Event generators predict multiparticle event configurations in HEP experiments

\[ P(x) \Rightarrow N \text{ performed using Monte Carlo integration} \]

- Estimate the total cross section
- Generate events one at a time

Relies on a computer’s ability to generate (pseudo) random numbers
Lecture 1

- Defining Event Generators
  - Modularity of HEP Events
- Monte Carlo Techniques
  - Calculating Integrals
  - Sampling Distributions
- Matrix Element Calculations
  - Applications
  - Limitations
- Parton Shower
  - Sudakov Form Factor
  - Coherence
  - Dipoles
- Summary
Phases of High Energy Collisions

- hard scattering
- initial/final state radiation
- partonic decays, $t \rightarrow bW$
- parton shower evolution

- nonperturbative phase
- colorless clusters
- cluster $\rightarrow$ hadrons
- hadronic decays
- backward parton evolution
- underlying event
Monte Carlo Basics

\[ I = \int_{x_1}^{x_2} dx f(x) = (x_2 - x_1) \langle f(x) \rangle \]

\[ \sigma = \int dx \frac{d\sigma}{dx} \]

\[ \simeq I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^{N} f(x_i) \]

\[ \simeq I_N \pm (x_2 - x_1) \sqrt{\frac{\langle f^2 \rangle - \langle f \rangle^2}{N}} \]

Non-uniform sampling can be more efficient:

\[ \int_{x_1}^{x_2} dx p(x) = 1 \Rightarrow I = \int_{x_1}^{x_2} dx p(x) \frac{f(x)}{p(x)} \]

\[ I = \left\langle \frac{f(x)}{p(x)} \right\rangle \pm \frac{1}{\sqrt{N}} \sqrt{\left( \left\langle \frac{f(x)^2}{p(x)^2} \right\rangle - \left\langle \frac{f(x)}{p(x)} \right\rangle^2 \right) \}} \]

Make \( f/p \) as flat as possible (reduce variance)
Stratified sampling: divide integration region into sub-volumes and sample according to variance
e.g., $\delta f(t)=0$ if sampling on intervals $T_1$ and $T_2$

Importance sampling:
choose $x_N$ based on $I_{N-1}$

VEGAS is an adaptive integrator that adjusts step functions to parallel integrand
Monte Carlo (cont)

Up to here, only considered MC as a numerical integration method

If function being integrated is a probability density (positive definite), can convert it to a simulation of physical process = an event generator

Simple example: \[ \sigma = \int_0^1 dx \frac{d\sigma}{dx} \]

Naive approach:

- pick events \( x \) with weights \( \frac{d\sigma}{dx} \)
- generate unweighted events by keeping them with probability \( \frac{1}{\sigma} \frac{d\sigma}{dx} \) and giving them all weight 1
- Events selected with same frequency as in nature

Often, more sophisticated sampling methods are employed
Given $f(x) > 0$ over $x_{\text{min}} \leq x \leq x_{\text{max}}$

Prob in $(x + dx, x)$ is $f(x) \, dx$

$$\int_{x_{\text{min}}}^{x} f(x) \, dx = R \int_{x_{\text{min}}}^{x_{\text{max}}} f(x) \, dx$$

$$x = F^{-1}(F(x_{\text{min}}) + R(F(x_{\text{max}}) - F(x_{\text{min}})))$$

- assumes $F(x), F^{-1}(x)$ are known
- fraction $R$ of area under $f(x)$ should be to the left of $x$

Realistic $f(x)$ are rarely this nice
If \( \text{max}[f(x)] \) is known, use **hit-or-miss**

1. select \( x = x_{\text{min}} + R(x_{\text{max}} - x_{\text{min}}) \)

2. if \( f(x)/f_{\text{max}} \leq (\text{new}) R \), reject \( x \) and \( \Rightarrow 1 \).

3. otherwise, keep \( x \)

Works because probability
\[
\frac{f(x)}{f_{\text{max}}} > R \propto f(x)
\]

Acceptable method if \( f(x) \)
does not fluctuate too wildly

Usually guess at \( \text{max}[f(x)] \)
and update if a “better” estimate is found in a run
Sampling Distributions: Method 3

Find \( g(x) \), with \( f(x) \leq g(x) \) over \( x \) range

- \( G(x) \) and its inverse \( G^{-1}(x) \) known
- e.g., \( \int_{\epsilon}^{z} dx \frac{1 + x^2}{1 - x} < \int_{\epsilon}^{z} dx \frac{2}{1 - x} = 2 \ln \left[ \frac{1 - \epsilon}{1 - z} \right] \)

1. select an \( x \) according to \( g(x) \), using Method 1
2. if \( f(x) / g(x) \leq \) (new) \( R \), reject \( x \) and \( \Rightarrow 1 \).
3. otherwise, keep \( x \)

first step selects \( x \) with a probability \( g(x) \)

second step retains this choice with probability \( f(x) / g(x) \)

total probability to pick a value \( x \) is then just the product of the two, i.e. \( f(x) dx \)
Radioactive Decay Problem

Know probability $f(t)$ that ‘something will happen’ (a nucleus decay, a parton branch) at time $t$

