Determining parton distribution functions PDFs

CTEQ School '06- Rhodes

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Oxford

- How do we determine them?- where does the information come from?
- What are the uncertainties? -experimental

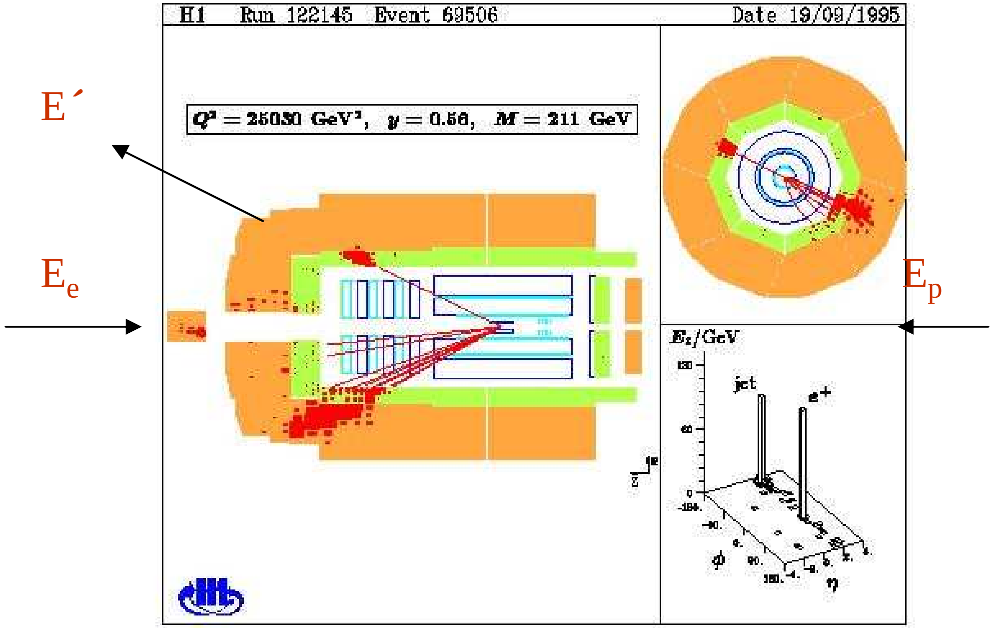
  -model

  -theoretical

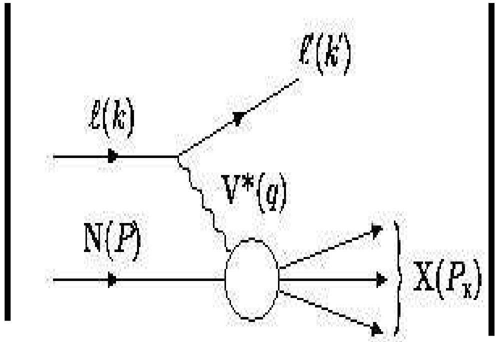
3. Why are they important?

PDFs were first investigated in deep inelastic lepton-hadron scattering -DIS

Candidate from NC sample



dσ ~



^{2} $L^{\mu\nu} W_{\mu\nu}$

Leptonic tensor - calculable

Hadronic tensor- constrained by Lorentz invariance

$q = k - k'$, $Q^2 = -q^2$

$P_x = p + q$, $W^2 = (p + q)^2$

$s = (p + k)^2$

$x = Q^2 / (2p.q)$

$y = (p.q)/(p.k)$

$W^2 = Q^2 (1/x - 1)$

$Q^2 = s\, x\, y$

$s = 4 E_e E_p$
$Q^2 = 4 E_e E' \sin^2\theta_e/2$
$y = (1 - E'/E_e \cos^2\theta_e/2)$
$x = Q^2/sy$

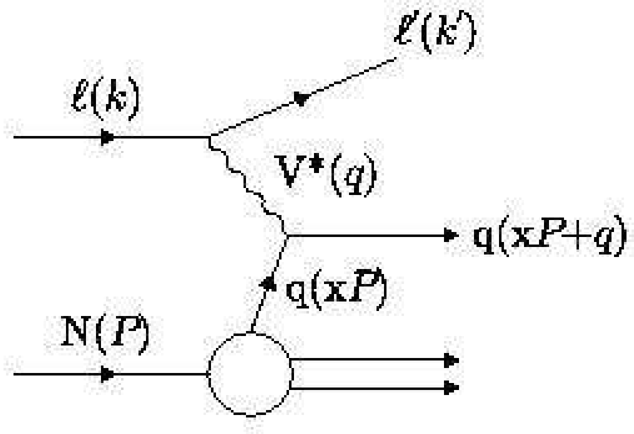
The kinematic variables are measurable

Completely generally the double differential cross-section for e-N scattering

$$\frac{d^2\sigma(e\pm N)}{dxdy} = \frac{2\pi\alpha^2 s}{Q^4}\left[Y_+ \mathbf{F_2}(x,Q^2) - y^2 \mathbf{F_L}(x,Q^2) \pm Y\_x\mathbf{F_3}(x,Q^2)\right], \quad Y\pm = 1 \pm (1-y)^2$$

Leptonic part

hadronic part



F_2, F_L and xF_3 are structure functions

which express the dependence of the cross-section

on the structure of the nucleon–

The Quark-Parton model interprets these structure functions as related to the momentum distributions of quarks or partons within the nucleon AND the measurable kinematic variable $x = Q2/(2p.q)$ is interpreted as the FRACTIONAL momentum of the incoming nucleon taken by the struck quark

$(xP+q)^2=x^2p^2+q^2+2xp.q \sim 0$

for massless quarks and $p^2 \sim 0$
so

$x = Q^2/(2p.q)$

The FRACTIONAL momentum of the incoming nucleon taken by the struck quark is the MEASURABLE quantity x

e.g. for charged lepton beams

F2$(x,Q^2) = \Sigma_i e_i^2(xq(x) + x\bar{q}(x))$ – Bjorken scaling

FL(x,Q2) = 0 - spin ½ quarks

xF3(x,Q2) = 0 - only γ exchange

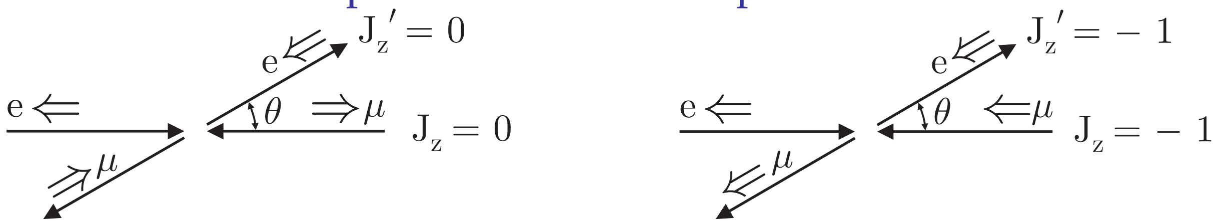
However for **neutrino beams**

xF3$(x,Q2)= \Sigma_i (xq(x) - x\bar{q}(x))$ ~ valence quark distributions of various flavours

Consider electron muon scattering

$$\frac{d\sigma}{dy} = \frac{2\pi\alpha^2}{Q^4} s\,[\,1 + (1-y)^2] \text{ , for elastic } e\mu$$

isotropic — non-isotropic



$$\frac{d\sigma}{dy} = \frac{2\pi\alpha^2 e_i^2}{Q^4} s\,[\,1 + (1-y)^2] \text{ ,}$$ so for elastic electron quark scattering, quark charge e_i e

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2 s}{Q^4}\,[\,1 + (1-y)^2]\, \Sigma_i\, e_i^2 (xq(x) + x\bar{q}(x))$$

so for eN, where eq has c. of m. energy^2 equal to xs, and q(x) gives probability that such a quark is in the Nucleon

Now compare the general equation to the QPM prediction to obtain the results

F_2 $(x,Q2) = \Sigma_i\, e_i^2(xq(x) + xq(x))$ – Bjorken scaling

F_L $(x,Q2) = 0$ - spin ½ quarks

xF_3 $(x,Q2) = 0$ - only γ exchange

Consider $\nu,\bar{\nu}$ scattering: neutrinos are handed

$$\frac{d\sigma(\nu)}{dy} = \frac{G_F^2 \, x\, s}{\pi} \qquad\qquad \frac{d\sigma(\bar{\nu})}{dy} = \frac{G_F^2 \, x\, s\,(1-y)^2}{\pi}$$

For ν q (left-left) | For $\bar{\nu}$ q (left-right)

$$\frac{d^2\sigma(\nu)}{dxdy} = \frac{G_F^2\, s}{\pi}\Sigma_i\, [xq_i(x) + (1-y)^2 x\bar{q}_i(x)]$$

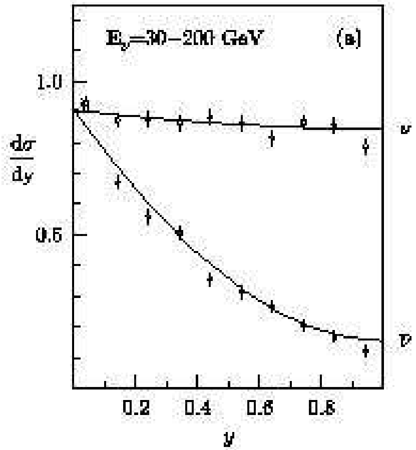
For νN

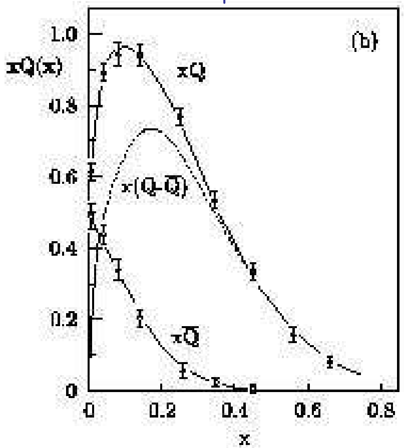
$$\frac{d^2\sigma(\bar{\nu})}{dxdy} = \frac{G_F^2\, s}{\pi}\Sigma_i\, [x\bar{q}_i(x) + (1-y)^2 xq_i(x)]$$

For $\bar{\nu}$N

Clearly there are antiquarks in the nucleon

3 Valence quarks plus a flavourless $q\bar{q}$ Sea





$q = q_{valence} + q_{sea}$ $\quad \bar{q} = \bar{q}_{sea}$ $\quad q_{sea} = \bar{q}_{sea}$

Compare to the general form of the cross-section for $\underline{\nu}/\nu$ scattering via $W_{+/-}$

$$F_L(x,Q2) = 0$$

$$xF_3(x,Q2) = 2\Sigma_i x(q_i(x) - \bar{q}_i(x))$$

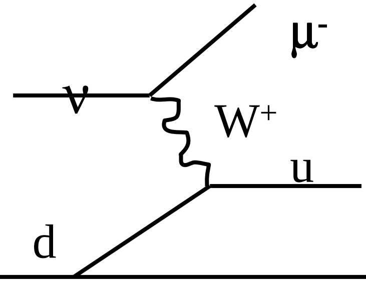
Valence

$$F_2(x,Q2) = 2\Sigma_i x(q_i(x) + \bar{q}_i(x))$$

Valence and Sea

And there will be a relationship between F_2^{eN} and $F_2^{\nu N}$

Also NOTE $\nu,\underline{\nu}$ scattering is FLAVOUR sensitive



W+ can only hit quarks of charge -e/3 or antiquarks -2e/3

$\sigma(\nu p) \sim (d + s) + (1-y)^2(\bar{u} + \bar{c})$

$\sigma(\bar{\nu} p) \sim (u + c)(1-y)^2 + (\bar{d} + \bar{s})$

So in $\nu,\bar{\nu}$ scattering the sums over q, qbar ONLY contain the appropriate flavours BUT-

high statistics $\nu,\bar{\nu}$ data are taken on isoscalar targets e.g. Fe $\Rightarrow$ (p + n)/2=N

d in proton = u in neutron

u in proton = d in neutron

$$xF_3(\nu,\bar{\nu}N) = x(u-\bar{u}+d-\bar{d}) = x(u_v+d_v)$$

$$\int \frac{xF_3}{x}dx = \int_0^1 (u_v + d_v)dx = 3$$

GLS sum rule

$$F_2(\nu,\bar{\nu}N) = x(u+\bar{u}+d+\bar{d}+s+\bar{s}+c+\bar{c})$$

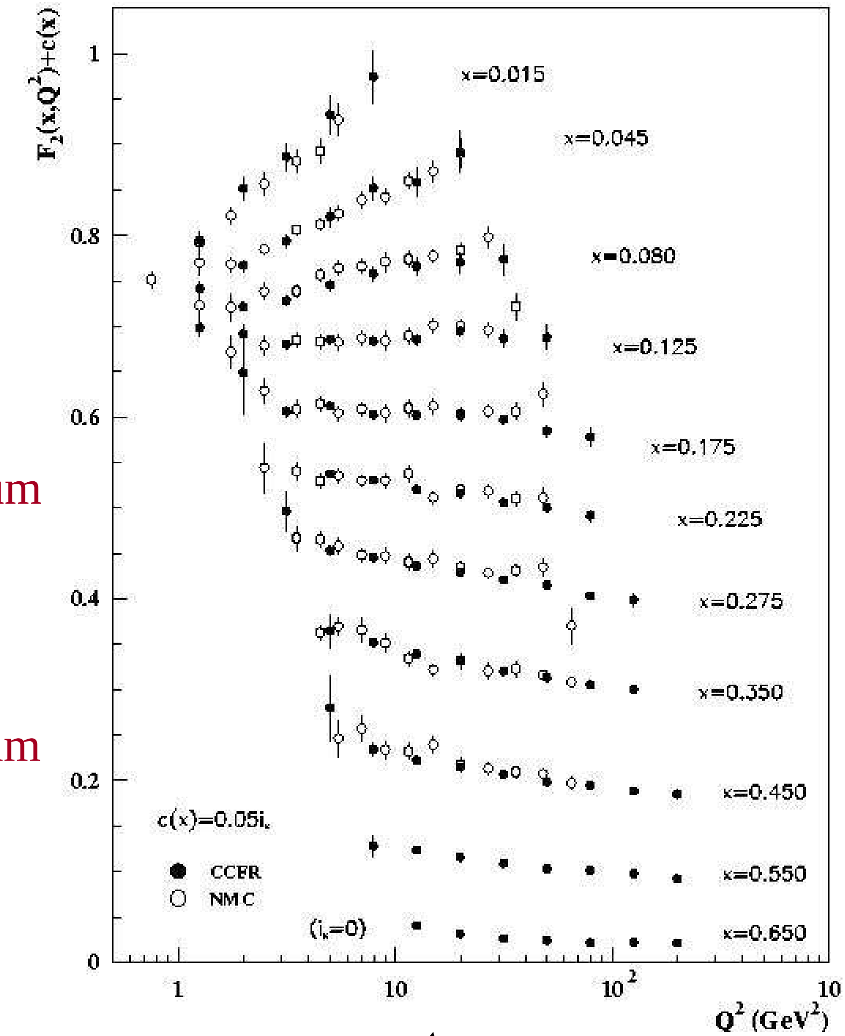
$$\int_0^1 F_2(\nu,\bar{\nu}N)dx = 1?$$

Total momentum of quarks

$$F_2(lp) = x(\frac{4}{9}(u+\bar{u})+\frac{1}{9}(d+\bar{d})+\frac{1}{9}(s+\bar{s})+\frac{4}{9}(c+\bar{c})$$

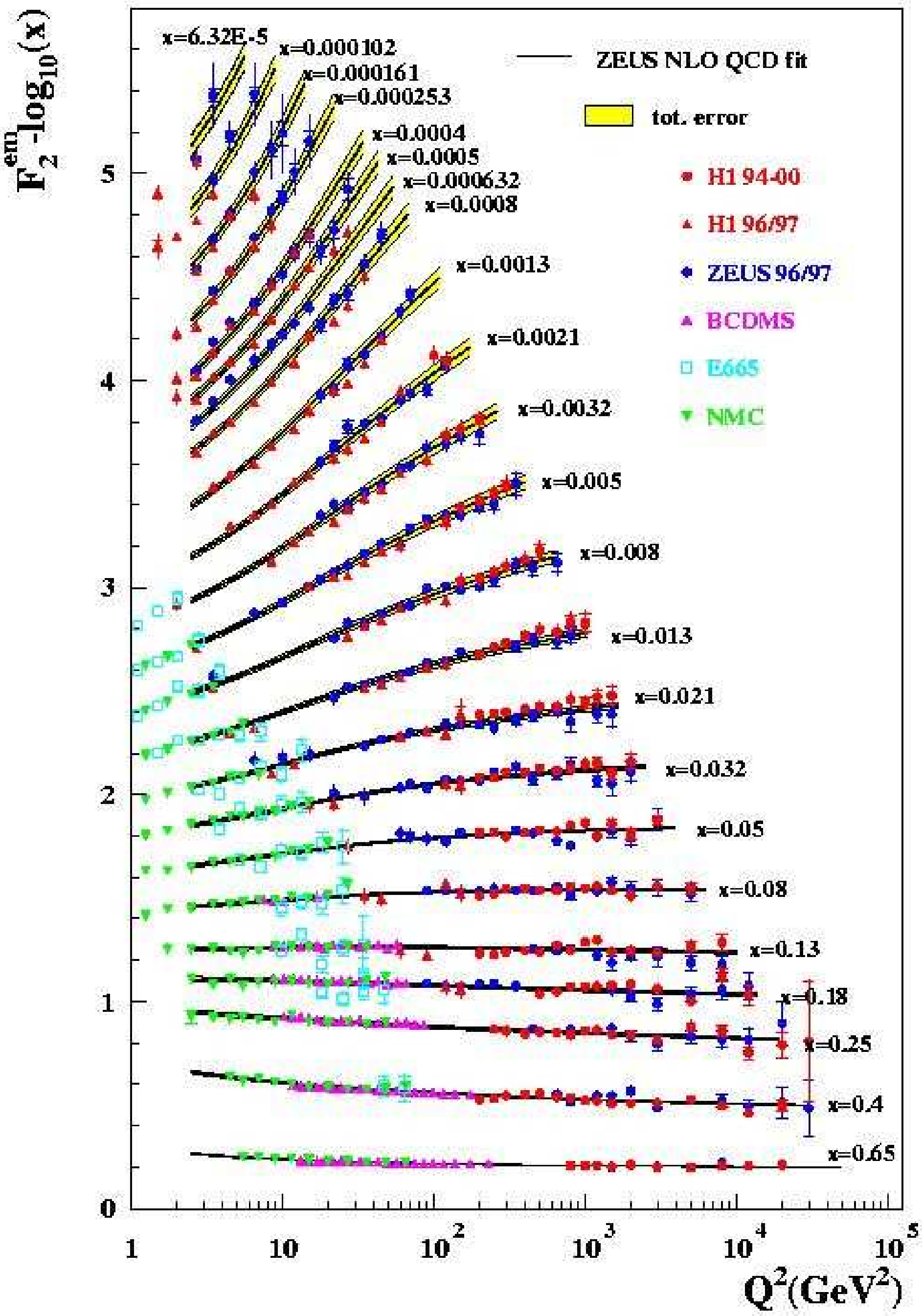
$$F_2(lN) = \frac{5}{18}x\left[u+\bar{u}+d+\bar{d}+\frac{2}{5}(s+\bar{s})+\frac{8}{5}(c+\bar{c})\right]$$

$$F_2(lN) \approx \frac{5}{18}F_2(\nu N)$$



A TRIUMPH

(and 20 years of understanding the c $\bar{c}$ contribution)



BUT –

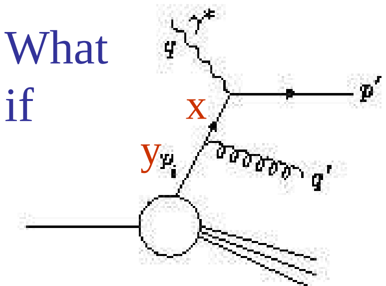
Bjorken scaling is broken – $\ln(Q^2)$

Particularly strongly at small x

–

QCD improves the Quark Parton Model

What if



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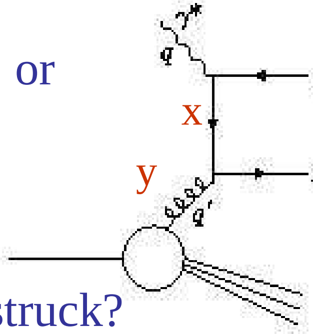


Before the quark is struck?

