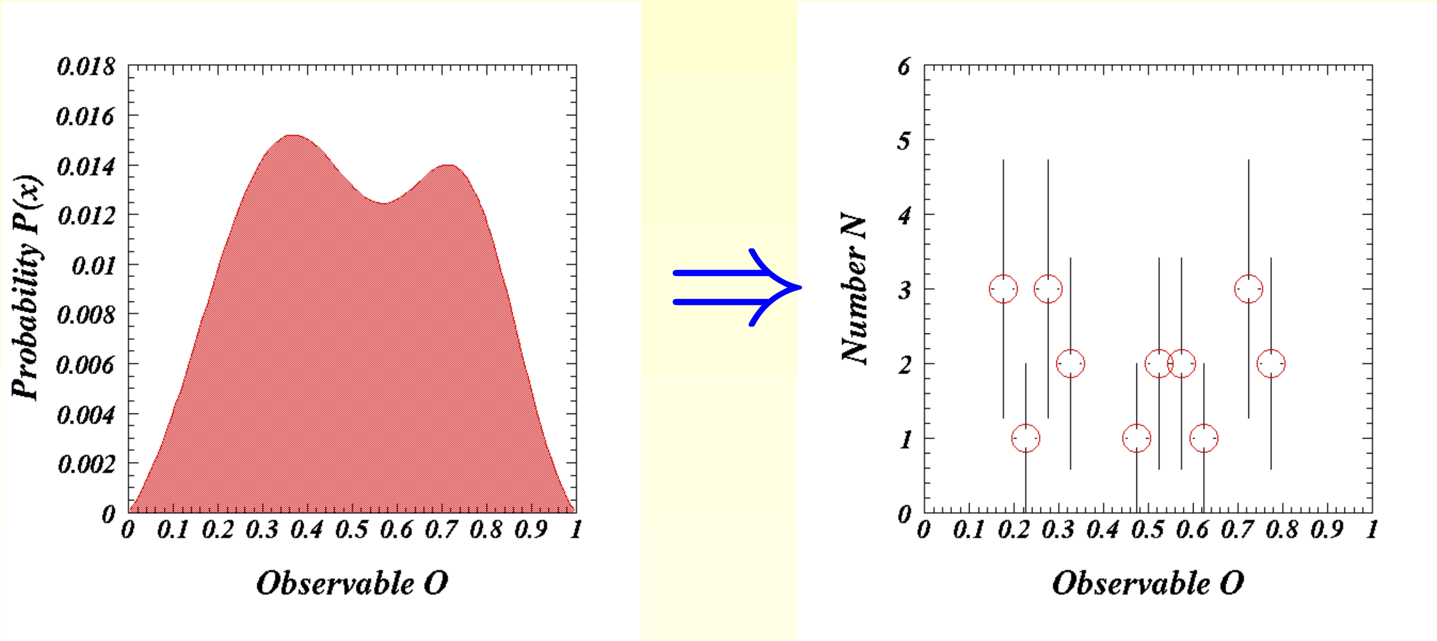
Event Generators CTEQSS04

Event generators predict multiparticle event configurations in HEP experiments



P(x) ⇒ N performed using Monte Carlo integration

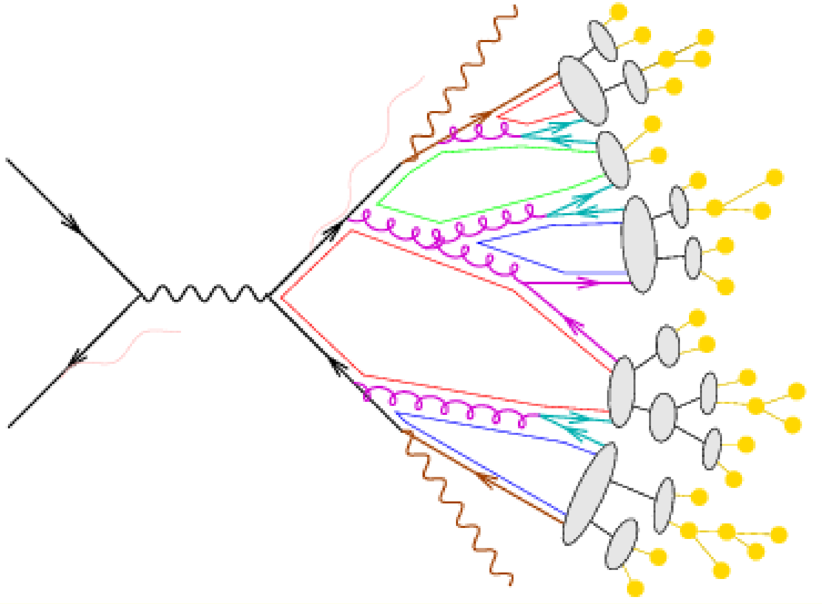
- Estimate the total cross section
- Generate events one at a time

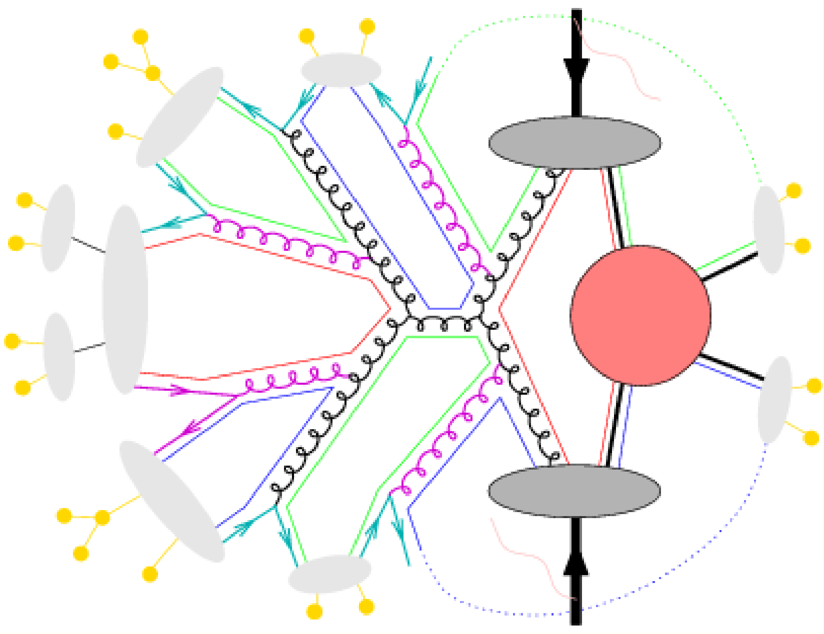
Relies on a computer's ability to generate (pseudo) random numbers

Lecture 1

- Defining Event Generators
  - Modularity of HEP Events
- Monte Carlo Techniques
  - Calculating Integrals
  - Sampling Distributions
- Matrix Element Calculations
  - Applications
  - Limitations
- Parton Shower
  - Sudakov Form Factor
  - Coherence
  - Dipoles
- Summary

Phases of High Energy Collisions





- **hard scattering**
- **initial/final state radiation**
- **partonic decays,** $t \rightarrow bW$
- **parton shower evolution**
- **nonperturbative phase**
- **colorless clusters**
- **cluster → hadrons**
- **hadronic decays**
- **backward parton evolution**
- **underlying event**

Monte Carlo Basics

$$
\begin{aligned}
I &= \int_{x_1}^{x_2} dx f(x) = (x_2 - x_1)\langle f(x)\rangle \qquad \left\{\sigma = \int dx \frac{d\sigma}{dx}\right\} \\
&\simeq I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x_i) \\
&\simeq I_N \pm (x_2 - x_1)\sqrt{\frac{(\langle f^2\rangle - \langle f\rangle^2)}{N}}
\end{aligned}
$$

Non-uniform sampling can be more efficient:

$$
\int_{x_1}^{x_2} dx p(x) = 1 \Rightarrow I = \int_{x_1}^{x_2} dx p(x) \frac{f(x)}{p(x)}
$$

$$
I = \left\langle \frac{f(x)}{p(x)} \right\rangle \pm \frac{1}{\sqrt{N}} \sqrt{\left(\left\langle \frac{f(x)^2}{p(x)^2} \right\rangle - \left\langle \frac{f(x)}{p(x)} \right\rangle^2\right)}
$$

Make f/p as flat as possible (reduce variance)

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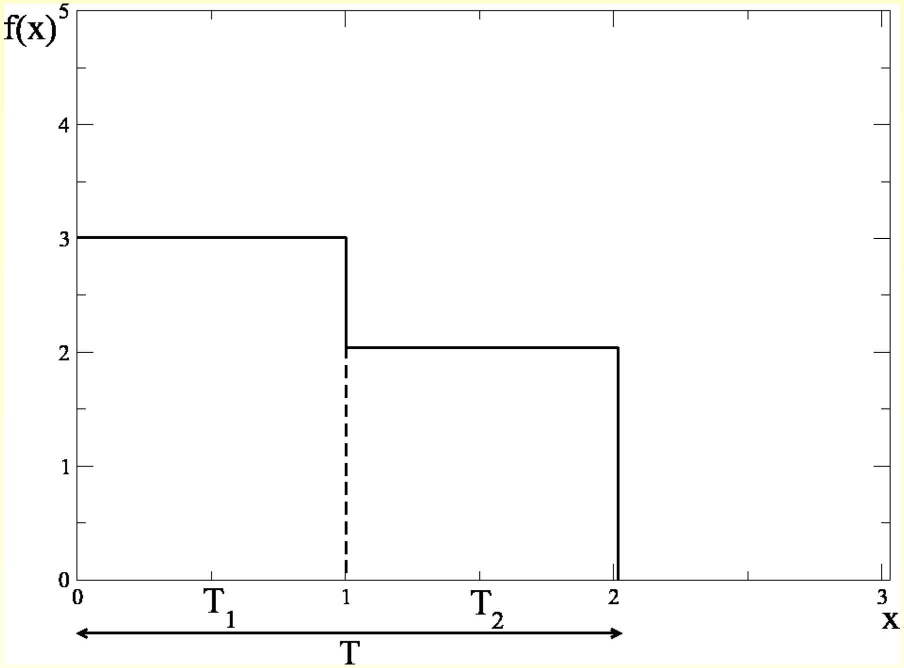
Monte Carlo (cont)

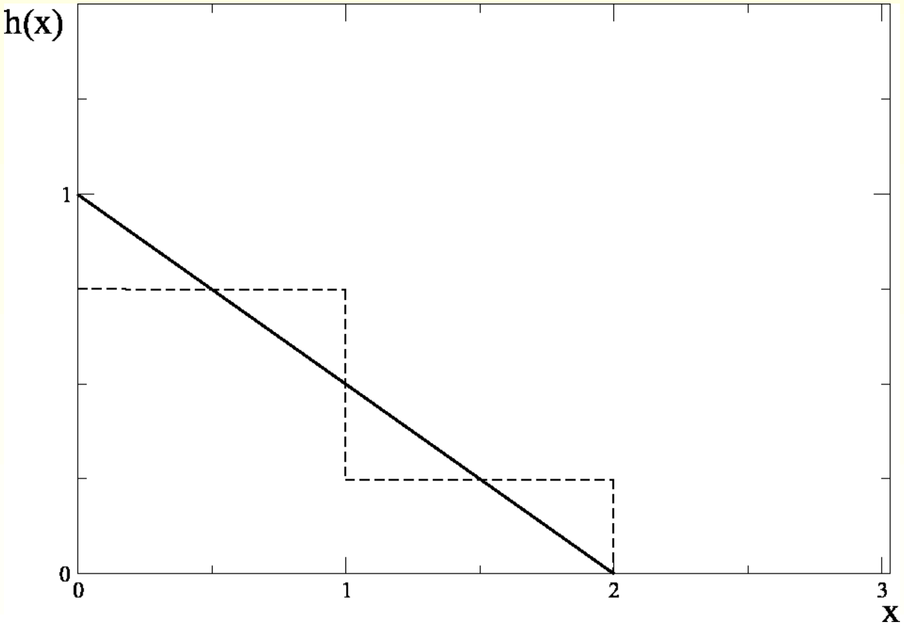
Stratified sampling: divide integration region into sub-volumes and sample according to variance

e.g., $\delta f(t) = 0$ if sampling on intervals T_1 and T_2

Importance sampling: choose x_N based on I_{N-1}

`VEGAS` is an adaptive integrator that adjusts step functions to parallel integrand





Monte Carlo (cont)

Up to here, only considered MC as a numerical integration method

If function being integrated is a probability density (positive definite), can convert it to a simulation of physical process = an event generator

Simple example: $\sigma = \int_0^1 dx \frac{d\sigma}{dx}$

Naive approach:

- pick events x with weights $\frac{d\sigma}{dx}$
- generate unweighted events by keeping them with probability $\frac{1}{\sigma}\frac{d\sigma}{dx}$ and giving them all weight 1
- Events selected with same frequency as in nature

