Photons in Large Momentum Transfer Processes

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Outline

1. Introduction
   - Why photons?
   - Some history
2. Theory overview and comparison to selected data
   - Direct Photon Production
   - Jet Photoproduction
3. Theoretical improvements
   - Resummation techniques
   - Some applications
4. A puzzle from two-photon production of jets and hadrons
5. Conclusions
6. Appendix: Notation, kinematics, and other useful information

Why Photons?

- Well understood electromagnetic interaction
- Well defined probe of strong interaction dynamics
- Classic examples
  - Deep inelastic scattering
  - Lepton pair production
  - $e^+e^- \rightarrow$ hadrons

Direct photons, photoproduction, two photon processes continue this history

- Calculations for the proposed Linear Collider will rely on a detailed understanding of the treatment of photons in high energy processes

Direct Photons - Some Early History

Late 70's - Early 80's

- Fixed target experiments, ISR (pp), prior to $\overline{p}p$ colliders
- Jet identification difficult - and controversial!
- Photons offered a way to directly probe the scattering subprocesses
  $\Rightarrow$ study parton-parton scattering without jet ID problems

Theoretically Interesting

- QCD hard scattering formalism not yet fully confirmed
- Competition from the Constituent Interchange Model
- γ/π ratio offered some interesting tests of QCD vs CIM - G. Farrar, Phys. Lett. **B67**, 337 (1977)
- QCD Compton process ($qg \rightarrow \gamma q$) proposed as a *test* of QCD - H. Fritzsch and P. Minkowski, Phys. Lett. **B69**, 316 (1977)

Early reviews: T. Ferbel and W.R. Molzen, Rev. Mod. Phys. **56**, 181 (1984); J.F. Owens, Rev. Mod. Phys. **59**, 465 (1987)

Time Passes . . .

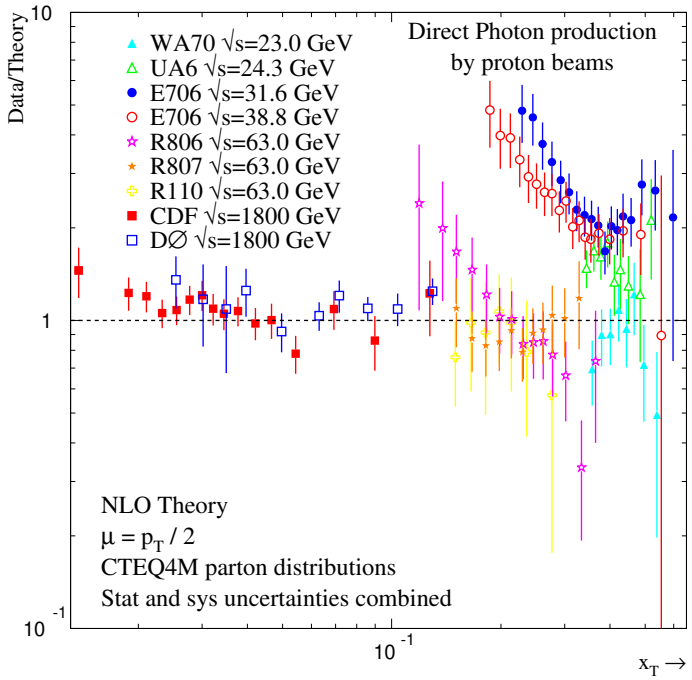
- Higher energies with development of $\overline{p}p$ colliders
- Greater reach in transverse momentum
- Jet production, detection became routine
- QCD hard scattering expectations confirmed
  - Event structure with two jets, balancing in p_T observed
  - Di-jet angular distribution confirmed
  - QCD expectations for direct photons also confirmed - with some exceptions
- Focus shifted somewhat from *testing* QCD to *using* QCD to constrain parton distribution functions and to perform searches for new physics.

QCD Compton subprocess involves an initial state gluon

$\Rightarrow$ direct photon production is sensitive to the gluon distribution especially in processes where the QCD Compton subprocess dominates, *e.g.*, pp scattering

Today

- Wide range of data available
  - Data review by W. Vogelsang and M.R. Whalley, J. Phys. **G23**, Suppl. 7A, A1-A69 (1997)
  - Online database at http://durpdg.dur.ac.uk/HEPDATA
- Usual direct photon talk emphasizes
  - problems describing p_T distributions



perhaps suggesting that there is something wrong with direct photon theory!

But direct photon theory is QCD . . .

The situation is really more complex . . .

Key predictions of QCD concerning direct photon production include

- Event structure
  1. Where are the photons produced?
  2. How are they correlated with jets?
- Energy dependence of the cross section
- Absolute cross section

Key element underlying each of these is the set of parton-level subprocesses involved.

Can one measure the properties of the subprocesses?

Yes!

Plan

- Review basic theoretical ingredients

- Compare theory and data using observables which directly test the underlying production mechanisms

  - γ-jet mass distribution

  - γ-jet angular distributions

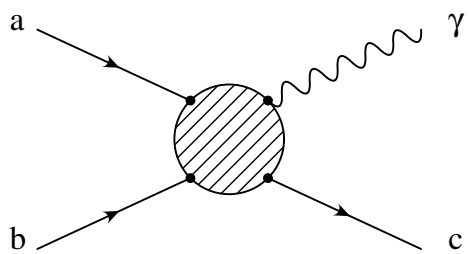
  - Event structure - correlations with produced hadrons

- Compare/contrast with jet photoproduction - which uses the same subprocesses

- Finally - examine the predictions for the γ p_T, rapidity, and center-of-mass energy dependence and compare to other examples of single particle inclusive production.

Theory Overview

- Lowest Order: $\mathcal{O}(\alpha\alpha_s)$
  1. $qg \to \gamma g$ QCD Compton
  2. $q\overline{q} \to \gamma g$ annihilation
- The single photon invariant cross section is given by a convolution with the beam and target parton distribution functions

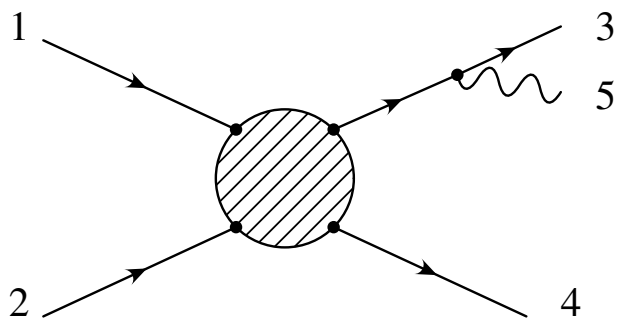


$$d\sigma(AB \to \gamma + X) = G_{a/A}(x_a, \mu_F)\, dx_a$$
$$G_{b/B}(x_b, \mu_F)\, dx_b \frac{1}{2\hat{s}} \overline{\sum_{ab}} |M(ab \to \gamma c)|^2 d^2PS$$

- d^2PS denotes two-body phase space
- μ_F is the factorization scale
- Expect an isolated photon recoiling against a jet
- See the appendix for more details about variables and four-vectors
- Also see the Handbook on Perturbative QCD at the CTEQ web site http://www.cteq.org. The appendix has additional information on how to calculate cross sections for hadronic processes starting at the parton level.

- Next to Leading Order: $\mathcal{O}(\alpha\alpha_s^2)$
  1. one-loop virtual contributions
  2. $q\overline{q} \to \gamma gg$
  3. $gq \to \gamma qg$
  4. $qq' \to \gamma qq'$ plus related subprocesses

- In next order one sees a new configuration
  - consider the subprocess $q(1)q(2) \to q(3)q(4)\gamma(5)$
  - examine the region where $s_{35} = (p_3 - p_5)^2 \approx 0$



$$\overline{\sum}|M(qq \to qq\gamma)|^2 \approx$$
$$\frac{\alpha}{2\pi}\, P_{\gamma q}(z)\, \frac{1}{s_{35}} \overline{\sum}|M(qq \to qq)|^2$$

- An internal quark line is going on-shell signalling long distance physics effects

- New configuration
  - photon is accompanied by jet fragments on the *same* side
  - high$-p_T$ jet recoils on the away side
- Factorize the singularity and include it in the bare *photon fragmentation function*
- Sum large logs with modified Altarelli-Parisi equations

$$Q^2 \frac{dD_{\gamma/q}(x,Q^2)}{dQ^2} = \frac{\alpha}{2\pi} P_{\gamma q} + \frac{\alpha_s}{2\pi} \left[D_{\gamma/q} \otimes P_{qq} + D_{\gamma/g} \otimes P_{gq} \right]$$

$$Q^2 \frac{dD_{\gamma/g}(x,Q^2)}{dQ^2} = \frac{\alpha_s}{2\pi} \left[\sum_q D_{\gamma/q} \otimes P_{qg} + D_{\gamma/g} \otimes P_{gg} \right]$$

- The situation has become more complex
- Expect to see two classes of events
  1. Direct (or pointlike) - no hadrons accompanying the photon
  2. Fragmentation (or bremsstrahlung) - photon is a fragment of a high-p_T jet. Part of the fragmentation function is perturbatively calculable.
- Expect (1) to dominate at high-p_T since the energy is not shared with accompanying hadrons.

A nagging question

- The perturbative part of the photon fragmentation functions first showed up in our examples at the next to leading order in α_s, that is, one order beyond the Born term. Are they just higher order corrections then?

- Not really. When one examines solutions of the relevant evolution equations, it becomes apparent that the leading behavior of the perturbative part of the solutions goes as $\ln Q^2$. This is quite different from the usual hadronic behavior where we are used to seeing the distributions decrease at large x and slowly increase at small x. (See additional notes at the end of the lecture)

- The factor of $\ln Q^2$ effectively cancels out one factor of α_s, so that the fragmentation and direct contributions end up having similar dependences on Q^2. We should have included these pieces even at the lowest order!

So, to our list of contributions add those involving photon fragmentation functions

- $\mathcal{O}(\alpha\alpha_s) : \frac{d\sigma}{d\hat{t}}(ab \to cd) \otimes D_{\gamma/c}$

- $\mathcal{O}(\alpha\alpha_s^2) : \frac{d\sigma}{d\hat{t}}(ab \to cde) \otimes D_{\gamma/c}$

Next-to-leading-order Calculations

- Have to integrate over unobserved partons. There are regions of phase space where partons can become parallel to each other (collinear) or soft. Both regions are singular.

- Usually use dimensional regularization to regulate the divergences

Two types of programs exist

1. Phase space integrations done symbolically so expressions for the integrated parton-level subprocess cross sections exist. Integrations over the parton momentum fractions x_a, x_b, and z_c done numerically. This approach is suitable for the single photon inclusive cross section.

2. All integrations done via Monte Carlo
   - Phase space slicing method
   - Subtraction method

With Monte Carlo programs one can examine correlations between the photon and other partons in the final state.

Phase Space Slicing Monte Carlo

- See B. Harris and J.F. Owens hep-ph/0102128, Phys. Rev. **D65** 094032 (2002).

- Work in n=4-2ϵ dimensions using dimensional regularization

- Notation:
  - At the parton level: $p_1+p_2 \to p_3+p_4+p_5$
  - Let $s_{ij} = (p_i + p_j)^2$ and $t_{ij} = (p_i - p_j)^2$

- Partition $2 \to 3$ phase space into three regions
  1. Soft: gluon energy $E_g < \delta_s \sqrt{s_{12}}/2$
  2. Collinear: s_{ij} or $|t_{ij}| < \delta_c s_{12}$
  3. Finite: everything else

- In soft region use the soft gluon approximation to generate a simple expression for the squared matrix element which can be integrated by hand

- In the collinear region use the leading pole approximation to generate a simple expression which can be integrated by hand.

- Resulting expressions have explicit poles from soft and collinear singularities

- Factorize initial and final state mass singularities and absorb into the fragmentation and distribution functions

- Add soft and collinear integrated results to the $2 \to 2$ contributions – singularities cancel

- generate finite region contributions in 4 dimensions using usual Monte Carlo techniques

- End results is a set of two-body weights and a set of three-body weights.

- Both are finite and both depend on the cutoffs δ_s and δ_c

- Cutoff dependence cancels for sufficiently small cutoffs when the two sets of weights are added at the histogramming stage

Simple example

Consider the integral of a quantity which has a pole at $x = 0$. Using dimensional regularization, one has an integral of the form

$$F = \int_0^1 dx\, x^{-1-\epsilon} f(x).$$

For x very near zero, approximate $f(x)$ by $f(0)$ yielding

$$F \approx f(0) \int_0^\delta dx\, x^{-1-\epsilon} + \int_\delta^1 dx\, x^{-1-\epsilon} f(x).$$

The first integral can be done analytically. The second is finite and can be evaluated with $\epsilon = 0$.

$$F \approx -\frac{f(0)}{\epsilon} + f(0) \log \delta + \int_\delta^1 dx\, \frac{f(x)}{x}.$$

The second integral can be done numerically. The dependence on the cutoff δ cancels for sufficiently small values of δ

Short summary of two-body kinematics

The following relations are useful when trying to understand the regions of the parton variables which are important for specific observables. More information is available in the appendix.

Consider a photon and a jet produced with approximately balancing p_T and (pseudo)rapidities η_γ and η_{jet} in the hadron-hadron center-of-mass system.

γ-jet invariant mass M: $M^2 = x_a x_b s$

γ-jet rapidity Y: $Y = (\eta_\gamma + \eta_{jet})/2$

Scattering angle in the γ-jet rest frame:

$$\cos\theta^* = \tanh(\eta_\gamma - \eta_{jet})/2$$

Parton momentum fractions

$$\begin{aligned} x_a &= x_T(e^{\eta_\gamma} + e^{\eta_{jet}})/2 \\ x_b &= x_T(e^{-\eta_\gamma} + e^{-\eta_{jet}})/2 \\ x_a &= Me^{Y}/\sqrt{s} \\ x_b &= Me^{-Y}/\sqrt{s} \end{aligned}$$

Note: these relations show that for $Y \approx 0$ one is sensitive to $x_a \approx x_b \approx M/\sqrt{s}$. For a photon with transverse momentum p_T and $\eta_\gamma \approx 0$ one has $x_a \approx x_b \approx x_T$, but this is only a guide, since the inclusive photon cross section involves an integration over x_a and, in all cases, there are additional integrations in the higher order terms.

