III. FACTORIZATION AND EVOLUTION

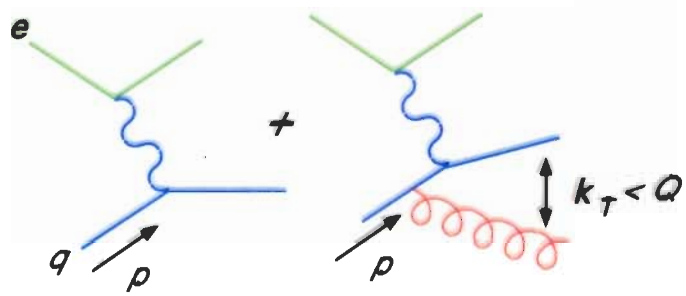
1. Factorization as a generalization of the Parton Model
2. Evolution
3. Hard hadron-hadron scattering
4. Prologue: QCD beyond the basic theorems

1. FACTORIZATION AS A GENERALZATION OF THE PARTON MODEL

- **Challenge: use AF in observables (cross sections (σ) (also some amplitudes ...)) that are** *not infrared safe*
- **Possible** *if:* σ **has a short-distance subprocess. Separate** *IR Safe* **from IR:** *this is factorization*
- *IR Safe part (short-distance) is calculable in pQCD*
- *Infrared part – example: parton distribution – measureable and universal*
- *Infrared safe – insensitive to soft gluon emission collinear rearrangements*
- **Just like Parton Model except in Parton Model the infrared safe part is** $\sigma_{\text{Born}} \Rightarrow \phi(x)$ **normalized uniquely**
- **In pQCD must define parton distributions more carefully – yet another scheme: the factorization scheme**

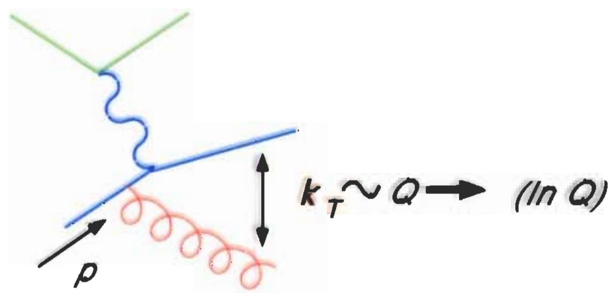
FIELD THEORY CORRECTIONS: DIS

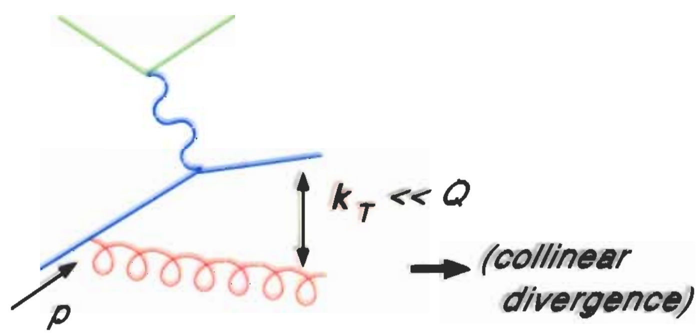
Basic observation: **virtual states not truly frozen. Some states fluctuate on scale $1/Q$:**



Short-lived states $\Rightarrow \ln(Q)$

Long-lived states $\Rightarrow$ **Collinear Logs (IR)**





RESULT: FACTORIZED DIS

$$F_2^{\gamma q}(x, Q^2) = \int_x^1 d\xi \, C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \times \phi_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

$$\equiv C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \otimes \phi_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

- ϕ **has** $\ln(\mu_F/\Lambda_{\rm QCD}) \cdots$
- C **has** $\ln(Q/\mu)$, $\ln(\mu_F/\mu)$
- **Often pick** $\mu = \mu_F$ **and often pick** $\mu_F = Q$. **So often see:**

$$F_2^{\gamma q}(x, Q^2) = C_2^{\gamma q}\left(\frac{x}{\xi}, \alpha_s(Q)\right) \otimes \phi_{q/q}(\xi, Q^2)$$

- **But we still need to specify what we** *really* **mean by factorization:** *scheme* **as well as** *scale*
- **For this, compute** $F_2^{\gamma q}(x, Q)$
- **Keep** $\mu = \mu_F$ **for simplicity**

"Compute quark-photon scattering" – *What does this mean?*

- **Must use an** *IR-regulated* **theory**
- **Extract the** *IR Safe part then* **take away the regularization**
- *Let's see how it works . . .*
- **At** *zeroth order – no interactions:*
- $C^{\gamma q_f (0)} = Q_f^2\ \delta(1 - x/\xi)$
  (Born cross section; parton model)
- $\phi^{(0)}_{q_f/q_{f'}}(\xi) = \delta_{ff'}\ \delta(1-\xi)$
  (at zeroth order, momentum fraction conserved)

$$F_2^{\gamma q_f\,(0)}(x,Q^2) = \int_x^1 d\xi\, C_2^{\gamma q_f\,(0)}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right) \times \phi^{(0)}_{q_f/q_f}(\xi,\mu_F,\alpha_s(\mu))$$