Often, more sophisticated sampling methods are employed

Sampling Distributions: Method 1

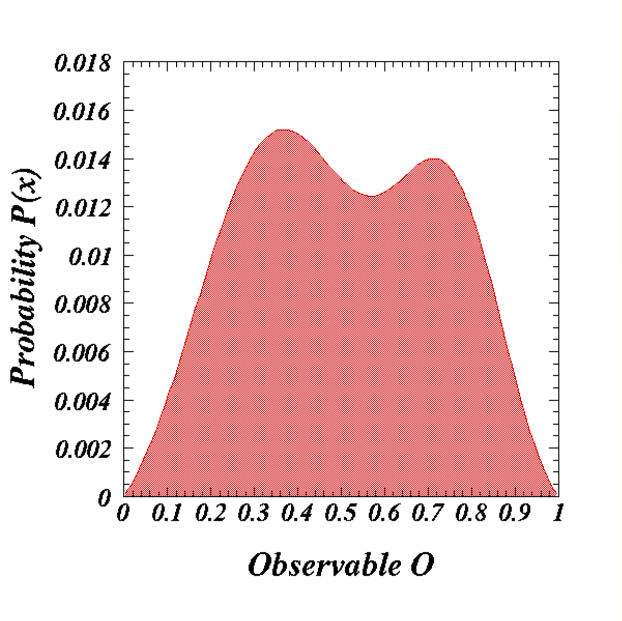
Given $f(x) > 0$ over $x_{\min} \le x \le x_{\max}$

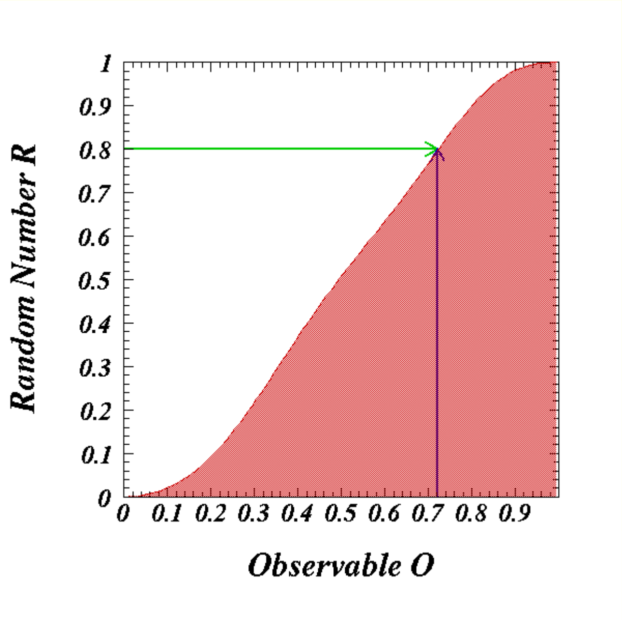
Prob in $(x + dx, x)$ is $f(x)dx$

$$\int_{x_{\min}}^{x} f(x)\,\mathrm{d}x = R \int_{x_{\min}}^{x_{\max}} f(x)\,\mathrm{d}x$$

$$x = F^{-1}(F(x_{\min}) + R(F(x_{\max}) - F(x_{\min})))$$

- assumes $F(x)$, $F^{-1}(x)$ are known
- fraction R of area under $f(x)$ should be to the left of x





Realistic $f(x)$ are rarely this nice

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Sampling Distributions: Method 2

If $\max[f(x)]$ is known, use **hit-or-miss**

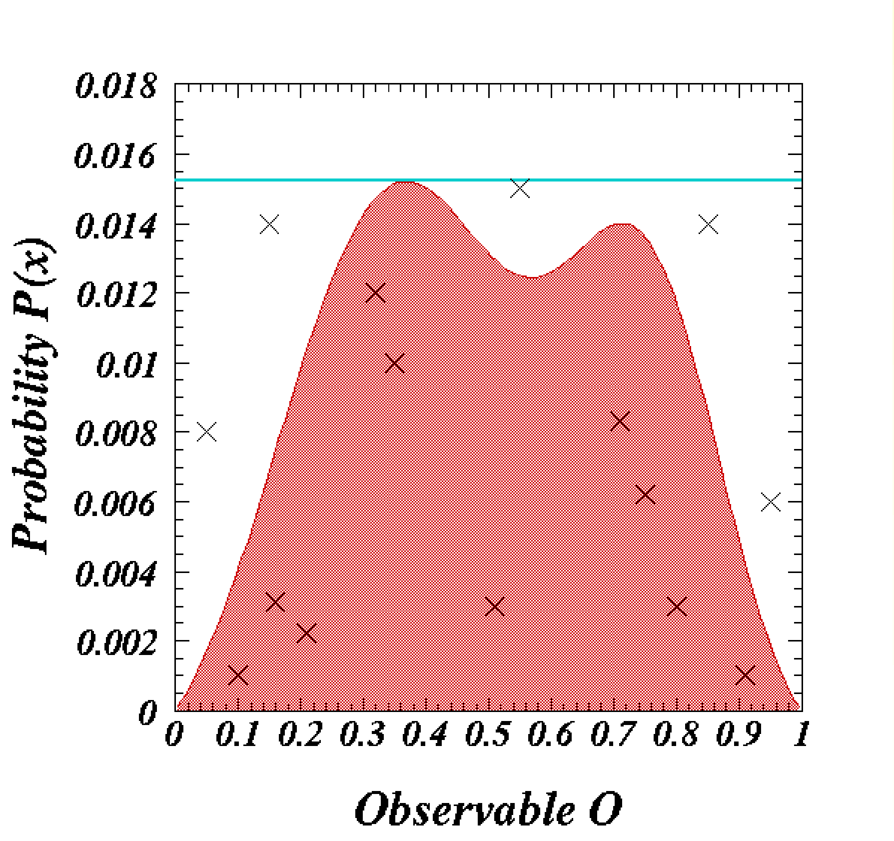
1. select $x = x_{\min} + R(x_{\max} - x_{\min})$
2. if $f(x)/f_{\max} \leq$ (new) R, reject x and $\Rightarrow 1$.
3. otherwise, keep x

Works because probability

$$f(x)/f_{\max} > R \propto f(x)$$

Acceptable method if $f(x)$ does not fluctuate too wildly

Usually guess at $\max[f(x)]$ and update if a “better” estimate is found in a run



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Sampling Distributions: Method 3

Find $g(x)$, with $f(x) \leq g(x)$ over x range

- $G(x)$ and its inverse $G^{-1}(x)$ known
- e.g., $\int_\epsilon^z dx \frac{1+x^2}{1-x} < \int_\epsilon^z dx \frac{2}{1-x} = 2\ln\left[\frac{1-\epsilon}{1-z}\right]$

1. select an x according to $g(x)$, using Method 1
2. if $f(x)/g(x) \leq$ (new) R, reject x and $\Rightarrow 1.$
3. otherwise, keep x

first step selects x with a probability $g(x)$

second step retains this choice with probability $f(x)/g(x)$

total probability to pick a value x is then just the product of the two, i.e. $f(x)\,\mathrm{d}x$

Radioactive Decay Problem

Know probability $f(t)$ that ‘something will happen’ (a nucleus decay, a parton branch) at time t

something happens at t **only** *if it did not happen at $t' < t$*

Equation for nothing $\mathcal{N}(t)$ to happen *up to time t* is ($\mathcal{N}(0) = 1$):

$$-\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}t} = f(t)\,\mathcal{N}(t) = \mathcal{P}(t)$$

$$\mathcal{N}(t) = \exp\left\{-\int_0^t f(t')\,\mathrm{d}t'\right\}$$

$$\mathcal{P}(t) = f(t)\exp\left\{-\int_0^t f(t')\,\mathrm{d}t'\right\}$$

- Naive answer modified by exponential suppression
- In the parton-shower language, this corresponds to the Sudakov form factor

Veto Algorithm

If $F(t)$ and $F^{-1}(t)$ exist:

$$\int_0^t \mathcal{P}(t')\,\mathrm{d}t' = \mathcal{N}(0) - \mathcal{N}(t) = 1 - \exp\left\{-\int_0^t f(t')\,\mathrm{d}t'\right\} = 1 - R$$

$$F(0) - F(t) = \ln R \quad \Longrightarrow \quad t = F^{-1}(F(0) - \ln R)$$

If not, use **veto algorithm**

1. start with `i = 0` and $t_0 = 0$
2. `++i` and select $t_i = G^{-1}(G(t_{i-1}) - \ln R)$
3. if $f(t_i)/g(t_i) \leq$ (new) R, $\Rightarrow 2.$
4. otherwise, keep t_i

- N vetos equivalent to probability of accepting first try times a partial sum of an exponential series in $f - g$

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MC Overview

- Use MC to perform integrals and sample distributions
- Technique generalizes to many dimensions

  Typical phase space $\sim d^3\vec{p} \times$ 100's particles
- Suitable for complicated integration regions

  Kinematic cuts or detector cracks
- Error scales as $1/\sqrt{N}$ vs $1/N^{2/d}, 1/N^{4/d}$ (trap,Simp)
- Only need a few points to estimate f
- Each additional point increases accuracy
- easy (non-rigorous) error estimate
- Can sample distributions where exact solutions cannot be found
- Veto algorithm applied to parton shower

Phase 1: Hard Scattering

Characterizes the rest of the event

Sets a high energy scale Q

Fixes a short time scale where partons are free objects

Allows use of perturbation theory

External partons can be treated as on the mass-shell

- Valid to $\max[\Lambda, m]/Q$
- Physics at scales below Q absorbed into parton distribution and fragmentation functions (Factorization Theorem)

Sets flow of Quantum numbers (Charge, Color)

- Note: Parton shower and hadronization models valid to $1/N_C$
- Gluon replaced by color-anticolor lines
- All color flows can be drawn on a piece of paper

Cross Sections and Decay Widths

Physics Quantities to calculate using Monte Carlo

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi_n(\sqrt{s}) \quad \Gamma = \frac{1}{2M}\int |\mathcal{M}|^2 d\Phi_n(M)$$

Phase Space:

$$d\Phi_n(M) = \prod_{i=1}^{n} \frac{d^3\vec{p}_i}{(2\pi)^3(2E_i)}(2\pi)^2\delta^{(4)}\left(p_0 - \sum_{i=1}^{n} p_i\right)$$

2-body: $2 \times 3 - 4$ integration variable ⇒ 2 angles $d\Omega$

N-body by Recursion:

$$d\Phi_n(M) = \frac{1}{2\pi}\int_0^{(M-m)^2} d\mu^2 d\Phi_2(M) d\Phi_{n-1}(\mu)$$

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Cross Sections

$$\begin{aligned}\sigma(s) &= \int_0^1 dx_1 f_1(x_1) \int_0^1 dx_2 f_2(x_2) \hat{\sigma}(x_1 x_2 s) \\ &= \int_0^1 \frac{d\tau}{\tau} \hat{\sigma}(\tau s) \int_\tau^1 \frac{dx}{x} x f_1(x) \frac{\tau}{x} f_2\left(\frac{\tau}{x}\right)\end{aligned}$$

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{1}{64\hat{s}} \frac{1}{|p_{1cm}^2|} |\mathcal{M}|^2, \hat{t} = (p_{\rm in} - p_{\rm out})^2$$

Considerations:

- $\hat{\sigma}$ may have resonant peaks (W or Z production)
- $f(x)$ may be steeply falling

Side-effect of Monte Carlo-ing:

- Can histogram distributions of produced particles/decay products
- Apply cuts

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Tree Level Calculation of Hard Scatter

- Read Feynman rules from $i\mathcal{L}_{int}$
- Use Wave Functions from Relativistic QM

  Propagators (Green functions) for internal lines
- Specify initial and final states

  Track spins/colors/etc. if desired
- Draw all valid graphs connecting them

  Tedious, but straight-forward
- Calculate (Matrix Element)^2

  Evaluate Amplitudes, Add and Square

  Symbolically Square, Evaluate

  Do something tricky
- Integrate over Phase Space

Learn by hand, then automate

Complications:

$|\mathcal{M}|^2$: Number of graphs grows quickly with number of external partons

$d\Phi_n$: Efficiency decreases with number of internal lines

Programs:

- `MadEvent`, `CompHep`, `Alpgen`, `Amegic++`
- Differ in methods of attack
- Most rely on `VEGAS` for MC integration

Limitations:

- Fixed number of partons
- No control of large logarithms as $E_g, \theta_{qg}, \theta_{gg} \to 0$

NOT event generators

- partonic jets: no substructure
- hard, wide-angle emissions only
- colored/fractionally charged states not suitable for detector simulation
- can guide physics analyses by revealing gross kinematic features

  Jacobian peak

Towards an Event Generator

HEP Events are modular:

- Events are transformations from $t = -\infty \to t = +\infty$
- Hard Interaction occurs over a short time scale $\Delta t \sim 10^{-2}\mathrm{GeV}^{-1}$
- Perturbation theory ($\alpha_s < \pi$) should work down to time $t = .1 - 1\mathrm{GeV}^{-1}$
- Hadronization on longer time scales
- Particle decays typically on longest time scales

Separation of time scales reduces the complex problem to manageable pieces (modules) which can be treated in series

