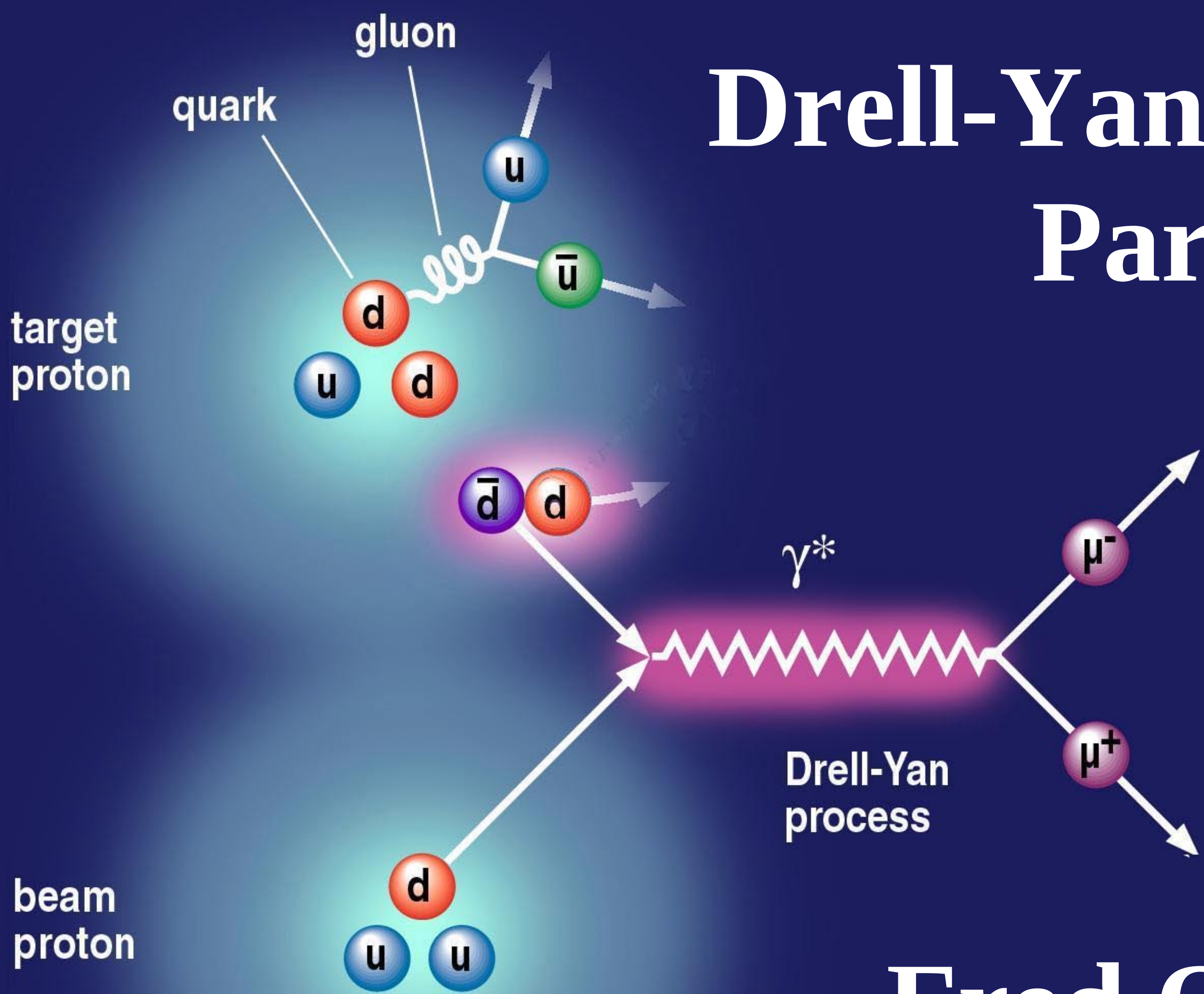
# Drell-Yan Process: Part I




## Fred Olness
## SMU

# Part I: Drell-Yan Process

**History:**

Discovery of J/ψ, Upsilon, W/Z, and "New Physics" ???

**Calculation of** $q\ q \to \mu^+\mu^-$ **in the Parton Model**

Scaling form of the cross section

Rapidity, longitudinal momentum, and $x_F$

**Comparison with data:**

NLO QCD corrections essential (the K-factor)

$\sigma(pd)/\sigma(pp)$ important for d-bar/ubar

W Rapidity Asymmetry important for slope of d/u at large x

**Where are we going?**

$P_T$ Distribution

W-mass measurement

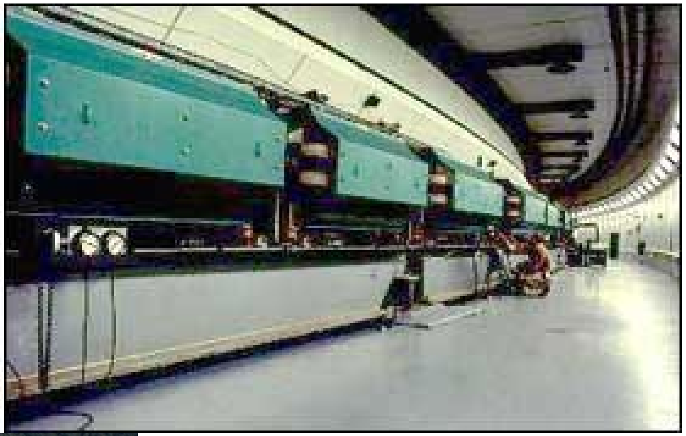
Resummation of soft gluons

# Historical

# Background

# Our story begins in the late 1960's

# Brookhaven National Lab
# Alternating Gradient Synchrotron

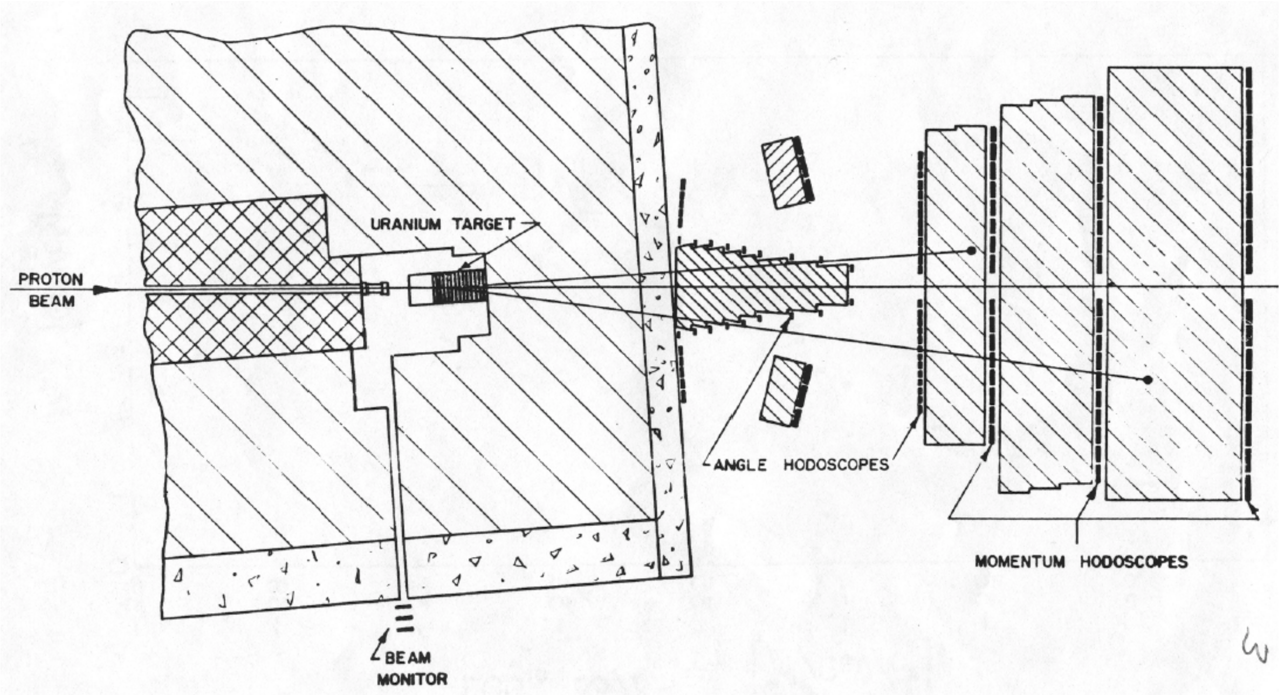


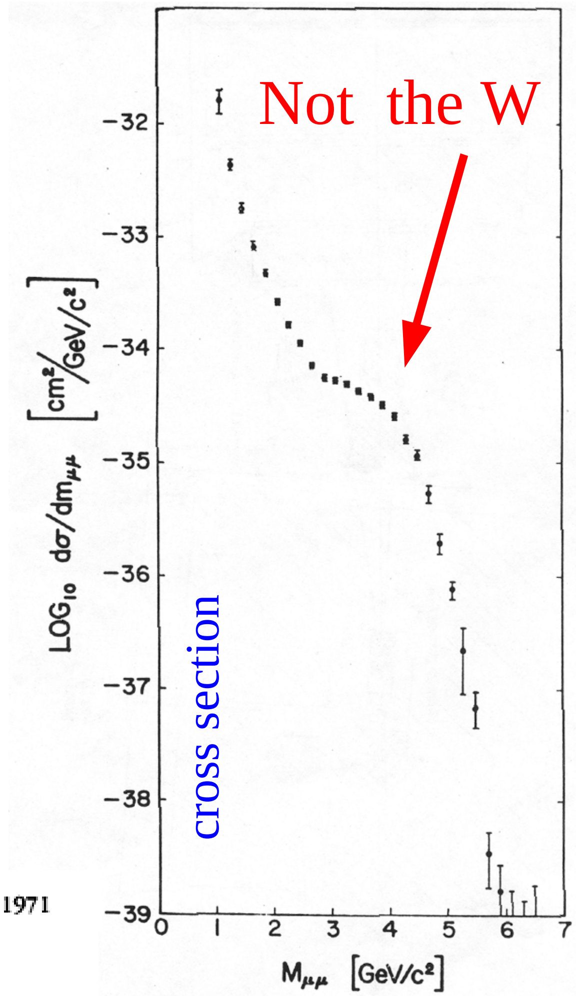
# An Early Experiment:

**The Goal:** $p + N \rightarrow W + X$

**They found:** $p + N \rightarrow \mu^+ \mu^- + X$

at BNL AGS




$M_{\mu\mu}$ GeV


VOLUME 27, NUMBER 11 PHYSICAL REVIEW LETTERS 13 SEPTEMBER 1971


**Production of Intermediate Bosons in Strong Interactions***


L. M. Lederman and B. G. Pope†
*Columbia University, New York, New York 10533*



(Received 14 June 1971)

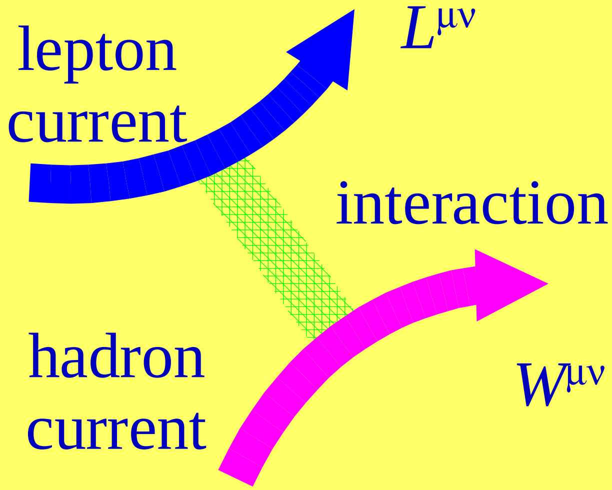

Several searches for the weak intermediate boson ($W$) have been carried out using the reaction

$$p + Z \rightarrow W + \text{anything}, \qquad (1)$$

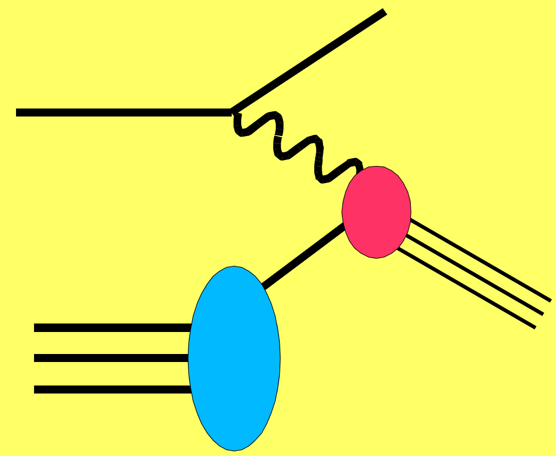
with the decay of the $W$ into muons as the signature.[1,2] Failure to observe a muon signal from any

# What is the explanation???

In DIS, we have two choices for an interpretation:



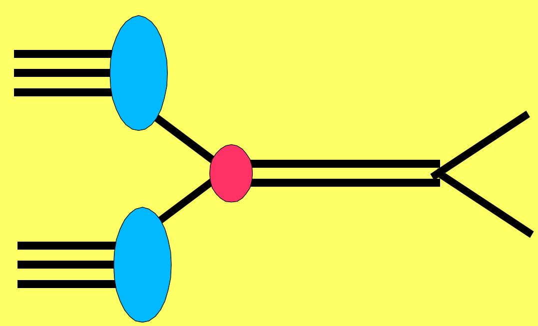

Conserved Current Interactions



The Parton Model

What about Drell-Yan???

? ? ?

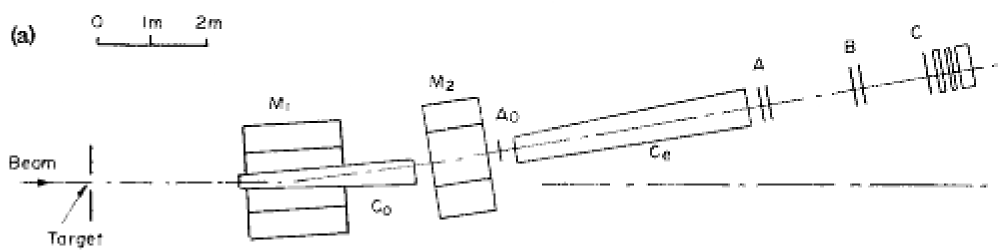


The Parton Model

# Discovery of the J/Psi Particle

## The Process: $p + Be \rightarrow e^+ e^- \ X$

very narrow width
$\Rightarrow$ long lifetime




at BNL AGS

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECE

### Experimental Observation of a Heavy Particle $J$†


J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen
J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan
*Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technolog*
*Cambridge, Massachusetts 02139*

and

Y. Y. Lee
*Brookhaven National Laboratory, Upton, New York 11973*


(Received 12 November 1974)

We report the observation of a heavy particle $J$, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \rightarrow e^+ + e^- + x$ by measuring the $e^+e^-$ mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

This experiment is part of a large program to ... daily with a thin Al foil. The beam spot

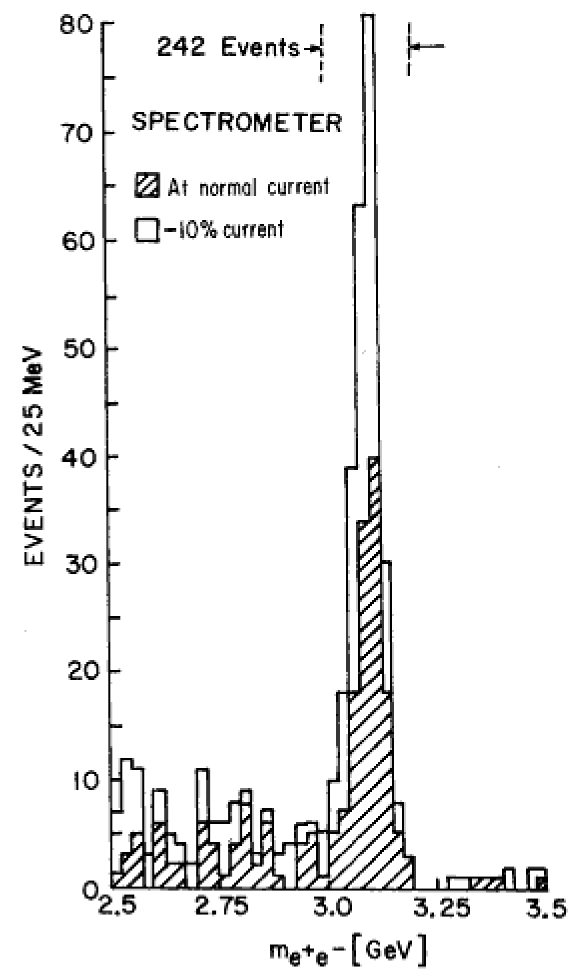
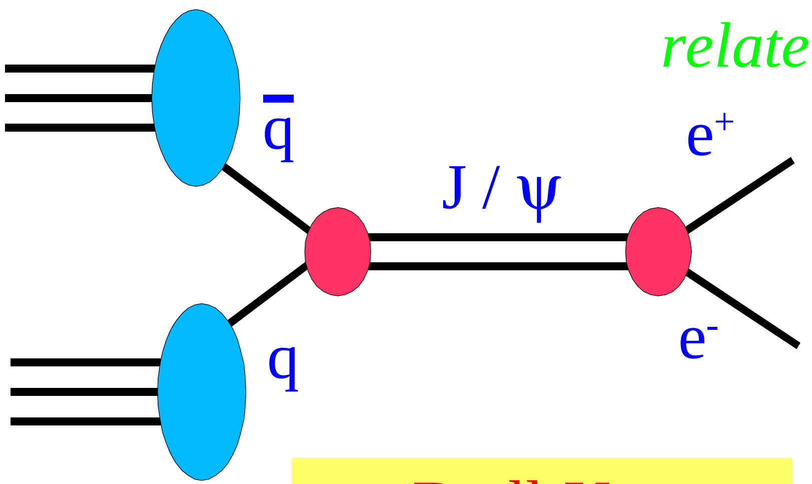
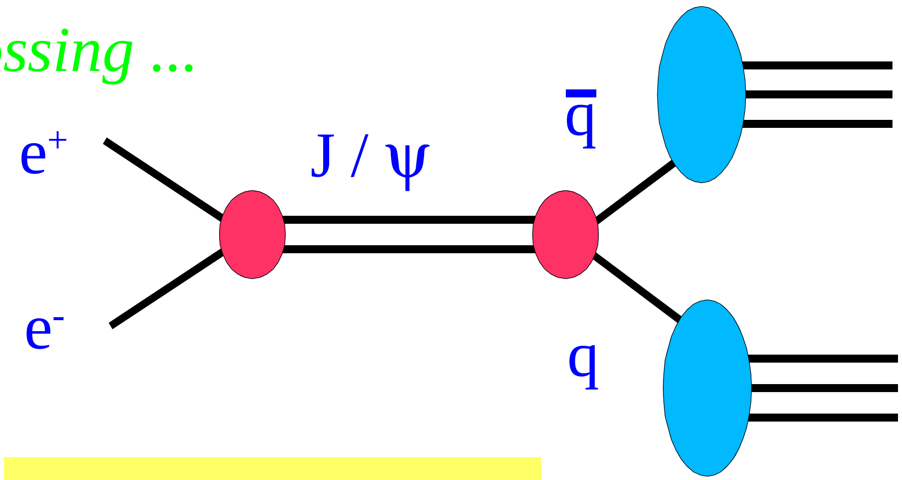



FIG. 2. Mass spectrum showing the existence of $J$. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

# The November Revolution

*related by crossing ...*




**Drell-Yan**
Brookhaven AGS

**$e^+e^-$ Production**
SLAC SPEAR
Frascati ADONE

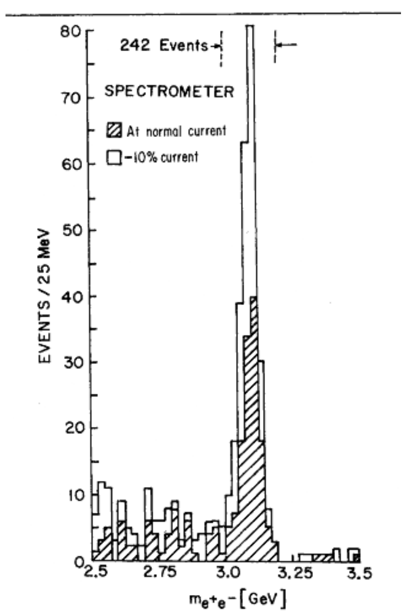
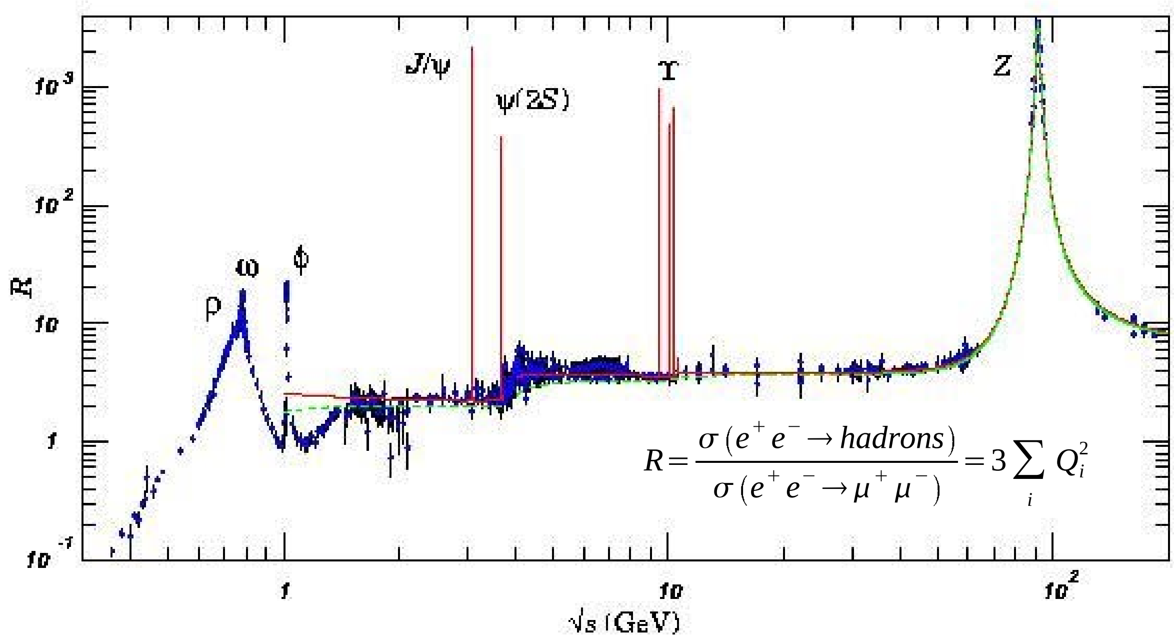



FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.