Comparison to Data

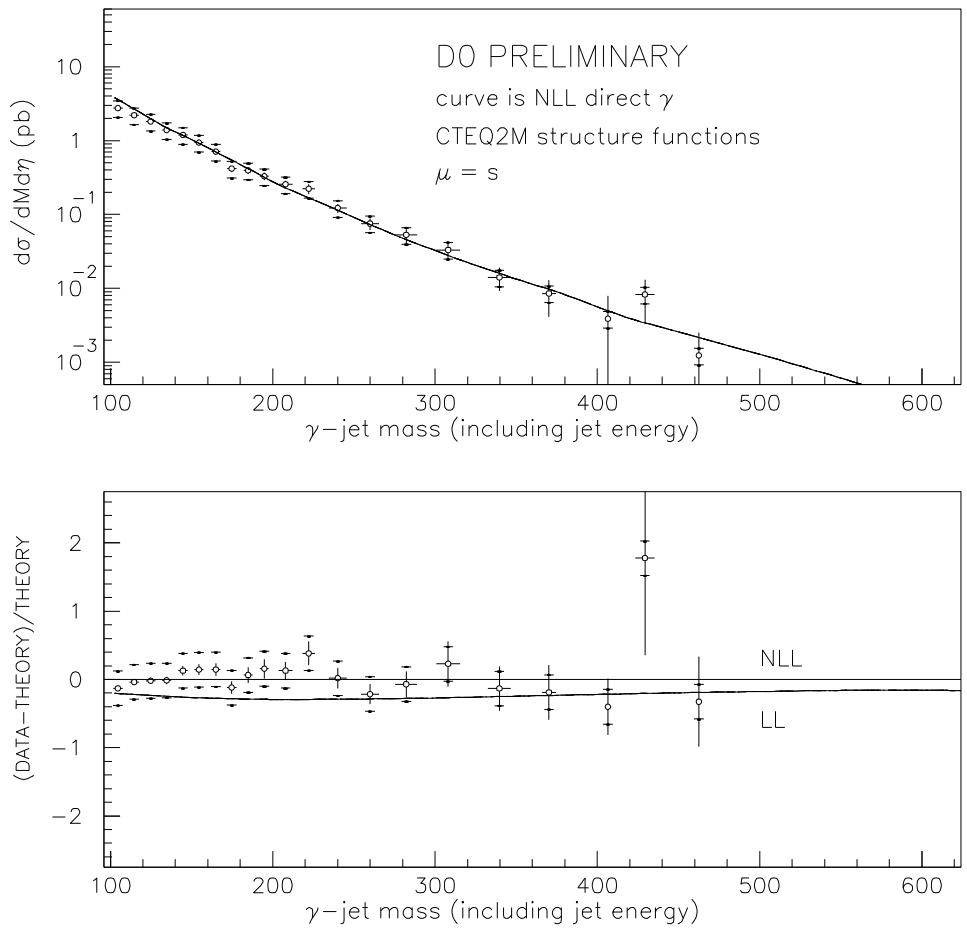
- Want to first examine data which yield information on the underlying parton subprocesses

- Need γ-jet observables
  - γ-jet invariant mass
  - γ-jet angular distributions

- Start with Tevatron data
  - Higher energies enable jet identification and reconstruction as compared to fixed target data
  - Essentially the only direct photon data with identified jets

γ-jet invariant mass distribution

- Preliminary data from DØ Run I



- Comparison suggests that the photons are being produced at the expected level, but doesn't, by itself, have much to say about the subprocesses

- Note that the relevant region of x in the pdfs is $x \approx M/\sqrt{s}$

γ-jet angular distributions

- QCD Compton and annihilation subprocess both behave as

$$\frac{d\sigma}{d\hat{t}} \sim (1 - \cos(\theta^*))^{-1} \text{ as } \cos(\theta^*) \to 1$$

- Other parton-parton scattering subprocesses ($qq \to qq, qg \to qg, gg \to gg$, etc.) behave as

$$(1 - \cos(\theta^*))^{-2}$$

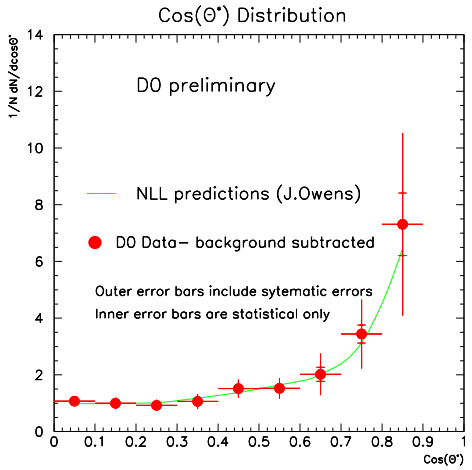
- This means that the γ-jet angular distribution should be flatter than that observed in jet-jet final states.

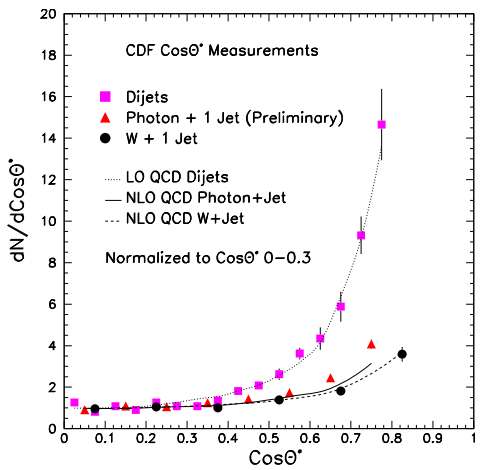
- See the appendix for a derivation of these relations

Direct Measurement of the γ-jet angular distribution

- Measuring both η_γ and η_{jet} allows one to reconstruct $\cos\theta^* = \tanh(\eta_\gamma - \eta_{jet})/2$

- Both DØ and CDF have measured the γ-jet angular distribution





- Both experiments observe a shape consistent with expectations

- Direct photon production is dominated by subprocesses which yield a flatter angular distribution than is observed for dijet production

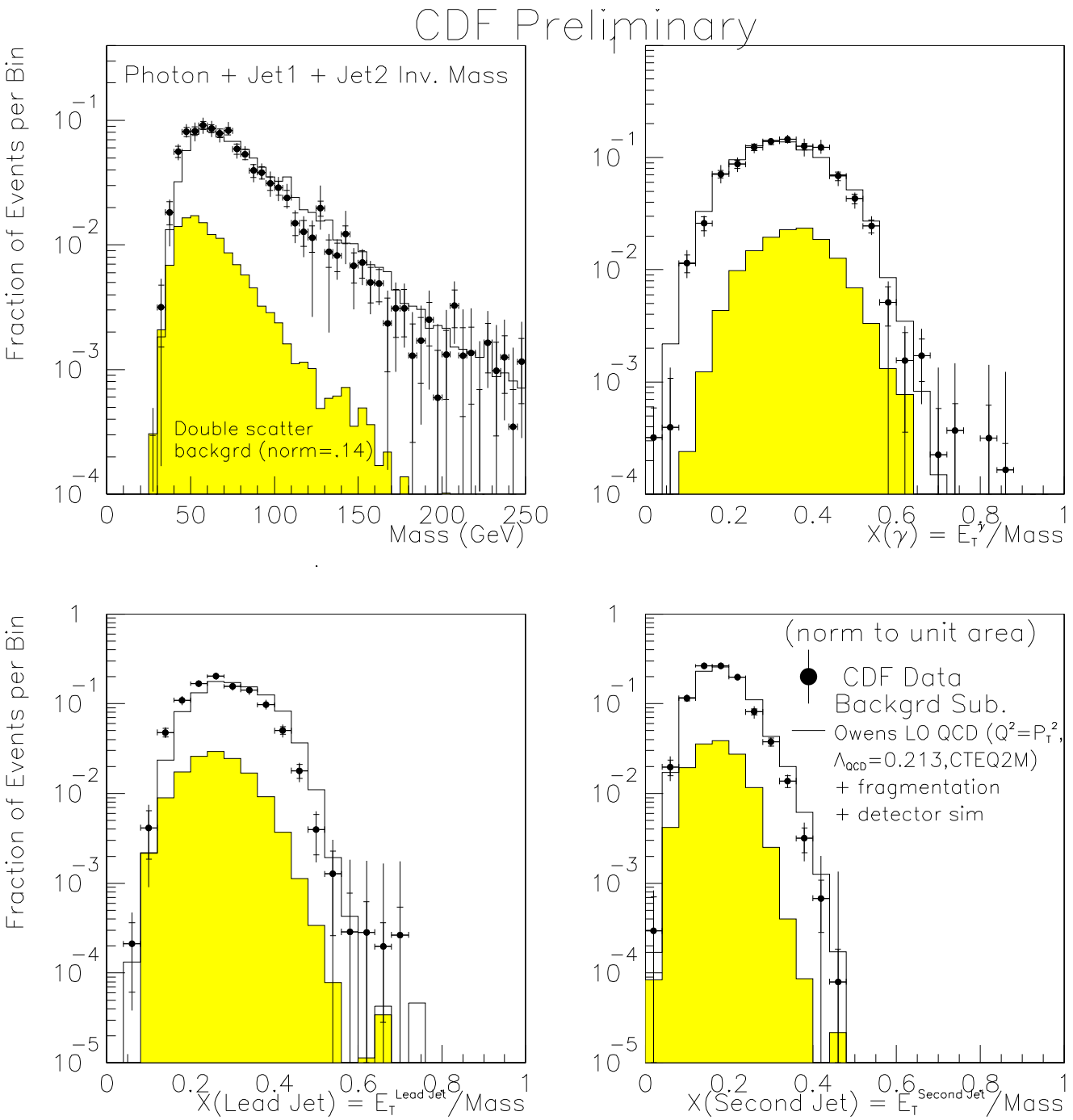
Three-body Final States

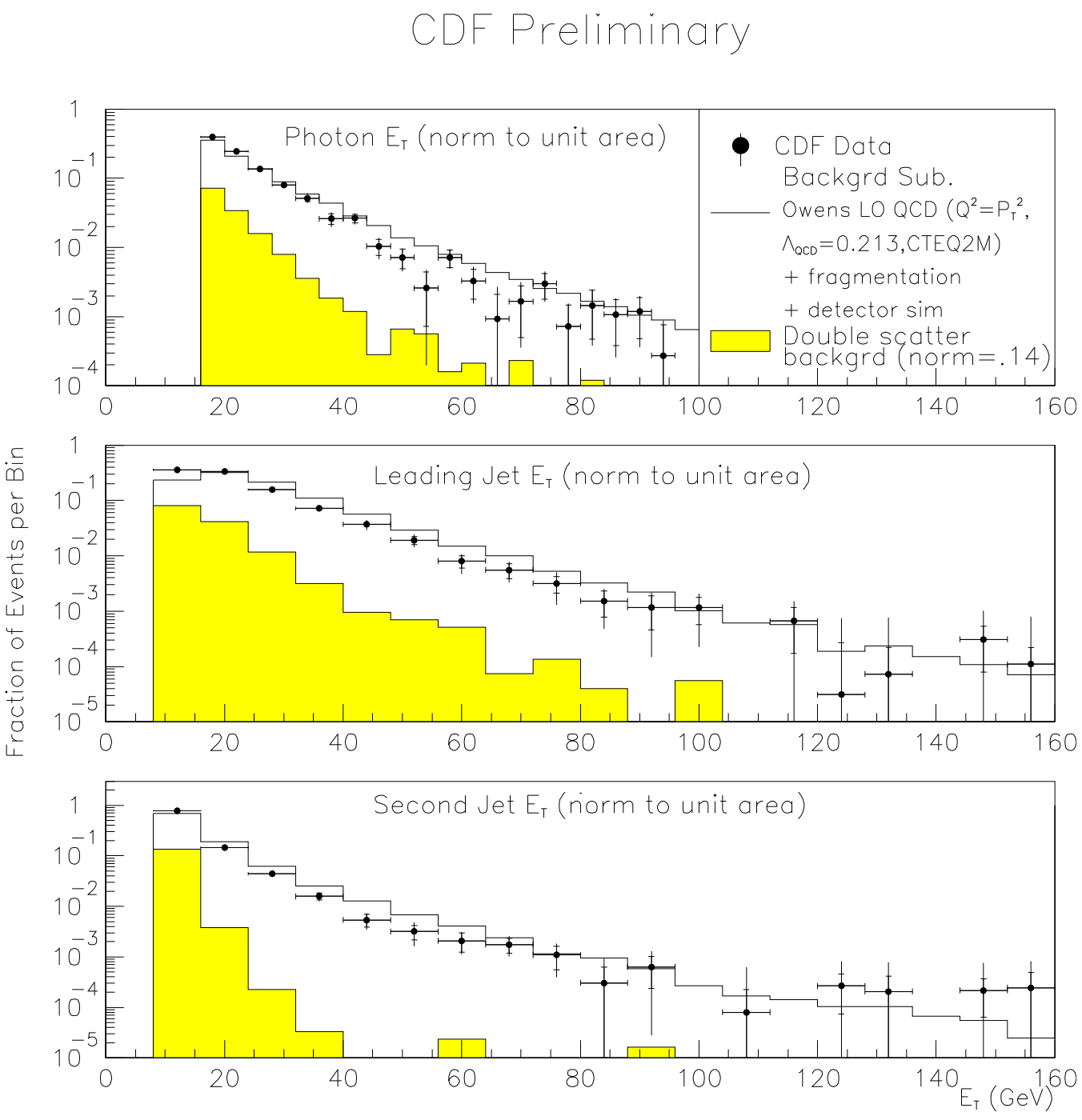
- The next-to-leading-order calculations involve the addition of γ+2 parton final states

- Should be able to examine correlations between the γ and the two final state jets in order to test the underlying subprocesses

- Such measurements have been done by CDF - Abe at al., Phys. Rev. **D57**, 67(1998)

- Caveat - these comparisons are to tree-level predictions based on $2 \rightarrow 3$ subprocesses, so they are not full next-to-leading-order predictions.


CDF Preliminary
Photon + Jet1 + Jet2 Inv. Mass
Fraction of Events per Bin
Double scatter
backgrd (norm=.14)
Mass (GeV)
X(γ) = E_T^γ/Mass
X(Lead Jet) = E_T^Lead Jet/Mass
X(Second Jet) = E_T^Second Jet/Mass
(norm to unit area)
CDF Data
Backgrd Sub.
Owens LO QCD (Q²=P_T²,
Λ_QCD=0.213,CTEQ2M)
+ fragmentation
+ detector sim


CDF Preliminary
Photon E_T (norm to unit area)
CDF Data
Backgrd Sub.
Owens LO QCD ($Q^2=P_T^2$,
Λ_{QCD}=0.213,CTEQ2M)
+ fragmentation
+ detector sim
Double scatter
backgrd (norm=.14)
Leading Jet E_T (norm to unit area)
Second Jet E_T (norm to unit area)
Fraction of Events per Bin
E_T (GeV)

Event Structure

- Lowest order Compton and annihilation subprocesses correspond to an isolated photon recoiling against a jet

- Fragmentation contributions add a component where the photon is accompanied by the hadronic fragments of the parent jet

- Expect to see fewer hadrons on the photon side of the event than would be the case with a hadronic trigger

- Results from UA-2 (R. Ansari et al., Z. Phys. **C41**, 395(1988)) show the reduced number of same side hadrons for photon triggers

- Note that the amount of hadronic activity for photon triggers is larger than that for W production, while being smaller than that for hadronic triggers

- Consistent with a superposition of direct and fragmentation components

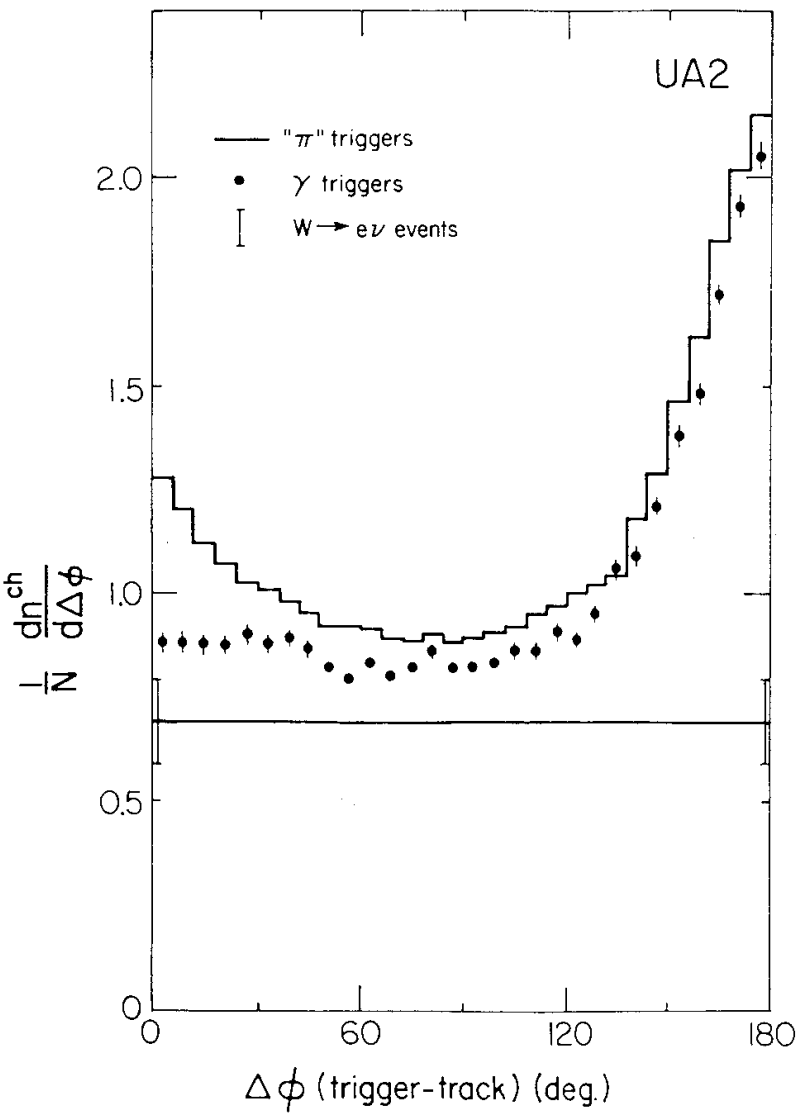
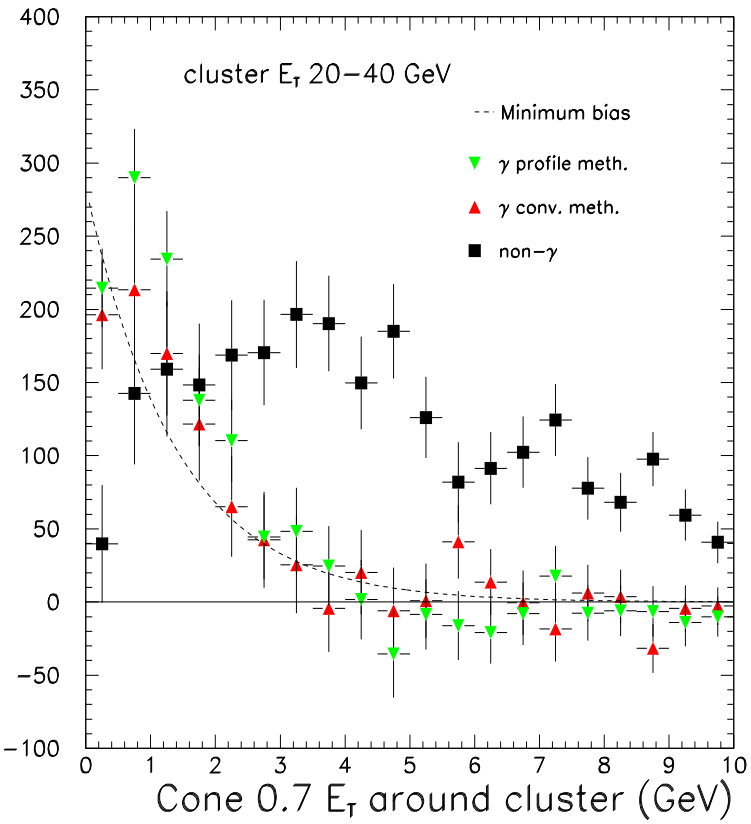

UA2
"π" triggers
γ triggers
W → eν events
2.0
1.5
1.0
0.5
0
0
60
120
180
$\frac{1}{N}\frac{dn^{ch}}{d\Delta\phi}$
Δφ (trigger-track) (deg.)