$something$ $happens$ $at$ $t$ $only$ $if$ $it$ $did$ $not$ $happen$ $at$ $t' < t$

Equation for nothing $\mathcal{N}(t)$ to happen $up$ $to$ $time$ $t$ is $(\mathcal{N}(0) = 1)$:

$$- \frac{d\mathcal{N}}{dt} = f(t) \mathcal{N}(t) = \mathcal{P}(t)$$

$$\mathcal{N}(t) = \exp \left\{ - \int_0^t f(t') \, dt' \right\}$$

$$\mathcal{P}(t) = f(t) \exp \left\{ - \int_0^t f(t') \, dt' \right\}$$

- Naive answer modified by exponential suppression
- In the parton-shower language, this corresponds to the Sudakov form factor
Veto Algorithm

If $F(t)$ and $F^{-1}(t)$ exist:

$$\int_0^t P(t') \, dt' = \mathcal{N}(0) - \mathcal{N}(t) = 1 - \exp \left\{ - \int_0^t f(t') \, dt' \right\} = 1 - R$$

$$F(0) - F(t) = \ln R \quad \Rightarrow \quad t = F^{-1}(F(0) - \ln R)$$

If not, use **veto algorithm**

1. start with $i = 0$ and $t_0 = 0$

2. $++i$ and select $t_i = G^{-1}(G(t_{i-1}) - \ln R)$

3. if $f(t_i)/g(t_i) \leq$ (new) $R$, $\Rightarrow$ 2.

4. otherwise, keep $t_i$

- $N$ vetos equivalent to probability of accepting first try times a partial sum of an exponential series in $f - g$
MC Overview

- Use MC to perform integrals and sample distributions
- Technique generalizes to many dimensions
  - Typical phase space $\sim d^3\vec{p} \times 100$’s particles
- Suitable for complicated integration regions
  - Kinematic cuts or detector cracks
- Error scales as $1/\sqrt{N}$ vs $1/N^{2/d}, 1/N^{4/d}$ (trap, Simp)
- Only need a few points to estimate $f$
- Each additional point increases accuracy
- Easy (non-rigorous) error estimate
- Can sample distributions where exact solutions cannot be found
- Veto algorithm applied to parton shower
Phase 1: Hard Scattering

Characterizes the rest of the event

Sets a high energy scale $Q$

Fixes a short time scale where partons are free objects

Allows use of perturbation theory

External partons can be treated as on the mass-shell

- Valid to $\max[\Lambda, m]/Q$
- Physics at scales below $Q$ absorbed into parton distribution and fragmentation functions (Factorization Theorem)

Sets flow of Quantum numbers (Charge, Color)

- Note: Parton shower and hadronization models valid to $1/N_C$
- Gluon replaced by color-anticolor lines
- All color flows can be drawn on a piece of paper
Cross Sections and Decay Widths

Physics Quantities to calculate using Monte Carlo

\[ \sigma = \frac{1}{2s} \int |M|^2 d\Phi_n(\sqrt{s}) \quad \Gamma = \frac{1}{2M} \int |M|^2 d\Phi_n(M) \]

Phase Space:

\[ d\Phi_n(M) = \prod_{i=1}^{n} \frac{d^3\vec{p}_i}{(2\pi)^3(2E_i)} (2\pi)^2 \delta^{(4)} \left( p_0 - \sum_{i=1}^{n} p_i \right) \]

2-body: 2 × 3 − 4 integration variable ⇒ 2 angles \( d\Omega \)

N-body by Recursion:

\[ d\Phi_n(M) = \frac{1}{2\pi} \int_{0}^{(M-m)^2} d\mu^2 d\Phi_2(M) d\Phi_{n-1}(\mu) \]
Cross Sections

\[ \sigma(s) = \int_0^1 dx_1 f_1(x_1) \int_0^1 dx_2 f_2(x_2) \hat{\sigma}(x_1 x_2 s) \]

\[ = \int_0^1 \frac{d\tau}{\tau} \hat{\sigma}(\tau s) \int_\tau^1 \frac{dx}{x} x f_1(x) \frac{\tau}{x} f_2 \left( \frac{\tau}{x} \right) \]

\[ \frac{d\hat{\sigma}}{d\hat{t}} = \frac{1}{64s} \frac{1}{p_{1cm}^2} |\mathcal{M}|^2, \hat{t} = (p_{in} - p_{out})^2 \]

Considerations:

- \( \hat{\sigma} \) may have resonant peaks (W or Z production)
- \( f(x) \) may be steeply falling

Side-effect of Monte Carlo-ing:

- Can histogram distributions of produced particles/decay products
- Apply cuts
Tree Level Calculation of Hard Scatter

- Read Feynman rules from $i\mathcal{L}_{\text{int}}$
- Use Wave Functions from Relativistic QM
  Propagators (Green functions) for internal lines
- Specify initial and final states
  Track spins/colors/etc. if desired
- Draw all valid graphs connecting them
  Tedious, but straight-forward
- Calculate (Matrix Element)$^2$
  Evaluate Amplitudes, Add and Square
  Symbolically Square, Evaluate
  Do something tricky
- Integrate over Phase Space
Learn by hand, then automate

