2002 CTEQ Summer School

Transverse momentum of W Boson

Effect of Multiple Gluon Emission:

Soft Gluon Resummation

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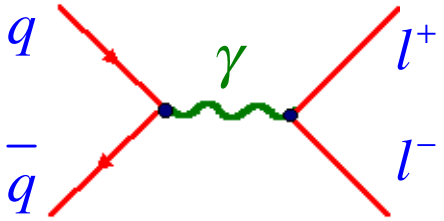
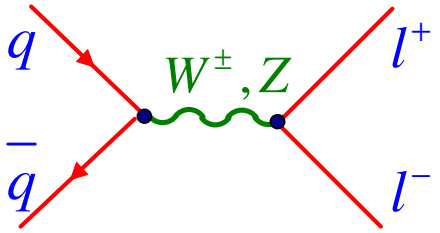
June 3-5, 2002, at Madison, Wisconsin

- Transverse momentum of

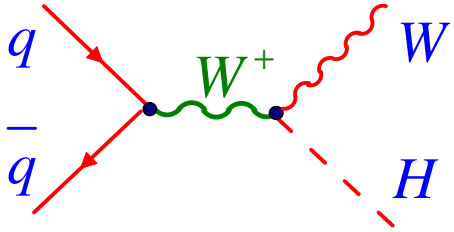
Drell-Yan



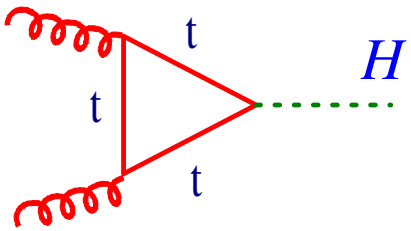
$W^{\pm}, Z$



W H

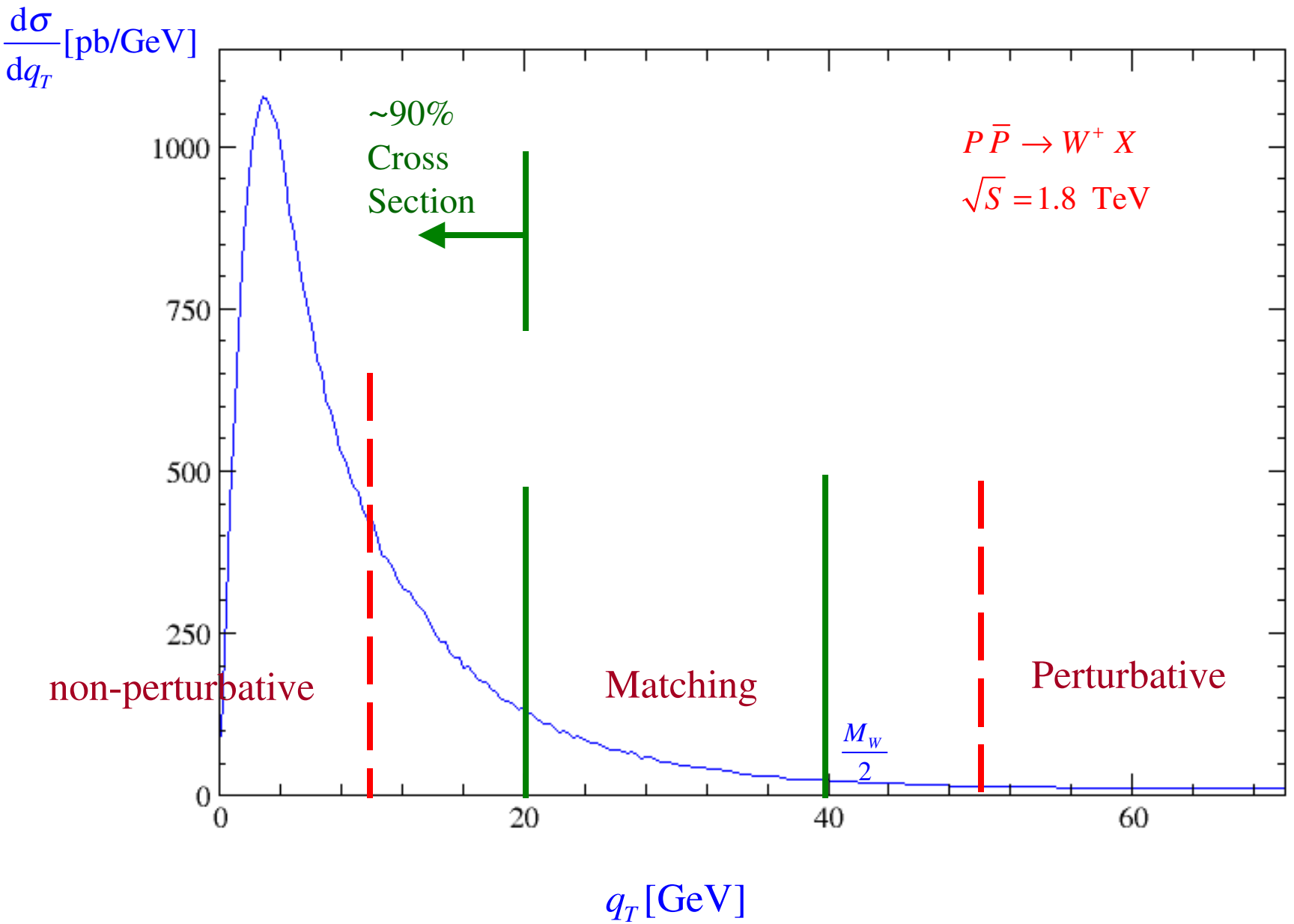


$g\ g \rightarrow H$



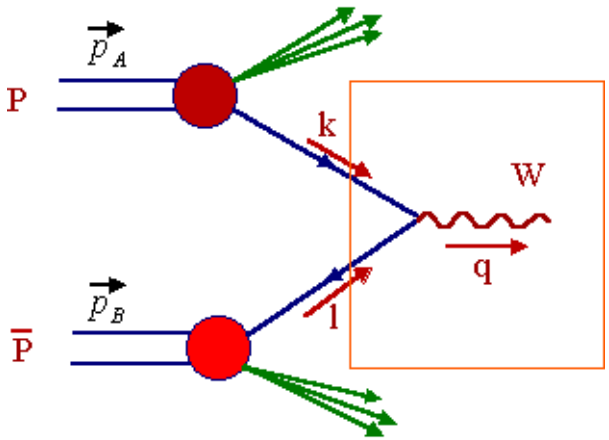
including QCD Resummations.

- Kinematics of Leptons from the decays


$\frac{d\sigma}{dq_T}$[pb/GeV]
~90%
Cross
Section
$P\bar{P} \rightarrow W^+ X$
$\sqrt{S} = 1.8$ TeV
1000
750
500
250
0
0
20
40
60
non-perturbative
Matching
Perturbative
$\frac{M_W}{2}$
q_T[GeV]

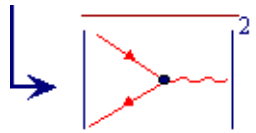
Fixed order pQCD prediction

$\alpha_S^{(0)}$



$$\sigma = \frac{1}{2S}\int \frac{d\xi_A}{\xi_A}\frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A,\mu) f_{i/B}(\xi_B,\mu)\cdot d\hat{\sigma}$$

$$d\hat{\sigma} = \underbrace{\overline{|M|^2}}(2\pi)^4\delta^{(4)}(q-k-l)\frac{d^3q}{(2\pi)^3 2q_0}$$

