Yesterday's lecture:

Phenomenological PDFs

- extracting PDFs from data
- nuclear effects in going from $F_2^d \to F_2^n$
- flavor asymmetries as evidence for non-perturbative physics

3.
Connection with low energy models

We know much...
We understand little...

One view: since PDFs are not "calculable", they can *only* be used to test self-consistency of experiments & QCD-parton model framework

Can one use PDFs to learn about low-energy nucleon structure?

Sometimes can get insight (even predictions!) into nucleon structure from PDFs especially sensitive to nonperturbative physics

e.g. $\bar{d} \neq \bar{u}$

$s \neq \bar{s}$ strange asymmetry

$u_p \neq d_n$ charge symmetry violation

$\Delta\bar{u} \neq \Delta\bar{d}$ spin dependent asymmetrieds

Can one use PDFs to learn about low-energy nucleon structure?

- Parisi & Petronzio (1976)
  - compute twist-2 PDFs at low scale μ (where valence quarks dominate), evolve to high Q^2 via DGLAP evolution
- Models studies
  - aim: use DIS data to discriminate between models
  - problem: how to set scale μ for given model, and how stable is evolution from μ?
  - select observables insensitive to μ (at least qualitatively)
- Gluck, Reya & Vogt
  - global fits of PDFs evolved from low scale

Dynamically generated PDFs

- input PDFs at low energy scale μ
  - evolve to higher energy scale using QCD evolution equations

- generates steep rise in sea below x ~ 0.01 purely perturbatively

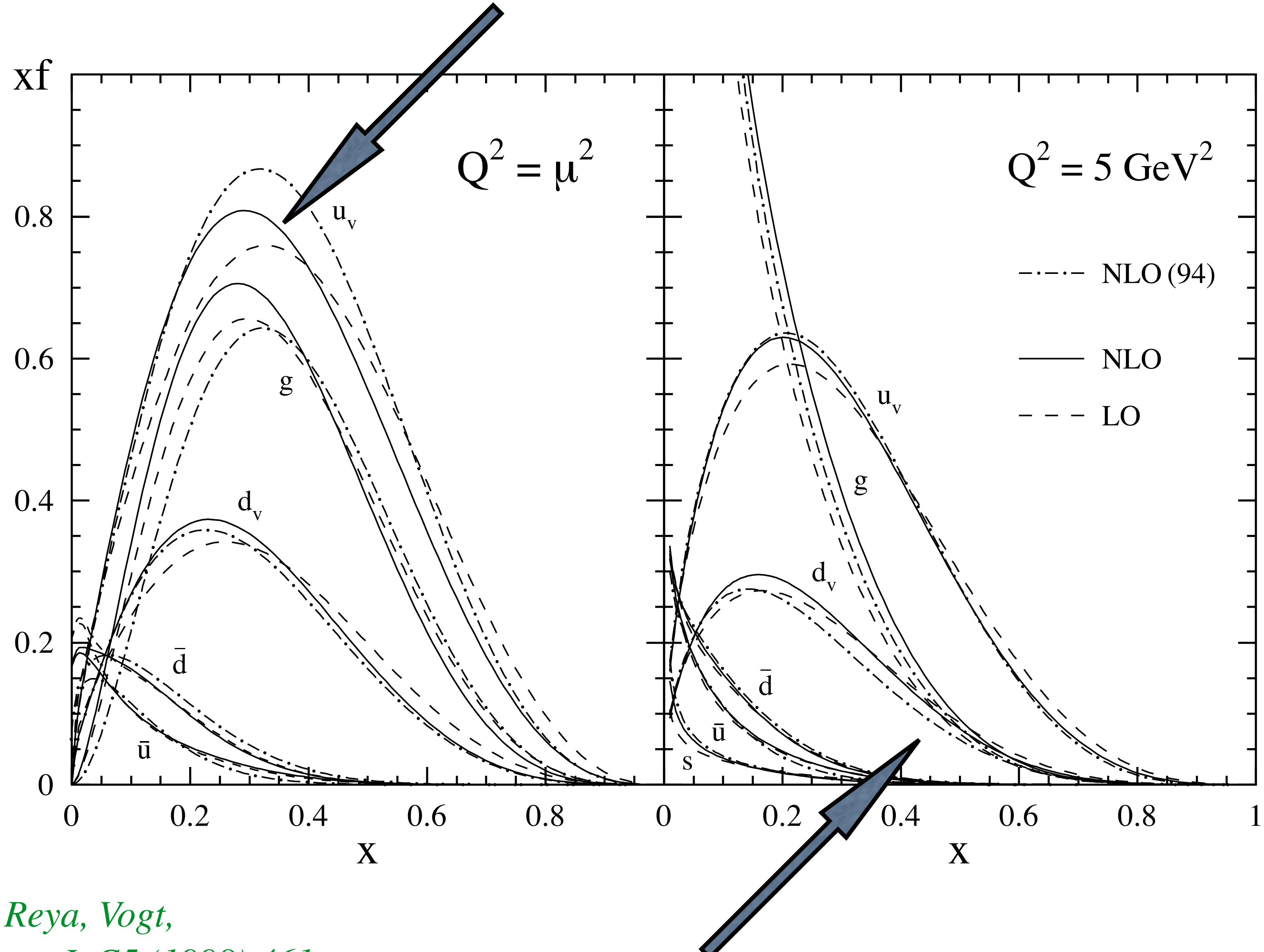
- valence-like input at scale μ
  - valence-like gluon, constrained by momentum conservation

$$\int_0^1 dx\; x \left(u_v(x,\mu^2) + d_v(x,\mu^2) + 2\bar{u}(x,\mu^2) + 2\bar{d}(x,\mu^2) + g(x,\mu^2) \right) = 1$$

- light flavor asymmetry $\bar{d}(x,\mu^2) \neq \bar{u}(x,\mu^2)$

- no strange sea

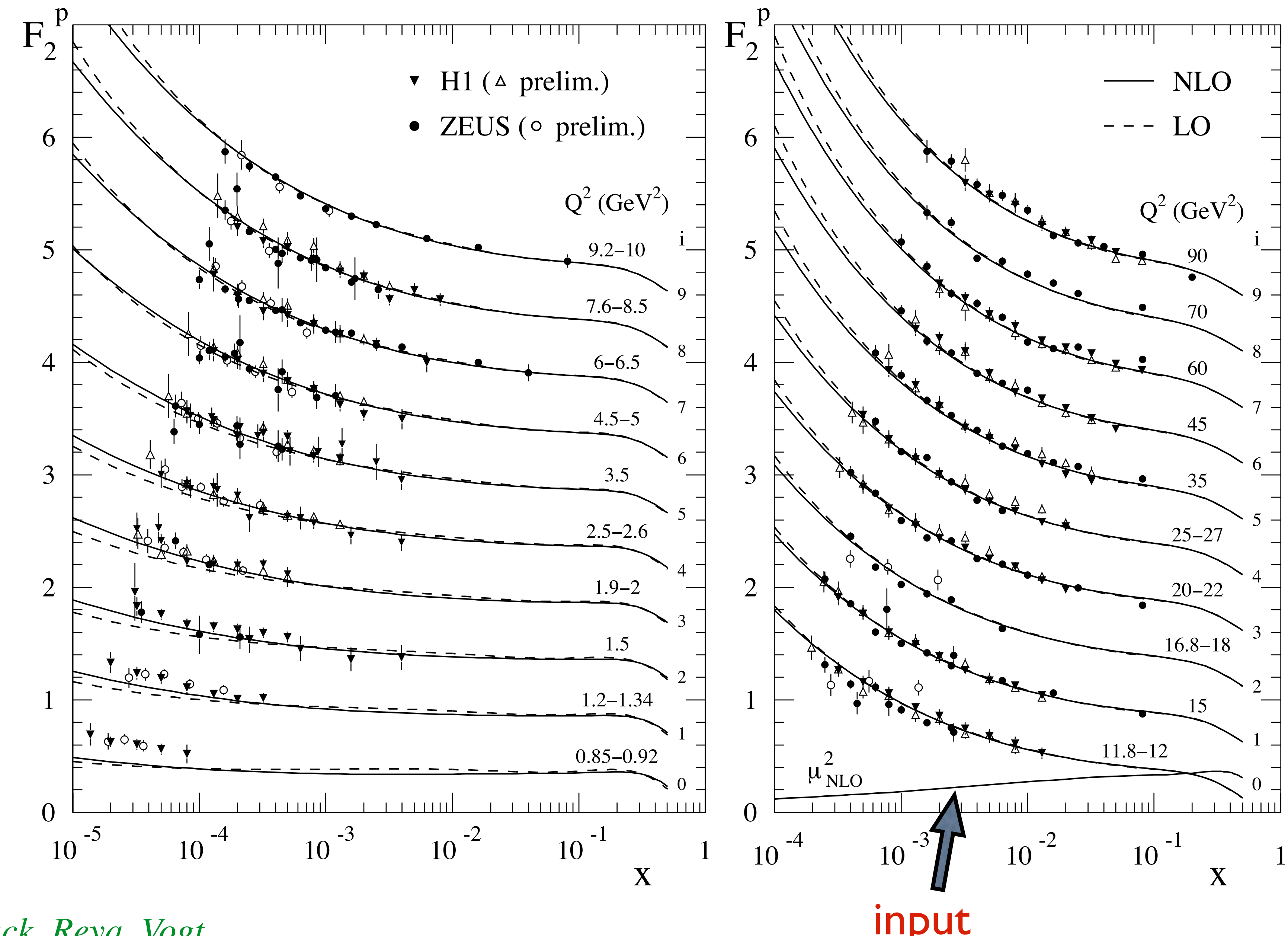
Input scale $\mu_{\mathrm{NLO}}^2 = 0.4\ \mathrm{GeV}^2$



Gluck, Reya, Vogt,
Eur. Phys. J. C5 (1998) 461

Evolved to higher scale

Dynamically generated PDFs



Gluck, Reya, Vogt,
Eur. Phys. J. C5 (1998) 461

But where do the input PDFs come from?

What is their relation with models of nucleon structure?