$y > x, \ z = x/y$

$$\frac{F_2(x)}{x} = \int dy dz \delta(x - zy) q(y) \left[\sigma^{POINT}(z) + \sigma(\gamma^* q \to qg)\right]$$

$$\sigma(\gamma^* q \to qg) = \frac{e_i^2 \alpha_s}{2\pi} P_{qq}(z) ln \frac{Q^2}{Q_0^2}$$

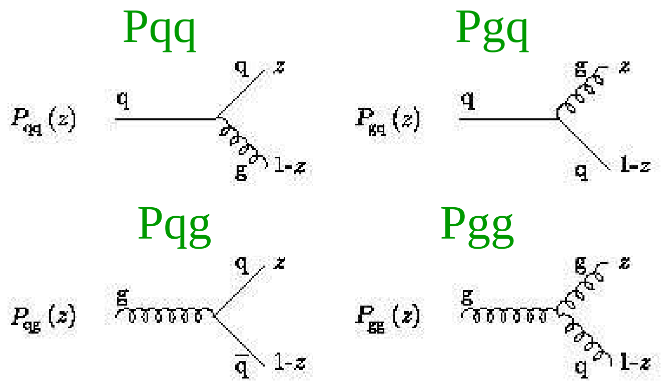
$$\frac{F_2(x, Q^2)}{x} = e_i^2 \left[q(x) + \Delta q(x, Q^2)\right] = e_i^2 q(x, Q^2)$$

$$\Delta q(x, Q^2) = \frac{\alpha_s}{2\pi} ln \frac{Q^2}{Q_0^2} \int_0^1 \frac{dy}{y} q(y) P_{qq}\left(\frac{x}{y}\right)$$

$$\frac{dq(x, Q^2)}{dlnQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 \frac{dy}{y} \left[Pqq(z) q(y, Q^2) + Pqg(z) g(y, Q^2)\right]$$

$$\frac{dg(x, Q^2)}{dlnQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 \frac{dy}{y} \left[\Sigma_q Pgq(z) q(y, Q^2) + Pgg(z) g(y, Q^2)\right]$$

The DGLAP parton evolution equations



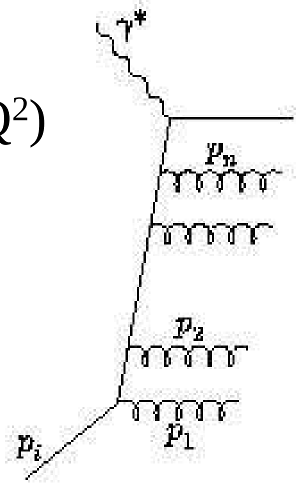
So $F_2(x,Q^2) = \Sigma_i e_i^2(xq(x,Q^2) + xq(x,Q^2))$

in LO QCD

The theory predicts the rate at which the parton distributions (both quarks and gluons) evolve with Q^2- (the energy scale of the probe) -BUT it does not predict their shape

Note $q(x,Q^2) \sim \alpha_s \ln Q^2$, but $\alpha_s(Q^2) \sim 1/\ln Q^2$, so $\alpha_s \ln Q^2$ is O(1), so we must sum all terms

$\alpha_s \rightarrow \alpha_s(Q^2)$



$\alpha_s^n \ln Q^{2n}$

Leading Log

Approximation

x decreases from target to probe

$x_{i-1} > x_i > x_{i+1} \ldots$

p_t^2 of quark relative to proton increases from target to probe

$p_t^2{}_{i-1} < p_t^2{}_i < p_t^2{}_{i+1}$

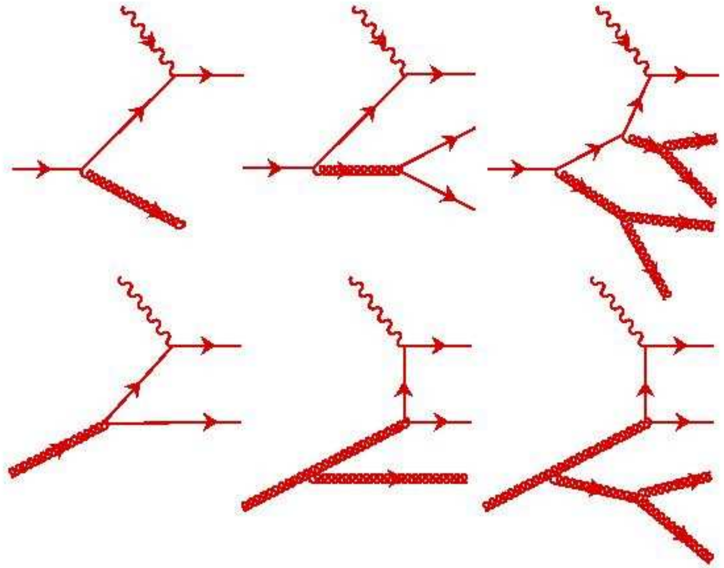
Dominant diagrams have STRONG p_t ordering

F2 is no longer so simply expressed in terms of partons -

convolution with coefficient functions is needed –

but these are calculable in QCD

What if higher orders are needed?



$$Pqq(z) = P^0qq(z) + \alpha_s P^1qq(z) + \alpha_s^2 P^2qq(z)$$

LO NLO NNLO

$$\frac{F_2(x,Q^2)}{x} = \int_0^1 \frac{dy}{y}\left[\Sigma_i C_2(z,\alpha_s) q_i(x,Q^2) + C_g(z,\alpha_s) g(y,Q^2)\right]$$

$$C_2(z,\alpha_s) = e_i^2\left[\delta(1-z) + \alpha_s f_2(z)\right]$$

$$C_g(z,\alpha_s) = \alpha_s f_g(z)$$

$$F_L(x,Q^2) = \frac{\alpha_s}{x}\left[\frac{4}{3}\int_0^1 \frac{dy}{y} z^2 F_2(y,Q^2) + 2\Sigma_i e_i^2 \int_0^1 \frac{dy}{y} z^2 (1-z) yg(y,Q^2)\right]$$

How do we determine Parton Distribution Functions ?

Parametrise the parton distribution functions (PDFs) at Q^2_0 (~1-7 GeV^2)- Use QCD to evolve these PDFs to Q2 >Q20

Construct the measurable structure functions and cross-sections by convoluting PDFs with coefficient functions: make predictions for ~2000 data points across the x,Q2 plane- Perform χ2 fit to the data

Formalism
NLO DGLAP
MSbar factorisation
Q_0^2
functional form @ Q_0^2
sea quark (a)symmetry
etc.

Data
DIS (SLAC, BCDMS, NMC, E665, CCFR, H1, ZEUS, …)
Drell-Yan (E605, E772, E866, …)
High E_T jets (CDF, D0)
W rapidity asymmetry (CDF)
νN dimuon (CCFR, NuTeV)
etc.

$f_i(x,Q^2) \pm \delta f_i(x,Q^2)$

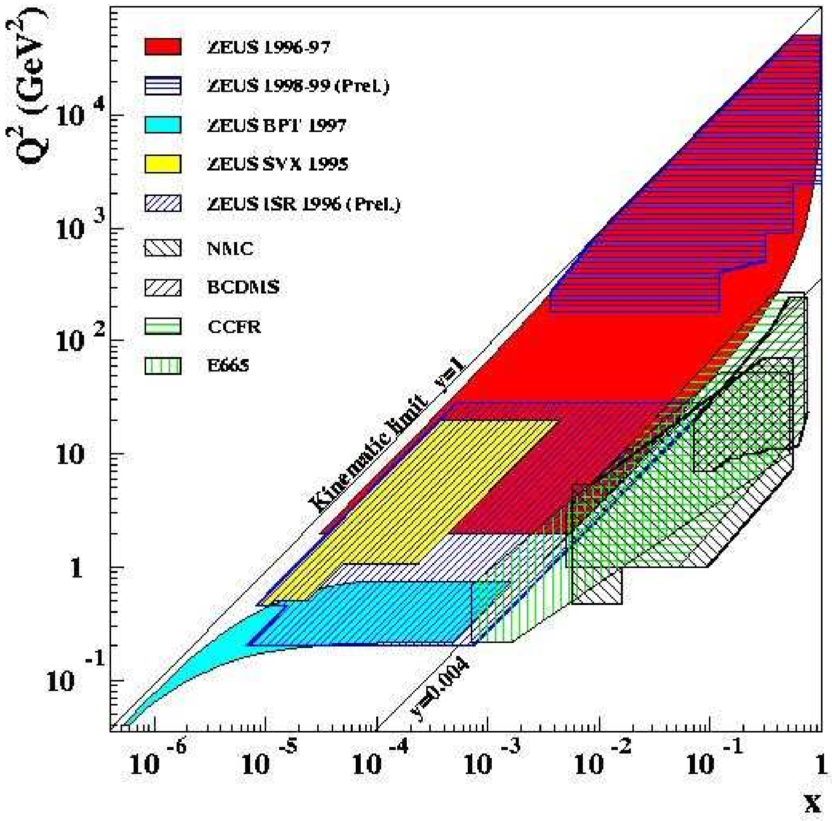
$\alpha_S(M_Z)$

Who?
Alekhin, CTEQ, MRST, GGK, Botje, H1, ZEUS, GRV, BFP, …

http://durpdg.dur.ac.uk/hepdata/

LHAPDFv5

The DATA – the main contribution is DIS data



Terrific expansion in measured range across the x, Q^2 plane throughout the 90's

HERA data

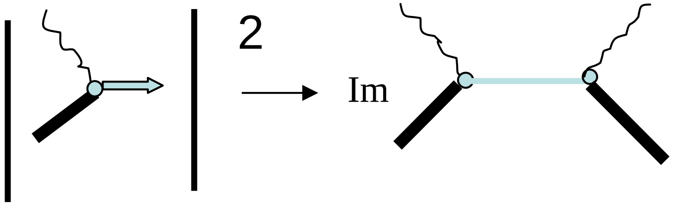
Pre HERA fixed target μp,μD NMC, BDCMS, E665 and ν,νbar Fe CCFR

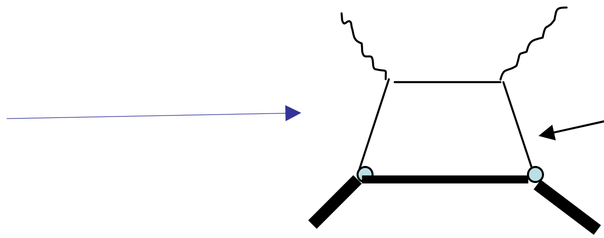
We have to impose appropriate kinematic cuts on the data so as to remain in the region when the NLO DGLAP formalism is valid

1. Q^2 cut : Q^2 > few GeV^2 so that perturbative QCD is applicable- $\alpha_s(Q^2)$ small
2. W^2 cut: to avoid higher twist terms- usual formalism is leading twist
3. x cut: to avoid regions where ln(1/x) resummation (BFKL) and non-linear effects may be necessary

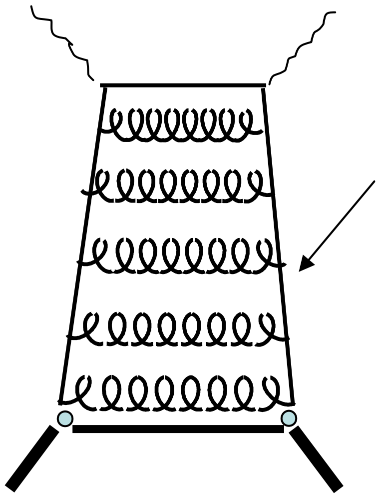
Need to extend the formalism?

Optical theorem





The handbag diagram- QPM



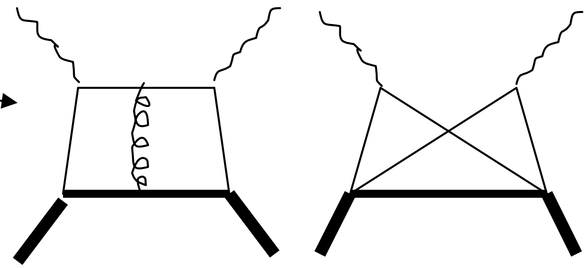
QCD at LL(Q^2)

Ordered gluon ladders ($\alpha_s^n \ln Q^{2\,n}$)

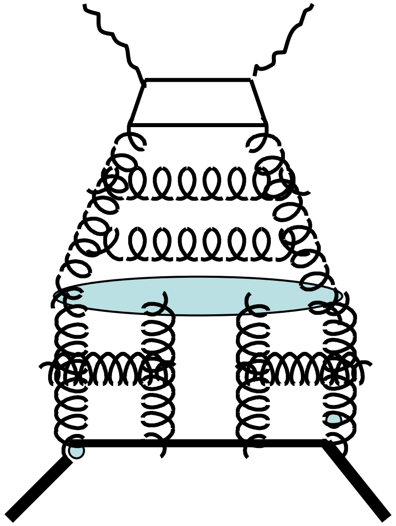
NLL(Q^2) one rung disordered $\alpha_s^n \ln Q^{2\,n-1}$

BUT what about completely disordered Ladders?
at small x there may be a need for BFKL ln(1/x) resummation?

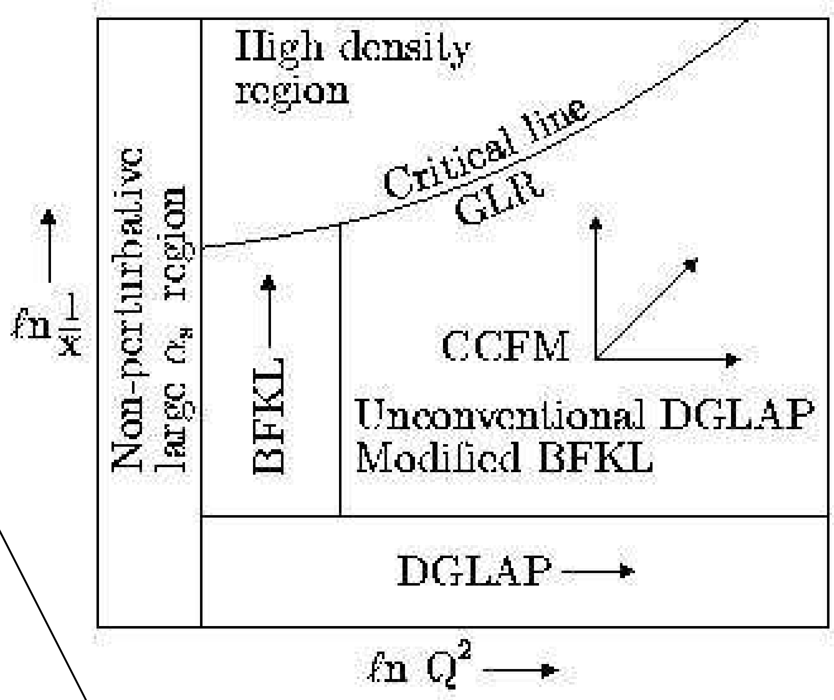
And what about Higher twist diagrams ?
Are they always subdominant?
Important at high x, low Q^2

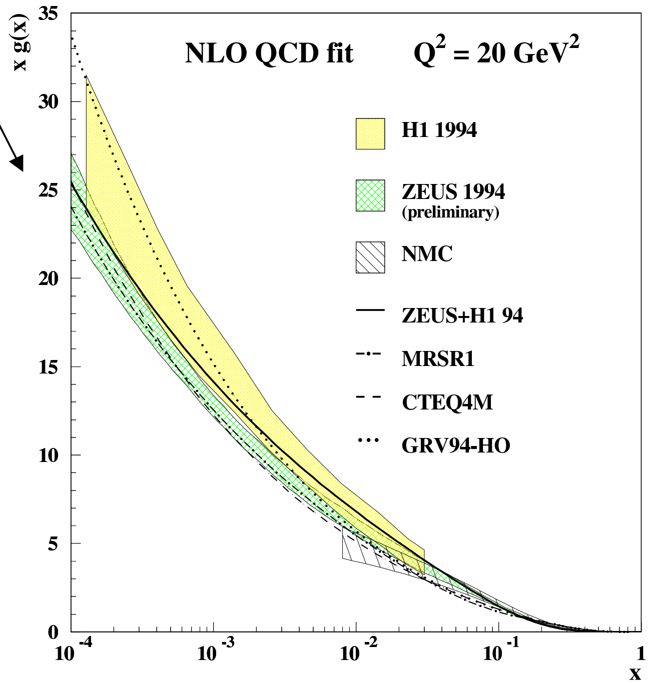


The strong rise in the gluon density at small-x leads to speculation that there may be a need for non-linear equations?- gluons recombining $gg \to g$



Non-linear fan diagrams form part of possible higher twist contributions at low x





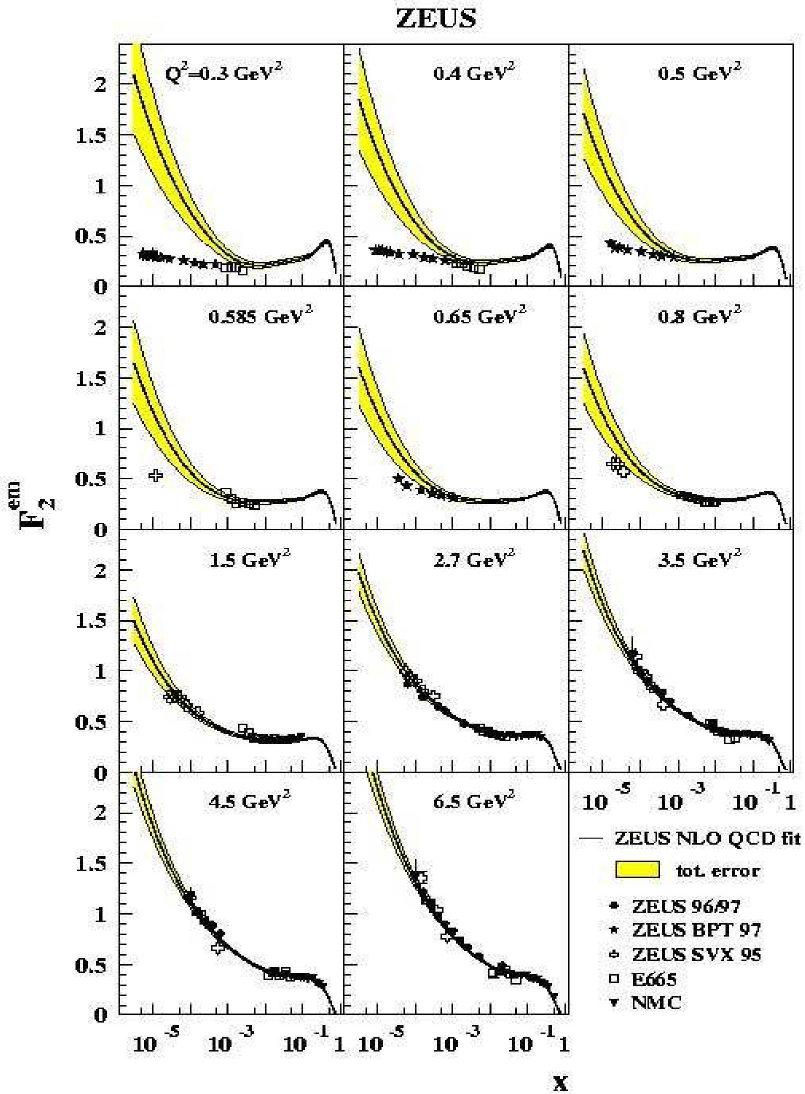
The CUTS

In practice it has been amazing how low in Q^2 the standard formalism still works- down to $Q^2 \sim 1$ GeV^2 : cut $Q^2 > 2$ GeV^2 is typical

It has also been surprising how low in x – down to x~ 10^{-5} : no x cut is typical

Nevertheless there are doubts as to the applicability of the formalism at such low-x..

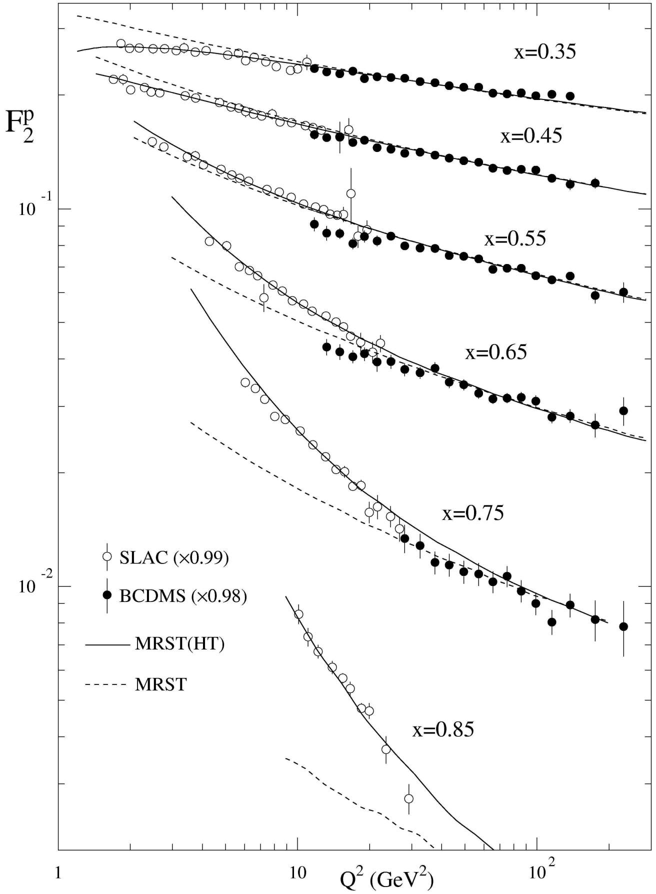
(See much later)
there **could be** ln(1/x) corrections and/or non-linear high density corrections for
x < 5 10 -3



Higher twist terms can be important at low-Q2 and high-x → this is the fixed target region (particularly SLAC).