Complications:

\[ |\mathcal{M}|^2: \text{Number of graphs grows quickly with number of external partons} \]

\[ d\Phi_n: \text{Efficiency decreases with number of internal lines} \]

Programs:

- MadEvent, CompHep, Alpgen, Amegic++
- Differ in methods of attack
- Most rely on VEGAS for MC integration

Limitations:

- Fixed number of partons
- No control of large logarithms as \( E_g, \theta_{qq}, \theta_{gg} \to 0 \)
NOT event generators

- partonic jets: no substructure
- hard, wide-angle emissions only
- colored/fractionally charged states not suitable for detector simulation
- can guide physics analyses by revealing gross kinematic features

Jacobian peak
Towards an Event Generator

HEP Events are modular:

- Events are transformations from $t = -\infty \rightarrow t = +\infty$
- Hard Interaction occurs over a short time scale $\Delta t \sim 10^{-2}\text{GeV}^{-1}$
- Perturbation theory ($\alpha_s < \pi$) should work down to time $t = .1 - 1\text{GeV}^{-1}$
- Hadronization on longer time scales
- Particle decays typically on longest time scales

Separation of time scales reduces the complex problem to manageable pieces (modules) which can be treated in series

- Previous step sets initial conditions for next one

Next piece is the parton shower
Matrix Element to Parton Shower: $\gamma^* \rightarrow q\bar{q}g$

$$d\sigma(q\bar{q}g) = \sigma_0 \frac{\alpha_s}{2\pi} dz \left\{ \frac{d s_{qg}}{s_{qg}} \left[ P_{q\rightarrow q}(z) - \frac{s_{qg}}{Q^2} \right] + \frac{d s_{\bar{q}g}}{s_{\bar{q}g}} \left[ P_{q\rightarrow q}(z) - \frac{s_{\bar{q}g}}{Q^2} \right] \right\}$$

$$\sigma_0 = \sigma(\gamma^* \rightarrow q\bar{q})$$

$$z = \frac{s_{qg}}{Q^2}, P_{q\rightarrow q}(z) = \frac{4}{3} \frac{1 + z^2}{1 - z}$$

$$s_{qg} = 2E_q E_g (1 - \cos \theta_{qg})$$

$s_{qg}, s_{\bar{q}g} \rightarrow 0$ when gluon is soft/collinear

$z \rightarrow 1$ when gluon is collinear

In soft/collinear limit, independent radiation from $q$ and $\bar{q}$
General Result

$|\mathcal{M}|^2$ involving $q \rightarrow qg$ (or $g \rightarrow gg$) strongly enhanced whenever emitted gluon is almost collinear

Propagator factors (internal lines)

\[
\frac{1}{(p_q + p_g)^2} \approx \frac{1}{2E_q E_g (1 - \cos \theta_{qg})} \rightarrow \frac{1}{E_q E_g \theta_{qg}^2}
\]

- soft+collinear divergences
- dominant contribution to the ME

Collinear factorization

\[
|\mathcal{M}_{p+1}|^2 d\Phi_{p+1} \approx |\mathcal{M}_p|^2 d\Phi_p \frac{dQ^2}{Q^2} \frac{\alpha_s}{2\pi} P(z) dz d\phi
\]

DGLAP kernels:

\[
P_{q \rightarrow q}(z) = C_F \frac{1 + z^2}{1 - z}, \quad P_{g \rightarrow g}(z) = N_C \frac{(1 - z(1 - z))^2}{z(1 - z)}
\]

⇒ Parton shower MC
Sudakov Form Factor

Variable \( t = \ln \left( \frac{Q^2}{\Lambda^2} \right) \), \( Q^2 \sim \frac{E_q E_g}{\theta_{qg}^2} \) is like a time-ordering

\[
d\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a \to bc}(z) \, dt \, dz
\]

\[
\mathcal{I}_{a \to bc}(t) = \int_{z_-(t)}^{z_+(t)} dz \frac{\alpha_{abc}}{2\pi} P_{a \to bc}(\hat{z})
\]

Probability for no emission in \( (t, t + \delta t) \):

\[
1 - \sum_{b,c} I_{a \to bc}(t) \, \delta t
\]

Over a longer time period, product of no-emission prob’s exponentiates:

\[
\mathcal{P}_{\text{no}}(t_0, t) = \exp \left\{ - \int_{t_0}^{t} dt' \sum_{b,c} \mathcal{I}_{a \to bc}(t') \right\} = S_a(t) = \frac{\Delta(t, t_c)}{\Delta(t_0, t_c)}
\]

Notation: \( S_a(t) \) for Pythia, \( \Delta(t, t_c) \) for Herwig
Sudakov Form Factor

Actual probability that a branching of $a$ occurs at $t$ is:

$$\frac{dP_a}{dt} = - \frac{dP_{no}(t_0, t)}{dt} = \left( \sum_{b,c} I_{a \rightarrow bc}(t) \right) \exp \left\{ - \int_{t_0}^{t} dt' \sum_{b,c} I_{a \rightarrow bc}(t') \right\}$$

Like Radioactive Decay! $S_a(t) = P_{no}(t_0, t)$ is referred to as the Sudakov form factor.