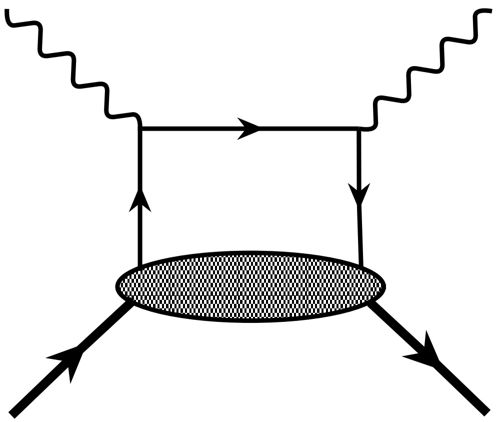
PDFs from models

Twist-2 quark distribution

$$\xi_- = \xi^0 - \xi^z$$

$$q(x) = \frac{1}{4\pi}\int d\xi_- e^{-iMx\xi_-}\langle N|\psi_+^\dagger(\xi^-)\psi_+(0)|N\rangle$$

$$\psi_+ = \frac{1}{2}(1+\gamma^0\gamma^z)\psi$$



PDFs from models

Twist-2 quark distribution

$$\xi_- = \xi^0 - \xi^z$$

$$q(x) = \frac{1}{4\pi}\int dz\, e^{-iMxz}\langle N|\psi_+^\dagger(\xi_-)\psi_+(0)|N\rangle$$

$$\psi_+ = \frac{1}{2}(1+\gamma^0\gamma^z)\psi$$

Insert complete set of intermediate states $|n(\vec{p})\rangle$

$$q(x) = \sum_n \int [dp]\, |\langle n(\vec{p})|\psi_+(0)|N\rangle|^2\, \delta(M(1-x) - p_n^+)$$

$$\frac{d^3p}{4E_p(2\pi)^3}$$

$$\sqrt{m_n^2 + \vec{p}^{\,2}} + p_z$$

R.L.Jaffe, "Relativistic Dynamics and Quark Nuclear Physics" (1985)

PDFs from models

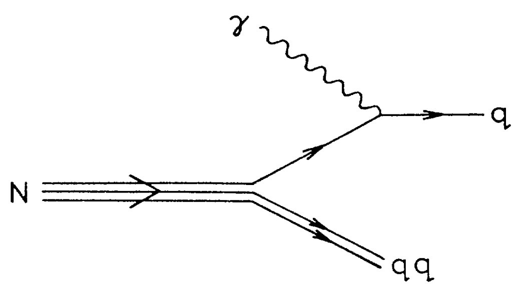
If nucleon = 3 constituent quarks at rest

- $p_n^+ = m_n$
- delta-function gives maximum in *q(x)* at

$$x = 1 - \frac{m_n}{M}$$

- largest contribution at large *x* from *lightest* intermediate state

$$|n\rangle = |qq\rangle$$



- expect *q(x)* to peak at *x* ~ 1/3

PDFs from models

Use specific model wave functions
to calculate valence PDFs

- Non-relativistic quark models
- Soliton models
- Bag models
- ......

see e.g. Thomas, Weise "The Structure of the Nucleon" (2001)

also nucl-th/9808008

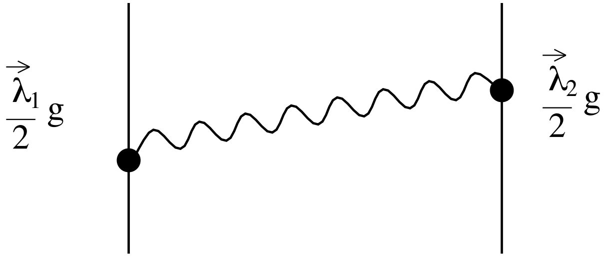
Non-relativistic quark models

Nucleon composed of 3 massive (non-relativistic) constituent quark "quasi-particles" bound in a confining potential

- may be viewed as "bare" valence quark dressed by "clouds" of $q\bar{q}$ pairs and gluons, giving mass $m_q \sim 330\ \mathrm{MeV}$

$$M_N \approx 3m_q,\ \ m_\rho \approx 2m_q$$

- residual one gluon exchange
  - strong attraction for S=0, I=0

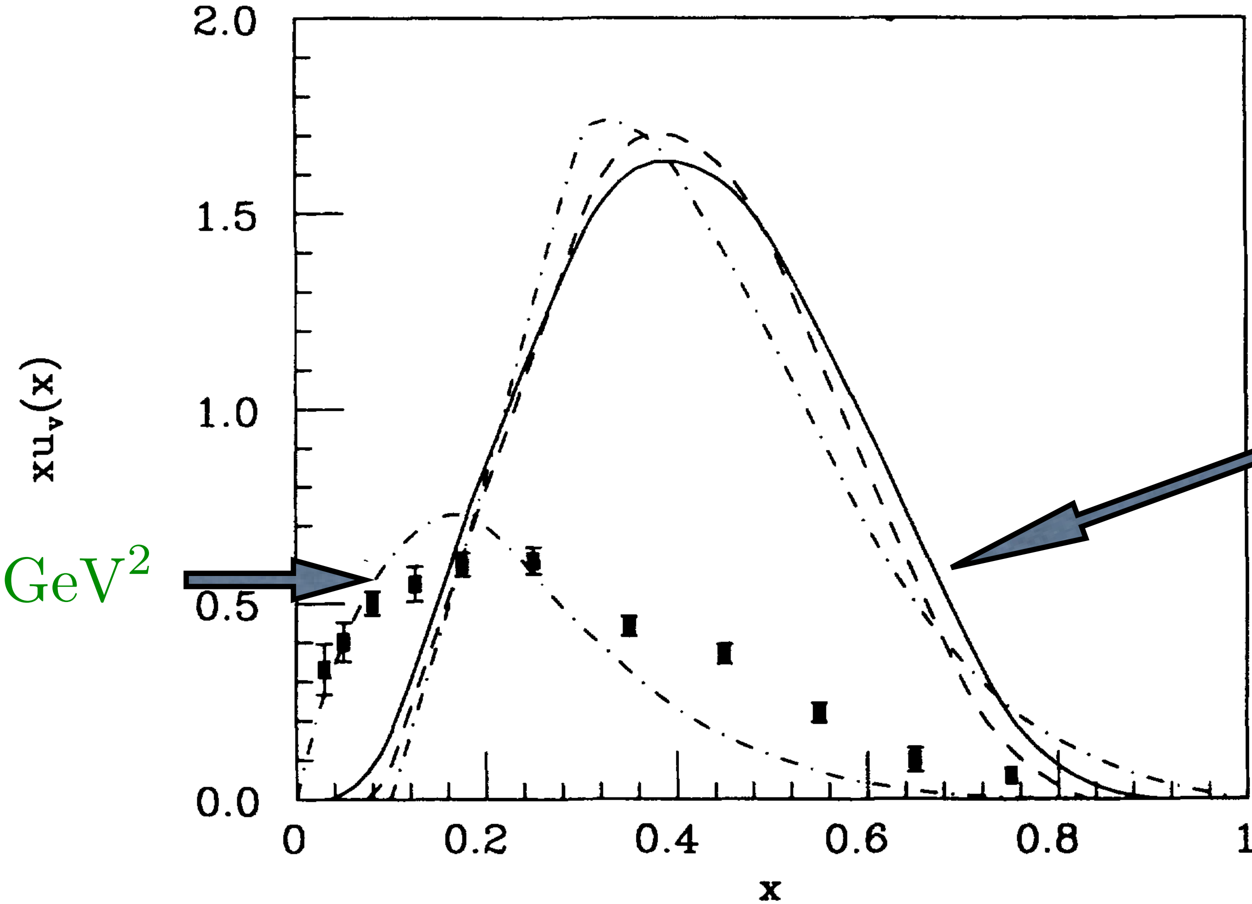


- rich and successful N^* spectroscopy

- classic prediction $\mu_n/\mu_p = -3/2$

Harmonic oscillator wave functions (Isgur-Karl)

$Q^2 = 10\ \mathrm{GeV}^2$



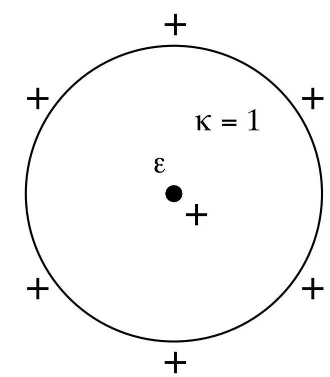
$\mu^2 = 0.34\ \mathrm{GeV}^2$

Weber, Phys. Rev. D49 (1994) 3160

Soliton models

QCD vacuum as a color dia-electric medium
(dielectric constant $\kappa_{\rm med} \ll 1$) (T. D. Lee)

color charge inserted into such
medium produces a "hole" with $\kappa = 1$
("perturbative vacuum")



QCD vacuum "excludes" color electric field

⇨ confinement of color charges

⇨ energy of charge in cavity (- free space) $\sim \dfrac{\kappa_{\rm med}^{-1} - 1}{R}$

perfect dia-electric $(\kappa_{\rm med} \to 0)$ is confining

Soliton models

Many variants of solitons models...

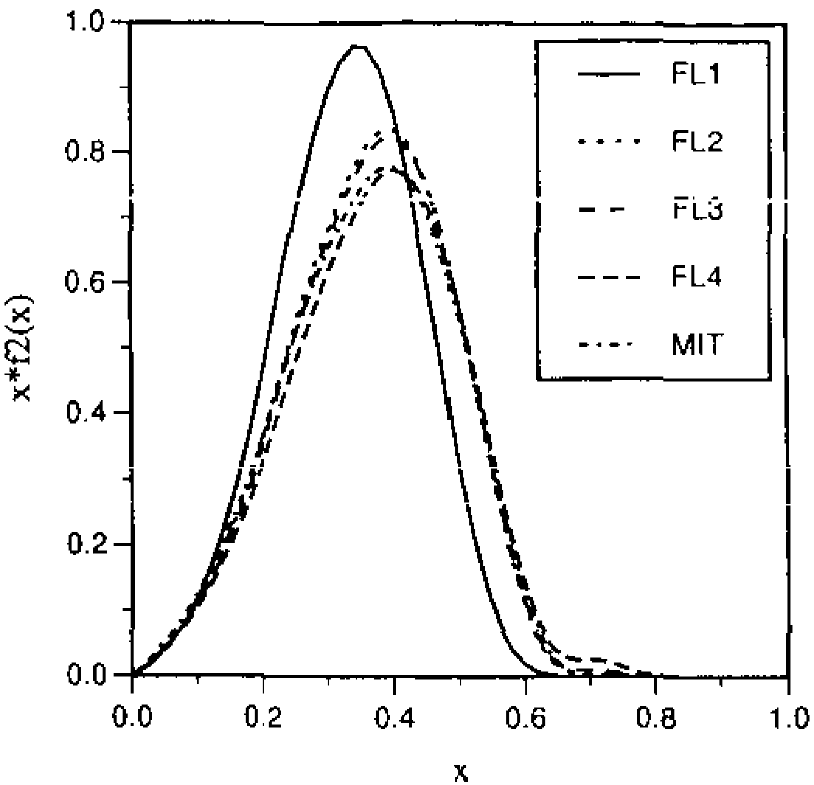
⇨ Non-Topological Soliton Model (Friedberg-Lee)
- scalar field, in which quarks "dig" a self-consistent "hole"