Kinematic target mass corrections and dynamic contributions ~ $1/Q^2$

$X \rightarrow 2x/(1 + \sqrt{(1+4m^2x^2/Q^2)})$



Fit with $F2=F2_{LT}\,(1 + D_2(x)/Q^2)$

Fits establish that higher twist terms are not needed if

$W^2 > 15$ GeV^2 – typical W2 cut

Also no sign of low-x higher twist effects in HERA kinematic region

x	$D_2(x)$ (GeV^2)
0 – 0.0005	0.0147
0.0005 – 0.005	0.0217
0.005 – 0.01	−0.0299
0.01 – 0.06	−0.0382
0.06 – 0.1	−0.0335
0.1 – 0.2	−0.121
0.2 – 0.3	−0.190
0.3 – 0.4	−0.242
0.4 – 0.5	−0.141
0.5 – 0.6	0.248
0.6 – 0.7	1.458
0.7 – 0.8	4.838
0.8 – 0.9	16.06

The form of the parametrisation

Parametrise the parton distribution functions (PDFs) at Q^2_0 (~1-7 GeV^2)

$xu_v(x) = A_u x^{au} (1-x)^{bu} (1+ \varepsilon_u \sqrt{x} + \gamma_u x)$

$xd_v(x) = A_d x^{ad} (1-x)^{bd} (1+ \varepsilon_d \sqrt{x} + \gamma_d x)$

$xS(x) = A_s x^{-\lambda s} (1-x)^{bs} (1+ \varepsilon_s \sqrt{x} + \gamma_s x)$

$xg(x) = A_g x^{-\lambda g} (1-x)^{bg} (1+ \varepsilon_g \sqrt{x} + \gamma_g x)$

These parameters control the low-x shape

These parameters control the middling-x shape

These parameters control the high-x shape

Alternative form for CTEQ

$xf(x) = A_0 x^{A1}(1-x)^{A2} e^{A3x} (1+e^{A4}x)^{A5}$

Parameters Ag, Au, Ad are fixed through momentum and number sum rules – explain other parameters may be fixed by model choices-

Model choices ⇒Form of parametrization at Q^2_0, value of Q^2_0,, flavour structure of sea, cuts applied, heavy flavour scheme → typically ~15 parameters

Use QCD to evolve these PDFs to $Q^2 > Q^2_0$

Construct the measurable structure functions by convoluting PDFs with coefficient functions: make predictions for ~1500 data points across the x,Q2 plane

Perform χ2 fit to the data

The fact that so few parameters allows us to fit so many data points established QCD as the THEORY OF THE STRONG INTERACTION and provided the first measurements of α_s (as one of the fit parameters)

The form of the parametrisation at Q^2_0

$x^a (1-x)^b$ ….. at one time (20 years ago?) we thought we understood it!

--------the high x power from counting rules ----$(1-x)^{2ns-1}$ - ns spectators

valence $(1-x)^3$, sea $(1-x)^7$, gluon $(1-x)^5$

--------the low-x power from Regge – low-x corresponds to high centre of mass energy for the virtual boson proton collision -----Regge theory gives high energy cross-sections as $s^{(\alpha-1)}$ -----------which gives x dependence $x^{(1-\alpha)}$, where α is the intercept of the Regge trajectory- different for singlet (no overall flavour) F2 ~x^0 and non-singlet (flavour- valence-like) xF3~$x^{0.5}$

But at what Q^2 would these be true? – Valence distributions evolve slowly but sea and gluon distributions evolve fast– we are just parametrising our ignorance -----and we need the arbitrary polynomial

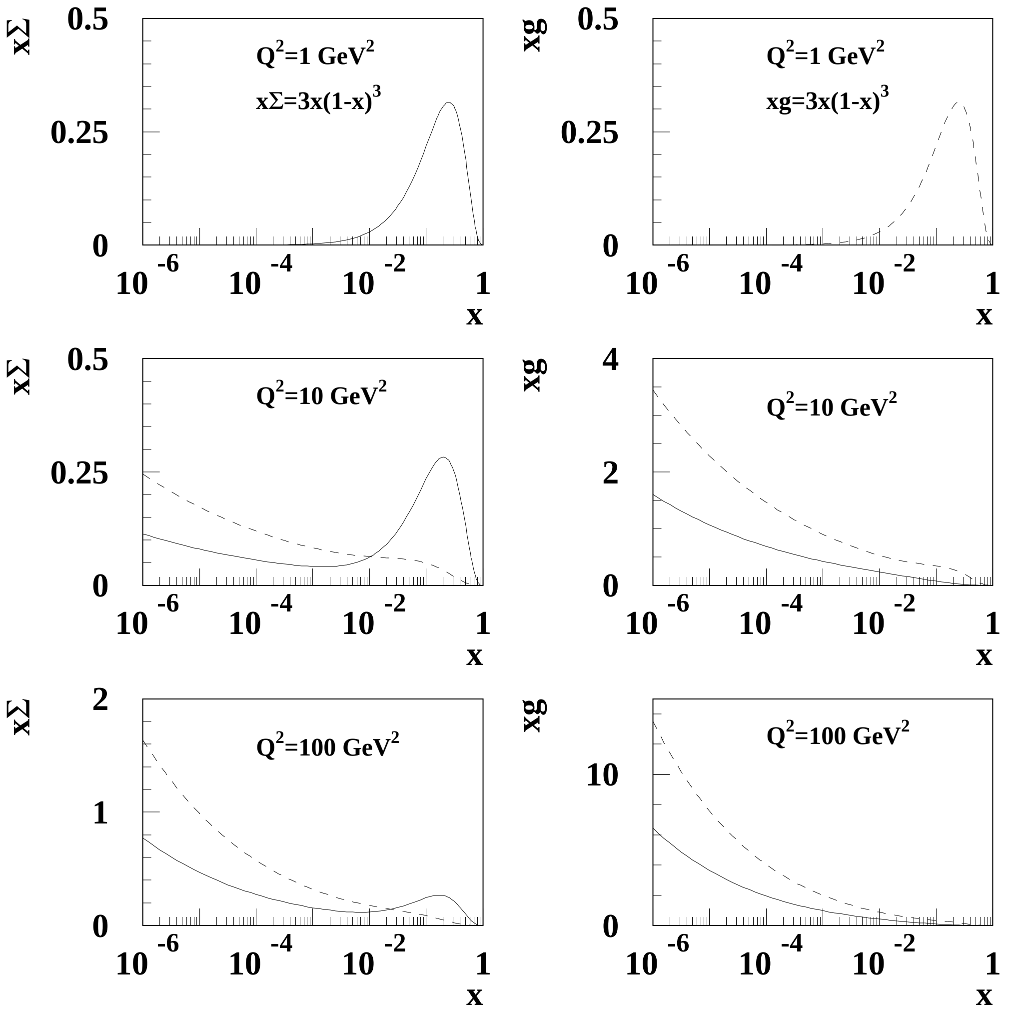
In any case the further you evolve in Q^2 the less the parton distributions look like the low Q^2 inputs and the more they are determined by QCD evolution

(In fact for the GRV partons one starts at very low-Q2 with valence-like input shapes, which →0 as x →0, so that all low-x sea and gluon PDFs are generated by QCD)

Example of parametrisation independence

Take sea=xΣ ----------valence-like and gluon - - - - zero at Q2=4

Take sea=xΣ ----------zero and gluon - - - - valence-like at Q2=4



But where is the information coming from?

Fixed target e/μ p/D data from **NMC, BCDMS, E665, SLAC**

F_2(e/μp)~ 4/9 x(u +ubar) +1/9x(d+dbar) + 4/9 x(c +cbar) +1/9x(s+sbar)

F_2(e/μD)~5/18 x(u+ubar+d+dbar) + 4/9 x(c +cbar) +1/9x(s+sbar)

Assuming u in proton = d in neutron – strong-isospin

Also use ν, νbar fixed target data from **CCFR** (Beware Fe target needs corrections)

F2(ν,νbar N) = x(u +ubar + d + dbar + s +sbar + c + cbar)
xF_3(ν,νbar N) = x(u_v + d_v) (provided s = sbar)
Valence information for 0< x < 1

Can get ~4 distributions from this: e.g. u, d, ubar, dbar – but need assumptions

like q=qbar for all flavours, sbar = 1/4 (ubar+dbar), dbar = ubar (wrong!) and need heavy quark treatment.

Note gluon enters only indirectly via DGLAP equations for evolution

Flavour structure

Historically an SU(3) symmetric sea was assumed

$u=u_v+u_{sea}$, $d=d_v+d_{sea}$

$u_{sea} = \text{ubar} = d_{sea} = \text{dbar} = s = \text{sbar} = K$ and $c=\text{cbar}=0$

Measurements of $\dfrac{F_2^{\mu n}}{F_2^{\mu p}} = \dfrac{u_v + 4d_v + 4/3K}{4u_v + d_v + 4/3K}$

Establish no valence quarks at small-x $F_2^{\mu n}/F_2^{\mu p} \rightarrow 0$

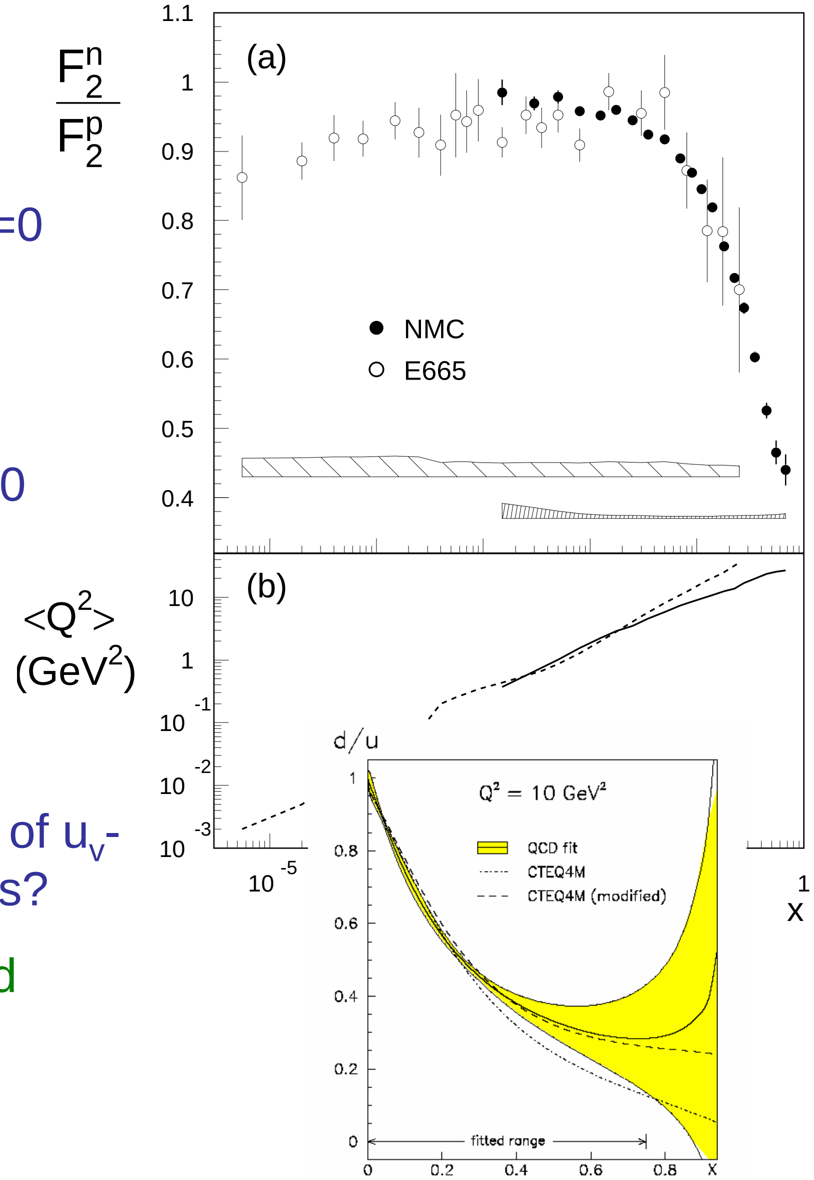
But $F_2^{\mu n}/F_2^{\mu p} \rightarrow 1/4$ as $x \rightarrow 1$

Not to 2/3 as it would for $d_v/u_v=1/2$,

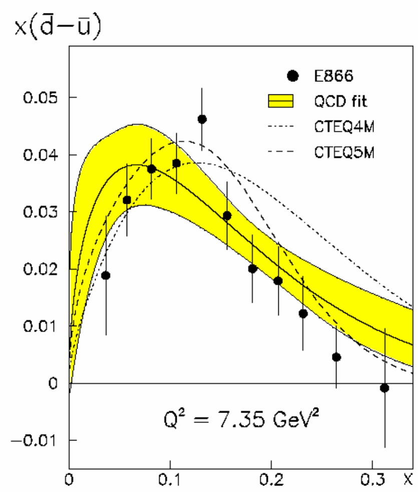
hence it look s as if $d_v/u_v \rightarrow 0$ as $x \rightarrow 1$

i.e the d_v momentum distribution is softer than that of u_v-
Why? Non-perturbative physics --diquark structures?

How accurate is this? Could $d_v/u_v \rightarrow 1/4$ (Farrar and Jackson)?



Flavour structure in the sea



dbar ≠ubar in the sea

Consider the Gottfried sum-rule (at LO)

$\int dx\ (F2p-F2n) = 1/3 \int dx\ (uv-dv) + 2/3 \int dx(ubar-dbar)$

If ubar=dbar then the sum should be 0.33

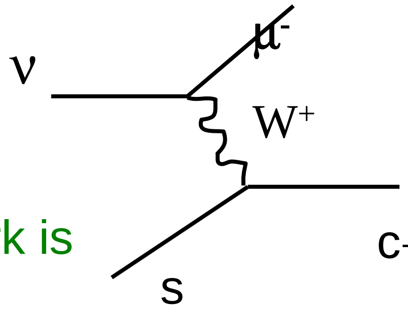
the measured value from NMC = 0.235 ± 0.026

Clearly dbar > ubar…why? low Q2 non-perturbative effects,

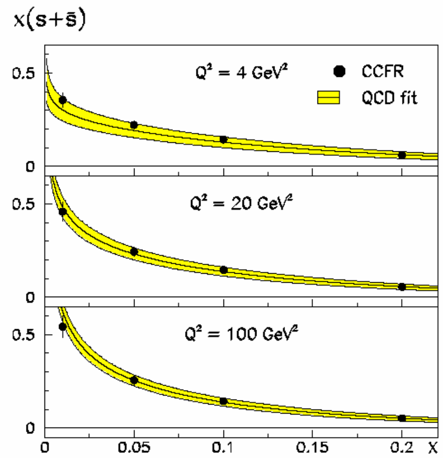
Pauli blocking, p $\rightarrow n\pi^+, p\pi^0, \Delta^{++}\pi^-$

sbar≠(ubar+dbar)/2,

in fact sbar ~ (ubar+dbar)/4



c→s $\mu^+\nu$



Why? The mass of the strange quark is larger than that of the light quarks

Evidence – neutrino opposite sign dimuon production rates

And even s≠sbar? Because of p→ΛK+

Heavy quark treatment – illustrate with charm

Massive quarks introduce another scale into the process, the approximation $m_q^2 \sim 0$ cannot be used

Zero Mass Variable Flavour Number Schemes (ZMVFNs) traditional

c=0 until $Q^2 \sim 4m_c^2$, then charm quark is generated by g→ c cbar splitting and treated as massless-- disadvantage incorrect to ignore m_c near threshold

Fixed Flavour Number Schemes (FFNs)

If $W^2 > 4m_c^2$ then c cbar can be produced by boson-gluon fusion and this can be properly calculated - disadvantage $\ln(Q^2/m_c^2)$ terms in the cross-section can become large- charm is never considered part of the proton however high the scale is.

General Mass variable Flavour Schemes (GMVFNs)

Combine correct threshold treatment with resummation of $\ln(Q^2/m_c^2)$ terms into the definition of a charm quark density at large Q^2

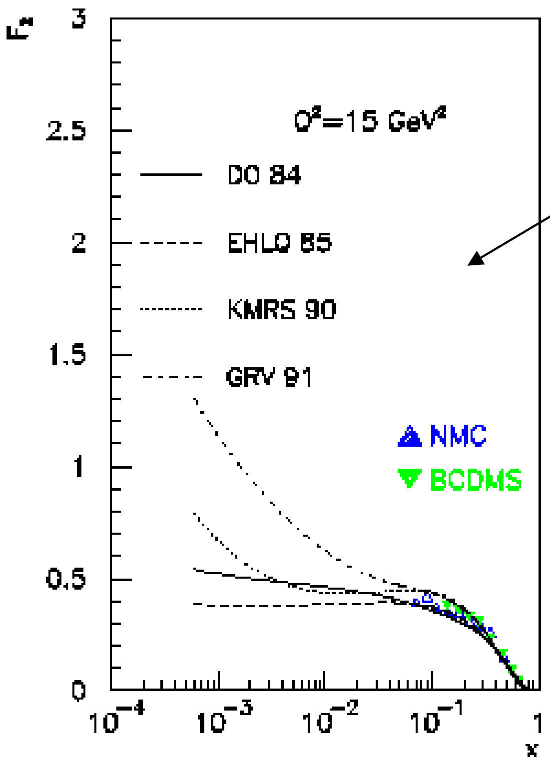
Arguments as to correct implementation but should look like FFN at low scale and like ZMVFN at high scale.

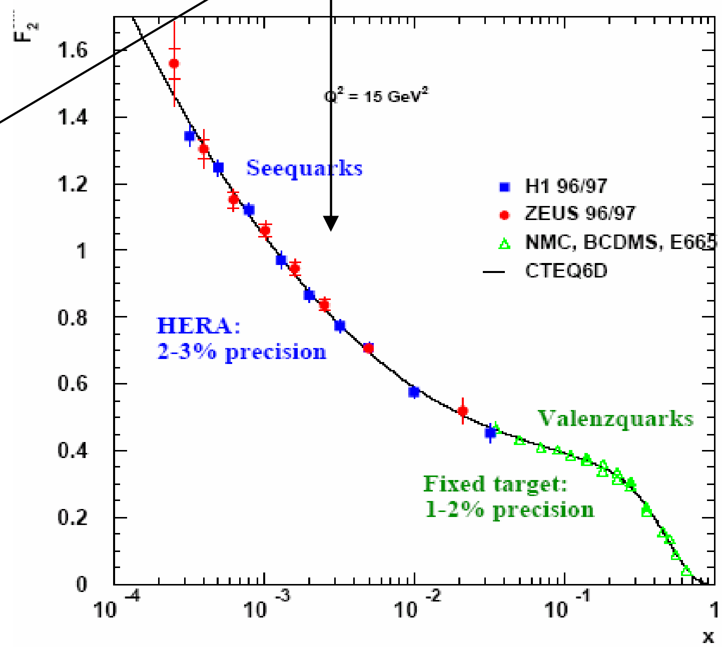
Additional complications for W exchange s→c threshold.

Low-x – within conventional NLO DGLAP

Before the HERA measurements most of the predictions for low-x behaviour of the structure functions and the gluon PDF were wrong

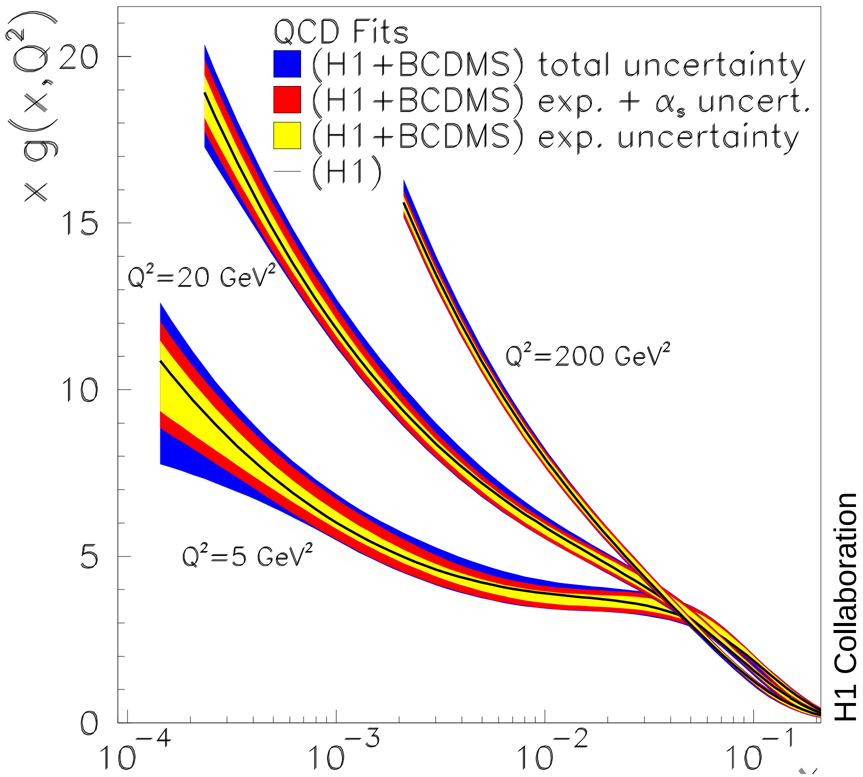
HERA ep neutral current (γ-exchange) data give much more information on the sea and gluon at small x…..