# More Discoveries with Drell-Yan

1974: The J/Psi (charm) discovery

$p+N \rightarrow J/\psi$

... *1976 Nobel Prize*

1977: The Upsilon (bottom) discovery

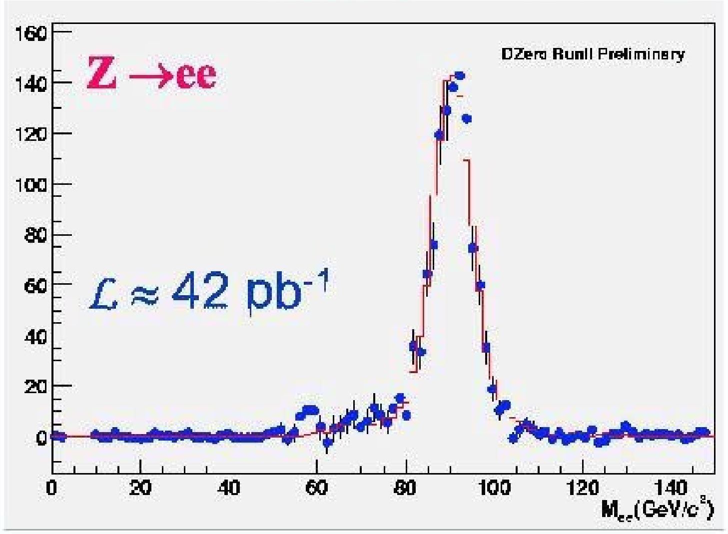
$p+N \rightarrow \Upsilon$

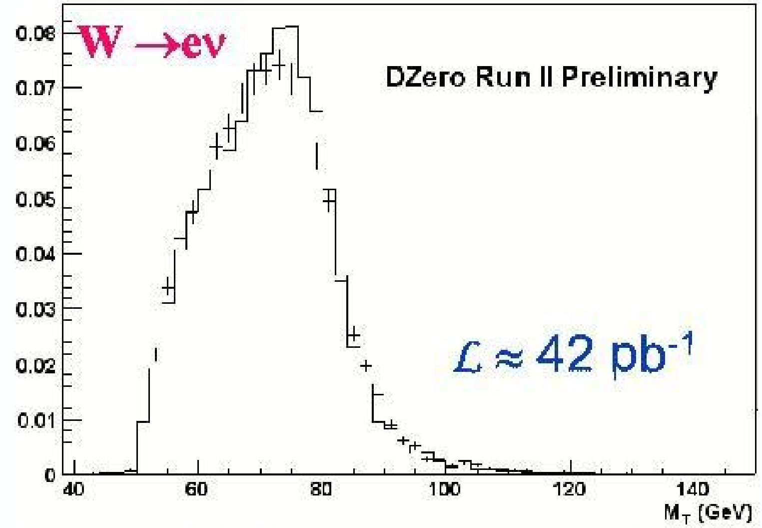
1983: The W and Z discovery

$p + \bar{p} \rightarrow W/Z$

... *1984 Nobel Prize*

# W/Z in the electron channel




- 1139 Z→ee candidates
  - $|\eta^e|<1.1$, $E_T>25$ GeV, no track match required
- $\varepsilon(Z) \approx 8\%$, bkgd ~ 18%

- 27370 W→eν candidates
  - $|\eta^e|<1.1$, $E_T$ & $\not{E}_T$ >25 GeV
- $\varepsilon(W) \approx 16\%$
- bkgd ~ 3% QCD, ~1.5% τ

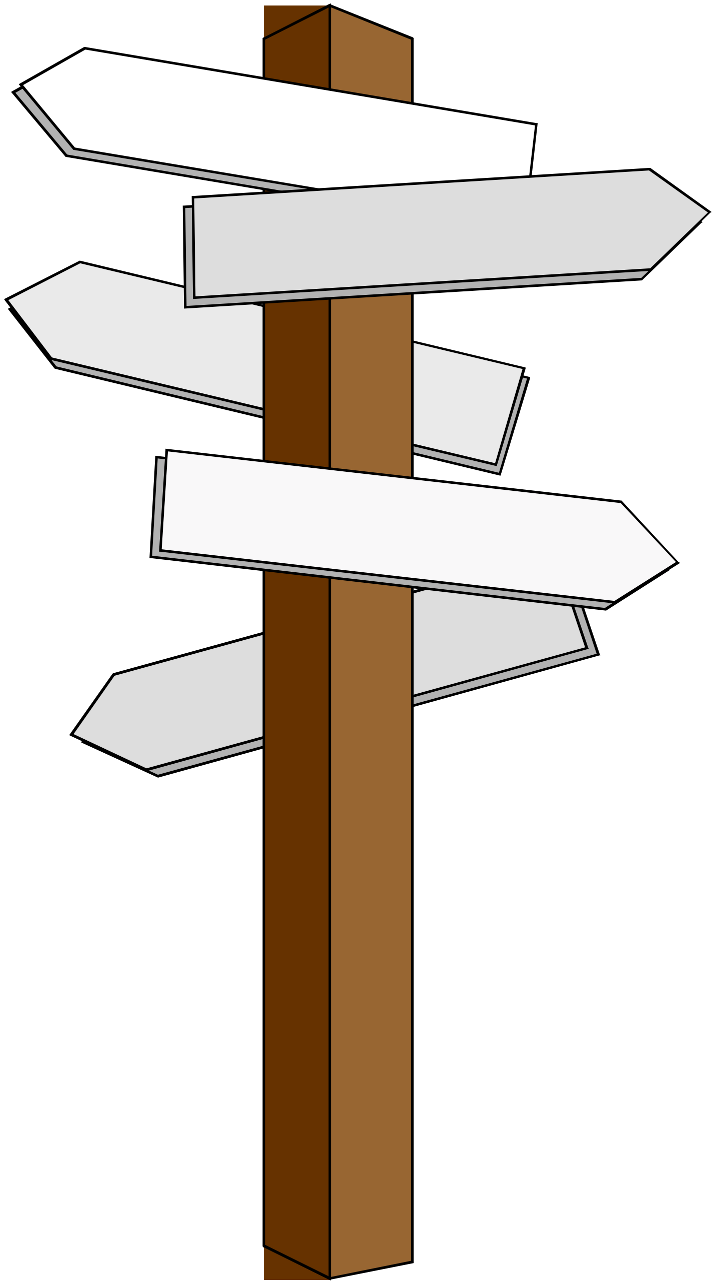
$$\sigma(W)Br(W \rightarrow e\nu) = 3054 \pm 100(N_W) \pm 86(sys) \pm 305(lumi) \quad pb$$

$$\sigma(Z)Br(Z \rightarrow ee) = 294 \pm 11(N_Z) \pm 8(sys) \pm 29(lumi) \quad pb$$

Where do we find

# New Physics??

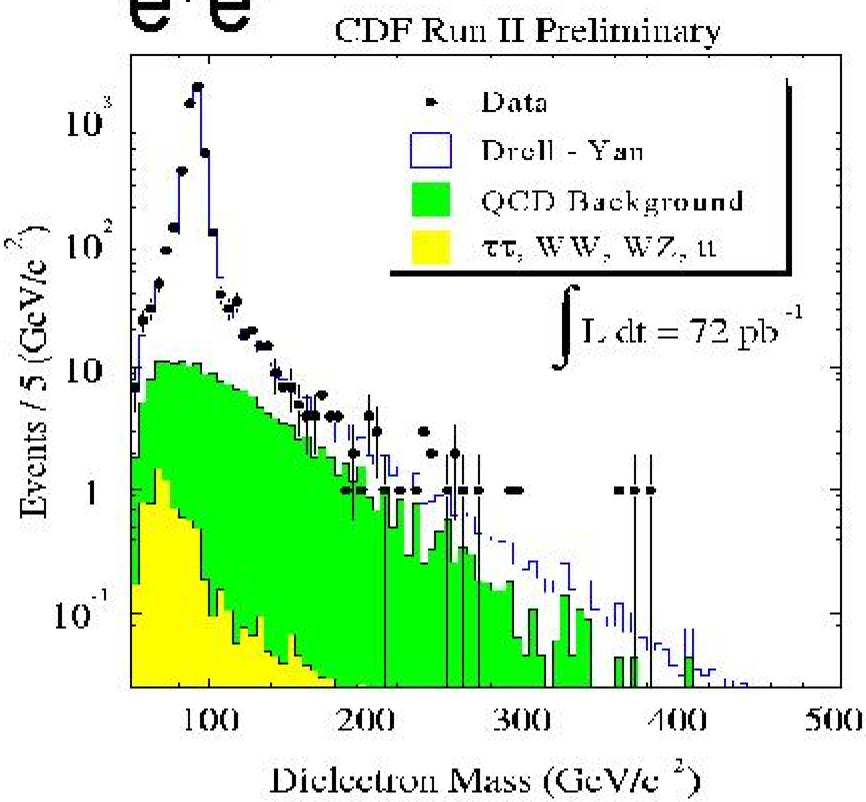
- New Higgs Bosons
- New W' or Z'
- SUSY
- ... unknown...



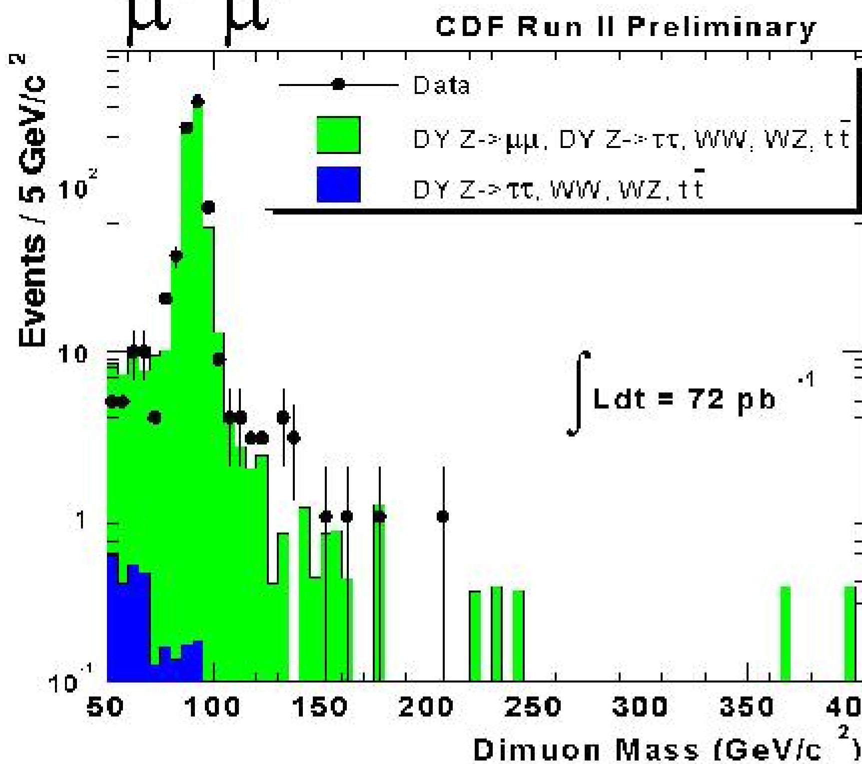
# Search in Drell-Yan Spectrum

- High Mass Dileptons
  - electrons & muons used
- Sensitive to Z' and Randall-Sundrum Graviton
- No excess observed

$e^+e^-$




$\mu^+ \mu^-$

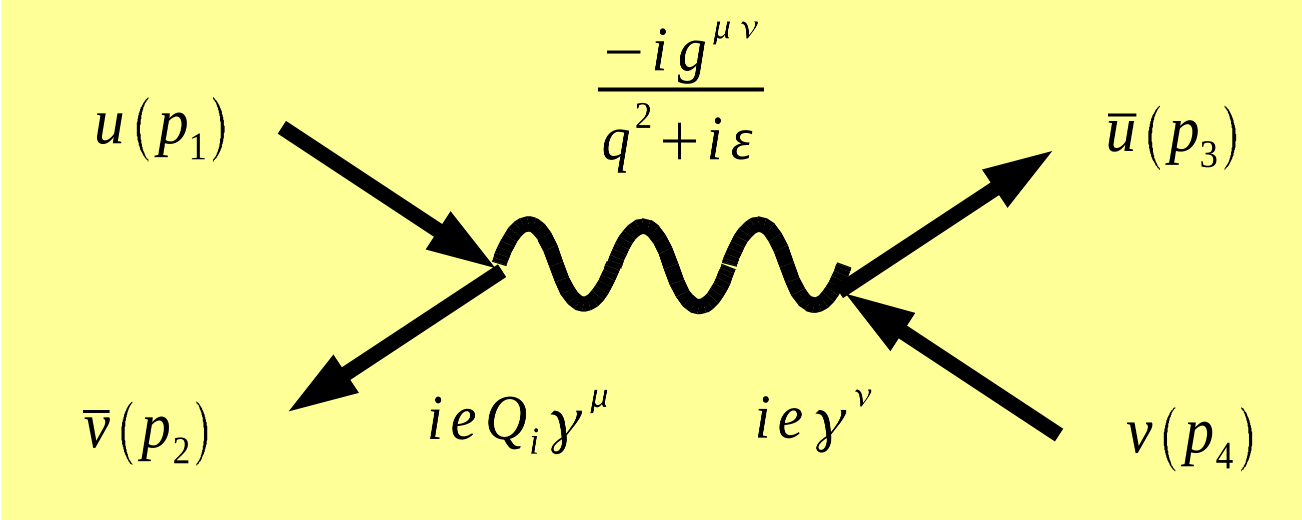


# Let's

# Calculate

First, we'll compute the partonic $\hat{\sigma}$ in the partonic CMS

## **Let's compute the Born process:** $q+\bar{q}\rightarrow e^{+}+e^{-}$



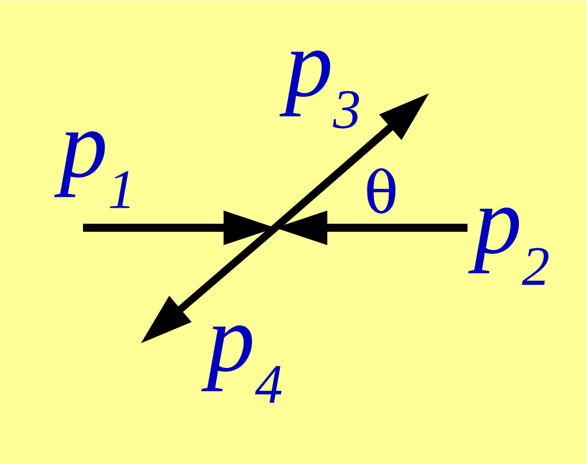

Gathering factors and contracting $g^{\mu\nu}$, we obtain:

$$-i M = i Q_i \frac{e^2}{q^2} \{\bar{v}(p_2)\gamma^{\mu} u(p_1)\} \; \{\bar{u}(p_3)\gamma_{\mu} v(p_4)\}$$

Squaring, and averaging over spin and color, ....

$$\overline{|M|^2} = \left(\frac{1}{2}\right)^2 3\left(\frac{1}{3}\right)^2 Q_i^2 \frac{e^4}{q^4} Tr\left[\not{p}_2 \gamma^{\mu} \not{p}_1 \gamma^{\nu}\right] \; Tr\left[\not{p}_3 \gamma_{\mu} \not{p}_4 \gamma_{\nu}\right]$$

# Let's work out some parton level kinematics




$$p_1 = \frac{\sqrt{\hat{s}}}{2}\left(1,0,0,+1\right)$$
$$p_2 = \frac{\sqrt{\hat{s}}}{2}\left(1,0,0,-1\right)$$
$$p_3 = \frac{\sqrt{\hat{s}}}{2}\left(1,+\sin(\theta),0,+\cos(\theta)\right)$$
$$p_4 = \frac{\sqrt{\hat{s}}}{2}\left(1,-\sin(\theta),0,-\cos(\theta)\right)$$

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

Defining the Mandelstam variables ...

$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$\hat{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$\hat{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

$$\hat{t} = -\frac{\hat{s}}{2}\left(1 - \cos(\theta)\right)$$
$$\hat{u} = -\frac{\hat{s}}{2}\left(1 + \cos(\theta)\right)$$

# We'll now compute the matrix element M

Manipulating the traces, we find ...

$$
\begin{aligned}
&Tr\left[\not{p}_2\gamma^{\mu}\not{p}_1\gamma^{\nu}\right]\ Tr\left[\not{p}_3\gamma_{\mu}\not{p}_4\gamma_{\nu}\right] \\
&= 4\left[p_1^{\mu}p_2^{\nu}+p_2^{\mu}p_1^{\nu}-g^{\mu\nu}(p_1\cdot p_2)\right]\times 4\left[p_3^{\mu}p_4^{\nu}+p_4^{\mu}p_3^{\nu}-g^{\mu\nu}(p_3\cdot p_4)\right] \\
&= 2^5\left[(p_1\cdot p_3)(p_2\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)\right] \\
&= 2^3\left[\hat{t}^2+\hat{u}^2\right]
\end{aligned}
$$

Where we have used:

$$p_1^2=p_2^2=p_3^2=p_4^2=0$$

$$
\begin{aligned}
\hat{s} &= 2(p_1\cdot p_2)=2(p_3\cdot p_4) \\
\hat{t} &= 2(p_1\cdot p_3)=2(p_2\cdot p_4) \\
\hat{u} &= 2(p_1\cdot p_4)=2(p_2\cdot p_3)
\end{aligned}
$$

Putting all the pieces together, we have:

$$\overline{|M|^2}=Q_i^2\,\alpha^2\,\frac{2^5\pi^2}{3}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)$$

with

$$q^2=(p_1+p_2)^2=\hat{s}$$

$$\alpha=\frac{e^2}{4\pi}$$

## ... and put it together to find the cross section

$$d\hat{\sigma} \simeq \frac{1}{2\hat{s}} \overline{|M|^2}\, d\Gamma$$

In the partonic CMS system

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) = \frac{d\cos(\theta)}{16\pi}$$

Recall,

$$\hat{t} = \frac{-\hat{s}}{2}\left(1 - \cos(\theta)\right) \quad and \quad \hat{u} = \frac{-\hat{s}}{2}\left(1 + \cos(\theta)\right)$$

so, the differential cross section is ...

$$\frac{d\hat{\sigma}}{d\cos(\theta)} = Q_i^2 \alpha^2 \frac{\pi}{6} \frac{1}{\hat{s}} \left(1 + \cos^2(\theta)\right)$$

and the total cross section is ...

$$\hat{\sigma} = Q_i^2 \alpha^2 \frac{\pi}{6} \frac{1}{\hat{s}} \int_{-1}^{1} d\cos(\theta) \left(1 + \cos^2(\theta)\right) = \frac{4\pi\alpha^2}{9\hat{s}} Q_i^2 \equiv \hat{\sigma}_0$$

# Some Homework:

#1) Show:

$$\frac{d^3 p}{(2\pi)^3 2E} = \frac{d^4 p}{(2\pi)^4} (2\pi)\, \delta^+(p^2 - m^2)$$

*This relation is often useful as the RHS is manifestly Lorentz invariant*

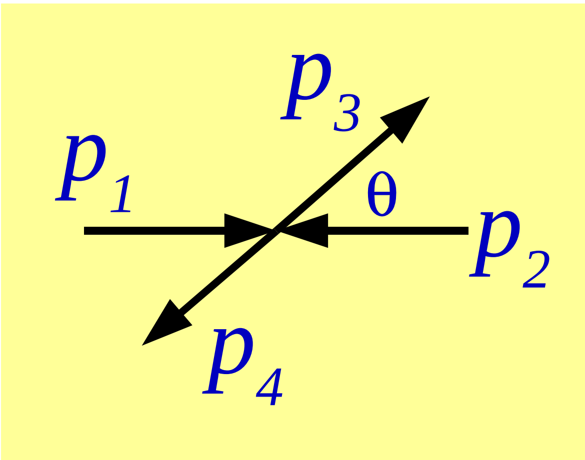
#2) Show that the 2-body phase space can be expressed as:

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4\, \delta(p_1 + p_2 - p_3 - p_4) = \frac{d\cos(\theta)}{16\pi}$$

*Note, we are working with massless partons, and* θ *is in the partonic CMS frame*

# Some More Homework:

#3) Let's work out the general 2→2 kinematics for general masses.




a) Start with the incoming particles.
Show that these can be written in the general form:

$$p_1 = (E_1, 0, 0, +p) \qquad p_1^2 = m_1^2$$
$$p_2 = (E_2, 0, 0, -p) \qquad p_2^2 = m_2^2$$

... with the following definitions:

$$E_{1,2} = \frac{\hat{s} \pm m_1^2 \mp m_2^2}{2\sqrt{\hat{s}}} \qquad p = \frac{\Delta(\hat{s}, m_1^2, m_2^2)}{2\sqrt{\hat{s}}}$$

$$\Delta(a,b,c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)}$$

*Note that $\Delta(a,b,c)$ is symmetric with respect to its arguments, and involves the only invariants of the initial state: s, $m_1^2$, $m_2^2$.*

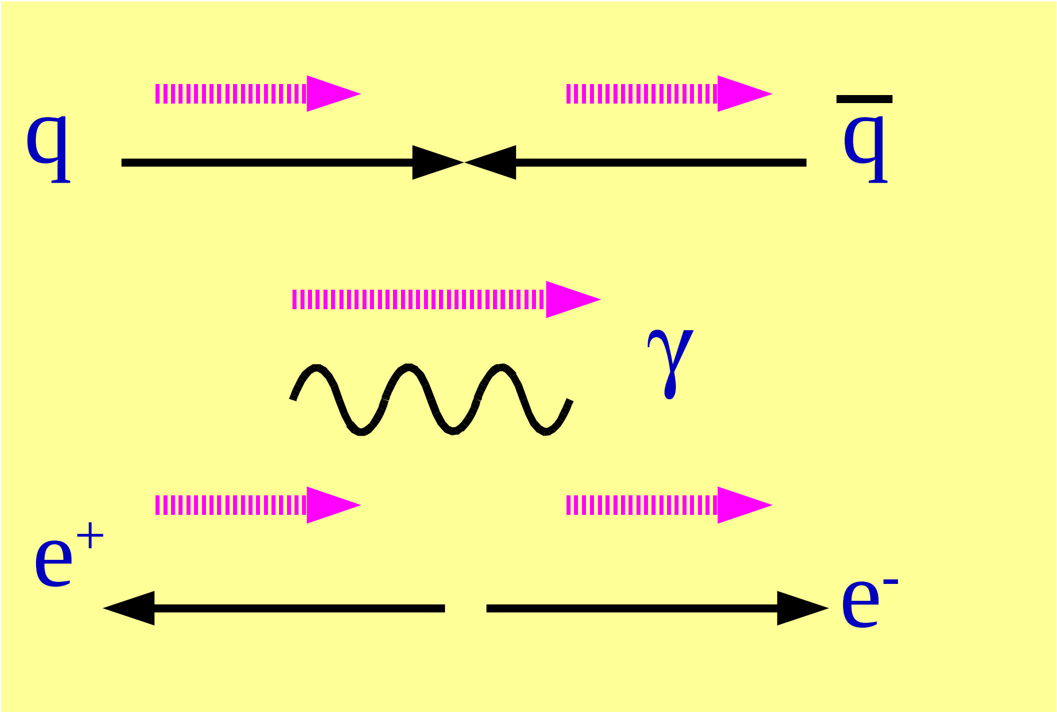
b) Next, compute the general form for the final state particles, $p_3$ and $p_4$. Do this by first aligning $p_3$ and $p_4$ along the z-axis (as $p_1$ and $p_2$ are), and then rotate about the y-axis by angle $\theta$.

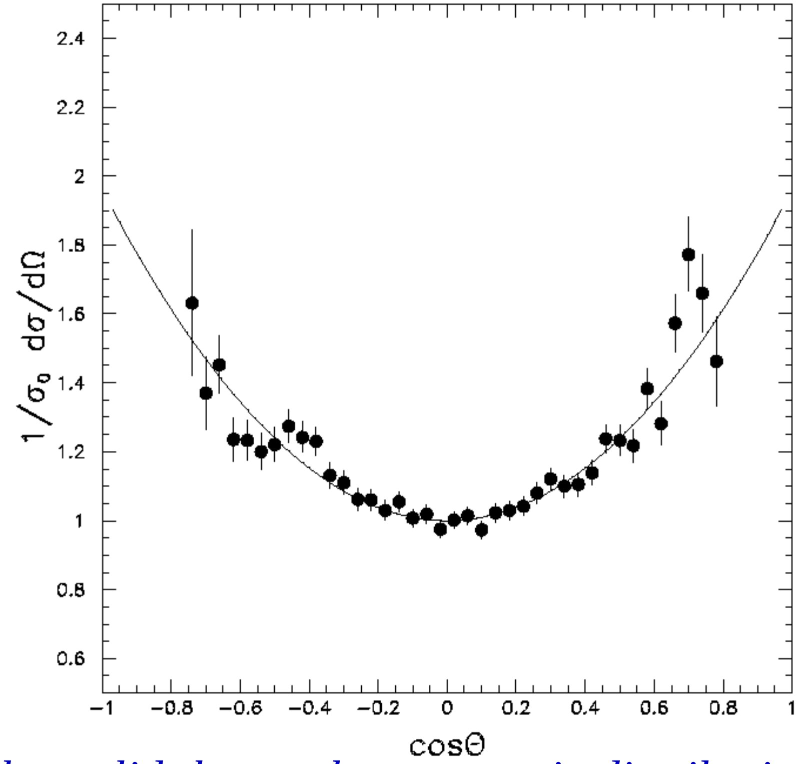
# What does the angular dependence tell us?