⇨ Color Dielectric Model (Nielseon-Patkos)

⇨ Chiral Soliton Models
- non-linear interacting systems of quarks and pions

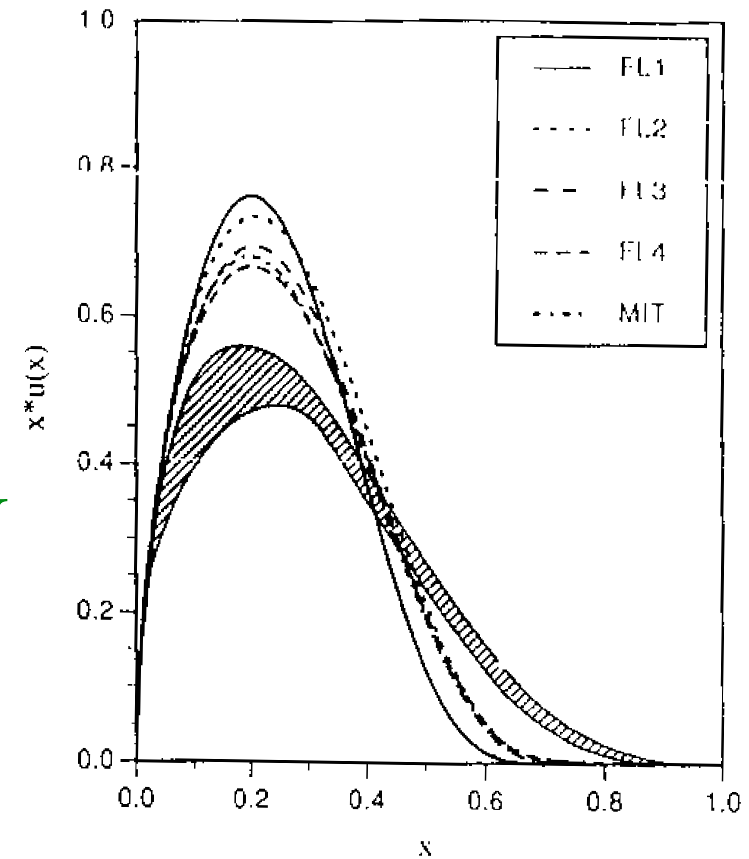
⇨ Topological Soliton (Skyrmion) Model
- baryon number identified with topological winding number associated with pion field

Friedberg-Lee soliton model

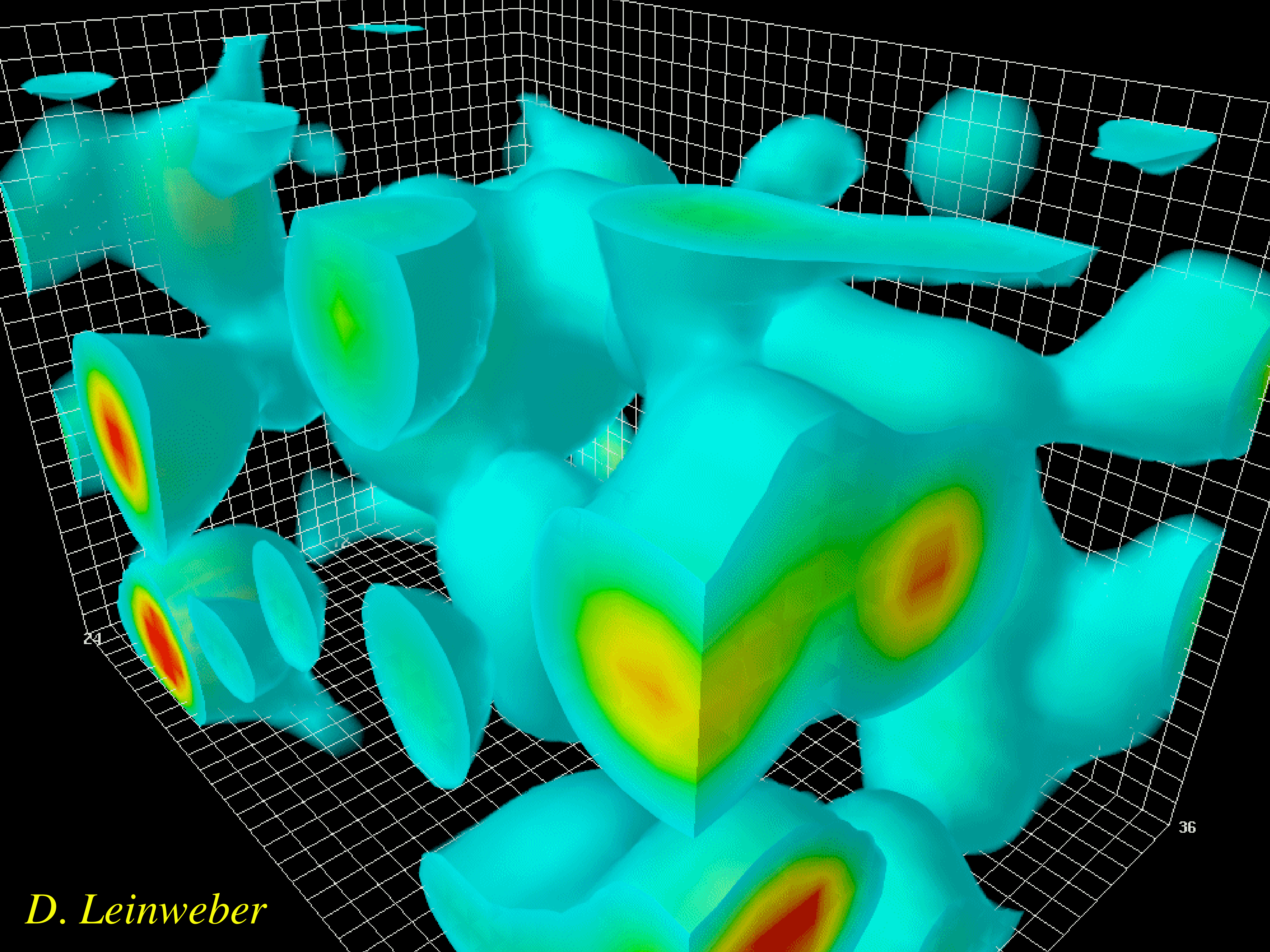


evolved from

$\mu \sim 0.5 \text{ GeV}$



Bate, Signal, J. Phys. G 18 (1992) 1875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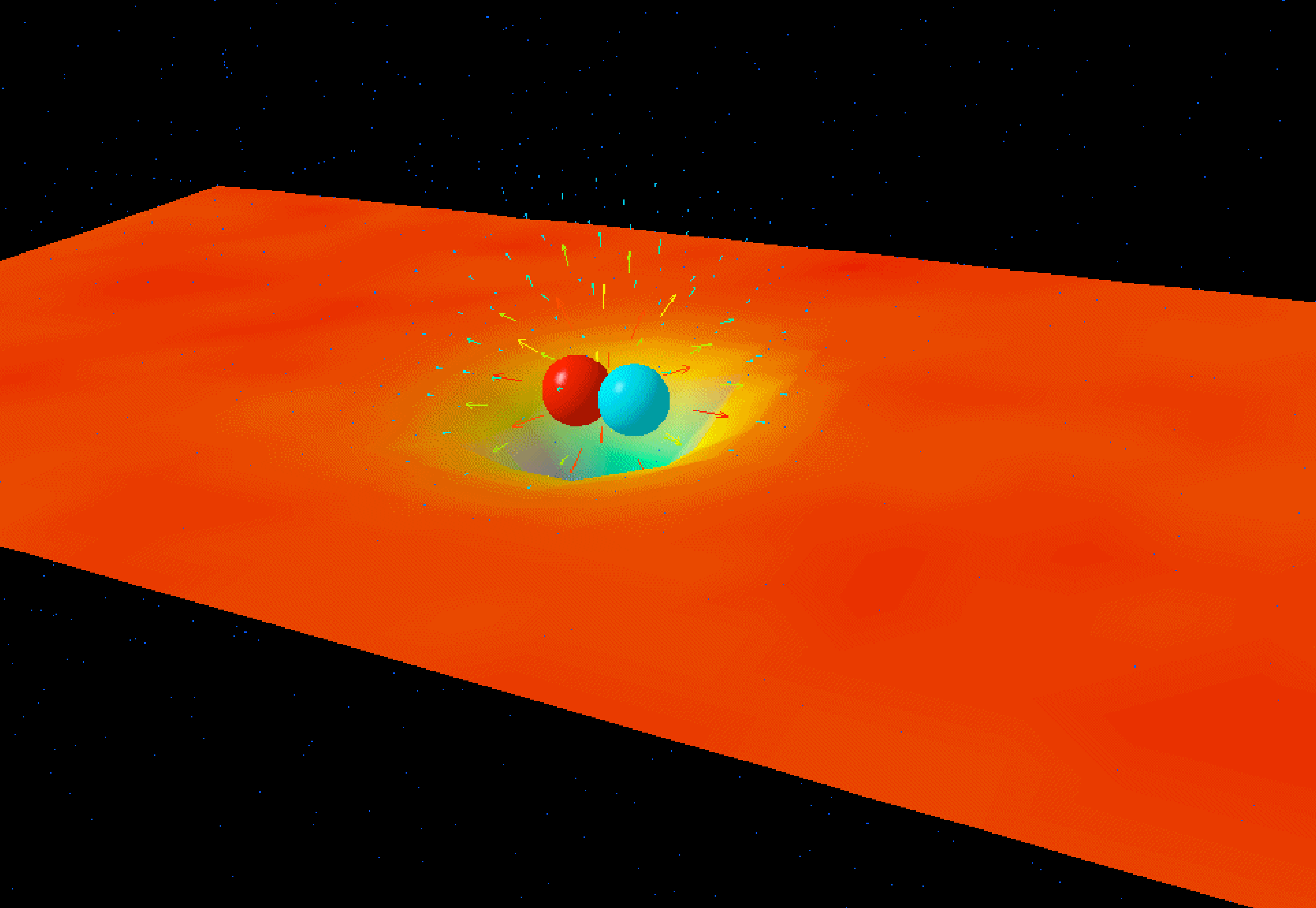
D. Leinweber