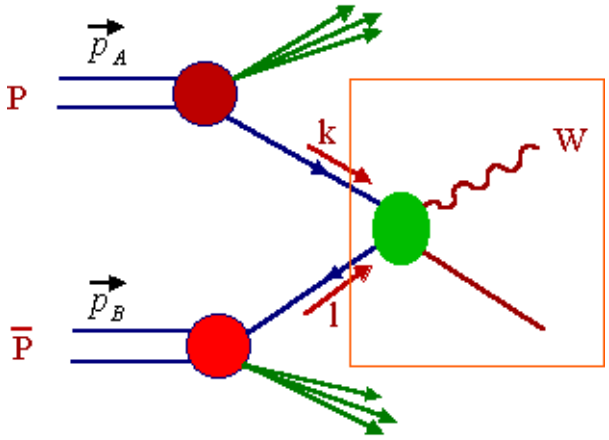


$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2\mathrm{d}y\mathrm{d}Q^2} = \frac{1}{S}\int \frac{\mathrm{d}\xi_A}{\xi_A}\frac{\mathrm{d}\xi_B}{\xi_B} f_{i/A}(\xi_A,\mu) f_{i/B}(\xi_B,\mu)$$
$$\cdot\left(\frac{\pi^2}{Q^2}\right)\cdot\overline{|M|^2}\cdot\delta\left(1-\frac{x_A}{\xi_A}\right)\cdot\delta\left(1-\frac{x_B}{\xi_B}\right)$$
$$\cdot\delta\left(q_T^2\right)\cdot\delta\left(Q^2-M_W^2\right)$$

$$s = (p_A + p_B)^2$$
$$k = \xi_A p_A$$
$$l = \xi_B p_B$$

$$Q \equiv \sqrt{Q^2} = \sqrt{q^2},\ \mu = Q = M_W,\ x_A = \frac{Q}{\sqrt{S}}e^{y},\ x_B = \frac{Q}{\sqrt{S}}e^{-y}$$

$\alpha_S^{(1)}$



$$\frac{d\sigma}{dq_T^2 dy dQ^2} = \int \frac{d\xi_A}{\left(\xi_A S + U - Q^2\right)} \left(\frac{\hat{s} d\hat{\sigma}}{d\hat{t}} \right) \cdot f_{i/A}\left(\xi_A, \mu\right)$$

$$\cdot f_{j/B}\left(\xi_B = \frac{-Q^2 - \xi_A \left(T - Q^2\right)}{\xi_A S + U - Q^2}, \mu \right) \cdot \delta\left(Q^2 - M_W^2\right)$$

$$+ \int \frac{d\xi_B}{\left(\xi_B S + T - Q^2\right)} \left(\frac{\hat{s} d\hat{\sigma}}{d\hat{t}} \right) \cdot f_{j/B}\left(\xi_B, \mu\right)$$

$$\cdot f_{i/A}\left(\xi_A = \frac{-Q^2 - \xi_B \left(U - Q^2\right)}{\xi_B S + T - Q^2}, \mu \right) \cdot \delta\left(Q^2 - M_W^2\right)$$

$$T = Q^2 - \sqrt{q_T^2 + Q^2}\sqrt{S}\, e^{-y},$$

$$U = Q^2 - \sqrt{q_T^2 + Q^2}\sqrt{S}\, e^{y},$$

$$\hat{s} = \xi_A \xi_B S$$

$$\hat{t} = \xi_A \left(T - Q^2\right) + Q^2$$

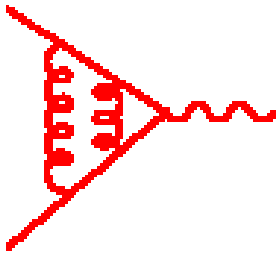
$$\frac{\hat{s}\, d\hat{\sigma}}{d\hat{t}} = \frac{1}{16\pi^2} \overline{|M|^2}$$

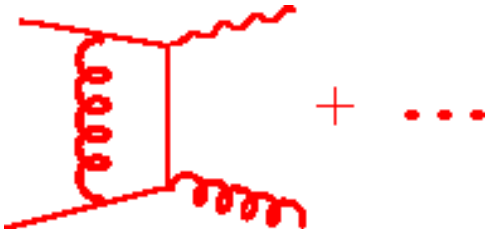
M =

(For simplicity, only consider qq→Wg)

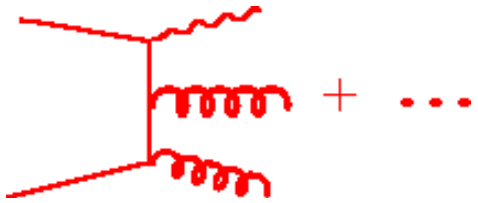
$\alpha_s^{(2)}$

- Virtual Corrections





- Real emission contributions



Perturbative Part:

- Higher order in $\alpha_S^{(n)}$
  ⟹ Less sensitive to Factorization Scale μ

- High q_T and smaller y (i.e. more central)
  ⟹ PDF (parton distribution function) better known

- With larger Luminosity
  ⟹ Test QCD in one large scale problem (i.e. $q_T \sim Q$)

- Up to now, most of the Data used in Testing QCD were
  One large scale observables, e.g., Jet-P_T.

- Observables involving Multiple Scales, e.g., q_T of W-Boson with mass M_W, can only be accurately described in QCD after including effects of Resummation.

What's QCD Resummation?

- Perturbative expansion

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \alpha_S \left\{ 1 + \alpha_S + \alpha_S^2 + \cdots \right\}$$

- The singular pieces, as $\frac{1}{q_T^2}$ (1 or log's)

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^{(n)} \ln^{(m)} \left(\frac{Q^2}{q_T^2} \right)$$

$$\sim \frac{1}{q_T^2} \{ \quad \alpha_S \left(\underline{L+1} \right)$$

$$+ \alpha_S^2 \left(\underline{L^3 + L^2} + \underline{L+1} \right)$$

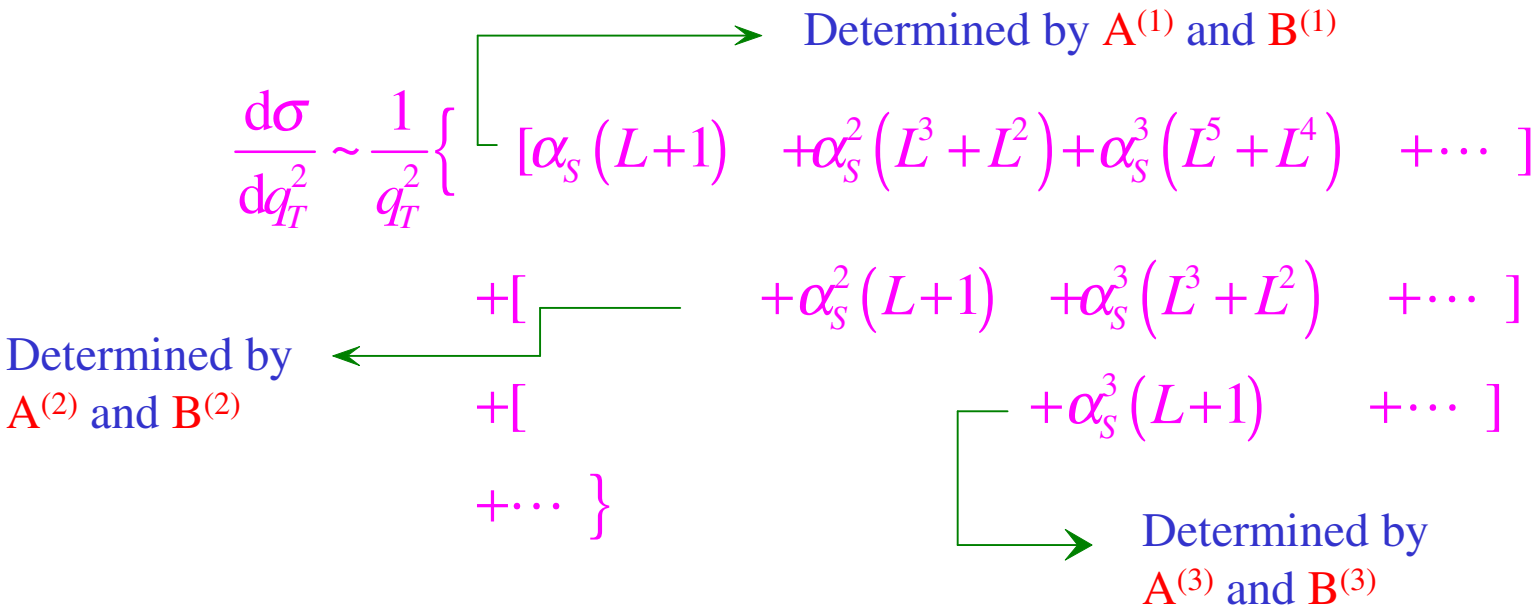
$$+ \alpha_S^3 \left(\underline{L^5 + L^4} + \underline{L^3 + L^2} + \underline{L+1} \right)$$

$$+ \cdots \quad \}$$

$$L \equiv \ln \left(\frac{Q^2}{q_T^2} \right)$$

Resummation is to reorganize the results in terms of the large Log's.