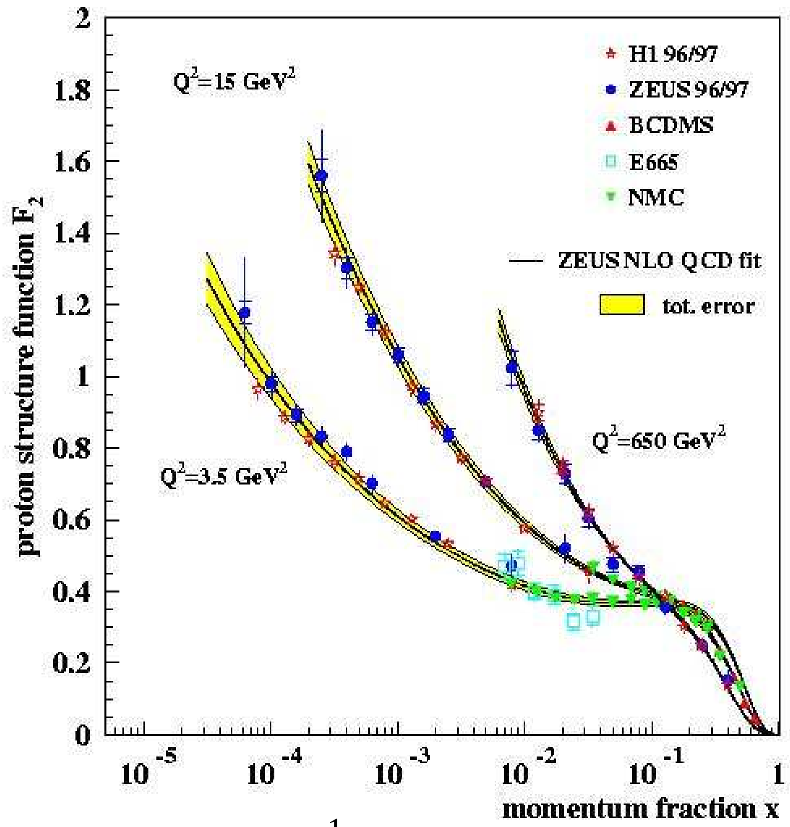


xSea directly from F_2, $F_2 \sim xq$

xGluon from scaling violations $dF_2 / d\ln Q^2$ – the relationship to the gluon is much more direct at small-x, $dF_2/d\ln Q^2 \sim$ Pqg xg



Low-x



$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\left[\Sigma_q Pgq(z)q(y,Q^2) + Pgg(z)g(y,Q^2)\right]$$

At small x,

small z=x/y

$$Pgq \to \frac{C_F}{z}, \qquad Pgg \to \frac{2C_A}{z}$$

Gluon splitting functions become singular

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\frac{6}{z}g(y,Q^2)$$

$xg(x,Q^2) \sim x^{-\lambda g}$

$$\lambda_g = \left(\frac{12\ln(t/t_0)}{\beta_0\ln(1/x)}\right)^{\frac{1}{2}},$$

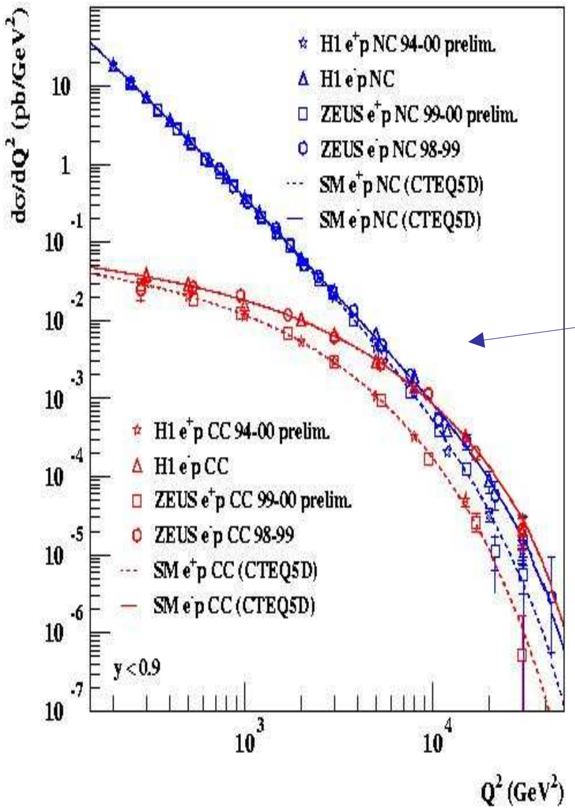
$t = \ln Q^2/\Lambda^2$

$\alpha_s \sim 1/\ln Q^2/\Lambda^2$

A flat gluon at low Q^2 becomes very steep AFTER Q^2 evolution AND F_2 becomes gluon dominated

$F_2(x,Q^2) \sim x^{-\lambda s}$, $\lambda s = \lambda g - \varepsilon$

High Q^2 HERA data



HERA data have also provided information at high $Q^2 \rightarrow Z^0$ and $W^{+/-}$ become as important as γ exchange $\rightarrow$ NC and CC cross-sections comparable

For NC processes

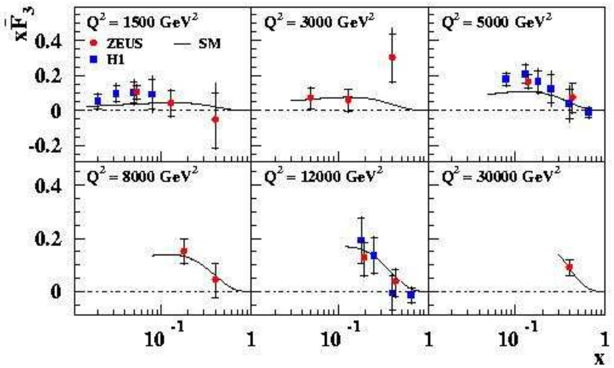
$$F_2 = \Sigma_i\ A_i(Q^2)\ [xq_i(x,Q^2) + x\bar{q}_i(x,Q^2)]$$

$$xF_3 = \Sigma_i\ B_i(Q^2)\ [xq_i(x,Q^2) - x\bar{q}_i(x,Q^2)]$$

$$A_i(Q^2) = e_i^2 - 2\ e_i\ v_i\ v_e\ P_Z + (v_e^2+a_e^2)(v_i^2+a_i^2)\ P_Z^2$$

$$B_i(Q^2) = \quad -2\ e_i\ a_i\ a_e\ P_Z + \quad 4a_i\ a_e\ v_i\ v_e \quad P_Z^2$$

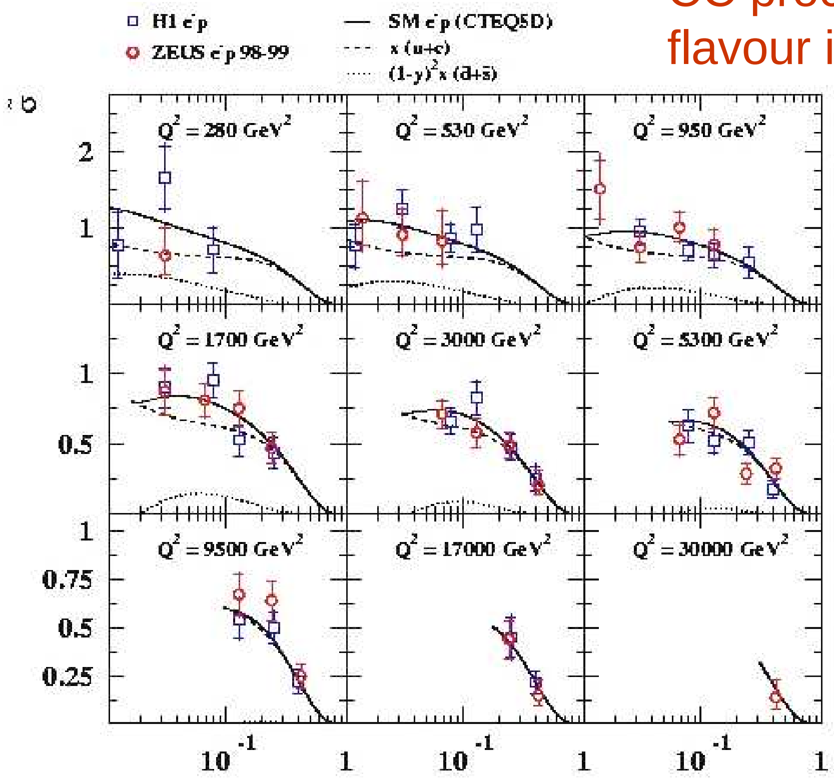
$$P_Z^2 = Q^2/(Q^2 + M^2_Z)\ 1/\sin^2\theta_W$$

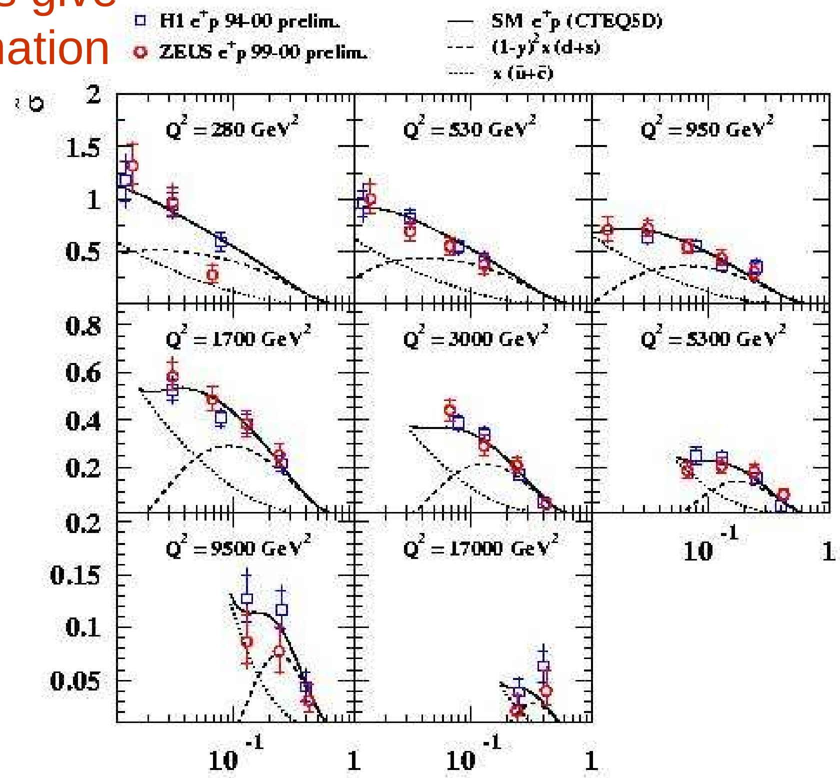


$\rightarrow$a new valence structure function xF_3 due to Z exchange is measurable from low to high x- on a pure proton target $\rightarrow$ no heavy target corrections- no assumptions about strong isospin

$\rightarrow$ e- running at HERA-II is already improving this measurement

CC processes give flavour information





$$\frac{d^2\sigma(e^-p)}{dxdy} = \frac{G_F^2\, M^4_W}{2\pi x(Q^2+M^2_W)^2}\,[x\,(u+c) + (1-y)^2 x\,(\bar{d}+\bar{s})]$$

$$\frac{d^2\sigma(e^+p)}{dxdy} = \frac{G_F^2\, M^4_W}{2\pi x(Q^2+M^2_W)^2}\,[x\,(\bar{u}+\bar{c}) + (1-y)^2 x\,(d+s)]$$

u_v at high x

M_W information

d_v at high x

Measurement of high-x d_v on a pure proton target

d is not well known because u couples more strongly to the photon. Historically information has come from deuterium targets –but even Deuterium needs binding corrections. Open questions: does u in proton = d in neutron?,
does dv/uv ⇒ 0, as x ⇒ 1?

Parton distributions are transportable to other processes

Accurate knowledge of them is essential for calculations of cross-sections of any process involving hadrons. Conversely, some processes have been used to get further information on the PDFs

E.G

DRELL YAN – p N $\rightarrow \mu^+\mu^-$ X, via q qbar $\rightarrow \mu^+\mu^-$, gives information on the Sea

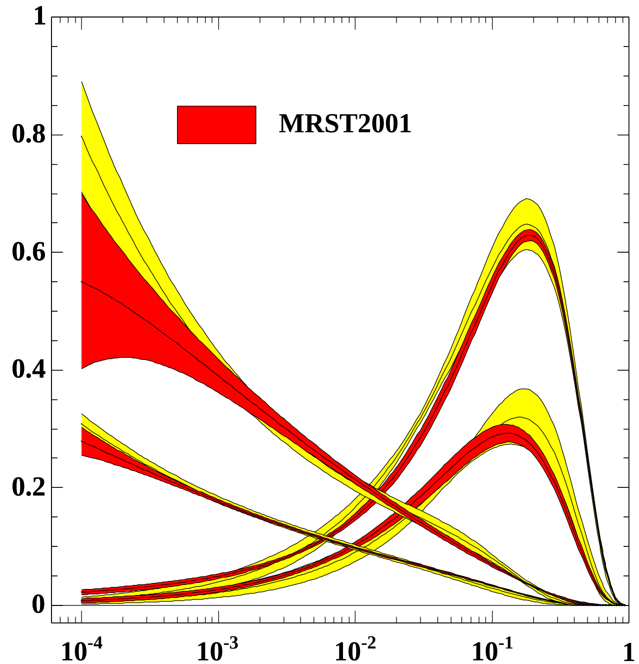
Asymmetry between pp $\rightarrow \mu^+\mu^-$ X and pn $\rightarrow \mu^+\mu^-$ X gives more information on dbar - ubar difference

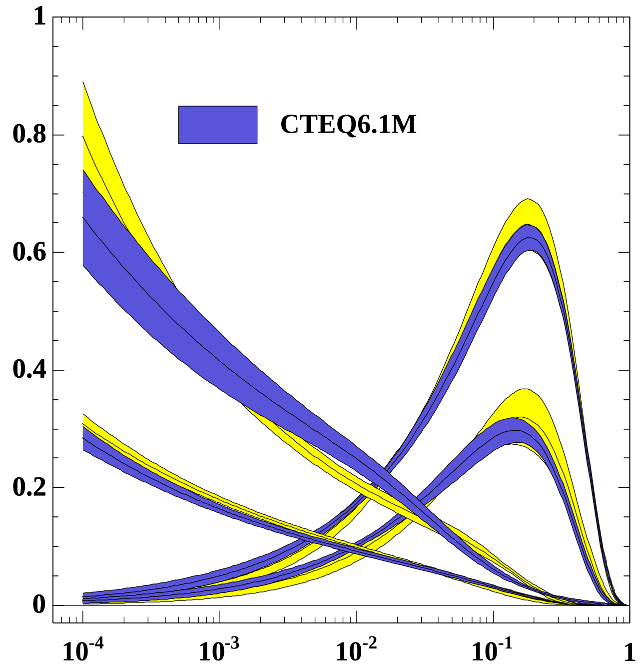
W PRODUCTION- p pbar $\rightarrow W^+(W^-)$ X, via u dbar $\rightarrow W^+$, d ubar $\rightarrow W^-$ gives more information on u, d differences

PROMPT g - p N $\rightarrow$ g X, via g q $\rightarrow$ g q gives more information on the gluon

(but there are current problems concerning intrinsic pt of initial partons)

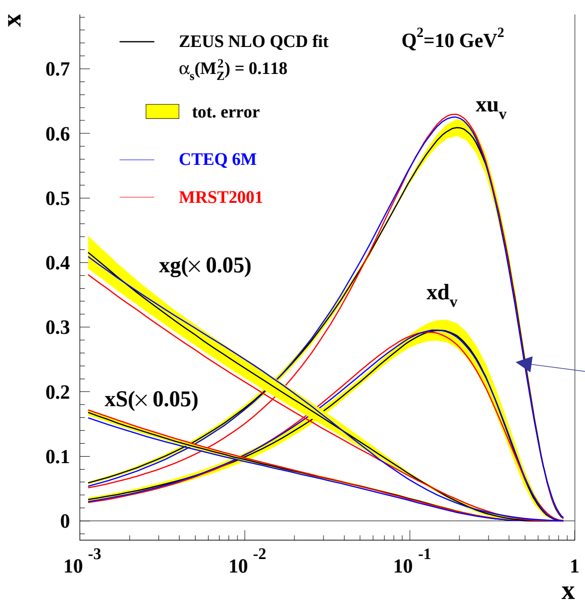
HIGH E_T INCLUSIVE JET PRODUCTION – p p $\rightarrow$ jet + X, via g g, g q, g qbar subprocesses gives more information on the gluon





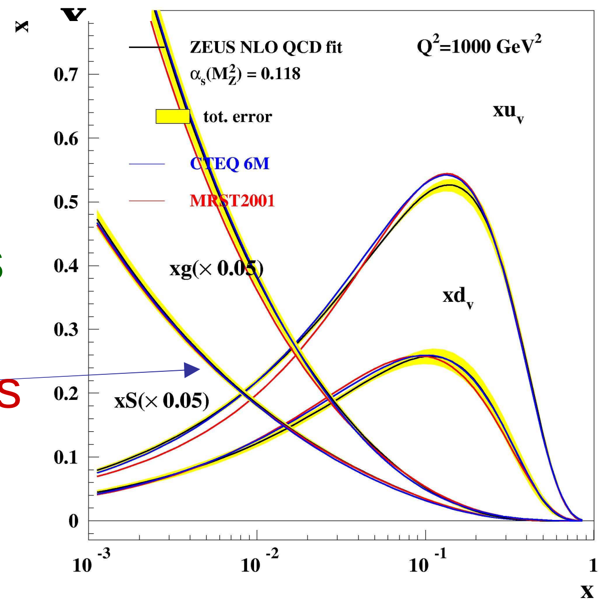
New millenium PDFs

With uncertainty estimates



–Valence distributions evolve slowly

Sea/Gluon distributions evolve fast

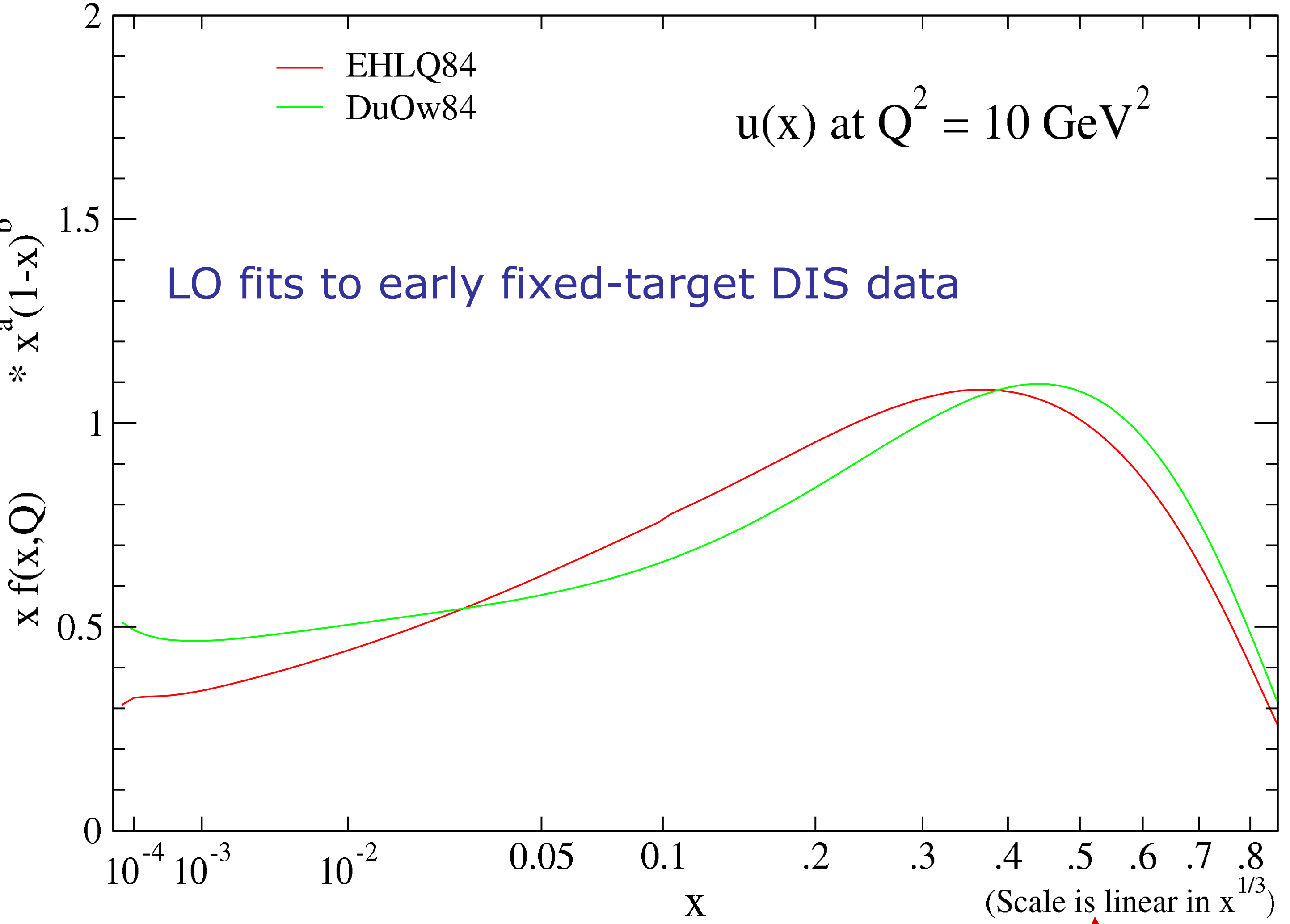


So how certain are we? First, some quantitative measure of the progress made over 20 years of PDF fitting (thanks to Wu-ki Tung)