Observe, the angular dependence: $q+\bar{q}\rightarrow e^{+}+e^{-}$

$$\frac{d\hat{\sigma}}{d\cos(\theta)} = Q_i^2\,\alpha^2\,\frac{\pi}{6}\,\frac{1}{\hat{s}}\left(1+\cos^2(\theta)\right)$$

Characteristic of scattering of spin ½ constitutients by a spin 1 vector



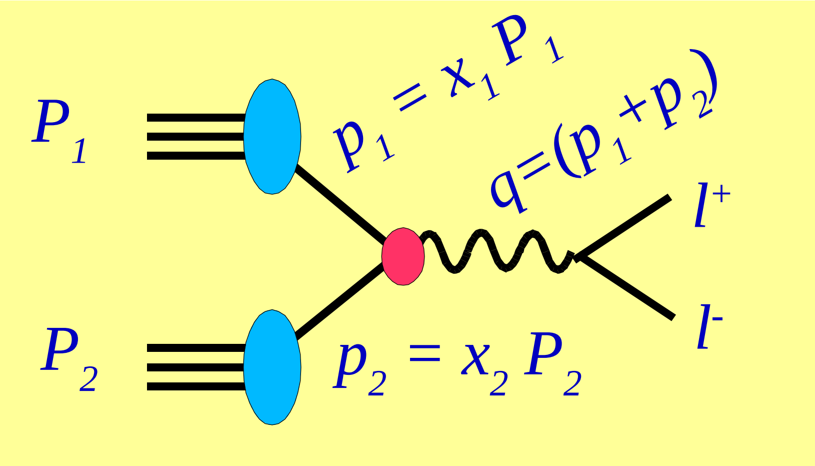




*Note, for the photon, the mirror image of the above is also valid; hence the symmetric distribution.*

*The W has V-A couplings, so we'll find: $(1+\cos\theta)^2$*

Next, we'll compute the hadronic CMS

# Kinematics in the Hadronic Frame




$$P_1 = \frac{\sqrt{s}}{2}(1,0,0,+1) \qquad P_1^2 = 0$$

$$P_2 = \frac{\sqrt{s}}{2}(1,0,0,-1) \qquad P_2^2 = 0$$

$$s = (P_1 + P_2)^2 = \frac{\hat{s}}{x_1 x_2} = \frac{\hat{s}}{\tau}$$

Therefore

$$\tau = x_1 x_2 = \frac{\hat{s}}{s} \equiv \frac{Q^2}{s}$$

Fractional energy² between partonic and hadronic system

$$\frac{d\sigma}{dQ^2} = \sum_{q,\bar{q}} \int dx_1 \int dx_2 \left\{ q(x_1)\bar{q}(x_2) + \bar{q}(x_1) q(x_2) \right\} \hat{\sigma}_0 \, \delta(Q^2 - \hat{s})$$

Hadronic cross section

Parton distribution functions

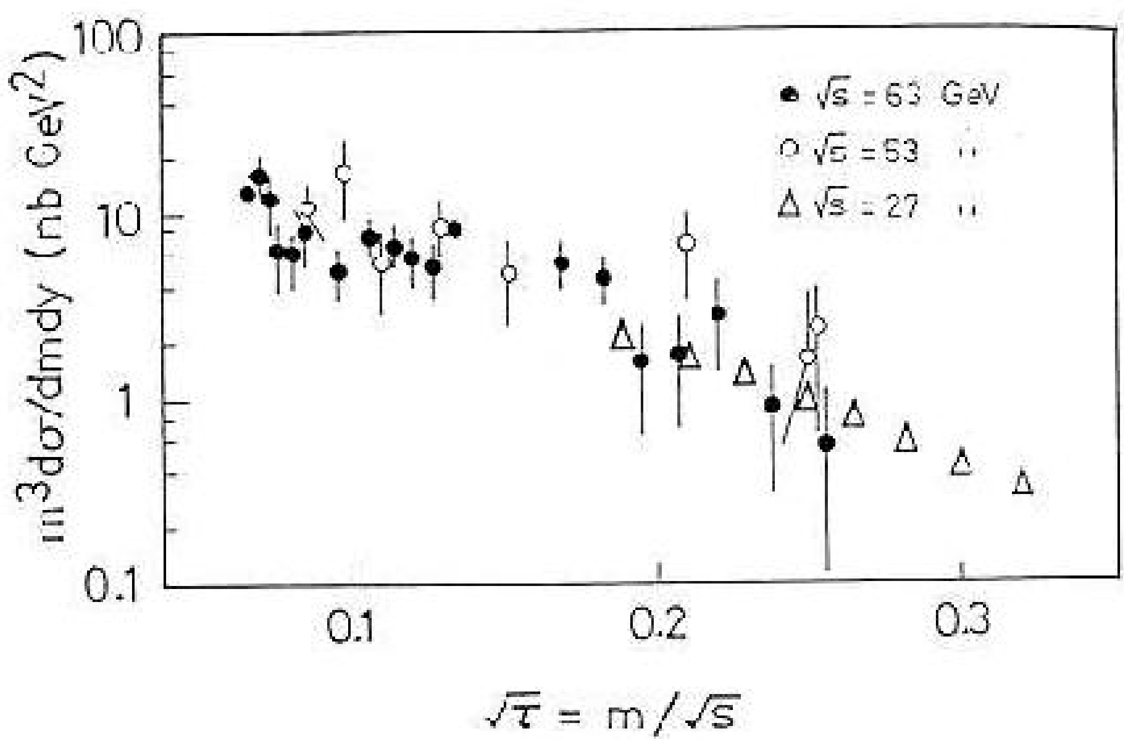
Partonic cross section

# Scaling form of the Drell-Yan Cross Section

Using: $\hat{\sigma}_0 = \dfrac{4\pi\alpha^2}{9\hat{s}} Q_i^2$ and $\delta(Q^2 - \hat{s}) = \dfrac{1}{s\,x_1}\,\delta\left(x_2 - \dfrac{\tau}{x_1}\right)$

we can write the cross section in the scaling form:

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_{q,\bar{q}} Q_i^2 \int_\tau^1 \frac{dx_1}{x_1}\, \tau \left\{ q(x_1)\bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1) \right\}$$



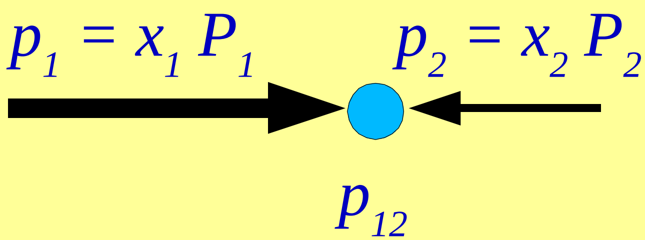

Notice the RHS is a function of only τ, not Q.

This quantity should lie on a universal scaling curve.

Cf., DIS case, & scattering of point-like constituents

# Longitudinal Momentum Distributions

Partonic CMS has longitudinal momentum w.r.t. the hadron frame




$$p_{12} = (p_1 + p_2) = (E_{12}, 0, 0, p_L)$$

$$E_{12} = \frac{\sqrt{s}}{2}(x_1 + x_2)$$

$$p_L = \frac{\sqrt{s}}{2}(x_1 - x_2) \equiv \frac{\sqrt{s}}{2} x_F$$

*$x_F$ is a measure of the longitudinal momentum*

The rapidity is defined as:

$$y = \frac{1}{2} \ln\left\{\frac{E_{12} + p_L}{E_{12} - p_L}\right\} = \frac{1}{2} \ln\left\{\frac{x_1}{x_2}\right\}$$

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y}$$

$$d x_1 \, d x_2 = d\tau \, dy$$

$$d Q^2 dx_F = dy \, d\tau \; s \; \sqrt{x_F^2 + 4\tau}$$

$$\frac{d\sigma}{dQ^2 \, dx_F} = \frac{4\pi\alpha^2}{9Q^4} \frac{1}{\sqrt{x_F^2 + 4\tau}} \tau \sum_{q,\bar{q}} Q_i^2 \left\{ q(x_1)\bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1) \right\}$$

So, we're ready to compare with data

*(or so we think...)*

# Let's compare data and theory

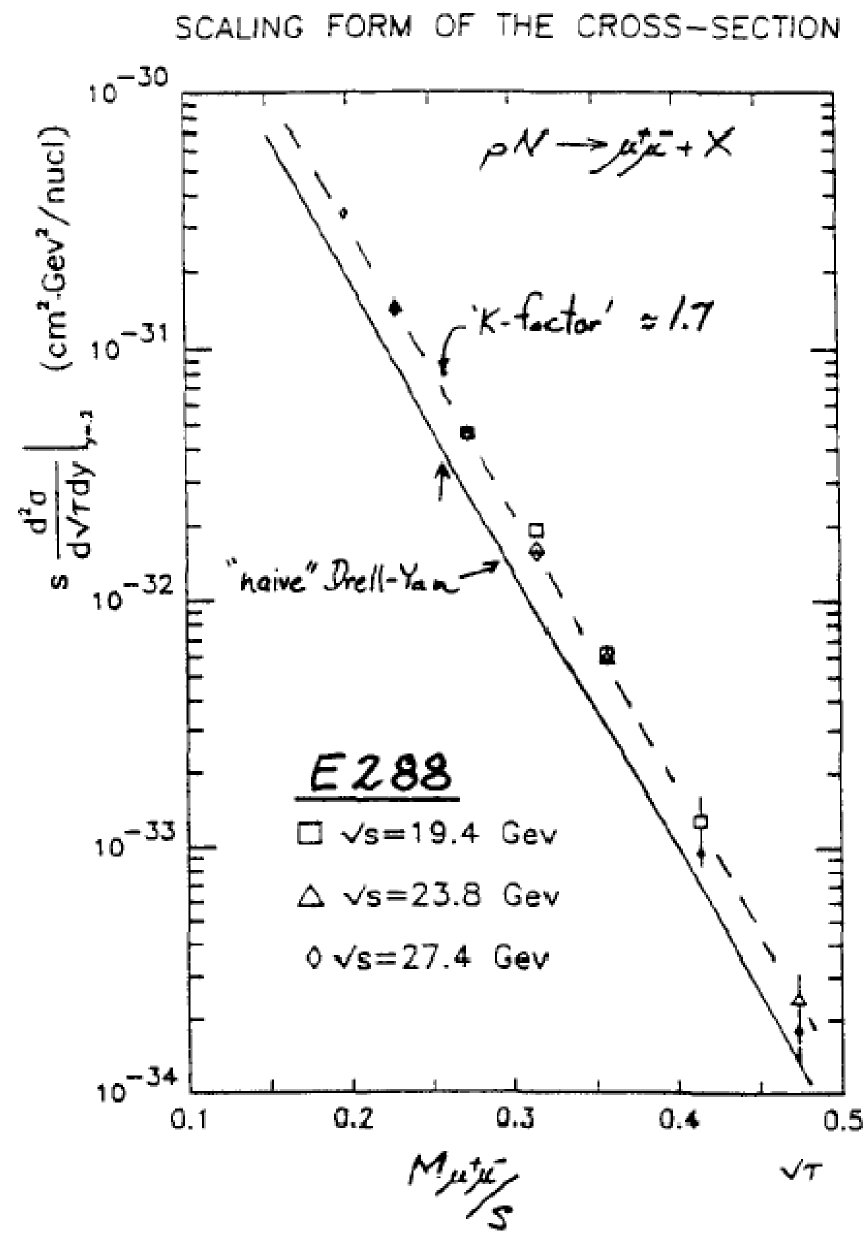



Table 1.2: Experimental $K$-factors.

| Experiment | Interaction | Beam Momentum | $K = \sigma_{\text{meas.}}/\sigma_{\text{DY}}$ |
|---|---|---|---|
| E288 [Kap 78] | $p\ Pt$ | 300/400 GeV | $\sim 1.7$ |
| WA39 [Cor 80] | $\pi^{\pm}\ W$ | 39.5 GeV | $\sim 2.5$ |
| E439 [Smi 81] | $p\ W$ | 400 GeV | $1.6 \pm 0.3$ |
| NA3 [Bad 83] | $(\bar{p}$ - $p)Pt$ | 150 GeV | $2.3 \pm 0.4$ |
| | $p\ Pt$ | 400 GeV | $3.1 \pm 0.5 \pm 0.3$ |
| | $\pi^{\pm}\ Pt$ | 200 GeV | $2.3 \pm 0.5$ |
| | $\pi^{-}\ Pt$ | 150 GeV | $2.49 \pm 0.37$ |
| | $\pi^{-}\ Pt$ | 280 GeV | $2.22 \pm 0.33$ |
| NA10 [Bet 85] | $\pi^{-}\ W$ | 194 GeV | $\sim 2.77 \pm 0.12$ |
| E326 [Gre 85] | $\pi^{-}\ W$ | 225 GeV | $2.70 \pm 0.08 \pm 0.40$ |
| E537 [Ana 88] | $\bar{p}\ W$ | 125 GeV | $2.45 \pm 0.12 \pm 0.20$ |
| E615 [Con 89] | $\pi^{-}\ W$ | 252 GeV | $1.78 \pm 0.06$ |

J. C. Webb, Measurement of continuum dimuon production in 800-GeV/c proton nucleon collisions, arXiv:hep-ex/0301031.

*Oooops,*

*we need the*

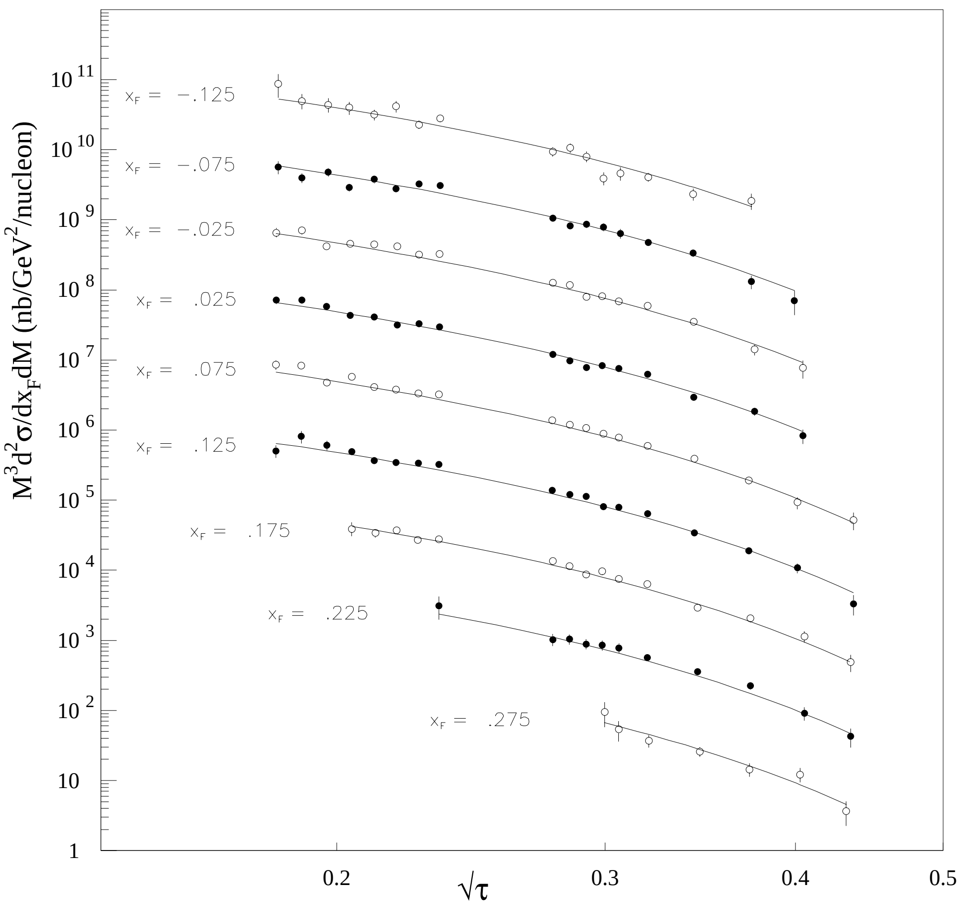
*QCD corrections*

$$K = 1 + \frac{2\pi\alpha_s}{3}(\ldots) + \ldots = ? = e^{2\pi\alpha_s/3}$$

# Excellent agreement between data and theory

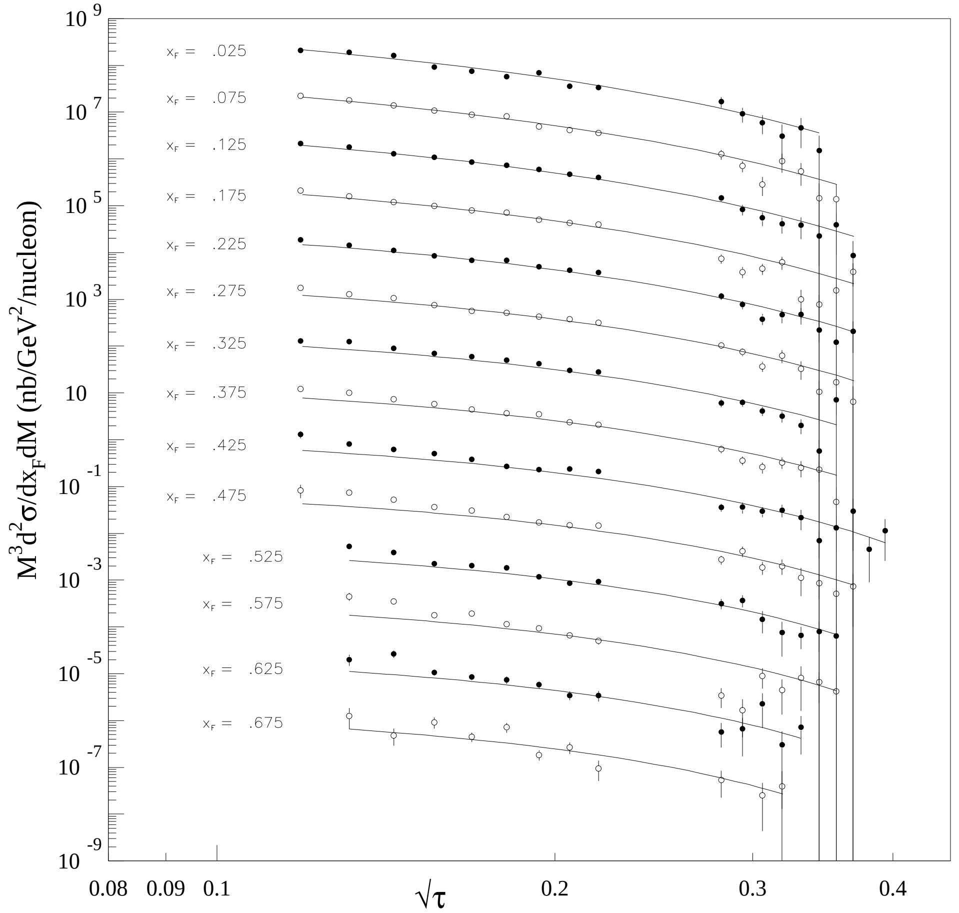
## p + Cu at 800 GeV

E605 (p Cu $\rightarrow \mu^+\mu^-$ X) $p_{LAB}$ = 800 GeV




## p + d at 800 GeV

E772 (p d $\rightarrow \mu^+\mu^-$ X) $p_{LAB}$ = 800 GeV




pp & pN processes sensitive to anti-quark distributions


A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne,
Eur. Phys. J. C23, 73 (2002);
Eur. Phys. J. C14, 133 (2000);
Eur. Phys. J. C4, 463 (1998)

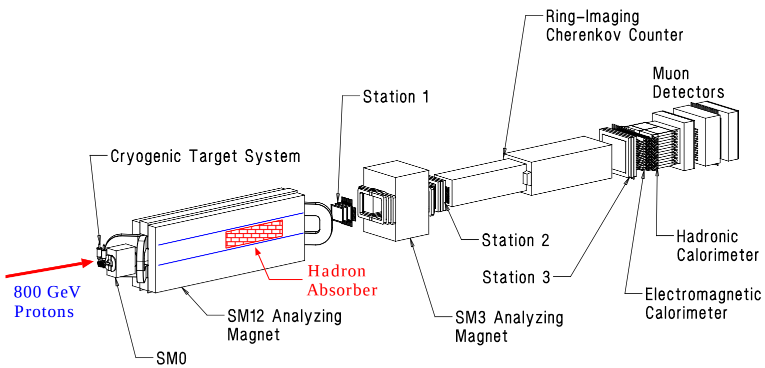
Drell-Yan can give us unique and detailed information about PDF's.

We'll now examine two examples:

1) Ratio of pp/pd cross section

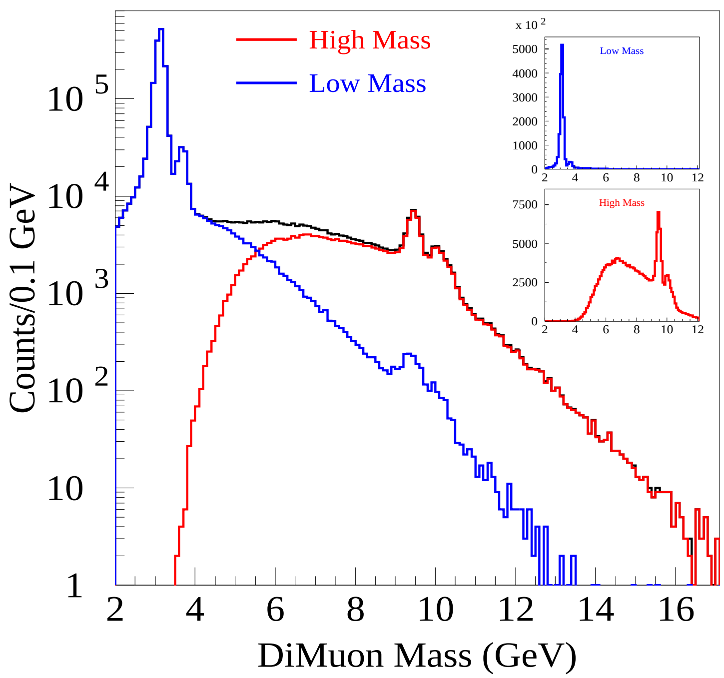
2) W Rapidity Asymmetry

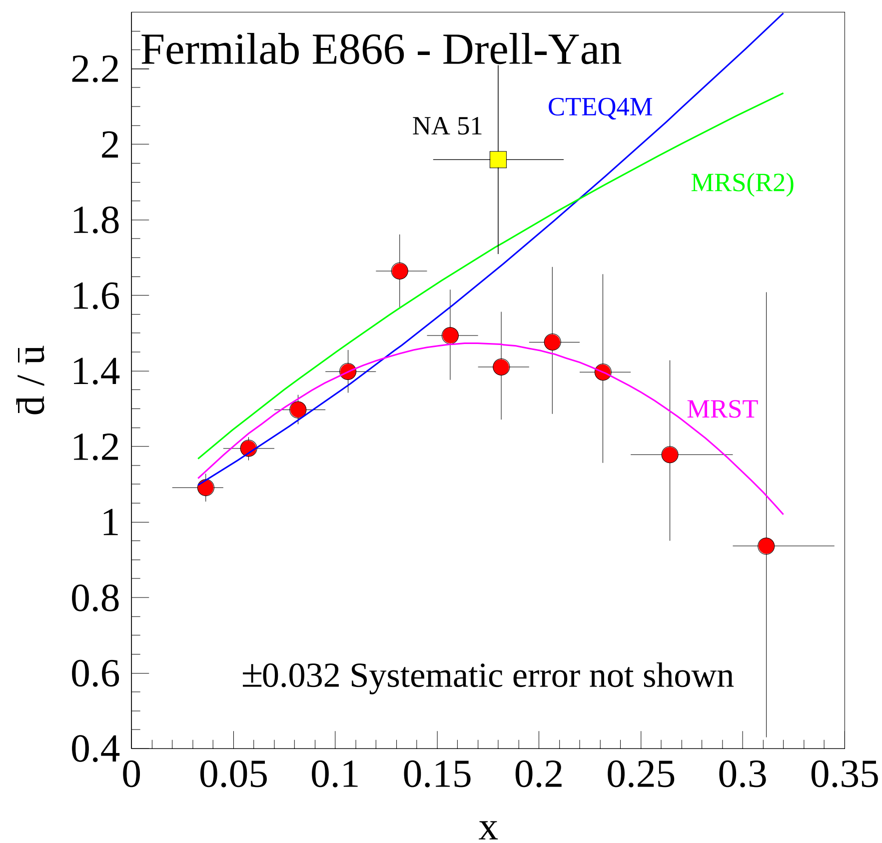
# A measurement of $\bar{d}(x)/\bar{u}(x)$
# Antiquark asymmetry in the Nucleon Sea
# FNAL E866/NuSea




ACU, ANL, FNAL, GSU, IIT, LANL, LSU,
NMSU, UNM, ORNL, TAMU, Valpo.

$$800 \text{ GeV } p+p \text{ and } p+d \to \mu^+\mu^- X$$



# Cross section ratio of pp *vs.* pd

Obtain the neutron PDF via isospin symmetry:

$$u \Leftrightarrow d$$
$$\bar{u} \Leftrightarrow \bar{d}$$

In the limit $x_1 >> x_2$:

$$\sigma^{pp} \propto \frac{4}{9} u(x_1)\bar{u}(x_2) + \frac{1}{9} d(x_1)\bar{d}(x_2)$$
$$\sigma^{pn} \propto \frac{4}{9} u(x_1)\bar{d}(x_2) + \frac{1}{9} d(x_1)\bar{u}(x_2)$$