Figure 12

Similar result seen in preliminary CDF data



E_T inside a cone of radius 0.7 about the photon looks just like minimum bias case - and not at all like that for hadronic events

Difficult to identify fragmentation component due to background from beam fragments

Recap

- Thus far we have seen that
  - Photons are produced with the expected γ-jet angular distribution which is flatter than that for dijets
  - Photon-jet rapidity correlations are as expected
  - Photon+2jet events appear to behave as expected
  - Photons appear to be accompanied by fewer hadrons than for purely hadronic triggers, although more than for W events
- All of these items suggest that the basic mechanism for producing high-p_T photons is as expected from QCD
- Is there any other way to probe the dynamics of photon-parton interactions that tests these same mechanisms?

Yes! Jet Photoproduction

- The same subprocesses are involved, but with the photon crossed to the initial state

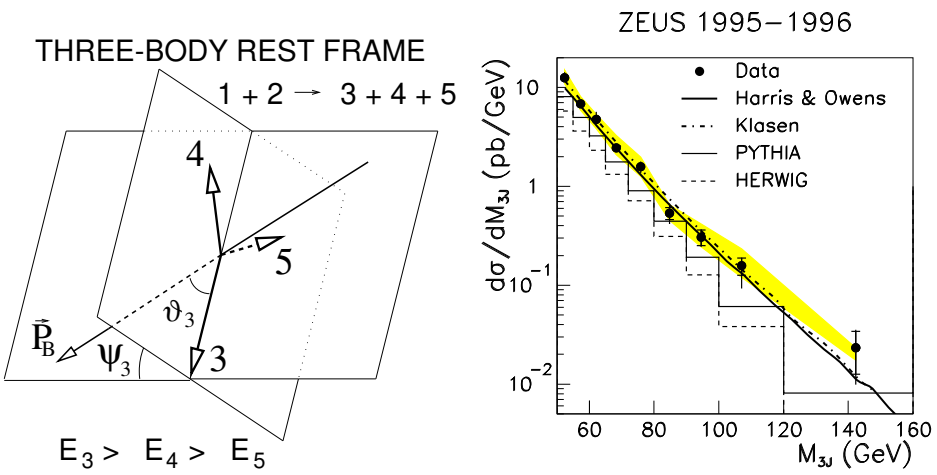
$$qg \to \gamma q \quad \Rightarrow \quad \gamma q \to qg \quad \text{etc.}$$

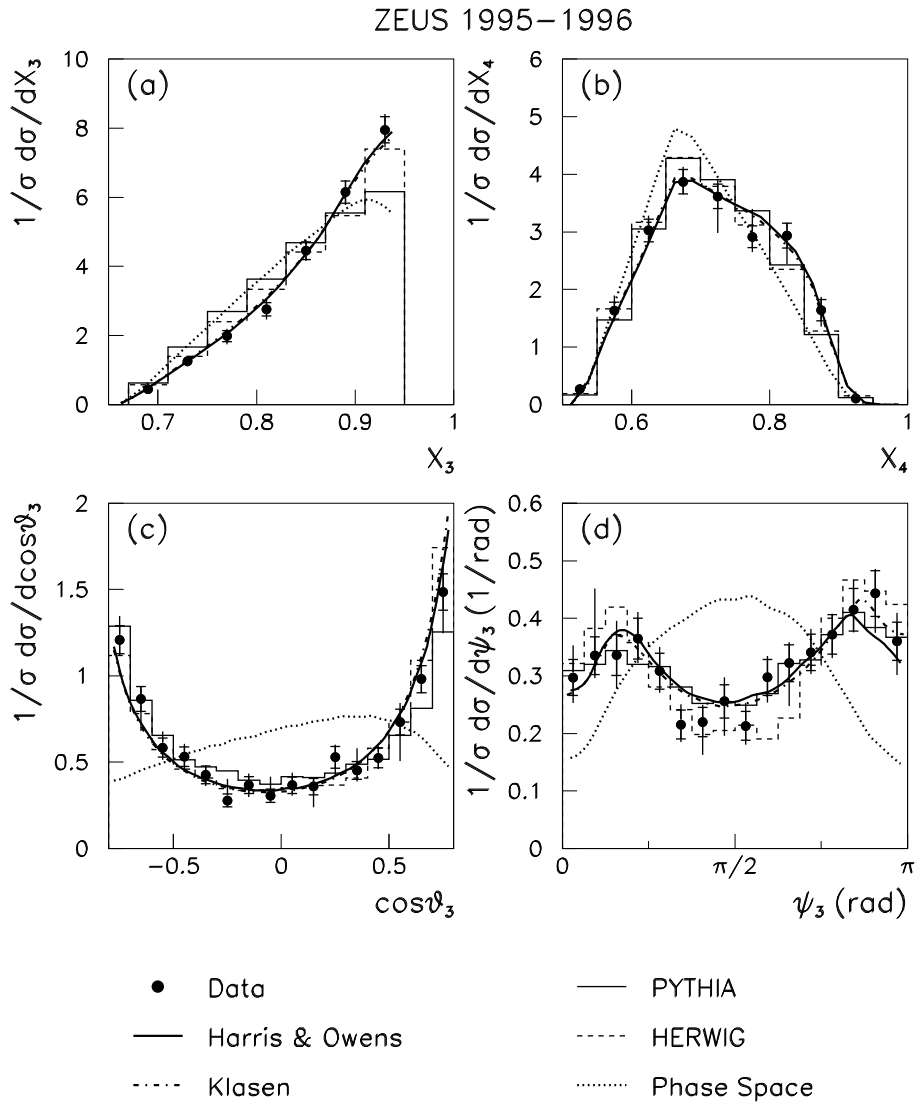
- Role of photon fragmentation is now in the initial state where it is treated using a photon pdf
- Details may be found in my CTEQ 2001 summer school lecture at www.cteq.org

Examples

- Consider several different processes in order of increasing complexity concerning the interactions of the photons

- Photoproduction of three-jet final states
  - Calculation based on $\mathcal{O}(\alpha\alpha_s^2)$ subprocesses
    1. $\gamma q \to qgg$
    2. $\gamma g \to q\overline{q}g$
    3. $\gamma q \to qq'\overline{q}'$
  - Consider three identified high-p_T jets with high three-jet mass
  - Only looking at the pointlike interaction of the photon - no resolved contribution
  - Calculation at this level is lowest order (LO) and also leading-log (LL)
  - Compare to data from the ZEUS Collaboration (hep-ex/9810046)
    * three-jet mass M_{3j}
    * energy fractions: jets ordered in energy with $E_3 > E_4 > E_5$ and energy fractions defined as $x_i = 2E_i/M_{3j}$
    * jet angular distributions





Observe good agreement between matrix element calculations and the data and also between shower Monte Carlo calculations and the data

Single jet and Dijet Photoproduction

- As noted earlier, there will be two components
  - direct or pointlike - photon contributes directly to the hard scattering
  - resolved - both photon and proton parton distributions convoluted with parton-parton scattering subprocesses
  - at $\mathcal{O}(\alpha\alpha_s^2)$ and beyond, specific diagrams can contribute to both. Need an experimental definition to distinguish between the two classes. For example the ZEUS Collaboration has used the following for dijet final states:

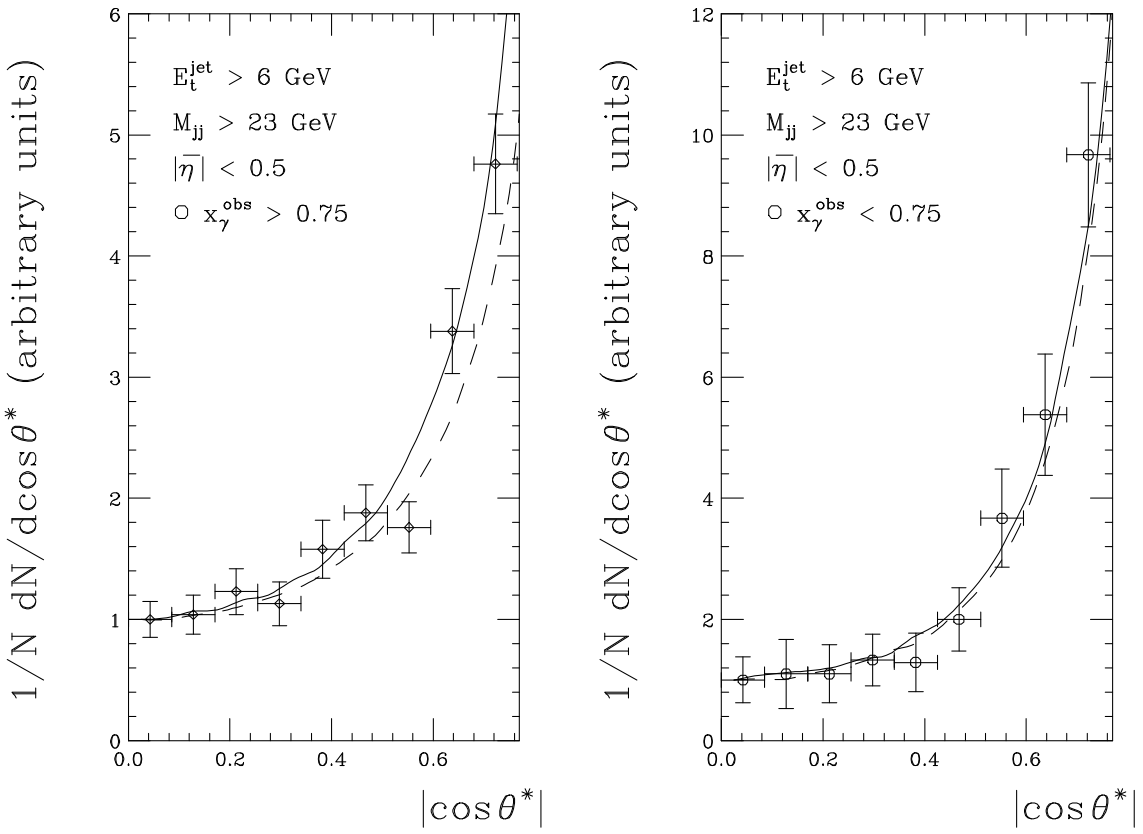
$$x_\gamma^{obs} = (E_T^{jet_1} e^{-\eta_{jet_1}} + E_T^{jet_2} e^{-\eta_{jet_2}})/2E_\gamma$$

  - positive η corresponds to the direction of the proton
  - consider two samples with $x_\gamma^{obs} > .75$ (mostly direct) and $x_\gamma^{obs} < .75$ (mostly resolved)

- Dijet angular distribution
  - $\gamma q \to qg$ and $\gamma g \to q\overline{q}$ are fermion exchange processes which have an angular distribution going as $1/(1-\cos\theta)$ in the dijet center of mass system
  - typical boson exchange processes in the resolved part $qq \to qq, qg \to qg, gg \to gg$ etc., behave as $1/(1-\cos\theta)^2$ which is much steeper

Compare to data from the ZEUS Collaboration (M. Derrick *et al.*, Phys./ Lett. B**384**, 401 (1996).)

Curves from Harris and Owens, Phys. Rev. D**56**, 4007 (1997). Dashed lines are LO and solid are NLO.



Note the different scales on these normalized distributions - the dominantly resolved sample has a much steeper distribution

Aside

Why is it possible to separate the direct and resolved components in jet photoproduction, yet it is difficult to isolate the fragmentation or bremsstrahlung component in direct photon production?

- In jet photoproduction one has a sample with two identified jets in the final state. These can be used to reconstruct x_γ and x_p directly.

- Higher order effects and smearing due to beam fragments entering the jet cones smear out the direct component peak at $x_\gamma = 1$, but one can still define two samples of events which are dominantly resolved or dominantly direct.

- In contrast, for direct photon production
  - Some of the fragmentation contribution is removed by isolation cuts used in the trigger (see later)
  - For fragmentation events, one must reconstruct x_a, x_b, and the fragmentation variable z_c. There is not enough information.

- To put it another way, as shown earlier, the direct photon events at the TeVatron have accompanying hadronic energy that is indistinguishable from the minimum bias case.

- This means that one can not use the accompanying hadronic energy as a flag for fragmentation events.

Lessons Learned

- Photoproduction data support the view that photons interact with partons as expected in QCD

- Two-component picture of the dynamics
  - Direct (point-like) and resolved components in jet photoproduction - resolved piece uses a photon parton distribution to resum large logs coming from configurations where partons are produced collinear with the incoming photon
  - Direct (point-like) and fragmentation components in direct photon production - fragmentation piece uses a photon fragmentation function to resum large logs coming from configurations where the photon is a fragment of a jet and there are partons which are produced collinear with the outgoing photon.

- So, to this point it seems that the theory of direct photon production is supported by the data.

Then, what is all the fuss about?

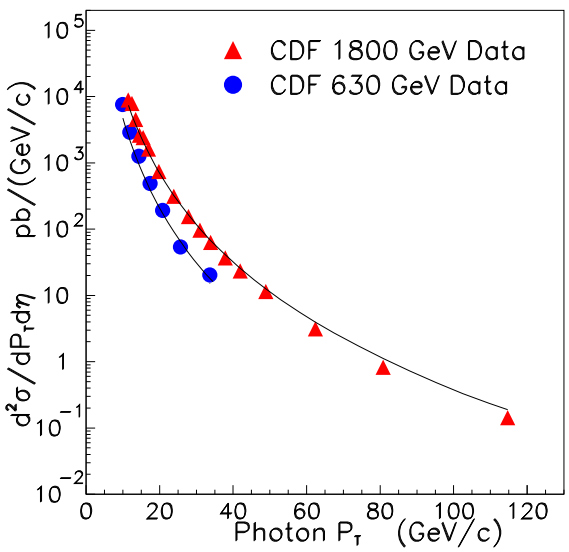
Single Photon Inclusive Cross Section

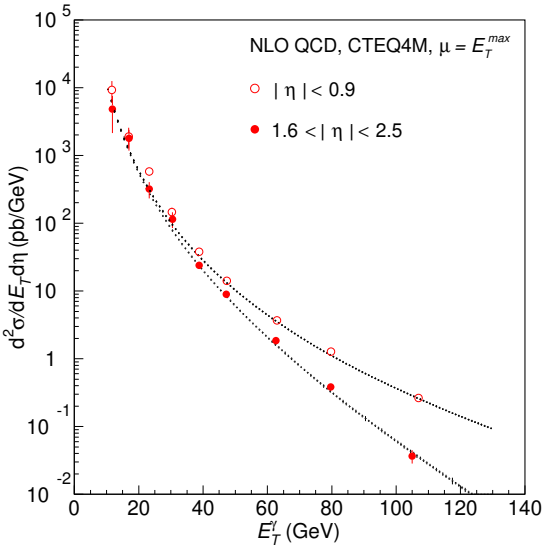
- Integrate over all partons, leaving only the photon as being observed
- In some sense this is the hardest measurement to interpret since if there is a disagreement between theory and experiment one has few clues as to the origin - all other details have been integrated out
- On the other hand, the inclusive nature of the observable makes the calculation easier - you don't have to model the distributions that are integrated over. Rather, you only have to get the integral over the distribution correct.
- Can measure - and calculate - the photon p_T and η distributions
- p_T spectrum is steeply falling so that it is especially susceptible to resolution smearing effects on the experimental side and approximations made in the theoretical calculations
- Calculations use the same techniques as described previously
  - Analytic calculations result in faster running programs, since many of the integrations have already been done and only the convolutions with pdfs and fragmentation functions need to be done.
  - Monte Carlo based programs are able to investigate the effects of the photon isolations cuts to be discussed below, although approximation techniques exist so that the analytic programs can also invoke these cuts.