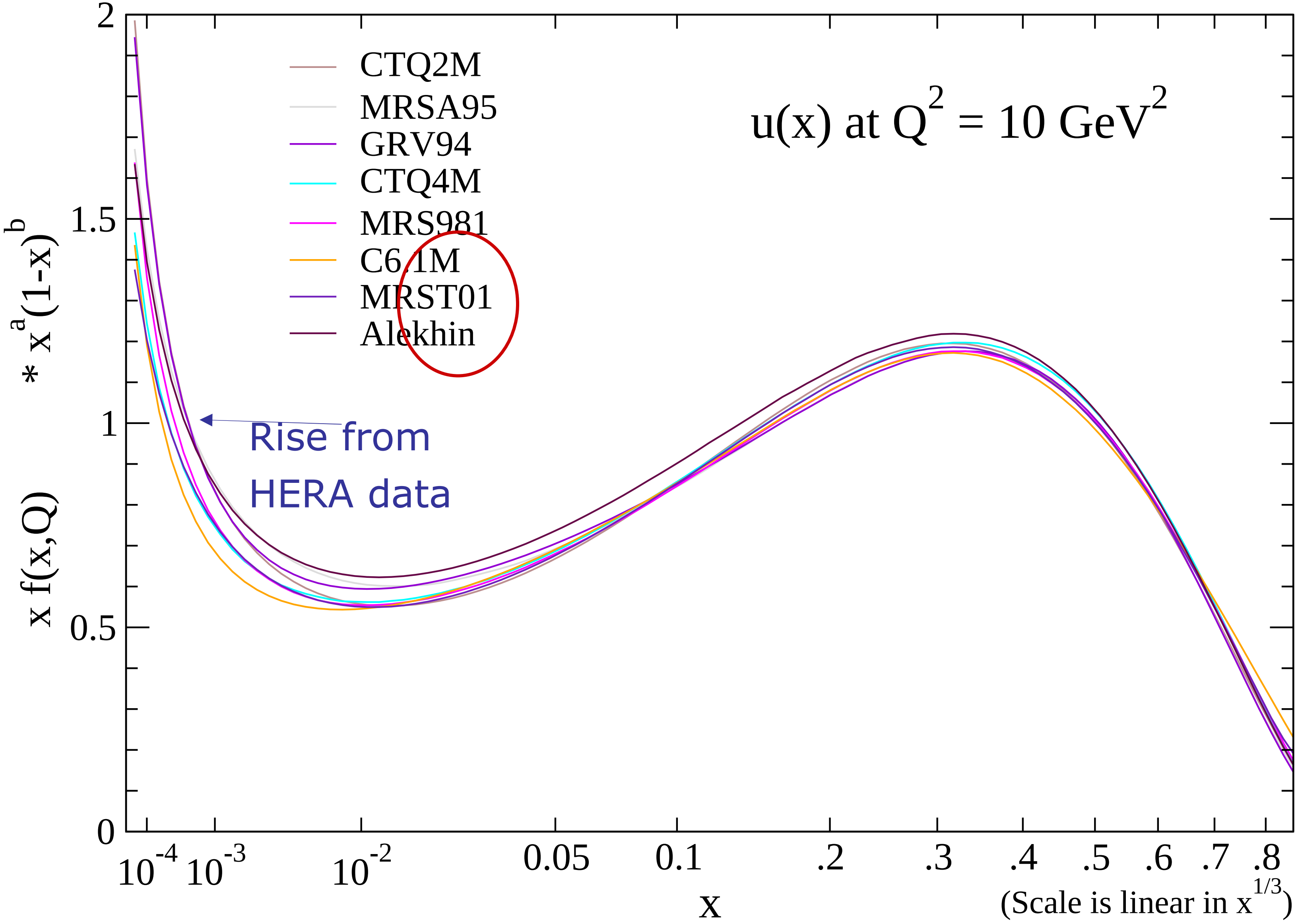
	Fixed-tgt	HERA	DY-W	Jets	Total
# Expt pts.	1070	484	145	123	1822
EHLQ ‘84	11475	7750	2373	331	21929
DuOw ‘84	8308	5005	1599	275	15187
MoTu ~‘90	3551	3707	857	218	8333
KMRS ~‘90	1815	7709	577	280	10381
CTQ2M ~’94	1531	1241	646	224	3642
MRSA ~’94	1590	983	249	231	3054
GRV94 ~’94	1497	3779	302	213	5791
CTQ4M ~’98	1414	666	227	206	2513
MRS98 ~’98	1398	659	111	227	2396
CTQ6M 02	1239	508	159	123	2029
MRST01/2	1378	530	120	236	2264

The u quark

To reveal the difference in both large and small x regions

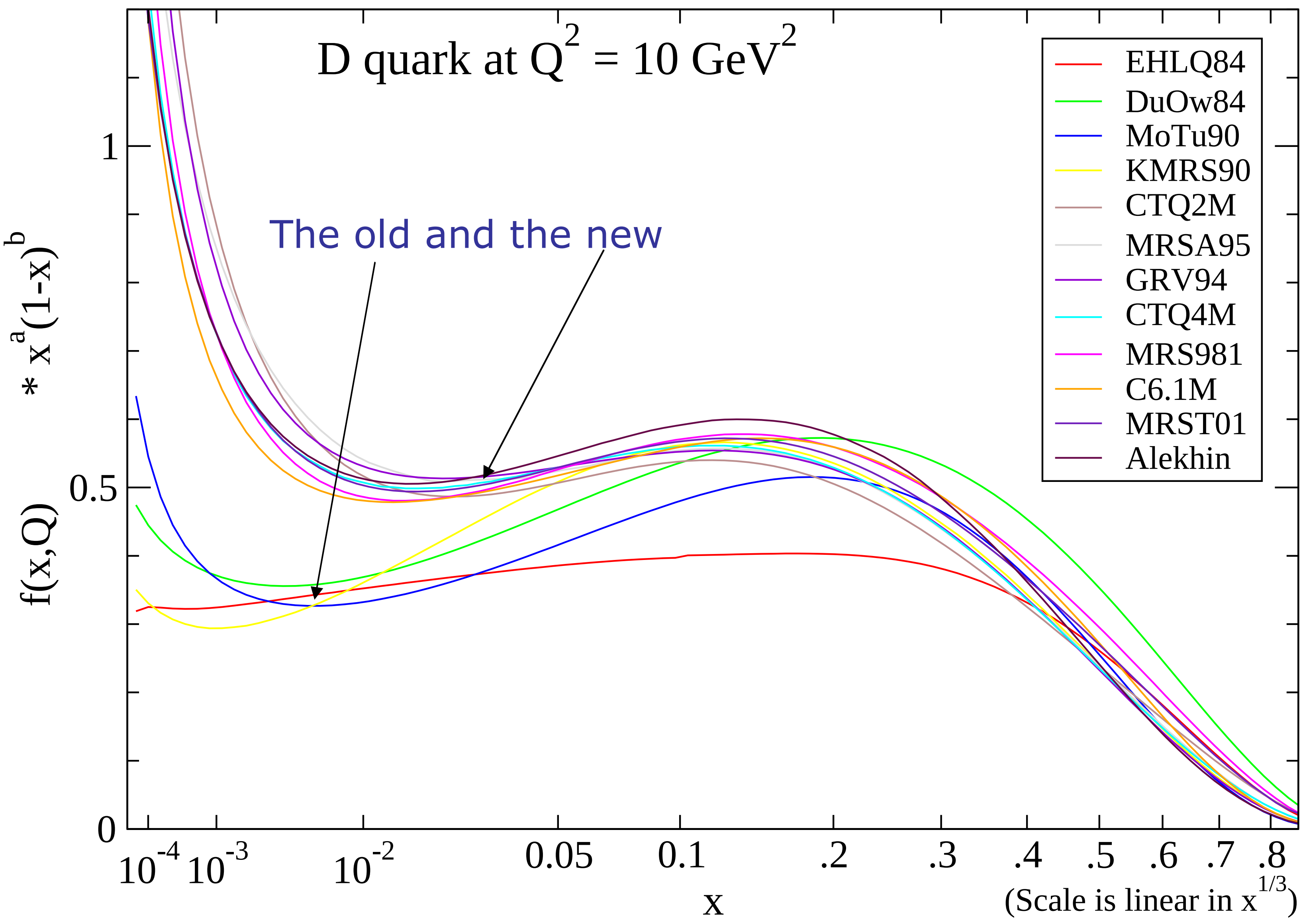


To view small and large x in one plot



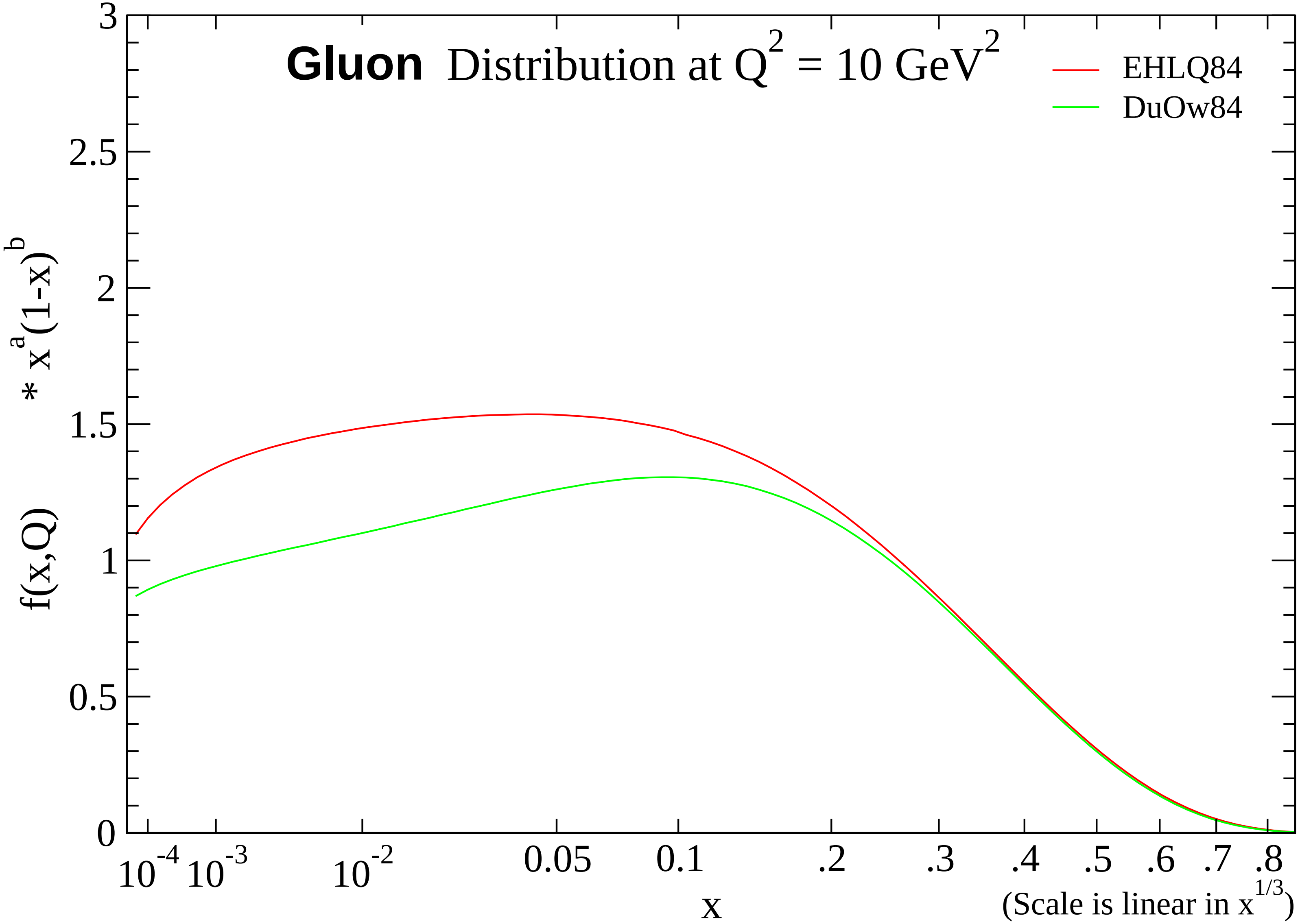
CTQ2M
MRSA95
GRV94
CTQ4M
MRS981
C6.1M
MRST01
Alekhin
u(x) at Q^2 = 10 GeV^2
Rise from
HERA data
x f(x,Q) * $x^a(1-x)^b$
0
0.5
1
1.5
2
10^{-4} 10^{-3} 10^{-2} 0.05 0.1 .2 .3 .4 .5 .6 .7 .8
x
(Scale is linear in $x^{1/3}$)

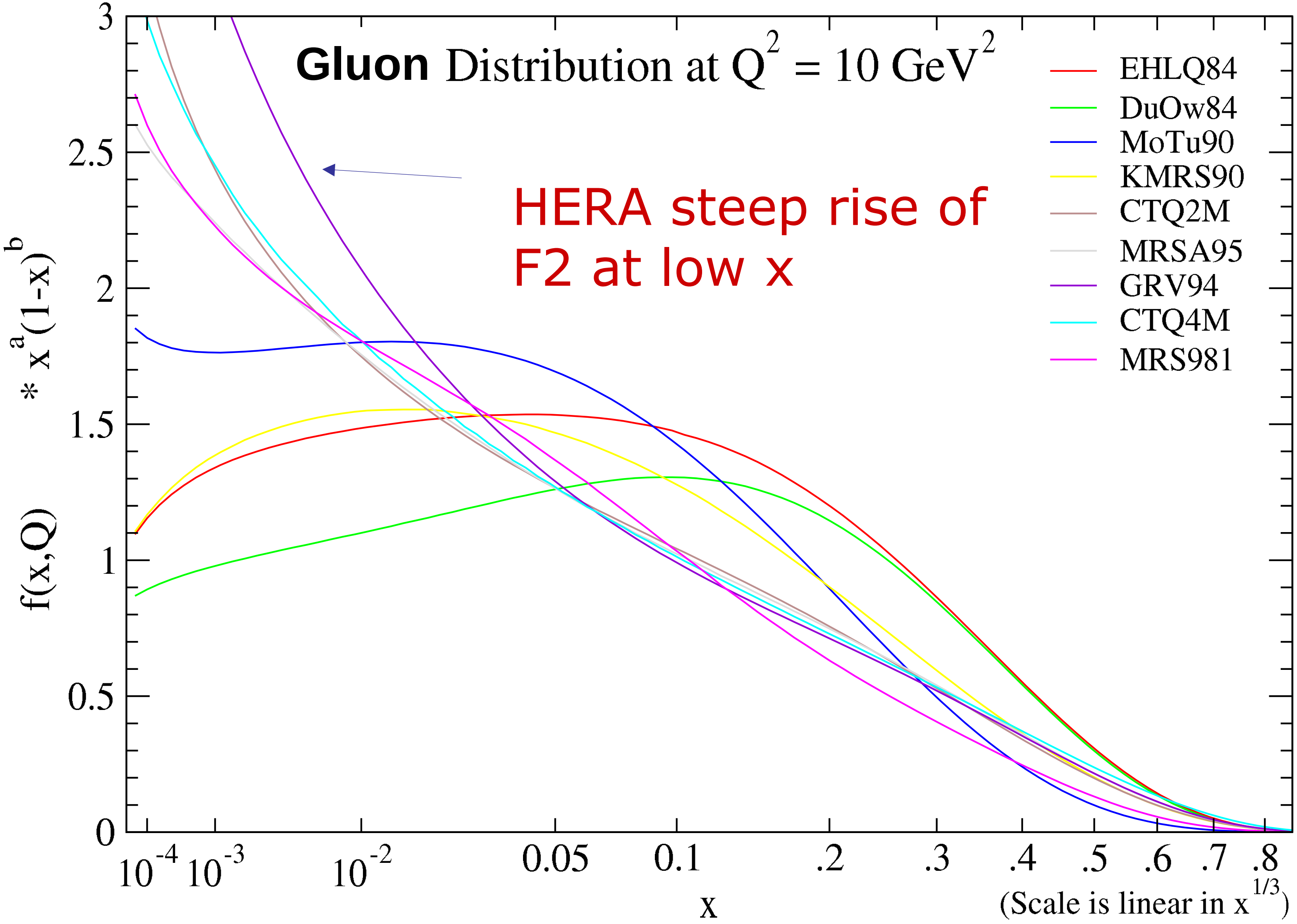
d quark



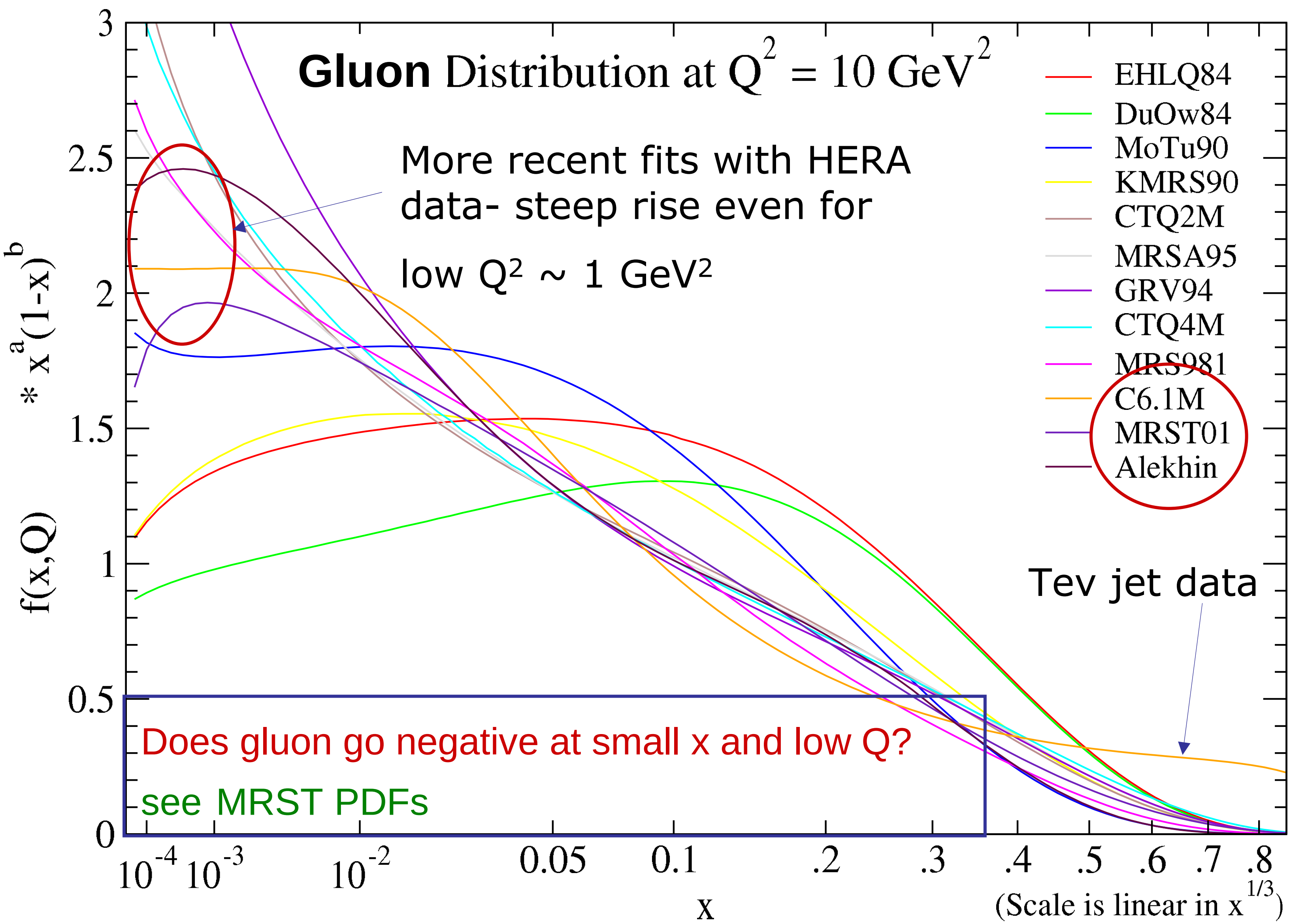
D quark at $Q^2 = 10\ GeV^2$
The old and the new
$f(x,Q) * x^a (1-x)^b$
1
0.5
0
10^{-4} 10^{-3} 10^{-2} 0.05 0.1 .2 .3 .4 .5 .6 .7 .8
x
(Scale is linear in $x^{1/3}$)
EHLQ84
DuOw84
MoTu90
KMRS90
CTQ2M
MRSA95
GRV94
CTQ4M
MRS981
C6.1M
MRST01
Alekhin