- Previous step sets initial conditions for next one

Next piece is the parton shower

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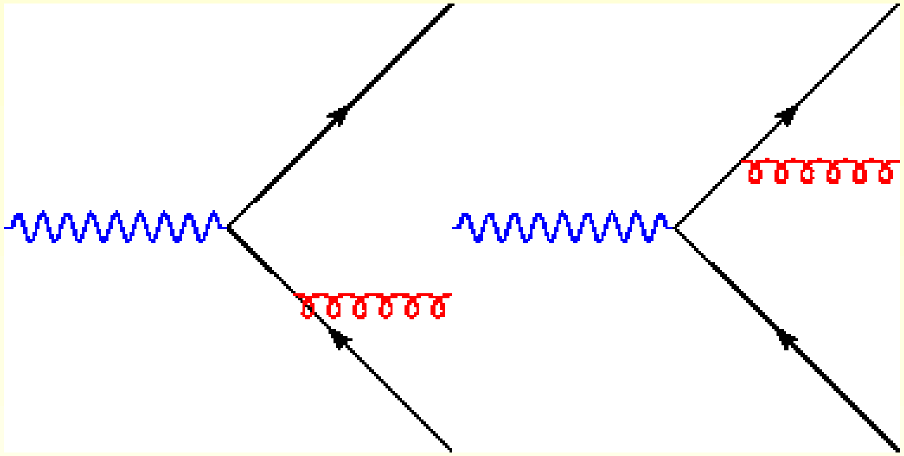
Matrix Element to Parton Shower: $\gamma^* \longrightarrow q\bar{q}g$

$$d\sigma(q\bar{q}g) = \sigma_0 \frac{\alpha_s}{2\pi} dz \left\{ \frac{ds_{qg}}{s_{qg}} \left[P_{q\to q}(z) - \frac{s_{qg}}{Q^2} \right] + \frac{ds_{\bar{q}g}}{s_{\bar{q}g}} \left[P_{q\to q}(z) - \frac{s_{\bar{q}g}}{Q^2} \right] \right\}$$



$$\sigma_0 = \sigma(\gamma^* \longrightarrow q\bar{q})$$

$$z = \frac{s_{q\bar{q}}}{Q^2}, P_{q\to q}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$s_{qg} = 2E_q E_g (1 - \cos\theta_{qg})$$

$s_{qg}, s_{\bar{q}g} \longrightarrow 0$ when gluon is soft/collinear

$z \longrightarrow 1$ when gluon is collinear

In soft/collinear limit, independent radiation from q and $\bar{q}$

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General Result

$|\mathcal{M}|^2$ involving $q \to qg$ (or $g \to gg$) strongly enhanced whenever emitted gluon is almost collinear

Propagator factors (internal lines)

$$\frac{1}{(p_q + p_g)^2} \approx \frac{1}{2E_qE_g(1-\cos\theta_{qg})} \to \frac{1}{E_qE_g\theta_{qg}^2}$$

- soft+collinear divergences
- dominant contribution to the ME

Collinear factorization

$$|\mathcal{M}_{p+1}|^2 d\Phi_{p+1} \approx |\mathcal{M}_p|^2 d\Phi_p \frac{dQ^2}{Q^2}\frac{\alpha_s}{2\pi}P(z)dzd\phi$$

DGLAP kernels:

$$P_{q\to q}(z) = C_F\frac{1+z^2}{1-z}, P_{g\to g}(z) = N_C\frac{(1-z(1-z))^2}{z(1-z)}$$

$\Rightarrow$ Parton shower MC

Sudakov Form Factor

Variable $t = \ln(Q^2/\Lambda^2)$, $Q^2 \sim E_q E_g/\theta_{qg}^2$ is like a time-ordering

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}t\,\mathrm{d}z$$

$$\mathcal{I}_{a\to bc}(t) = \int_{z_-(t)}^{z_+(t)} \mathrm{d}z\, \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)$$

Probability for no emission in $(t, t+\delta t)$: $1 - \sum_{b,c} \mathcal{I}_{a\to bc}(t)\,\delta t$

Over a longer time period, product of no-emission prob's exponentiates:

$$\mathcal{P}_{\mathrm{no}}(t_0, t) = \exp\left\{ -\int_{t_0}^{t} \mathrm{d}t' \sum_{b,c} \mathcal{I}_{a\to bc}(t') \right\} = S_a(t) = \frac{\Delta(t, t_c)}{\Delta(t_0, t_c)}$$

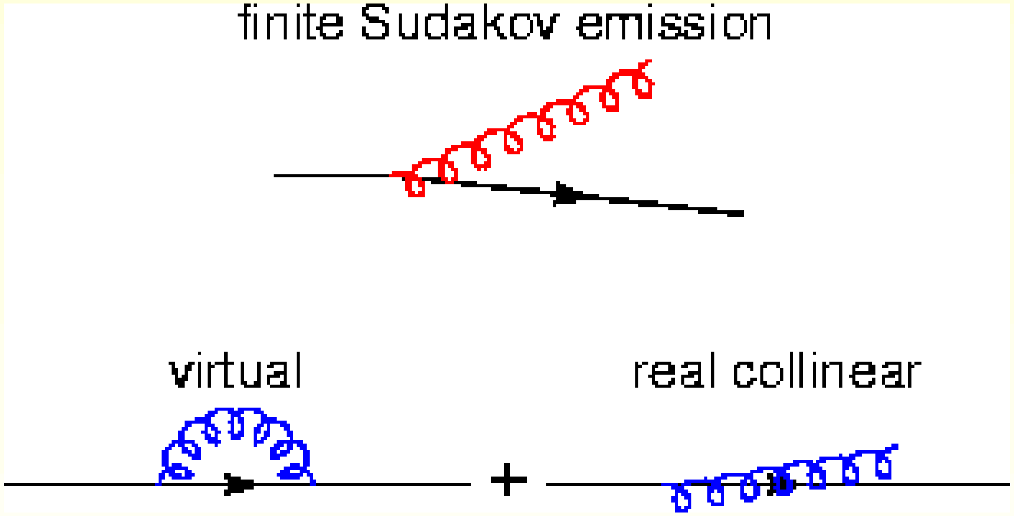
Notation: $S_a(t)$ for **`Pythia`**, $\Delta(t, t_c)$ for **`Herwig`**

Sudakov Form Factor

Actual probability that a branching of a occurs at t is:

$$\frac{\mathrm{d}\mathcal{P}_a}{\mathrm{d}t} = -\frac{\mathrm{d}\mathcal{P}_{\rm no}(t_0,t)}{\mathrm{d}t} = \left(\sum_{b,c}\mathcal{I}_{a\to bc}(t)\right)\exp\left\{-\int_{t_0}^{t}\mathrm{d}t'\sum_{b,c}\mathcal{I}_{a\to bc}(t')\right\}$$

Like Radioactive Decay! $S_a(t) = \mathcal{P}_{\rm no}(t_0,t)$ is referred to as the Sudakov form factor



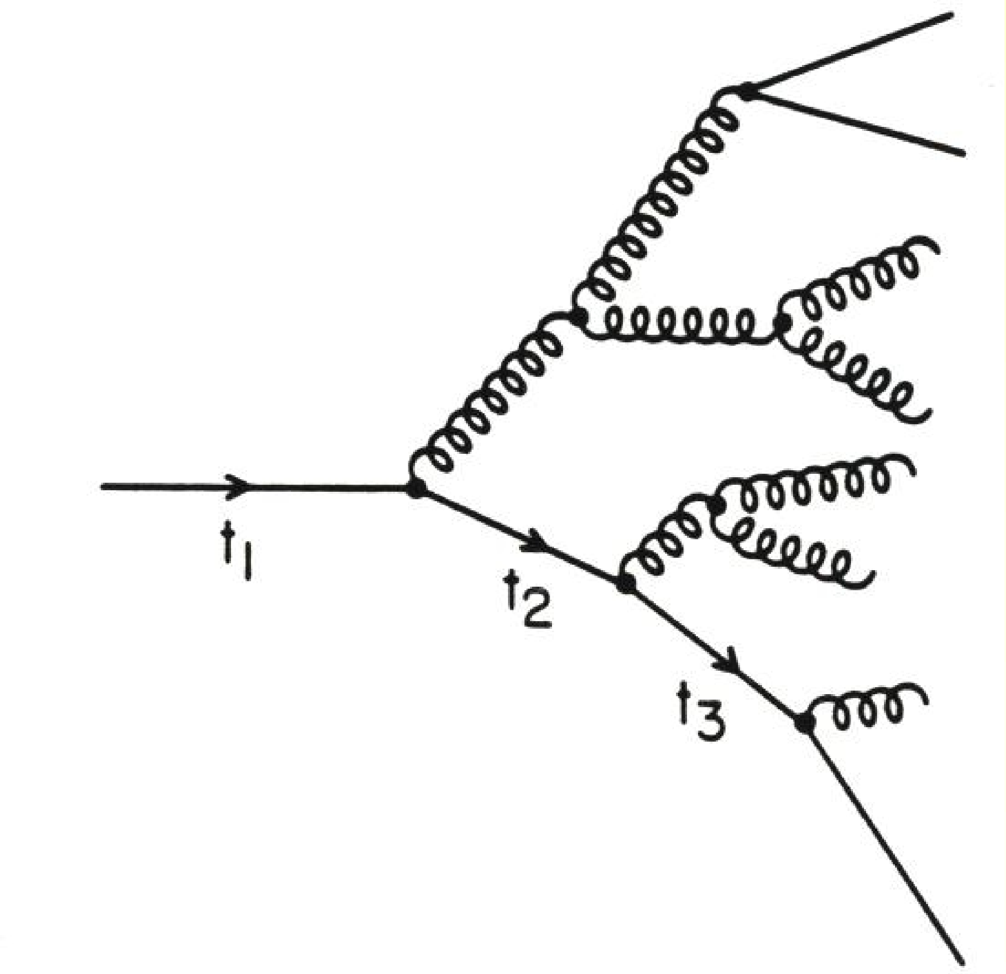
- Leading Logarithms
- Collinear splitting prob.
- Exact E-p

Interplay between real and virtual emissions

Below resolution scale, cancellation of singularities, leaving a finite remnant

Evolution of the parton shower

Continue emissions with decreasing t down to the cutoff scale $\sim \Lambda_{\rm QCD}$



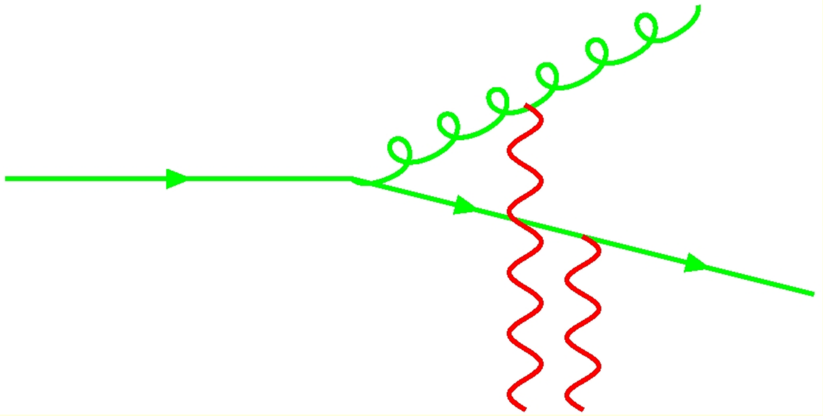
$t_1 > t_2 > t_3 > t_c$

$t_c \rightarrow \Lambda_{\rm QCD}$

Make transition to a model of hadronization at $\Lambda_{\rm QCD}$

Color Coherence

In previous discussion of PS, interference effects were ignored, but they can be relevant



Add a soft gluon to a shower of N almost collinear gluons

- incoherent emission: couple to all gluons

$$|\mathcal{M}_{N+1}|^2 \sim N \times \alpha_s \times N_C$$

- coherent emission: soft (=long wavelength) resolves only overall color charge (that of initial gluon)

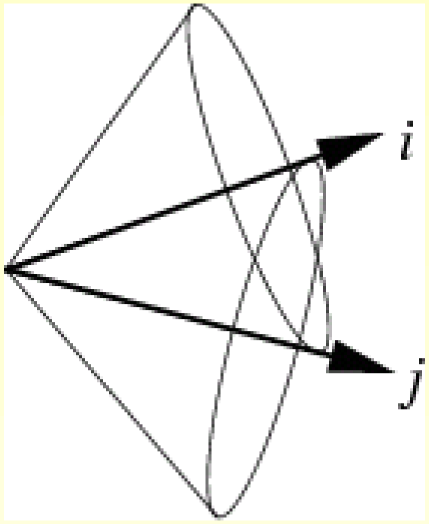
$$|\mathcal{M}_{N+1}|^2 \sim 1 \times \alpha_s \times N_C$$

Color Coherence Realized

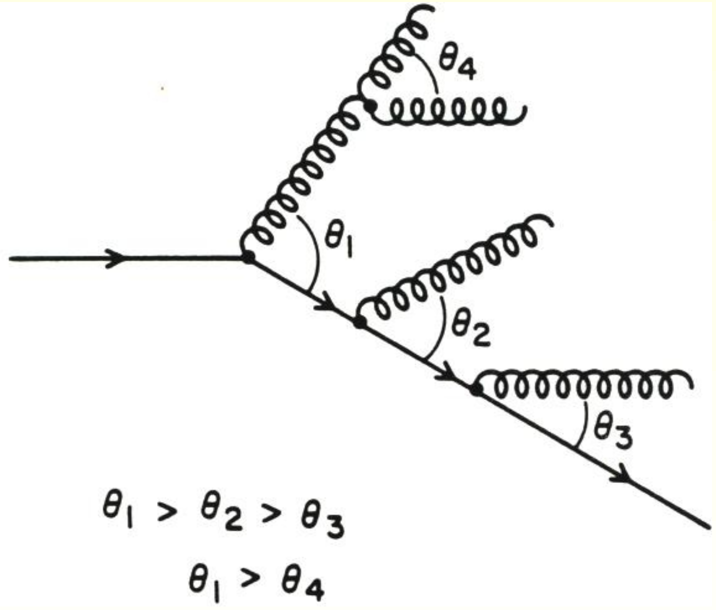
Showers should be Angular-Ordered

Keep same picture but change $Q^2 \to E^2\zeta$

$$\zeta = \frac{p_i \cdot p_j}{E_i E_j} = (1 - \cos\theta_{ij}) \sim \theta_{ij}^2/2$$