Tevatron data

CDF: $\sqrt{s} = 1800, 630$ GeV, $|\eta| < 0.9$

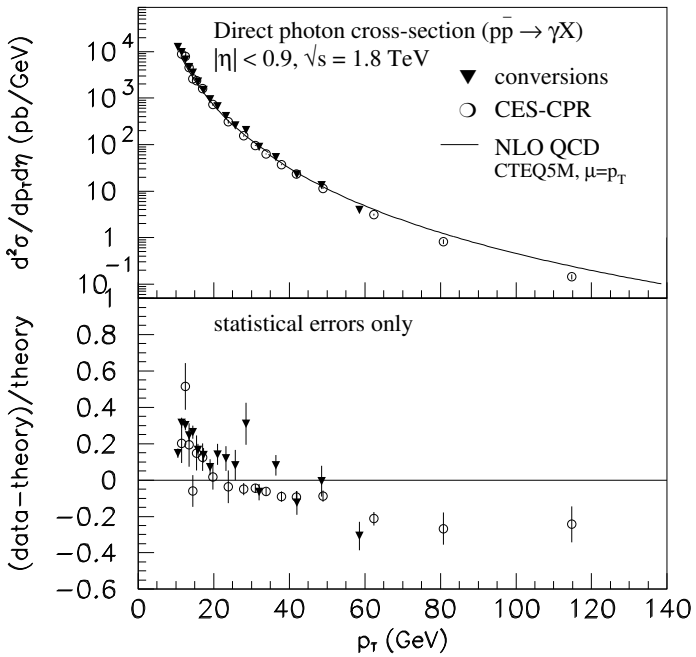
DØ: $\sqrt{s} = 1800, 630$ GeV, $|\eta| < 0.9, 1.6 < |\eta| < 2.5$

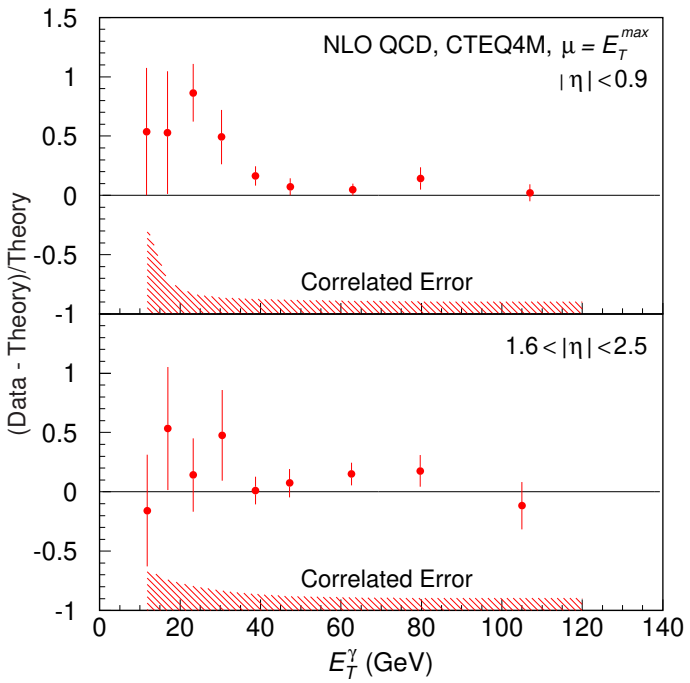




- Agreement looks good when plotted on a logarithmic scale
- Confirms expectation that the QCD description of direct photon production is correct
- But what if we look closer . . . ?

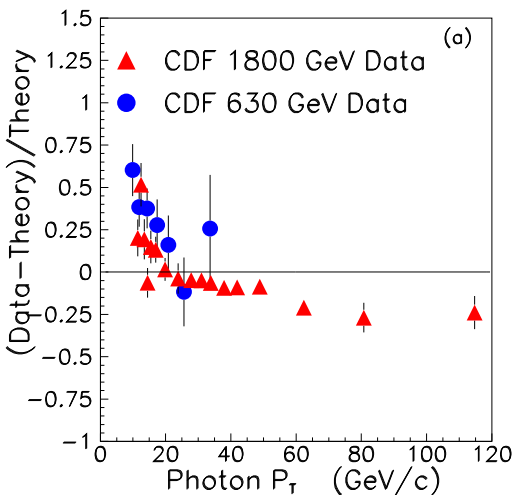
Look on a linear scale . . .

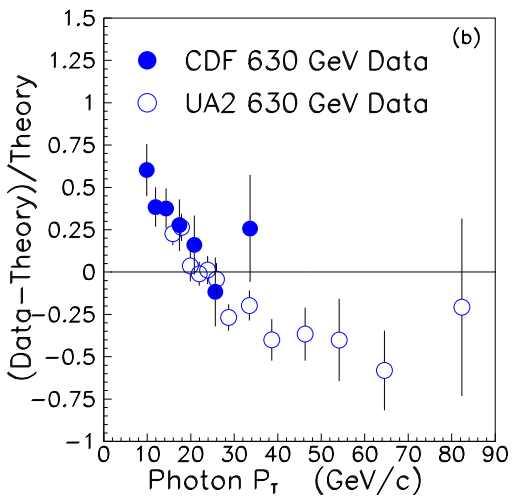


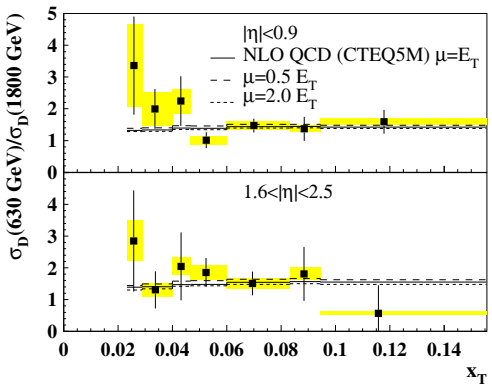


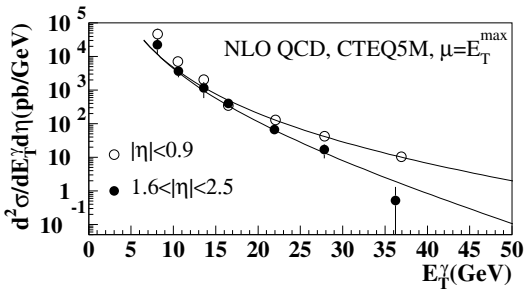
Both CDF and DØ see an excess at the low p_T end in the central region

- Problem seen by CDF at both 1800 and 630 GeV
- Excess occurs at low p_T, not at fixed x_T, so the solution can not be a simple adjustment of the pdfs
- Effect also seen by UA-2









- at low p_T also seen by DØ at 630 GeV
- somewhat reduced for the forward rapidity region

Isolation Cuts

- So, what can explain the disparity between shapes of the experimental and theoretical distributions?

- One item to consider is that these are not truly inclusive measurements. Rather, they are measurements of inclusive cross sections for *isolated* photons.

- Algorithms are different for each experiment (UA-2, CDF, DØ) but all limit the allowed hadronic energy in the vicinity of the electromagnetic trigger

  $\Rightarrow$ fragmentation component is reduced

- Expect fragmentation to be most important at the low p_T end of the distribution

- Trigger bias effect - fragmentation provides only a portion of the available energy to the photon. The pointlike or direct subprocesses are more efficient and so dominate at high values of p_T

- Errors on modeling the effects of the isolation cuts could especially affect the low p_T end of the spectrum

Specific Example - CDF cone algorithm

- Require that there be less than 1 GeV of hadronic transverse energy in a cone of radius

$$\mathcal{R} = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < 0.4$$

about the direction of the photon.

Theoretical modeling (phase space slicing method)

- For the two-body fragmentation component there is no dependence on $\mathcal{R}$ since the fragmentation functions are inclusive quantities.
  - Work in the collinear approximation (all emitted partons or photons are collinear with the parent parton)
  - parent parton transverse momentum is $p_{T part}$
  - photon transverse momentum is $p_{T\gamma} = z p_{T part}$
  - hadronic E_T is $(1-z)p_{T part} = (1-z)p_{t\gamma}/z$.

  Requiring that the hadronic E_T is less than $E_{T cut}$ results in

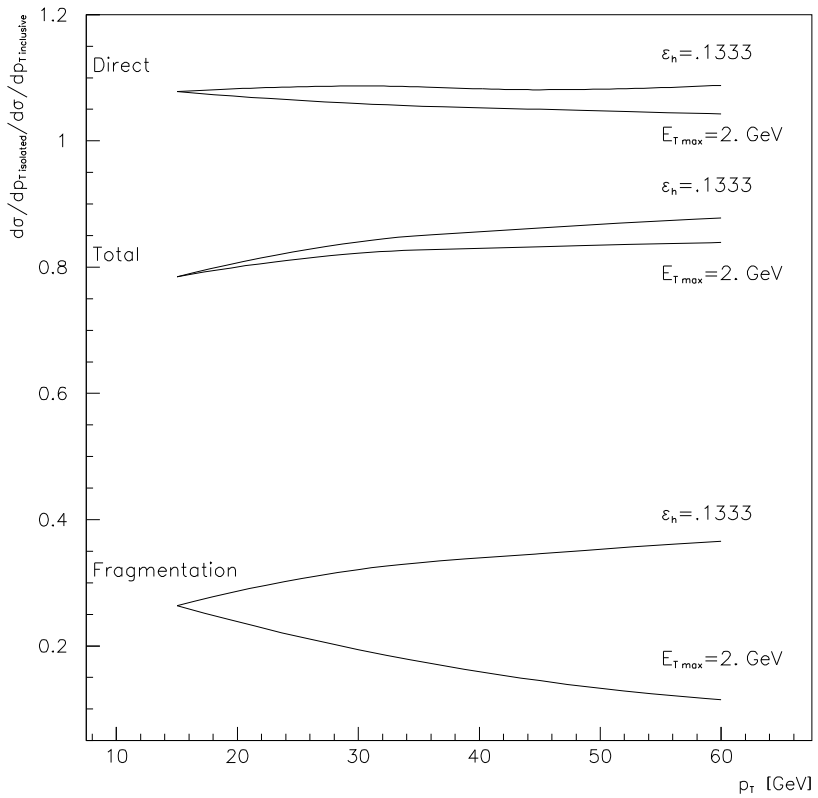
$$z > \frac{1}{1 + E_{T cut}/p_{T\gamma}}.$$

- For the $2 \to 3$ pointlike subprocesses, one can explicitly enforce the isolation condition on an event-by-event basis in the Monte Carlo.

- One can also enforce a similar isolation condition on the $2 \to 3$ fragmentation component

Example

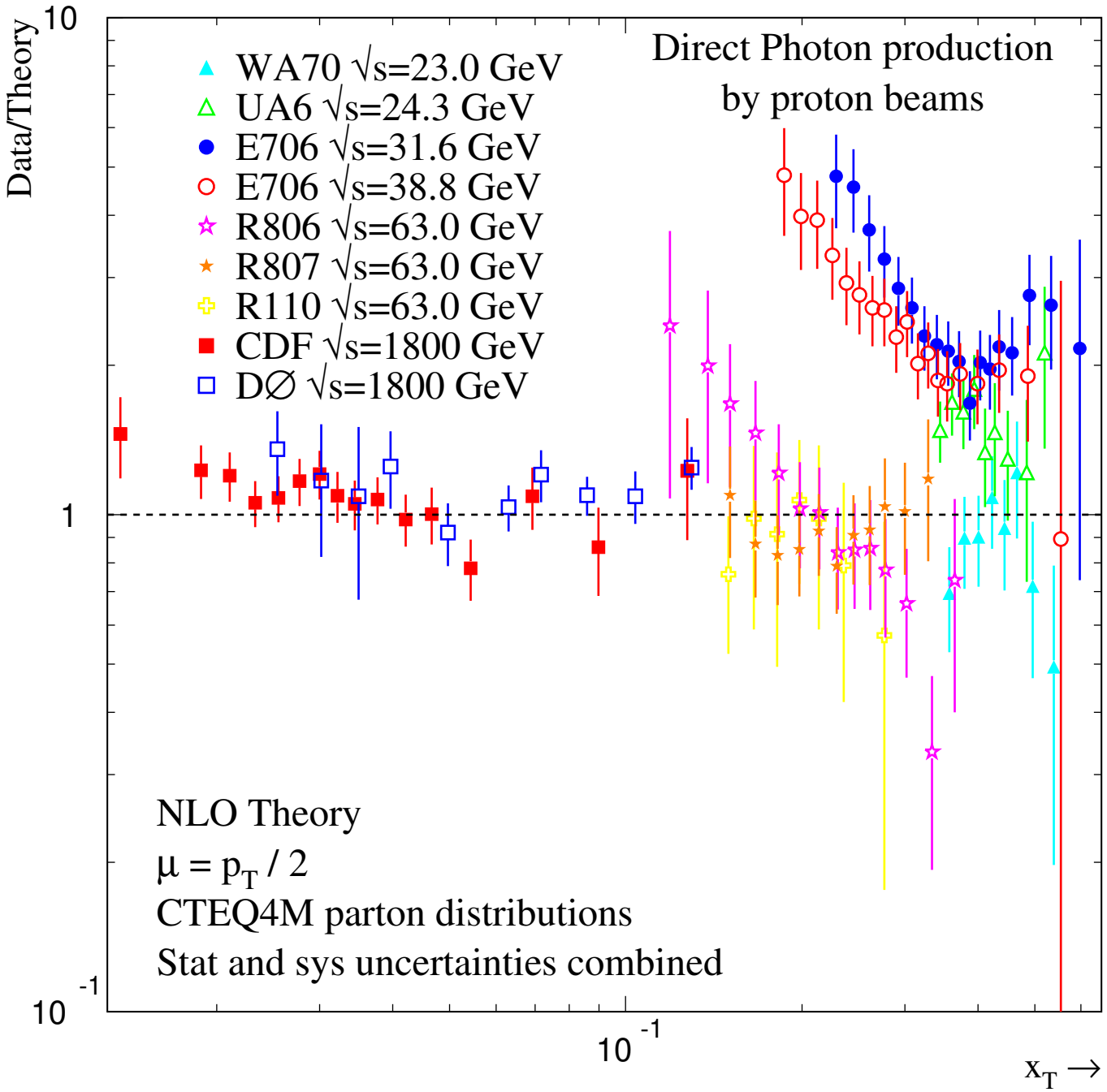
Study of the behavior of the isolated photon cross section by Catani, Fontannaz, Guillet, and Pilon, hep-ph/0204023



- Look at the "Total" curve for $E_{Tmax} = 2$ GeV. The cone radius is $\mathcal{R} = .7$ (similar to $E_{Tmax} = 1$ GeV in a cone of radius .4)
- $\sim 22\%$ reduction due to isolation at p_T=15 GeV
- Expected effect of the isolation slowly decreases as p_T increases
- Even if the isolation was *totally removed*, the cross section would only increase by 28 %.
- Strongly suggests that the modelling of the isolation cuts is not responsible for the discrepancy between theory and experiment

Fixed Target and Lower Energy Collider Data

It is now time to consider the situation for the inclusive cross section at lower energies