The story about the gluon is more complex





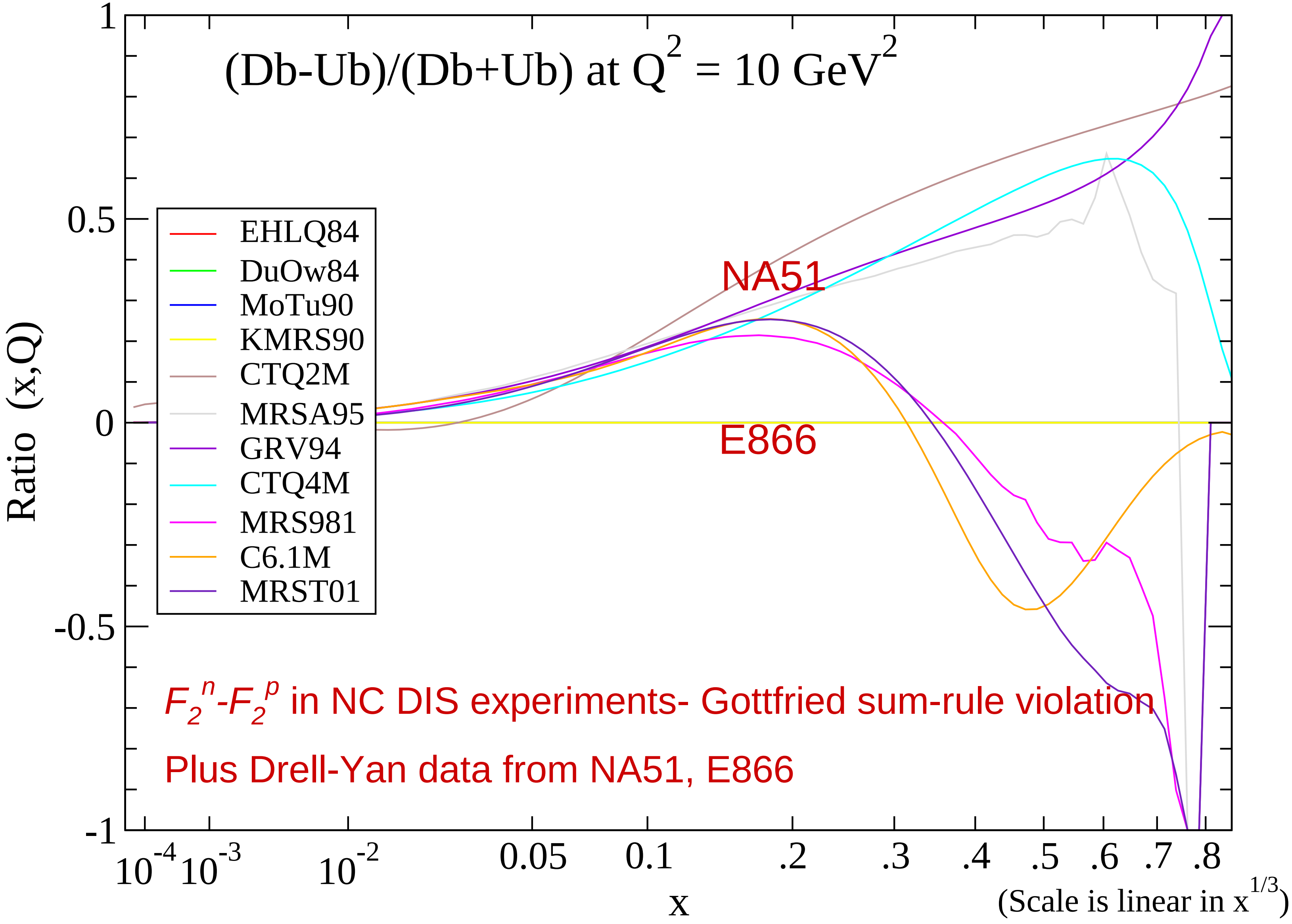
Gluon Distribution at $Q^2 = 10\ GeV^2$
EHLQ84
DuOw84
MoTu90
KMRS90
CTQ2M
MRSA95
GRV94
CTQ4M
MRS981
HERA steep rise of F2 at low x
$f(x,Q) * x^a (1-x)^b$
x
(Scale is linear in $x^{1/3}$)
0
0.5
1
1.5
2
2.5
3
10^{-4}
10^{-3}
10^{-2}
0.05
0.1
.2
.3
.4
.5
.6
.7
.8



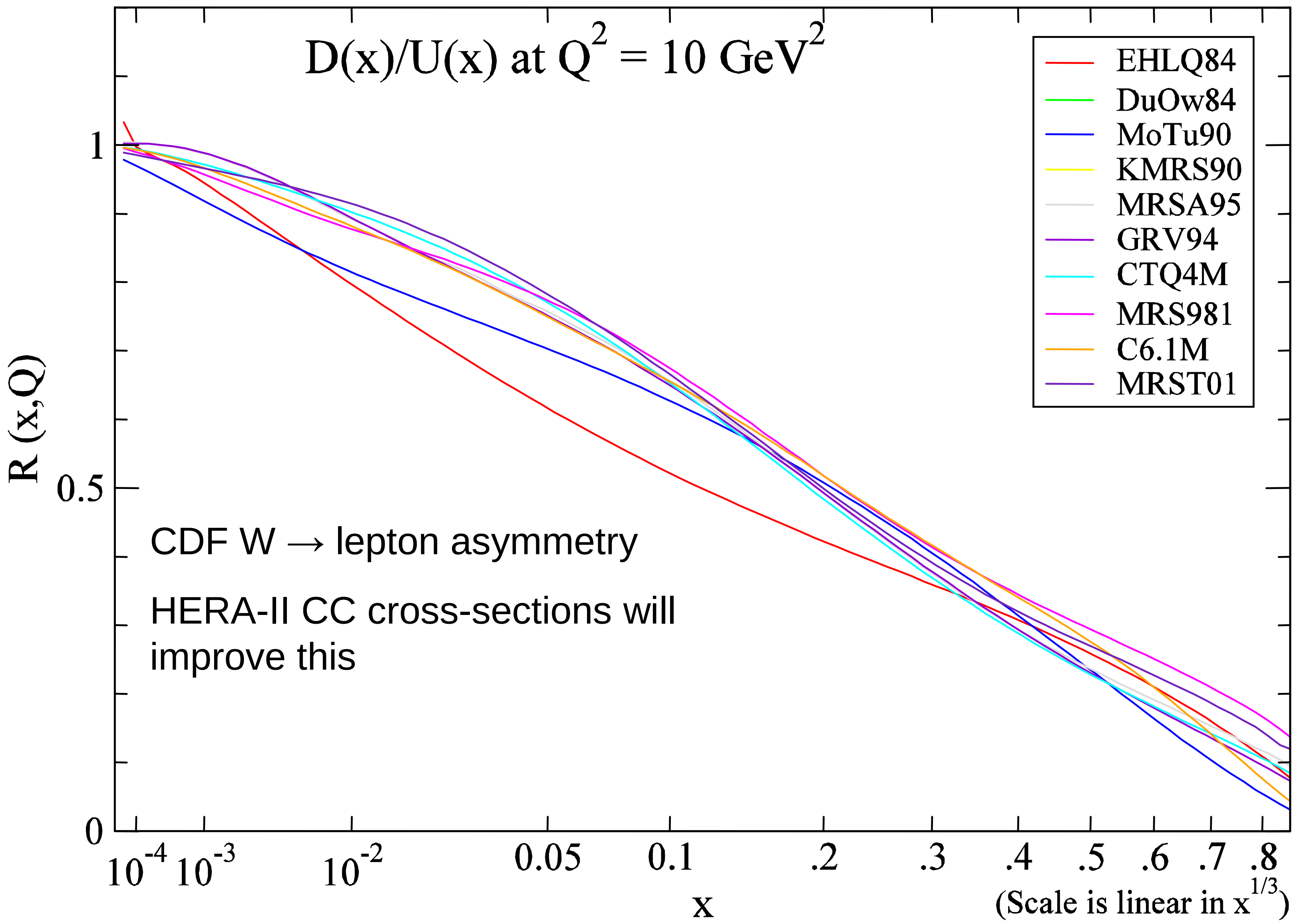
Gluon Distribution at Q² = 10 GeV²
More recent fits with HERA data- steep rise even for low Q² ~ 1 GeV²
EHLQ84
DuOw84
MoTu90
KMRS90
CTQ2M
MRSA95
GRV94
CTQ4M
MRS981
C6.1M
MRST01
Alekhin
Tev jet data
Does gluon go negative at small x and low Q?
see MRST PDFs
f(x,Q) * x^a (1-x)^b
3
2.5
2
1.5
1
0.5
0
10^-4 10^-3 10^-2 0.05 0.1 .2 .3 .4 .5 .6 .7 .8
x
(Scale is linear in x^1/3)

The non-strange sea quarks:

do not observe isospin symmetry



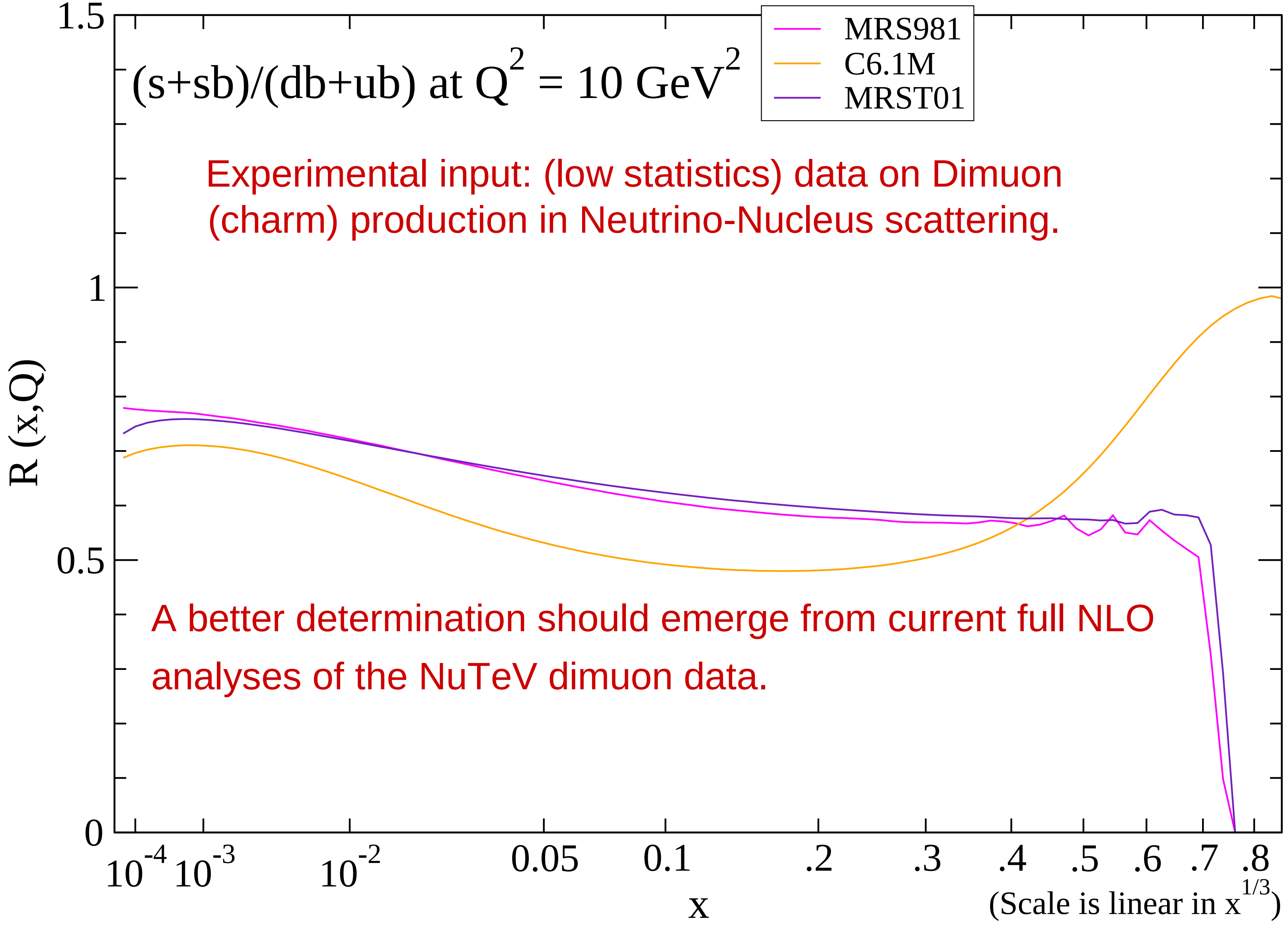
The high-x shape of d/u



CDF W → lepton asymmetry

HERA-II CC cross-sections will improve this

Strange Content of the Nucleon



Experimental input: (low statistics) data on Dimuon (charm) production in Neutrino-Nucleus scattering.

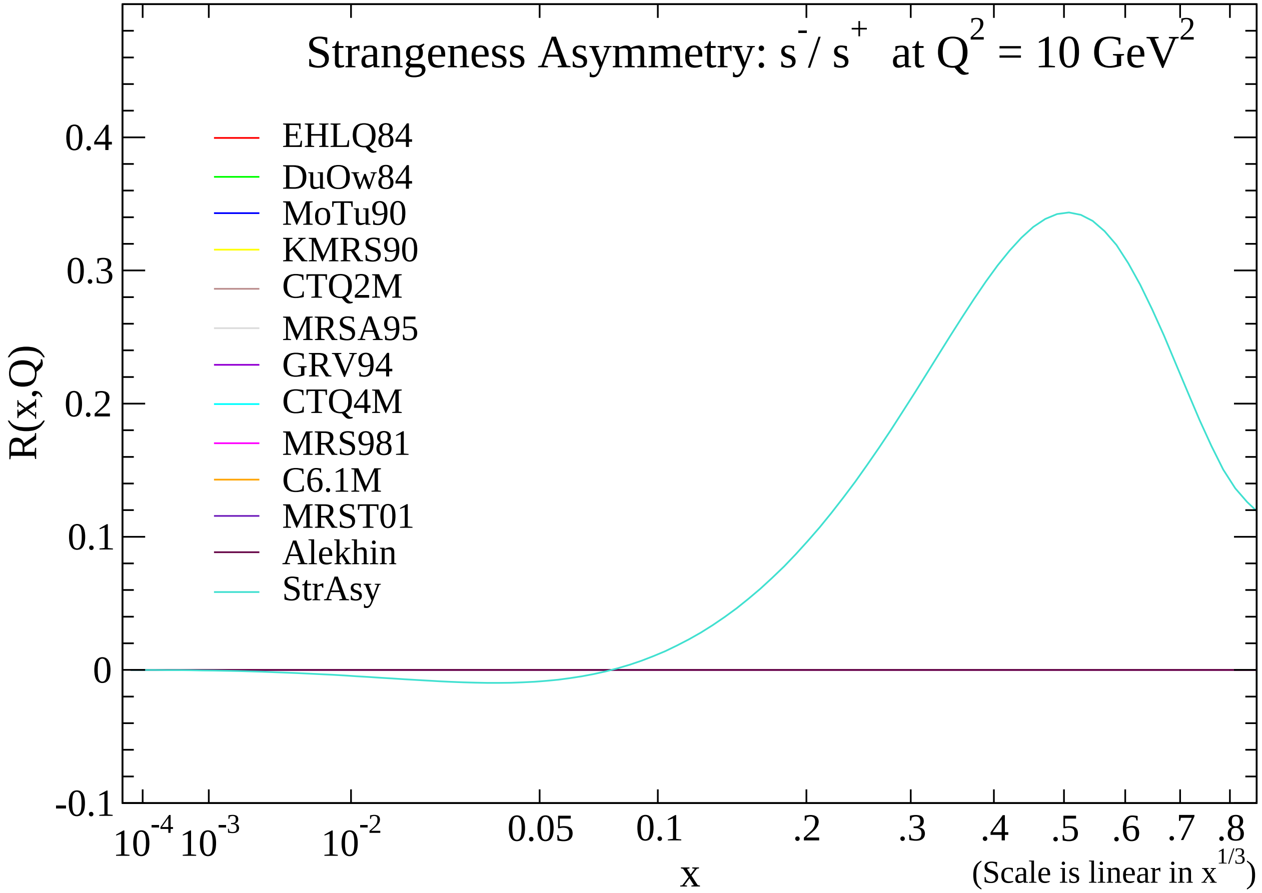
A better determination should emerge from current full NLO analyses of the NuTeV dimuon data.

Is the strangeness sector charge symmetric?- is this the cause of the NuTeV $\sin^2\theta_W$ anomaly?

No good data, but how much asymmetry can be tolerated?

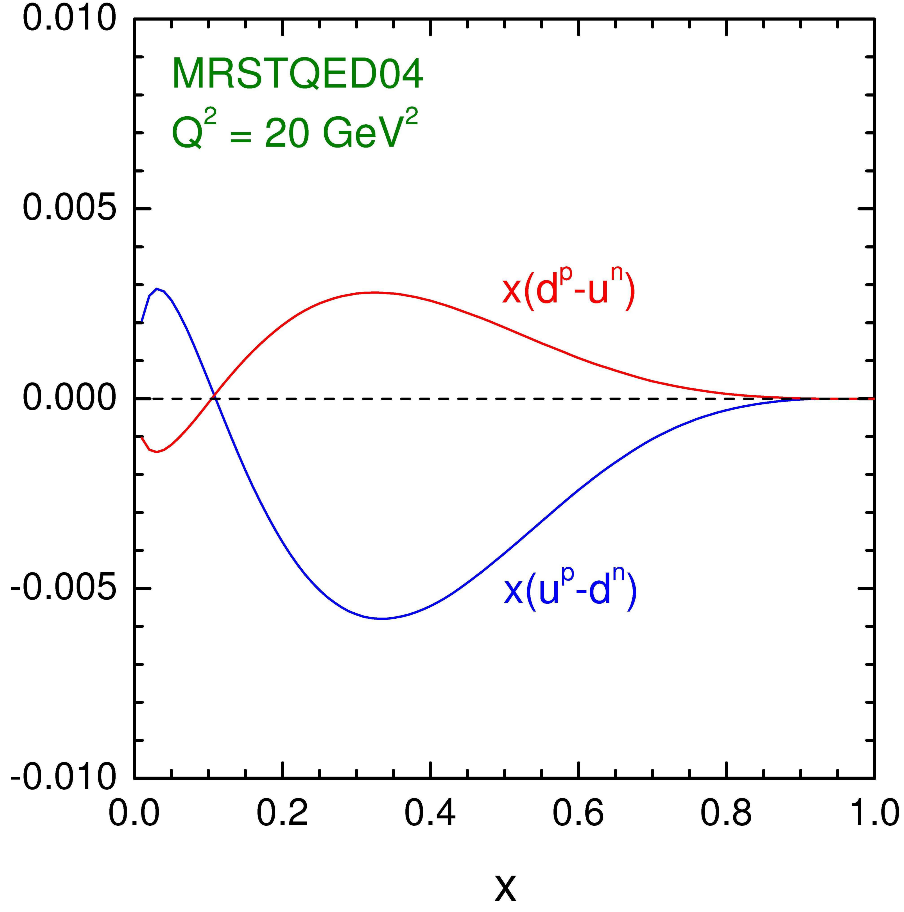
The

dete



Is it true that u in proton = d in neutron

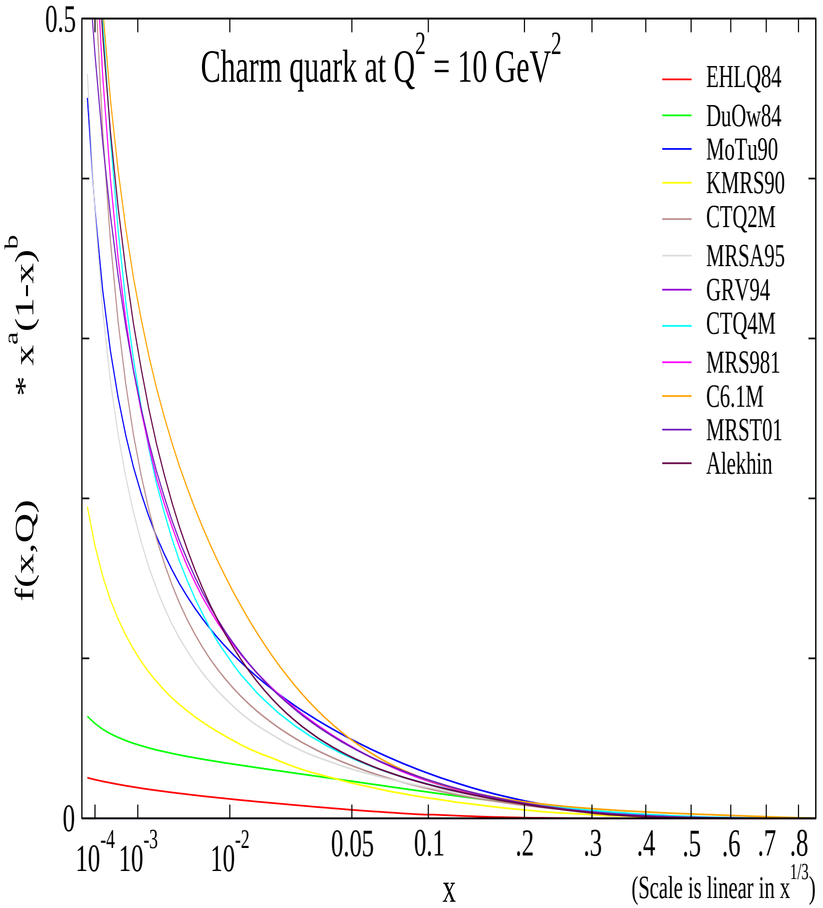
NOT if QED corrections are incorporated in the analysis- is this the cause of the NuTeV $\sin^2\theta_W$ anomaly?