D. Leinweber

Bag models

Bogoliubov (1967)

quarks in spherical cavity (radius R),
with attractive scalar field $V_s = -\Theta(R-r)\ m$

$\Longrightarrow$ mimics asymptotic freedom ($r > R$)
and confinement ($r < R$, $m \to \infty$)

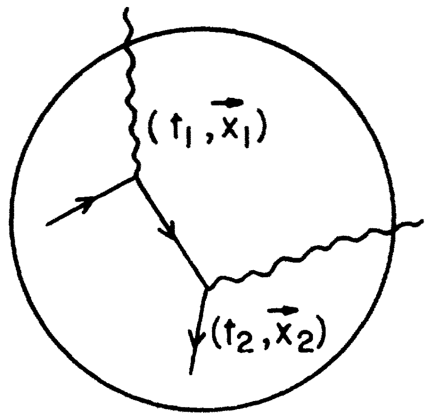
MIT bag model

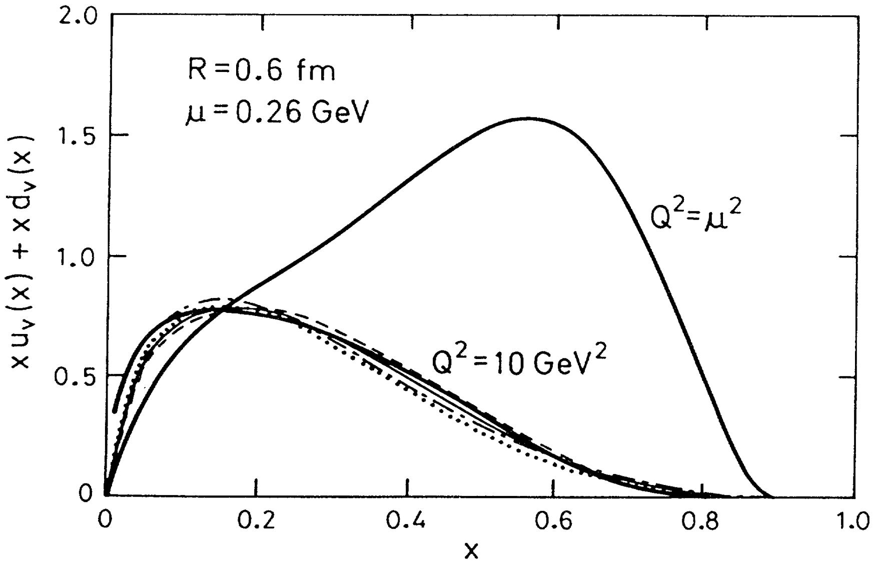
solve Dirac equation for relativistic quarks inside cavity

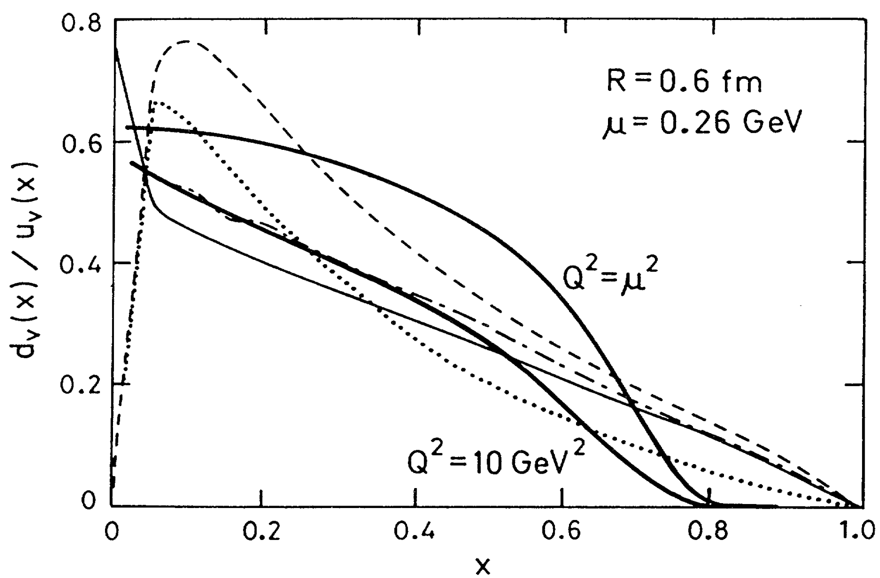
confinement achieved by requiring no quark current flow through surface of bag

get quark and antiquark distributions functions

DIS from the "bag"







d(x) softer than *u(x)* due to OGE between "spectator" quarks

⇨ raises energy of *S=1 qq* pair relative to *S=0 qq* pair

Feynman, 1972

Schreiber, Signal, Thomas, Phys. Rev. D44 (1991) 2653

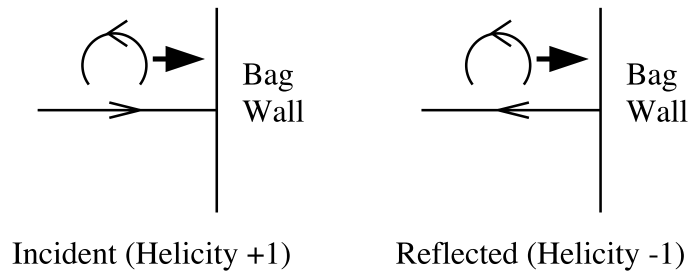
Chiral symmetry

In massless quark limit, QCD has exact
$SU(2)_L \times SU(2)_R$ symmetry

Symmetry broken spontaneously

⟹ appearance of pesudoscalar Goldstone bosons
- pions - which play important role in hadronic physics

Reflection of quark at bag surface changes helicity



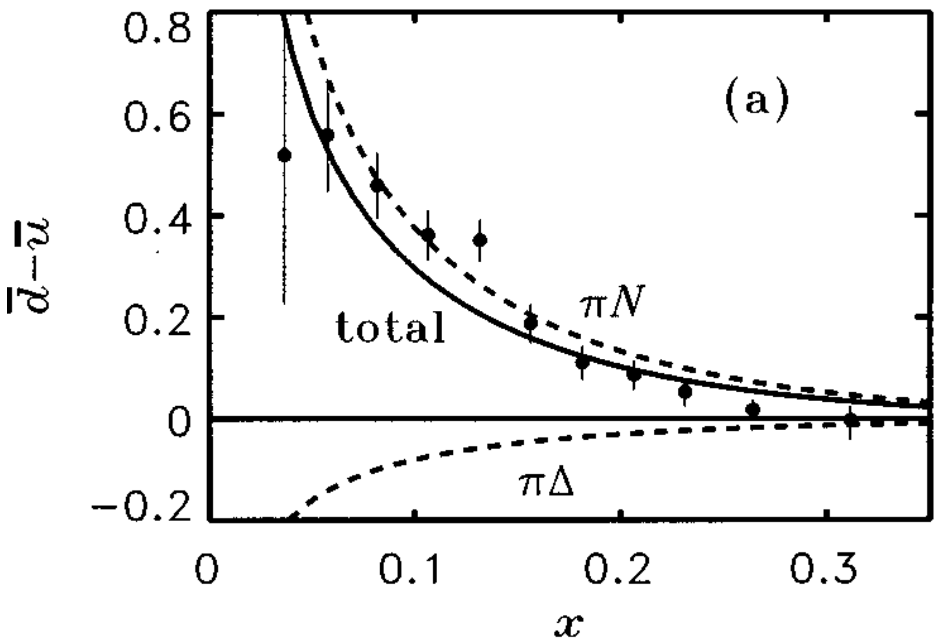
⟹ violates chiral symmetry!

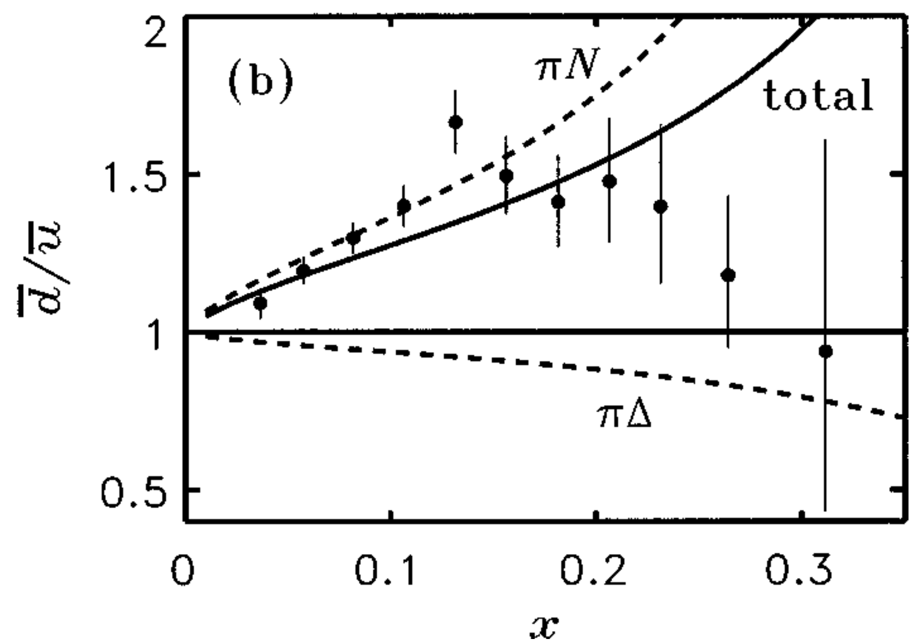
⟹ add pion field to bag model to restore chiral symmetry

⟹ chiral quark models, e.g. Cloudy Bag Model

Thomas, Theberge, Miller, Phys. Rev. D24 (1981) 216

Pion cloud contributions to flavor asymmetry in proton sea





WM, Speth, Thomas,
Phys. Rev. D59 (1998) 014033

Can get reasonable description at $x < 0.2$

difficult to understand downturn at large x

Which is the “true” model ?

⟹ mimic different aspects of QCD at low-energy

⟹ the models are not “systematic”

It might be right, but it’s not systematic...

It might be wrong, but at least it’s systematic...