Soft gluon radiation off color lines $i,\ j$



$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega}\frac{d\Omega}{2\pi}\frac{\alpha_s}{2\pi}C_{ij}W_{ij}$$

$$W_{ij} = \frac{1 - \cos\theta_{ij}}{(1 - \cos\theta_{iq})(1 - \cos\theta_{jq})}$$

$$W_{ij} = W^{[i]} + W^{[j]}$$

$$\int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} W^{[i]} = \frac{1}{1 - \cos\theta_{iq}}\theta[\theta_{ij} - \theta_{iq}]$$

Running coupling should depend on $k_T^2 \sim z(1-z)Q^2$

Generalised Dipoles

Alternative picture: dipole radiation from color charges

$$dn = \alpha_{\rm eff} \cdot \frac{dk_\perp^2}{k_\perp^2} \cdot dy \cdot \text{ (Polarization Sum)}$$

Kinematic Constraints

$$k_\perp \cosh(y) \leq \frac{\sqrt{s}}{2} \quad (\sqrt{s} \text{ is dipole mass})$$

rapidity range $\Delta y \approx ln \left(\frac{s}{k_\perp^2} \right)$

- emission of a photon leaves the electromagnetic current unchanged except for small recoil effects
- emission of a gluon changes the current
- However,

$$dn(q, g_1, g_2, \bar{q}) = dn(q, g_1, \bar{q}) \left[dn(q, g_2, g_1) + dn(g_1, g_2, \bar{q}) - \epsilon \right]$$

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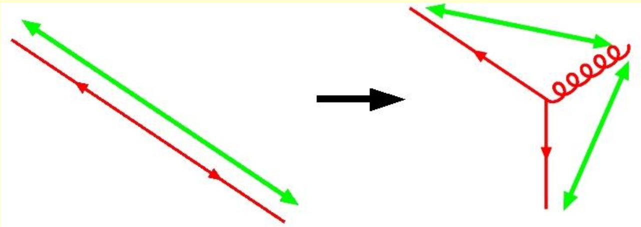
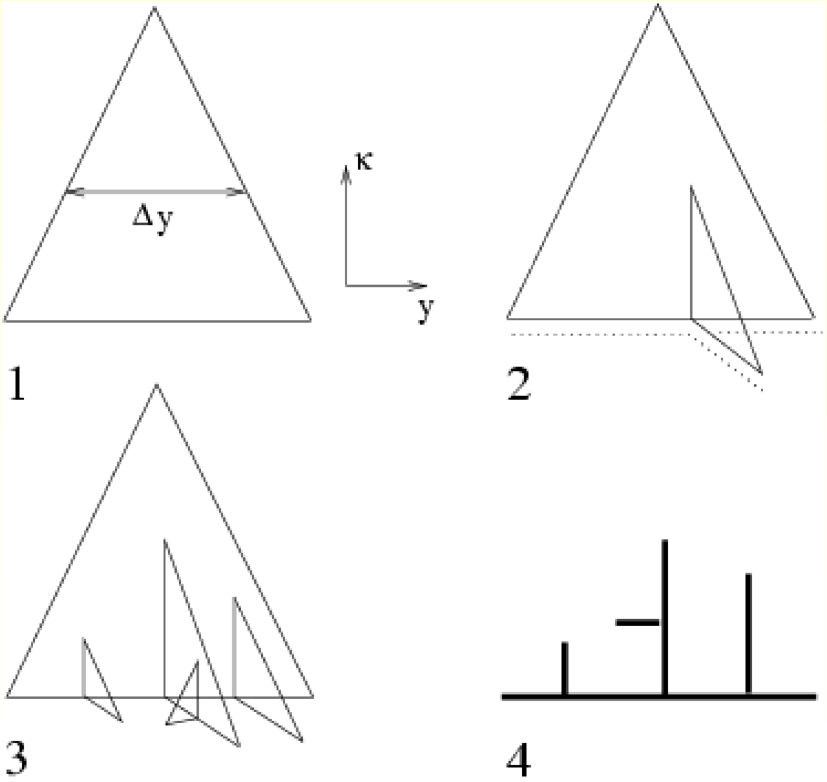
Generalised Dipoles

The emission of the first gluon splits the original color dipole into two dipoles which radiate independently



Shower can be traced in origami diagram:



$\kappa = \ln(k_T^2)$

1. Before emission
2. 1st emission at κ_1
3. After several emissions
4. Bottom view

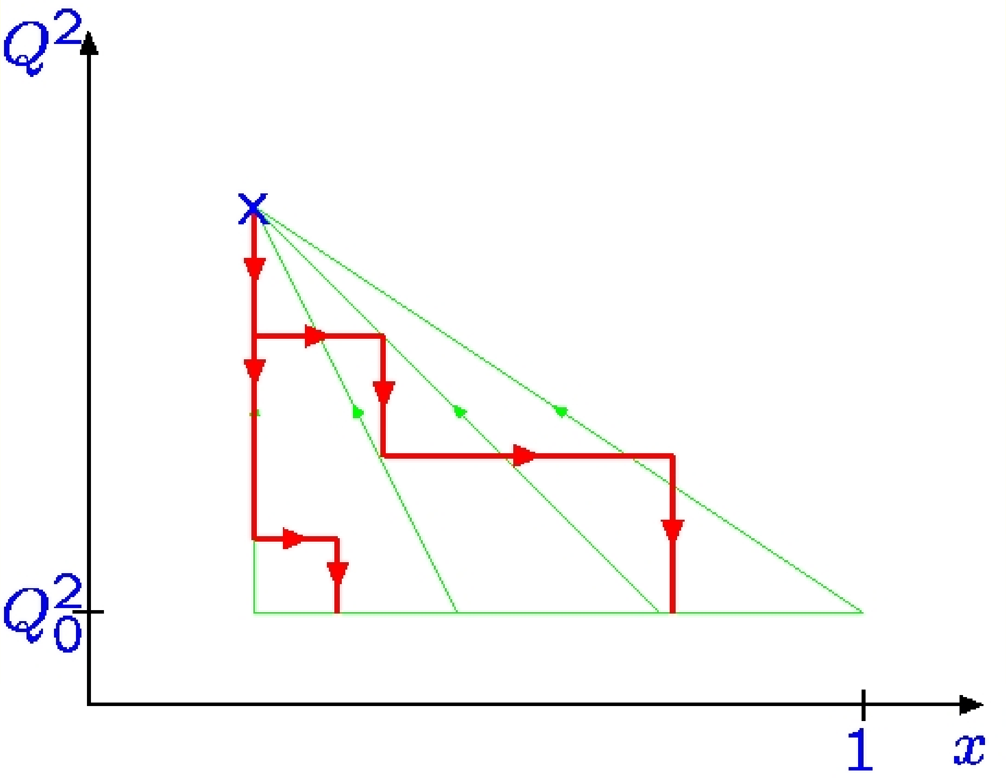
Initial State Radiation

In hadronic collisions, incoming partons can also radiate

$p_1 \rightarrow p_2 + k, p_1^2 = p_2^2 = 0 \Rightarrow k^2 = (p_1 - p_2)^2 = -2p_1 \cdot p_2 < 0$

Backwards (from hard scatter) evolution of partons with virtualities increasing $\rightarrow 0$

Since backwards, must normalize to the incoming flux of partons (PDF)



- Collinear parton shower obeys DGLAP evolution
- Weight Sudakov: $\dfrac{f_i(x, Q^2_{\mathrm{lo}})}{f_i(x, Q^2_{\mathrm{hi}})}$

NLL Jet Structure

Typically, parton shower has complicated constraints/integrands and are evaluated numerically

However, with some simplifications, can calculate NLL Sudakovs

Evolution in $k_{Tij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$

Analytic Sudakov form factors:

$$\begin{aligned}\Delta_q(Q_1, Q) &= \exp\left(-\int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\right)\\ \Delta_g(Q_1, Q) &= \exp\left(-\int_{Q_1}^{Q} dq\,\left[\Gamma_g(q,Q) + \Gamma_f(q)\right]\right)\end{aligned}$$

$$\begin{aligned}\Gamma_q(q,Q) &= \frac{2C_F}{\pi}\frac{\alpha_s(q)}{q}\left(\ln\frac{Q}{q} - \frac{3}{4}\right)\\ \Gamma_g(q,Q) &= \frac{2C_A}{\pi}\frac{\alpha_s(q)}{q}\left(\ln\frac{Q}{q} - \frac{11}{12}\right) \quad \Gamma_f(q) = \frac{N_f}{3\pi}\frac{\alpha_s(q)}{q}\end{aligned}$$

Jet Rates starting from $Z \to q\bar{q}$

2 jet rate: neither $q\bar{q}$ radiates

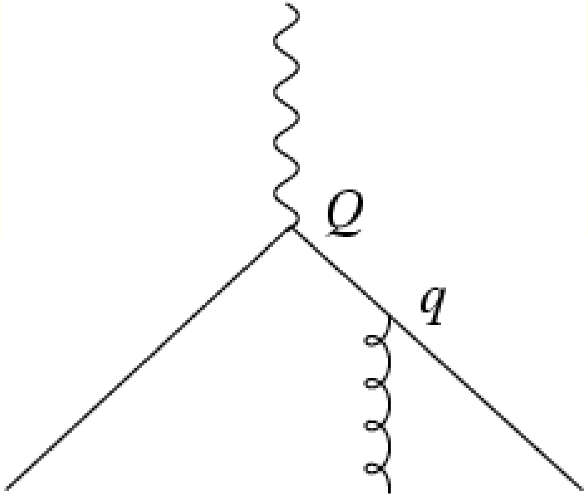
$$R_2(Q_1, Q) = [\Delta_q(Q_1, Q)]^2$$

3 jet rate: one intermediate state

$$R_3(Q_1, Q) = 2\,[\Delta_q(Q_1, Q)]^2 \int_{Q_1}^{Q} dq\,\Gamma_q(q, Q)\Delta_g(Q_1, q)$$

The overall NLL probability is

$$\begin{aligned} \Delta_q(Q_1, Q)\frac{\Delta_q(Q_1, Q)}{\Delta_q(Q_1, \tilde{q})}\Gamma_q(q, Q)\Delta_q(Q_1, \tilde{q})\Delta_g(Q_1, q) \\ = \Gamma_q(q, Q)\,[\Delta_q(Q_1, Q)]^2\,\Delta_g(Q_1, q) \end{aligned}$$



Add $\bar{q}$ contribution and integrate over $Q_1 < q < Q$ to get R_3

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Jet Rates starting from $Z \to q\bar{q}$

- N limited only by phase space
- All topologies generated in Parton Shower
- Rates given by dominant soft/collinear regions
- Contrast with ME

  One topology must be specified

  No soft/collinear approximations, valid for large E, θ

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NOT an Event Generator

By the end of the parton shower, we have nearly exhausted our ability to apply perturbation theory

This is still not enough

- \+ Have a description of jet structure
- \+ Can ask questions about energy flow and isolation
- \+ See if kinematic features survive
- – Don't know response of detector to a soft quark/gluon
- – Cannot tag a b quark
- – Can't ask about charged tracks or neutrals

Next step is into the Brown Muck

Parton Shower Summary

Modern PS models are very sophisticated implementations of perturbative QCD

Derived from factorization theorems of full gauge theory

- Accelerated charges radiate (QED/QCD)
- Gluons have color and radiate as well
- Parton Shower development encoded in Sudakov FF
- Performed to LL and some sub-LL accuracy with exact kinematics
- Color coherence leads to angular ordering of shower
- Still need hadronization models to connect with data
- Shower evolves virtualities of partons to a low enough values where this connection is possible

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Lecture 2

- Hadronization
  - string
  - cluster
- Underlying Event
  - parametrizations
  - multiple-interactions
- The Event Generator Programs
- New Developments

Hadronization

QCD partons are free only on a **very short time scale**

Hadrons are the physical states of the strong interaction

Need a description of how partons are confined

Lacking a theory, we need a model

- **enough** variables to fit data
- **few enough** that there is some predictability
- start related to the end of the parton shower
- Use **basic** understanding of QCD

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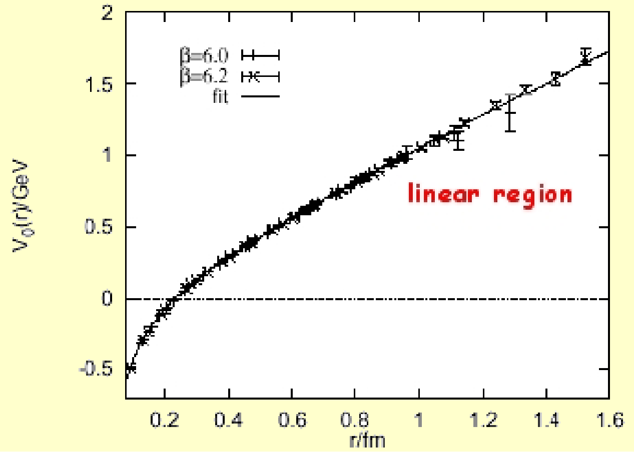
QCD is a confining theory

- Linear potential $V_{\rm QCD}(r) \sim kr$

  Confirmed by Lattice, Spectroscopy, Regge Trajectories

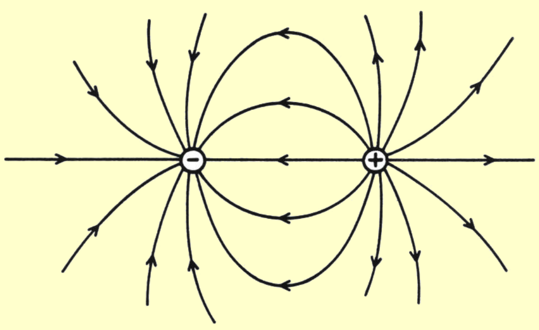


- Gluons are self-coupling

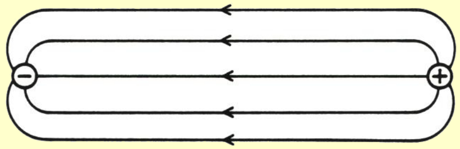
  Field lines contract into Flux-tubes

  Analogy with field behavior inside of superconductors

QED FIELD LINES



QCD FIELD LINES (?)