Resummed results:



QCD Resummation

In the formalism by Collins-Soper-Sterman, in addition to these perturbative results, the effects from physics beyond the leading twist is also implemented as

[non-perturbative functions].

CSS Resummation Formalism

$$\frac{d\sigma}{dq_T^2\, dy\, dQ^2} = \frac{\pi}{S}\sigma_0 \delta\left(Q^2 - M_W^2\right)\cdot$$

$$\left\{ \frac{1}{(2\pi)^2}\int d^2b \quad e^{i\vec{q}_T\cdot\vec{b}}\tilde{W}\left(b,Q,x_A,x_B\right)\cdot \text{[Non-perturbative functions]} \right.$$

$$\left. + Y\left(q_T,y,Q\right)\right\}$$

$$\tilde{W} = e^{-S(b)}\cdot C\otimes f\left(x_A\right)\cdot C\otimes f\left(x_B\right)$$

$$C\otimes f\left(x_A\right) = \sum_j \int_{x_A}^1 \frac{d\xi_A}{\xi_A} C_{qj}\left(\frac{x_A}{\xi_A},b,\mu\right)\cdot f_{j/A}\left(\xi_A,\mu\right)$$

$$C\otimes f\left(x_B\right) = \sum_k \int_{x_B}^1 \frac{d\xi_B}{\xi_B} C_{qk}\left(\frac{x_A}{\xi_A},b,\mu\right)\cdot f_{k/B}\left(\xi_B,\mu\right)$$

Sudakov form factor $S(b) = \int_{\left(\frac{b_0}{b}\right)^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right)A\left(\bar{\mu}\right) + B\left(\bar{\mu}\right)\right]$

[Non-perturbative functions] are functions of (b, Q, x_A, x_B) which include QCD effects beyond Leading Twist.

- For $W^{\pm}$

$$\sigma_0 = \left(\frac{4\pi^2\alpha}{3} \sum_{jj'} Q_{jj'}^{(W)} \right), \qquad Q_{jj'}^{(W)} = \frac{1}{4\sin^2\theta_W} \left(kM\right)_{jj'}^2$$

- For Z

$$\sigma_0 = \left(\frac{4\pi^2\alpha}{3} \sum_{jj'} Q_{jj'}^{(Z)} \right), \qquad Q_{jj'}^{(Z)} = \frac{1+\left[1-4\left|e_q\right|\sin^2\theta_W\right]^2}{16\sin^2\theta_W\cos^2\theta_W}\delta_{jj'}$$

- For Drell-Yan pair ($\ell^+\ell^-$ production)

$$\sigma_0\,\delta\left(q^2-Q^2\right) = \frac{4\pi\alpha^2}{9Q^2}$$

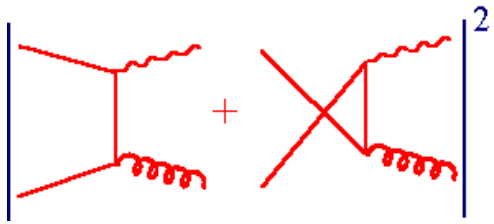
Note:

$$A \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot A^{(n)}, \qquad B \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot B^{(n)},$$

$$C \equiv \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot C^{(n)}$$

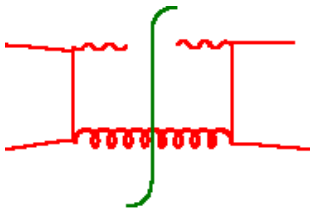
Find $\left.\dfrac{d\sigma}{dQ^2\,dy\,dq_T^2}\right|_{q_T\to 0}$

Consider

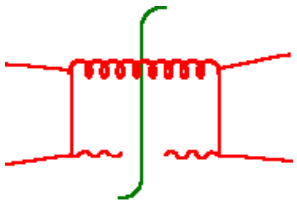


$$\cdot\delta\left(s+t+u-Q^2\right)$$

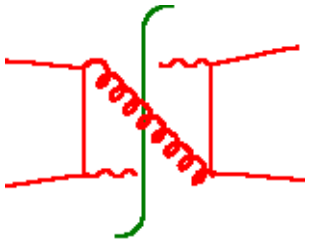
In ‘tHooft-Feynman gauge



$$\sim \frac{t\,u}{t^2}$$



$$\sim \frac{t\,u}{u^2}$$



$$\sim \frac{2s\,Q^2}{t\,u}$$

$$s=\xi_A\xi_B S$$

$$t=Q^2\left[1-\frac{\xi_A}{x_A}\sqrt{1+\frac{q_T^2}{Q^2}}\right]$$

$$u=Q^2\left[1-\frac{\xi_B}{x_B}\sqrt{1+\frac{q_T^2}{Q^2}}\right]$$

$$x_A=\frac{Q}{\sqrt{S}}e^{y},\quad x_B=\frac{Q}{\sqrt{S}}e^{-y},$$

$$t\,u=s\,q_T^2$$

Taking the singular part, as $\frac{1}{q_T^2}$ (1 or log's),

$$\left.\frac{2sQ^2}{tu}\delta\left(s+t+u-Q^2\right)\right|_{\text{singular}}$$

$$=\frac{1}{q_T^2}\left(\frac{3}{4}\right)\left\{\delta(1-z_A)\delta(1-z_B)\cdot\left[2\left(\frac{4}{3}\right)\ln\left(\frac{Q^2}{q_T^2}\right)+2(-2)\right]\right.$$

$$+P_{qq}^{(1)}(z_A)\delta(1-z_B)+\delta(1-z_A)P_{qq}^{(1)}(z_B)$$

$$\left.+\frac{4}{3}\left[-(1-z_A)\delta(1-z_Ba)-\delta(1-z_A)(1-z_B)\right]\right\}$$

(The last term is canceled by contributions from $\frac{u}{t}$ and $\frac{t}{u}$.)

$$P_{qq}^{(1)}=C_F\left(\frac{1+z}{1-z}\right)_+,\quad C_F=\frac{4}{3},\quad z_A=\frac{x_A}{\xi_A},\quad z_B=\frac{x_B}{\xi_B}$$

As $q_T \to 0$

$$\left.\frac{d\sigma}{dq_T^{2\backslash} dy dQ^2}\right|_{q_T\to 0}$$

$$= \left(\frac{\pi}{s}\sigma_0\right)\cdot\delta\left(Q^2 - M_W^2\right)\cdot\left(\frac{1}{2\pi q_T^2}\right)\left(\frac{\alpha_s(Q)}{\pi}\right)$$

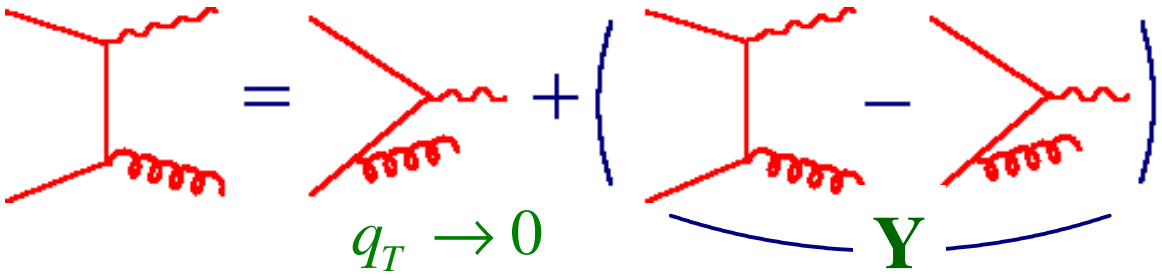
$$\cdot\Big\{ f_{q/A}(x_A,Q)\left[P_{\bar{q}\leftarrow\bar{q}}\otimes f_{\bar{q}}\right]_{x_B,Q}$$

$$+\left[P_{q\leftarrow q}\otimes f_q\right]_{x_A,Q} f_{\bar{q}/B}(x_B,Q)$$

$$+ f_{q/A}(x_A,Q) f_{\bar{q}/B}(x_B,Q)\cdot\left[2\left(\frac{4}{3}\right)\ln\left(\frac{Q^2}{q_T^2}\right)+2(-2)\right]\Big\}$$

$$\sigma_0 = \frac{4\pi^2\alpha}{3}\frac{1}{4\sin^2\theta_W}\left(KM\right)^2_{jj'}$$

$$\left[P_{q\leftarrow q}\otimes f_q\right]_{x,Q} \equiv \int_x^1 \frac{dz}{z} P_{q\leftarrow q}(z) f_q\left(\frac{x}{z},Q\right)$$