What about something which might be right,
AND is systematic?

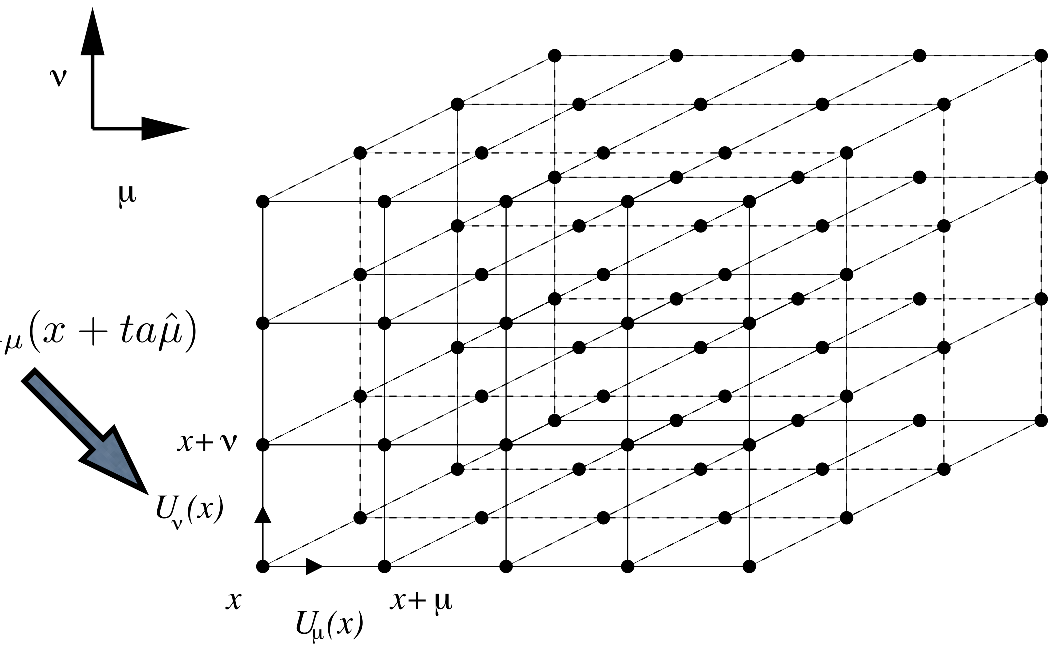
4.
PDFs from the lattice

Lattice QCD (in a nutshell)

Solve QCD equations of motion *numerically* on discretized space-time grid

Wilson (1974)

$$U_\mu(x) = \exp i g\, a \int_0^1 dt\, A_\mu(x + ta\hat{\mu})$$



- quarks on lattice nodes
- gluons as links between nodes

Observables calculated from path integrals in Euclidean space

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U\, O(U) e^{-S_G(U)}$$

generating functional

$$Z = \int \mathcal{D}U \;\det M(U)\, e^{-S_G(U)}$$

Fermion mass matrix

$$M(x,y,U) = m\,\delta_{x,y} + \frac{1}{2}\sum_{\mu} \gamma_\mu \left(U_\mu(x)\delta_{y,x+\hat{\mu}} - U_\mu^\dagger(x-\hat{\mu})\delta_{y,x-\hat{\mu}} \right)$$

Approximations

- finite lattice spacing a $(\to 0)$
- finite lattice volume V $(\to \infty)$
- large quark mass m_q $(\to m_q^{\text{phys}})$ ⟸ cost $\propto m_q^{-4}$
- "quenching" - suppression of background $q\bar{q}$ loops ⟸ OK only at large m_q

PDFs from Lattice QCD

Cannot calculate x-distribution on lattice

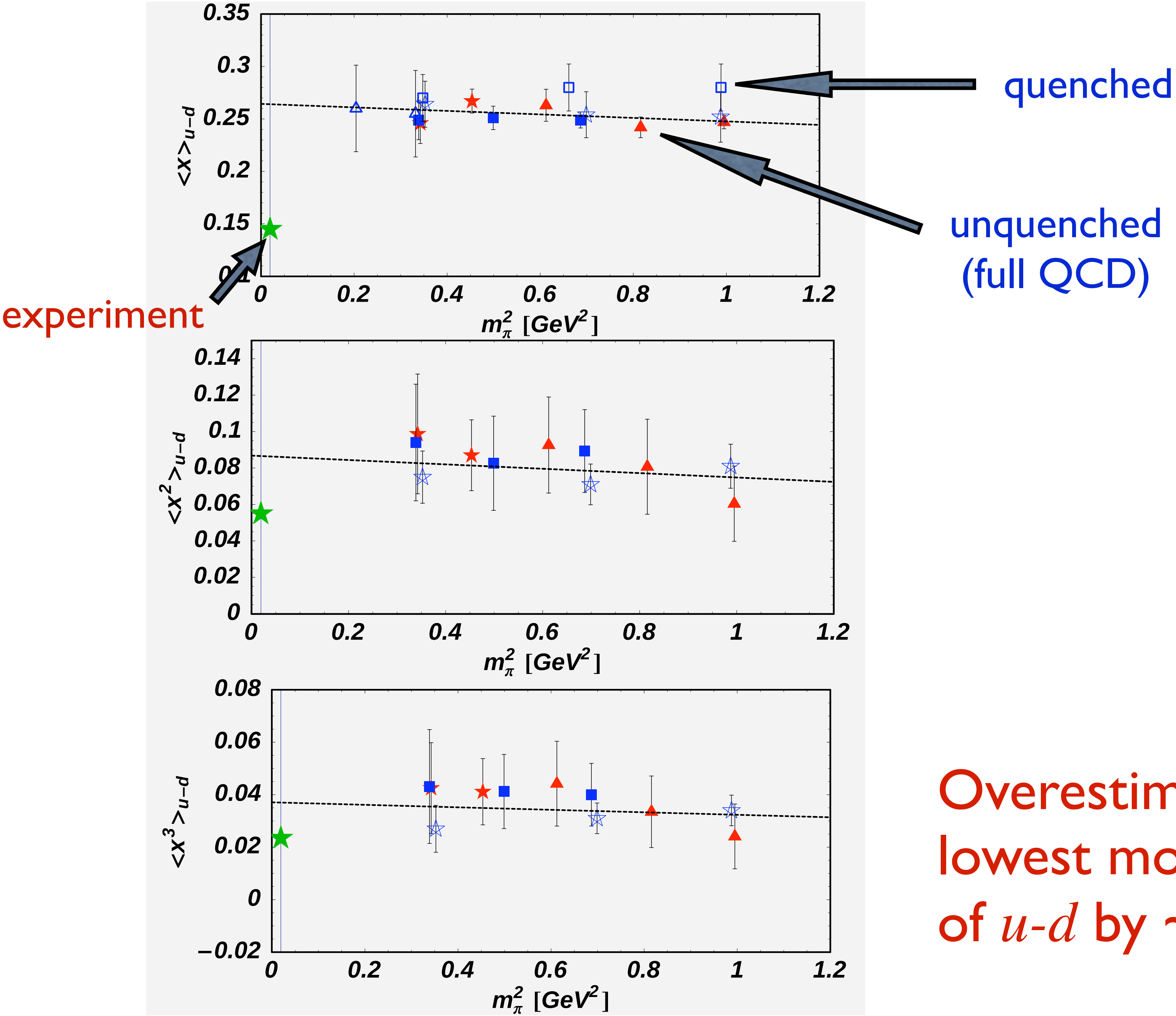
(no light-cone in Euclidean space) - *only moments*

$$\langle x^n \rangle_q = \int_0^1 dx\ x^n\ \Big(q(x) + (-1)^{n+1}\bar{q}(x)\Big)$$

Use operator product expansion to relate *moments* of PDFs to *matrix elements of local operators*

$$\langle x^n \rangle\ p_{\mu_1} \cdots p_{\mu_{n+1}} = \langle N|\ \mathcal{O}_{\{\mu_1 \cdots \mu_{n+1}\}} |N\rangle$$

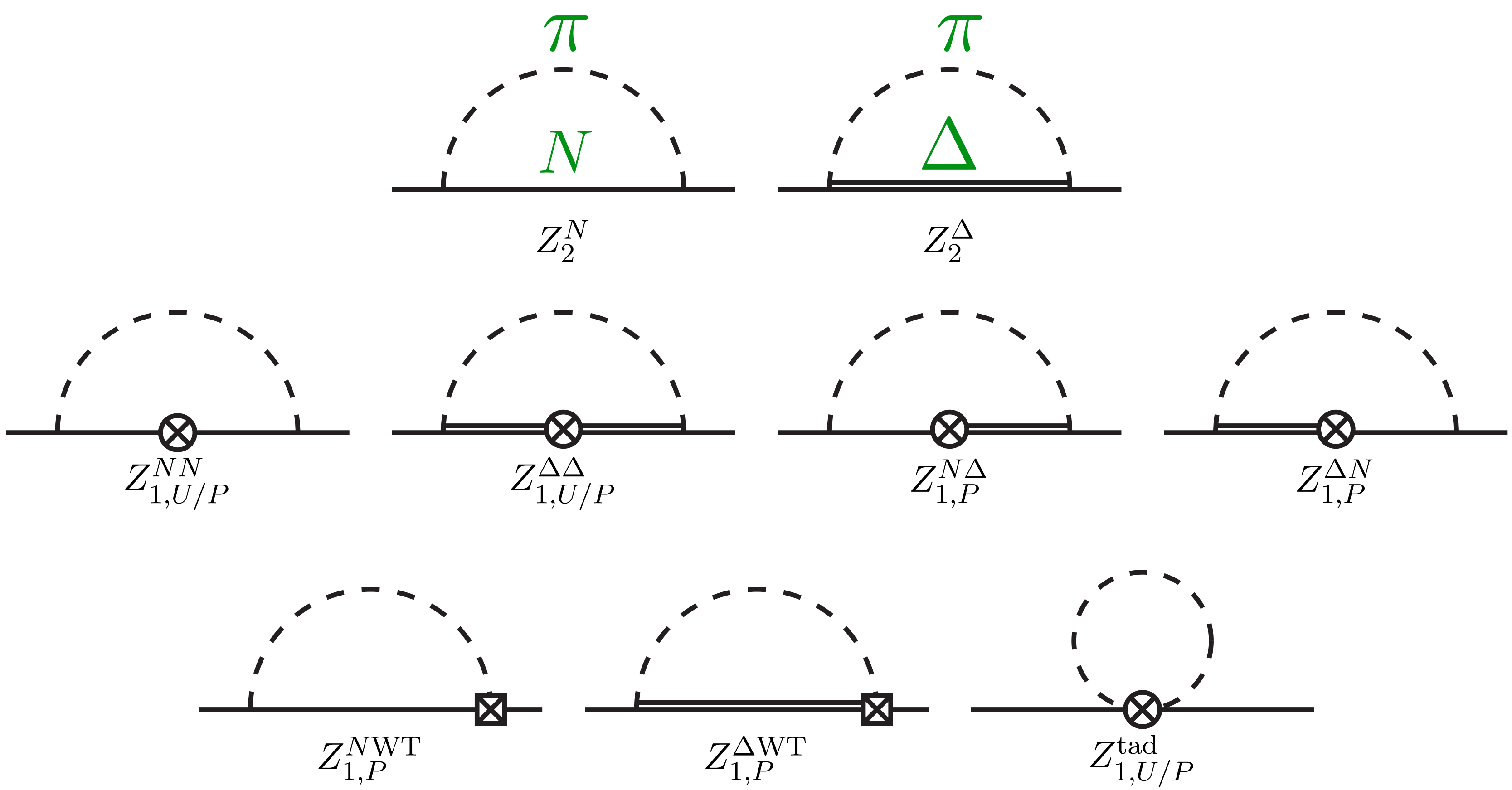
twist-2 operators



Overestimates lowest moment of u-d by ~50% !