- Over time, 2 phenomenological models have survived

  cluster

  Lund string

- Not exactly Orthogonal, Exhaustive

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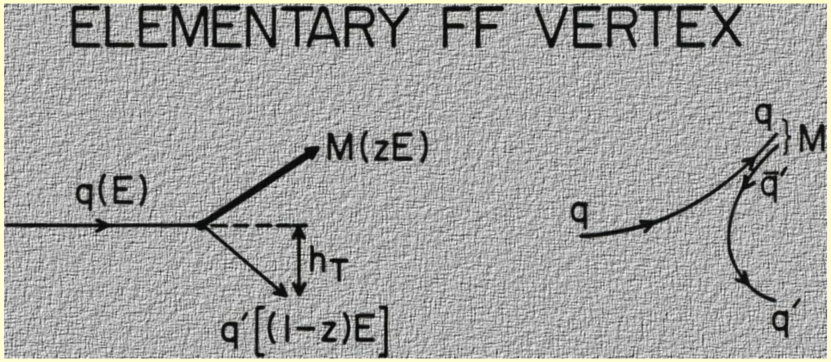
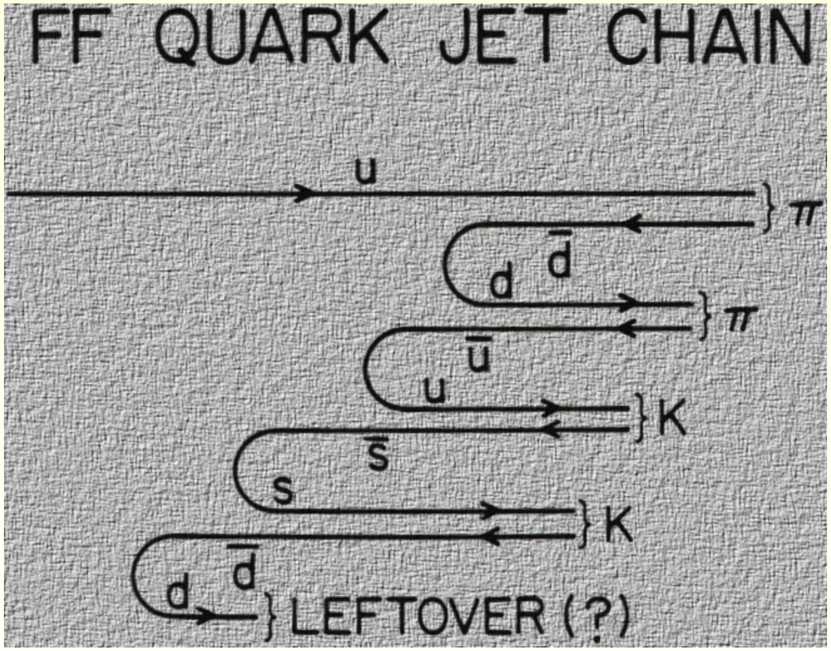
Necrologos: Independent Fragmentation





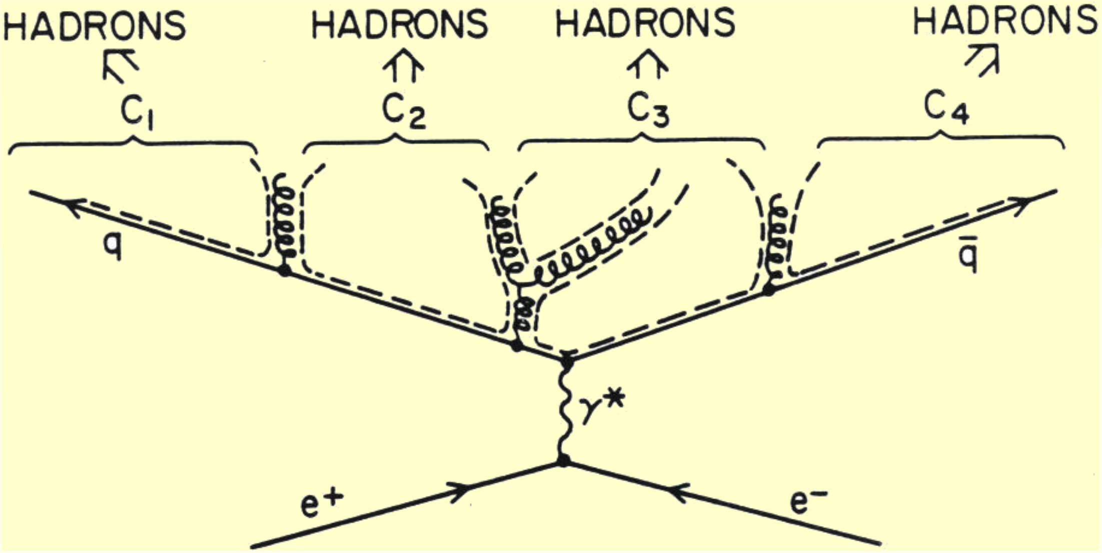
- FF = Feynman-R. Field
- pure phenomenological model
- imagine $q\bar{q}$ pairs tunnel from the vacuum to dress bare quark
- $f_{q\to h}(z)$ is probability $q \to h$ with fraction z of some E/p variable
- $f_{g\to h}(z)$? $g \to q\bar{q}$?
- Lorentz invariant? (E_q)
- Useful for its time

FF: $f(z) = 1 - a + 3a(1-z)^2$

Preconfinement

Perturbative evolution of quarks and gluons organizes them into clumps of color-singlet clusters

In PS, color-singlet pairs end up close in phase space



- Cluster model takes this view to the extreme
- Non-perturbative splitting:

  gluon ⇒ color-anticolor pair
- Parton shower **cutoff** is a critical parameter

Cluster hadronization in a nutshell

- Nonperturbative $g \to q\bar{q}$ splitting $(q = uds)$ isotropically
  Here, $m_g \approx 750\,\mathrm{MeV} > 2m_q$.

- Cluster formation, universal spectrum

- Cluster fission until

$$M^p < M^p_{\mathrm{fiss}} = M^p_{\mathrm{max}} + (m_{q1} + m_{q2})^p$$

  where masses are chosen from

$$M_i = \left[\left(M^P - (m_{qi} + m_{q3})^P\right) r_i + (m_{qi} + m_{q3})^P\right]^{1/P}$$

  with additional phase space constraints

- Cluster decay
  isotropically into pairs of hadrons
  simple rules for spin, species

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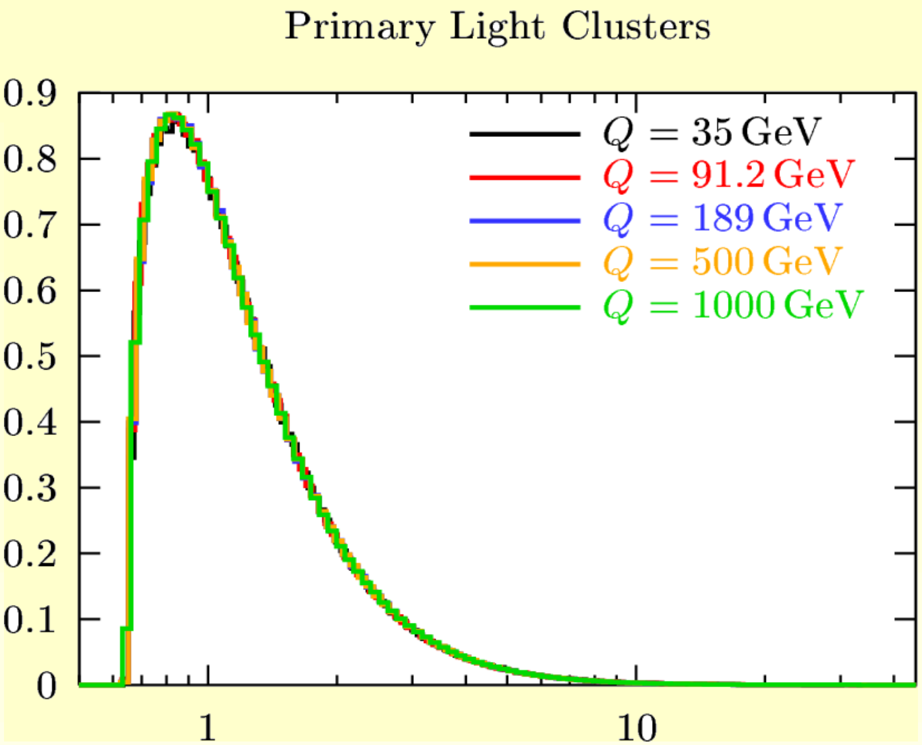
Cluster Fission



- Mass spectrum of color-singlet pairs asymptotically independent of energy, production mechanism
- Peaked at low mass
- Broad tail at large mass

Small fraction of clusters heavier than typical

$\Rightarrow$ Cluster fission (string-like)

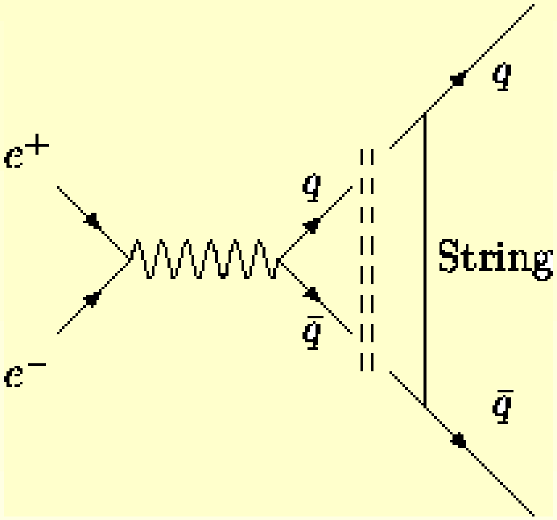
Fission threshold becomes crucial parameter

15% of primary clusters split

produces 50% of hadrons

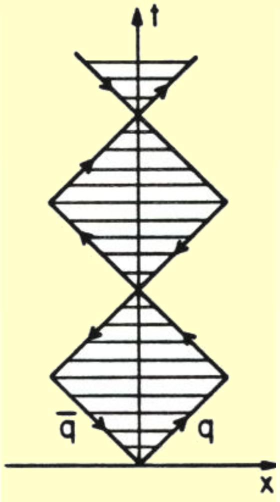
Lund String Model

String=color flux tube is stretched between q and $\bar{q}$



Classical string will oscillate in space-time

Endpoints $q, \bar{q}$ exchange momentum with the string



Quantum Mechanics: string energy can be converted to $q\bar{q}$ pairs (tension $\kappa \sim$ 1 GeV/fm)

dProb/dx/dt = (constant)$\exp(-\pi m^2/\kappa)$ **[WKB]**

- $u : d : s : qq = 1 : 1 : 0.35 : 0.1$

Area (swept out by string) Law

$$dP_n(\{p_j\}; P_{tot}) = \prod_{j=1}^{n} N_j d^2 p_j \delta({p_j}^2 - {m_j}^2)\delta(\sum_{j=1}^{n} p_j - P_{tot}) \exp(-bA)$$

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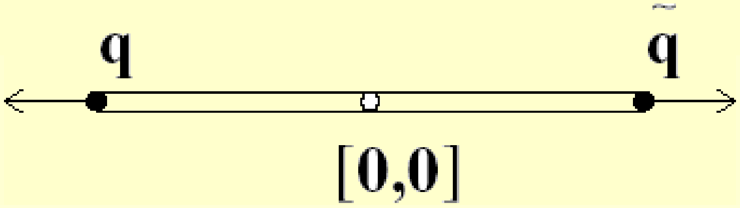
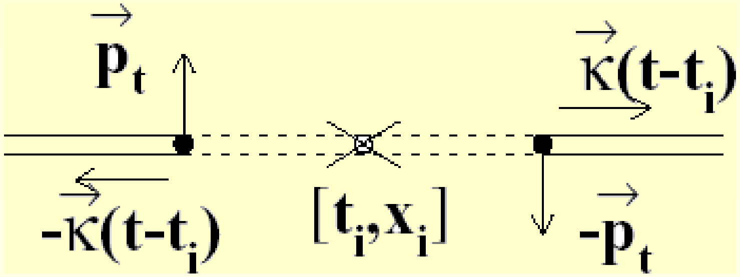
Hadron Formation