As $q_T \to 0$

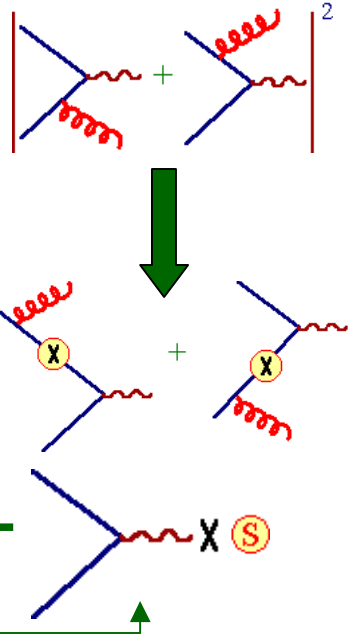
$$\left.\frac{d\sigma}{dq_T^{2\backslash} dy dQ^2}\right|_{q_T \to 0}$$

$$= \left(\frac{\pi}{s}\sigma_0\right) \cdot \delta\left(Q^2 - M_W^2\right) \cdot \left(\frac{1}{2\pi q_T^2}\right)\left(\frac{\alpha_s(Q)}{\pi}\right)$$

$$\cdot \Big\{ f_{q/A}(x_A, Q)\left[P_{\bar{q}\leftarrow\bar{q}} \otimes f_{\bar{q}}\right]_{x_B, Q} + \left[P_{q\leftarrow q} \otimes f_q\right]_{x_A, Q} f_{\bar{q}/B}(x_B, Q)$$

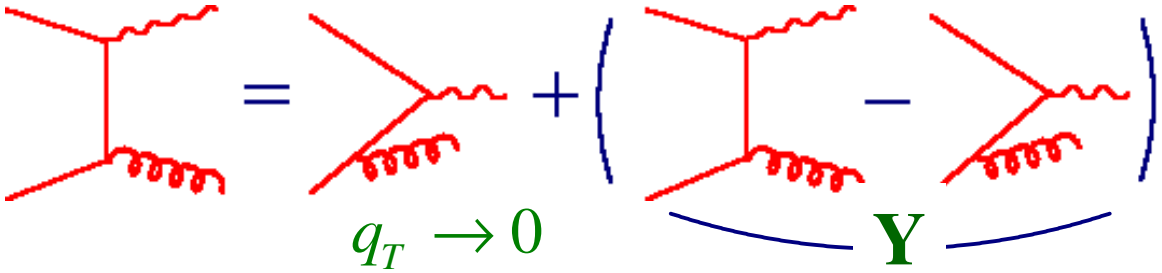
$$+ f_{q/A}(x_A, Q) f_{\bar{q}/B}(x_B, Q) \cdot \left[2\left(\frac{4}{3}\right)\ln\left(\frac{Q^2}{q_T^2}\right) + 2(-2)\right] \Big\}$$

Diagramatically,



Exponentiate

To preserve transverse momentum conservation, we have to go to the impact parameter space (b-space) to perform resummation.



To exponentiate , use CSS (Collins-Soper-Sterman) formalism

$$\frac{d\sigma}{dq_T^2 dy dQ^2} = \frac{\pi}{S}\sigma_0 \cdot \delta\left(Q^2 - M_W^2\right) \cdot \left\{ \frac{1}{2\pi} \int d^2 b e^{i\vec{q}_T \cdot \vec{b}} \right.$$

Impact parameter space

$$\cdot \int_{x_A}^1 \frac{d\xi_A}{\xi_A} \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{i/A}\left(\xi_A, \frac{1}{b}\right) f_{j/B}\left(\xi_B, \frac{1}{b}\right)$$

$$\cdot e^{-S(b)}$$

Sudakov form factor

$$\left. \cdot \delta\left(1 - \frac{x_A}{\xi_A}\right) \delta\left(1 - \frac{x_B}{\xi_B}\right) \right\},$$

$$S(b) = \int_{\left(\frac{b_0}{b}\right)^2}^{Q^2} d\ln\left(\bar{\mu}^2\right) \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) \frac{\alpha_s(\bar{\mu})}{\pi} \left(\frac{4}{3}\right) + \frac{\alpha_s(\bar{\mu})}{\pi}(-2) \right]$$

to resum the “Remnant” of soft & collinear cancellations

Find $\left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}q_T^2}\right|_{q_T\to 0}$ from Resummed formula up to $O(\alpha_s)$, use the formula

$$\frac{1}{(2\pi)^2}\int_0^\infty \mathrm{d}^2 b e^{i\vec{q}_T\cdot\vec{b}} = \frac{1}{(2\pi)}\int_0^\infty b\mathrm{d}bJ_0(q_T b) = \frac{1}{2\pi q_T^2}\int_0^\infty z\mathrm{d}zJ_0(z)$$

and

$$\int_0^\infty \mathrm{d}z\cdot z\cdot J_0(z)F(z) = -\int_0^\infty \mathrm{d}z\cdot z\cdot J_1(z)\frac{\mathrm{d}F(z)}{\mathrm{d}z}$$

get

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}q_T^2}\right|_{q_T\to 0} = \left(\frac{\pi}{S}\sigma_0\right)\cdot\delta\left(Q^2-M_W^2\right)\cdot\left(\frac{1}{2\pi q_T^2}\right)$$
$$\cdot\left\{-\int_0^\infty \mathrm{d}z\cdot z\cdot J_1(z)\frac{\mathrm{d}}{\mathrm{d}z}\left[e^{-S(b,Q,b_0)}f_{q/A}(x_A,\mu)f_{\bar{q}/B}(x_B,\mu)\right]\right\}$$

With $\mu\sim\frac{1}{b}$.

Use

$$\frac{\mathrm{d}}{\mathrm{d}z} f\left(x,\frac{1}{b}\right)=\left(\frac{-2}{z}\right)\cdot\left(\frac{\alpha_S(q)}{2\pi}\right)\cdot\left[P_{qj}^{(1)}\otimes f_j(x,Q)+P_{qg}^{(1)}\otimes f_g(x,Q)\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}z} e^{-S(b,Q,b_0)}=\left(\frac{-2}{z}\right)\cdot\left(\frac{\alpha_S(Q)}{2\pi}\right)\cdot\left[2\ln\left(\frac{Q^2z^2}{b_0^2q_T^2}\right)A^{(1)}+2B^{(1)}\right]$$

and

$$\int_0^\infty \mathrm{d}z\cdot J_1(z)=1$$

$$\int_0^\infty \mathrm{d}z\cdot J_1(z)\ln^{(m)}\left(\frac{z}{b_0}\right)=0,\ \text{for } m=1 \text{ or } 2$$

With $b_0=2e^{-\gamma}$, then

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}q_T^2}\right|_{q_T\to 0}=\left(\frac{\pi}{S}\sigma_0\right)\cdot\delta\left(Q^2-M_W^2\right)\cdot\left(\frac{1}{2\pi q_T^2}\right)\left(\frac{\alpha_S(Q)}{\pi}\right)$$

$$\cdot\left\{f_{q/A}(x_A,Q)\cdot\left[P_{qj}^{(1)}\otimes f_j+P_{qg}^{(1)}\otimes f_g\right](x_B,Q)+(A\leftrightarrow B)\right.$$

$$\left.+f_{q/A}(x_A,Q)\,f_{\bar{q}/B}(x_B,Q)\cdot\left[2A^{(1)}\ln\left(\frac{Q^2}{q_T^2}\right)+2B^{(1)}\right]\right\}$$