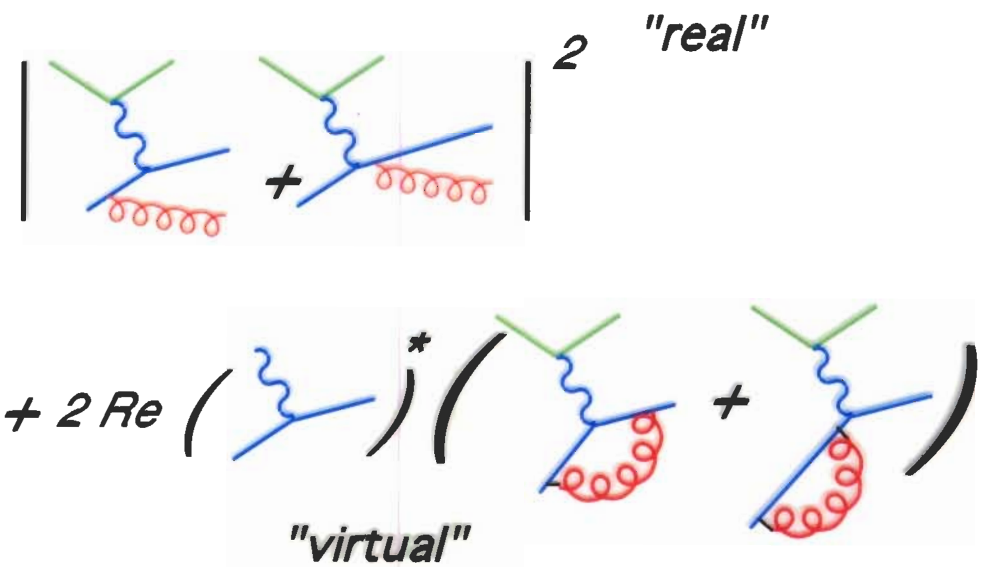
$$= Q_f^2 \int_x^1 d\xi\ \delta(1-x/\xi)\ \delta(1-\xi)$$

$$= Q_f^2\ x\,\delta(1-x)$$

- **On to one loop . . .**

$F^{\gamma q}$ @ AT ONE LOOP: FACTORIZATION SCHEMES

Start with F_2 for a *quark*:



Have to combine final states with different phase space

$$\Downarrow$$

“Plus Distributions”:

$$\int_0^1 dx \, \frac{f(x)}{(1-x)_+} \quad \equiv \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)}$$

$$\int_0^1 dx \, f(x)\left(\frac{\ln(1-x)}{1-x}\right)_+ \quad \equiv \int_0^1 dx \, \left(f(x)-f(1) \right) \, \frac{\ln(1-x)}{(1-x)}$$

and so on . . . where

- $f(x)$ **will be parton distributions**
- $f(x)$ **term: real gluon, with momentum fraction** $1-x$
- $f(1)$ **term: virtual, with elastic kinematics**

A Special Distribution
DGLAP “evolution kernel” = “splitting function”

$$P_{qq}^{(1)}(x) = C_F \, \frac{\alpha_s}{\pi} \, \left[\frac{1+x^2}{1-x}\right]_+$$

- **Will see:** P_{qq} **a probability per unit log** k_T

Expansion and Result:

$$F_2^{\gamma q}(x, Q^2) = \int_x^1 d\xi \; C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \times \phi_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

$$F_2^{\gamma q_f}(x, Q^2) = C_2^{(0)} \; \phi^{(0)} + \frac{\alpha_s}{2\pi} \; C^{(1)} \; \phi^{(0)} + \frac{\alpha_s}{2\pi} \; C^{(0)} \; \phi^{(1)} + \dots$$

$$F_2^{\gamma q_f}(x, Q^2) = Q_f^2 \; \Big\{ \; x \; \delta(1-x) + \frac{\alpha_s}{2\pi} \; C_F \left[\frac{1+x^2}{1-x}\left(\frac{\ln(1-x)}{x}\right) + \frac{1}{4}\,(9-5x)\right]_+ + \frac{\alpha_s}{2\pi} \; C_F \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \; \left[\frac{1+x^2}{1-x}\right]_+ \; \Big\} + \dots$$

$$F_1^{\gamma q_f}(x, Q^2) = \frac{1}{2x} \; \left\{F_2^{\gamma q_f}(x, Q^2) - C_F \, \alpha \, \frac{\alpha_s}{\pi^2} \, 2x \; \right\}$$

Factorization Schemes

$\overline{\text{MS}}$

$$\phi^{(1)}_{q/q}(x,\mu^2) = \frac{\alpha_s}{\pi^2}\; P_{qq}(x)\;\int_0^{\mu^2} \frac{dk_T^2}{k_T^2}$$

With k_T-integral "IR regulated".

Advantage: technical simplicity; not tied to process.

$C^{(1)}(x)_{\overline{\text{MS}}} = (\alpha_s/2\pi)\; P_{qq}(x)\ln(Q^2/\mu^2)+$ μ**-independent**

DIS:

$$\phi_{q/q}(x,\mu^2) = \frac{\alpha_s}{\pi^2}\; F^{\gamma q_f}(x,\mu^2)$$

Absorbs all uncertainties in DIS into a PDF.

Closer to experiment for DIS.

$C^{(1)}(x)_{\overline{DIS}} = (\alpha_s/2\pi)\;\; P_{qq}(x)\ln(Q^2/\mu^2) + 0$

Using the Regulated Theory
and
Getting Parton Distributions for Real Hadrons

IR-regulated QCD is not *REAL* QCD

***BUT* it only differs at low momenta**

***THUS* we can use it for IR Safe functions: $C_2^{\gamma q}$, etc.**

This enables us to get PDFs for real hadrons:

- **Compute $F_2^{\gamma q}$, $F_2^{\gamma G}$ . . .**
- **Define factorization scheme; find IR Safe C's**
- **Use factorization in the full theory**

$$F_2^{\gamma N} = \sum_{a=q_f,\bar{q}_f,G} C^{\gamma a} \otimes \phi_{a/N}$$

- **Measure F_2; then use the known C's to derive $\phi_{a/N}$**
- **Multiple flavors and cross sections complicate technicalities; not logic (Global Fits)**

NOW HAVE $\phi_{a/N}(\xi, \mu^2)$

USE IT IN ANY OTHER PROCESS THAT FACTORIZES

2. EVOLUTION

- **Q^2-dependence**
  - **In general, Q^2/μ^2 dependence still in $C_a\left(x/\xi, Q^2/\mu^2,\right.$**

