(\mu)}{\pi}\right)\,\left\{2(1+z^2)\,\left[\frac{\ln(1+z^2)}{1-z}\right]_+ \right.$$
$$\left. -\frac{[(1+z^2)\ln\,z\,]}{(1-z)} + \left(\frac{\pi^2}{3}-4\right)\,\delta(1-z)\right\}$$

- $\sigma_{\rm Born}$ describes Electroweak annihilation of $q\,\bar{q}$
- Compared to data at high energy (γ and Z) with forward-backward asymmetry in $\sigma_{\rm Born}$



- This way, detect “new” physics in the hard scattering

- *Factorized hadron-hadron scattering*
  "Collinear factorized"

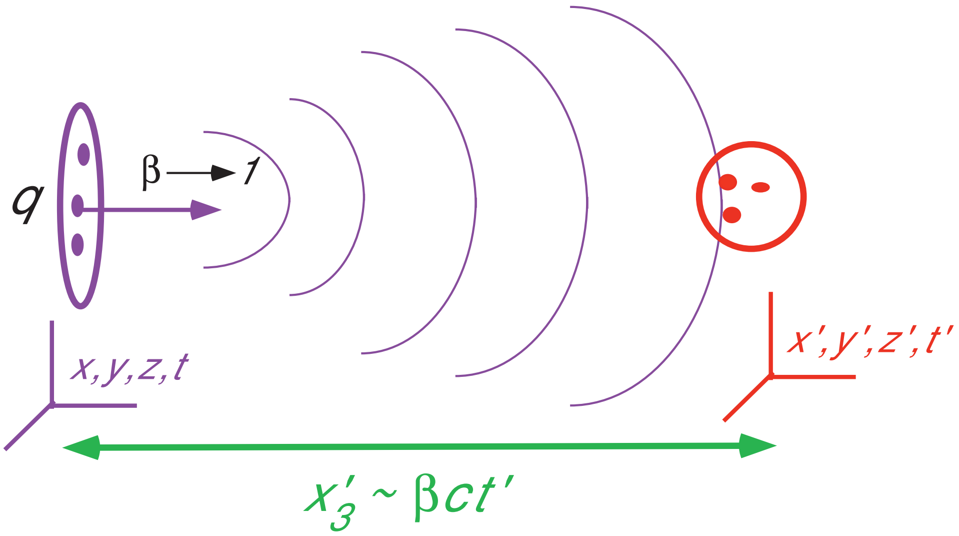
$$\sigma_{AB\to F(Q)} \;=\; \hat{\phi}_{a/A} \otimes \hat{\sigma}_{ab\to F} \;\otimes \hat{\phi}_{b/B}$$

- Requirements: same as for IR Safety in e^+e^- cross sections
- Insensitivity to soft emission, collinear rearrangement
- Continuous dependence on particle momenta
- Numerous Cases

$$\begin{array}{ll} p+p\to & \text{high } p_T \text{ jets} \\ & \text{high } p_T \; \textit{particles}, \text{ heavy particles :} \\ & W,\ Z\ldots \\ & H,\ \tilde{q},\ \ldots \\ & Q\,\bar{Q}\ldots \end{array}$$

- PDFs ϕ's same as in DIS (experiment) same evolution
- PT $\to$ $\hat{\sigma}_{ab}$'s (IR-regulated theory)
- 'New physics' in $\hat{\sigma}_{ab}$'
- Evolution $\to$ ability to extrapolate to new energy scales
- Predictions in hadron-hadron cross sections

Why Factorization? Heuristic, classical argument:



field	x frame	x' frame
scalar	$\frac{q}{\lvert\vec{x}\rvert}$	$\frac{q\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
		$\rightarrow 1/\gamma \sim$ '*ruler*'
gauge $(-)$	$A^-(x) = \frac{q}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{q\gamma(1+\beta)\Delta}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
		$\rightarrow 1/\gamma \quad \gg$ '*ruler*'
field strength	$E_3(x) = \frac{q}{\lvert\vec{x}\rvert^2}$	$E_3'(x') = \frac{-q\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
		$\rightarrow 1/\gamma^2 \quad \ll$ '*ruler*'

- *Lorentz contracted fields of incident particles do not overlap until the moment of the scattering!*

- *Initial-state interaction disappear at high enough energies!*

- $\Rightarrow$ *Repalced by parton distributions*

- *Interactions after the scattering are too late to affect large momentum transfer, creation of heavy particle, etc.*

- $\Rightarrow$ *Cross section for hard scattering is IR safe*

$$\Downarrow$$

$$\sigma_{AB\to F(Q)} = \hat{\phi}_{a/A} \otimes \hat{\sigma}_{ab\to F} \otimes \hat{\phi}_{b/B}$$

4. PROLOGUE: PQCD BEYOND THE BASIC THEOREMS

Background themes:

- QCD as servant and troublemaker in the search for new physics
  Huston,Lykken,Klasen,Zeppenfeld,Richman...
- QCD as a testing ground for mathematical physics
  Lykken,Nagle,Giele...
- QCD as the exemplary quantum field theory
  all

Current Melodies:

- The search for 'precision QCD'
  - *NLO applications & NNLO development*
    Repond,Yuan,Flaugher,Womersley,Reina ...
  - *'resummed' cross sections: 'evolution plus'*
    Yuan,Mrenna ...
  - *event generators: reliable extrapolations*
    Mrenna...
  - *PDF uncertainties*
    Giele

- From short distances to long in QCD
  - *Small-x QCD: soft partons & diffraction*
    Repond,Brandt,...
  - *Large-x QCD: narrow jets & power corrections*
    Repond,Flaugher,Gary,...
  - *Jet structure; multiplicities*
    Flaugher,Gary...
  - *Heavy quarks: rare decays & new physics*
    Olness,Klasen,Richman...
  - *Top physics*
    Willenbrock,Roser
  - *'Light' hadron structure, spin*
    Giele,Nagle
  - *QCD in extreme conditions*
    Nagle