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\sum_{n-1}^{\infty}\sum_{m=0}^{2n-1}\alpha_S^{(n)}\ln^{(m)}\left(\frac{Q^2}{q_T^2}\right)_{(n)}F_{(m)}$$

$${}_1F_1 = \left[2A^{(1)}\right] f_{a/A} f_{b/B}$$

$${}_1F_0 = \left[2B^{(1)}\right] f_{a/A} f_{b/B} + \sum_i P_{ai} \otimes f_{i/A} \cdot f_{b/B} + f_{a/A} \cdot \sum_i P_{bi} \otimes f_{i/B}$$

$${}_2F_3 = -\left[A^{(1)}\right]^2 f_{a/A} f_{b/B}$$

$${}_2F_2 = -\frac{3}{2}\left[A^{(1)}\right]\left(\sum_i P_{ai} \otimes f_{i/A} \cdot f_{b/B} + f_{a/A} \cdot \sum_i P_{bi} \otimes f_{i/B}\right)$$

$$+ f_{a/A} f_{b/B} \cdot \left(-3\left[A^{(1)}B^{(1)}\right] + 2\beta_1\left[A^{(1)}\right]\right)$$

$$\cdots$$

$$A \equiv \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \cdot A^{(n)}, \quad \text{etc.}$$

$$\frac{\alpha_S(\mu)}{\pi} = \frac{\alpha_S(Q)}{\pi} - \beta_1\left(\frac{\alpha_s(Q)}{\pi}\right)^2 \cdot \ln\left(\frac{\mu^2}{Q^2}\right), \qquad \beta_1 = \frac{33 - 2N_F}{12}$$

N_F is the number of light quark flavor

Leading Log Resummation (DDT formalism)

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\left\{\left(\frac{\alpha_S}{\pi}\right)\cdot\ln\left(\frac{Q^2}{q_T^2}\right)\cdot\left[2A^{(1)}\right]\right.$$
$$+\left(\frac{\alpha_S}{\pi}\right)^2\cdot\ln^3\left(\frac{Q^2}{q_T^2}\right)\cdot\left[-\left(A^{(1)}\right)^2\right]$$
$$\left.+\left(\frac{\alpha_S}{\pi}\right)^3\cdot\ln^5\left(\frac{Q^2}{q_T^2}\right)\cdot\left[\frac{1}{4}\left(A^{(1)}\right)^3\right]+\cdots\right\}$$

$$\sim \frac{2}{q_T^2}\cdot\left(\frac{\alpha_S}{\pi}\right)\cdot\ln\left(\frac{Q^2}{q_T^2}\right)\cdot A^{(1)}\cdot e^{-\frac{1}{2}A^{(1)}\cdot\left(\frac{\alpha_S}{\pi}\right)\ln^2\left(\frac{Q^2}{q_T^2}\right)}$$

$$\frac{d\sigma}{dq_T^2} \sim f_{a/A}\, f_{b/B}\cdot\frac{2}{q_T^2}\cdot\left(\frac{\alpha_S}{\pi}\right)\cdot\ln\left(\frac{Q^2}{q_T^2}\right)\cdot A^{(1)}\cdot e^{-\frac{1}{2}A^{(1)}\cdot\left(\frac{\alpha_S}{\pi}\right)\ln^2\left(\frac{Q^2}{q_T^2}\right)}$$

Diagramatically, Resummation is doing

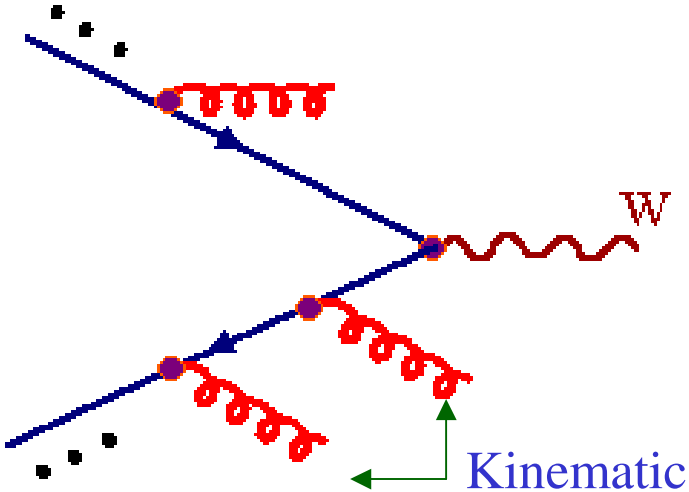
$\Rightarrow$ Resum large $\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)$ terms

$$\left.\frac{d\sigma}{dq_T^2 dy}\right|_{q_T \to 0} \sim \frac{1}{q_T^2}\sum_{n=1}^{\infty}\sum_{m=0}^{2n-1}\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)\cdot C_m^n$$

Monte-Carlo programs ISAJET, PYTHIA, HERWIG contain these physics.

(Note: Arbitrary cut-off scale in these programs to affect the amount of Backward radiation , i.e. Initial state radiation.)

Monte-Carlo Approach



Backward Radiation
(Initial State Rad.)

Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off.
(In contrast to perform integration in impact parameter space, i.e., b space.)

→ The shape of q_T (w) is generated. But, the integrated rate remains the same as at Born level (finite virtual correction is not included).

* Recently, there are efforts to include part of higher order effect in the event generator.

Note that the integrated rate is the same as the Born level rate ($\alpha_s^{(0)}$) even though the q_T – distribution is different (i.e., not $\delta\left(q_T^2\right)$ any more).

$$\sigma = \int \mathrm{d}q_T^2 \frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \int \mathrm{d}^2 b \underbrace{\int e^{i\bar{q}_T \cdot \bar{b}} \mathrm{d}^2 q_T}_{\delta^2(b)} \sigma_0 e^{-S(b)}$$

$$= \int \mathrm{d}^2 b \delta^2(b) \cdot \sigma_0 \cdot e^{-S(b)} \longrightarrow 1 \text{ at b=0}$$

$$= \sigma_0$$

For C-Function = $\delta\left(1 - \frac{x_A}{\xi_B}\right)$

To recover the "K-factor" in the NLO total rate

→ To include the C-Functions

$$\frac{d\sigma}{dQ^2 dy} \sim \left| + + \cdots \right|^2$$

Finite + $\left| + \right|^2$

$$\frac{d\sigma}{dq_T d\, y dQ^2}$$

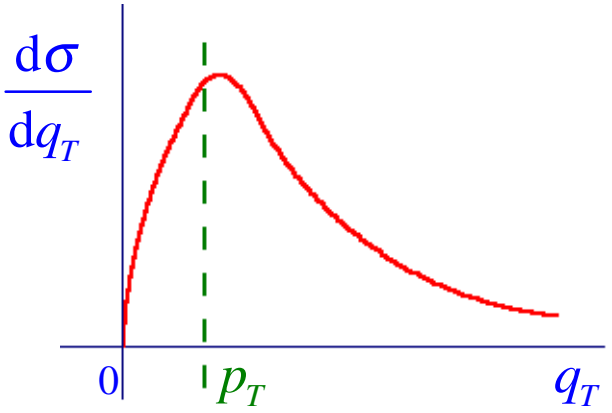
$$\frac{1}{q_T^2}\ln\left(\frac{Q^2}{q_T^2}\right), \quad \frac{1}{q_T^2}$$

Singular

q_T

$\delta\left(q_T^2\right)$

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y}\right|_{q_T^2=0}^{q_T^2=p_T^2} = \int_0^{p_T^2} \mathrm{d}q_T^2 \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}q_T^2}$$



$$= \frac{1}{S}\sigma_0 \delta\left(Q^2 - M_W^2\right)$$

$$\cdot\left\{ f_{q/A}(x_A,Q) f_{\bar{q}/B}(x_B,Q)\cdot\left(1-\left(\frac{\alpha_S(Q)}{\pi}\right)\ln\left(\frac{Q^2}{p_T^2}\right)\left[\frac{1}{2}\ln\left(\frac{Q^2}{p_T^2}\right)\left(\frac{4}{3}\right)+(-2)\right]\right)\right.$$

$$-\left(\frac{\alpha_S(Q)}{\pi}\right)\frac{1}{2}\ln\left(\frac{Q^2}{p_T^2}\right)\left[f_{q/A}(x_A,Q)\left[P\otimes f\right]_{x_B} + \left[P\otimes f\right]_{x_A} f_{\bar{q}/B}(x_B,Q)\right]$$

$$\left.+\left(\frac{\alpha_S(Q)}{\pi}\right)\left[f_{q/A}(x_A,Q)\left[C\otimes f\right]_{x_B} + \left[C\otimes f\right]_{x_A} f_{\bar{q}/B}(x_B,Q)\right]\right\}$$

→ The area under the q_T – curve will reproduce the total rate at the order $\alpha_S^{(1)}$ if Y term is calculated to $\alpha_S^{(1)}$ as well.