For the ratio we have:

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2} \frac{\left(1+\frac{1}{4}\frac{d_1}{u_1}\right)}{\left(1+\frac{1}{4}\frac{d_1}{u_1}\frac{\bar{d}_2}{\bar{u}_2}\right)} \left(1+\frac{\bar{d}_2}{\bar{u}_2}\right) \approx \frac{1}{2}\left(1+\frac{\bar{d}_2}{\bar{u}_2}\right)$$

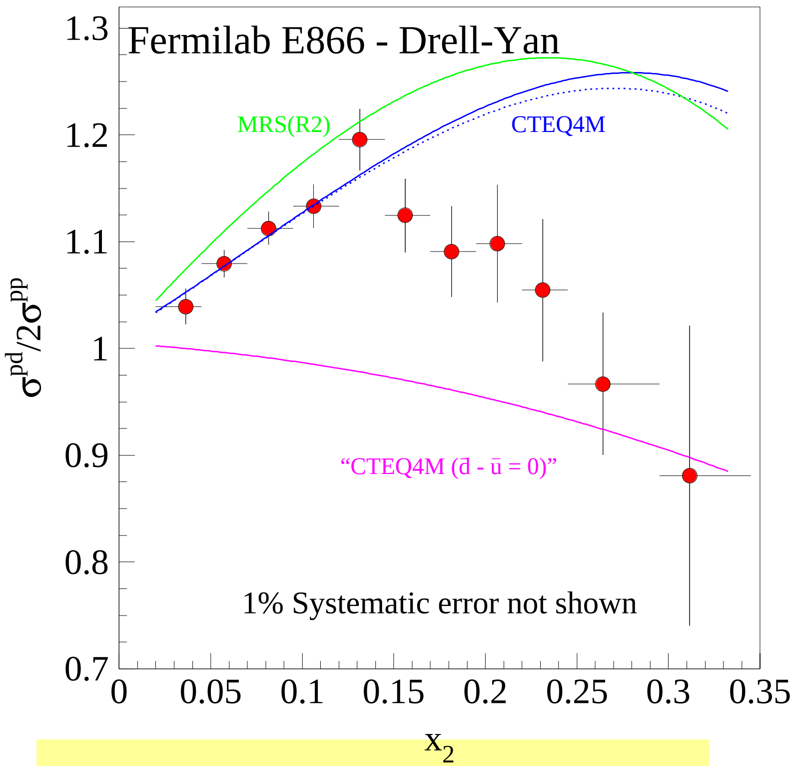
As promised, this provides information about the sea-quark distributions

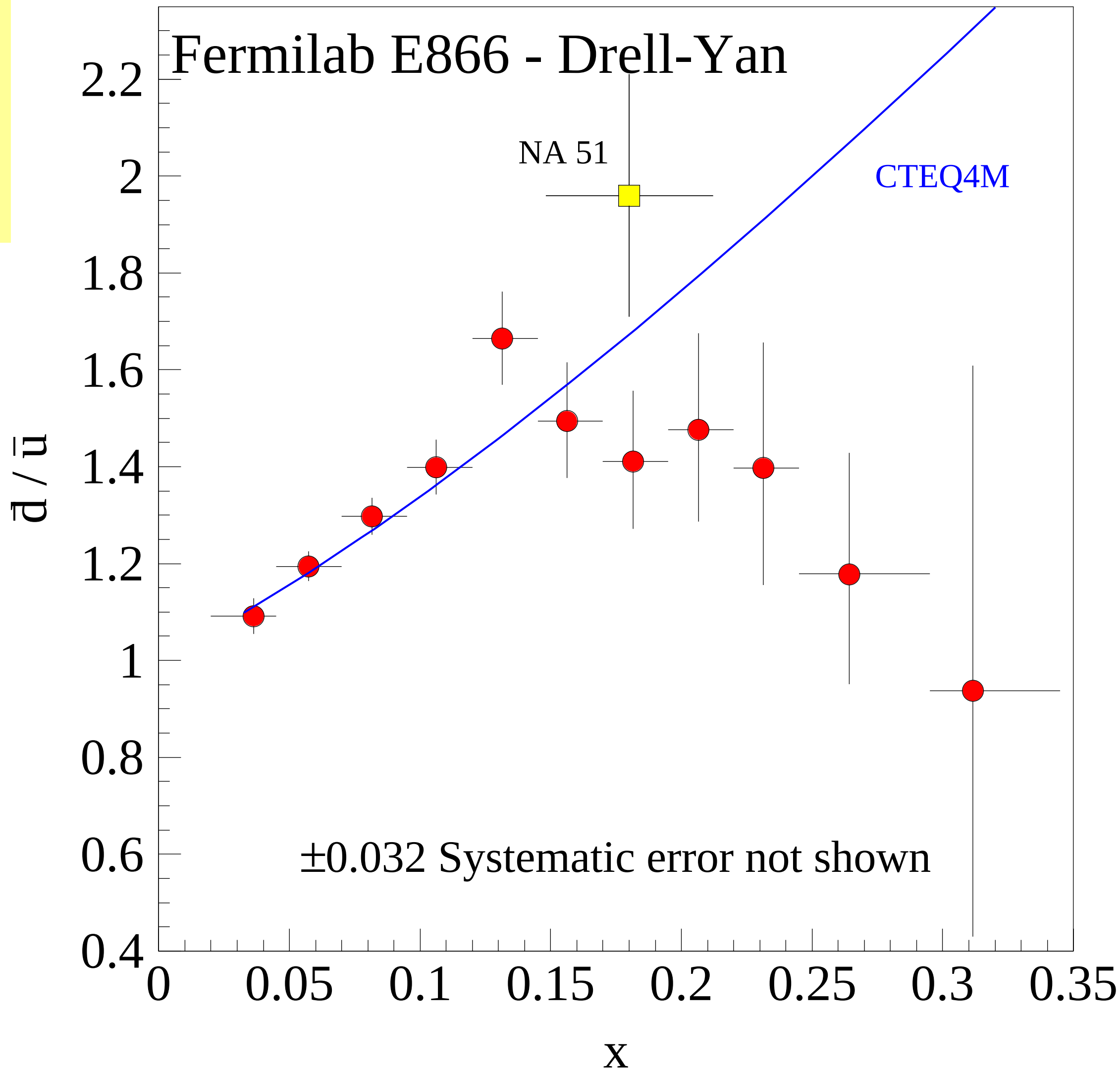
$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1+\frac{\bar{d}_2}{\bar{u}_2}\right)$$

EXERCISE: Verify the above.

# Does the theory match the data???

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1+\frac{\bar{d}_2}{\bar{u}_2}\right)$$



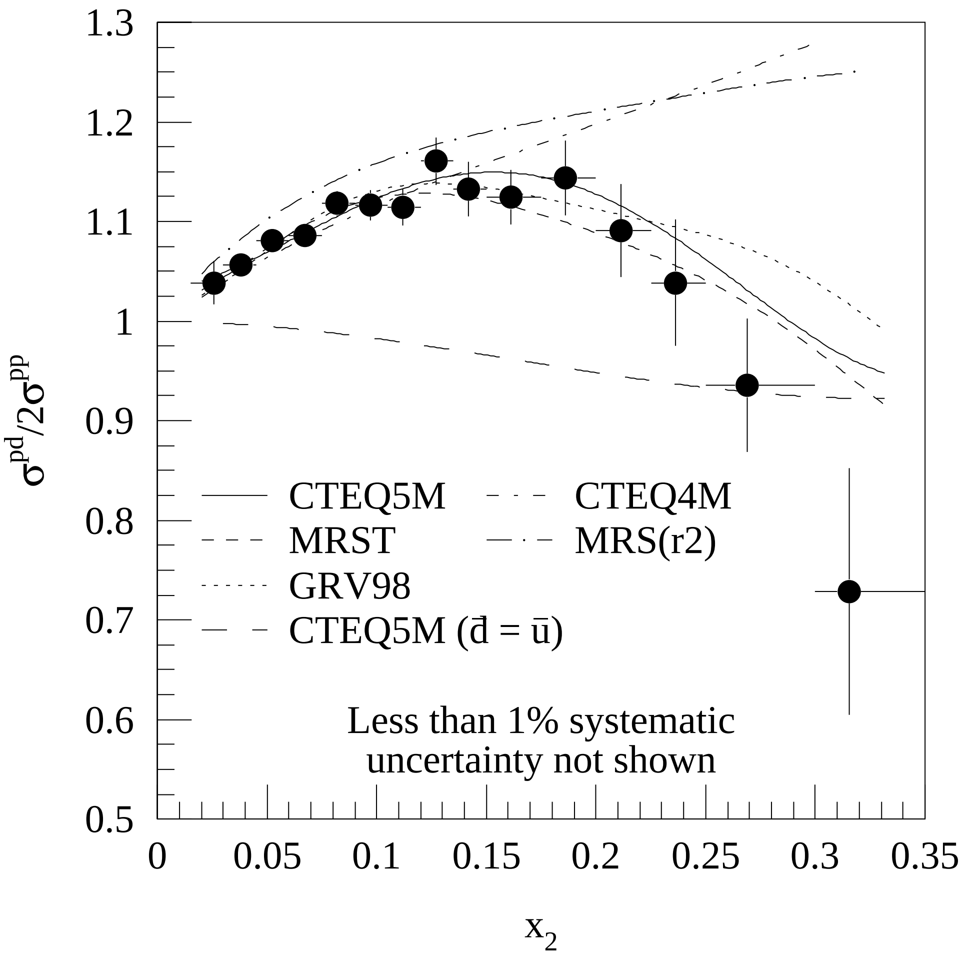



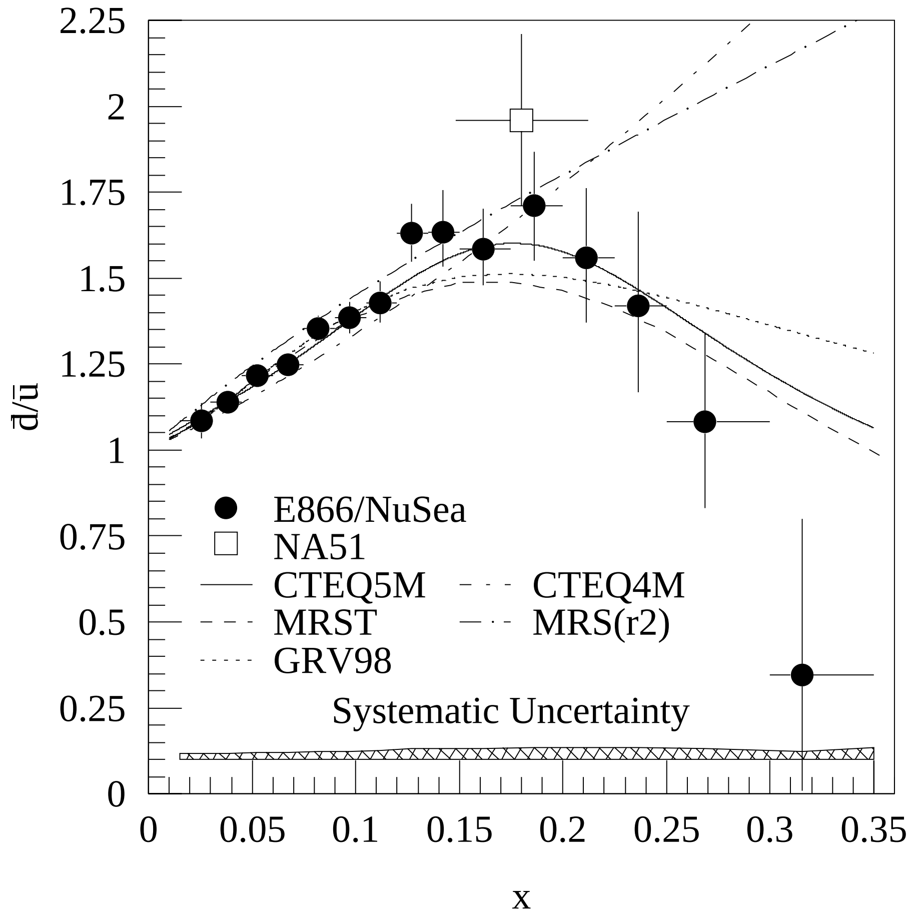

Implies R<1 for large x:

$$\bar{d} \ll \bar{u}$$

E.A. Hawker, et al. [FNAL E866/NuSea Collaboration], Measurement of the light antiquark flavor asymmetry in the nucleon sea, PRL 80, 3715 (1998)

# E866 required significant changes in the hi-x sea distributions




With increased flexibility in the parameterization of the sea-quark distributions, good fits are obtained


E.A. Hawker, et al. [FNAL E866/NuSea Collaboration], Measurement of the light antiquark flavor asymmetry in the nucleon sea, PRL 80, 3715 (1998)

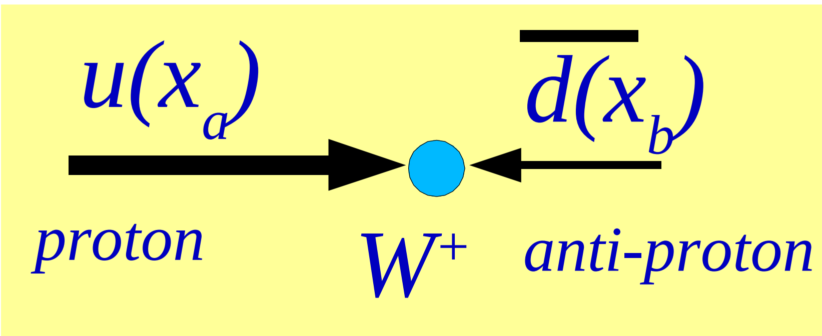
H. L. Lai, et al.} [CTEQ Collaboration], Global {QCD} analysis of parton structure of the nucleon: CTEQ5 parton distributions, EPJ C12, 375 (2000)

# Next ...

## 2) W Rapidity Asymmetry

# Where do the W's and Z's come from ???

$$\frac{d\sigma}{dy}(W^{\pm}) = \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\sum_{q\bar{q}} |V_{q\bar{q}}|^2 \left[q(x_a)\,\bar{q}(x_b) + q(x_b)\,\bar{q}(x_a)\right]$$




For anti-proton:

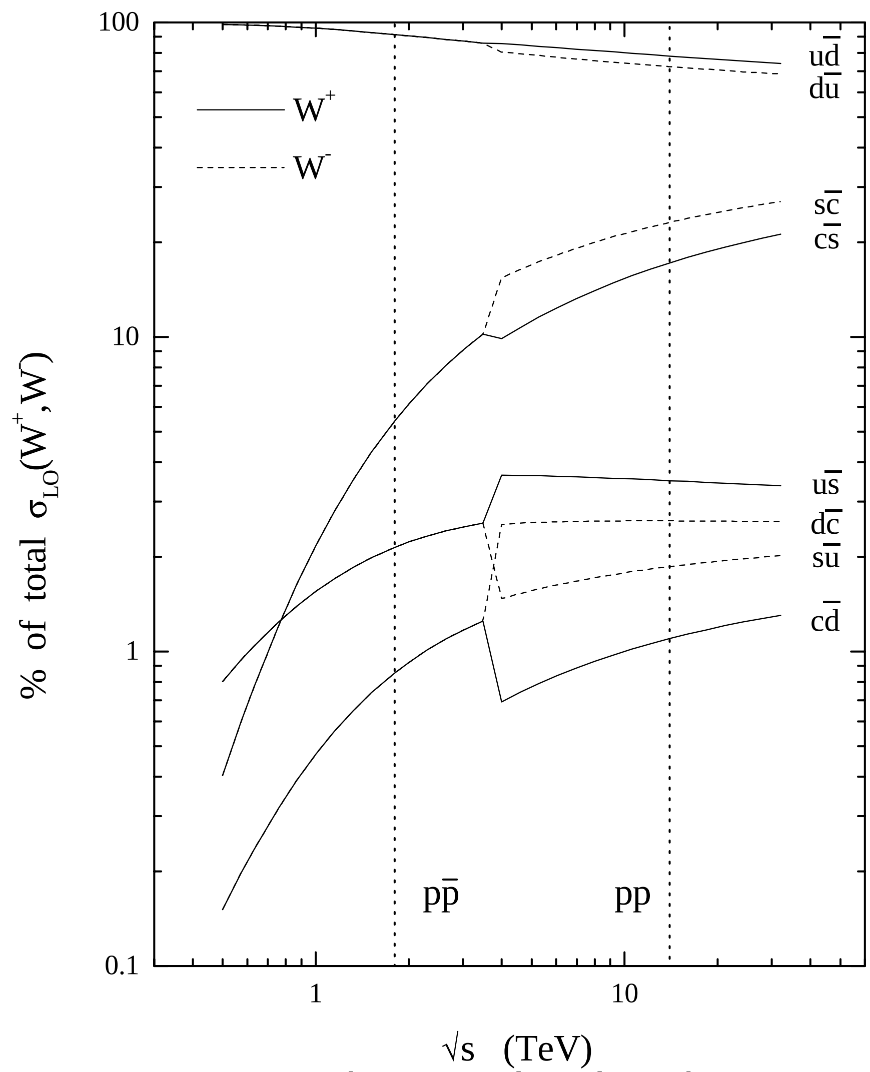
$$u(x) \Leftrightarrow \bar{u}(x) \qquad d(x) \Leftrightarrow \bar{d}(x)$$

Therefore

$$\frac{d\sigma}{dy}(W^{+}) \approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\left[u(x_a)\,d(x_b)\right]$$

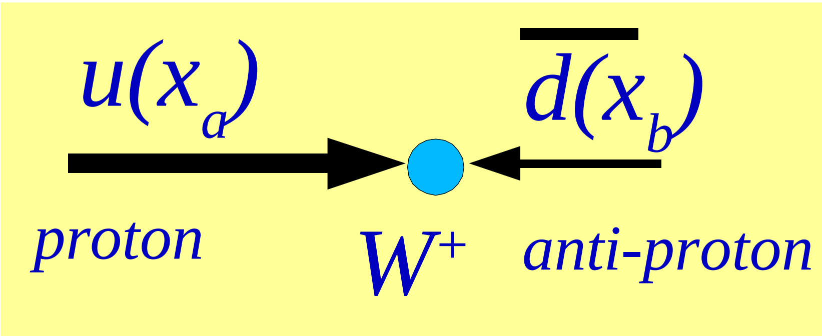
$$\frac{d\sigma}{dy}(W^{-}) \approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\left[d(x_a)\,u(x_b)\right]$$

**flavour decomposition of W cross sections**




A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne,
Eur. Phys. J. C23, 73 (2002); Eur. Phys. J. C4, 463 (1998)

# A bit of calculation




$$A(y) = \frac{\frac{d\sigma}{dy}(W^+) - \frac{d\sigma}{dy}(W^-)}{\frac{d\sigma}{dy}(W^+) + \frac{d\sigma}{dy}(W^-)}$$

With the previous approximation,

$$A \approx \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)} = \frac{R_{du}(x_b) - R_{du}(x_a)}{R_{du}(x_b) + R_{du}(x_a)}$$

where $\quad R_{du}(x) = \dfrac{d(x)}{u(x)}$

We can make Taylor expansions:

$$x_{1,2} = x_0 e^{\pm y} \simeq x_0(1 \pm y)$$

$$R_{du}(x_{1,2}) \approx R_{du}(x_0) \pm y\, x_0\, R'_{du}(\sqrt{\tau})$$

Thus, the asymmetry is:

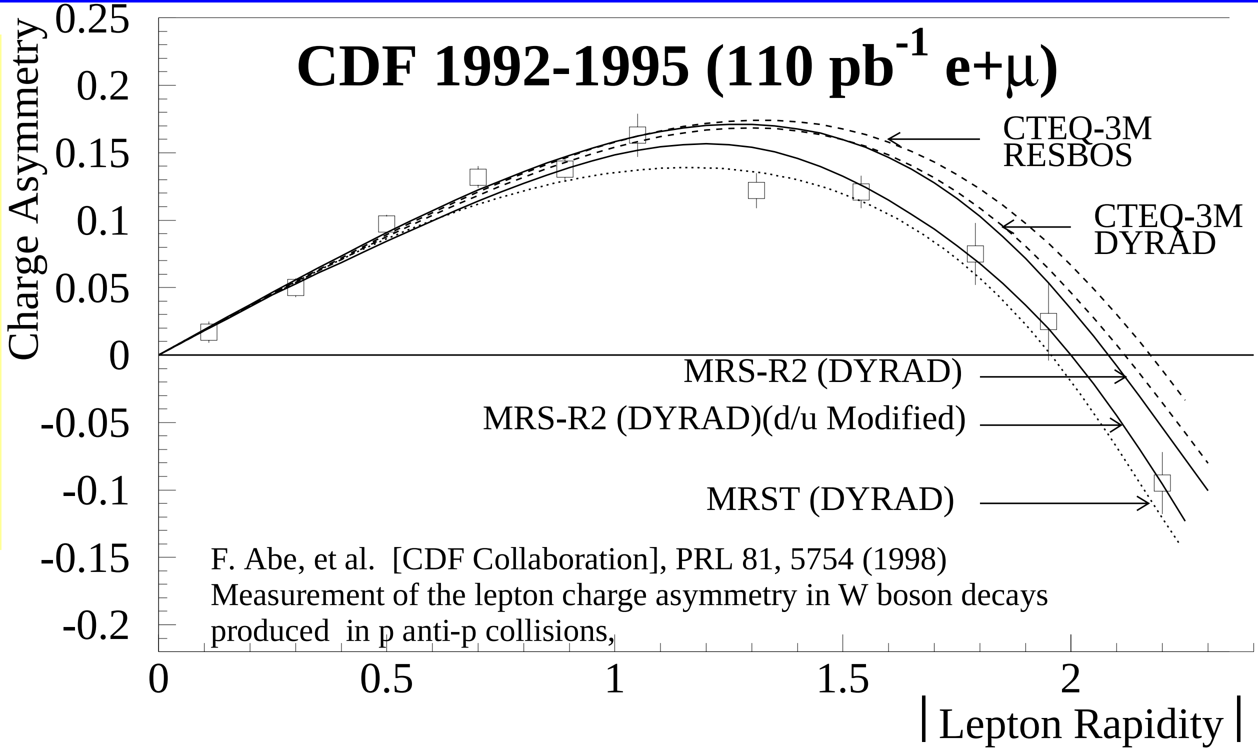
$$\boxed{A(y) = -y\, x_0 \frac{R'_{du}(x_0)}{R_{du}(x_0)}}$$

EXERCISE: Verify the above.

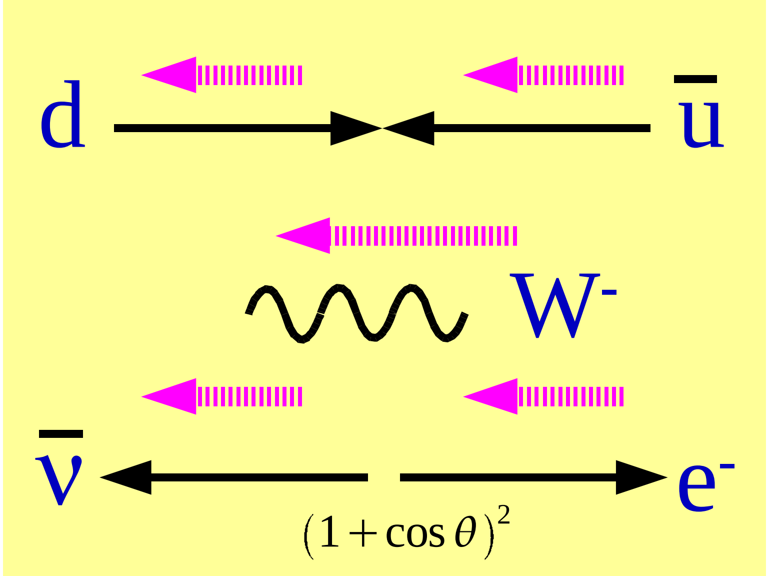
# Charged Lepton Asymmetry

Unfortunately,
we don't measure the W
directly since W→eν.