    Choose $\mu = Q$

    $$F_2^{\gamma A}(x, Q^2) = \sum_a \int_x^1 d\xi\, C_2^{\gamma a}\left(\frac{x}{\xi}, 1, \alpha_s(Q)\right) \phi_{a/A}(\xi, \mu^2)$$

    $Q \gg \Lambda_{\rm QCD} \to$ compute C's in PT.

    $$C_2^{\gamma a}\left(\frac{x}{\xi}, 1, \alpha_s(Q)\right) = \sum_n \left(\frac{\alpha_s}{\pi}\right)^n C_2^{\gamma a(n)}\left(\frac{x}{\xi}\right)$$

    But still need PDFs at $\mu = Q$: $\phi_{a/A}(\xi, Q^2)$

  - **Remarkable result: EVOLUTION**

    Can use $\phi_{a/A}(x, Q_0^2)$
    to determine $\phi_{a/A}(x, Q^2)$ and hence $F_{1,2,3}(x, Q^2)$
    for any Q !
    So long at $\alpha_s(Q)$ is still small

- **Illustrate by a 'nonsinglet' distribution**

$$F_a^{\gamma \mathrm{NS}} = F_a^{\gamma p} - F_a^{\gamma n}$$

$$F_2^{\gamma \mathrm{NS}}(x, Q^2) = \sum_a \int_x^1 d\xi \; C_2^{\gamma \mathrm{NS}}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \alpha_s(\mu)\right) \phi_{\mathrm{NS}}(\xi, \mu^2)$$

Gluons, antiquarks cancel

At one loop: $C_2^{\mathrm{NS}} = C_2^{\gamma N}$

- **'Mellin' Moments and Anomalous Dimensions**

$$\bar{f}(N) = \int_0^1 dx \; x^{N-1} \; f(x)$$

- **Reduces convolution to a product**

$$f(x) = \int_x^1 dy \; g\left(\frac{x}{y}\right) \; h(y) \rightarrow \bar{f}(N) = \bar{g}(N) \; \bar{h}(N+1)$$

- **Moments applied to NS structure function:**

$$\bar{F}_2^{\gamma \rm NS}(N,Q^2) = \bar{C}_2^{\gamma \rm NS}\left(N, \frac{Q}{\mu}, \alpha_s(\mu)\right)\, \bar{\phi}_{\rm NS}(N,\mu^2)$$

(Note $\phi_{\rm NS}(N,\mu^2) \equiv \int_0^1 d\xi \xi^N \phi(\xi,\mu^2)$ **here.)**

- $\bar{F}_2^{\gamma \rm NS}(N,Q^2)$ **is PHYSICAL**

$$\Rightarrow \quad \mu\frac{d}{d\mu}\, \bar{F}_2^{\gamma \rm NS}(N,Q^2) = 0$$

- **'Separation of variables'**

$$\mu\frac{d}{d\mu} \ln \bar{\phi}_{\rm NS}(N,\mu^2) = -\gamma_{\rm NS}(N,\alpha_s(\mu))$$

$$\gamma_{\rm NS}(N,\alpha_s(\mu)) = \mu\frac{d}{d\mu} \ln \bar{C}_2^{\gamma \rm NS}(N,\alpha_s(\mu))$$

- **Because α_s is the only variable held in common!**

$$\mu \frac{d}{d\mu} \ln \bar{\phi}_{\rm NS}(N,\mu^2) = -\gamma_{\rm NS}(N,\alpha_s(\mu))$$
$$\gamma_{\rm NS}(N,\alpha_s(\mu)) = \mu \frac{d}{d\mu} \ln \bar{C}_2^{\gamma^{\rm NS}}(N,\alpha_s(\mu))$$

- **Only need to know C's $\Rightarrow \gamma_n$ from IR regulated theory!**

$\Downarrow$

Q-DEPENDENCE DETERMINED BY PT

EVOLUTION

THIS WAS HOW WE FOUND OUT QCD IS 'RIGHT'

THIS IS HOW QCD PREDICTS PHYSICS AT NEW SCALES

$\gamma_{\rm NS}$ AT ONE LOOP

$$
\begin{aligned}
\gamma_{\rm NS}(N,\alpha_s) &= \mu\frac{d}{d\mu}\ \ln \bar{C}_2^{\gamma^{\rm NS}}(N,\alpha_s(Q)) \\
&= \mu\frac{d}{d\mu}\ \{\ (\alpha_s/2\pi)\ \bar{P}_{qq}(N)\ln(Q^2/\mu^2) + \mu \text{ indep.}\ \} \\
&= -\frac{\alpha_s}{\pi}\ \int_0^1 dx\ x^{N-1}\ P_{qq}(x) \\
&= -\frac{\alpha_s}{\pi}C_F\ \int_0^1 dx\ \left[\left(x^{N-1}-1\right)\frac{1+x^2}{1-x}\right] \\
&= -\frac{\alpha_s}{\pi}C_F\ \left[\ 4\sum_{m=2}^{N}\frac{1}{m} - 2\frac{2}{N(N+1)} + 1\ \right] \\
&\equiv -\frac{\alpha_s}{\pi}\ \gamma_{\rm NS}^{(1)}
\end{aligned}
$$