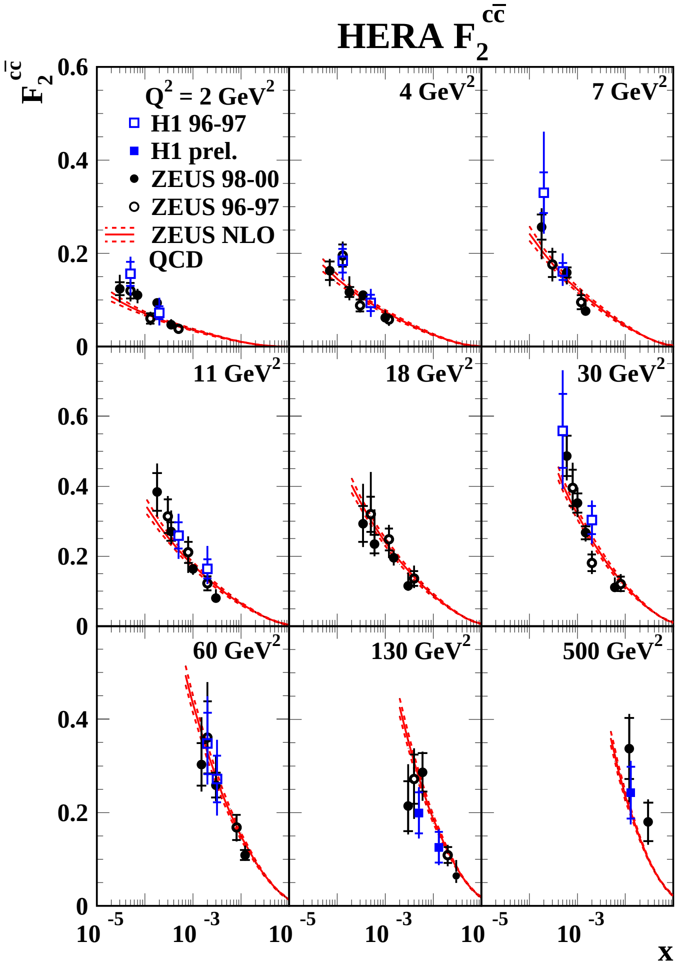


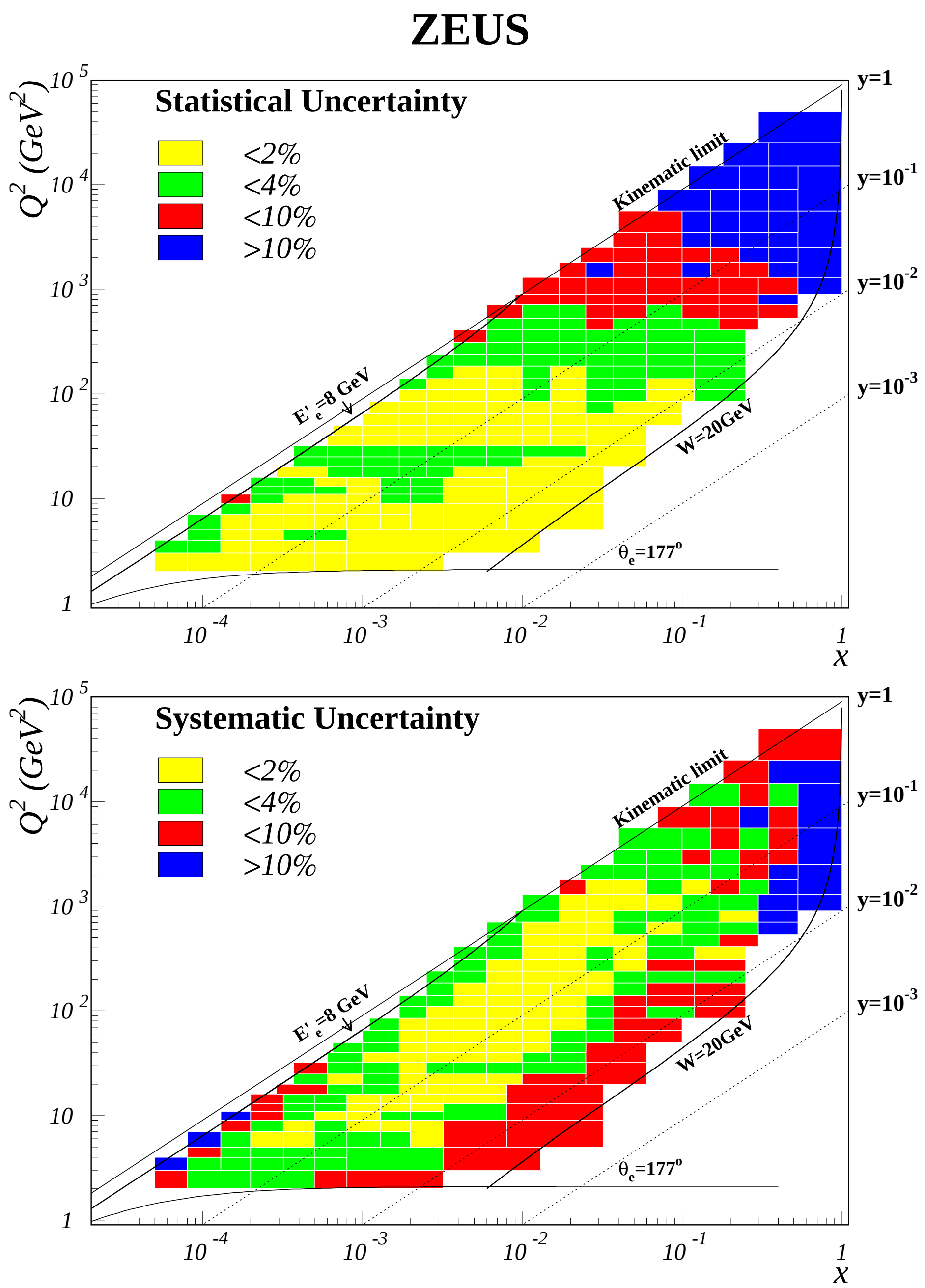
Heavy quarks

Heavy quark distributions in fits are dynamically generated from g→c cbar

Results depend on the “scheme” chosen to handle heavy quark effects in pQCD–fixed-flavor-number (FFN) vs. variable-flavor-number (VFN) schemes







Modern analyses assess PDF uncertainties within the fit

Clearly errors assigned to the data points translate into errors assigned to the fit parameters --

and these can be propagated to any quantity which depends on these parameters— **the parton distributions or the structure functions and cross-sections which are calculated from them**

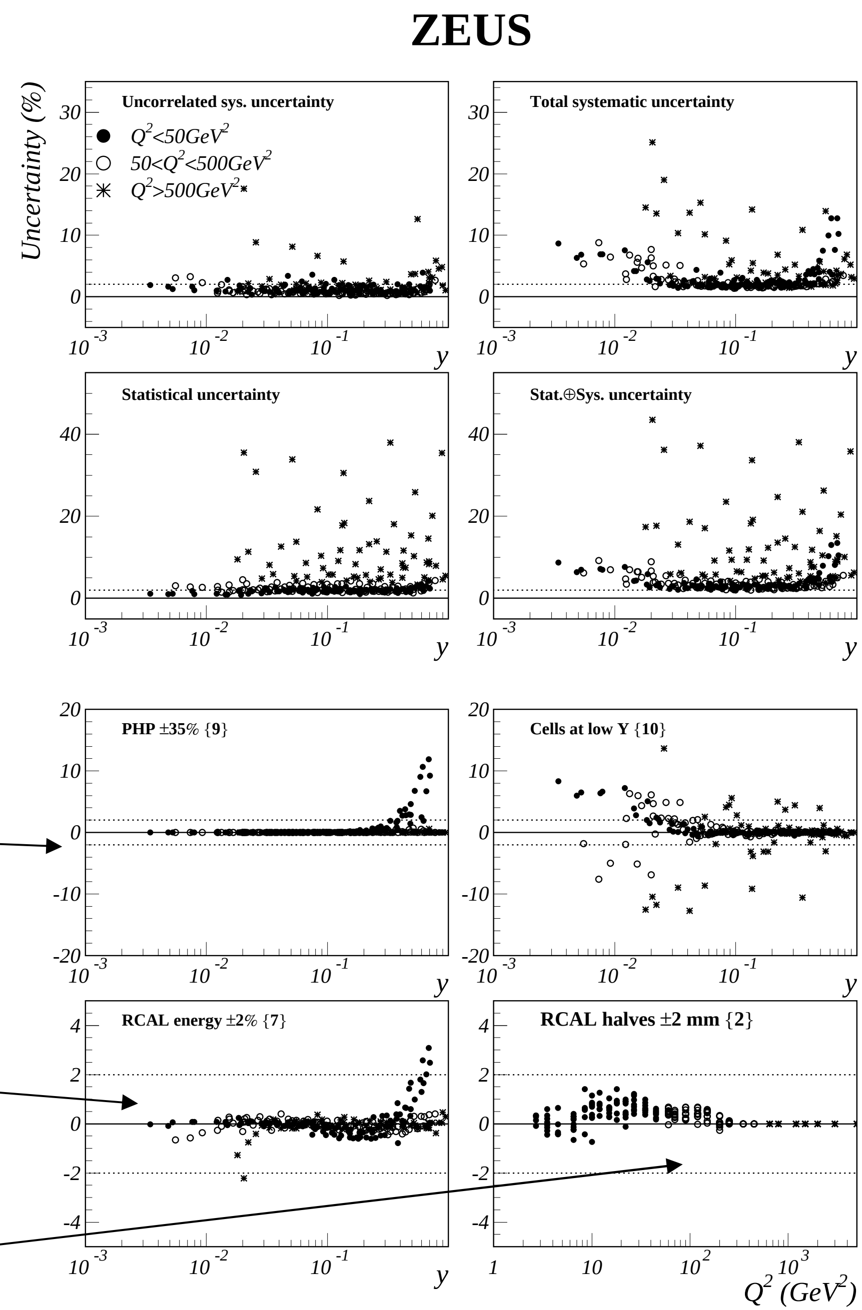
$$< \sigma^2 F > = \Sigma_j \Sigma_k \frac{\partial F}{\partial p_j} V_{jk} \frac{\partial F}{\partial p_k}$$

The errors assigned to the data are both statistical and systematic **and for much of the kinematic plane the size of the point-to-point correlated systematic errors is ~3 times the statistical errors.**

What are the sources of correlated systematic errors?

Normalisations are an obvious example

BUT there are more subtle cases- e.g. **Calorimeter energy scale/angular resolutions** can move events between x,Q2 bins and thus **change the shape** of experimental distributions



Vary the estimate of the photo-production background

Vary energy scales in different regions of the calorimeter

Vary position of the RCAL halves

Why does it matter?

Treatment of correlated systematic errors

$$\chi 2 = \Sigma_i \left[\frac{F_i^{QCD}(p) - F_i^{MEAS}}{(\sigma_i^{STAT})^2 + (\Delta_i^{SYS})^2} \right]^2$$

Errors on the fit parameters, p, evaluated from $\Delta\chi 2 = 1$,

THIS IS NOT GOOD ENOUGH if experimental systematic errors are correlated between data points-

$$\chi^2 = \Sigma_i \Sigma_j [F_i^{QCD}(p) - F_i^{MEAS}] V_{ij}^{-1} [F_j^{QCD}(p) - F_j^{MEAS}]$$

$$V_{ij} = \delta_{ij}(\sigma_i^{STAT})^2 + \Sigma_\lambda \Delta_{i\lambda}^{SYS} \Delta_{j\lambda}^{SYS}$$

Where $\Delta_{i\lambda}^{SYS}$ is the correlated error on point **i** due to systematic error source **λ**

It can be established that this is equivalent to

$$\chi^2 = \Sigma_i \frac{[F_i^{QCD}(p) - \Sigma_\lambda s_\lambda \Delta_{i\lambda}^{SYS} - F_i^{MEAS}]^2}{(\sigma_i^{STAT})^2} + \Sigma s_\lambda^2$$

Where s_λ are systematic uncertainty fit parameters of zero mean and unit variance

This has modified the fit prediction by each source of systematic uncertainty

CTEQ, ZEUS, H1, MRST have all adopted this form of χ2 – but use it differently in the OFFSET and HESSIAN methods …hep-ph/0205153

End lecture -1

Treatment of correlated systematic errors

$$\chi 2 = \Sigma_i \left[\frac{F_i^{QCD}(p) - F_i^{MEAS}}{(\sigma_i^{STAT})^2 + (\Delta_i^{SYS})^2} \right]^2$$

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$$\chi^2 = \Sigma_i \ \Sigma_j \left[F_i^{QCD}(p) - F_i^{MEAS} \right] V_{ij}^{-1} \left[F_j^{QCD}(p) - F_j^{MEAS} \right]$$

$$V_{ij} = \delta_{ij}(\sigma_i^{STAT})^2 + \Sigma_\lambda \Delta_{i\lambda}^{SYS} \Delta_{j\lambda}^{SYS}$$

Where $\Delta_{i\lambda}^{SYS}$ is the correlated error on point **i** due to systematic error source λ

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Where s_λ are systematic uncertainty fit parameters of zero mean and unit variance

This has modified the fit prediction by each source of systematic uncertainty

CTEQ, ZEUS, H1, MRST have all adopted this form of $\chi 2$ – but use it differently in the OFFSET and HESSIAN methods …hep-ph/0205153

How do experimentalists usually proceed: OFFSET method

1. Perform fit without correlated errors ($s_\lambda = 0$) for central fit
2. Shift measurement to upper limit of one of its systematic uncertainties ($s_\lambda = +1$)
3. Redo fit, record differences of parameters from those of step 1
4. Go back to 2, shift measurement to lower limit ($s_\lambda = -1$)
5. Go back to 2, repeat 2-4 for next source of systematic uncertainty
6. Add all deviations from central fit in quadrature (positive and negative deviations added in quadrature separately)
7. This method does not assume that correlated systematic uncertainties are Gaussian distributed

A1

Fortunately, there are smart ways to do this (Pascaud and Zomer LAL-95-05, Botje hep-ph-0110123)

Slide 47

A1 Cooper-Sarkar, 15/03/2004

Fortunately, there are smart ways to do this (Pascaud and Zomer LAL-95-05)

Define matrices

$$M_{jk} = \frac{1}{2}\frac{\partial^2 \chi^2}{\partial p_j \, \partial p_k} \qquad C_{j\lambda} = \frac{1}{2}\frac{\partial^2 \chi^2}{\partial p_j \, \partial s_\lambda}$$

Then M expresses the variation of χ2 wrt the theoretical parameters, accounting for the statistical errors, and C expresses the variation of χ2 wrt theoretical parameters and systematic uncertainty parameters.

Then the covariance matrix accounting for statistical errors is $V^p = M^{-1}$ and the covariance matrix accounting for correlated systematic uncertainties is $V^{ps} = M^{-1} C C^T M^{-1}$. The total covariance matrix $V^{tot} = V^p + V^{ps}$ is used for the standard propagation of errors to any distribution F which is a function of the theoretical parameters

$$< \sigma^2_F > = T \sum_j \sum_k \frac{\partial F}{\partial pj} V_{jk}^{tot} \frac{\partial F}{\partial pk}$$

Where T is the χ2 tolerance, T = 1 for the OFFSET method.

This is a conservative method which gives predictions as close as possible to the central values of the published data. It does not use the full statistical power of the fit to improve the estimates of s_{λ}, since it chooses to distrust that systematic uncertainties are Gaussian distributed.

There are other ways to treat correlated systematic errors- HESSIAN method (covariance method)

Allow s_λ parameters to vary for the central fit. The total covariance matrix is then the inverse of a single Hessian matrix expressing the variation of $\chi 2$ wrt both theoretical and systematic uncertainty parameters.

If we believe the theory why not let it calibrate the detector(s)? Effectively the theoretical prediction is not fitted to the central values of published experimental data, but allows these data points to move collectively according to their correlated systematic uncertainties

The fit determines the optimal settings for correlated systematic shifts such that the most consistent fit to all data sets is obtained. In a global fit the systematic uncertainties of one experiment will correlate to those of another through the fit

The resulting estimate of PDF errors is much smaller than for the Offset method for $\Delta\chi 2 = 1$

We must be very confident of the theory to trust it for calibration– but more dubiously **we must be very confident of the model choices** we made in setting boundary conditions

We must check that $|s_\lambda|$ values are not >>1, so that data points are not shifted far outside their one standard deviation errors - Data inconsistencies!

We must check that superficial changes of model choice (values of Q^2_0, form of parametrization…) do not result in large changes of s_λ

Technically, fitting many s_λ parameters can be cumbersome

CTEQ have given an analytic method CTEQ hep-ph/0101032,hep-ph/0201195

$$\chi 2 = \sum_i \frac{[F_i^{QCD}(p) - F_i^{MEAS}]^2}{(\sigma_i^{STAT})^2} - B\,A^{-1}B$$

where

$$B_\lambda = \sum_i \Delta_{i\lambda}^{sys} \frac{[F_i^{QCD}(p) - F_i^{MEAS}]}{(\sigma_i^{STAT})^2}, \quad A_{\lambda\mu} = \delta_{\lambda\mu} + \sum_i \frac{\Delta_{i\lambda}^{sys}\,\Delta_{i\mu}^{sys}}{(\sigma_i^{STAT})^2}$$

such that the contributions to χ2 from statistical and correlated sources can be evaluated separately.

The problem of large systematic shifts to the data points now becomes manifest as a large value of $BA^{-1}B$ – the correlated systematic error's contribution to the χ2. A small overall value of χ2 can be obtained by the cancellation of two large numbers.

Is this acceptable? What can be done about this?

.Some data sets incompatible/only marginally compatible?