Still the lepton contains
important information




$$A(y) = \frac{\frac{d\sigma}{dy}(l^+) - \frac{d\sigma}{dy}(l^-)}{\frac{d\sigma}{dy}(l^+) + \frac{d\sigma}{dy}(l^-)}$$

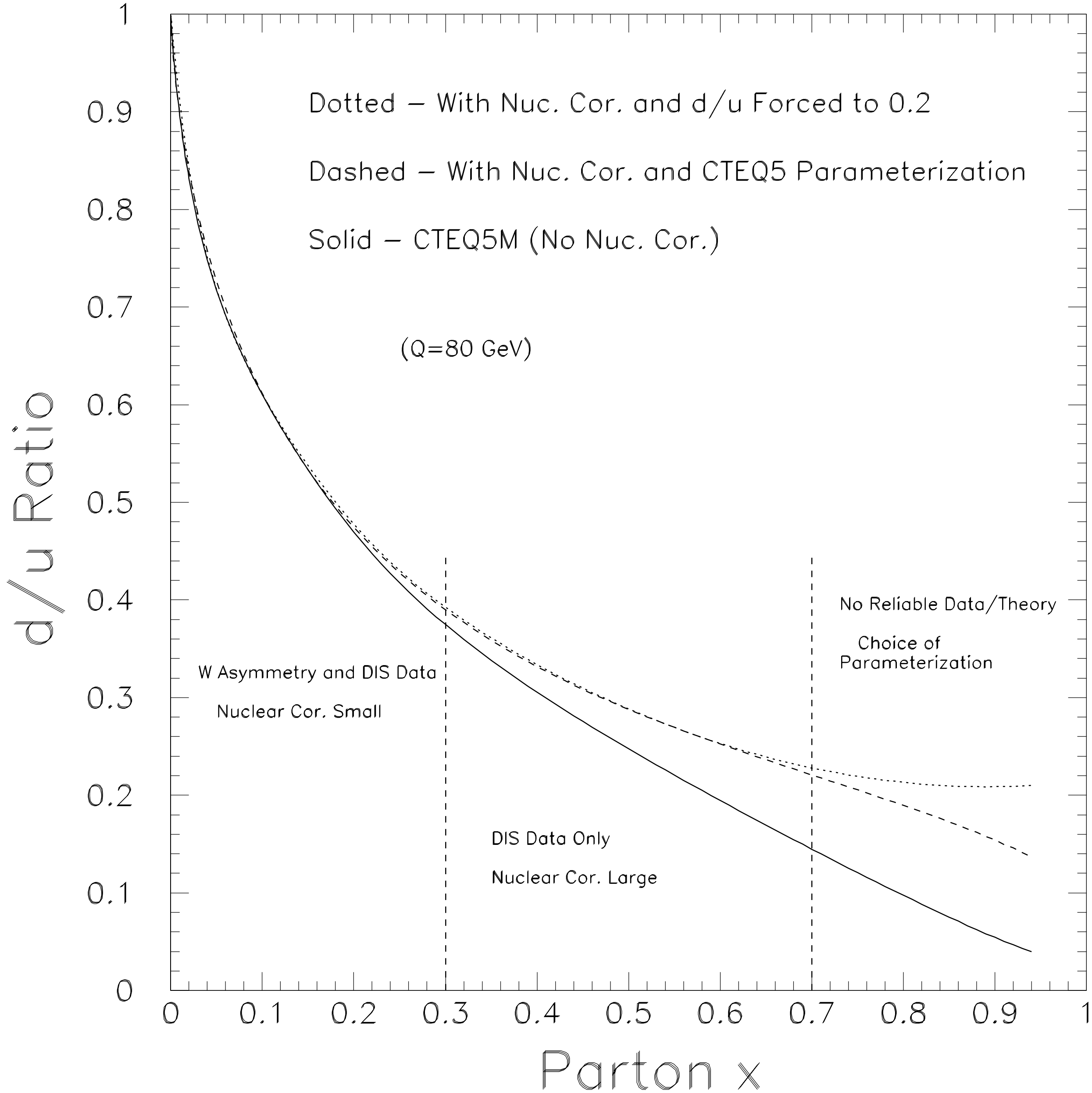


# d/u Ratio at High-x

The form of the
d/u ratio at large x
as a function of

1) Parameterization

2) Nuclear Corrections




S. Kuhlmann, et al., Large-x parton distributions, PL B476, 291 (2000)

# End of Part I: Where have we been???

**History:**

Discovery of J/ψ, Upsilon, W/Z, and "New Physics" ???

**Calculation of $q\, q \to \mu^+\mu^-$ in the Parton Model**

Scaling form of the cross section

Rapidity, longitudinal momentum, and $x_F$

**Comparison with data:**

NLO QCD corrections essential (the K-factor)

$\sigma(pd)/\sigma(pp)$ important for d-bar/ubar

W Rapidity Asymmetry important for slope of d/u at large x

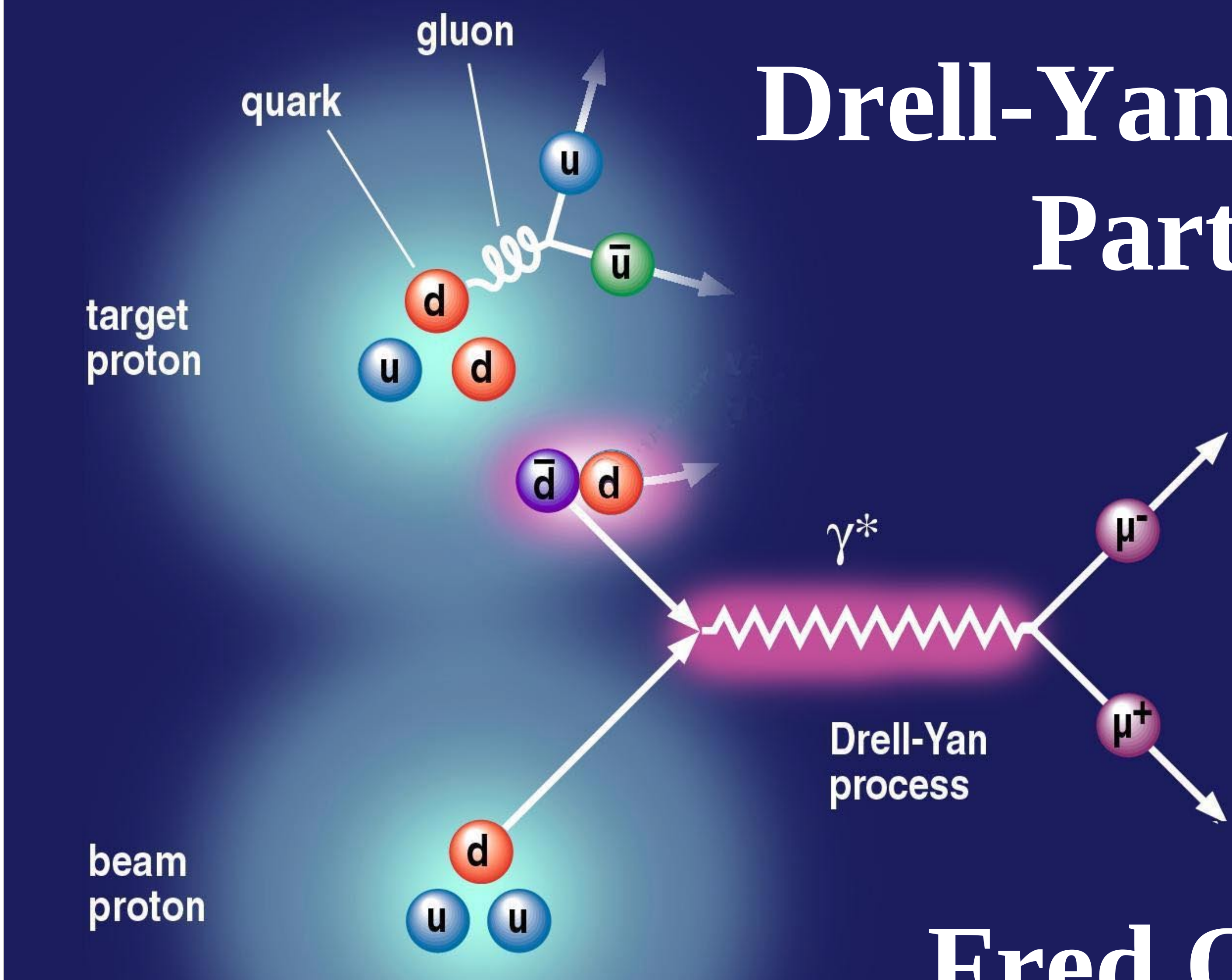
**Where are we going?**

$P_T$ Distribution

W-mass measurement

Resummation of soft gluons

# Drell-Yan Process: Part II




## Fred Olness
## SMU

# Part II: W Boson Production as an example

## Finding the W Boson Mass:

- The Jacobian Peak, and the W Boson $P_T$
- Multiple Soft Gluon Emissions
- Single Hard Gluon Emission

## Road map of Resummation

- Summing 2 logs per loop: multi-scale problem $(Q,q_T)$
- Correlated Gluon Emission
- Non-Perturbative physics at small $q_T$.

## Transverse Mass Distribution:

- Improvement over $P_T$ distribution

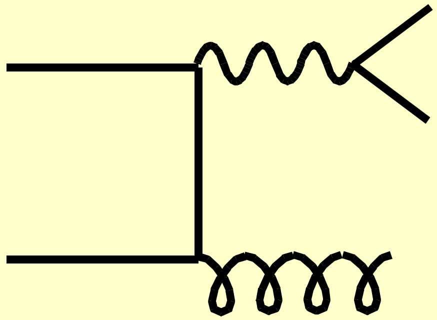
## What can we expect in future?

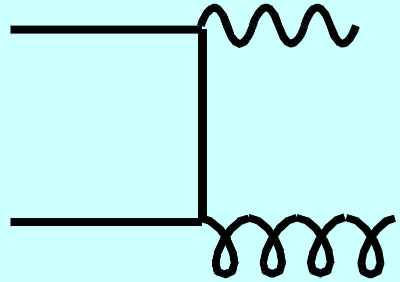
- Tevatron Run II
- LHC

## Side Note: From *pp*→γ/Z/W, we can obtain *pp*→γ/Z/W→$l^+l^-$

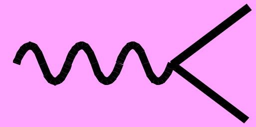
Schematically:

$$d\sigma(q\bar{q}\to l^+ l^- g) \;=\; d\sigma(q\bar{q}\to\gamma^* g) \;\times\; d\sigma(\gamma^*\to l^+ l^-)$$





$\otimes$



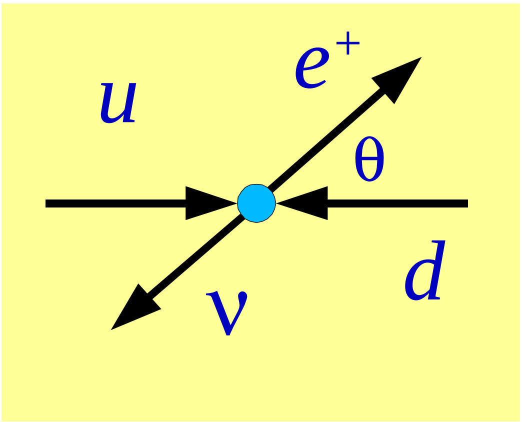
For example:

$$\frac{d\sigma}{dQ^2\, d\hat{t}}(q\bar{q}\to l^+ l^- g) \;=\; \frac{d\sigma}{d\hat{t}}(q\bar{q}\to\gamma^* g) \;\times\; \frac{\alpha}{3\pi Q^2}$$

# Part II: W Boson Production as an example

**How do we measure the W-boson mass?**

$$u + \bar{d} \rightarrow W^{+} \rightarrow e^{+}\nu$$

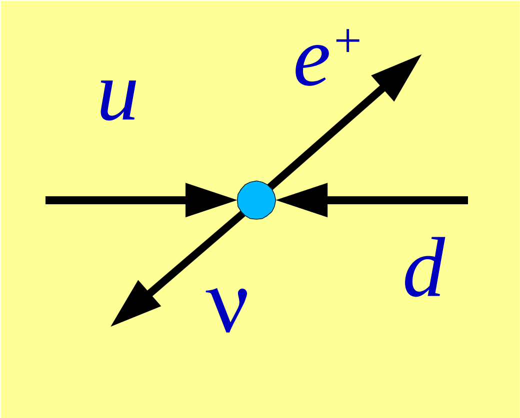



- Can't measure W directly
- Can't measure $\nu$ directly
- Can't measure longitudinal momentum

**We can measure the $P_T$ of the lepton**

**How can we use this to extract the W-Mass???**

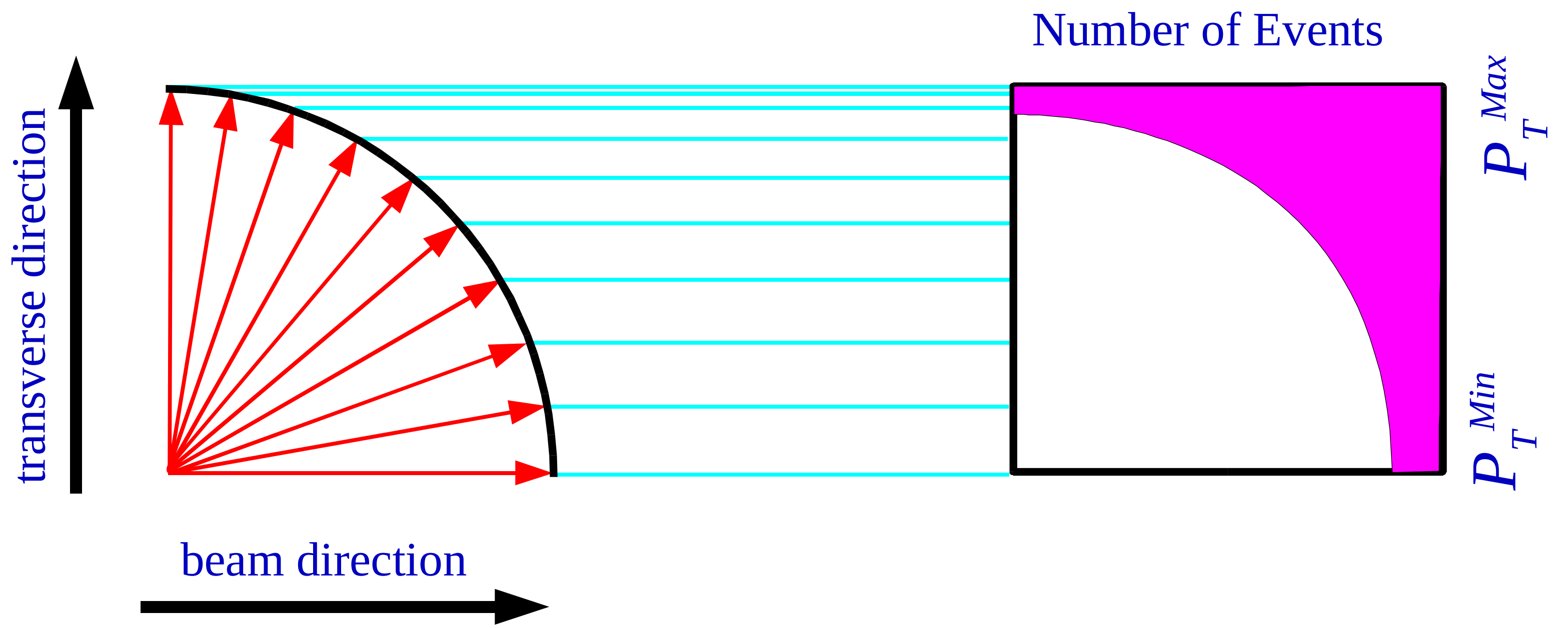
# The Jacobian Peak




Suppose lepton distribution is uniform in θ

*The dependence is actually $(1+\cos\theta)^2$, but we'll take care of that later*

What is the distribution in $P_T$?




We find a peak at $P_T^{max} \approx M_W/2$

# The Jacobian Peak

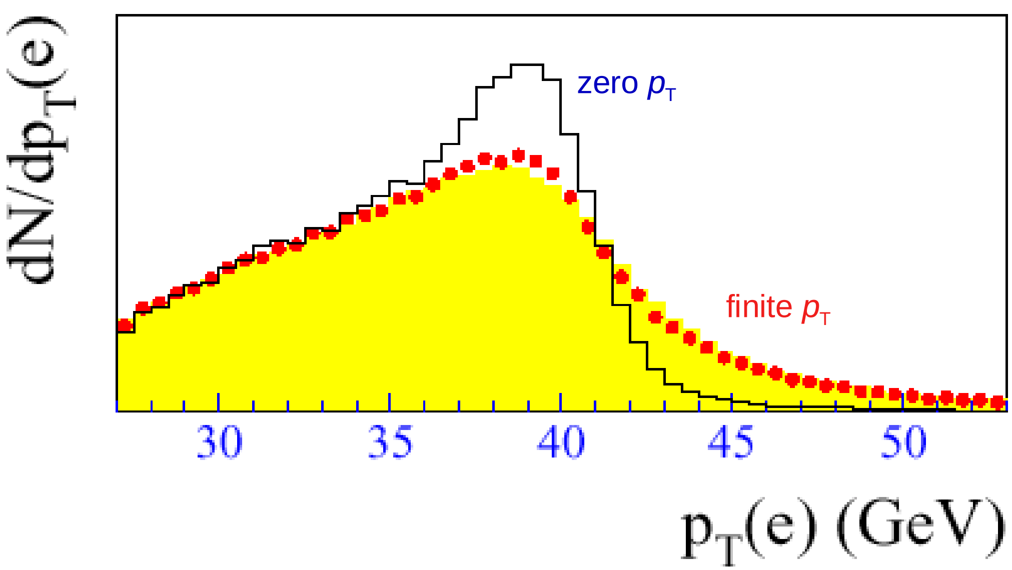
Now that we've got the picture, here's the math ... *(in the W CMS frame)*

$$p_T^2 = \frac{\hat{s}}{4}\sin^2\theta \qquad \cos\theta = \sqrt{1-\frac{4p_T^2}{\hat{s}}} \qquad \frac{d\cos\theta}{dp_T^2} = \frac{2}{\hat{s}}\,\frac{1}{\cos\theta}$$

So we discover the $P_T$ distribution has a singularity at cosθ=0, or θ=π/2

$$\frac{d\sigma}{dp_T^2} = \frac{d\sigma}{d\cos\theta}\times\frac{d\cos\theta}{dp_T^2} \approx \frac{d\sigma}{d\cos\theta}\times\frac{1}{\cos\theta}$$

**singularity!!!**



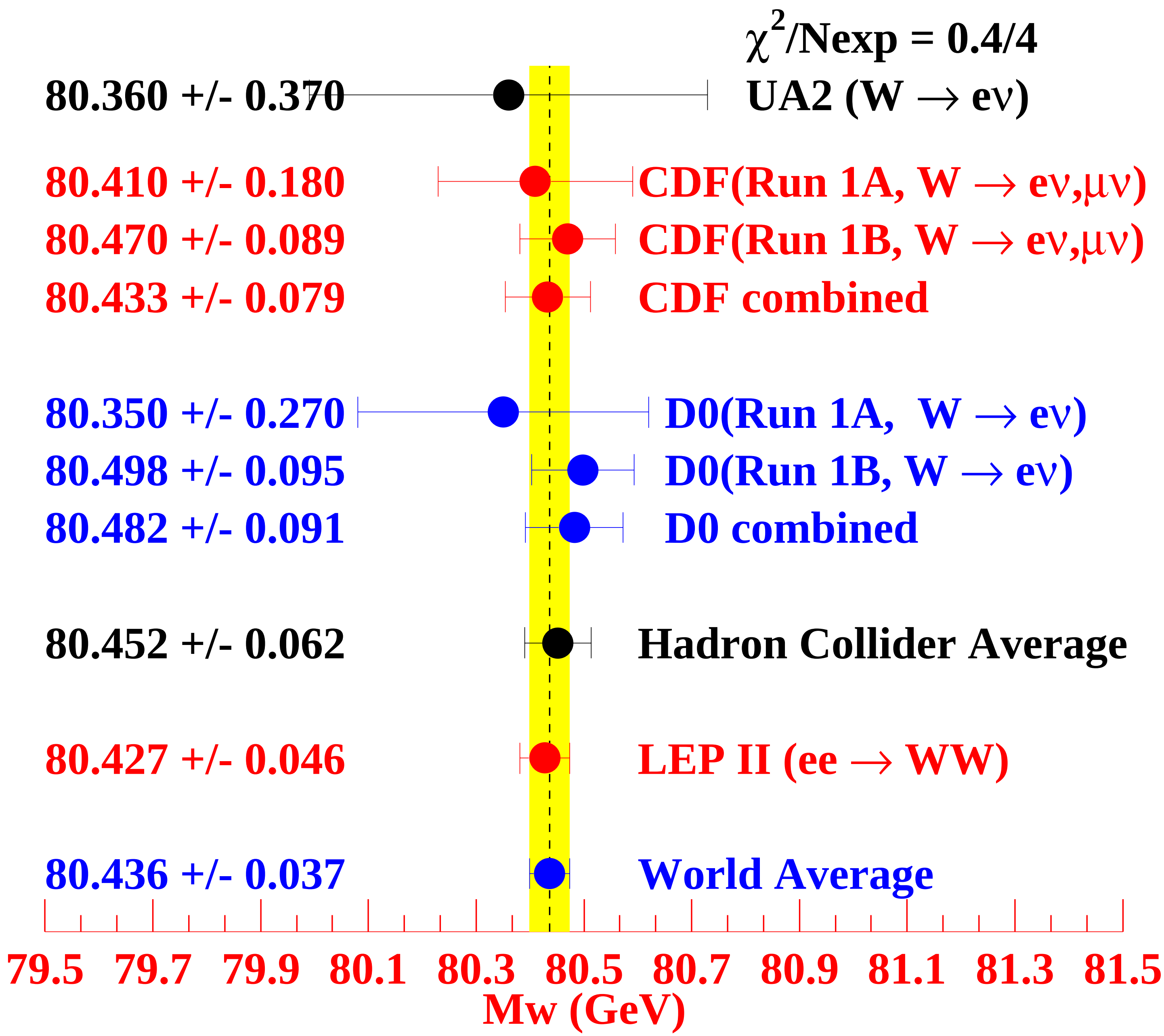

# BUT !!!

Measuring the Jacobian peak is complicated if the W boson has finite $P_T$.