- Leading Logarithms
- Collinear splitting prob.
- Exact E-p

Interplay between real and virtual emissions

Below resolution scale, cancellation of singularities, leaving a finite remnant
Evolution of the parton shower

Continue emissions with decreasing $t$ down to the cutoff scale $\sim \Lambda_{QCD}$

$t_1 > t_2 > t_3 > t_c$

$t_c \rightarrow \Lambda_{QCD}$

Make transition to a model of hadronization at $\Lambda_{QCD}$
Color Coherence

In previous discussion of PS, interference effects were ignored, but they can be relevant.

Add a soft gluon to a shower of $N$ almost collinear gluons:

- incoherent emission: couple to all gluons
  
  $|\mathcal{M}_{N+1}|^2 \sim N \times \alpha_s \times N_C$

- coherent emission: soft (=long wavelength) resolves only overall color charge (that of initial gluon)
  
  $|\mathcal{M}_{N+1}|^2 \sim 1 \times \alpha_s \times N_C$
Color Coherence Realized

Showers should be Angular-Ordered

Keep same picture but change $Q^2 \rightarrow E^2 \zeta$

$$\zeta = \frac{p_i \cdot p_j}{E_i E_j} = \left(1 - \cos \theta_{ij}\right) \sim \frac{\theta_{ij}^2}{2}$$

Soft gluon radiation off color lines $i, j$

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{d\Omega}{2\pi} \alpha_s C_{ij} W_{ij}$$

$$W_{ij} = \frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{iq})(1 - \cos \theta_{jq})}$$

$$W_{ij} = W^{[i]} + W^{[j]}$$

$$\int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} W^{[i]} = \frac{1}{1 - \cos \theta_{iq}} \theta[\theta_{ij} - \theta_{iq}]$$

Running coupling should depend on $k_T^2 \sim z(1 - z)Q^2$
Generalised Dipoles

Alternative picture: dipole radiation from color charges

\[ dn = \alpha_{\text{eff}} \cdot \frac{dk^2}{k^2} \cdot dy \cdot (\text{Polarization Sum}) \]

Kinematic Constraints

\[ k_\perp \cosh(y) \leq \frac{\sqrt{s}}{2} \quad (\sqrt{s} \text{ is dipole mass}) \]

rapidity range \( \Delta y \approx \ln \left( \frac{s}{k^2_\perp} \right) \)

- emission of a photon leaves the electromagnetic current unchanged except for small recoil effects
- emission of a gluon changes the current
- However,

\[ dn(q, g_1, g_2, \bar{q}) = dn(q, g_1, \bar{q}) \left[ dn(q, g_2, g_1) + dn(g_1, g_2, \bar{q}) - \epsilon \right] \]
Generalised Dipoles

The emission of the first gluon splits the original color dipole into two dipoles which radiate independently.

Shower can be traced in origami diagram:

\[ \kappa = \ln(k^2_T) \]

1. Before emission
2. 1st emission at \( \kappa_1 \)
3. After several emissions
4. Bottom view
Initial State Radiation

In hadronic collisions, incoming partons can also radiate

\[ p_1 \rightarrow p_2 + k, \quad p_1^2 = p_2^2 = 0 \Rightarrow k^2 = (p_1 - p_2)^2 = -2p_1 \cdot p_2 < 0 \]

Backwards (from hard scatter) evolution of partons with virtualities increasing \( \rightarrow 0 \)

Since backwards, must normalize to the incoming flux of partons (PDF)

- Collinear parton shower obeys DGLAP evolution
- Weight Sudakov:

\[
\frac{f_i(x, Q_{lo}^2)}{f_i(x, Q_{hi}^2)}
\]
NLL Jet Structure

Typically, parton shower has complicated constraints/integrands and are evaluated numerically.

However, with some simplifications, can calculate NLL Sudakovs.

Evolution in \( k_{Tij}^2 = 2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij}) \)

Analytic Sudakov form factors:

\[
\Delta_q(Q_1, Q) = \exp \left( -\int_{Q_1}^Q dq \, \Gamma_q(q, Q) \right) \\
\Delta_g(Q_1, Q) = \exp \left( -\int_{Q_1}^Q dq \, [\Gamma_g(q, Q) + \Gamma_f(q)] \right)
\]

\[
\Gamma_q(q, Q) = \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q} \left( \ln \frac{Q}{q} - \frac{3}{4} \right) \\
\Gamma_g(q, Q) = \frac{2CA}{\pi} \frac{\alpha_s(q)}{q} \left( \ln \frac{Q}{q} - \frac{11}{12} \right) \\
\Gamma_f(q) = \frac{N_f}{3\pi} \frac{\alpha_s(q)}{q}
\]
Jet Rates starting from $Z \rightarrow q\bar{q}$