Hint:

$$(1-x^2)/(1-x) = 1 + x \ldots (1-x^k)/(1-x) = \sum_{i=0}^{k-1} x^k$$

SOLUTION: SCALE BREAKING

$$\mu \frac{d}{d\mu}\ \bar{\phi}_{\rm NS}(N,\mu^2) = -\gamma_{\rm NS}(N,\alpha_s(\mu))\ \bar{\phi}_{\rm NS}(N,\mu^2)$$

$$\begin{aligned}\bar{\phi}_{\rm NS}(N,\mu^2) &= \bar{\phi}_{\rm NS}(N,\mu_0^2) \\ &\quad \times \exp\left[-\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\ \gamma_{\rm NS}(N,\alpha_s(\mu)) \right]\end{aligned}$$

$$\Downarrow$$

$$\bar{\phi}_{\rm NS}(N,Q_0^2) = \bar{\phi}_{\rm NS}(N,Q_0^2) \left(\frac{\ln(Q^2/\Lambda^2_{\rm QCD})}{\ln(Q_0^2/\Lambda^2_{\rm QCD})} \right)^{-2\gamma_N^{(1)}/\beta_0}$$

Hint:

$$\alpha_s(Q) = \frac{4\pi}{\beta_0\ \ln(Q^2/\Lambda^2_{\rm QCD})}$$

So also:

$$\bar{\phi}_{\rm NS}(N,Q^2) = \bar{\phi}_{\rm NS}(N,Q_0^2) \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{-2\gamma_N^{(1)}/\beta^{(1)}}$$

$$\bar{\phi}_{\rm NS}(N, Q^2) = \bar{\phi}_{\rm NS}(N, Q_0^2) \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{-2\gamma_N^{(1)}/\beta^{(1)}}$$

- Mild' scale breaking
- For $\alpha_s \to \alpha_0 \neq 0$, get a power Q-dependence:

$$\left(Q^2\right)^{\gamma^{(1)}\frac{\alpha_s}{2\pi}}$$

- QCD's consistency with the Parton Model (73-74)

again:

$$\mu\frac{d}{d\mu}\ \bar{\phi}_{\rm NS}(N, \mu^2) = -\gamma_N(\alpha_s(\mu))\ \bar{\phi}_{\rm NS}(N, \mu^2)$$

$$\Downarrow$$

$$\mu\frac{d}{d\mu}\ \bar{\phi}_{\rm NS}(N, \mu^2) = \int_x^1 \frac{d\xi}{\xi}\ P_{\rm NS}(\xi, \alpha_s(\mu))\ \bar{\phi}_{\rm NS}(\xi, \mu^2)$$

Splitting function $\leftrightarrow$ Moments

$$\int_0^1 dx\ x^{N-1}\ P_{qq}(x, \alpha_s) = \gamma_{qq}(N, \alpha_s)$$

BEYOND NONSINGLET

COUPLED EVOLUTION

$$\mu\frac{d}{d\mu}\,\bar{\phi}_{q/A}(N,\mu^2) = \sum_{b=q,\bar{q},G}\int_x^1 \frac{d\xi}{\xi}\,P_{\rm ab}(\xi,\alpha_s(\mu))\,\bar{\phi}_{b/A}(\xi,\mu^2)$$

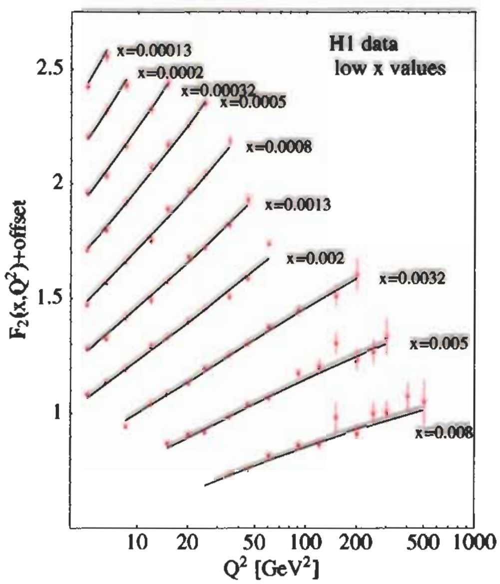
Physical Contxt of Evolution

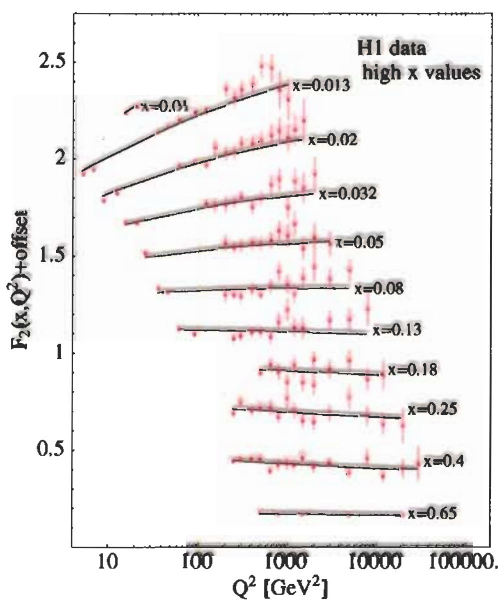
- **Parton Model:** $\phi_{a/A}(x)$ **density of parton** a **with momentum fraction** x, **assumed independent of** Q
- **PQCD:** $\phi_{a/A}(x,\mu)$: **same density, but with transverse momentum** $\leq \mu$
- **If there** *were* **a maximum transverse momentum** Q_0, $\phi(x,Q_0)$ **would freeze for** $\mu \geq Q_0$
- *Not so* **in renormalized PT**
- Scale breaking measures the change in the density as maximum transverse momentum increases

- Cross sections we compute still depend on our choice of μ through uncomputed "higher orders" in C and evolution

But in an asymptotically free theory, "higher order" means "smaller".