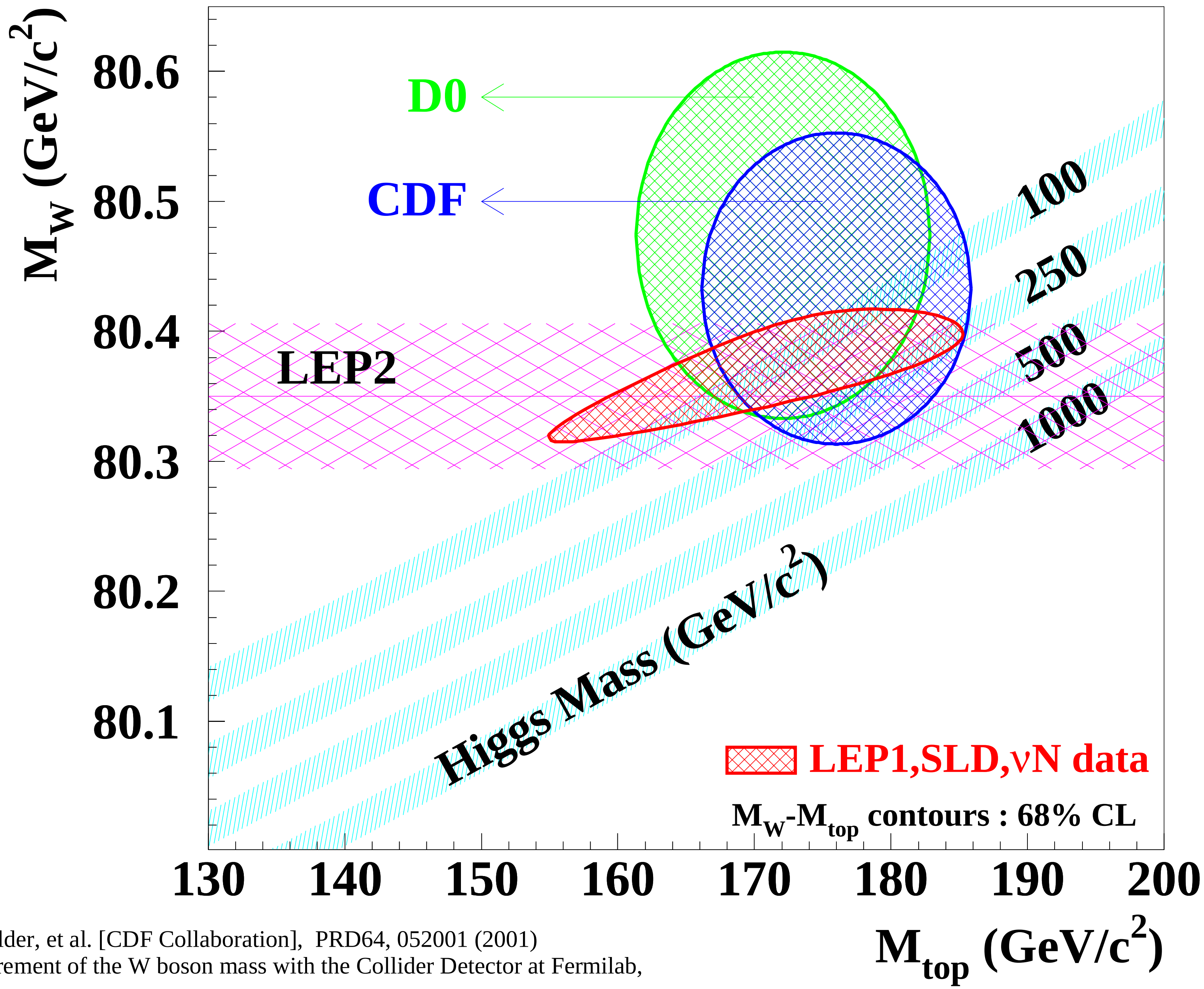
1) The W-mass is important fundamental quantity of the Standard Model

2) $P_T$ Distribution is important for measuring the W-mass

# The W-Mass is an important fundamental quantity



# The W-Mass is an important fundamental quantity




T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
Measurement of the W boson mass with the Collider Detector at Fermilab,

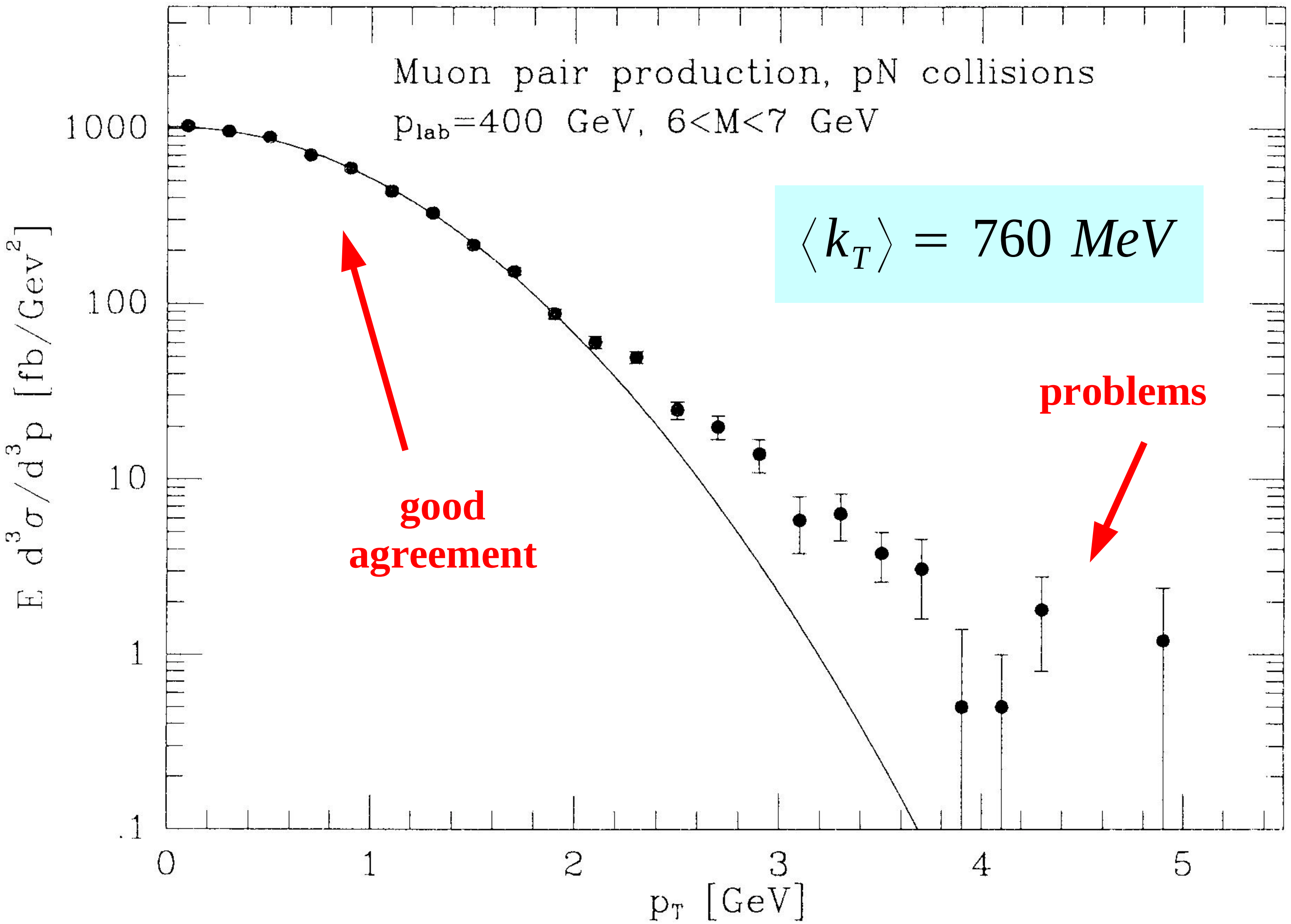
What gives the W

$P_T$ ???

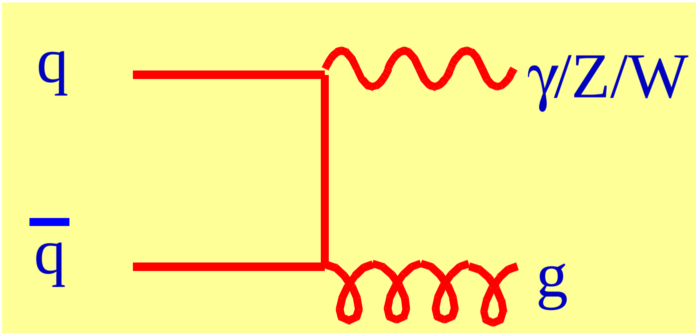
# What about the intrinsic $k_T$ of the partons?

Assume a Gaussian form:

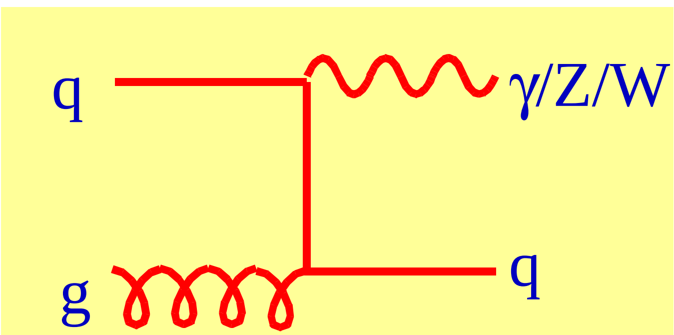
$$\frac{d^2\sigma}{d^2 p_T} \approx \sigma_0 \, e^{-p_T^2}$$



# For high $P_T$, we need a hard parton emission




annihilation




Compton

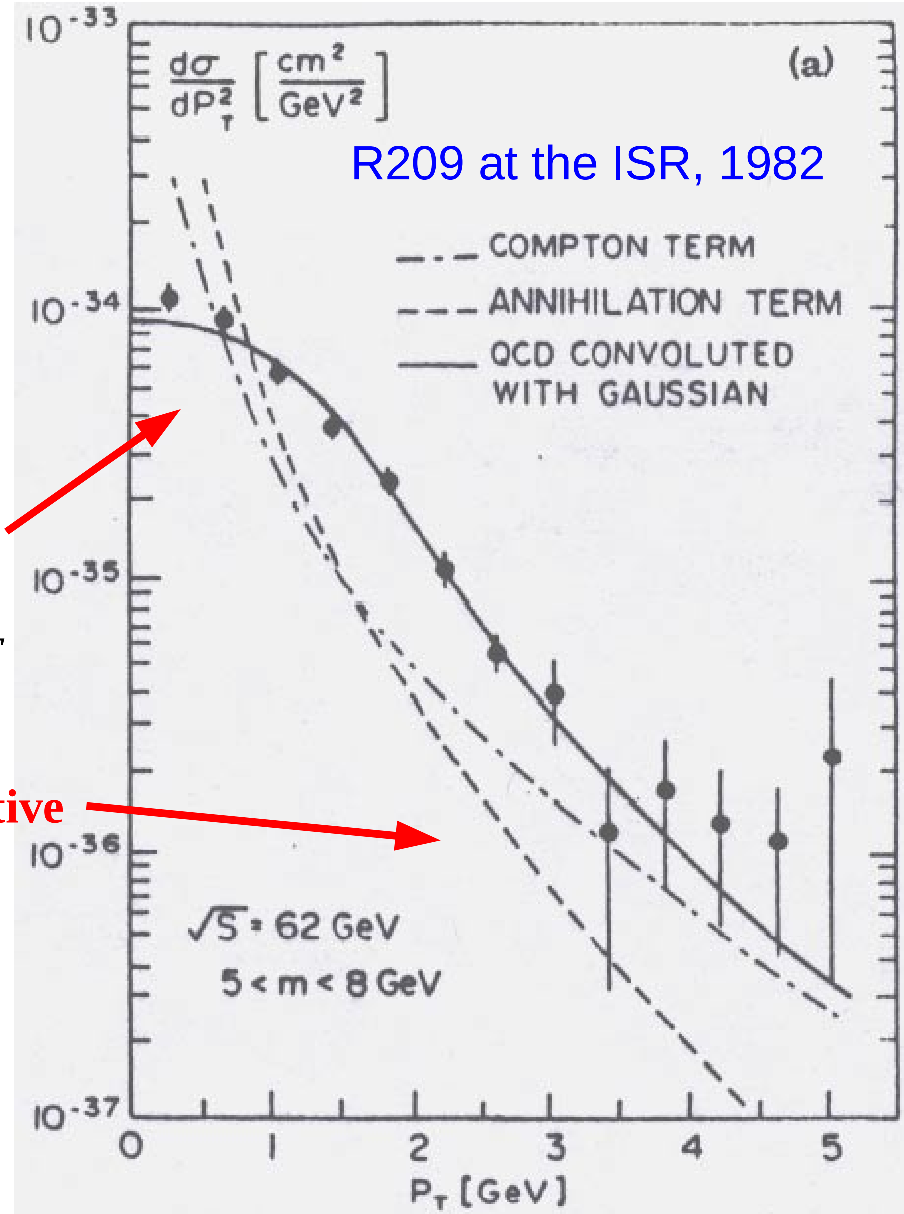
Combination of Gaussian & QCD corrections

**Gaussian**

$$e^{-p_T^2}$$

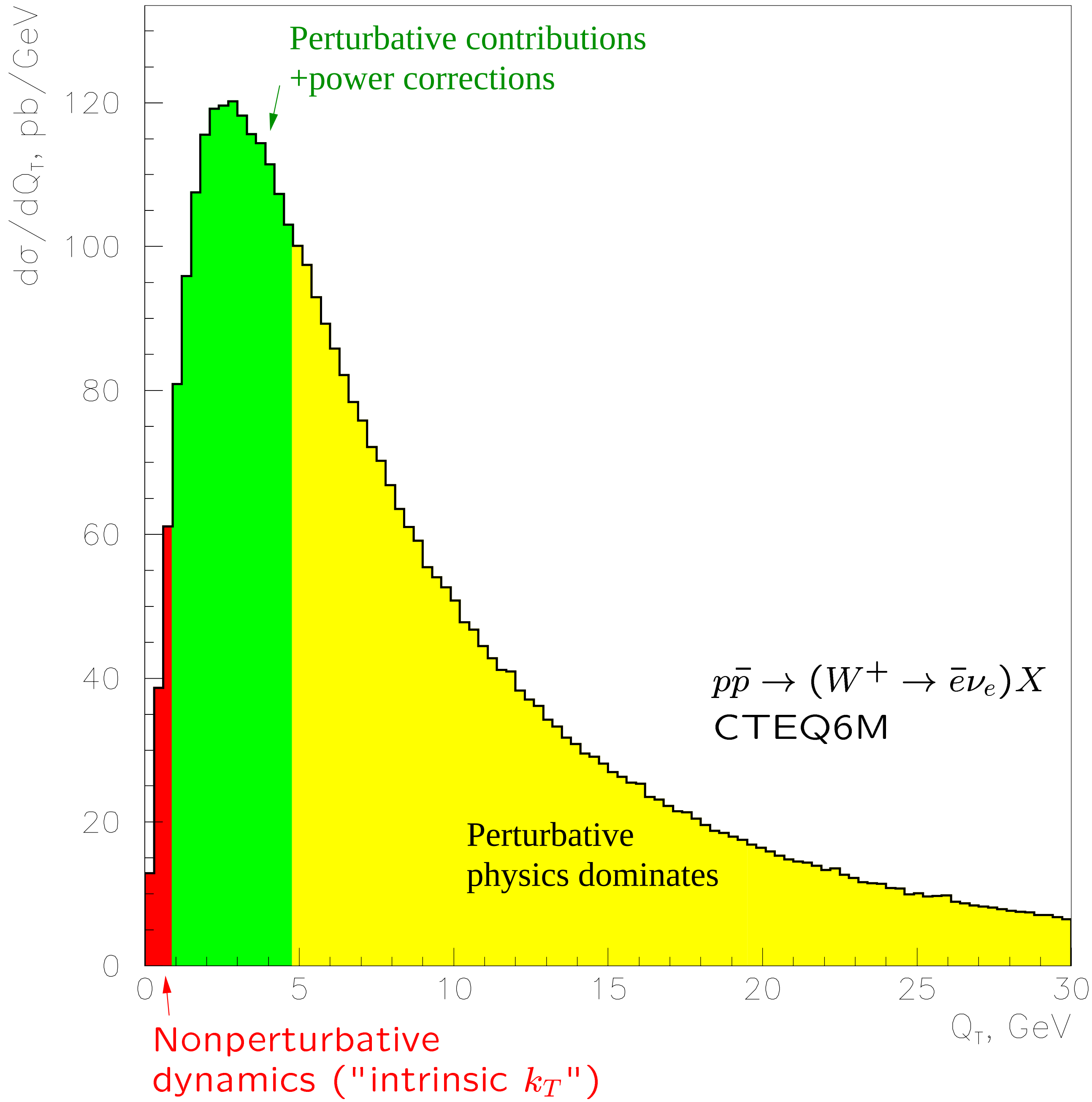
**Perturbative**

$$\frac{1}{p_T^2}$$



# The complete $P_T$ spectrum for the W boson

The full $P_T$ spectrum for the W-boson showing the different theoretical regions



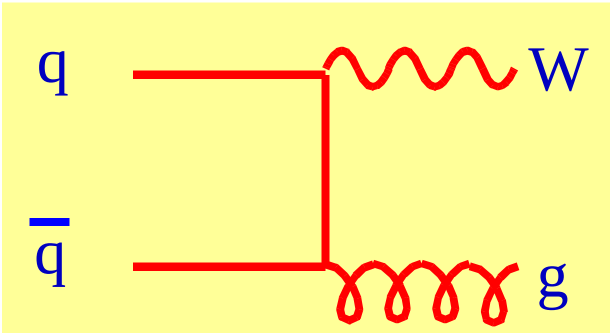
# Road map for Resummation

$\Rightarrow$

BEFORE

AFTER

# NLO $P_T$ distribution for the W boson




In the limit $P_T \to 0$

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \frac{4\alpha_s}{3\pi}\,\frac{\ln s/p_T^2}{p_T^2}$$

singular

finite

$$\int_0^s \frac{d\sigma}{d\tau\, dy\, dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} + O(\alpha_s)$$

$$\int_0^{p_T^2} \frac{d\sigma}{d\tau\, dy\, dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \left\{ 1 - \int_{p_T^2}^{s} \frac{4\alpha_s}{3\pi}\,\frac{\ln s/p_T^2}{p_T^2}\, d\,p_T^2 \right\}$$

$$= \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \left\{ 1 - \frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2} \right\}$$

effect of gluon emission

$$= \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \exp\left\{ \frac{-2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2} \right\}$$

assume this exponentiates

Parisi & Petronzio, NP B154, 427 (1979)
Dokshitzer, D'yakanov, Troyan, Phy. Rep. 58, 271 (1980)

# Resummation of soft gluons: Step #1

Differentiating the previous expression for $d^2\sigma/d\tau\, dy$

Sudakov
Form Factor

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \frac{4\alpha_s}{3\pi}\,\frac{\ln s/p_T^2}{p_T^2} \times \exp\left\{-\frac{2\alpha_s}{3\pi}\ln^2\frac{s}{p_T^2}\right\}$$

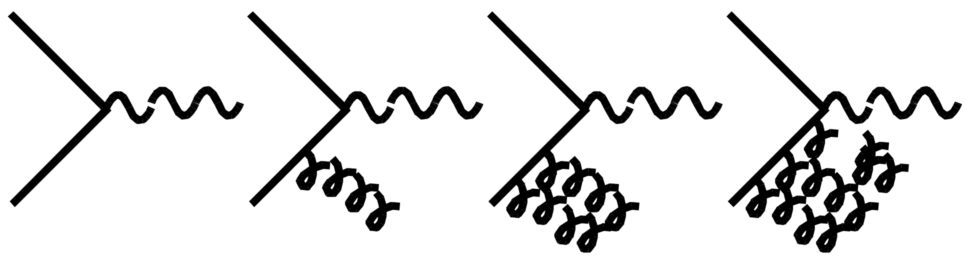
*finite at $p_T$=0*

We just resummed (exponentiated) an infinite series of soft gluon emissions

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2!} - \frac{(\alpha_s L^2)^3}{3!} + \ \ldots$$

$$L = \ln\frac{s}{p_T^2}$$

Soft gluon emissions
treated as uncorrelated



*I've skipped over some details ..*


Parisi & Petronzio, NP B154, 427 (1979)
Dokshitzer, D'yakanov, Troyan, Phy. Rep. 58, 271 (1980)

Curci, Greco, Srivastava, PRL 43, 834 (1979); NP B159, 451 (1979)
Jeff Owens, 2000 CTEQ Summer School Lectures

# We skipped over a few details ...

1) We summed only the leading logarithmic singularity, $\alpha_s L^2$.

We'll need to do better to ensure convergence of perturbation series

2) We assumed exponentiation; proof of this is non-trivial.

The existence of two scales $(Q,p_T) \equiv (Q,q_T)$ yields 2 logs per loop

3) Gluon emission was assumed to be uncorrelated.

This leads to too strong a suppression at $P_T=0$.

Will need to impose momentum conservation for $P_T$.

4) In the limit $P_T \to 0$, terms of order $\alpha_S(\mu=P_T) \to \infty$;

Must handle this Non-Perturbative region.

# 1) We summed only the leading logarithmic singularity

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2}\left\{1 + \alpha_s^1 L^2 + \alpha_s^2 L^4 + \ldots\right\}$$

we resum these terms

$$\frac{\alpha_s L}{q_T^2}\left\{\quad + \alpha_s^1 L^1 + \alpha_s^2 L^3 + \alpha_s^3 L^5 + \ldots\right\}$$

we miss these terms

$$L = \ln \frac{s}{p_T^2}$$

The terms we are missing are suppressed by $\alpha_s L$, not $\alpha_s$!

*If (somehow) we could sum the sub-leading log ...*

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2} e^{\alpha_s (L^2 + L)}$$

we resum these terms

we miss these terms

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\left\{\alpha_s^1 (L^1 + 1) + \alpha_s^2 (L^3 + L^2) + \alpha_s^3 (L^5 + L^4) + \ldots\right\}$$

$$\frac{1}{q_T^2}\left\{\quad + \alpha_s^2 (L^1 + 1) + \alpha_s^3 (L^3 + L^2) + \alpha_s^4 (L^5 + L^4) + \ldots\right\}$$

Now, the terms we are missing are suppressed only by $\alpha_s$!

## 2) We assumed exponentiation; proof is non-trivial

Review where the logs come from

Review one-scale problem *(Q)*

resummation via RGE

Review two-scale problem $(Q,q_T)$

resummation via RGE+ Gauge Invariance

# Where do the Logs come from?

# Total Cross Section: $\sigma(e^+e^-)$ at 3 Loops

$$\sigma(Q^2)=\sigma_0\Bigg\{1+\frac{\alpha_s(Q^2)}{4\pi}(3C_F)+\left[\frac{\alpha_s(Q^2)}{4\pi}\right]^2\left[-C_F^2\left(\frac{3}{2}\right)+C_FC_A\left(\frac{123}{2}-44\zeta(3)\right)-C_FTn_f(-22+16\zeta(3))\right]$$

$$+\left[\frac{\alpha_s(Q^2)}{4\pi}\right]^3\Bigg[C_F^3\left(-\frac{69}{2}\right)+C_F^2C_A(-127-572\zeta(3)+880\zeta(5))$$

$$+C_FC_A^2\left(\frac{90445}{54}-\frac{10948}{9}\zeta(3)+\frac{440}{3}\zeta(5)\right)$$

$$+C_F^2Tn_f(-29+304\zeta(3)-320\zeta(5))+C_FC_ATn_f\left(-\frac{31040}{27}+\frac{7168}{9}\zeta(3)+\frac{160}{3}\zeta(5)\right)$$

$$+C_FT^2n_f^2\left(\frac{4832}{27}-\frac{1216}{9}\zeta(3)\right)-C_F\pi^2\left(\frac{11}{3}C_A-\frac{4}{3}Tn_f\right)^2+\frac{\left(\sum_f Q_f\right)^2}{\left(N\sum_f Q_f^2\right)}\frac{D}{16}\left(\frac{176}{3}-128\zeta(3)\right)\Bigg]\Bigg\}. \tag{5.1}$$

Rev. Mod. Phys., Vol. 67, No. 1, January 1995

One mass scale: $Q^2$. No logarithms!!!

# Drelly-Yan at 2 Loops:



$$
\begin{aligned}
H^{(2),S+V}_{q\bar q}(z) = {} & \left[\frac{\alpha_s}{4\pi}\right]^2 \delta(1-z)\Bigg\{ C_A C_F \left[ \left[\tfrac{193}{3} - 24\zeta(3)\right]\ln\left[\frac{Q^2}{M^2}\right] - 11\ln^2\left[\frac{Q^2}{M^2}\right] - \tfrac{12}{5}\zeta(2)^2 + \tfrac{592}{9}\zeta(2) + 28\zeta(3) - \tfrac{1535}{12} \right] \\
& + C_F^2 \left[ [18 - 32\zeta(2)]\ln^2\left[\frac{Q^2}{M^2}\right] + [24\zeta(2) - 176\zeta(3) - 93]\ln\left[\frac{Q^2}{M^2}\right] \right. \\
& \left. + \tfrac{8}{5}\zeta(2)^2 - 70\zeta(2) - 60\zeta(3) + \tfrac{511}{4} \right] \\
& + n_f C_F \left[ 2\ln^2\left[\frac{Q^2}{M^2}\right] - \tfrac{34}{3}\ln\left[\frac{Q^2}{M^2}\right] + 8\zeta(3) - \tfrac{112}{9}\zeta(2) + \tfrac{127}{6} \right]\Bigg\} \\
& + C_A C_F \left[ -\tfrac{44}{3}\mathcal{D}_0(z)\ln^2\left[\frac{Q^2}{M^2}\right] + \left\{\left[\tfrac{536}{9} - 16\zeta(2)\right]\mathcal{D}_0(z) - \tfrac{176}{3}\mathcal{D}_1(z)\right\}\ln\left[\frac{Q^2}{M^2}\right] \right. \\
& \left. - \tfrac{176}{3}\mathcal{D}_2(z) + \left[\tfrac{1072}{9} - 32\zeta(2)\right]\mathcal{D}_1(z) + \left[56\zeta(3) + \tfrac{176}{3}\zeta(2) - \tfrac{1616}{27}\right]\mathcal{D}_0(z) \right] \\
& + C_F^2 \left[ [64\mathcal{D}_1(z) + 48\mathcal{D}_0(z)]\ln^2\left[\frac{Q^2}{M^2}\right] + \{192\mathcal{D}_2(z) - 96\mathcal{D}_1(z) - [128 + 64\zeta(2)]\mathcal{D}_0(z)\}\ln\left[\frac{Q^2}{M^2}\right] \right. \\
& \left. + 128\mathcal{D}_3(z) - (128\zeta(2) + 256)\mathcal{D}_1(z) + 256\zeta(3)\mathcal{D}_0(z) \right] \\
& + n_f C_F \left[ \tfrac{8}{3}\mathcal{D}_0(z)\ln^2\left[\frac{Q^2}{M^2}\right] + \left[\tfrac{32}{3}\mathcal{D}_1(z) - \tfrac{80}{9}\mathcal{D}_0(z)\right]\ln\left[\frac{Q^2}{M^2}\right] + \tfrac{32}{3}\mathcal{D}_2(z) - \tfrac{160}{9}\mathcal{D}_1(z) - \left[\tfrac{224}{27} - \tfrac{32}{3}\zeta(2)\right]\mathcal{D}_0(z) \right].
\end{aligned}
\tag{7.14}
$$

Two mass scales: $\{Q^2, M^2\}$. Logarithms!!!