2 jet rate: neither $q\bar{q}$ radiates

$$R_2(Q_1, Q) = [\Delta_q(Q_1, Q)]^2$$

3 jet rate: one intermediate state

$$R_3(Q_1, Q) = 2 [\Delta_q(Q_1, Q)]^2 \int_{Q_1}^{Q} dq \, \Gamma_q(q, Q) \Delta_g(Q_1, q)$$

The overall NLL probability is

$$\Delta_q(Q_1, Q) \frac{\Delta_q(Q_1, Q)}{\Delta_q(Q_1, \bar{q})} \Gamma_q(q, Q) \Delta_q(Q_1, \bar{q}) \Delta_g(Q_1, q)$$

$$= \Gamma_q(q, Q) [\Delta_q(Q_1, Q)]^2 \Delta_g(Q_1, q)$$

Add $\bar{q}$ contribution and integrate over $Q_1 < q < Q$ to get $R_3$
Jet Rates starting from $\mathcal{Z} \rightarrow q\bar{q}$

- $N$ limited only by phase space
- All topologies generated in Parton Shower
- Rates given by dominant soft/collinear regions
- Contrast with ME
  - One topology must be specified
  - No soft/collinear approximations, valid for large $E, \theta$
**NOT an Event Generator**

By the end of the parton shower, we have nearly exhausted our ability to apply perturbation theory. This is still not enough:

+ Have a description of jet structure
+ Can ask questions about energy flow and isolation
+ See if kinematic features survive
- Don’t know response of detector to a soft quark/gluon
- Cannot tag a $b$ quark
- Can’t ask about charged tracks or neutrals

**Next step is into the Brown Muck**
Parton Shower Summary

Modern PS models are very sophisticated implementations of perturbative QCD

Derived from factorization theorems of full gauge theory

- Accelerated charges radiate (QED/QCD)
- Gluons have color and radiate as well
- Parton Shower development encoded in Sudakov FF
- Performed to LL and some sub-LL accuracy with exact kinematics
- Color coherence leads to angular ordering of shower
- Still need hadronization models to connect with data
- Shower evolves virtualities of partons to a low enough values where this connection is possible
Lecture 2

• Hadronization
  string
  cluster

• Underlying Event
  parametrizations
  multiple-interactions

• The Event Generator Programs

• New Developments
Hadronization

QCD partons are free only on a **very short time scale**

**Hadrons** are the physical states of the strong interaction

Need a description of how partons are confined

Lacking a theory, we need a model

- **enough** variables to fit data
- **few enough** that there is some predictability
- start related to the end of the parton shower
- Use **basic** understanding of QCD
QCD is a confining theory

- Linear potential $V_{\text{QCD}}(r) \sim kr$
  - Confirmed by Lattice, Spectroscopy, Regge Trajectories
- Gluons are self-coupling
  - Field lines contract into Flux-tubes
  - Analogy with field behavior inside of superconductors
- Over time, 2 phenomenological models have survived
  - cluster
  - Lund string
- Not exactly Orthogonal, Exhaustive
Necrologos: Independent Fragmentation

- FF = Feynman-R. Field
- pure phenomenological model
- imagine $q\bar{q}$ pairs tunnel from the vacuum to dress bare quark
- $f_{q\rightarrow h}(z)$ is probability $q \rightarrow h$ with fraction $z$ of some $E/p$ variable
- $f_{g\rightarrow h}(z)$? $g \rightarrow q\bar{q}$?
- Lorentz invariant? ($E_q$)
- Useful for its time

**FF:** $f(z) = 1 - a + 3a(1 - z)^2$
**Preconfinement**

**Perturbative** evolution of quarks and gluons organizes them into clumps of color-singlet clusters.

In PS, color-singlet pairs end up close in phase space.

- Cluster model takes this view to the extreme.
- Non-perturbative splitting:
  \[ \text{gluon} \Rightarrow \text{color-anticolor pair} \]
- Parton shower **cutoff** is a critical parameter.
Cluster hadronization in a nutshell

- Nonperturbative $g \rightarrow q\bar{q}$ splitting ($q = uds$) isotropically
  Here, $m_g \approx 750 \text{ MeV} > 2m_q$.

- Cluster formation, universal spectrum

- Cluster fission until

  $$M^p < M_{\text{fiss}}^p = M_{\text{max}}^p + (m_{q1} + m_{q2})^p$$

  where masses are chosen from

  $$M_i = 
  [(M^P - (m_{qi} + m_{q3})^P) r_i + (m_{qi} + m_{q3})^P]^{1/P}$$

  with additional phase space constraints

- Cluster decay

  isotropically into pairs of hadrons

  simple rules for spin, species
Cluster Fission

- Mass spectrum of color-singlet pairs asymptotically independent of energy, production mechanism
  - Peaked at low mass
  - Broad tail at large mass

Small fraction of clusters heavier than typical

⇒ Cluster fission (string-like)

Fission threshold becomes crucial parameter

15% of primary clusters split
produces 50% of hadrons
Lund String Model

String = color flux tube is stretched between $q$ and $\bar{q}$

Classical string will oscillate in space-time

Endpoints $q, \bar{q}$ exchange momentum with the string

**Quantum Mechanics:** string energy can be converted to $q\bar{q}$ pairs (tension $\kappa \sim 1$ GeV/fm)

$$d\text{Prob}/dx/dt = (\text{constant})\exp(-\pi m^2/\kappa) \quad [\text{WKB}]$$