• Evolution in DIS (with CTEQ6 fits)





3. HADRON HADRON SCATTERING

- **Same generalization from Parton Model as DIS**
- **For final state system F, mass Q**

$$\sigma_{AB\to F(Q)} = \sum_{a,b=q,\bar{q},G} \int_0^1 d\xi\, d\eta\, \phi_{a/A}(x,\mu)\ \phi_{b/B}(\eta,\mu) \\ \times\hat{\sigma}_{ab\to F(Q)}(xp_A, yp_B, Q, \mu, \alpha_s(\mu)) \\ \times\theta(\xi\eta p_A\cdot p_B - Q^2)$$

Convolution form:

$$\sigma_{AB\to F(Q)} = \hat{\phi}_{a/A}\otimes\hat{\sigma}_{ab\to F}\ \otimes\hat{\phi}_{b/B}$$

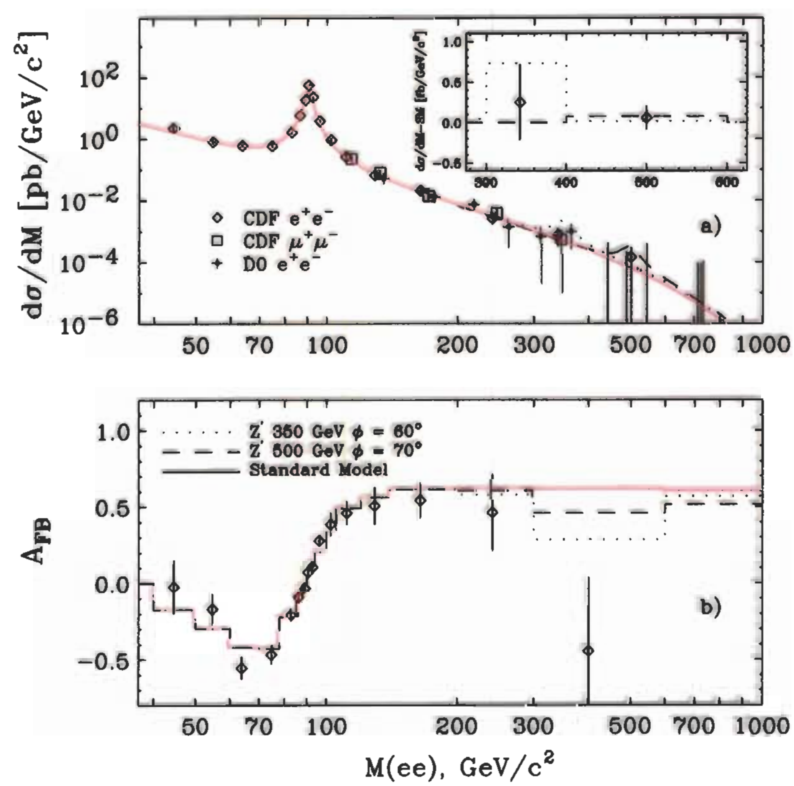
- **Depends on choice of μ through uncomputed "higher orders" in $\hat{\sigma}$ and evolution**

- **A typical $\mathcal{O}(\alpha_s)$ (NLO) hard-scattering cross section at one loop: $\hat{\sigma})_{\bar{q}q}$ for Drell-Yan**

$$\hat{\sigma}_{\bar{q}q} = \sigma_{\rm Born}(Q^2) \left(\frac{\alpha_s(\mu)}{\pi}\right) \{2(1+z^2) \left[\frac{\ln(1+z^2)}{1-z}\right]_+ - \frac{[(1+z^2)\ln z\,]}{(1-z)} + \left(\frac{\pi^2}{3} - 4\right) \delta(1-z)\}$$

$C + P \ln Q^2/\mu^2$

- **$\sigma_{\rm Born}$ describes Electroweak annihilation of $q\,\bar{q}$**
- **Compared to data at high energy (γ and Z) with forward-backward asymmetry in $\sigma_{\rm Born}$**



- **This way, detect "new" physics in the hard scattering**

- **Factorized hadron-hadron scattering "Collinear factorized"**

$$\sigma_{AB\to F(Q)} = \hat{\phi}_{a/A} \otimes \hat{\sigma}_{ab\to F} \otimes \hat{\phi}_{b/B}$$

- **Requirements: same as for IR Safety in** e^+e^- **cross sections**
- **Insensitivity to soft emission, collinear rearrangement**
- **Continuous dependence on particle momenta**
- **Numerous Cases**

$$\begin{aligned} p + p \to \quad & \text{high } p_T \text{ jets} \\ & \text{high } p_T \textit{ particles}, \text{ heavy particles}: \\ & W,\ Z \ldots \\ & H,\ \tilde{q},\ \ldots \\ & Q\,\bar{Q} \ldots \end{aligned}$$

- **PDFs ϕ's same as in DIS (experiment) same evolution**
- **PT $\to$ $\hat{\sigma}_{ab}$'s (IR-regulated theory)**
- **'New physics' in $\hat{\sigma}_{ab}$'**

- **Evolution → ability to extrapolate to new energy scales**
- **Predictions in hadron-hadron cross sections**

- Lorentz contracted fields of incident particles do not overlap until the moment of the scattering!

- Initial-state interaction disappear at high enough energies!

- $\Rightarrow$ **Repalced by parton distributions**

- Interactions after the scattering are too late to affect large momentum transfer, creation of heavy particle, etc.

- $\Rightarrow$ **Cross section for hard scattering is IR safe**

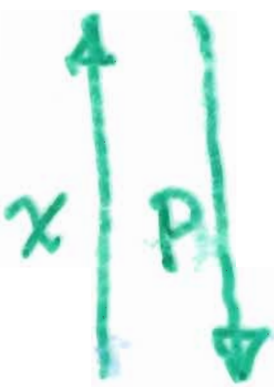
$$\Downarrow$$

$$\sigma_{AB\to F(Q)} = \hat{\phi}_{a/A} \otimes \hat{\sigma}_{ab\to F} \otimes \hat{\phi}_{b/B}$$

4. PROLOGUE
(PQCD & BEYOND)

A. Lectures by x and p

Melnitchouk Boden Tung	Hadron Structure
Kowalski Venugopalan	Low x Saturation
Bass Mrenna Soper	Transport Hadronization Factorization
Stankus	photons
Owens Qiu Lecompte	heavy quarks
Ellis Campbell Olness Jacak Varelas	vector bosons jets
Gary Huston Kilgore Han	QCD → beyond std model