Chiral extrapolation of lattice moments

Even though structure functions are measured at high energies
→ their moments have chiral expansion



Chiral extrapolation of lattice moments

Even though structure functions are measured at high energies
➡ their moments have chiral expansion

$$\langle x^n \rangle_{u-d} \;=\; a_n \left(1 + c_{\rm LNA} m_\pi^2 \log \frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) \;+\; b_n \frac{m_\pi^2}{m_\pi^2 + m_{b,n}^2}$$

Detmold et al, Phys. Rev. Lett. 87 (2001) 172001
also Arndt, Savage (2001), Ji, Chen (2001)

Leading non-analytic coefficient

$$c_{\rm LNA} = -(1 + 3 g_A^2)/(4\pi f_\pi)^2$$

calculated from χPT

PDF in heavy quark limit

$$u(x) - d(x) \xrightarrow{m_q \to \infty} \delta(x - \frac{1}{3})$$

Moment

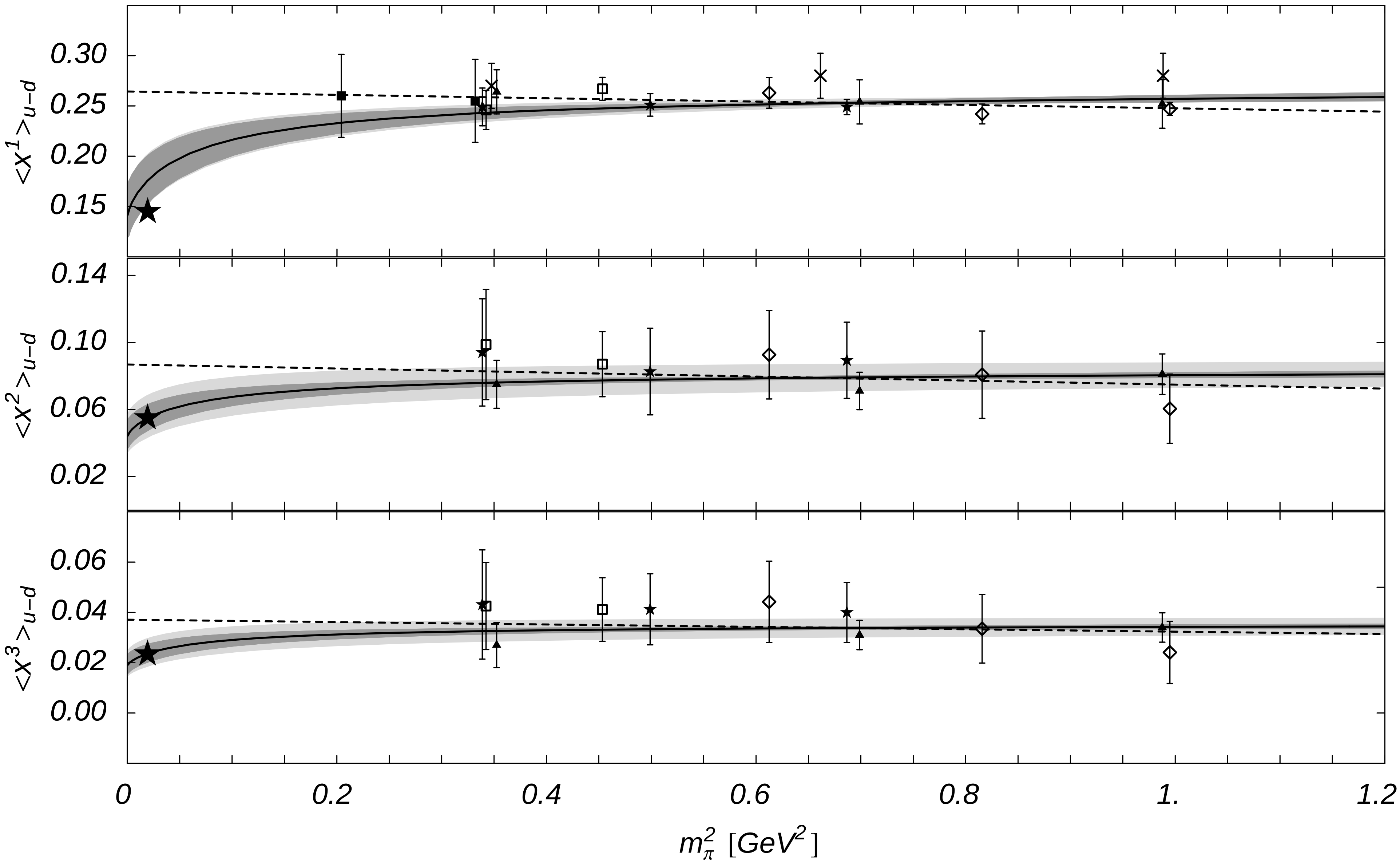
$$\langle x^n \rangle_{u-d} \xrightarrow{m_q \to \infty} \frac{1}{3^n}$$

Coefficient ensures correct $m_\pi \to \infty$ behavior

$$b_n = \frac{1}{3^n} - a_n \left(1 - \mu^2 c_{\mathrm{LNA}}\right)$$

Parameter μ determines amount of curvature at low m_π^2 $(m_\pi^2 \propto m_q)$

Detmold, WM, Thomas, Mod. Phys. Lett. A18 (2003) 2681



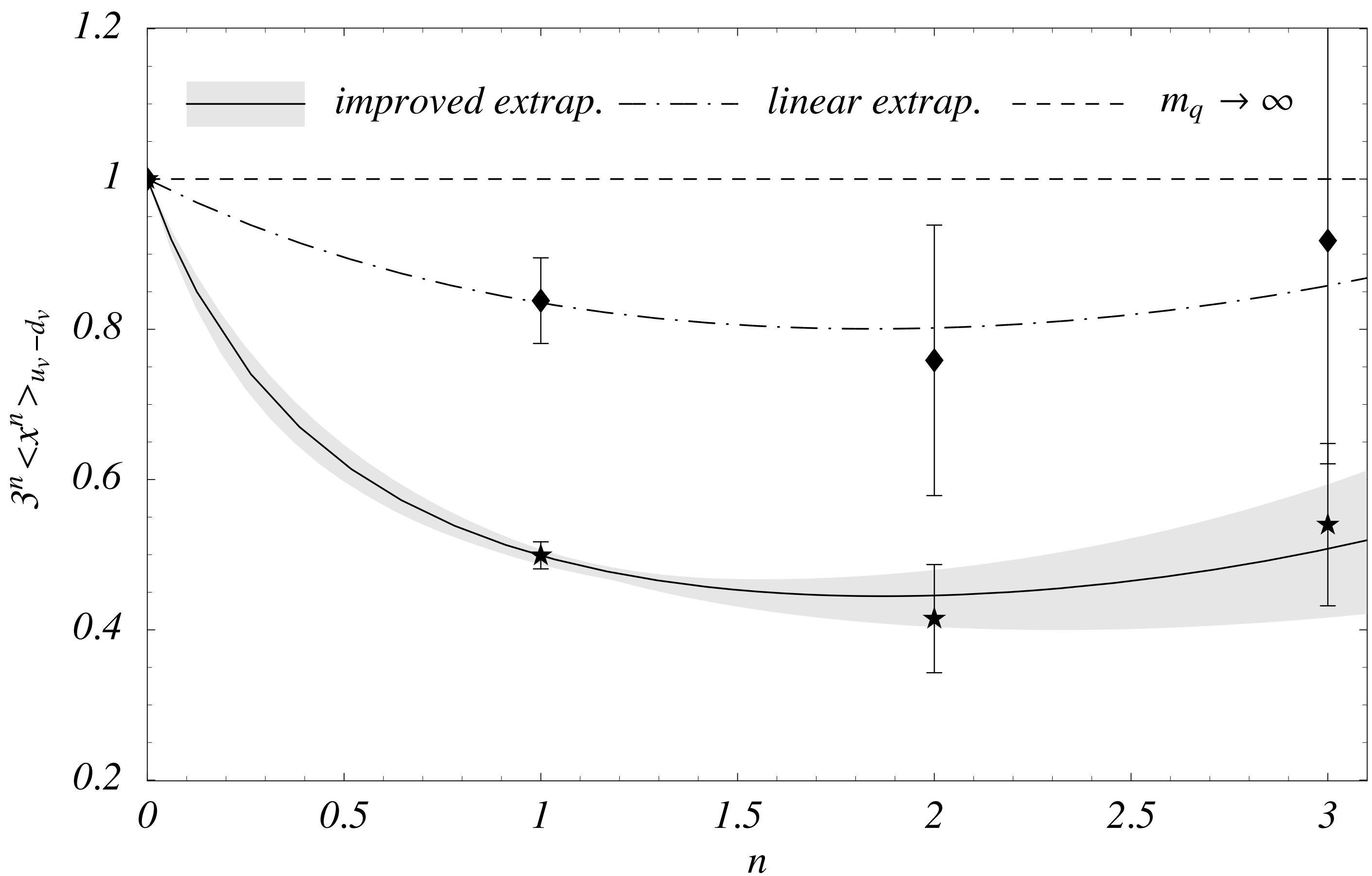
Chiral physics *vital* for understanding lattice data