[non-perturbative function] is a function of (b, Q, x_A, x_B), implemented to include effects beyond Leading Twist.

Until we know how to calculate QCD non-perturbatively, (Lattice Gauge Theory?), these functions can only be parameterized. However, the same functions should describe Drell-Yan, $W^{\pm}$, Z^0 data.

- Test QCD in problems involving multiple scales.
- Measuring these non-perturbative functions may help in understanding the non-perturbative part of QCD.

[non-perturbative functions] , dependent of Q, b, x_A, x_B, is necessary to describe q_T – distribution of Drell-Yan, $W^{\pm}$,Z^0 events.

Fits:
- F. Landry ('01), MSU Ph.D Thesis

$$\exp\left[-g_1 b^2 - g_2 b^2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 b^2 \ln\left(100 x_A x_B\right)\right]$$

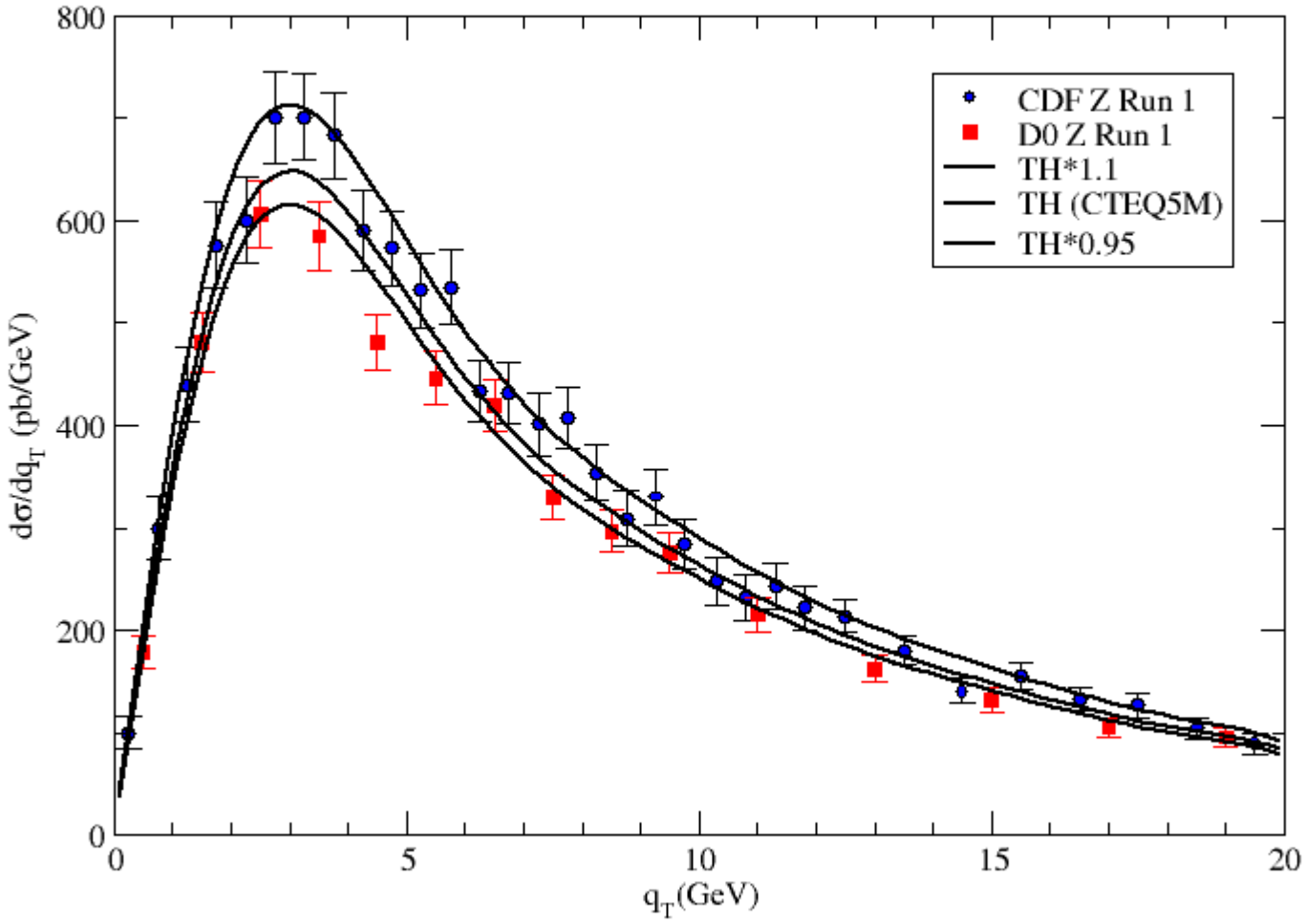
New term with x-dependence

$$Q_0 = 1.6 \text{ GeV}$$
$$g_1 = 0.21^{+0.01}_{-0.01} \text{ GeV}^2$$
$$g_2 = 0.68^{+0.02}_{-0.02} \text{ GeV}^2$$
$$g_3 = -0.60^{+0.05}_{-0.04}$$
$$\left(b_{\max} = 0.5 \text{ GeV}^{-1} \right)$$

Experiments included in global fits

Experiment	CM Energy (GeV)	τ Range	Boson Mass (GeV)	Process
R209	62.0	0.007-0.031	5-11	$p + p \rightarrow \mu^+\mu^- + X$
CDF Z	1800.0	0.003	91.19	$p + \bar{p} \rightarrow Z$
DØ Z	1800.0	0.003	91.19	$p + \bar{p} \rightarrow Z$
E288	27.4	0.033-0.110	5-9	$p + Cu \rightarrow \mu^+\mu^- + X$
E605	38.8	0.033-0.054 0.073-0.22	7-9 10.5-18	$p + Cu \rightarrow \mu^+\mu^- + X$

$P\bar{P} \rightarrow Z^0$



We discussed the high q_T ($q_T \sim O(M_W)$) and the low q_T ($q_T \ll M_W$) regions. To obtain the complete spectrum of q_T of W, we need to include the Matching term Y(q_T , y, Q) in the CSS Resummation formalism.

Y term (the matching term) is designed to improve QCD Resummation in the medium q_T ($q_T < M_W$) region.

However, there is No Perfect Matching .

Why matching fails?

Because the scale in evaluating the PDF is different as $q_T \sim Q$.

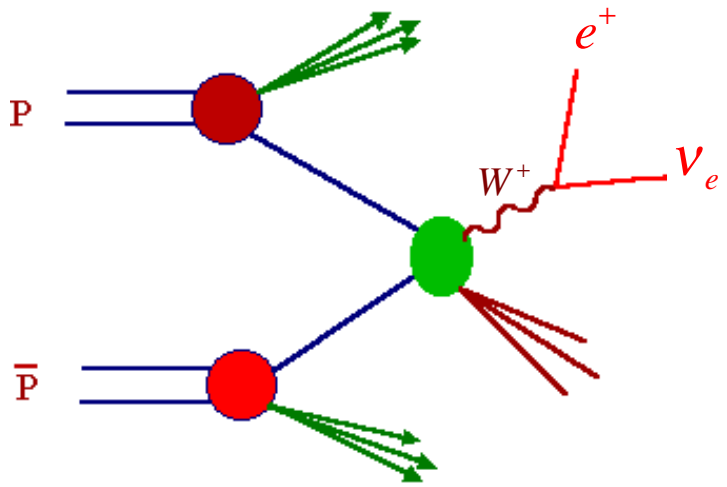
Exponentiate
(scale is x_A, x_B)
➡ Independent of q_T
(include "all order" effects)

ξ_A, ξ_B depend on Q & q_T
(fixed order)