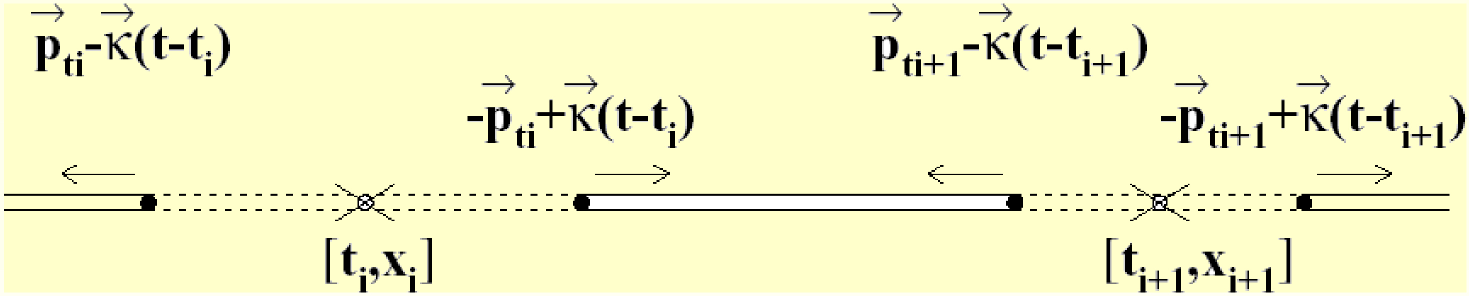
Original String



Tunnelling



Fragmentation



Adjacent breaks form a hadron

$$E_{had} = \kappa|x_i - x_{i+1}| \quad \vec{p}_{had} = \vec{p}_T + \vec{\kappa}(t_i - t_{i+1})$$

$m^2_{had} \propto$ area swept out by string

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Iterative Solution

String breaking and hadron formation can be treated as an iterative process

Use light-cone coordinates $x^{\pm} = x \pm t$

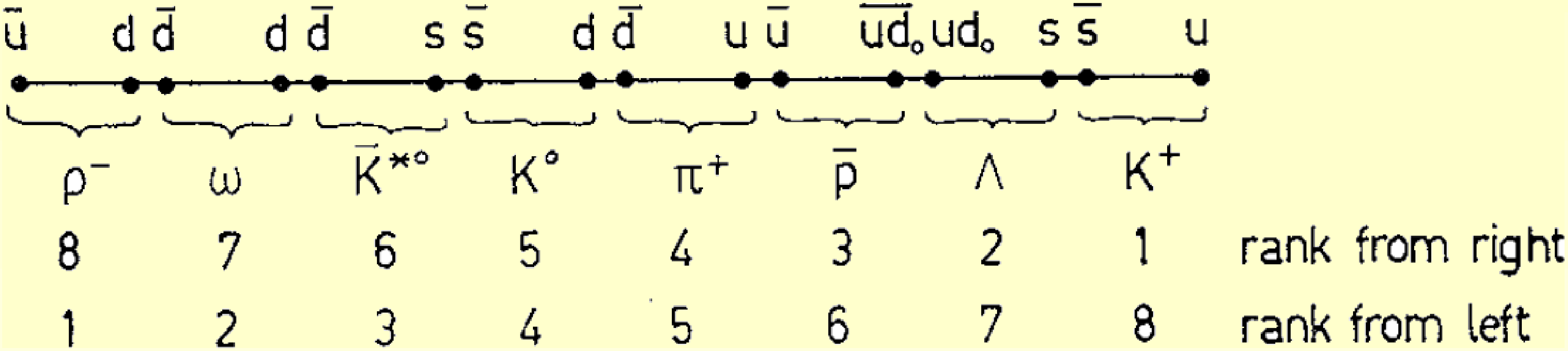
Boundary Conditions:

$$x_0^+ = 2E_0/\kappa, x_{n+1}^- = 2\bar{E}_0/\kappa, x_0^- = x_{n+1}^+ = 0$$

1. select z_i according to $f(z)dz$
    - $f^h(z, p_T) \sim \frac{1}{z}(1-z)^a \exp\left[-\frac{b(m_h^2 + p_T^2)}{z}\right]$
2. $\Delta x^+ = (x_{i-1}^+ - x_i^+) = z_i x_{i-1}^+$
3. $\Delta x^- = (x_{i-1}^- - x_i^-) = \frac{-m_i^2}{\kappa^2 \Delta x^+}$
    - mass^2 of hadron $\propto \Delta x^+ \Delta x^-$
4. Continue until string is consumed

Example Break-Up

Tunnelling of pairs leads to a string of hadrons



Simple rules for spin, isospin, *etc.*

Note diquarks

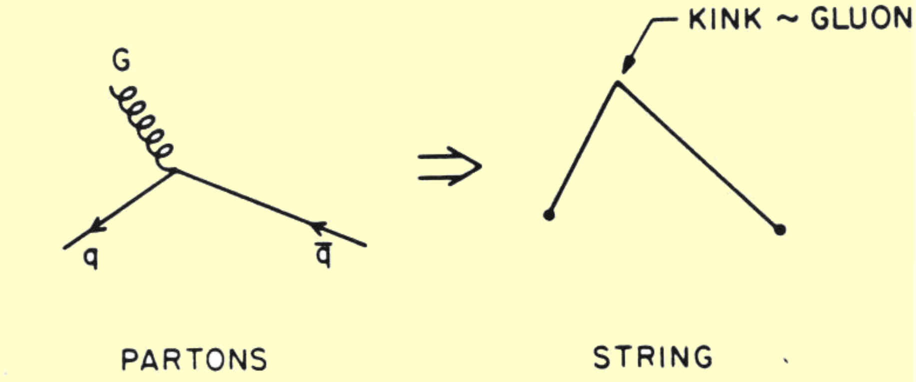
Algorithm can start from either end of string

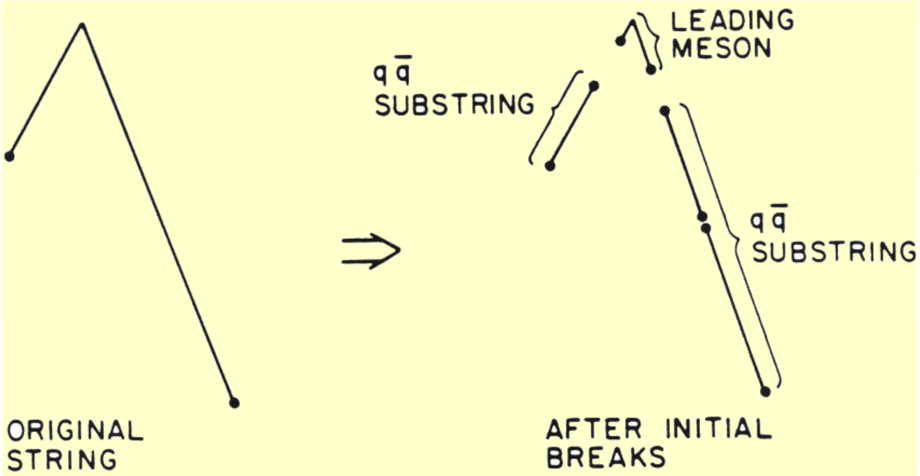
Inclusion of Gluon Radiation

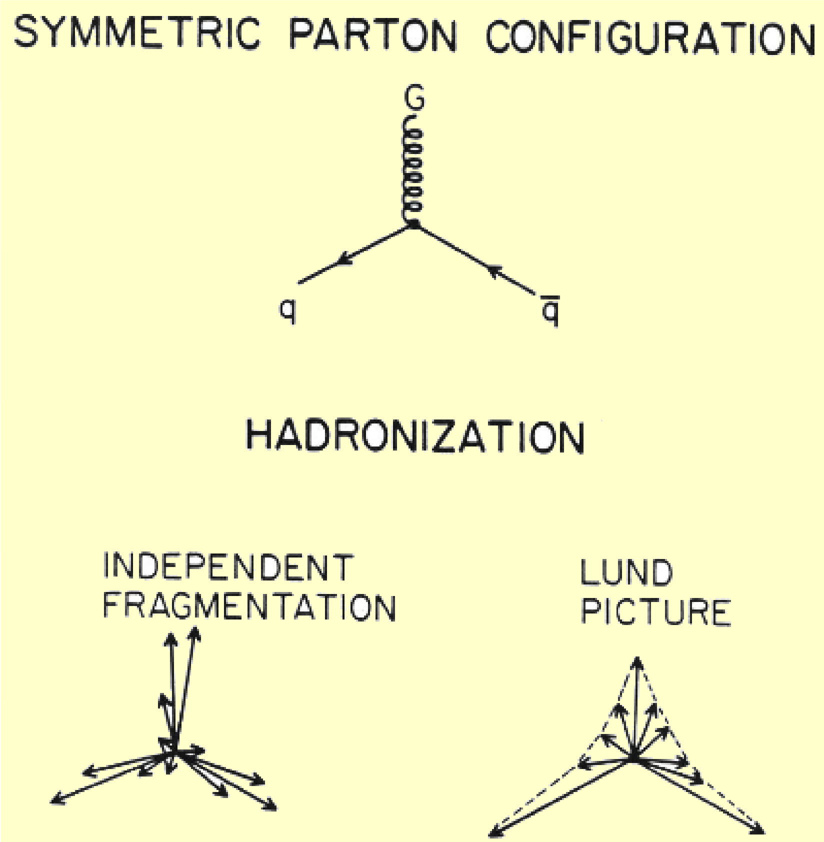
Perturbative Parton Shower generates gluons
(color-anticolor pair + correction in $1/N_C$ expansion)

Gluon = kink on string, i.e. some motion to system

String effect $\Rightarrow$ particles move in direction of kink

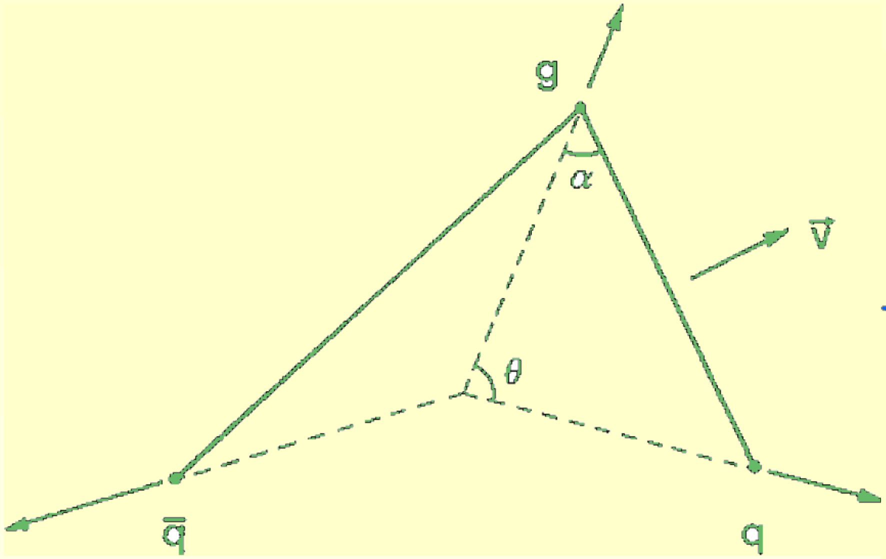






Infrared Stability

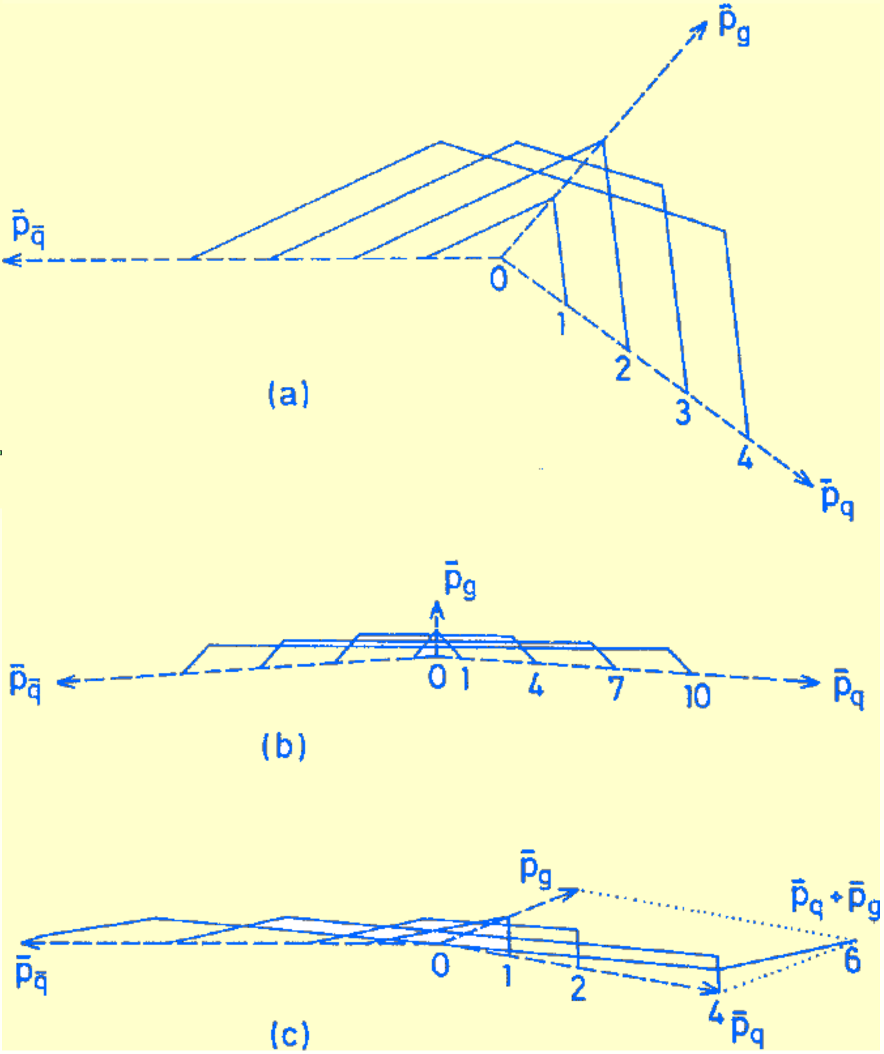
“Hard” gluon imparts motion to string system



q and $\bar{q}$ give bits of momentum to g and vice versa

Parton that gives up all its energy is still dragged along

Soft or collinear gluons collapse into simpler string



Hadronization Overview

Clusters (Herwig)