- $u : d : s : qq = 1 : 1 : 0.35 : 0.1$

**Area** (swept out by string) Law

$$dP_n(\{p_j\}; P_{tot}) = \prod_{j=1}^{n} N_j d^2 p_j \delta (p_j^2 - m_j^2) \delta (\sum_{j=1}^{n} p_j - P_{tot}) \exp(-bA)$$
Hadron Formation

Adjacent breaks form a hadron

\[ E_{\text{had}} = \kappa |x_i - x_{i+1}| \quad \vec{p}_{\text{had}} = \vec{p}_T + \vec{\kappa}(t_i - t_{i+1}) \]

\[ m_{\text{had}}^2 \propto \text{area swept out by string} \]
Iterative Solution

String breaking and hadron formation can be treated as an iterative process

Use light-cone coordinates \( x^\pm = x \pm t \)

Boundary Conditions:

\[
x_0^+ = 2E_0/\kappa, \quad x_{n+1}^- = 2\bar{E}_0/\kappa, \quad x_0^- = x_{n+1}^+ = 0
\]

1. select \( z_i \) according to \( f(z)dz \)
   - \( f^h(z, p_T) \sim \frac{1}{z}(1 - z)^a \exp\left[-\frac{b(m_h^2 + p_T^2)}{z}\right] \)

2. \( \Delta x^+ = (x_{i-1}^+ - x_i^+) = z_i x_{i-1}^+ \)

3. \( \Delta x^- = (x_{i-1}^- - x_i^-) = \frac{-m_i^2}{\kappa^2 \Delta x^+} \)
   - mass\(^2\) of hadron \( \propto \Delta x^+ \Delta x^- \)

4. Continue until string is consumed
Example Break-Up

Tunnelling of pairs leads to a string of hadrons

\[ \bar{u} \quad d \quad \bar{d} \quad d \quad \bar{d} \quad s \quad \bar{s} \quad d \quad \bar{d} \quad u \quad \bar{u} \quad \bar{u}d_0 \quad ud_0 \quad s \quad \bar{s} \quad u \]

\[ \rho^- \quad \omega \quad \bar{K}^{*0} \quad K^0 \quad \pi^+ \quad \bar{p} \quad \Lambda \quad K^+ \]

Simple rules for spin, isospin, etc.

Note diquarks

Algorithm can start from either end of string
Perturbative Parton Shower generates gluons 
(color-anticolor pair $\dagger$ correction in $1/N_C$ expansion)

Gluon = kink on string, i.e. some motion to system

String effect $\Rightarrow$ particles move in direction of kink
Infrared Stability

“Hard” gluon imparts motion to string system

Soft or collinear gluons collapse into simpler string

$q$ and $\bar{q}$ give bits of momentum to $g$ and vice versa

Parton that gives up all its energy is still dragged along
## Hadronization Overview

<table>
<thead>
<tr>
<th>Clusters (Herwig)</th>
<th>Strings (Pythia, Ariadne)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• perturbation theory can be applied down to low scales if the coherence is treated correctly</td>
<td>• dynamics of the non-perturbative phase must be treated correctly</td>
</tr>
<tr>
<td>• There must be non-perturbative physics, but it should be very simple</td>
<td>• Model includes some non-perturbative aspect of color (interjet) coherence (string effect)</td>
</tr>
<tr>
<td>• Improving data has meant successively making non-pert phase more string-like</td>
<td>• Improving data has meant successively making non-pert phase more cluster-like</td>
</tr>
</tbody>
</table>
Underlying Event

- Hadrons (protons) are extended objects
- Remnant remains after hard partons scatter
- Need a description of how partonic remnants are confined

Two Approaches

1. Soft parton-parton collisions dominate
2. Semi-Hard parton-parton cross section is large and can be calculated even at low $p_T$
Soft Underlying Event

UA5 Monte Carlo

- hadron-hadron scattering produces two leading clusters and several central ones
- parametrize $N_{ch}$ and sample
- clusters given $p_T$ and $y$ from some distribution
  \[
  \frac{dN}{dp_T^2} \sim e^{-bp_T}, \frac{1}{(p_T + p_0)^n}
  \]
  \[y \sim \text{flat with Gaussian tails}\]
- $p_L = m \sinh(y)$

Herwig adds in their cluster model

UE model is a mechanism for producing the objects used in description of hadronization
Consequence of composite nature of hadrons!