Comments

1. Data/Theory plotted versus x_T

2. Data plotted at same x_T but different $\sqrt{s}$ correspond to different p_T's

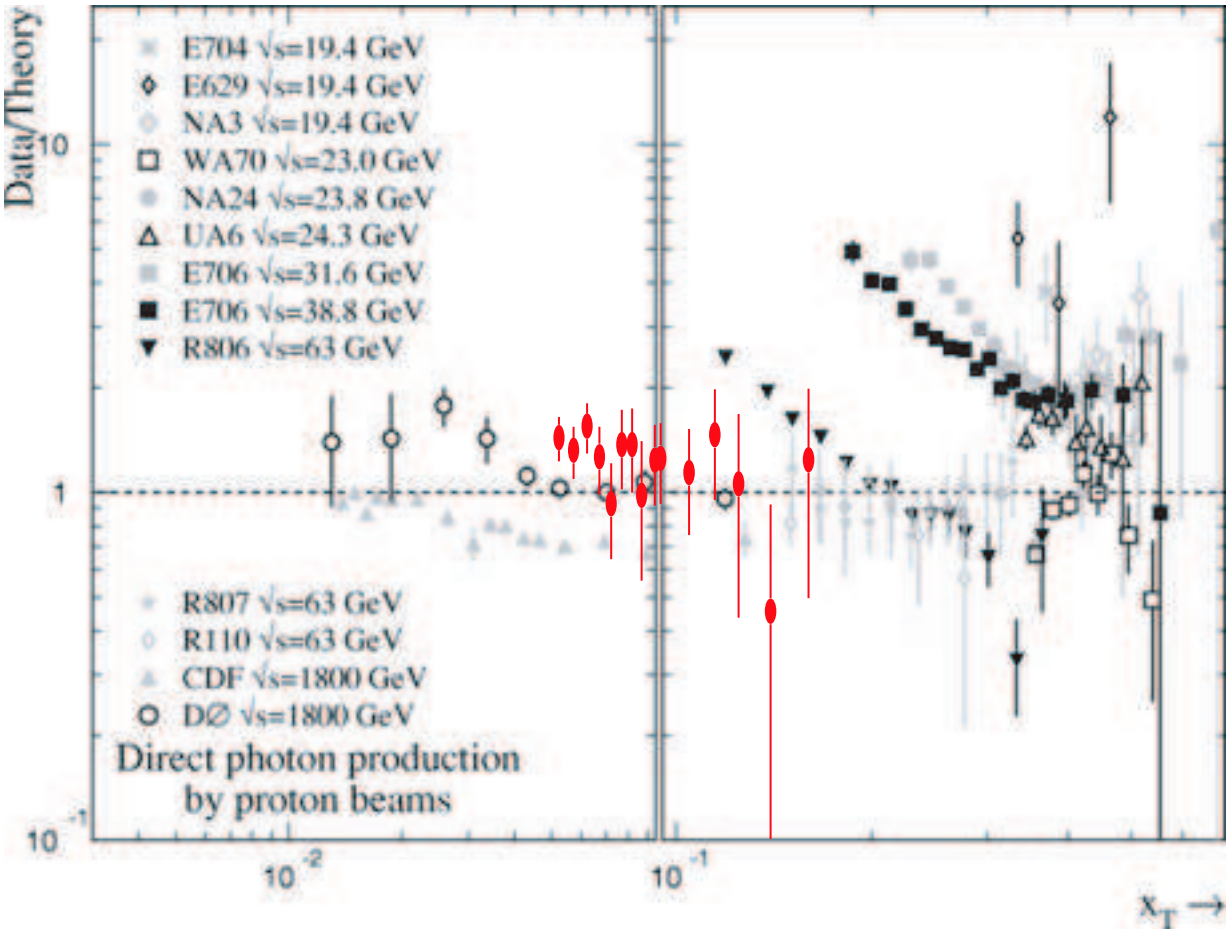
3. E706 higher than theory, UA6 somewhat above theory, WA70 and theory agree

   - Likely that there is some experimental inconsistency here since the range in $\sqrt{s}$ is relatively small and it would take a significant modification of the theory to explain all three sets simultaneously.

4. See some shape disagreements among the ISR experiments

5. Plotted on this scale, the previously noted deviations of the theory from the CDF and DØ data look pretty darned small!

- Similar plot showing new PHENIX data (nucl-ex/0504013)

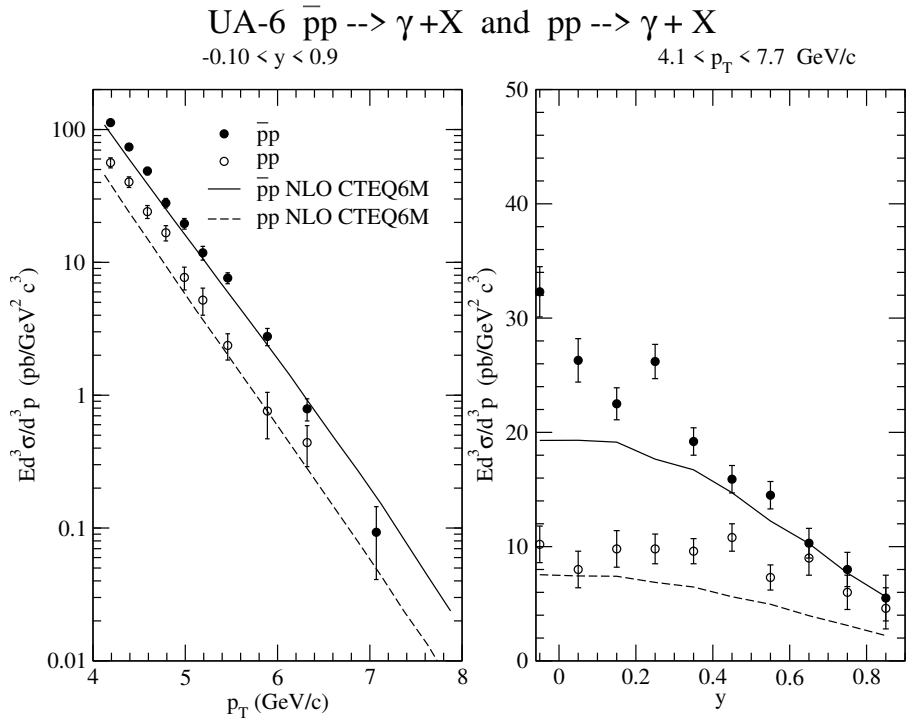


- Agreement is comparable to that for other collider experiments and better than for the fixed target regime

Example - UA-6

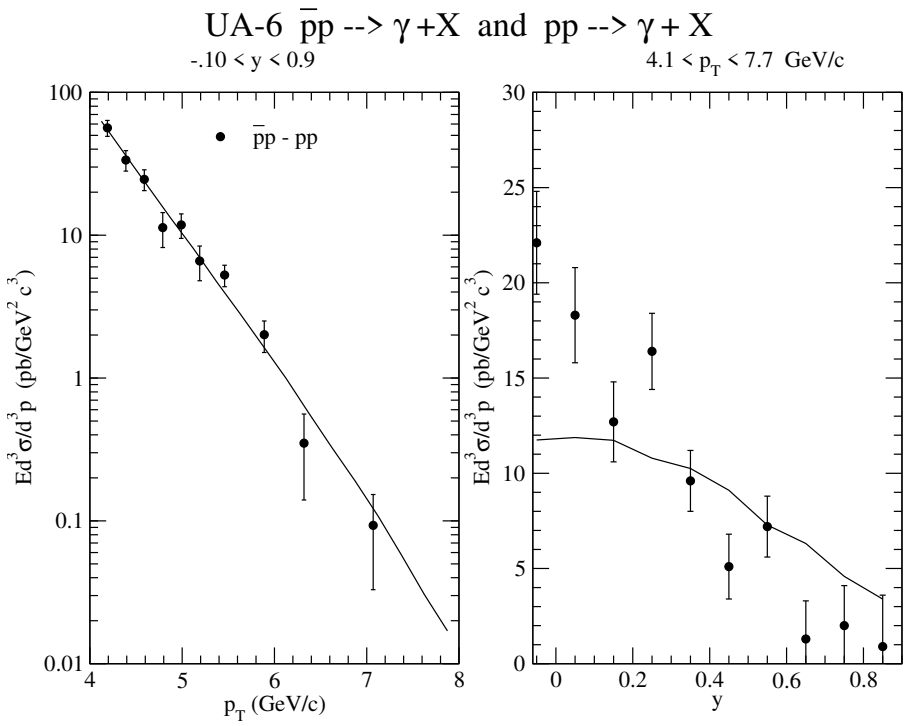
- Measured both pp and $\overline{p}p$ at $\sqrt{s} = 24.3 GeV/c$

- Initial state gluon and gluon fragmentation contributions cancel in the $\overline{p}p - pp$ difference



- Theory below the data at the lower end of the p_T range

- Rapidity theory curves are flatter than the data



- Cross section difference cancels contributions from gluons

- p_T difference is well described

- Rapidity theory curve is somewhat flatter than the data

- So, the situation is mixed - the $\overline{p}p$ and pp curves are individually below the data, the p_T difference is well described, while the rapidity difference curve is a bit too flat

So what is going on?

- Theory and data have different shapes for $\sqrt{s} = 630$, 1800 GeV with the theory being flatter than the data

- Some of the lower energy experiments show this same behavior to an even larger degree - others do not

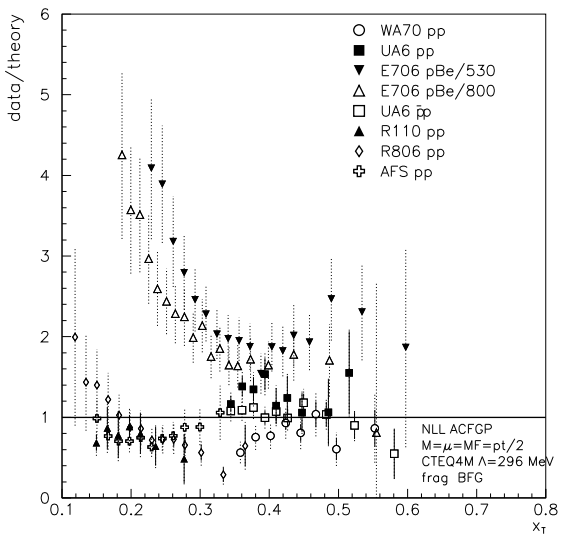
- Critical review of the situation for the lower energy experiments: Aurenche, Fontannaz, Guillet, Kniehl, Pilon, and Werlen, hep-ph/9811382

Is this behavior seen for any other processes?

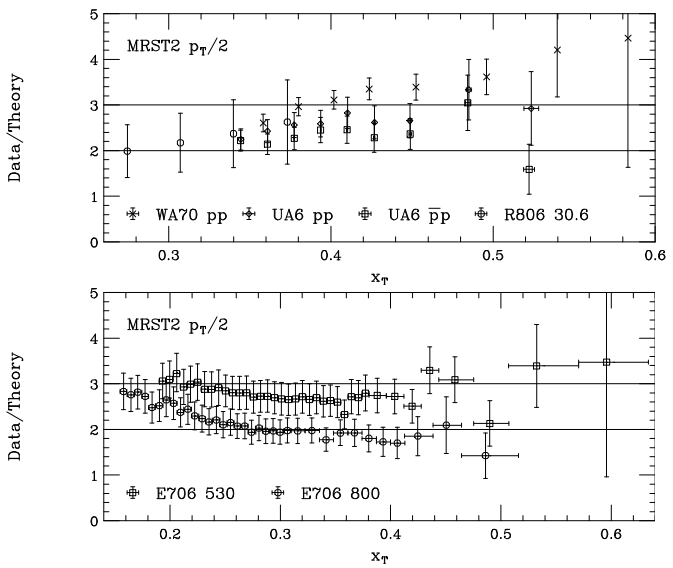
Yes - Inclusive single hadron production!

- Situation reviewed by Aurenche, Fontannaz, Guillet, Kniehl, and Werlen, hep-ph/9910252

Example plot for direct photon production

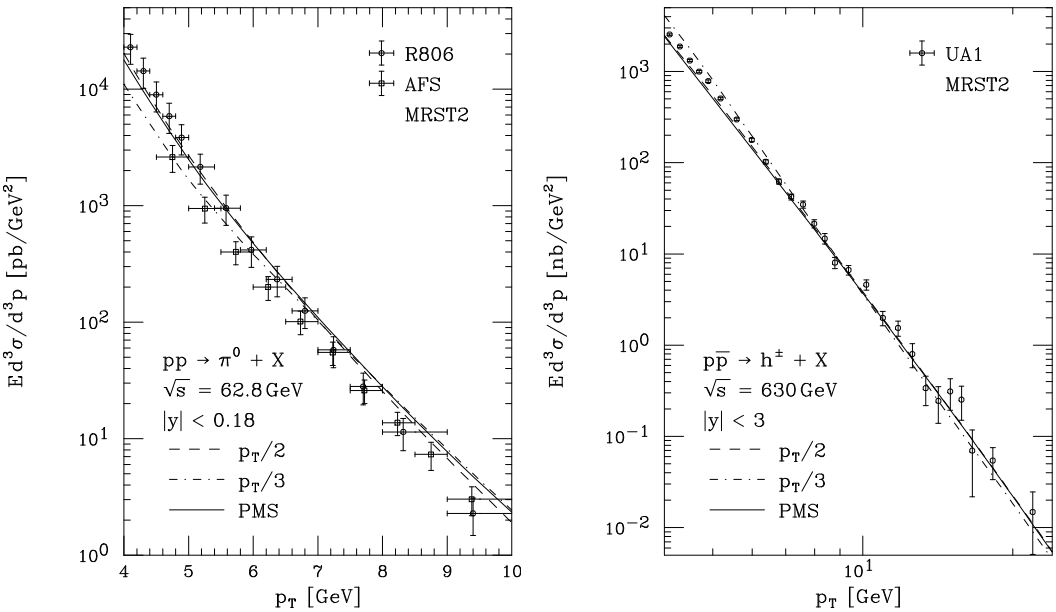


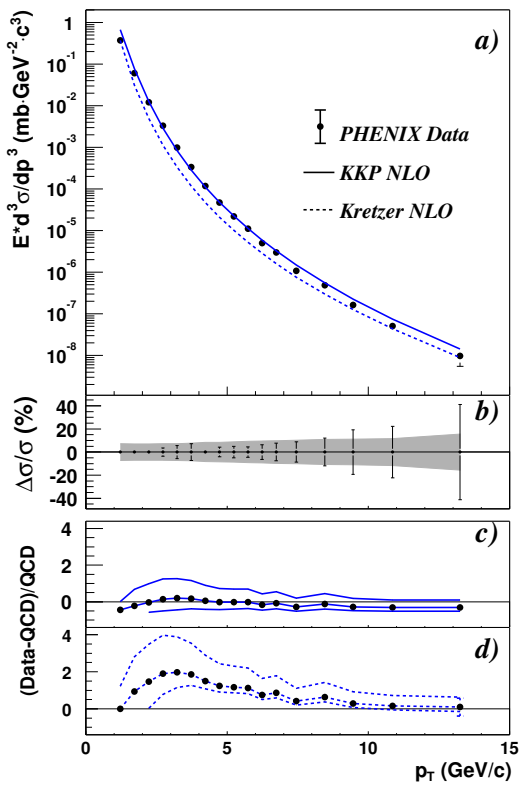
And two for π^0 production



See similar excesses of data over theory

But a strange thing happens as one goes up in energy. . .





Agreement between theory and data gets better at higher energies *and at lower p_T's!*

So where do we stand?

1. Situation with fixed target direct photon production is confused by some disagreement between experiments
   - ▷ See Apanasevich et al., hep-ph/0007191 for a discussion of the systematics of γ/π^0 ratios and consistency between experiments

2. All experiments see an excess of data over theory for single hadron inclusive production at fixed target energies

3. Agreement between theory and data for single hadron production improves with increasing energy and is excellent by $\sqrt{s} = 200$ GeV

4. Likely that we need an *improved method of calculating single particle inclusive cross sections* in the fixed target energy range - one that would improve agreement for both photon and hadron production

5. A reassessment of systematic errors on the existing fixed target photon experiments might also help resolve the discrepancies between data sets

6. This still leaves the question of why the higher energy hadroproduction data are well described by theory at the low p_T end of the spectrum but the photon data are not. . .