- perturbation theory can be applied down to low scales if the coherence is treated correctly
- There must be non-perturbative physics, but it should be very simple
- Improving data has meant successively making non-pert phase more string-like

Strings (Pythia, Ariadne)

- dynamics of the non-perturbative phase must be treated correctly
- Model includes some non-perturbative aspect of color (interjet) coherence (string effect)
- Improving data has meant successively making non-pert phase more cluster-like

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Underlying Event

- Hadrons (protons) are extended objects
- Remnant remains after hard partons scatter
- Need a description of how partonic remnants are confined

Two Approaches

1. Soft parton-parton collisions dominate
2. Semi-Hard parton-parton cross section is large and can be calculated even at low p_T

Soft Underlying Event

UA5 Monte Carlo

- hadron-hadron scattering produces two leading clusters and several central ones
- parametrize N_{ch} and sample
- clusters given p_T and y from some distribution

  $$\frac{dN}{dp_T^2} \sim e^{-bp_T}, \frac{1}{(p_T + p_0)^n}$$

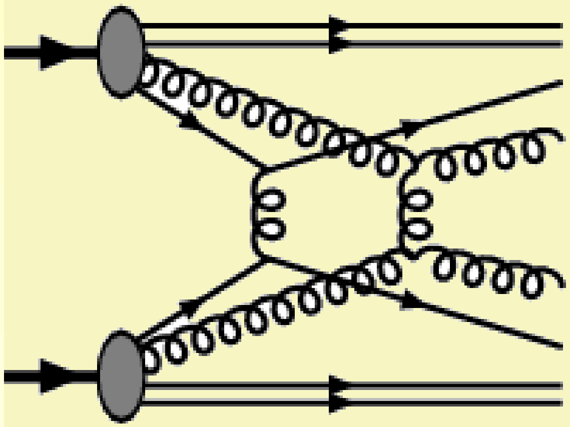
  $y \sim$ flat with Gaussian tails
- $p_L = m \sinh(y)$

`Herwig` adds in their cluster model

UE model is a mechanism for producing the objects used in description of hadronization

Multiple Interaction Model

Consequence of composite nature of hadrons!

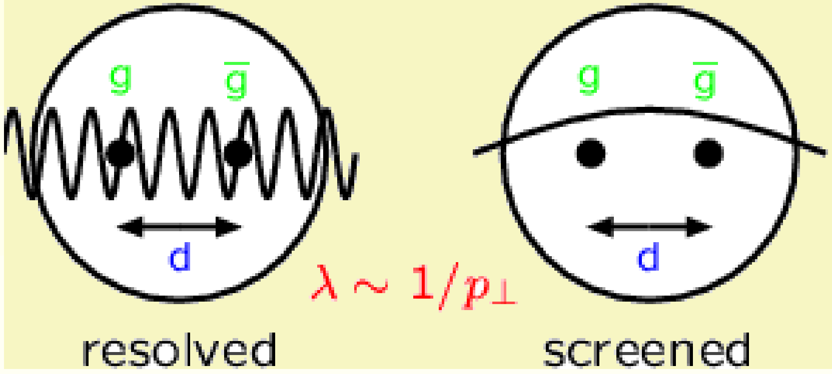


Evidence:

- direct observation: AFS, UA1, CDF
- implied by width of multiplicity distribution + jet universality: UA5
- forward–backward correlations: UA5
- pedestal effect: UA1, H1, CDF

One new free parameter: $p_{\perp\min}$

$$\frac{1}{2}\sigma_{\text{jet}} = \int_{p^2_{\perp\min}}^{s/4} \frac{d\sigma}{dp_\perp^2}\, dp_\perp^2 \quad \Longleftarrow \quad \int_0^{s/4} \frac{d\sigma}{dp_\perp^2}\, \frac{p_\perp^4}{(p^2_{\perp 0} + p^2_\perp)^2}\, dp_\perp^2$$



Measure of colour screening length d in hadron:

$p_{\perp\min}\,\langle d\rangle \approx 1 (= \hbar)$

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Multiple Interaction Model

$\bar{n} = \sigma_{\mathrm{hard}}(p_{\perp\min})/\sigma_{\mathrm{nd}}(s) > 1$

Not a violation of unitarity! σ_{hard} is inclusive

naive estimate: no E-p conservation

- must consider interactions of all partons

On average, $\bar{n}$ semi-hard interactions **in one hard collision**

Collisions ranked in $x_\perp = 2p_\perp/E_{\mathrm{cm}}$, produced with prob

$$f(x_\perp) = \frac{1}{\sigma_{\mathrm{nd}}(s)} \frac{\mathrm{d}\sigma}{\mathrm{d}x_\perp}$$

The probability that the hardest interaction is at $x_{\perp 1}$:

$$f(x_{\perp 1}) \exp\left\{ - \int_{x_{\perp 1}}^{1} f(x'_\perp)\, \mathrm{d}x'_\perp \right\}$$

- like radioactive decay

Multiple Interaction Model

generate a chain of scatterings $1 > x_{\perp 1} > x_{\perp 2} > \cdots > x_{\perp i}$

using $x_{\perp i} = F^{-1}(F(x_{\perp i-1}) - \ln R_i)$

- $F(x_\perp) = \int_{x_\perp}^{1} f(x'_\perp)\, \mathrm{d}x'_\perp = \frac{1}{\sigma_{\mathrm{nd}}(s)} \int_{s x_\perp^2/4}^{s/4} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp^2}\, \mathrm{d}p_\perp^2$

Refinements

Parton x_i for PDF evaluation is rescaled $x'_i = \frac{x_i}{\sum_{j=1}^{i-1} x_j}$

Almost no experimental information on correlated PDFs

Include “Matter” distributed in hadrons

$$\mathcal{O}(b) \propto \int \mathrm{d}t \int \mathrm{d}^3x\, \rho(x,y,z)\, \rho(x+b, y, z+t)$$

$$\langle \mathrm{N}_{\mathrm{int}} \rangle \sim \mathrm{k}\ \mathcal{O}(b)$$

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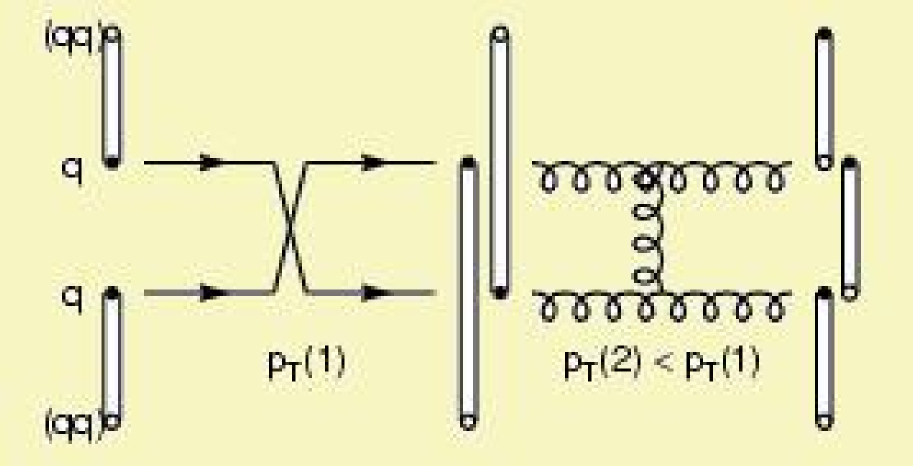
Strings and the UE



Each additional interaction adds more color flow

- Color information encoded in strings
- Subsequent interactions mainly reconnect to pre-existing strings
- Fits prefer a minimization of total string length

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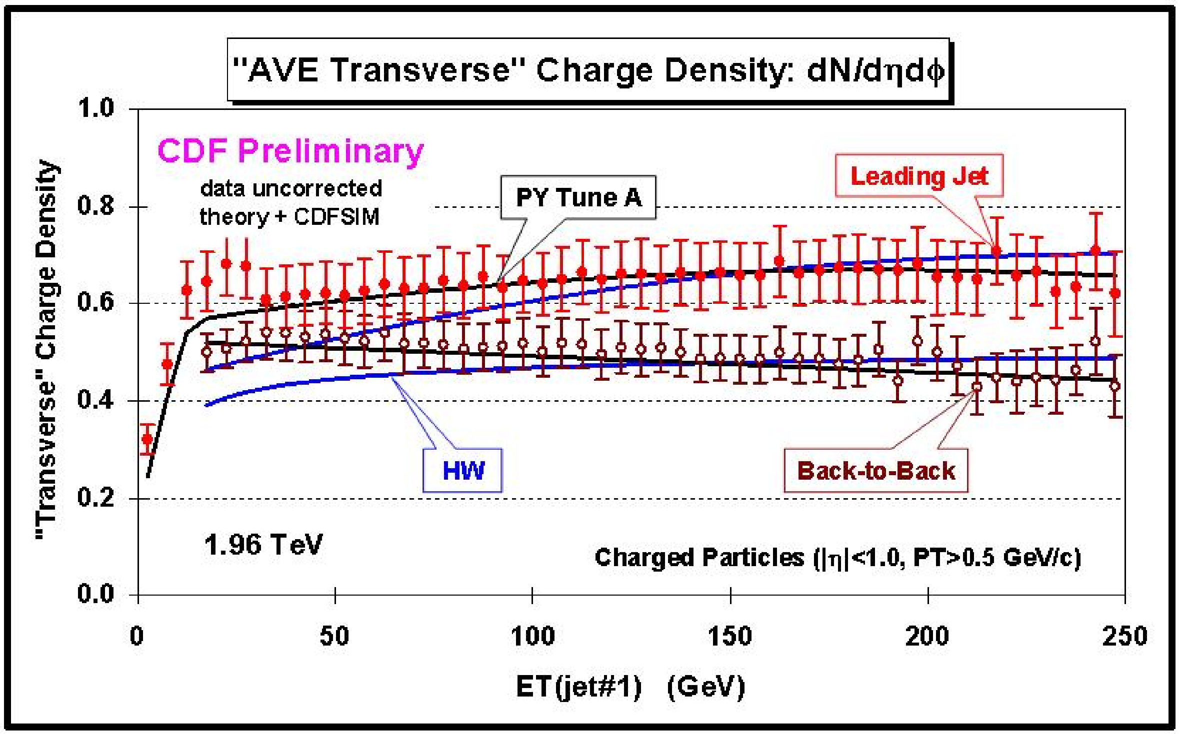
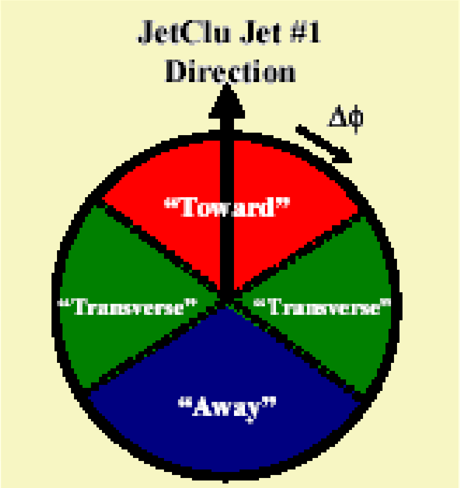
Pythia Options

```
MSTP(82) :
    (D=1) structure of multiple interactions. For QCD processes, used down to
values below , it also affects the choice of structure for the
one hard/semi-hard interaction.

    = 0 :
        simple two-string model without any hard interactions. Toy model only!
    = 1 :
        multiple interactions assuming the same probability in all events,
with an abrupt cut-off at PARP(81). (With a slow energy dependence given by
PARP(89) and PARP(90).)
    = 2 :
        multiple interactions assuming the same probability in all events,
with a continuous turn-off of the cross section at PARP(82). (With a slow
energy dependence given by PARP(89) and PARP(90).)
    = 3 :
        multiple interactions assuming a varying impact parameter and a
hadronic matter overlap consistent with a Gaussian matter distribution, with
a continuous turn-off of the cross section at PARP(82). (With a slow energy
dependence given by PARP(89) and PARP(90).)
    = 4 :
        multiple interactions assuming a varying impact parameter and a
hadronic matter overlap consistent with a double Gaussian matter distribution
given by PARP(83) and PARP(84), with a continuous turn-off of the cross
section at PARP(82). (With a slow energy dependence given by PARP(89) and
PARP(90).)
```

Pythia at Run2: Underlying Event





PYTHIA 6.206 and CDF Tune A (CTEQ5L)			
Parameter	Default	Tune	Description
PARP(67)	1.0	4.0	Scale factor for ISR
MSTP(82)	1.0	4	Double Gaussian matter distribution
PARP(82)	1.9	2.0	Cutoff (GeV) for MPIs
PARP(83)	0.5	0.5	Warm Core with % of matter
PARP(84)	0.2	0.4	within a given radius
PARP(85)	0.33	0.9	Prob. that two gluons have NNC
PARP(86)	0.66	0.95	gg versus $q\bar{q}$
PARP(89)	1000.0	1800.0	Reference energy (GeV)
PARP(90)	0.16	0.25	Power of Energy scaling for cutoff

The Parton Shower Programs

	Pythia	Herwig	Ariadne
PS Ordering	Mass angle veto	Angle	k_T
Hadronization	String	Cluster	String
Underlying Event	Mult. Int	UA6/(Jimmy)	LDCM