Odds and evens

- For unpolarized parton distributions
  - n even $\Longrightarrow$ total $q + \bar{q}$
  - n odd $\Longrightarrow$ valence $q - \bar{q}$
- If have sufficient number of moments
  - fit odd and even moments separately to obtain both valence and total
  - subtract 2 x empirical sea from odd moments

$$q_v \equiv q - \bar{q} = q + \bar{q} - 2\bar{q}$$

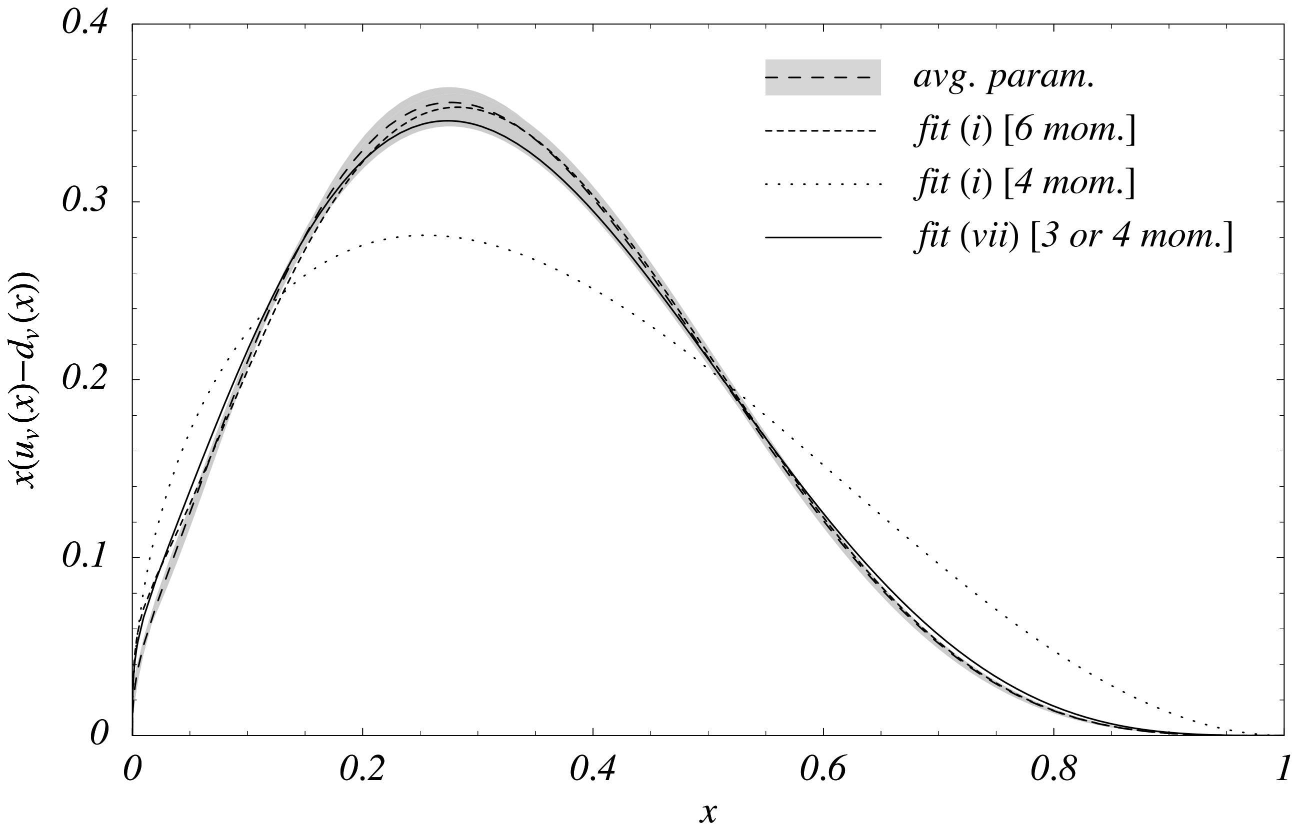
Chiral extrapolation of valence moments



Moments of $u_v - d_v$ (scaled by 3^n)

How well can one reconstruct PDFs from a few moments?

Test case:

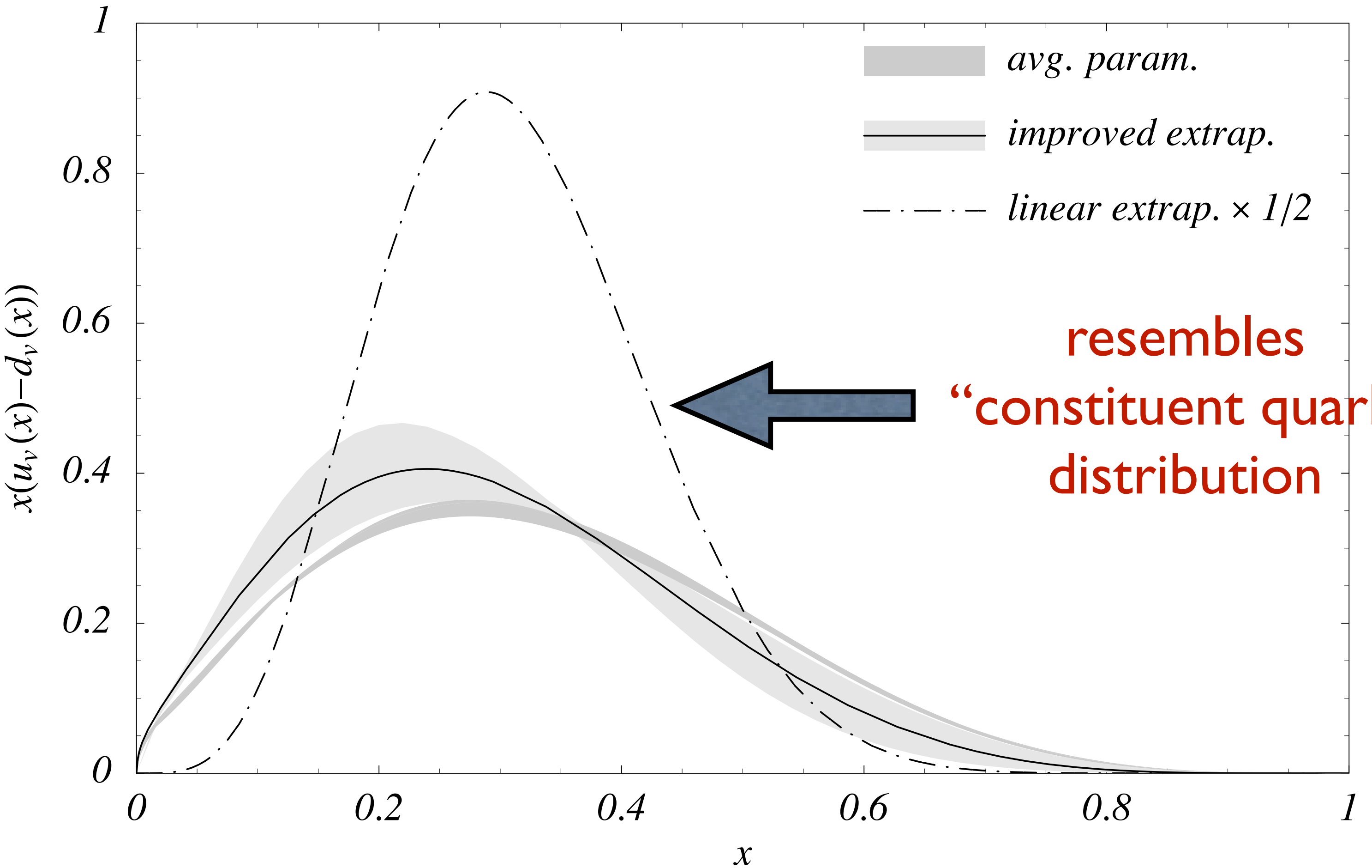


$$xq(x) = ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)$$

⇨ $fit(i)$: 4 unconstrained parameters (b, c, ϵ, γ)

⇨ $fit(vii)$: 2 unconstrained parameters (b, c)