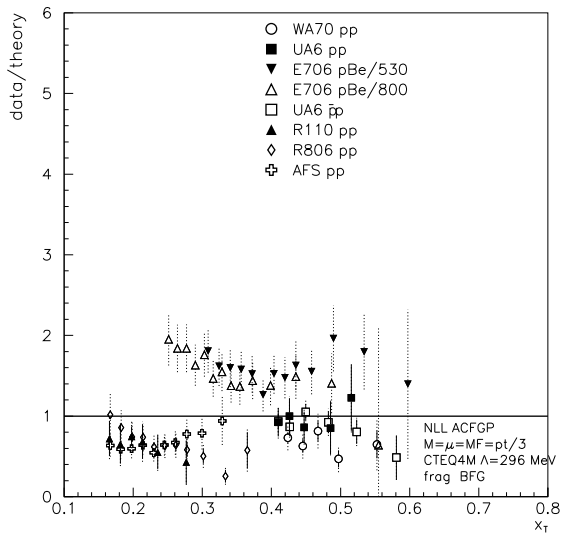
Theoretical Ideas and Scenarios

Summarizing the analysis of Aurenche, Fontannaz, Guillet, Kniehl, Pilon, and Werlen, hep-ph/9811382

- Consider only pp collisions in order to avoid the question of isolation cuts at the colliders

- Only consider data for which the chosen factorization scale exceed the starting scale Q_0 for the pdfs

- Resulting comparison suggests that the only problem is one of normalization of the data, at least if the scales are chosen as $p_T/3$ - perhaps the systematic errors were underestimated?



- This still leaves the question of the collider data - perhaps a better job needs to be done on modeling the isolation cuts?

If the direct photon calculations are ok, then what about the single hadron inclusive cross sections?

- The steeply falling spectra in the fixed target region force the fragmentation variable z to be near one. As one goes up in energy the distributions flatten somewhat and $< z >$ decreases

  1. Fragmentation functions are not well constrained by data at high values of z

  2. As $z \rightarrow 1$ large logarithms of $(1 - z)$ should be resummed

- In this view, the theoretical problems are largely due to issues related to fragmentation:

  – High-z problems for hadron production

  – Effects of isolation cut s for photon production

Resummation Techniques

Threshold Resummation

Basic Physics -

- For inclusive calculations singularities from soft real gluon emission cancel against infrared singularities from virtual gluon emission

- Limitations on real gluon emission imposed by phase space constraints can upset this cancellation

- Singular terms still cancel, but there can be large logarithmic remainders

- Classic example is thrust distribution in $e^+e^- \rightarrow jets$

High-p_T/high-mass particle production

- Phase space for gluon emission is limited near threshold in the parton-parton scattering subprocess

- Steeply falling pdf's constrain real gluon emission when high-mass or high-p_T is required

Define $v = 1 + t/s$ and $w = -u/(s+t)$.

Threshold occurs at $w = 1 \quad (s + t + u = 0)$

$\frac{d\sigma}{dvdw}$ contains terms like $\alpha_s^m \left(\frac{\ln^n(1-w)}{1-w}\right)_+$

Can sum leading logs ($n = 2m - 1$),

next-to-leading-logs ($n = 2m - 2$), etc.

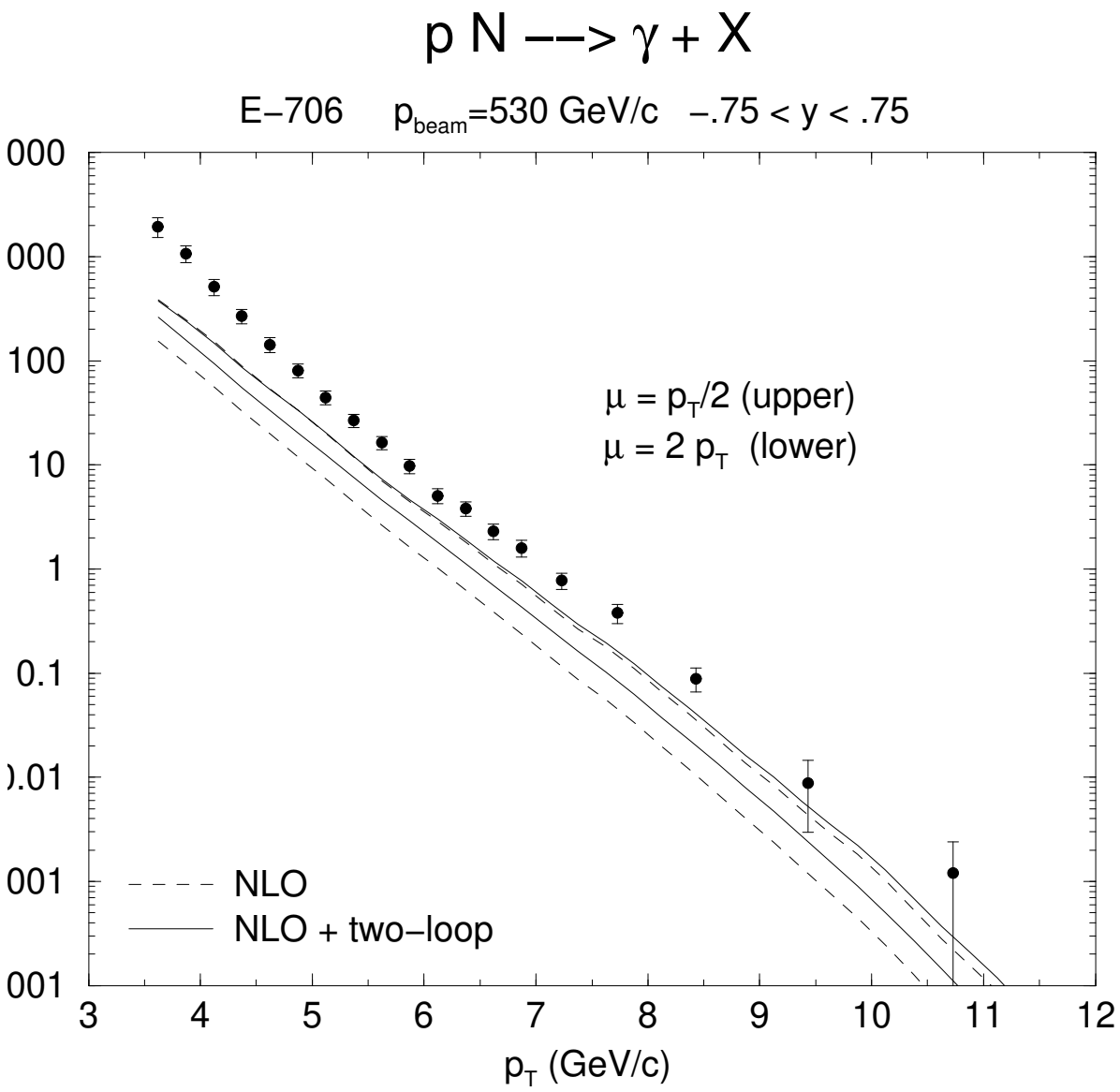
Results

- Corrections large as $x_T \to 1$
- Variations with respect to scale(s) reduced
- Corrections with $\mu = p_T/2$ small at intermediate x_T

Threshold resummation can't solve the current dilemma, but the results are interesting nonetheless.

For more details see:

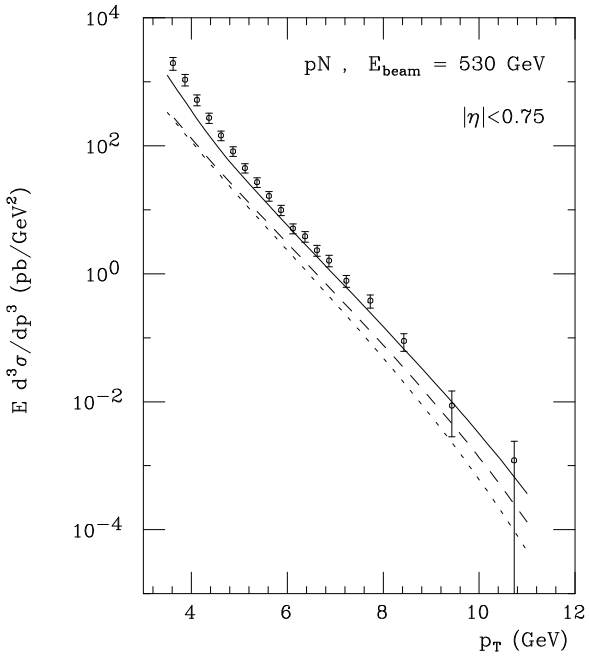
1. E. Laenen, G. Oderda, and G. Sterman, hep-ph/9806467, Phys. Lett. B438, 173 (1998)
2. S. Catani, M.L. Mangano, and P. Nason, hep-ph/9806484, JHEP 9807 (1998) 024; S. Catani, *et al.*, hep-ph/9903436, JHEP 9903 (1999) 025.
3. N. Kidonakis and J.F. Owens, hep-ph/9912388, Phys. Rev. D61,094004, 2000; hep-ph/0307352, J. Mod. Phys. A19:149, 2004


p N −−> γ + X
E−706 p_{beam}=530 GeV/c −.75 < y < .75
000
000
100
10
1
0.1
).01
001
001
μ = p_T/2 (upper)
μ = 2 p_T (lower)
NLO
NLO + two−loop
3
4
5
6
7
8
9
10
11
12
p_T (GeV/c)

New formalism - Combine threshold and recoil resummation formalisms

E. Laenen, G. Sterman, and W. Vogelsang, hep-ph/0002078, Phys. Rev. Lett. **84**:4296 (2000); hep-ph/0010080, Phys. Rev. **D63**, 114018, 2001

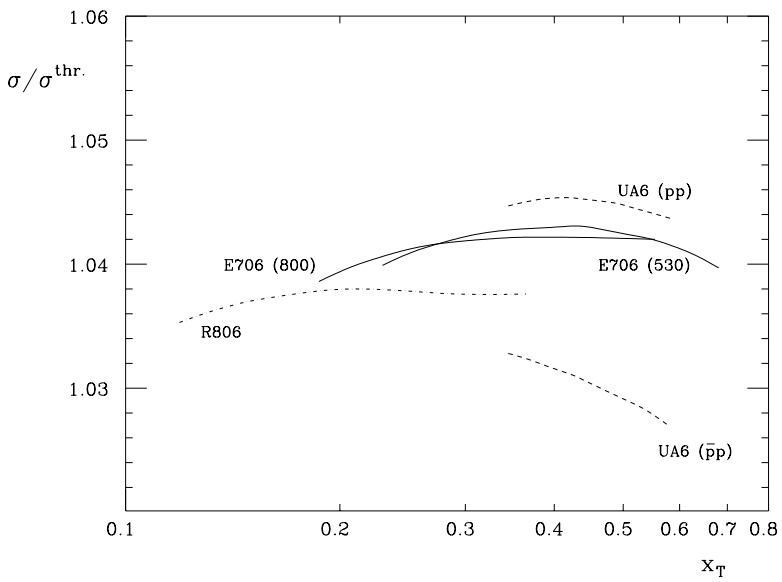
- Retain the physics of the threshold resummation *and* include recoil effects of initial state gluon emission
- Conserve both energy and momentum (longitudinal and transverse)
- Early results are encouraging

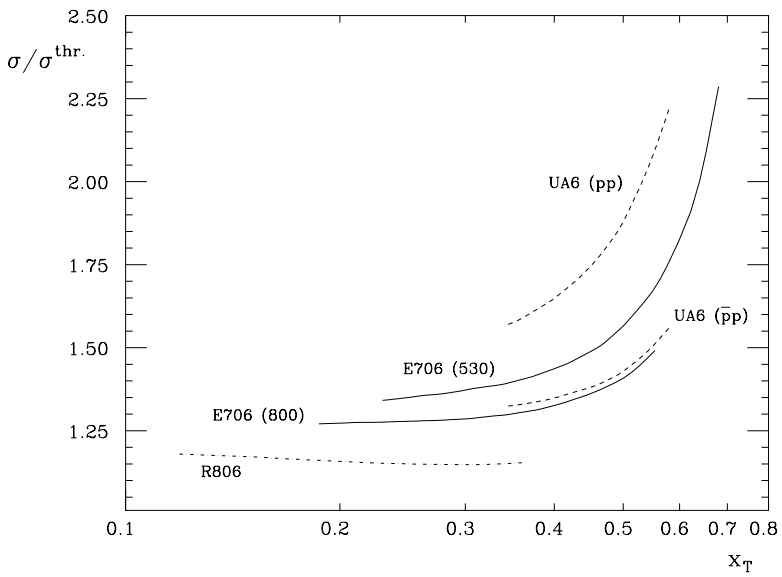


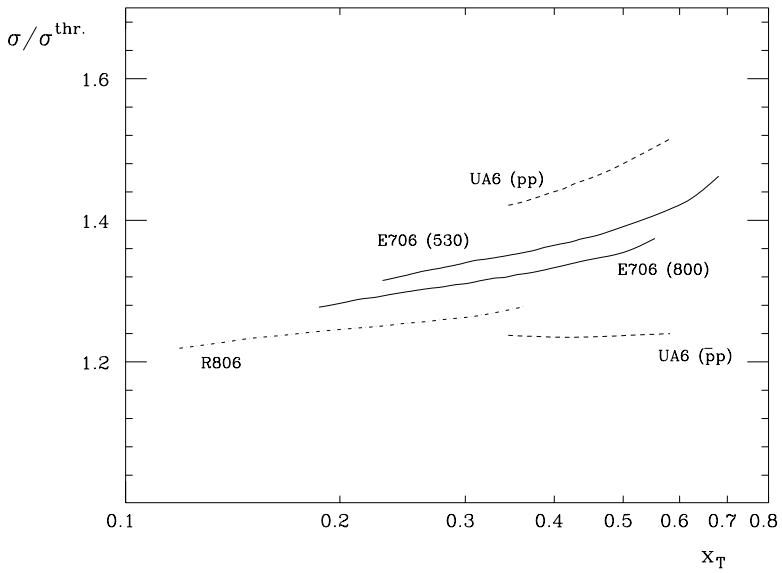
However,

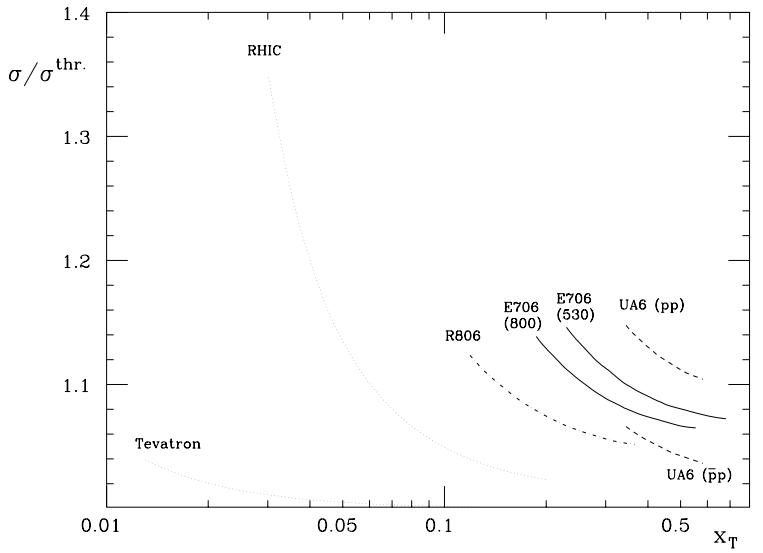
- Still need a nonperturbative parameter to describe parton k_T at the starting point of the shower evolution
- Need to develop a matching algorithm to tie on to perturbative result for large k_T for the γ-jet pair, *i.e.*, need to incorporate full NLO terms

- Recent modification by Sterman and Vogelsang (hep-ph/0904234) avoids kinematic singularities for large parton recoil (when most of the observed transverse momentum would come from recoil effects and not the "hard scatter.")
- Results depend on the parametrization chosen for subleading recoil terms









- Possible to get a decent "fit" to a wide variety of data, but the nonperturbative parameters are unnaturally large.

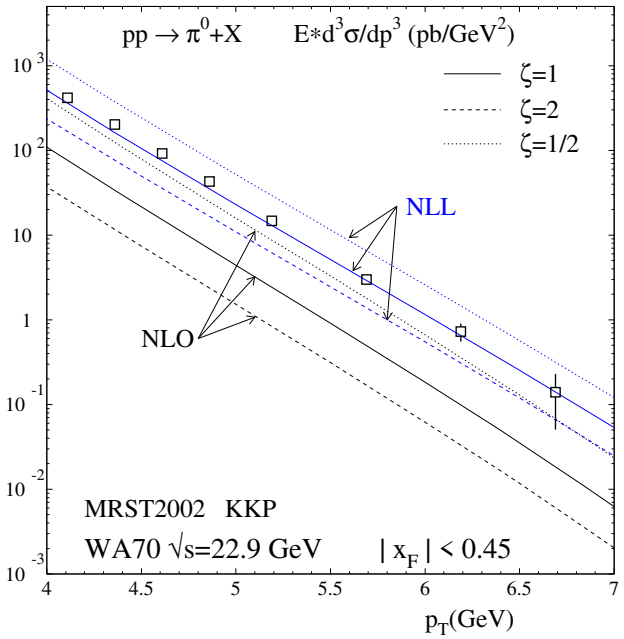
- See hep-ph/0904234 for additional details.