Finding them: http://cepa.fnal.gov/mrenna/generator.html

- http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html
- http://hepwww.rl.ac.uk/theory/seymour/herwig/
- http://www.thep.lu.se/~leif/ariadne/

Fortran codes

http://www.ibiblio.org/pub/languages/fortran/ch1-1.html

"The Future" =`C++`=`ThePEG`

Toolkit for high energy Physics Event Generation

`http://www.thep.lu.se/ThePEG/`

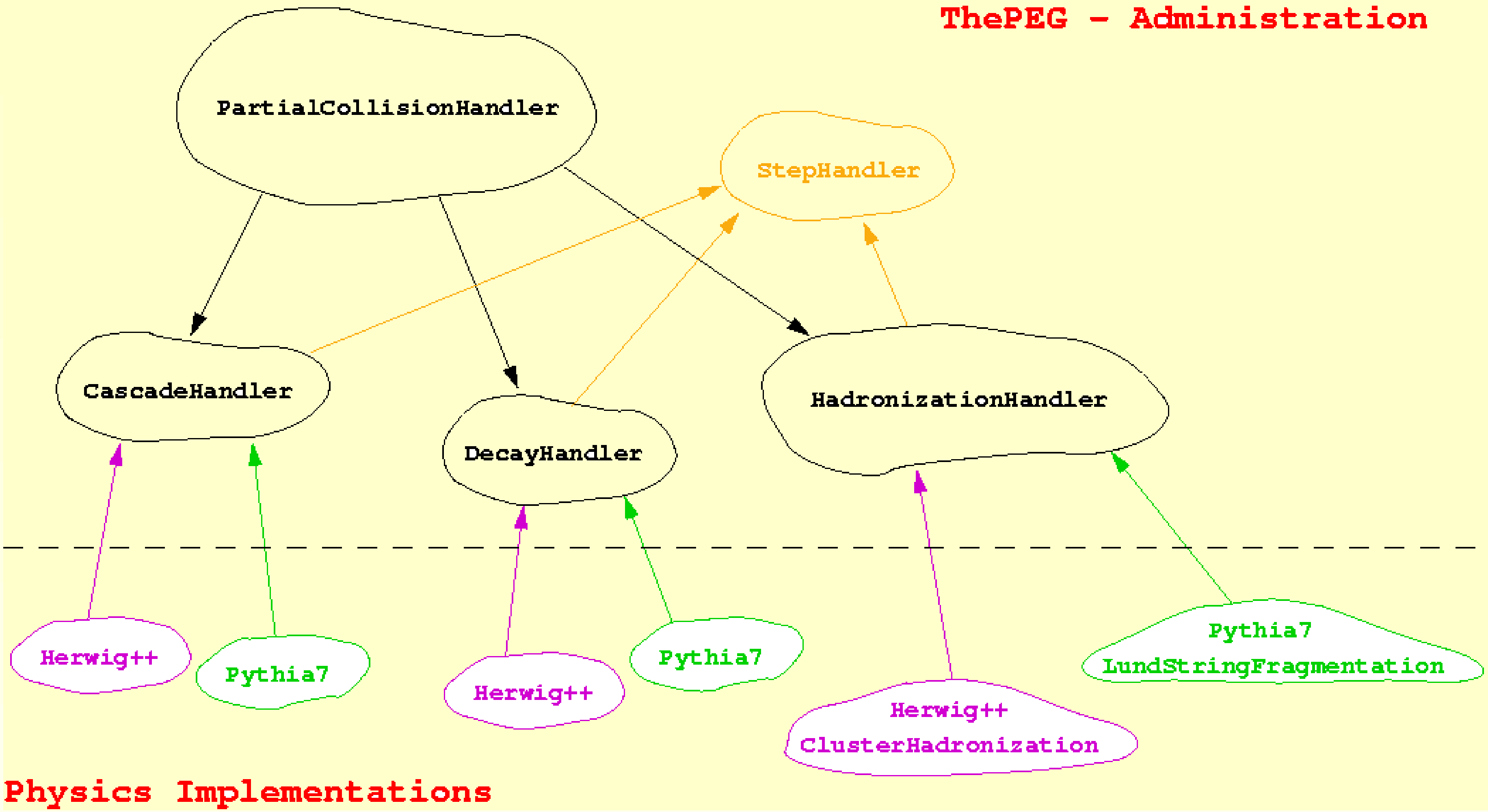
Share administrative overhead common to event generators

Independent *physics* implementation

Common basis for `Pythia7/Herwig++`

- Lack of independence.
- Less possiblity to test codes against each other.
- Physics is still independent.
- Beneficial for the user to have the same framework.
- Use `Herwig++` with String Fragmentation from Pythia7

PartialCollisionHandlers



`Sherpa` is also `C++` event generator in a different framework

Developments

- Improved showers for `Pythia` and `Herwig`
- More detailed models of Underlying Event
- New programs for automating Tree-Level calculations
- Event generators for NLO calculations
- Matrix Element and Parton Shower Matching

New ME Programs

Automatically calculate code needed for a given HEP process and generate events

List of those actively supporting hadron colliders

- `Alpgen` `http://m.home.cern.ch/m/mlm/www/alpgen/`
- `CompHep` `http://theory.sinp.msu.ru/comphep`
- `Grace` `http://atlas.kek.jp/physics/nlo-wg/grappa.html`
- `MadEvent` `http://madgraph.hep.uiuc.edu/index.html`
- `Sherpa/Amegic++` `http://141.30.17.181/`

Advantages and disadvantages of each

An impressive improvement from several years ago

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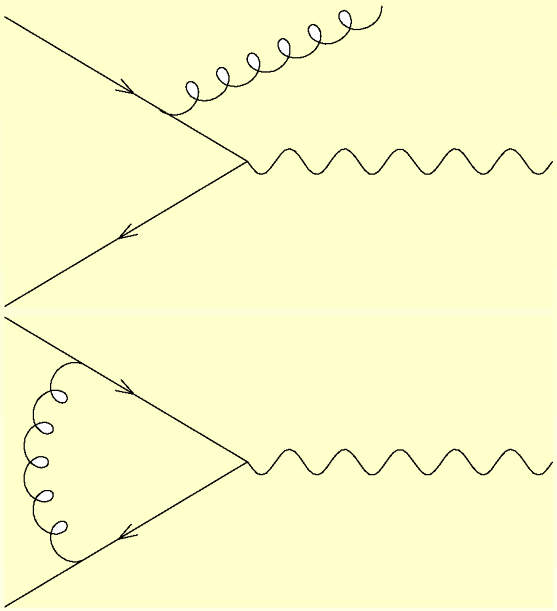
Event Generator At NLO



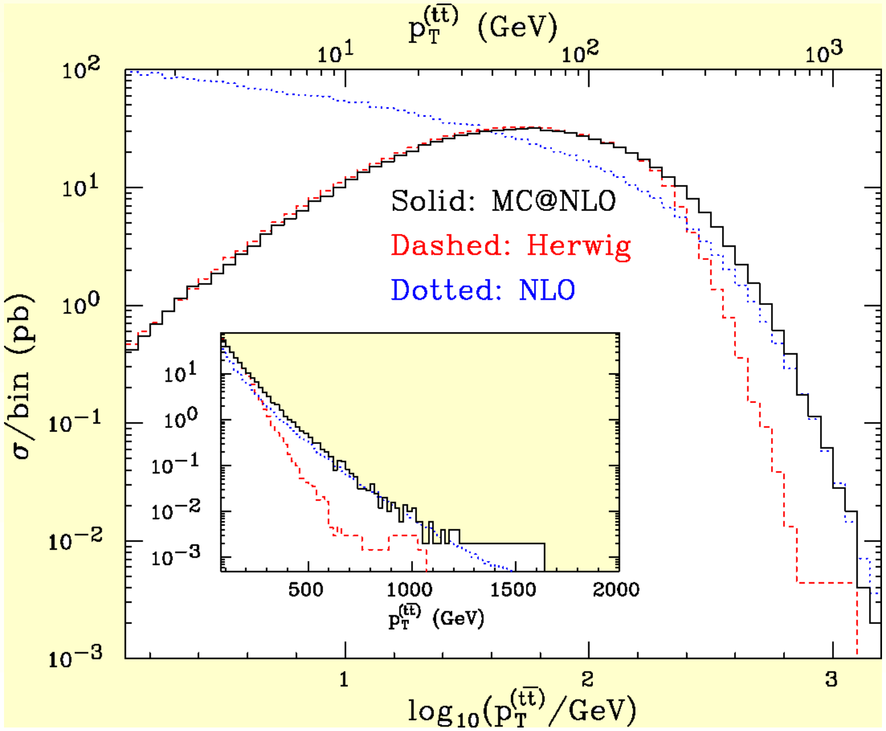
Next-to-Leading-Order

Calculations give an improved description of the hard kinematics and cross sections

not event generators

Solution (`MC@NLO`):

Remove divergences by adding and subtracting the Monte Carlo result for one emission



Event Generators for Many Legs

Want to exploit tools for generating tree level diagrams

Each topology (e.g. $W+0,1,2,3,4$ partons) has no soft/collinear approximation

How do I add a parton shower to each topology with no double counting?

Solution (`CKKW`):

1. Make the $|\mathcal{M}|^2$ result “look” like a parton shower down to a reasonable cutoff scale ($k_T^{\text{cut}}/Q_{\text{hard}} \sim .1$)
2. Add on ordinary parton shower below K_T^{cut}

$$k_T^2 = 2\text{min}(E_i, E_j)^2(1-\cos\theta_{ij}) \sim \text{min}(E_i/E_j, E_j/E_i)m^2$$

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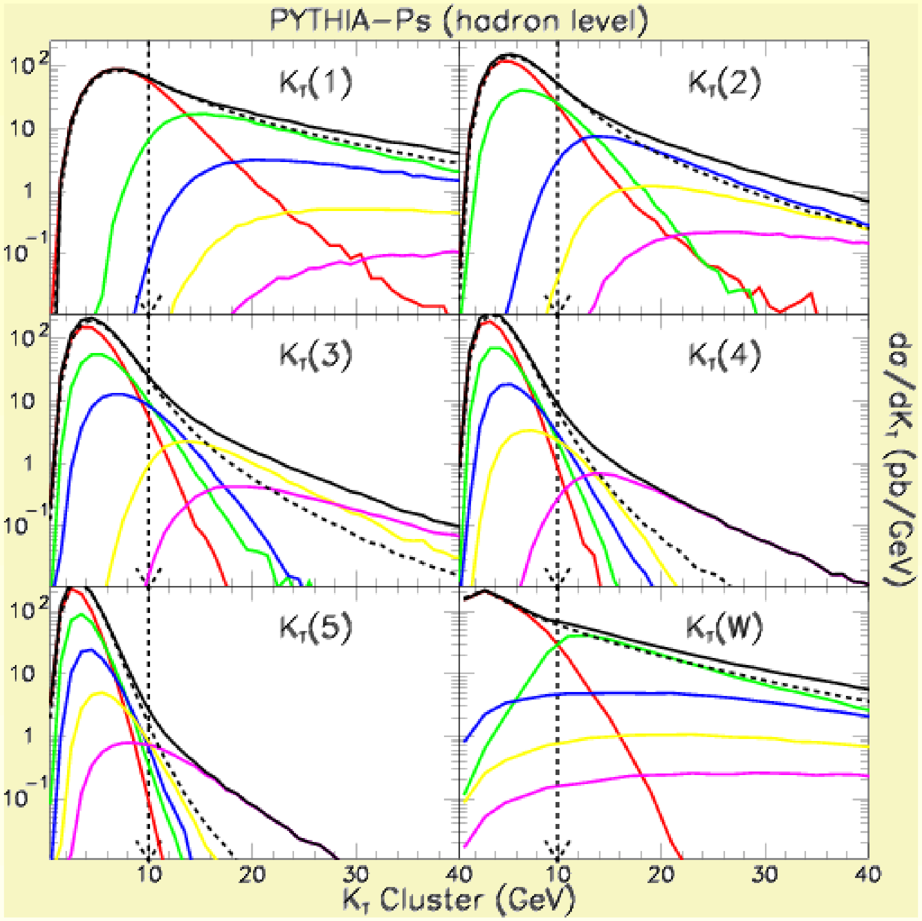
W+0 $\oplus \cdots \oplus$ W+4 hard partons



Dashed is `Pythia` with default (ME) correction

Solid is Pseudoshower result

Combines ME contributions (0, 1, 2, 3, 4 partons)

Overall Summary

- Event Generators accumulate our understanding of the Standard Model into one package
- Apply perturbation theory whenever possible

  hard scattering, parton showering, decays
- Rely on models or parametrizations when present calculational methods fail

  hadronization, underlying event, beam remnants
- Out of the box, they give reliable estimates of the full, complicated structure of HEP events
- Attentive users will find more flexibility & applications
- Understanding the output can lead to a broader understanding of the Standard Model (and physics beyond)

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MC4Run2: MC/ME Tuning

`http://cepa.fnal.gov/patriot/mc4run2`

- Forum for CDF, DØ, & Theorists
  - LHC participation is welcome/encouraged
- Discussions/Presentations on:
  - Underlying Event Tunes
  - B Production Tunes
  - Parton Shower Matching
  - PDF systematics
    - $*$ LHAPDF interface ala PDFLIB
  - Herwig/Pythia Problems with Photon-Jet Balancing
  - Non-perturbative corrections to Jet Cross Sections
  - ResBos-A
  - ...

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