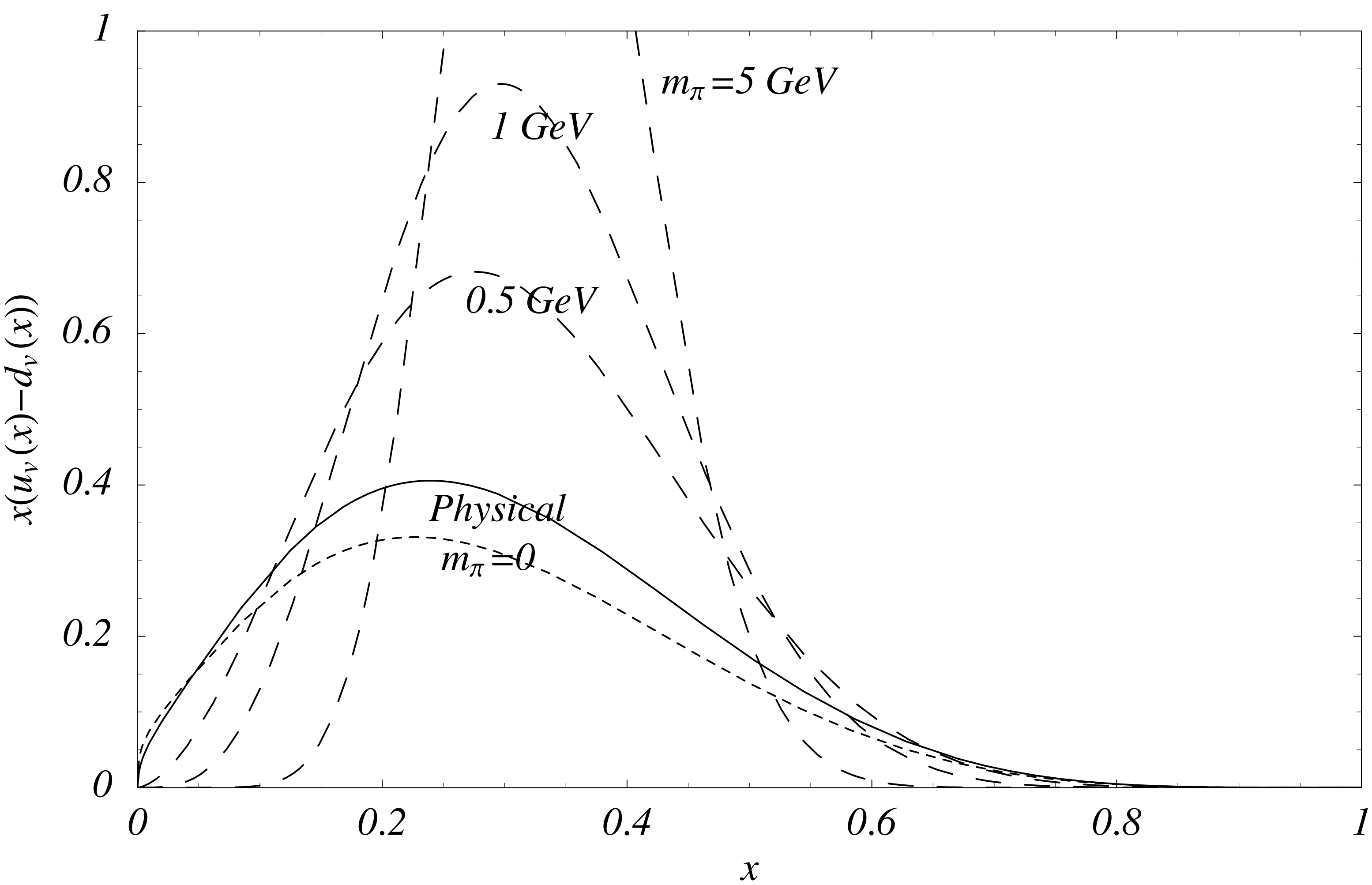
Reconstructed distribution



resembles
"constituent quark"
distribution

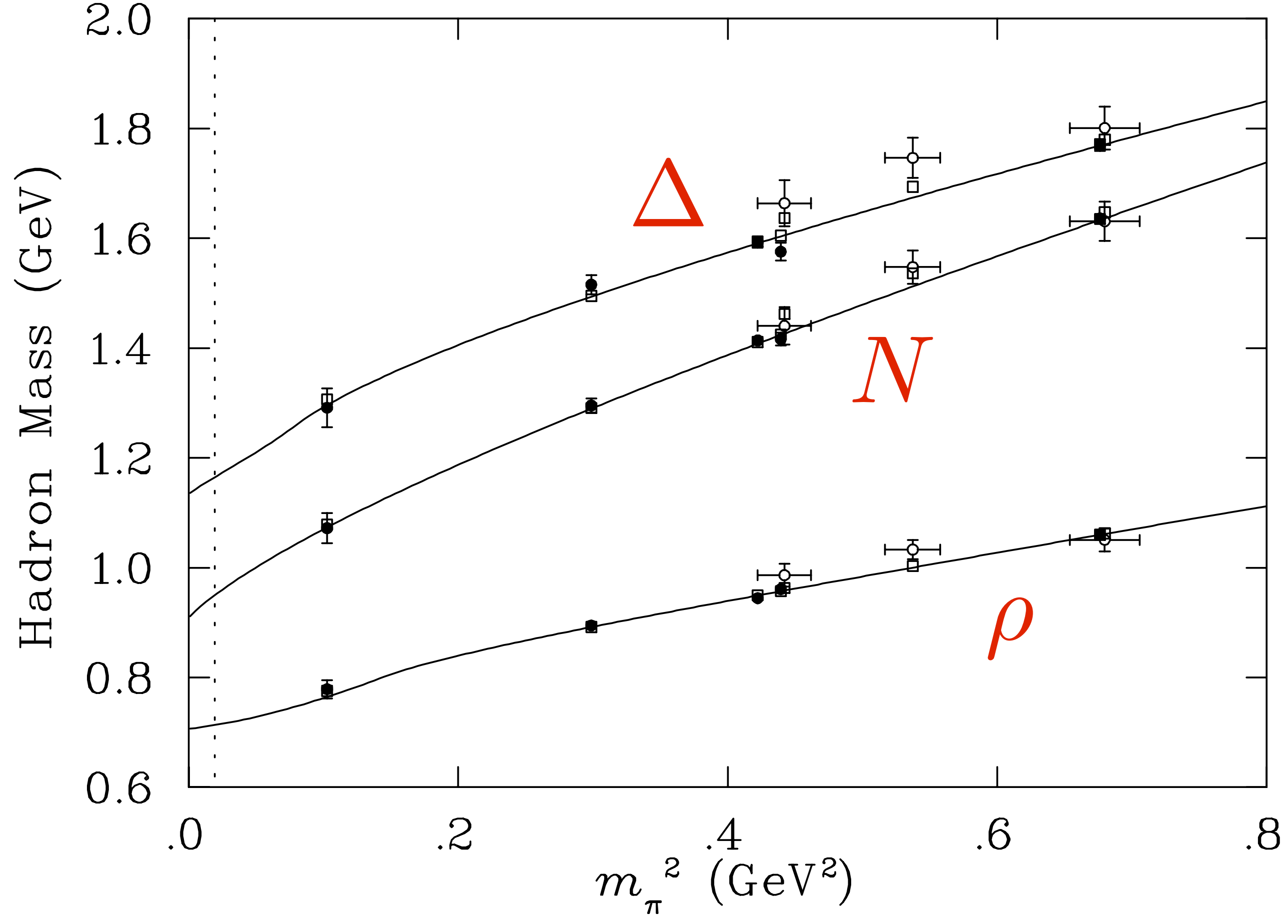
$$xq(x) = ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)$$

Quark mass dependence of PDFs



Looks like "constituent quark" distribution in heavy quark limit !

Wright, Leinweber, Thomas, Tsushima, hep-lat/0111053



slope in m_π^2 $(\sim m_q)$ for baryons
~ 3/2 x for mesons at large m_π^2

⟹ as in “constituent quark” picture

Connecting models with lattice QCD

At large quark masses, observables display "constituent quark" behavior

$$M_{\text{baryon}} \sim 3m_q$$
$$M_{\text{meson}} \sim 2m_q$$

Suggests new approach to modeling QCD

- construct "constituent quark" model at large quark masses
- extrapolate to physical quark mass using known chiral behavior

Cloet, Leinweber, Thomas, Phys. Rev. C65 (2002) 062201

Outlook

- Wealth of information contained in PDFs on nonperturbative structure of nucleon

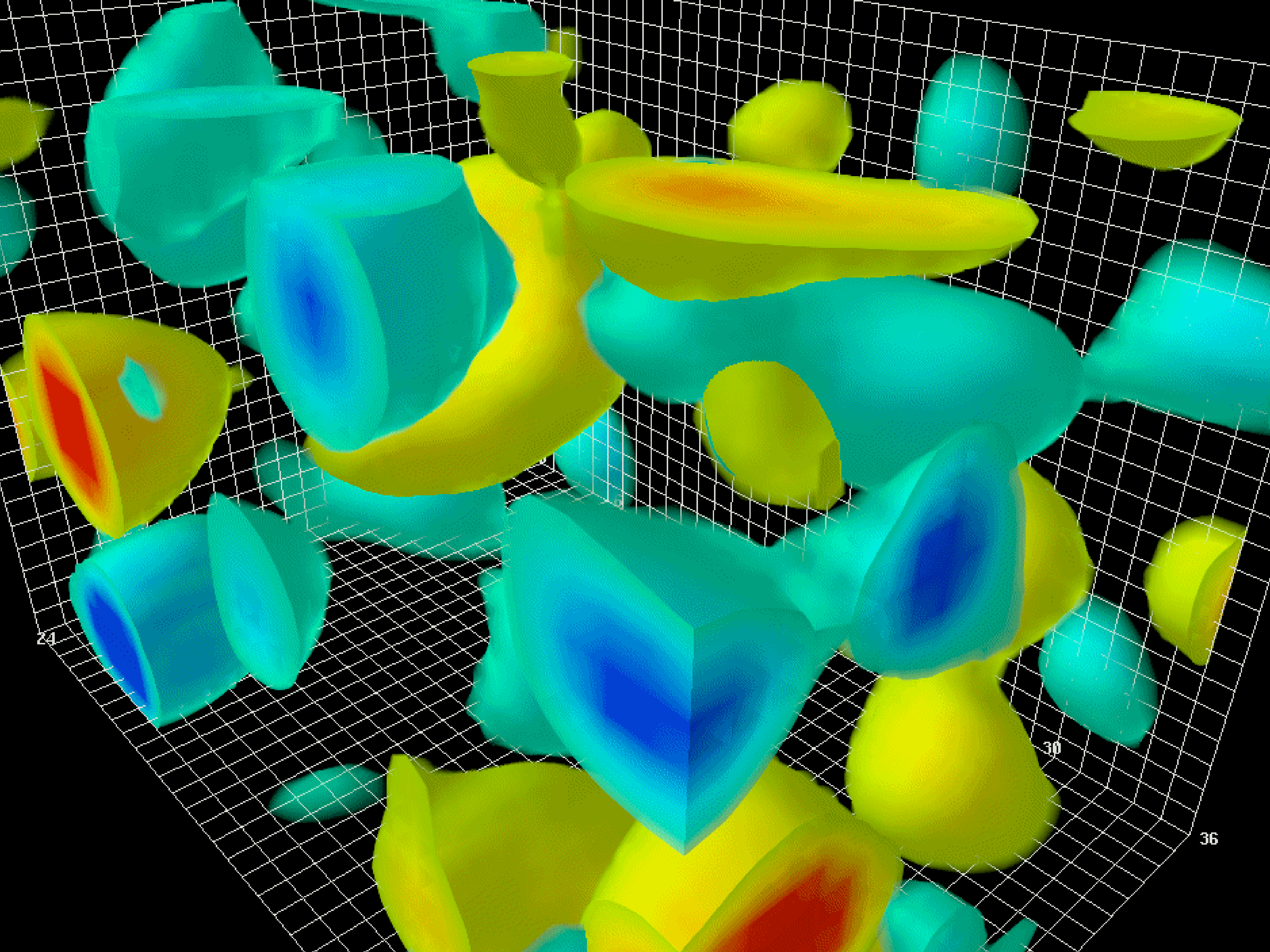
- Importance of high-energy/nuclear "frontier"

  - understanding nuclei is inevitable for complete reconstruction of PDFs

- Other issues:

  - higher twists (nonperturbative quark-gluon correlations in nucleon)

  - quark-hadron duality (interface between deep inelastic (scaling) and resonance regions)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