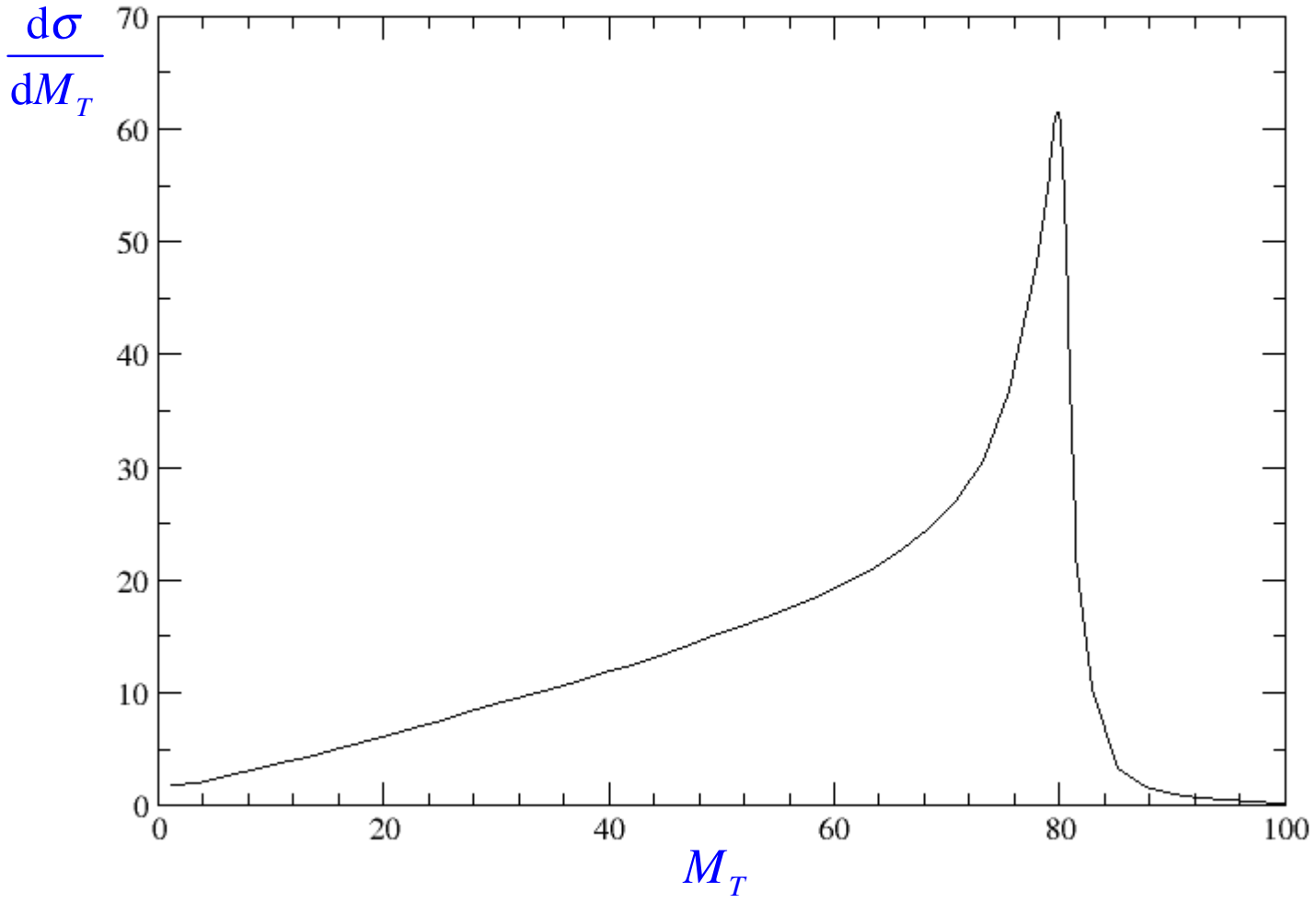
Some More Remarks on

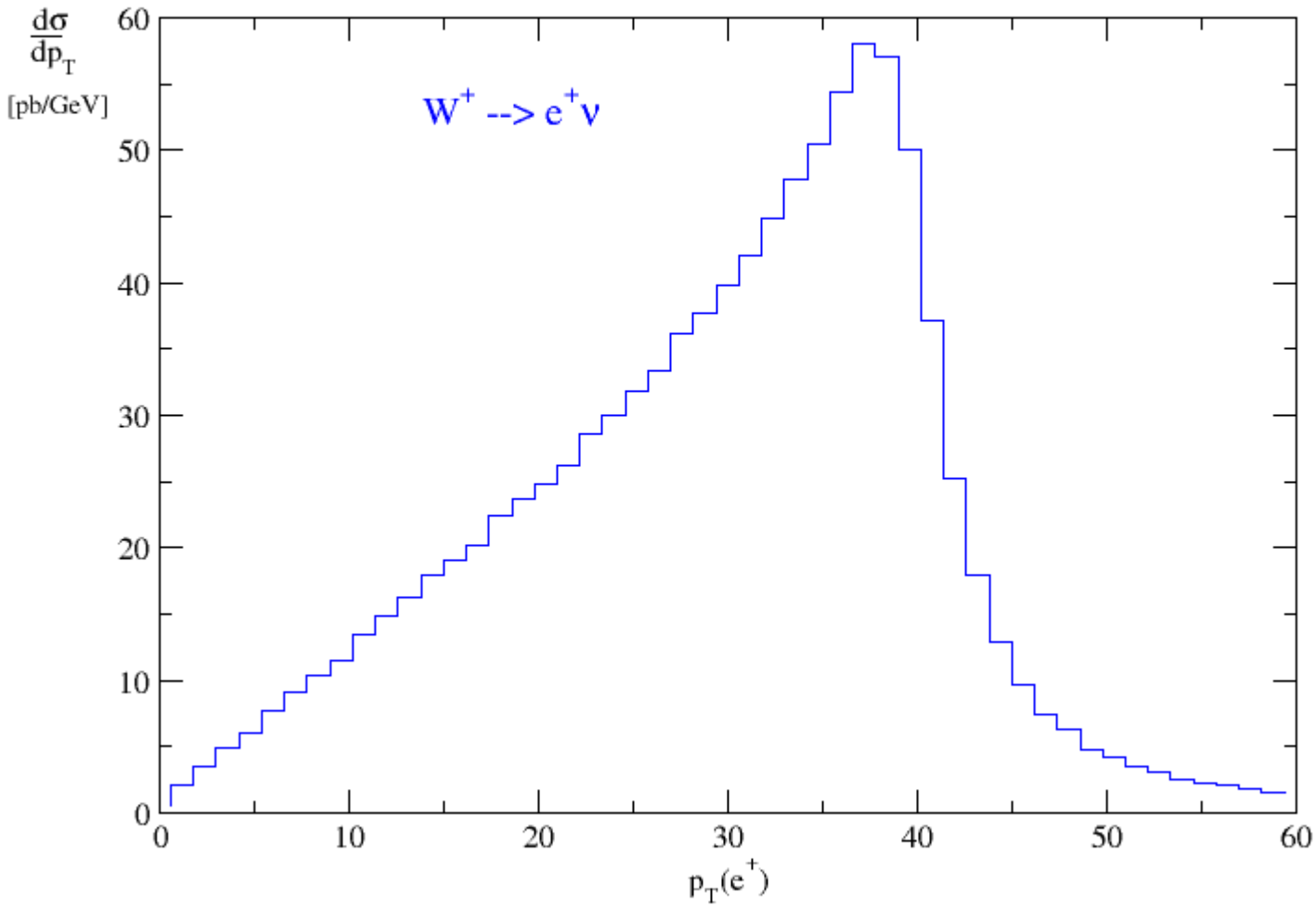
- Kinematics (y, q_T) of W-boson
- Kinematics $\left(y_e,\ p_T^e,\ m_T^{ev}\right)$ of lepton