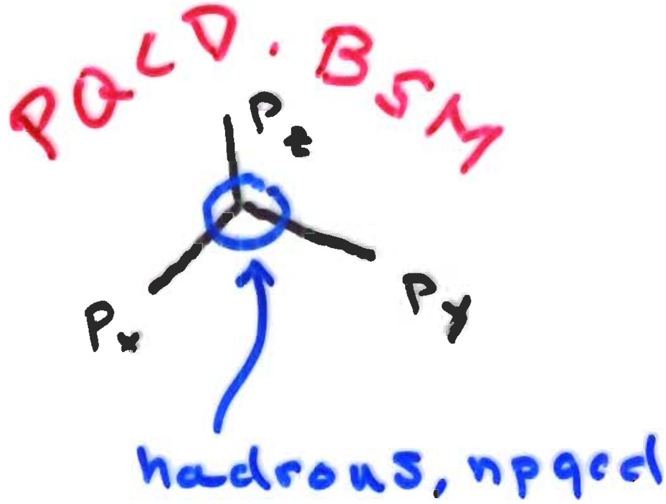
B. Connections to some non-perturbative topics...

Low-x

Chiral Symmetry Breaking (χSB)

Evolution in high-density AA collisions

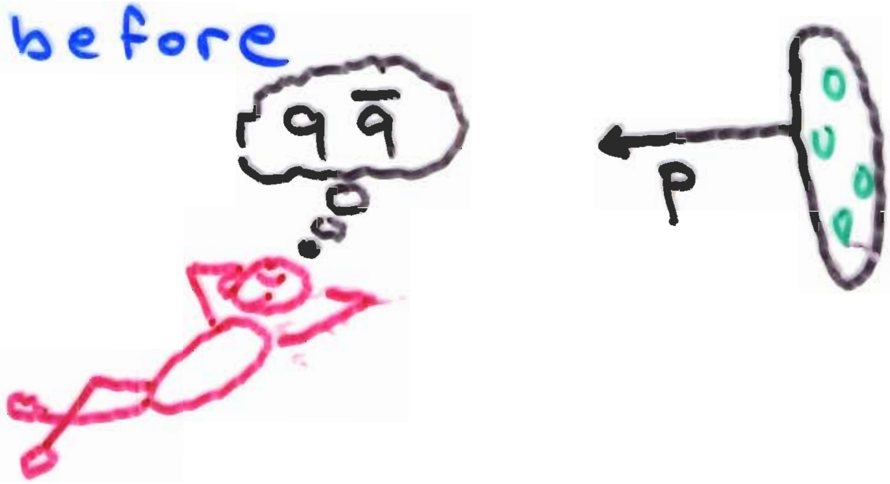
Phases in QCD



Low-x in the electron rest frame

'The electron's quantum dreams come true'

i) before

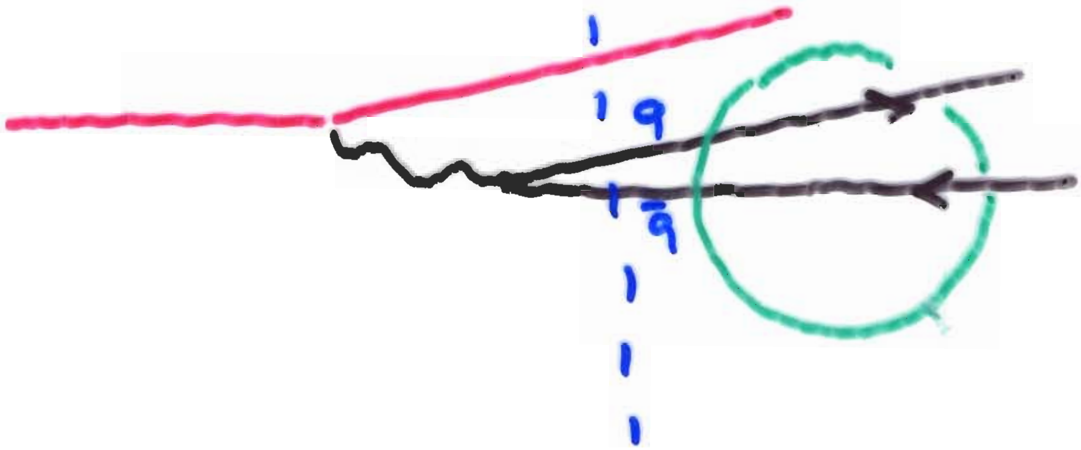


ii) after

"Infinite momentum frame"