Evidence:
- direct observation: AFS, UA1, CDF
- implied by width of multiplicity distribution + jet universality: UA5
- forward–backward correlations: UA5
- pedestal effect: UA1, H1, CDF

One new free parameter: \( p_{\perp \text{min}} \)

\[
\frac{1}{2} \sigma_{\text{jet}} = \int_{p_{\perp \text{min}}}^{s/4} \frac{d\sigma}{dp_{\perp}^2} dp_{\perp}^2 \quad \leftrightarrow \quad \int_{0}^{s/4} \frac{d\sigma}{dp_{\perp}^2} \frac{p_{\perp}^4}{(p_{\perp 0}^2 + p_{\perp}^2)^2} dp_{\perp}^2
\]

Measure of colour screening length \( d \) in hadron:
\[ p_{\perp \text{min}} \langle d \rangle \approx 1 (= \bar{h}) \]
Multiple Interaction Model

\[ \bar{n} = \sigma_{\text{hard}}(p_{\perp \text{min}})/\sigma_{\text{nd}}(s) > 1 \]

Not a violation of unitarity! \( \sigma_{\text{hard}} \) is inclusive

naive estimate: no E-p conservation

- must consider interactions of all partons

On average, \( \bar{n} \) semi-hard interactions in one hard collision

Collisions ranked in \( x_{\perp} = 2p_{\perp}/E_{\text{cm}} \), produced with prob

\[ f(x_{\perp}) = \frac{1}{\sigma_{\text{nd}}(s)} \frac{d\sigma}{dx_{\perp}} \]

The probability that the hardest interaction is at \( x_{\perp 1} \):

\[ f(x_{\perp 1}) \exp \left\{ - \int_{x_{\perp 1}}^{1} f(x_{\perp}') \, dx_{\perp}' \right\} \]

- like radioactive decay
Multiple Interaction Model

generate a chain of scatterings \( 1 > x_{\perp 1} > x_{\perp 2} > \cdots > x_{\perp i} \)
using \( x_{\perp i} = F^{-1}(F(x_{\perp i-1}) - \ln R_i) \)

- \( F(x_\perp) = \int_{x_\perp}^{1} f(x_\perp') \, dx_\perp' = \frac{1}{\sigma_{nd}(s)} \int_{sx_{\perp i}^2/4}^{s/4} \frac{d\sigma}{dp_{\perp}^2} \, dp_{\perp}^2 \)

**Refinements**

Parton \( x_i \) for PDF evaluation is rescaled \( x_i' = \frac{x_i}{\sum_{j=1}^{i-1} x_j} \)

Almost no experimental information on correlated PDFs

Include "Matter" distributed in hadrons

\( \mathcal{O}(b) \propto \int dt \int d^3x \, \rho(x, y, z) \rho(x + b, y, z + t) \)

\( \langle N_{int} \rangle \sim k \, \mathcal{O}(b) \)
Strings and the UE

Each additional interaction adds more color flow

- Color information encoded in strings
- Subsequent interactions mainly reconnect to pre-existing strings
- Fits prefer a minimization of total string length
Pythia Options

MSTP(82) :

(D=1) structure of multiple interactions. For QCD processes, used down to values below, it also affects the choice of structure for the one hard/semi-hard interaction.

= 0 :
  simple two-string model without any hard interactions. Toy model only!
= 1 :
  multiple interactions assuming the same probability in all events, with an abrupt cut-off at PARP(81). (With a slow energy dependence given by PARP(89) and PARP(90).)
= 2 :
  multiple interactions assuming the same probability in all events, with a continuous turn-off of the cross section at PARP(82). (With a slow energy dependence given by PARP(89) and PARP(90).)
= 3 :
  multiple interactions assuming a varying impact parameter and a hadronic matter overlap consistent with a Gaussian matter distribution, with a continuous turn-off of the cross section at PARP(82). (With a slow energy dependence given by PARP(89) and PARP(90).)
= 4 :
  multiple interactions assuming a varying impact parameter and a hadronic matter overlap consistent with a double Gaussian matter distribution given by PARP(83) and PARP(84), with a continuous turn-off of the cross section at PARP(82). (With a slow energy dependence given by PARP(89) and PARP(90).)
Pythia at Run2: Underlying Event

**Table: Parameters and Tuning**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default</th>
<th>Tune</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARP(67)</td>
<td>1.0</td>
<td>4.0</td>
<td>Scale factor for ISR</td>
</tr>
<tr>
<td>MSTP(82)</td>
<td>1.0</td>
<td>4</td>
<td>Double Gaussian matter distribution</td>
</tr>
<tr>
<td>PARP(82)</td>
<td>1.9</td>
<td>2.0</td>
<td>Cutoff (GeV) for MPIs</td>
</tr>
<tr>
<td>PARP(83)</td>
<td>0.5</td>
<td>0.5</td>
<td>Warm Core with % of matter within a given radius</td>
</tr>
<tr>
<td>PARP(84)</td>
<td>0.2</td>
<td>0.4</td>
<td>Prob. that two gluons have NNC</td>
</tr>
<tr>
<td>PARP(85)</td>
<td>0.33</td>
<td>0.9</td>
<td>gg versus qq</td>
</tr>
<tr>
<td>PARP(86)</td>
<td>0.66</td>
<td>0.95</td>
<td>Reference energy (GeV)</td>
</tr>
<tr>
<td>PARP(89)</td>
<td>1000.0</td>
<td>1800.0</td>
<td>Power of Energy scaling for cutoff</td>
</tr>
</tbody>
</table>