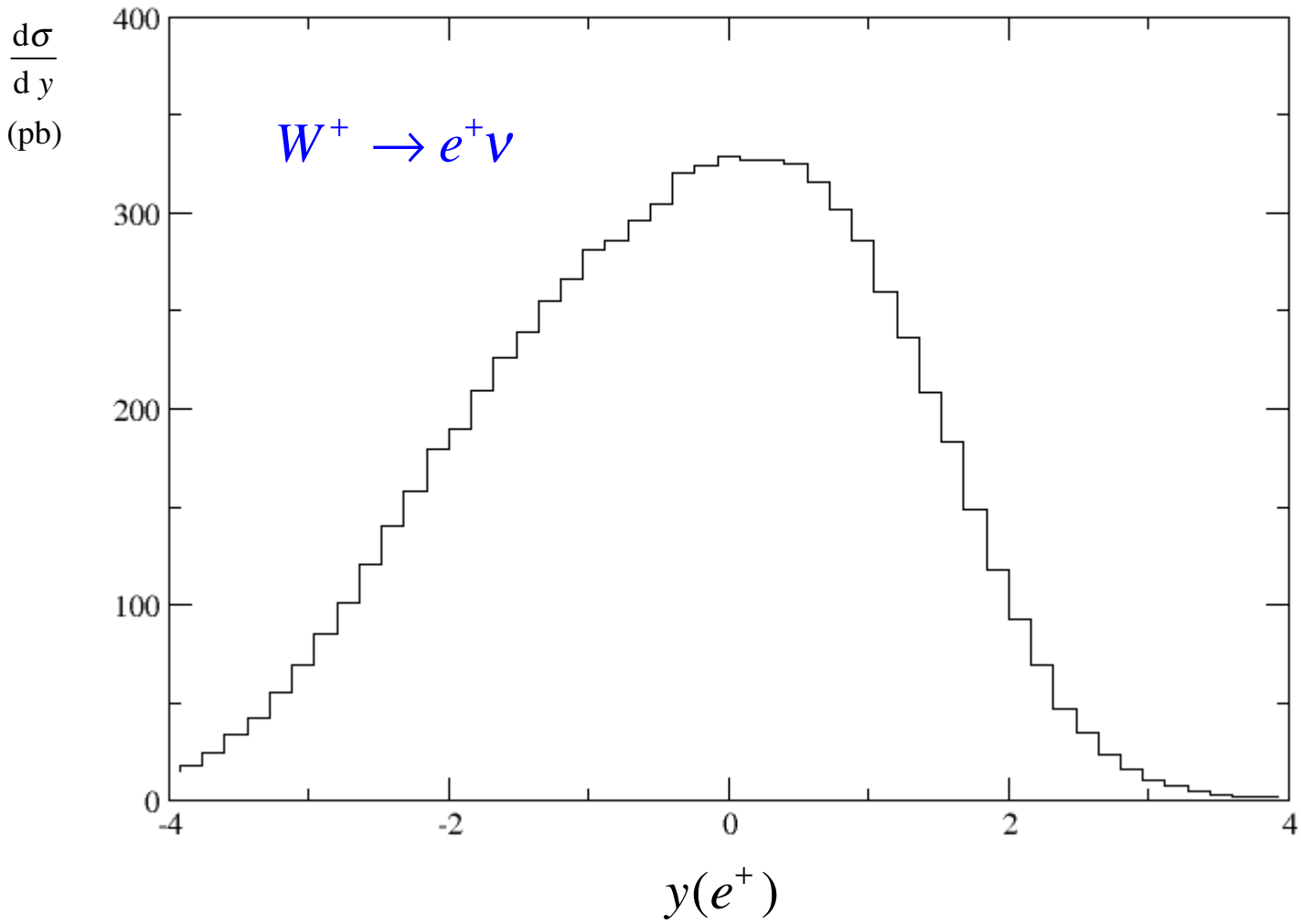
$$m_T^{ev} = 2\, p_T^e \not{E}_T \left[1 - \cos\left(\phi_2 - \phi_1\right)\right]$$



Transverse mass of W




dσ/dp_T
[pb/GeV]
W+ --> e+ν
0
10
20
30
40
50
60
p_T(e+)


$\frac{d\sigma}{d\,y}$ (pb)
400
300
200
100
0
-4
-2
0
2
4
$W^+ \to e^+\nu$
$y(e^+)$

What can we learn from the q_T of Z-Boson at the Tevatron

- QCD

  * Calibrate q_T of W-boson

    ⇒ Improve the measurement of M_W.

  * Measure PDF

    Small x, Large Q

    (~ 10^{-3}) (~100 GeV)

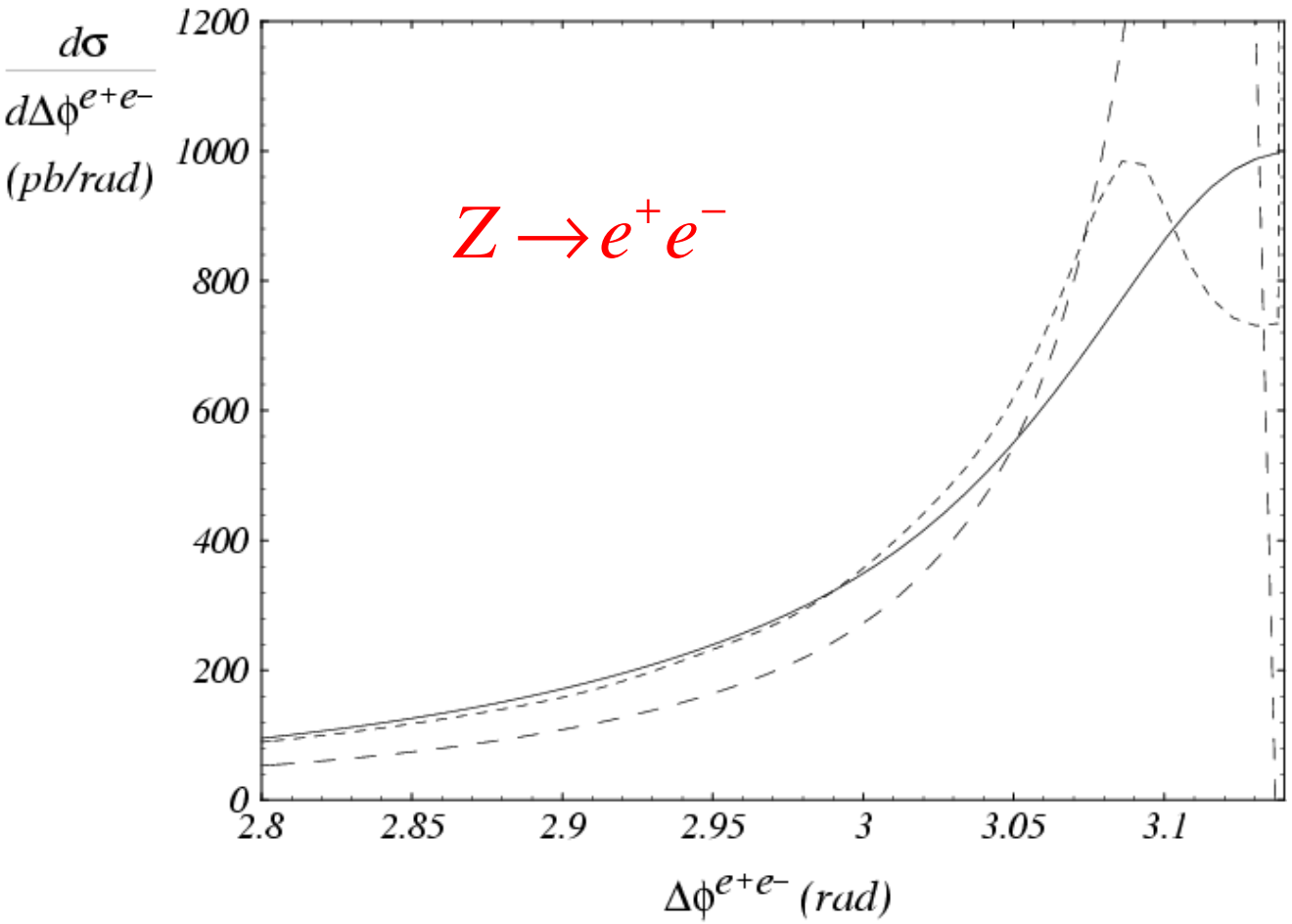
- Electroweak

  Forward-Backward Asymmetry of lepton

  on- or off- Resonance

  ⇒ $\sin^2\theta_W$, Z', etc.

Azimuthal angle correlation



Solid: CSS (2,2,1)

Dashed curves: Pert (1) , with two different prescriptions.

Conclusions:

With 2 fb^{-1} luminosity @ Tevatron Run 2,

W-Boson Physics can provide

- Testing one-scale QCD physics
- Testing two-scale QCD physics
- Probing non-perturbative QCD physics

Some New Physics may be associated with $W^{\pm}$ and Z.