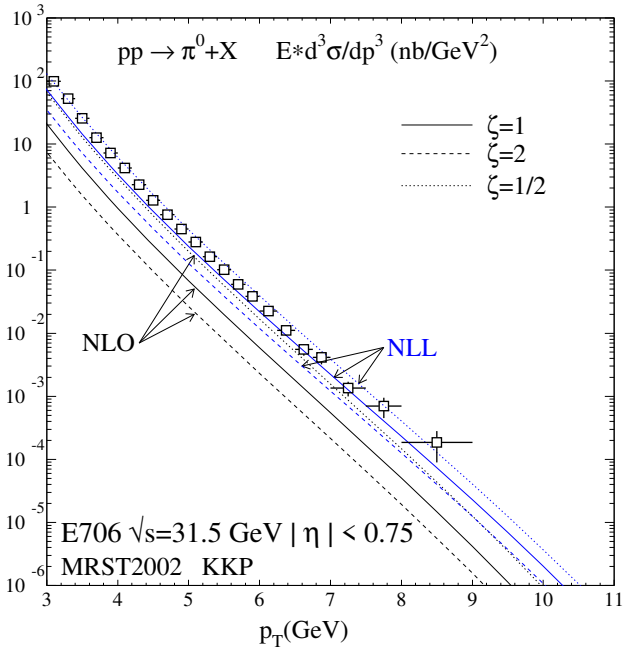
What about the π^0 cross sections?

- Recent paper by de Florian and Vogelsang (hep-ph/0501258) applies threshold resummation to π^0 production

- Large values of the fragmentation variable z relevant for fixed target energies leads to large threshold resummation corrections there.

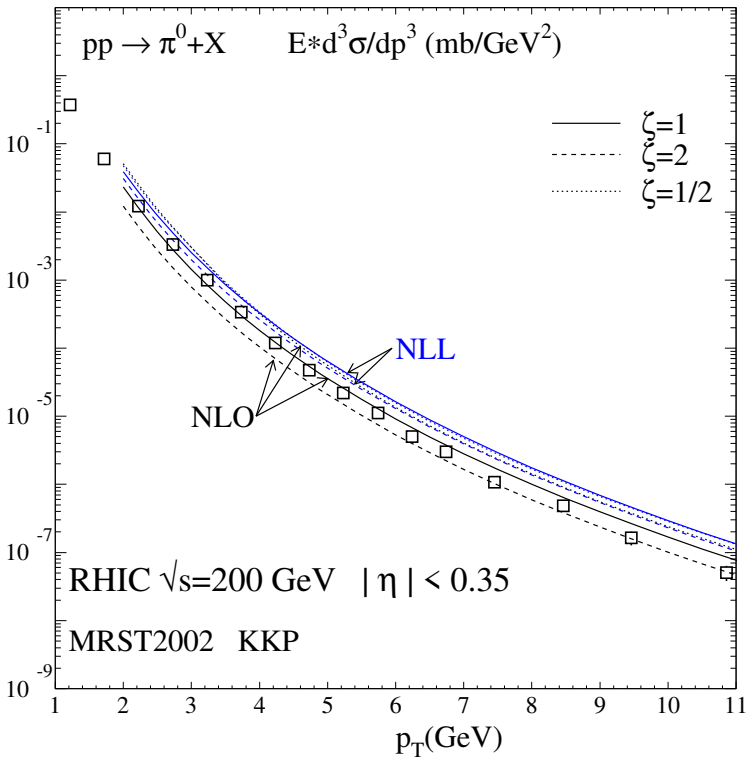
- Enhancement is strongly energy dependent since the relevant values of z decrease as one goes to higher energies at fixed p_T.





- Blue curves include the resummation corrections properly matched to an existing NLO calculation in order to avoid double counting.

- Note the reduced scale dependence of the resummed results.



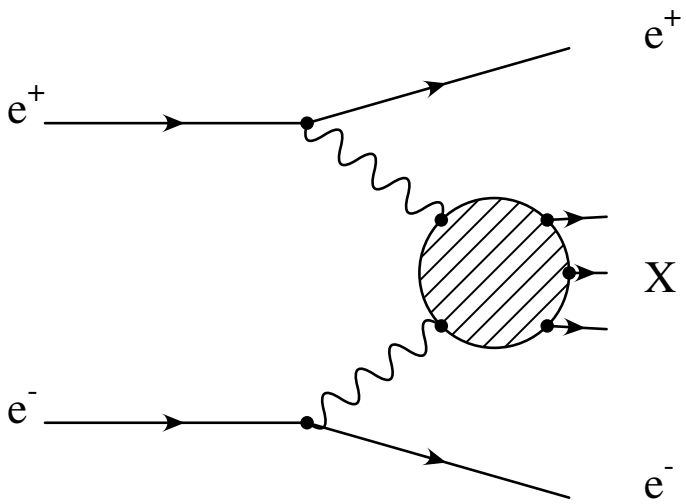
- Note reduced enhancement at RHIC energy compared to the previous fixed target results
- Previous results for direct photons were only applied to the direct part.
- The fragmentation part should get large corrections as in the π^0 case
- Fragmentation plays a reduced role at fixed target energies, but enhancements of factors of 3 or so to this component could have a significant effect.
- Perhaps threshold resummation applied to both the direct and fragmentation components could result in an improved description of direct photon production

Another Approach

- Implementing recoil effects requires:
  - Nonperturbative input to describe parton k_T at some starting point of evolution
  - Assumptions about the kinematics imposed on the hard scattering subprocess (use of on-shell kinematics for partons with non-zero k_T, kinematic limits for shower generated k_T's. etc
- Have to make choices, but one only has the photon p_T distribution as a guide
- Another approach is to handle the parton showering via Monte Carlo generation and then correct the hard scattering to include the NLO matrix element effects, *i.e.*, build a next-to- leading-order shower Monte Carlo
- Make kinematic choices and choose parameter values to fit the entire event structure including the $\gamma-$jet transverse momentum, etc.
- Much work on this approach is currently underway, primarily for electroweak processes at this point
  1. MC@NLO: S. Frixione and B. Webber, hep-ph/0402116; hep-ph/0204244, JHEP 0206:029, 2002.
  2. Higgs: G. Corcella and S. Moretti, hep-ph/0402146.
  3. DIS: J. Collins and X. Zu, hep-ph/0212053; hep-ph/0204127, JHEP 0206:018, 2002.

Another Process with photons

- Consider two photon hadroproduction in $e^+e^- \to e^+e^- + \mathrm{X}$



- Several new complicating factors
  - Either photon can be (nearly) on-shell or space-like.
  - As in the photoproduction case, there are point-like contributions and resolved contributions which involve photon pdf's.
  - For (nearly) on-shell photons there will be three classes of contributions:
    1. direct-direct
    2. direct-resolved
    3. resolved-resolved

Examples of Subprocesses

- Direct-direct
  - $\gamma\gamma \to q\overline{q}(+g)$
- Direct-resolved
  - $\gamma q \to qg(+g)$
  - $\gamma g \to q\overline{q}(+g)$
  - $\gamma q \to qq'\overline{q}'$
  - etc.
- Resolved-resolved
  - all $2 \to 2$ and $2 \to 3$ partonic subprocesses

Two photon QCD processes contain all the contributions that one would expect to find in hadron-hadron and photon-hadron collisions together with the direct-direct two photon terms!

- Understanding QCD at a linear collider will require mastering all the lessons from existing colliders!

- Data on dijet production from the OPAL Collaboration at LEP with $\sqrt{s} = 189 - 209$ GeV

- Eur. Phys. J. C31, 307 (2003), hep-ex/0301013

  - $\cos\theta^* = \tanh(\frac{\eta_1^{jet} - \eta_2^{jet}}{2})$

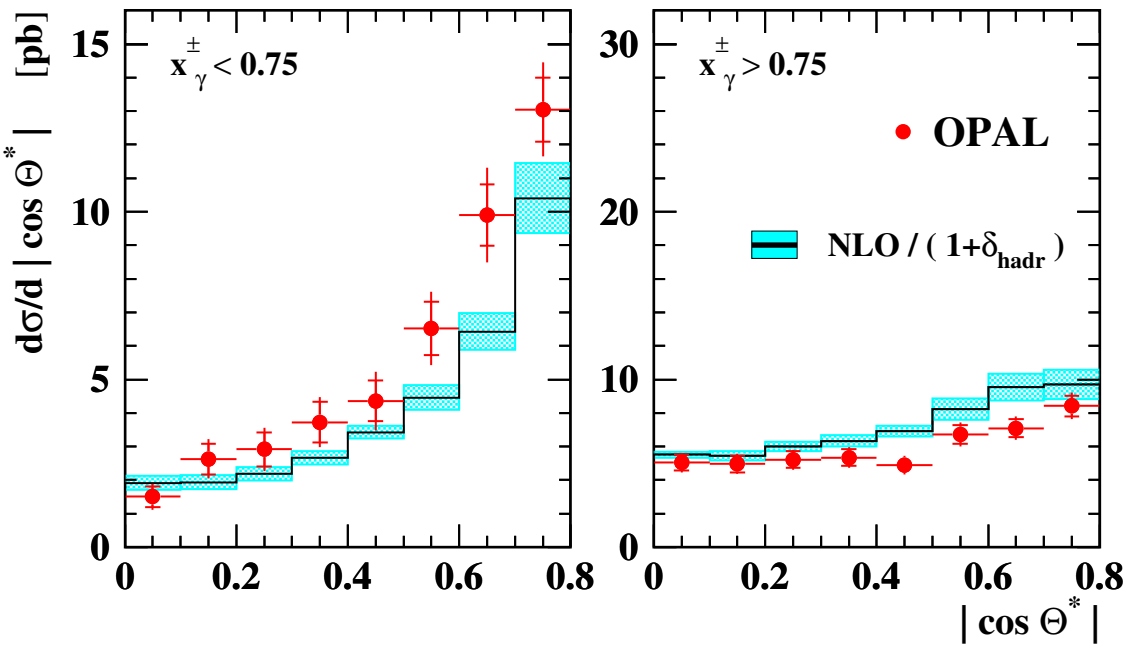
  - $|\eta_{1,2}^{jet}| < 2$

  - $\frac{|E_{T,1}^{jet} - E_{T,2}^{jet}|}{E_{T,1}^{jet} + E_{T,2}^{jet}} < \frac{1}{4}$

- Compared to NLO calculations

  - Klasen, Kleinwort, and Kramer, Eur. Phys. J. Direct C1, 1 (1998)

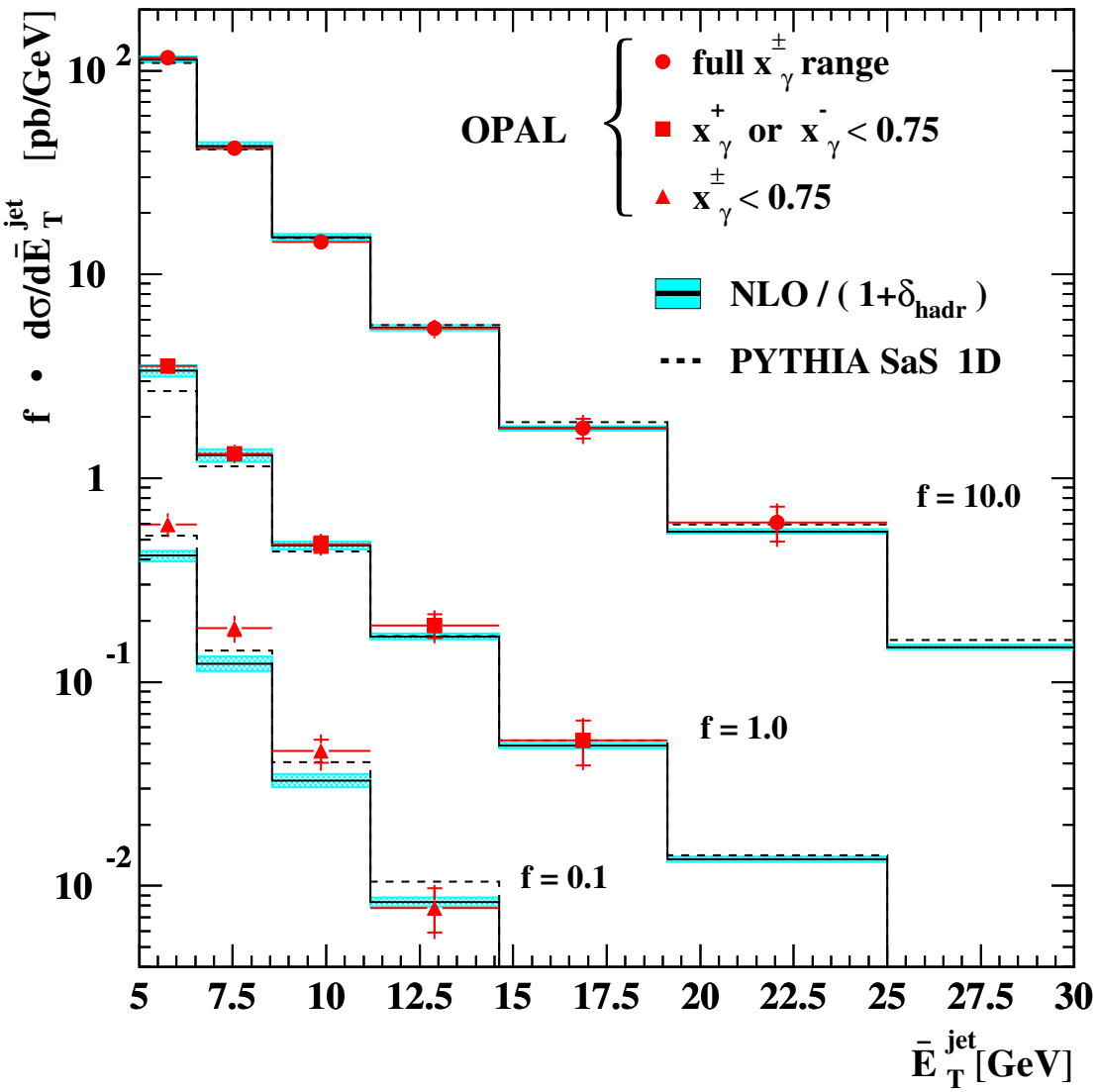
  - B. Pötter, Eur. J. Phys. Direct C5, 1 (1999)