# Renormalization Group Equation

More Differential Quantities ⇒ More Mass Scales ⇒ More Logs!!!

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right)$$

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right) \quad and \quad \ln\left(\frac{q_T^2}{\mu^2}\right)$$

How do we resum logs? Use the Renormalization Group Equation

For a physical observable R:

$$\mu \frac{dR}{d\mu} = 0$$

Using the chain rule:

$$\left\{\mu^2 \frac{\partial}{\partial \mu^2} + \underbrace{\left[\mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2}\right]}_{\beta(\alpha_s(\mu))} \frac{\partial}{\partial \alpha_s(\mu^2)}\right\} R(\mu^2, \alpha_s(\mu^2)) = 0$$

Solution⇒ $\ln\left(\frac{Q^2}{\mu^2}\right) = \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{dx}{\beta(x)}$

# Renormalization Group Equation: *OVER SIMPLIFIED!*

$$\left\{\mu^2 \frac{\partial}{\partial \mu^2} + \beta\left(\alpha_s(\mu)\right) \frac{\partial}{\partial \alpha_s(\mu^2)}\right\} R(\mu^{2,}\alpha_s(\mu^2)) = 0$$

If we expand R in powers of $\alpha_s$, and we know $\beta$, we then know $\mu$ dependence of R.

$$R(\mu, Q, \alpha_s(\mu^2)) = R_0 + \alpha_s(\mu^2)\, R_1\left[\ln\left(Q^2/\mu^2\right) + c_1\right] + \alpha_s^2(\mu^2)\, R_2\left[\ln^2\left(Q^2/\mu^2\right) + \ln\left(Q^2/\mu^2\right) + c_2\right] + O(\alpha_s^3(\mu^2))$$

Since $\mu$ is arbitrary, choose $\mu$=Q.

$$R(Q, Q, \alpha_s(Q^2)) = R_0 + \alpha_s(Q^2)\, R_1\left[0 + c_1\right] + \alpha_s^2(Q^2)\, R_2\left[0 + 0 + c_2\right] + \ldots$$

We just summed the logs

# Two-Scale Problems

For $R(\mu,Q,\alpha_s)$, we could resum $\ln(Q^2/\mu^2)$ by taking $Q=\mu$.

What about $R(\mu,Q,q_T,\alpha_s)$; how do we resum $\ln(Q^2/\mu^2)$ and $\ln(q_T^2/\mu^2)$.

Are we stuck? Can't have $\mu^2=Q^2$ and $\mu^2=q_T^2$ at the same time!

Solution: Use Gauge Invariance; cast in similar form to RGE

Use axial-gauge with axial vector $\xi$.

This enters the cross section in the form: $(\xi\bullet p)$.

$$\sigma\left(x, \frac{Q^2}{\mu^2}, \frac{(p\cdot\xi)^2}{\mu^2}, \ldots\right)$$

$$\frac{d\sigma}{d\mu^2}=0$$

RGE allows us to vary $\mu$ to resum logs

$$\frac{d\sigma}{d\,(p\cdot\xi)^2}=0$$

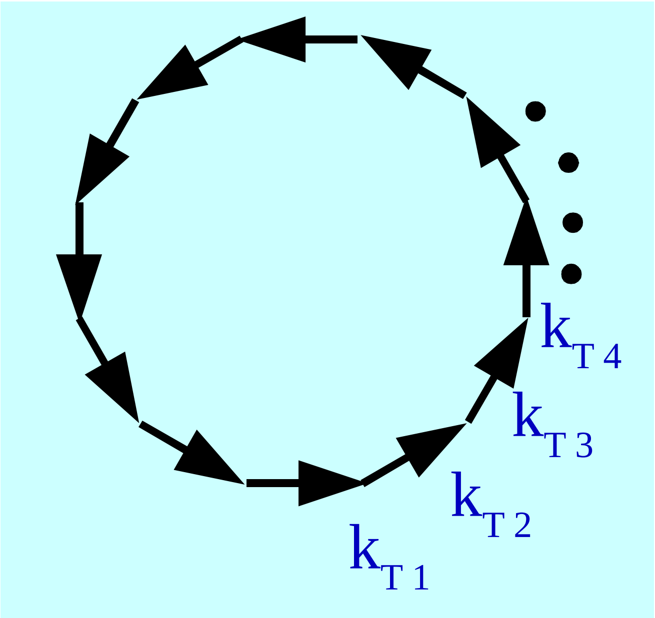
Gauge invariance allows us to vary $(\xi\bullet p)$ to resum logs

It is covenient to transform to impact parameter space (b-space) to implement this mechanism

The details will fill multiple lectures:
See Sterman TASI 1995; Soper CTEQ 1995

## 3) We assumed gluon emission was uncorrelated

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \frac{\ln s/p_T^2}{p_T^2} \times \exp\left\{-\frac{2\alpha_s}{3\pi}\ln^2\frac{s}{p_T^2}\right\}$$




This leads to too strong a suppression at $P_T$=0.
Need to impose momentum conservation for $P_T$.

A particle can receive finite $k_T$ kicks,
yet still have $P_T$=0

A convenient way to impose transverse momentum conservation is in impact parameter space (b-space) via the following relation:

$$\delta^{(2)}\left(\sum_{i=1}^{n}\vec{k}_{i\,T} - \vec{p}_T\right) = \frac{1}{(2\pi)^2}\int d^2b\, e^{-i\vec{b}\cdot\vec{p}_T}\prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{i\,T}}$$

# 4) We encounter Non-Perturbative Physics

$$S(b,Q) = \int_{\sim 1/b^2}^{\sim Q^2} \frac{d\mu^2}{\mu^2} \left\{ A(\alpha_s(\mu^2)) \ln\left(\frac{Q^2}{\mu^2}\right) + B(\alpha_s(\mu^2)) \right\}$$

as b $\to \infty$, $\alpha_S(\sim 1/b) \to \infty$. **PROBLEM!!!**

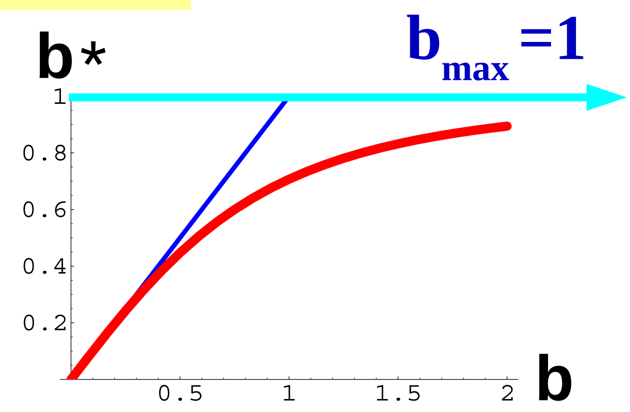
**Solution**: Use a Non-Perturbative Sudakov form factor ($S_{NP}$) in the region of large b (small $q_T$)

$$\tilde{\sigma}(b) \sim e^{S(b)} \to e^{S(b_*)} * e^{S_{NP}(b)}$$

with

$$b_* = \frac{b}{\sqrt{1+b^2/b_{max}^2}}$$

Note, as b $\to \infty$, $b_* \to b_{max}$.



# A Brief *(but incomplete)* History of Non-Perturbative Corrections

Original CSS: $$S_{NP}^{CSS}(b) = h_1(b,\xi_a)+h_2(b,\xi_b)+h_3(b)\ln Q^2$$

J. Collins and D. Soper, *Nucl.Phys.* **B193** 381 (1981);
erratum: **B213** 545 (1983); J. Collins, D. Soper, and G. Sterman, *Nucl. Phys.* **B250** 199 (1985).

Davies, Webber, and Stirling (DWS): $$S_{NP}^{DWS}(b) = b^2\left[g_1+g_2\ln(b_{max}Q^2)\right]$$

C. Davies and W.J. Stirling, Nucl. Phys. B244 337 (1984);
C. Davies, B. Webber, and W.J. Stirling, Nucl. Phys. B256 413 (1985).

Ladinsky and Yuan (LY): $$S_{NP}^{LY}(b) = g_1 b\left[b+g_3\ln(100\,\xi_a\xi_b)\right]+g_2 b^2\ln(b_{max}Q)$$

G.A. Ladinsky and C.P. Yuan, Phys. Rev. D50 4239 (1994);
F. Landry, R. Brock, G.A. Ladinsky, and C.P.Yuan, Phys. Rev. D63 013004 (2001).

“BLNY”: $$S_{NP}^{BLNY}(b) = b^2\left[g_1+g_1g_3\ln(100\,\xi_a\xi_b)+g_2\ln(b_{max}Q)\right]$$

F. Landry, “Inclusion of Tevatron Z Data into Global Non-Perturbative QCD Fitting”, Ph.D. Thesis, Michigan State University, 2001.
F. Landry, R. Brock, P. Nadolsky, and C.P.Yuan, PRD67, 073016 (2003)

“$q_T$ resummation”: $$\widetilde{F}^{NP}(q_T) = 1-e^{-\tilde{a}\,q_T^2}$$ *(not in b-space)*

R.K. Ellis, Sinisa Veseli, Nucl.Phys. B511 (1998) 649-669
R.K. Ellis, D.A. Ross, S. Veseli, Nucl.Phys. B503 (1997) 309-338

Functional Extrapolation:

J. Qui, X. Zhang, PRD63, 114011 (2001); E. Berger, J. Qiu, PRD67, 034023 (2003)

Analytical Continuation:

A. Kulesza, G. Sterman, W. vogelsang, PRD66, 014011 (2002)

# Recap: Where have we been???

1) We now summed the two leading logarithmic singularities, $\alpha_s(L^2+L)$.

2) We still assumed exponentiation; but sketched ingredients of proof.

The existence of two scales $(Q,p_T)\equiv(Q,q_T)$ yields 2 logs per loop

Use Renormalization Group + Gauge Invariance

Transformation to b-space

3) Gluon emission was assumed to be uncorrelated.

Impose momentum conservation for $P_T$. *(In b-space)*

4) Introduced Non-Perturbative function for small $q_T$ (large b) region.

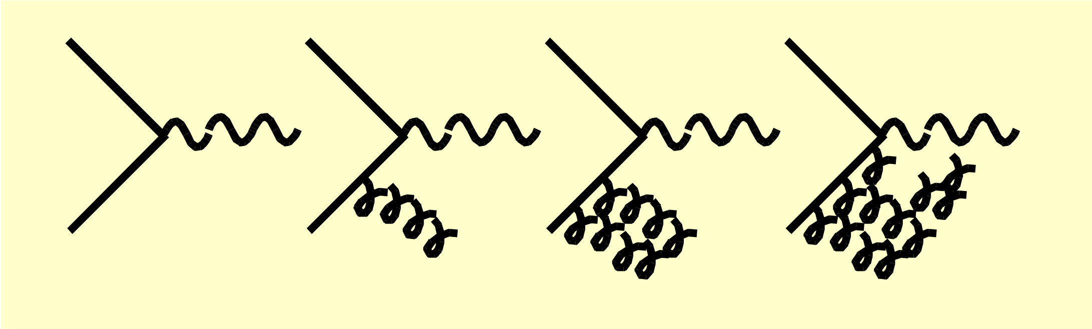
# What do we get for the cross section

$$\frac{d\sigma}{dy\,dQ^2\,dq_T^2} = \frac{1}{(2\pi)^2}\int_0^\infty d^2b\;e^{i\,b\cdot q_T}\,\widetilde{W}(b,Q)\;e^{-S(b_*,Q)+S_{NP}(b,Q)}$$

with

$$-S(b,Q) = -\int_{\sim 1/b^2}^{\sim Q^2}\frac{d\mu^2}{\mu^2}\left\{A\ln\left(\frac{Q^2}{\mu^2}\right) + B\right\}$$

where we have resummed the soft gluon contributions



*I've left out A LOT of material*

## Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

Let's expand out the resummed expression:

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2}\, e^{\alpha_s (L^2 + L)} \sim \frac{1}{q_T^2}\left\{\alpha_s L + \alpha_s^2 (L^3 + L^2) + \ldots\right\}$$

Compare the above with the perturbative and asymptotic results:

$$d\sigma_{resum} \sim \left\{\alpha_s L + \alpha_s^2 (L^3 + L^2 + 0 + 0) + \alpha_s^3 (L^5 + L^4) + \ldots\right\}$$

$$d\sigma_{pert} \sim \left\{\alpha_s L + \alpha_s^2 (L^3 + L^2 + L^1 + 1) + \alpha_s^3 (0 + 0)\right\}$$

$$d\sigma_{asym} \sim \left\{\alpha_s L + \alpha_s^2 (L^3 + L^2 + 0 + 0) + \alpha_s^3 (0 + 0)\right\}$$

Note that $\sigma_{ASYM}$ removes overlap between $\sigma_{RESUM}$ and $\sigma_{PERT}$.

We expect:

$\sigma_{RESUM}$ is a good representation for $q_T \sim 0$

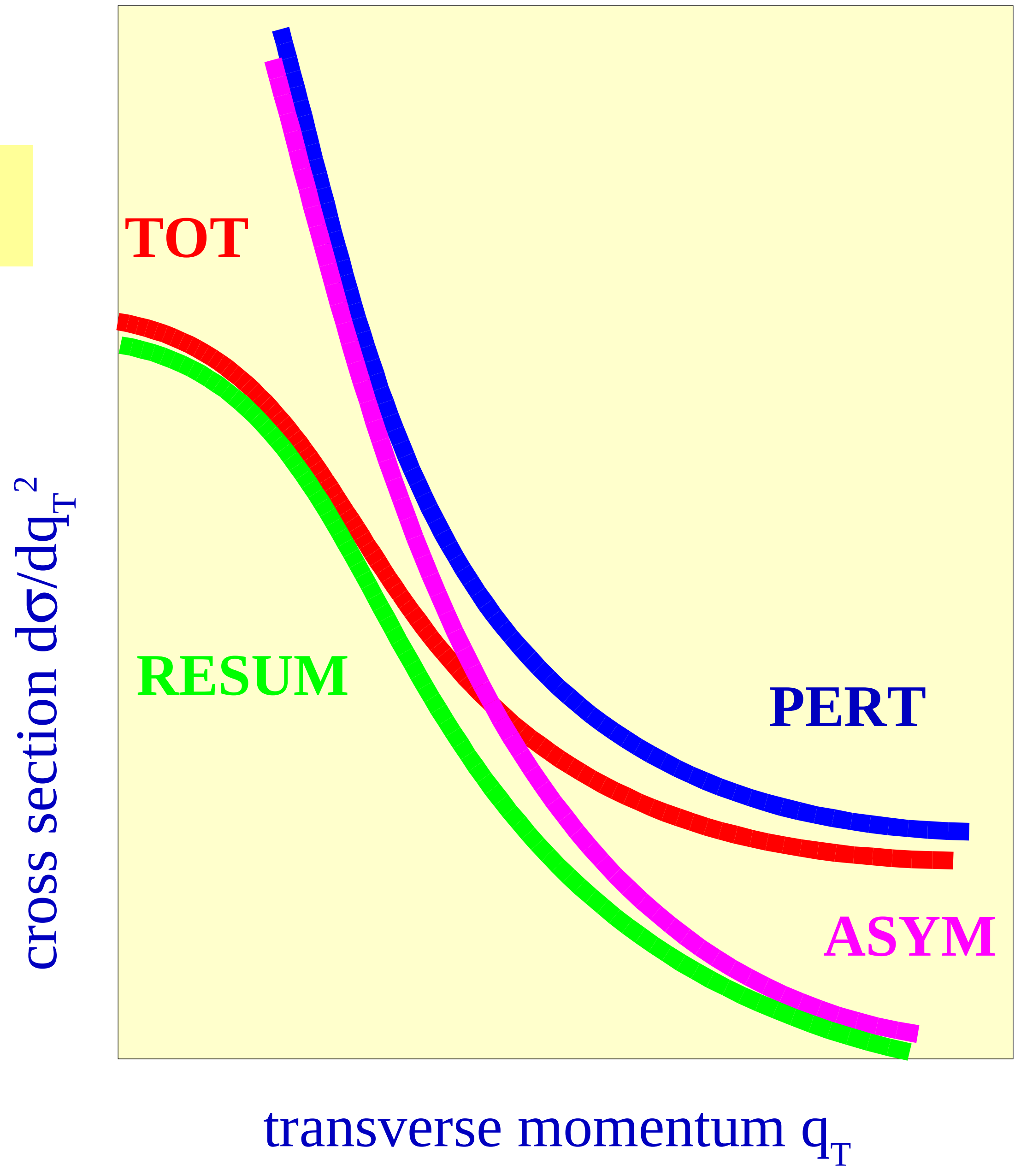
$\sigma_{PERT}$ is a good representation for $q_T \sim M_W$

# Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

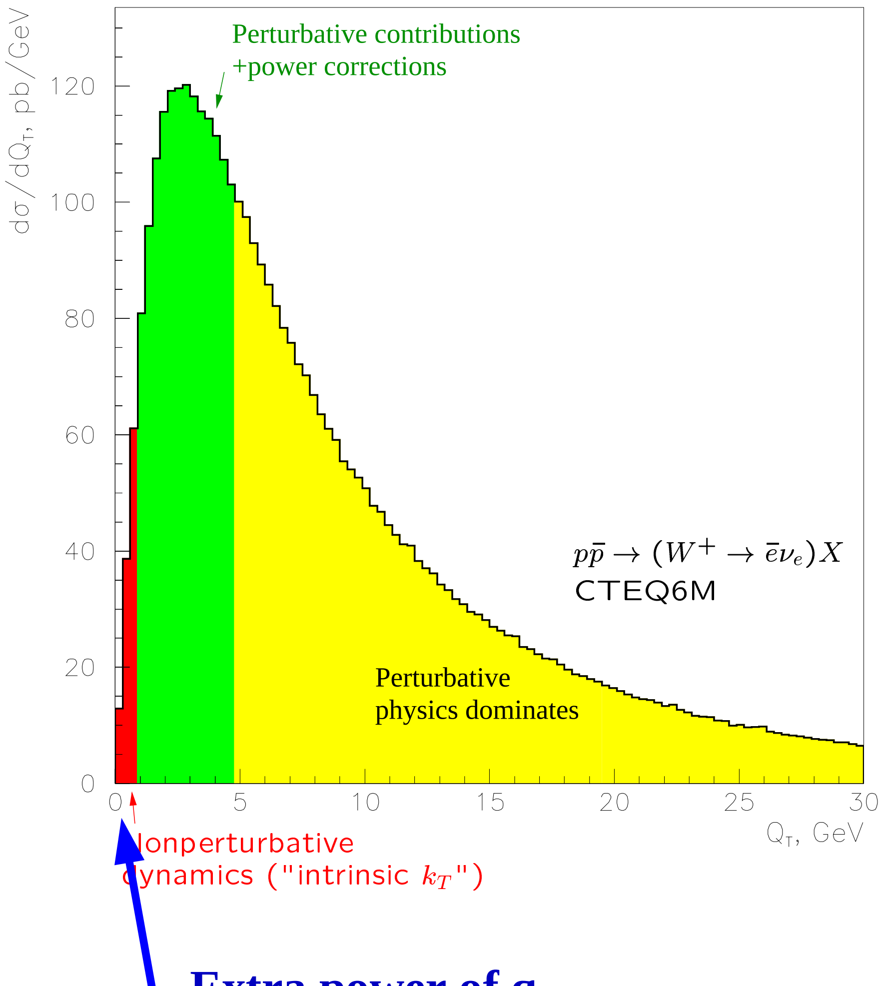
$$\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$$