Alternative

Target rest frame &
Dipole picture



long-lived
state

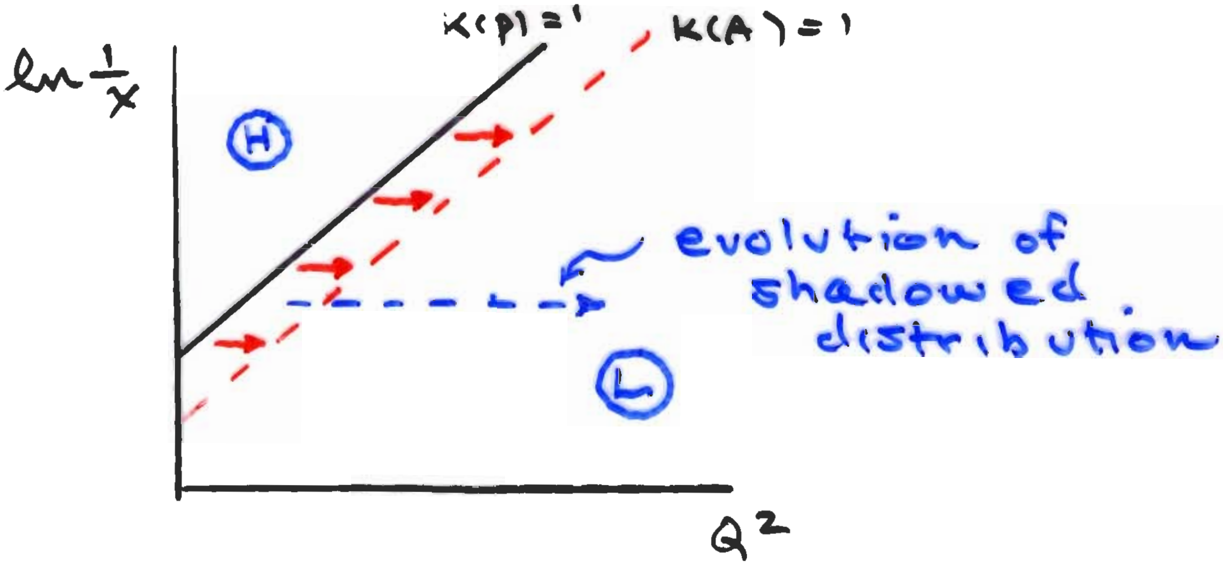
Here $q, \bar{q}$ are naturally
electron "constituents"

$\leftrightarrow$ very low x for hadron
target

Quantum mechanics implies
this generalization of
parton picture

(Low-x)

- The Evolution plane (GLR)



$$\kappa = \frac{3\pi^2\alpha_s}{2Q^2}\left(\frac{xG(x,Q^2)}{\pi R^2}\right) \begin{matrix} > 1 & (H) \\ < 1 & (L) \end{matrix}$$

( ) ~ $A^{1/3}$

$\kappa > 1$ all $\frac{G}{Q^2 R}$ power corrections contribute : new evolution

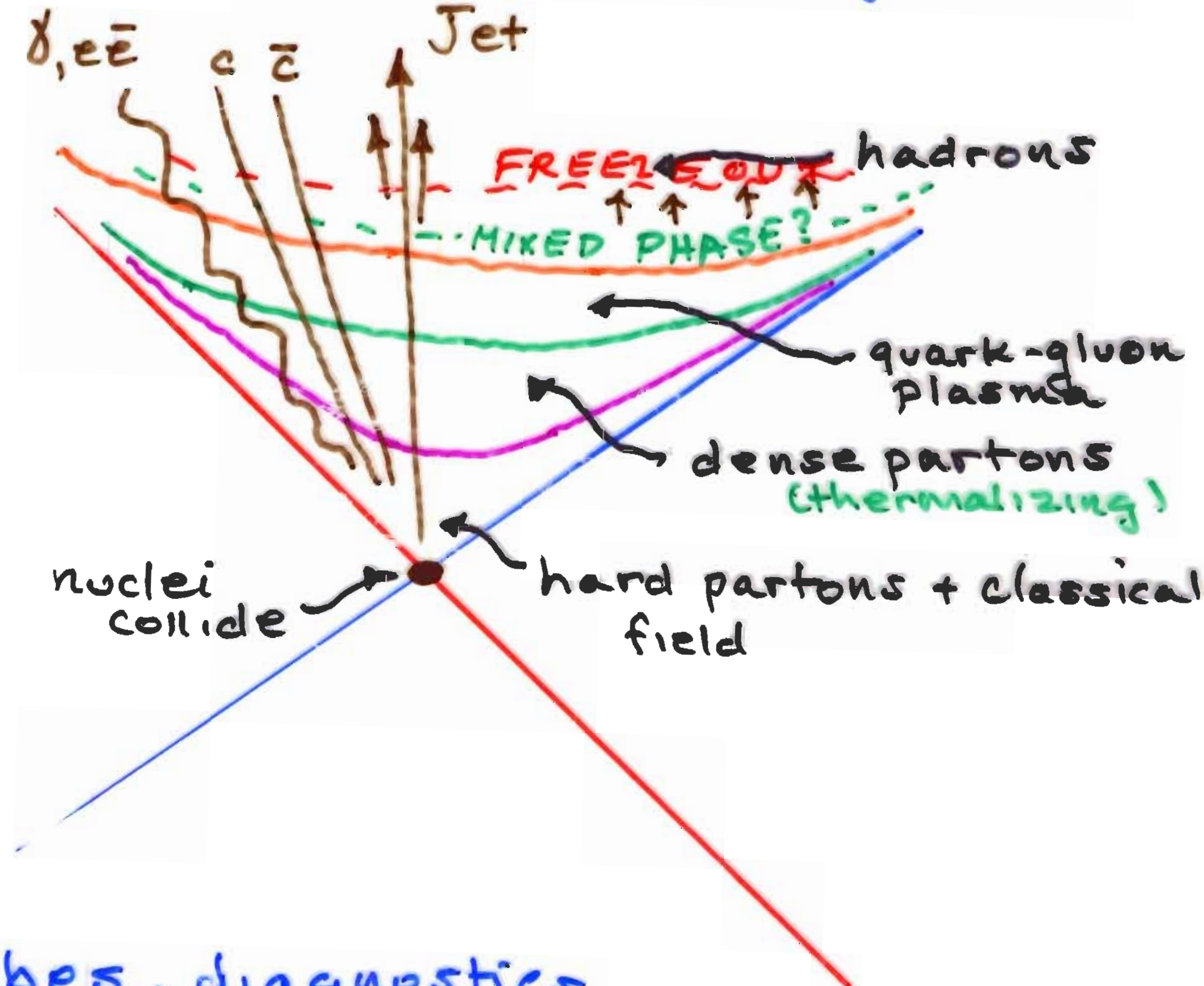
BFKL
Balitsky, Kovchegov
McLerran, Venugopalan
AGL