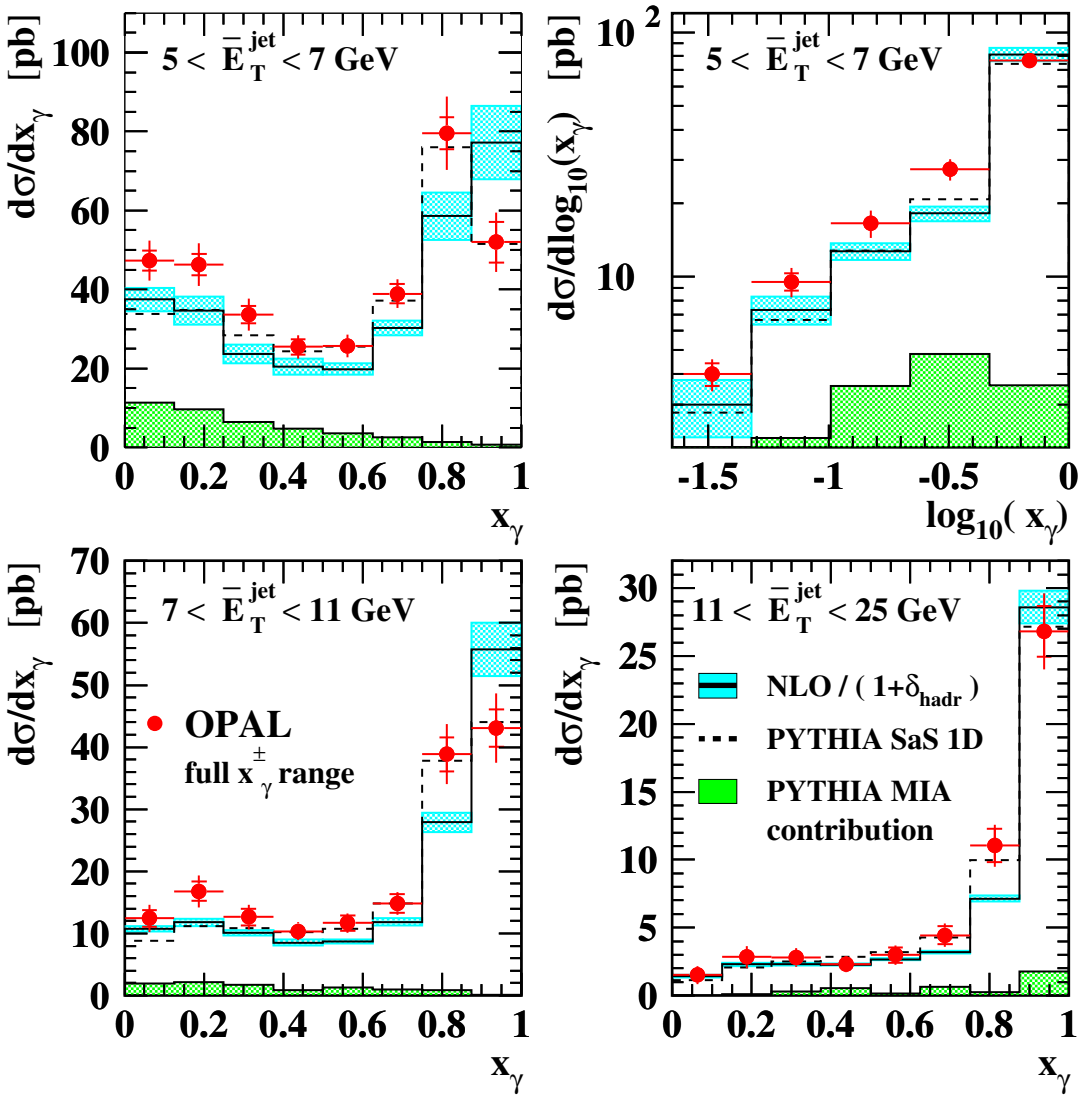
- $x_\gamma^\pm < .75$ isolates the resolved-resolved component which is dominated by subprocesses with gluon exchange which go as $(\cos\theta^*)^{-2}$

- $x_\gamma^\pm > .75$ isolates the direct-direct component which goes as $(\cos\theta^*)^{-1}$

- Expectations supported by the data



- Dijet $\overline{E}_T^{jet}$ distributions are well described by the theoretical curves



- x_γ distributions agree with the data

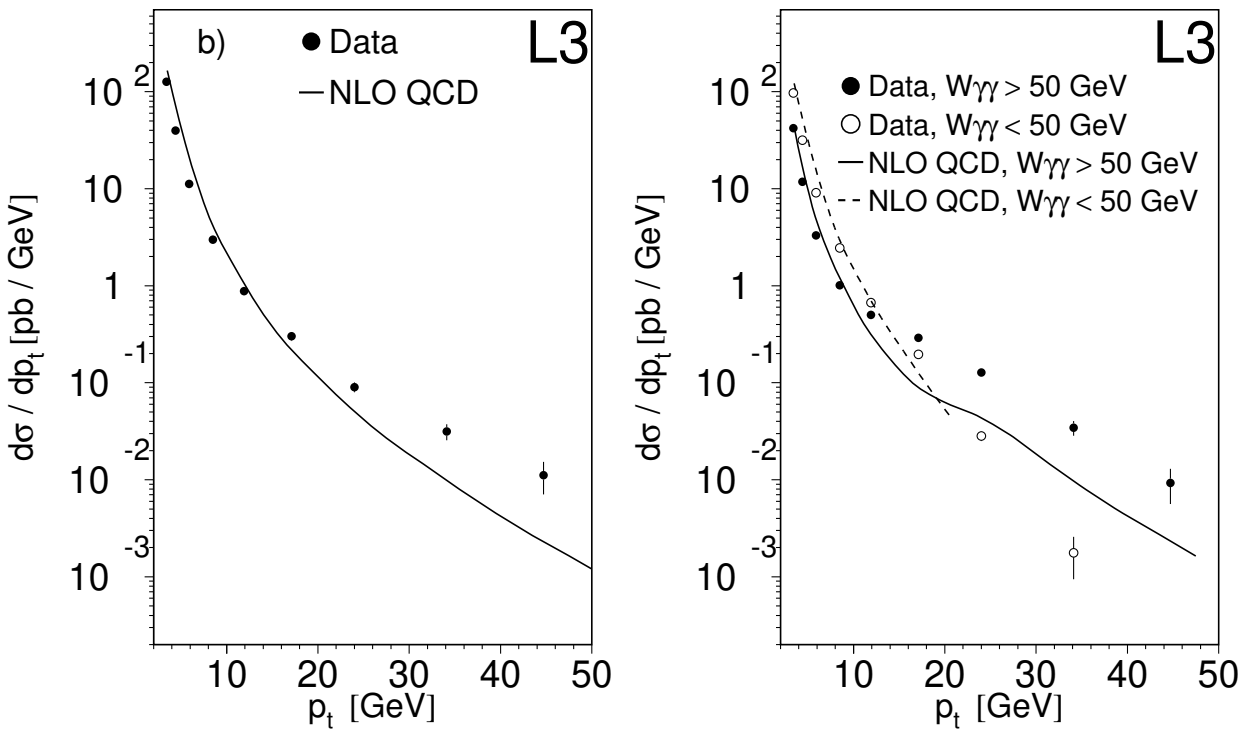
So all is well, right? Well, not so fast ...

Data from the L3 Collaboration on

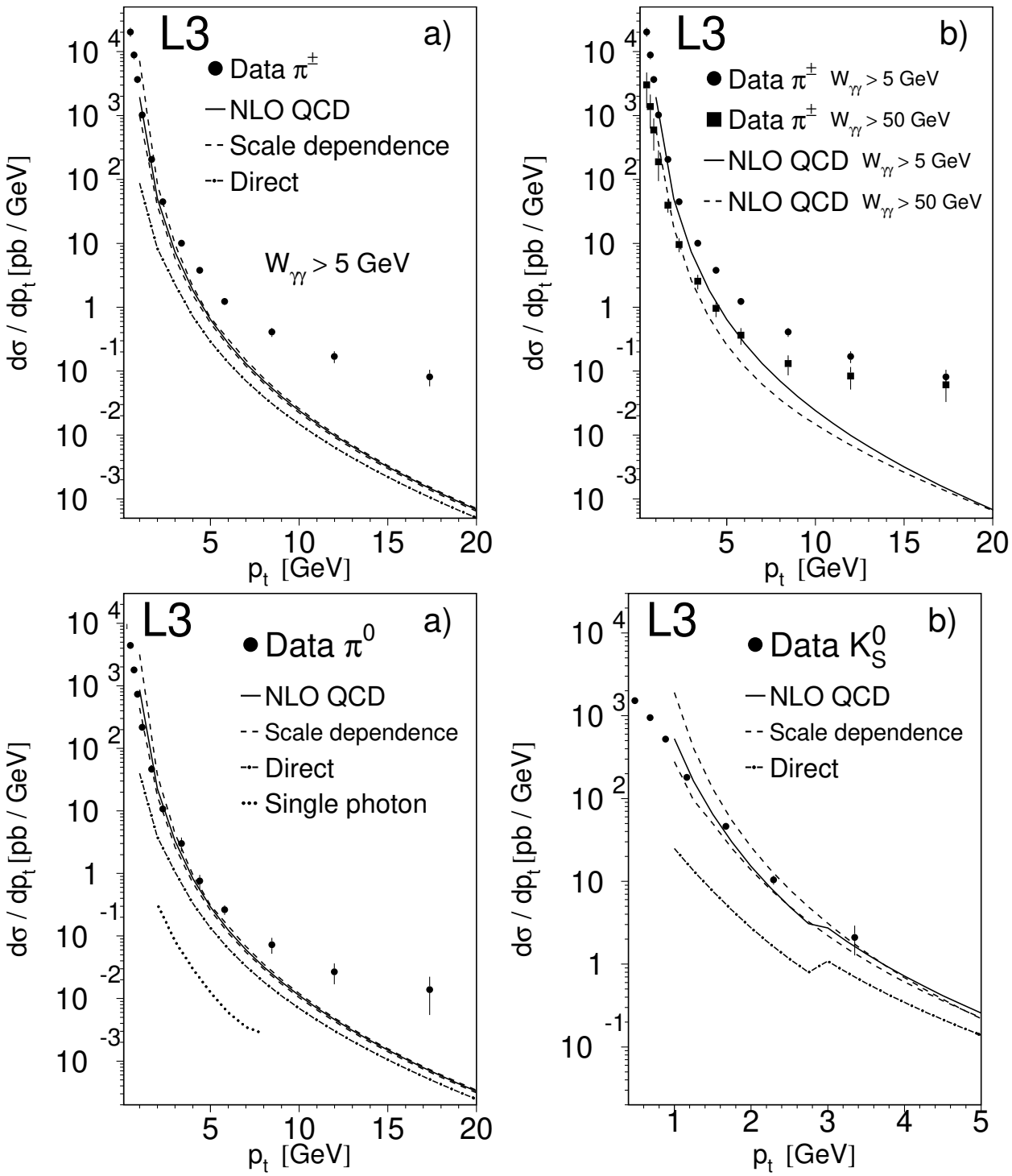
- jets - hep-ex/0410012
- charged hadrons - hep-ex/0301025
- π^0 and K^0_S - hep-ex/0109037

Compared to NLO calculations from

- L. Bertora hep-ph/0306167
- Binneweis, Kniehl, and Kramer, Phys. Rev. **D53** (1996)6110



- Agreement is ok at low p_T end, poor at the high end
- Discrepancy is worse for the higher slice of two photon mass ($W_{\gamma\gamma} > 50$ GeV)



See similar situation for inclusive single charged and neutral hadron production

Comments

- OPAL and L3 charged hadron data agree with each other and with QCD in the region where they overlap ($p_T < 10$ GeV.)
- L3 data extend to higher p_T values where the disagreements occur.
- Various NLO programs agree with each other.
- Disagreement occurs in a region which should be dominated by the direct-direct subprocesses which, for the jet case, would be free from photon pdf and fragmentation function uncertainties.
- Understanding these disagreements will be an important physics topic for a future linear collider program.

The situation is an interesting puzzle.

Summary and Conclusions

1. Examining γ-jet observables suggests that high-p_T photons are produced in accordance with the expectations based on QCD

2. Difficulties are associated primarily with the p_T distribution
   - Theory underestimates data somewhat at the low p_T end for collider energies, at least near the central region
   - Theory lies significantly below some data sets at fixed target energies

3. Problems may be in part due to experimental normalizations and/or systematic errors

4. May need to improve theoretical techniques for single particle inclusive production
   - Resummation (k_T and threshold)
   - NLO shower models

5. Photoproduction observables confirm that QCD matrix elements give a good description of photon interactions there, as well

6. Some interesting discrepancies have been observed in high-p_T two photon processes at LEP. These require more investigation.

Some miscellaneous and hopefully useful stuff

1. Check out the CTEQ web page at www.cteq.org
   - information on past summer schools, including transparencies of many of the lectures
   - information and links for parton distributions
   - CTEQ List of Challenges in Perturbative QCD
   - CTEQ Pedagogical Page
   - CTEQ Handbook of Perturbative QCD

2. Four-vectors and rapidity
   - rapidity is defined as $y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$. For massless particles this reduces to the pseudorapidity which is defined as $\eta = \ln \cot \theta/2$.
   - the four-vector for a massless particle with transverse momentum p_T and rapidity y may be conveniently expressed as

$$p^\mu = (p_T \cosh y, p_T, 0, p_T \sinh y)$$

3. Mandelstam variables
   - For a two-body process $p_1 + p_2 \to p_3 + p_4$ the three Mandelstam variables are defined as

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - p_3)^2 \\ u &= (p_1 - p_4)^2 \end{aligned}$$

- For processes with more particles one sometimes encounters variables such as $s_{ij} = (p_i + p_j)^2$ which is just the squared invariant mass of particles i and j and $t_{ij} = (p_i - p_j)^2$ which is the squared four-momentum transfer between particles i and j.

4. Another example: direct photon production $qg \to \gamma + q$
   - four-vectors in the hadron-hadron center-of-mass frame

$$\begin{aligned} p_q &= \frac{\sqrt{s}}{2} x_a (1, 0, 0, 1) \\ p_g &= \frac{\sqrt{s}}{2} x_b (1, 0, 0 - 1) \\ p_\gamma &= p_T (\cosh y, 1, 0, \sinh y) \end{aligned}$$

   - Substituting these four-vectors into the expressions for the Mandelstam variables above yields

$$\begin{aligned} \hat{s} &= x_a x_b \; s \\ \hat{t} &= -x_a \; p_T \sqrt{s} \; e^{-y} \\ \hat{u} &= -x_b \; p_T \sqrt{s} \; e^{y} \end{aligned}$$

   - A $\hat{}$ is often used to denote a variable at the parton level.

5. Convolutions
   - The symbol $\otimes$ is sometimes used to denote a convolution:

$$\begin{aligned} f \otimes g &= \int_0^1 dy \int_0^1 dz \; f(y) \; g(z) \; \delta(x - yz) \\ &= \int_x^1 \frac{dz}{z} \; f(x/z) \; g(z) \end{aligned}$$

Subprocesses and angular distributions

The two lowest order subprocesses for direct photon production are (in units of $\pi\alpha\alpha_s/\hat{s}^2$)

$$\begin{aligned}\frac{d\sigma}{d\hat{t}}(gq \to \gamma q) &= -\frac{e_q^2}{3}\left[\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}}\right] \\ \frac{d\sigma}{d\hat{t}}(q\overline{q} \to \gamma g) &= \frac{8}{9}e_q^2\left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}}\right]\end{aligned}$$

whereas the dominant parton-parton scattering subprocesses for hadroproduction are (in units of $\pi\alpha_s^2/\hat{s}^2$)

$$\begin{aligned}\frac{d\sigma}{d\hat{t}}(qq' \to qq') &= \frac{4}{9}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right] \\ \frac{d\sigma}{d\hat{t}}(qg \to qg) &= -\frac{4}{9}\left[\frac{\hat{s}}{\hat{u}} + \frac{\hat{u}}{\hat{s}}\right] + \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \\ \frac{d\sigma}{d\hat{t}}(gg \to gg) &= \frac{9}{2}\left[3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right]\end{aligned}$$

In the parton-parton center-of-mass frame, one can write

$$\begin{aligned}\hat{t} &= -\frac{\hat{s}}{2}(1 - \cos(\theta^*)) \\ \hat{u} &= -\frac{\hat{s}}{2}(1 + \cos(\theta^*))\end{aligned}$$

Therefore, as $\cos(\theta^*) \to 1(-1), \hat{t}\,(\hat{u}) \to 0$. Hence, in this limit, the first two subprocesses behave as $(1 - |\cos(\theta^*)|)^{-1}$ while the next three behave as $(1-|\cos(\theta^*)|)^{-2}$.

Center of mass scattering angle

Start in the parton-parton center of mass frame where one has

$$p_1 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \quad p_2 = \frac{\sqrt{\hat{s}}}{2}(1,0,0-1)$$

$$p_3 = \frac{\sqrt{\hat{s}}}{2}(1,\sin\theta^*,0,\cos\theta^*) \quad p_4 = \frac{\sqrt{\hat{s}}}{2}(1,-\sin\theta^*,0,-\cos\theta^*)$$

from which one can derive

$$\hat{t} = -\frac{\hat{s}}{2}(1-\cos(\theta^*)) \quad \hat{u} = -\frac{\hat{s}}{2}(1+\cos(\theta^*))\,.$$

Next, write the parton four-vectors in the hadron-hadron frame as

$$\begin{aligned} p_1 &= \frac{\sqrt{s}}{2}x_a(1,0,0,1) \\ p_2 &= \frac{\sqrt{s}}{2}x_b(1,0,0-1) \\ p_3 &= p_T(\cosh y_3, 1, 0, \sinh y_3) \\ p_4 &= p_T(\cosh y_4, -1, 0, \sinh y_4) \end{aligned}$$

which can be used to derive

$$\begin{aligned} \hat{t} &= -\sqrt{s}x_a p_T e^{-y_3} \\ \hat{u} &= -\sqrt{s}x_a p_T e^{-y_4}. \end{aligned}$$

From these two sets of expressions one can obtain

$$\frac{\hat{t}}{\hat{u}} = e^{-(y_3-y_4)} = \frac{1-\cos\theta^*}{1+\cos\theta^*}.$$

It then follows that

$$\cos\theta^* = \tanh\frac{y_3-y_4}{2}.$$