$\sigma_{RESUM}$ for $q_T \sim 0$

$\sigma_{PERT}$ for $q_T \sim M_W$

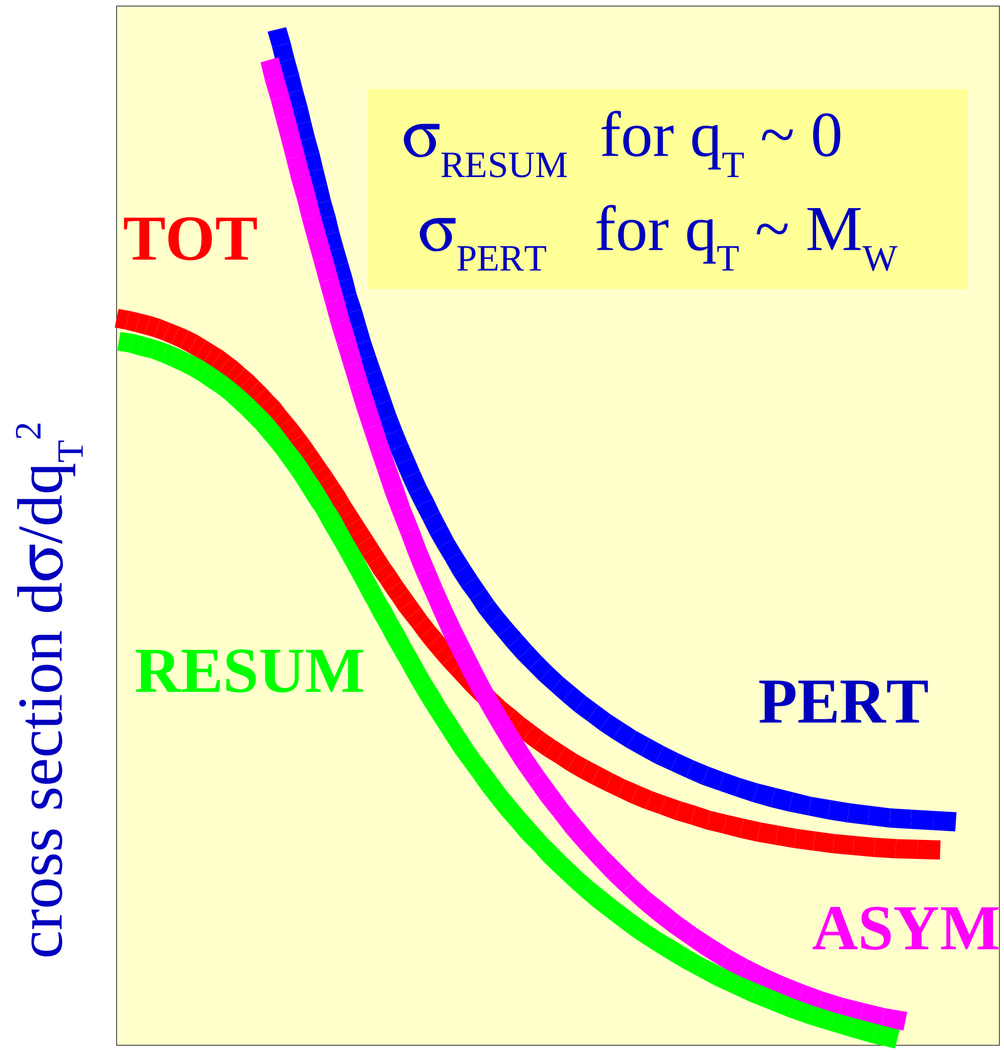


## Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$




**Extra power of $q_T$**

**$d\sigma/dq_T^{\,1}$**

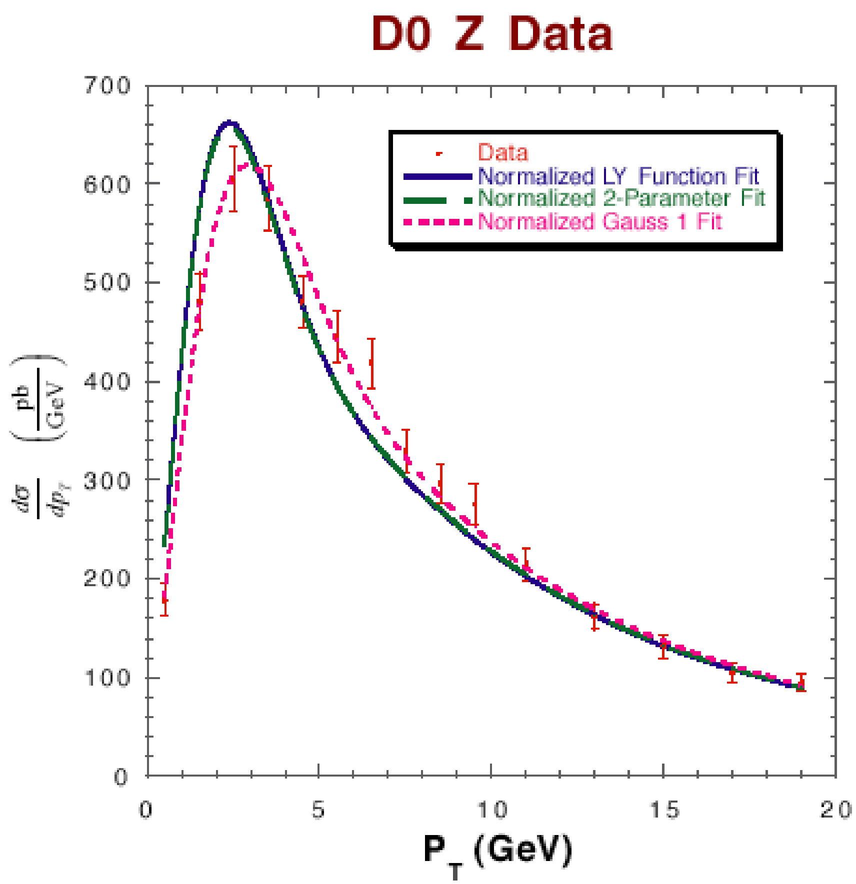


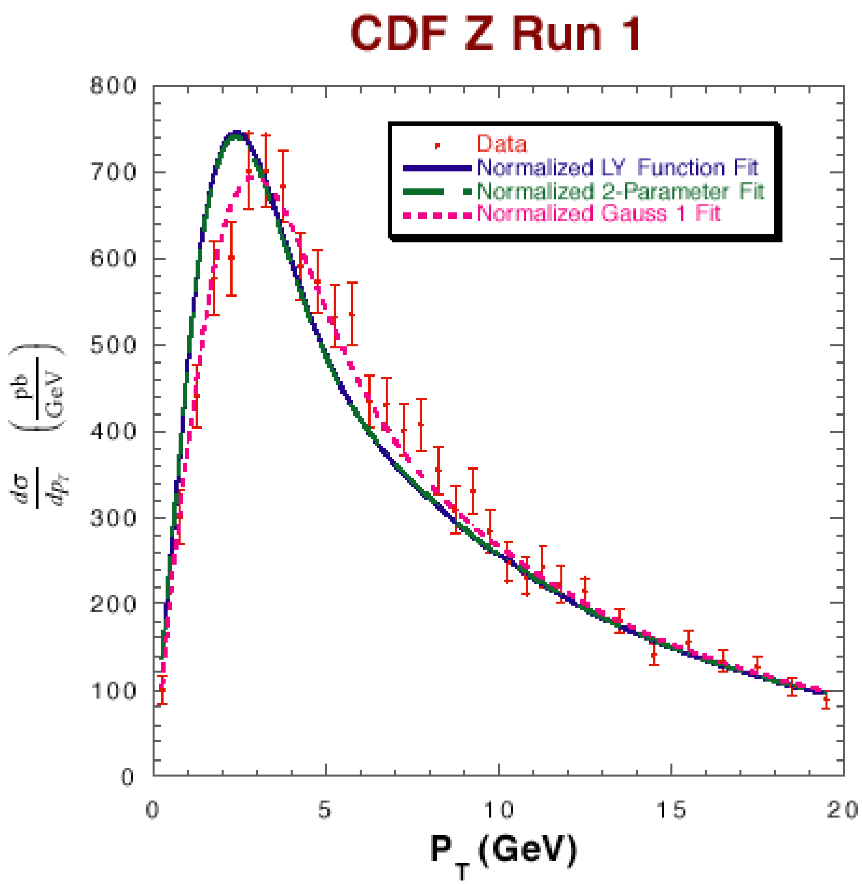

transverse momentum $q_T$

$\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

# Let's compare with some real results

We'll look at Z data where we can measure both leptons for $Z \to e^+e^-$




different $S_{NP}(b,Q)$ functions yield difference at small $q_T$.

Let's return

to the

measurement

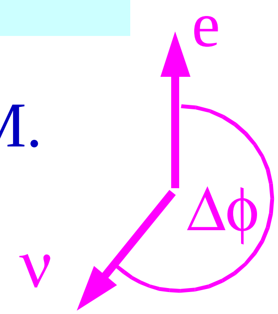
of $M_W$

## Transverse Mass Distribution

We can measure $d\sigma/dp_T$ and look for the Jacobian peak.
However, there is another variable that is relatively insensitive to $p_T(W)$.

Transverse Mass $\quad M_T^2(e,\nu) = \left(|\vec{p}_{eT}|+|\vec{p}_{\nu T}|\right)^2 - \left(\vec{p}_{eT}+\vec{p}_{\nu T}\right)^2$

Invariant Mass $\quad M^2(e,\nu) = \left(|\vec{p}_{e}|+|\vec{p}_{\nu}|\right)^2 - \left(\vec{p}_{e}+\vec{p}_{\nu}\right)^2$




In the limit of vanishing longitudinal momentum, $M_T \sim M$.
$M_T$ is invariant under longitudinal boosts.

$M_T$ can also be expressed as: $\quad M_T^2(e,\nu) = 2|\vec{p}_{eT}||\vec{p}_{\nu T}|\left(1-\cos\Delta\phi_{e\nu}\right)$

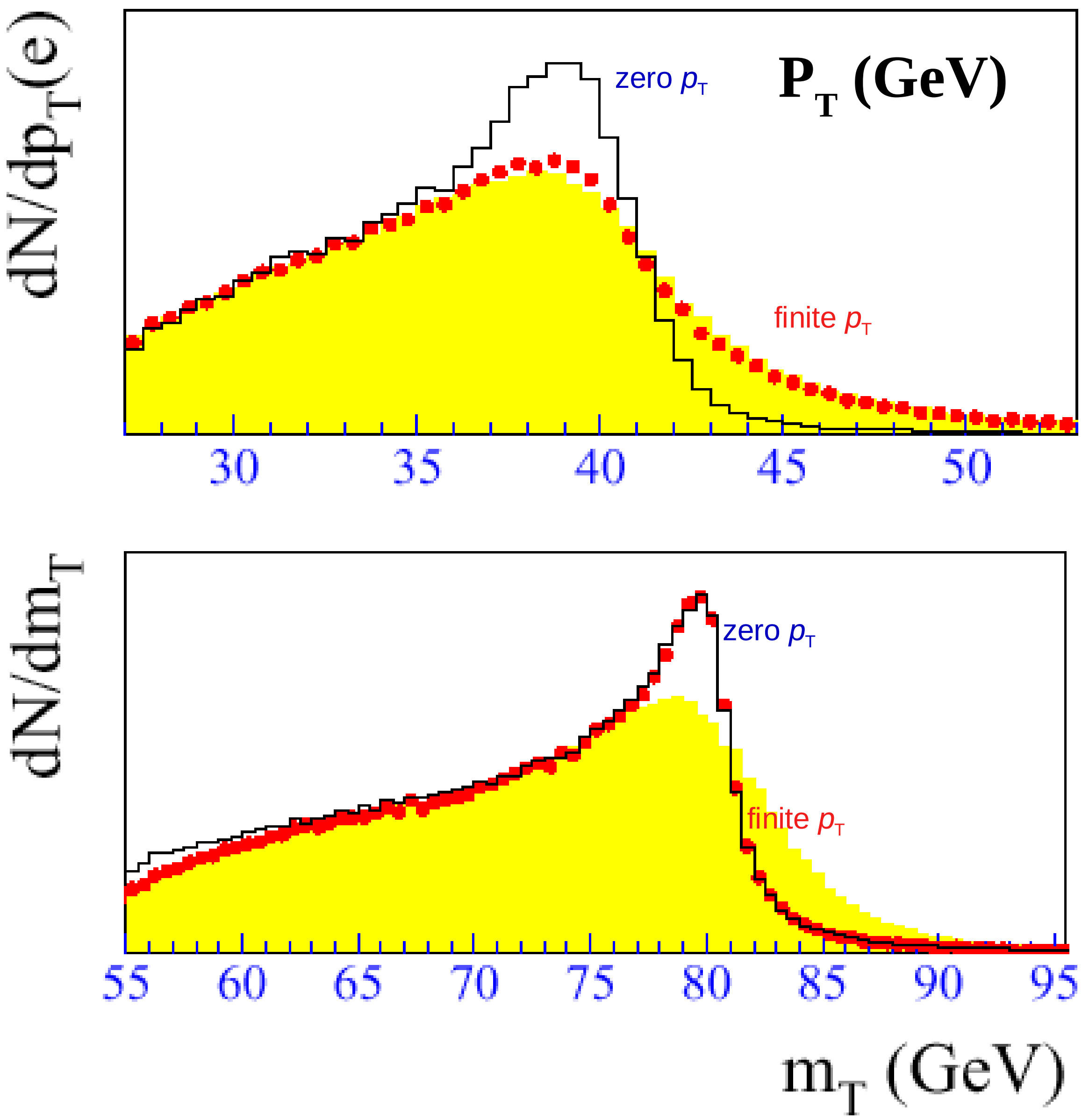
For small values of $P_T^W$, $M_T$ is invariant to leading order.

Exercise:

a) Verify the above definitions of $M_T$ are ≡.

b) For $p_{Te} = +p^* + p_T^W/2$ and $p_{T\nu} = -p^* + p_T^W/2$; verify $M_T$ is invariant to leading order in $p_T^W$.

# Compare $P_T$ and Transverse Mass Distribution




$M_T$ distribution is much less sensitive to $P_T$ of W

Still, we need $P_T$ distribution of W to extract mass and width with precision

PDF and $p_T$(W) uncertainties will need to be controlled: currently uncertainty: ~10-15 & 5-10 MeV/$c^2$

Statistical precision in Run II will be miniscule…placing an enormous burden on control of modeling uncertainties.

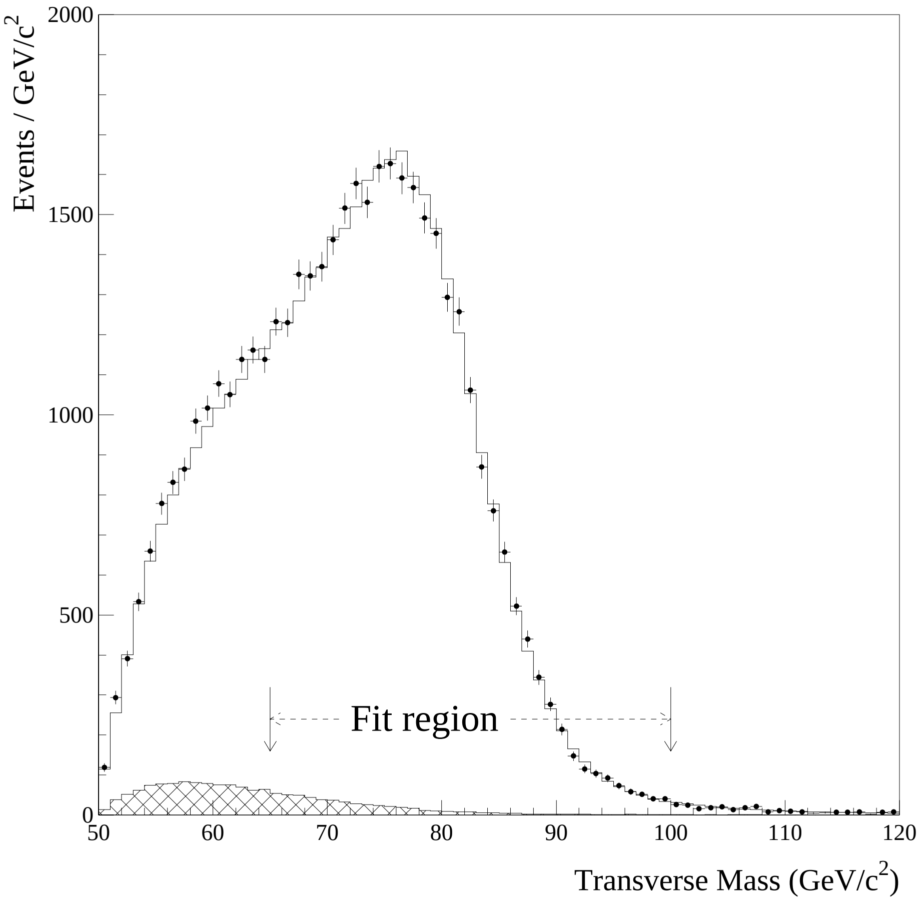
# The Future:

Tevatron Run II ... *happening now*

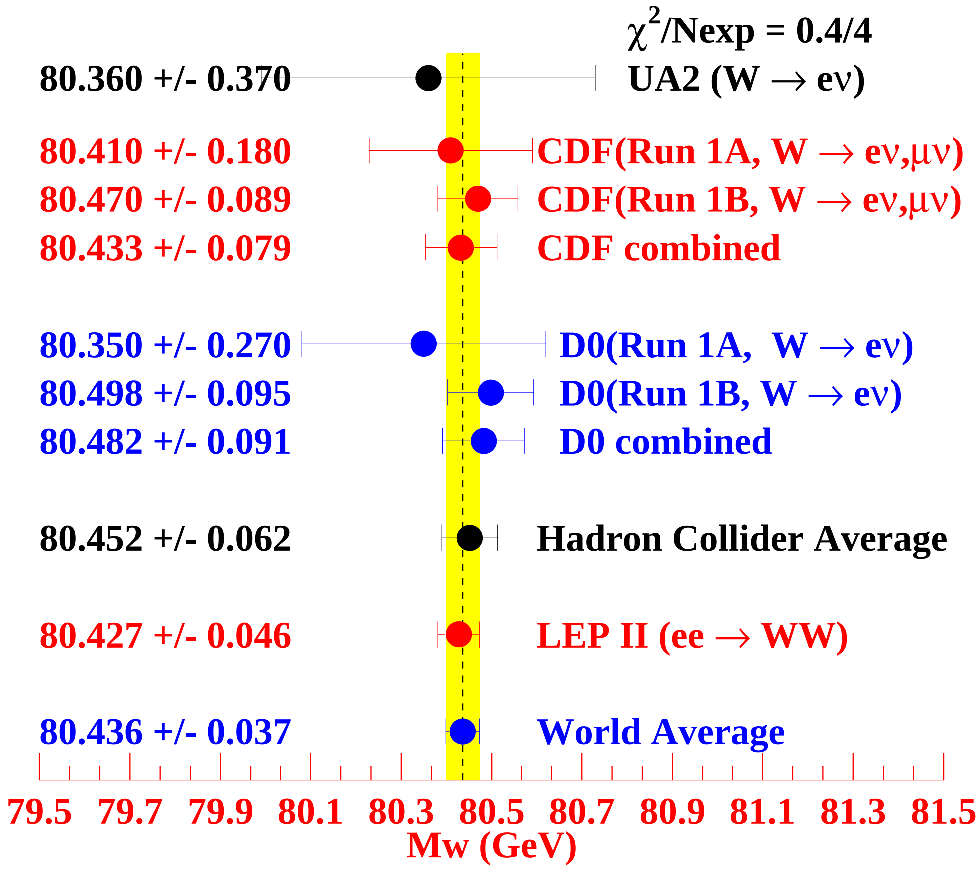
LHC ... *happening soon*

# Transverse Mass Distribution and $M_W$ Measurement

## Transverse Mass Distribution from CDF




## Combined World Measurements of $M_W$




T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
Measurement of the W boson mass with the Collider Detector at Fermilab,

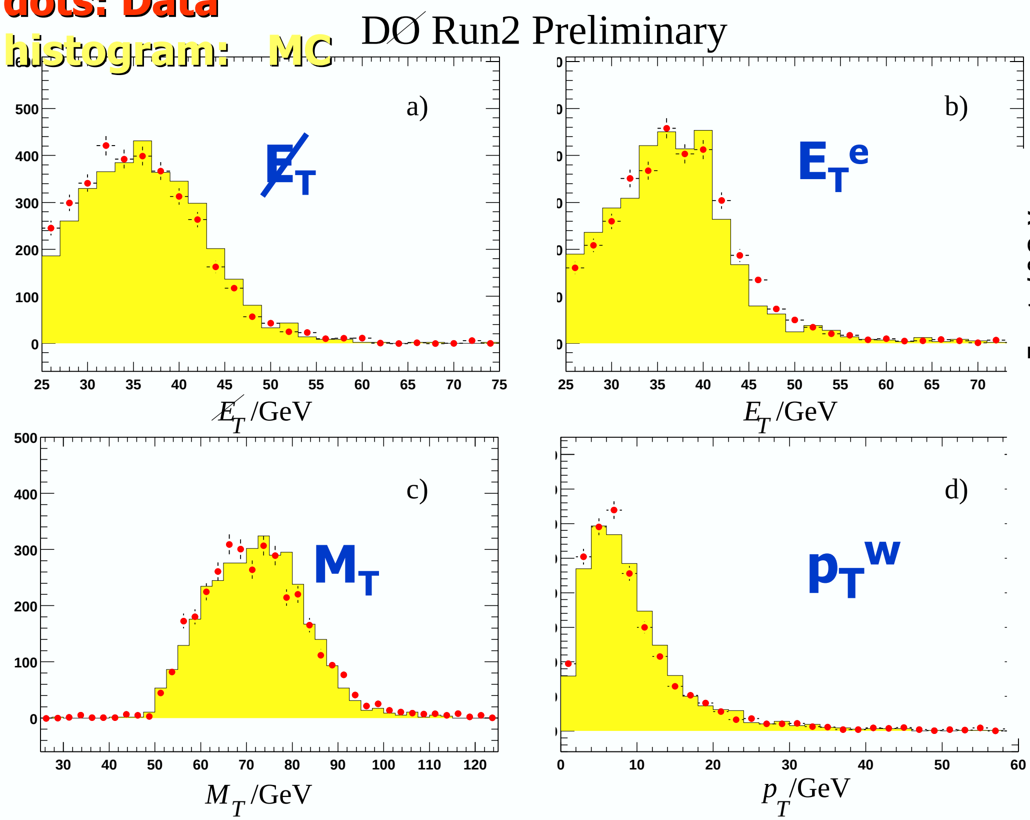
Preliminary
Run II
measurements

# Electroweak Physics

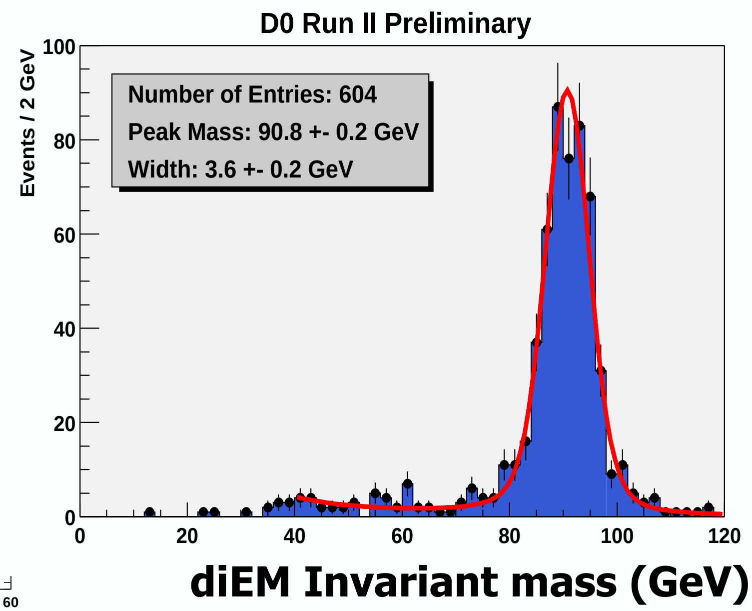
## High priority measurements

- $W \to e\nu$ cross-section

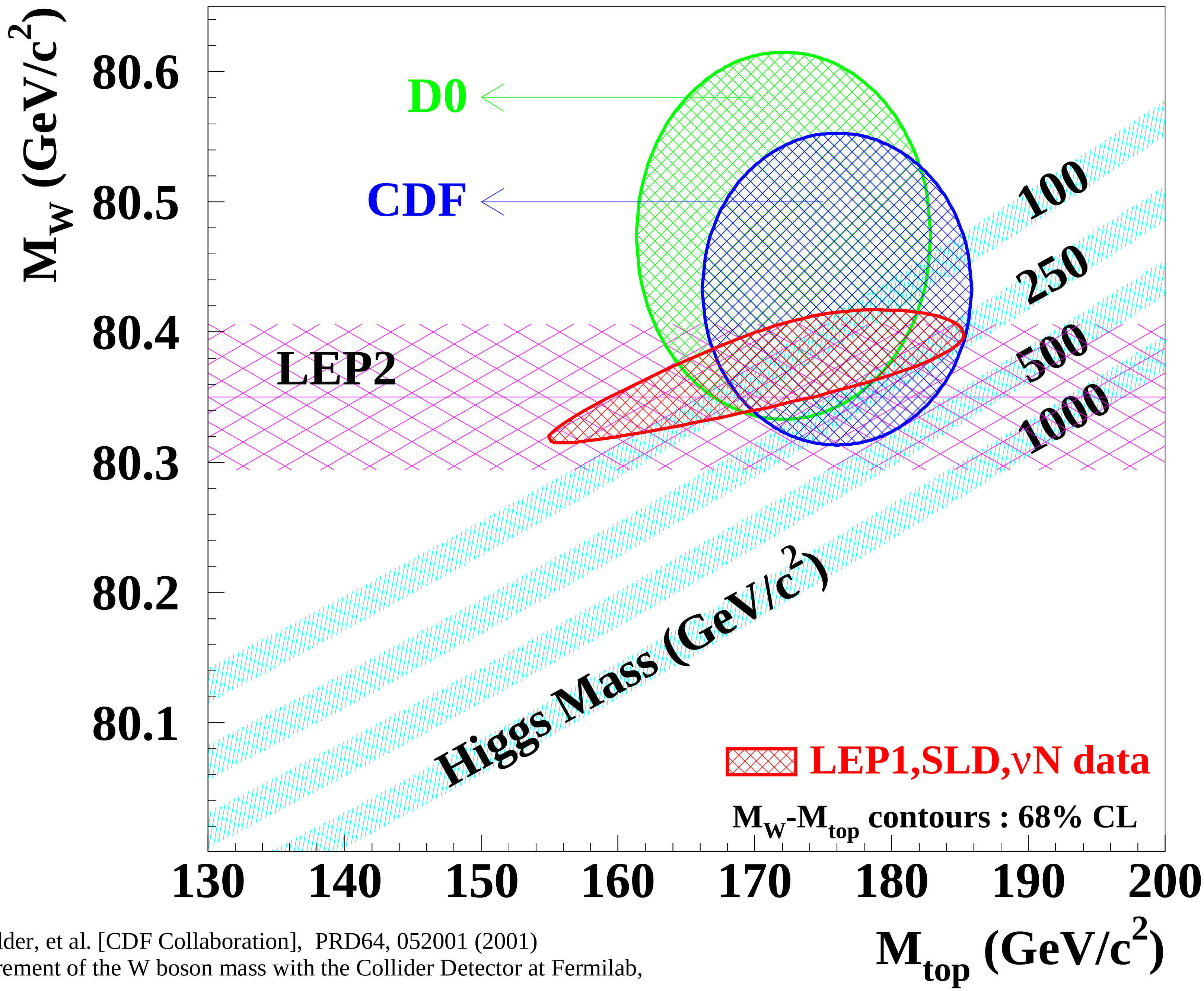
**dots: Data**
**histogram: MC**




- $Z \to e^+e^-$ cross-section



# The W-Mass is an important fundamental quantity




T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
Measurement of the W boson mass with the Collider Detector at Fermilab,

# Part II: Drell-Yan Process: Where have we been???

**Finding the W Boson Mass:**

The Jacobian Peak, and the W Boson $P_T$

Multiple Soft Gluon Emissions

Single Hard Gluon Emission

**Road map of Resummation**

Summing 2 logs per loop: multi-scale problem $(Q,q_T)$

Correlated Gluon Emission

Non-Perturbative physics at small $q_T$.

**Transverse Mass Distribution:**

Improvement over $P_T$ distribution

**What can we expect in future?**

Tevatron Run II

LHC

# Thanks to ...

Jeff Owens

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and my other CTEQ colleagues

*and the many web pages where I borrowed my figures ...*

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CTEQ Pedagogical Page:

CTEQ Lectures:
C.P. Yuan, 2002
Chip Brock, 2001
Jeff Owens, 2000

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