(H) $\kappa > 1$: Saturation

(L) $\kappa < 1$: Shadowing = influence of A-dependence of $\kappa=1$ line

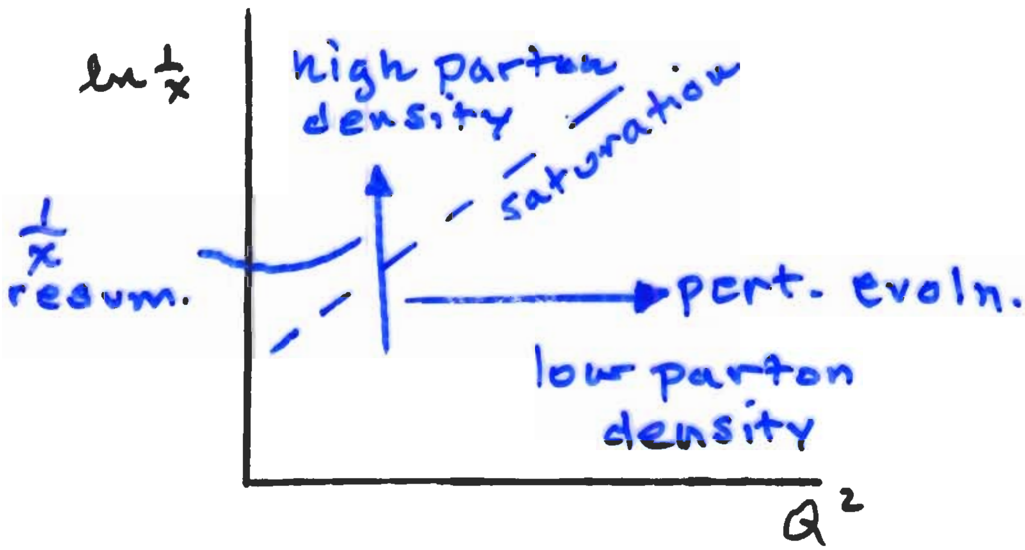
Mueller, Qiu

- Plausible history: probes of early times

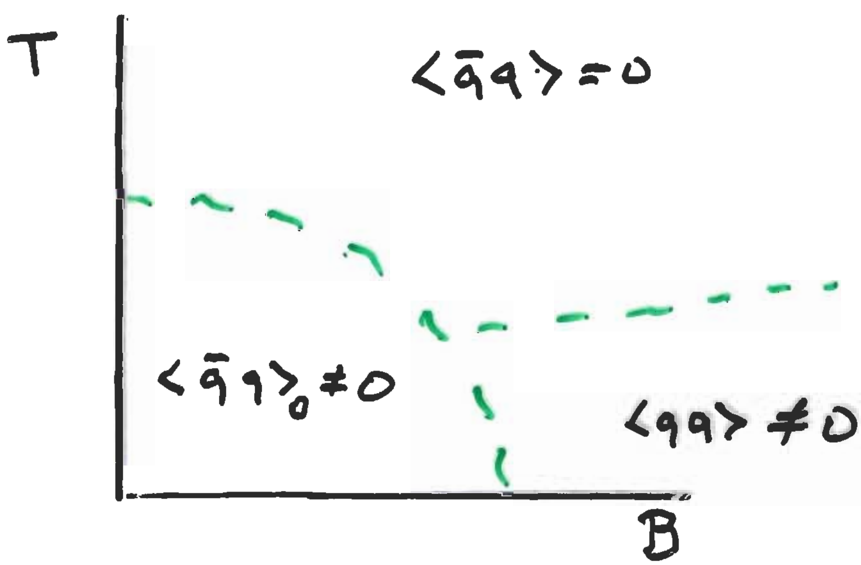


- Probes, diagnostics
  - jets: energy loss
    'LPM' analysis: multiple scattering
    - enhances radiation with short formation time
      $\Delta E \propto L^2$
    - supresses radiation with long formation time
  - $c\bar{c}$: J/ψ suppression
  - strange hadrons: enhanced by 'χ restoration'
  - γ $e\bar{e}$: charged particles at early times
  - hadrons: correlations, fluctuation at 'freeze-out'

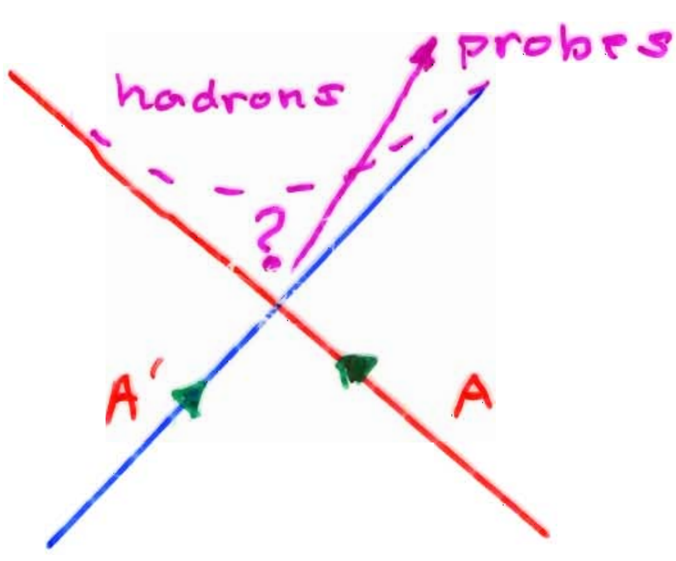
SUMMARY



$\ln \frac{1}{x}$
high parton density
saturation
$\frac{1}{x}$ resum.
pert. evoln.
low parton density
Q^2



T
$\langle \bar{q}q \rangle = 0$
$\langle \bar{q}q \rangle_0 \neq 0$
$\langle qq \rangle \neq 0$
B



probes
hadrons
?
A'
A