**Graph: Charge Density**

- AVE Transverse Charge Density: \( dN/d\eta d\phi \)
- 1.96 TeV
- Charged Particles \(|\eta|<1.0, PT>0.5\) GeV/c
- ET(jet#1) (GeV)

**Legend:**
- PY Tune A
- Leading Jet
- HW
- Back-to-Back

**Graph: Jet Clustering**

- Jet Clustering #1 Direction
- \( \Delta \phi \)
- Toward
- Transverse
- “Away”
# The Parton Shower Programs

## Programs

<table>
<thead>
<tr>
<th>PS Ordering</th>
<th>Mass</th>
<th>Angle</th>
<th>$k_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle veto</td>
<td></td>
<td></td>
<td></td>
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<tr>
<th>Hadronization</th>
<th>String</th>
<th>Cluster</th>
<th>String</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Underlying Event</th>
<th>Mult. Int</th>
<th>UA6/(Jimmy)</th>
<th>LDCM</th>
</tr>
</thead>
</table>

Finding them: [http://cepa.fnal.gov/mrenna/generator.html](http://cepa.fnal.gov/mrenna/generator.html)

- [http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html](http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html)
- [http://hepwww.rl.ac.uk/theory/seymour/herwig/](http://hepwww.rl.ac.uk/theory/seymour/herwig/)
- [http://www.thep.lu.se/~leif/ariadne/](http://www.thep.lu.se/~leif/ariadne/)

## Fortran codes

[http://www.ibiblio.org/pub/languages/fortran/ch1-1.html](http://www.ibiblio.org/pub/languages/fortran/ch1-1.html)
“The Future” ≡ C++ ≡ ThePEG

Toolkit for high energy Physics Event Generation
http://www.thep.lu.se/ThePEG/

Share administrative overhead common to event generators

Independent *physics* implementation

Common basis for Pythia7/Herwig++

- Lack of independence.
- Less possibility to test codes against each other.
- Physics is still independent.
- Beneficial for the user to have the same framework.
- Use Herwig++ with String Fragmentation from Pythia7
Sherpa is also C++ event generator in a different framework
Developments

- Improved showers for Pythia and Herwig
- More detailed models of Underlying Event
- New programs for automating Tree-Level calculations
- Event generators for NLO calculations
- Matrix Element and Parton Shower Matching
New ME Programs

Automatically calculate code needed for a given HEP process and generate events

List of those actively supporting hadron colliders

- Alpgen [http://m.home.cern.ch/m/mlm/www/alpgen/](http://m.home.cern.ch/m/mlm/www/alpgen/)
- CompHep [http://theory.sinp.msu.ru/comphep](http://theory.sinp.msu.ru/comphep)
- MadEvent [http://madgraph.hep.uiuc.edu/index.html](http://madgraph.hep.uiuc.edu/index.html)
- Sherpa/Amegic++ [http://141.30.17.181/](http://141.30.17.181/)

Advantages and disadvantages of each

An impressive improvement from several years ago
Next-to-Leading-Order Calculations give an improved description of the hard kinematics and cross sections not event generators

Solution (MC@NLO):
Remove divergences by adding and subtracting the Monte Carlo result for one emission
Event Generators for Many Legs

Want to exploit tools for generating tree level diagrams

Each topology (e.g. $W + 0, 1, 2, 3, 4$ partons) has no soft/collinear approximation

How do I add a parton shower to each topology with no double counting?

Solution (CKKW):

1. Make the $|M|^2$ result “look” like a parton shower down to a reasonable cutoff scale ($k_T^{\text{cut}}/Q_{\text{hard}} \sim .1$)

2. Add on ordinary parton shower below $K_T^{\text{cut}}$

$$k_T^2 = 2\min(E_i, E_j)^2(1 - \cos \theta_{ij}) \sim \min(E_i/E_j, E_j/E_i)m^2$$
W+0 ⊕ ⋯ ⊕ W+4 hard partons

Dashed is Pythia with default (ME) correction

Solid is Pseudoshower result

Combines ME contributions \((0, 1, 2, 3, 4\) partons)
Overall Summary

- Event Generators accumulate our understanding of the Standard Model into one package
- Apply perturbation theory whenever possible
  - hard scattering, parton showering, decays
- Rely on models or parametrizations when present calculational methods fail
  - hadronization, underlying event, beam remnants
- Out of the box, they give reliable estimates of the full, complicated structure of HEP events
- Attentive users will find more flexibility & applications
- Understanding the output can lead to a broader understanding of the Standard Model (and physics beyond)
MC4Run2: MC/ME Tuning

http://cepa.fnal.gov/patriot/mc4run2

• Forum for CDF, DØ, & Theorists
  – LHC participation is welcome/encouraged

• Discussions/Presentations on:
  – Underlying Event Tunes
  – $B$ Production Tunes
  – Parton Shower Matching
  – PDF systematics
    * LHAPDF interface ala PDFLIB
  – Herwig/Pythia Problems with Photon-Jet Balancing
  – Non-perturbative corrections to Jet Cross Sections
  – ResBos-A
  – ...