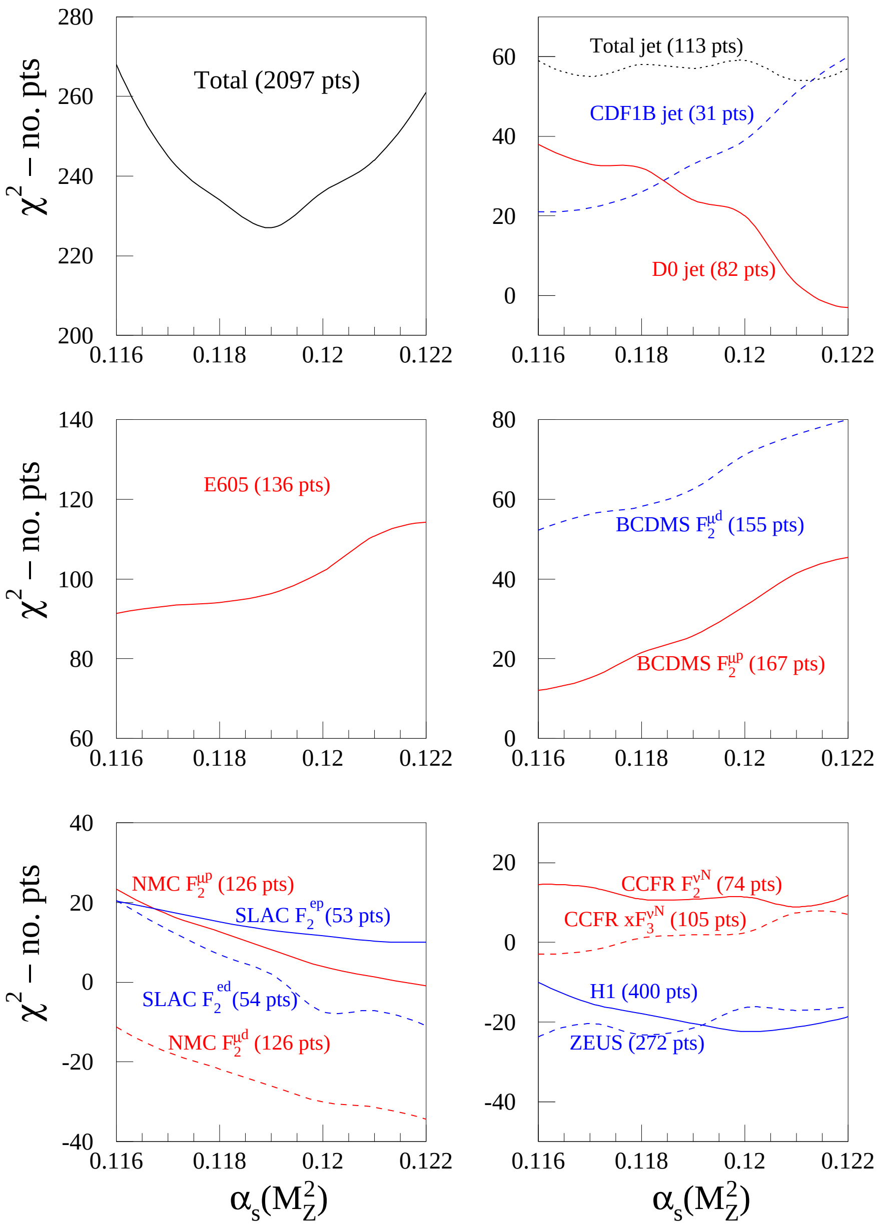
One could restrict the data sets to those which are sufficiently consistent that these problems do not arise – (H1, GKK, Alekhin)

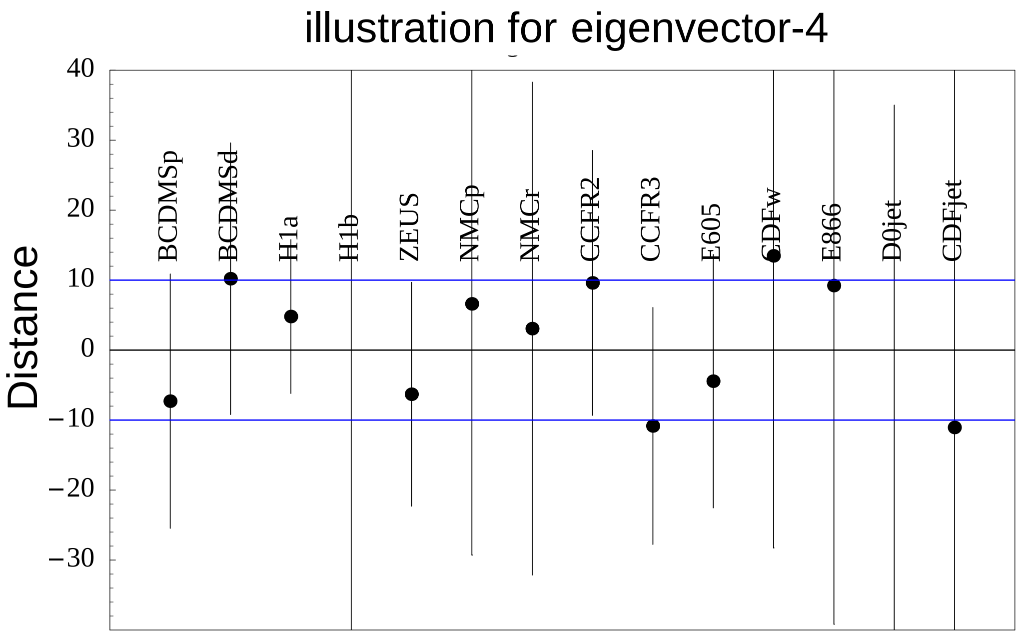
But one loses information since partons need constraints from many different data sets – no-one experiment has sufficient kinematic range / flavour info.

To illustrate: the $\chi 2$ for the MRST global fit is plotted versus the variation of a particular parameter (α_s).

The individual $\chi 2_e$ for each experiment is also plotted versus this parameter in the neighbourhood of the global minimum. Each experiment favours a different value of. α_s

PDF fitting is a compromise. Can one evaluate acceptable ranges of the parameter value with respect to the individual experiments?





CTEQ look at eigenvector combinations of their parameters rather than the parameters themselves. They determine the 90% C.L. bounds on the distance from the global minimum from $\int P(\chi_e^2, N_e)\, d\chi_e^2 = 0.9$ for each experiment

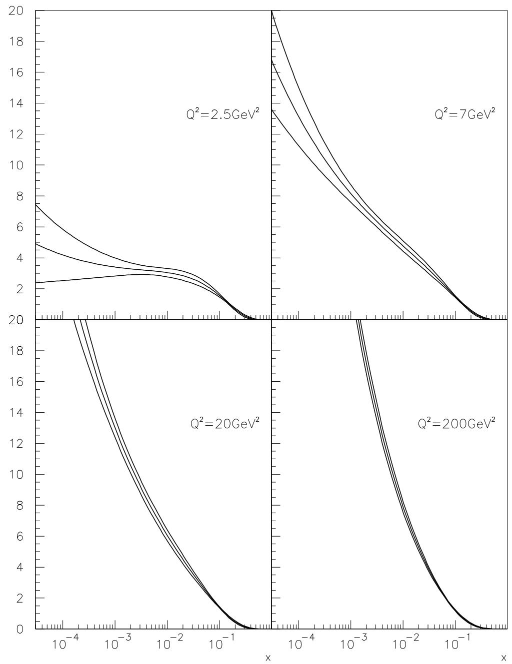
This leads them to suggest a modification of the χ2 tolerance, Δχ2 = 1, with which errors are evaluated such that $\Delta\chi 2 = T^2$, T = 10.

Why? Pragmatism. The size of the tolerance T is set by considering the distances from the χ2 minima of individual data sets from the global minimum for all the eigenvector combinations of the parameters of the fit.

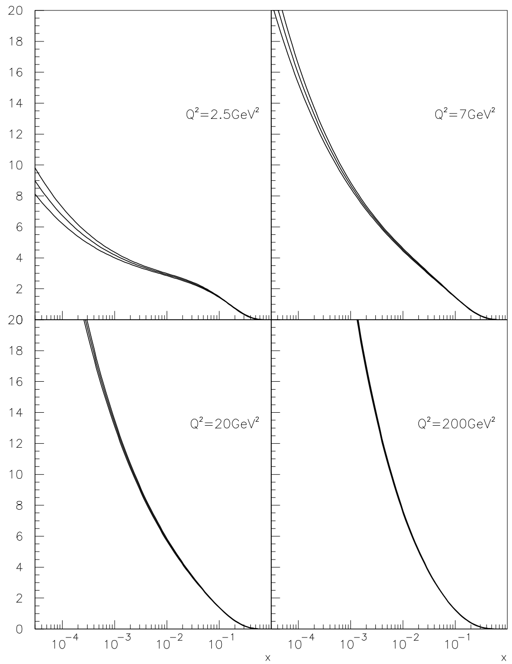
All of the world's data sets must be considered acceptable and compatible at some level, even if strict statistical criteria are not met, since the conditions for the application of strict statistical criteria, namely Gaussian error distributions are also not met.

One does not wish to lose constraints on the PDFs by dropping data sets, but the level of inconsistency between data sets must be reflected in the uncertainties on the PDFs.

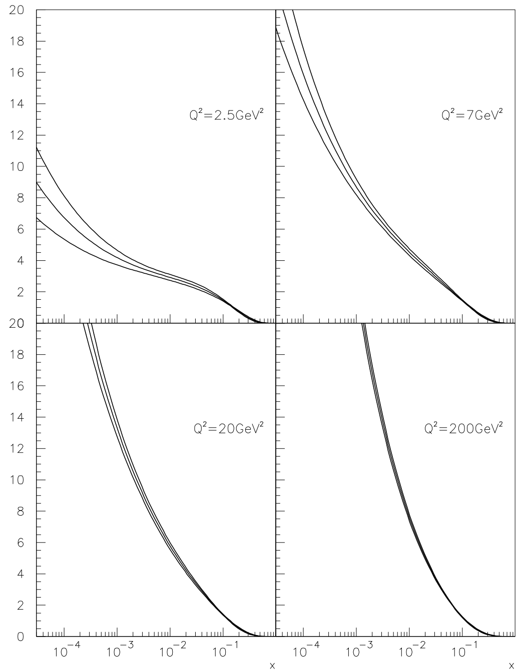
Compare gluon PDFs for Hessian and Offset methods for the ZEUS fit analysis



Offset method



Hessian method T=1



Hessian method T=7

The Hessian method gives comparable size of error band as the Offset method, when the tolerance is raised to T ~ 7 – (similar ball park to CTEQ, T=10)

Note this makes the error band large enough to encompass reasonable variations of model choice. (For the ZEUS global fit $\sqrt{2N}$=50, where N is the number of degrees of freedom)

Aside on model choices

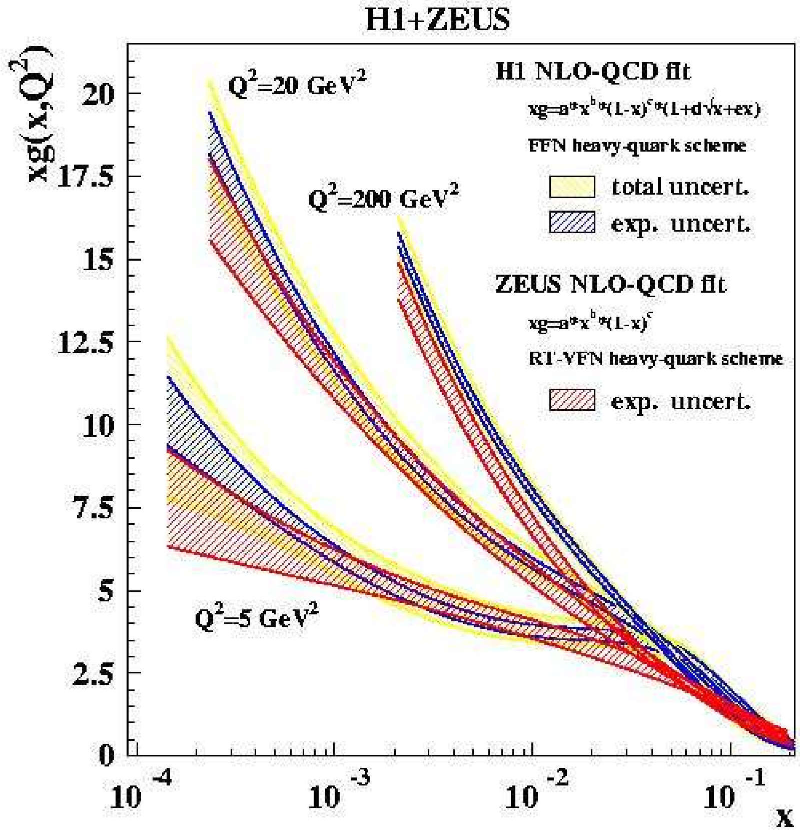
We trust NLO QCD– but are we sure about every choice which goes into setting up the boundary conditions for QCD evolution? – form of parametrization etc.

The statistical criterion for parameter error estimation within a particular hypothesis is $\Delta\chi 2 = T^2 = 1$. But for judging the acceptability of an hypothesis the criterion is that $\chi 2$ lie in the range $N \pm\sqrt{2N}$, where N is the number of degrees of freedom

There are many choices, such as the form of the parametrization at Q^2_0, the value of Q_0^2 itself, the flavour structure of the sea, etc., which might be considered as superficial changes of hypothesis, but the $\chi 2$ change for these different hypotheses often exceeds $\Delta\chi 2=1$, while remaining acceptably within the range $N \pm\sqrt{2N}$.

In this case the model error on the PDF parameters usually exceeds the experimental error on the PDF, if this has been evaluated using T=1, with the Hessian method.

If the experimental errors have been estimated by the Hessian method with T=1, then the model errors are usually larger. Use of restricted data sets also results in larger model errors. **Hence total error (model + experimental) can end up being in the same ball park as the Offset method** (or the Hessian method with T ~ 7-10).



Comparison of ZEUS (Offset) and H1(Hessian, T=1) gluon distributions –

Yellow band (total error) of H1 comparable to red band (total error) of ZEUS

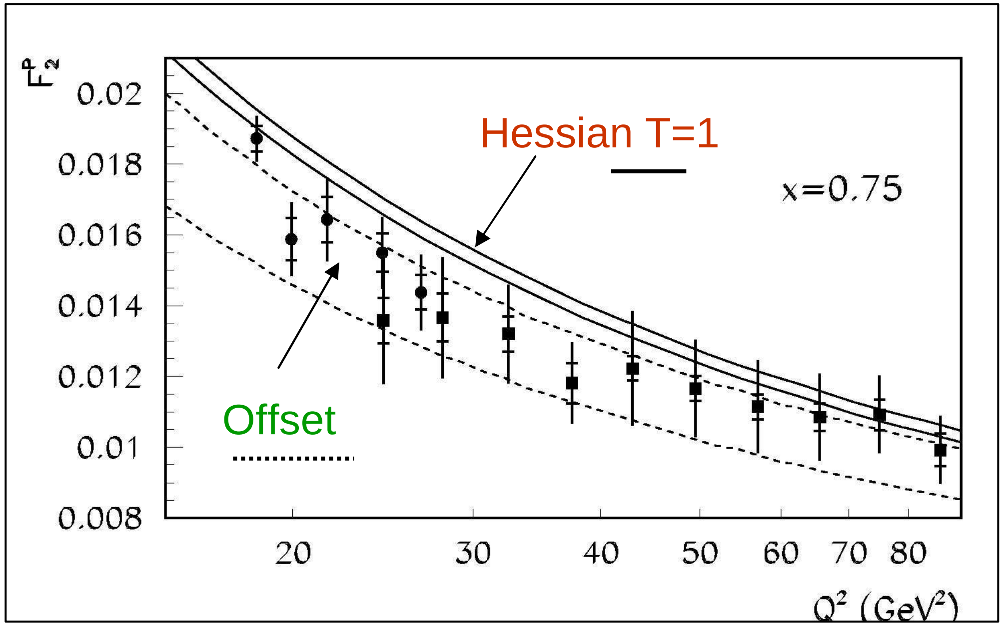
Swings and roundabouts

Last remarks on the Hessian versus the Offset method

As an experimentalist I am keenly aware that correlated systematic errors are rarely Gaussian distributed.

Further reasons to worry about the use of the Hessian method with T=1

1. Alekhin's plot hep-ph-0011002



Conclusion: an increased tolerance, $\boldsymbol{\Delta\chi^2 = T^2}$**, T = 10,** seems like a good idea!

2. It may be dangerous to let the QCD fit determine the optimal values for the systematic shift parameters.

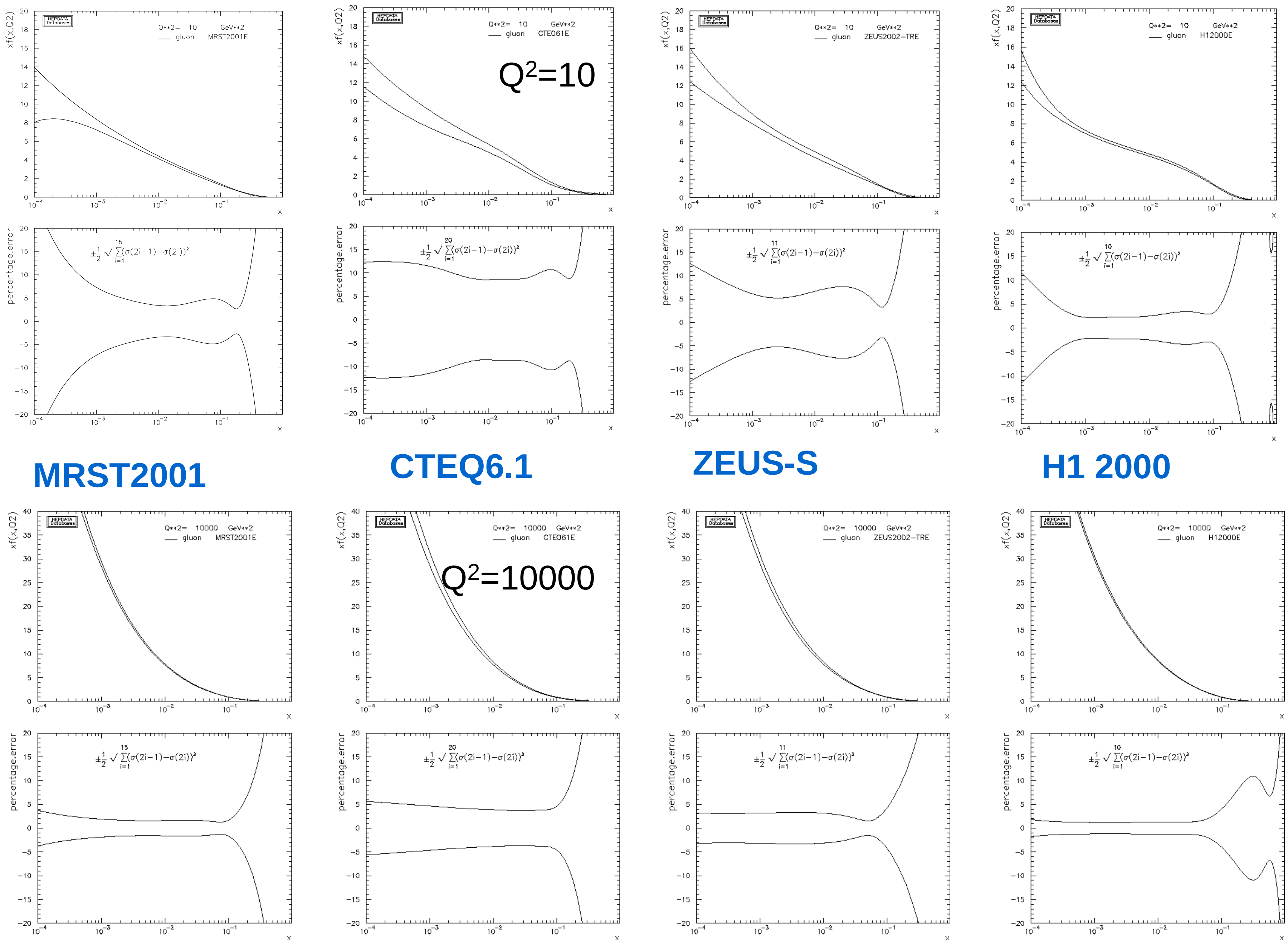
$s\lambda$ parameters are estimated as different for same data set when different combinations of data/models are used – different calibration of detector according to model

Comparison of $s\lambda$ values determined using a Hessian NLO QCD PDF fit to ZEUS and H1 data with $s\lambda$ values determined using a 'theory-free' Hessian fit to combine the data.

Using $\Delta\chi 2=1$ on the QCD fit to the separate data sets gives beautiful small PDF uncertainties but a central value which is far from that of a QCD fit to the theory free data combination.. So what are the real uncertainties? – Conclusion: an increased tolerance **$\Delta\chi 2 = T^2$, T ~ 10,** is a good idea!

Zeus $s\lambda$	HERA QCD fit	HERA no theory fit
1	1.65	0.31
2	-0.56	0.38
3	-1.26	-0.11
4	-1.04	0.97
5	-0.40	0.33
6	-0.85	0.39
7	1.05	-0.58
8	-0.28	0.83
9	-0.23	-0.42
10	0.27	-0.26

Diifferent uncertainty estimates on the gluon persist as Q^2 increases



The general trend of PDF uncertainties is that

The u quark is much better known than the d quark

The valence quarks are much better known than the gluon at high-x

The valence quarks are poorly known at small-x but they are not important for physics in this region

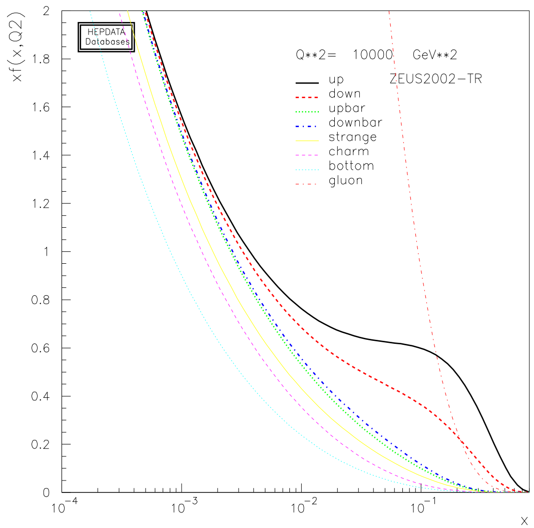
The sea and the gluon are well known at low-x

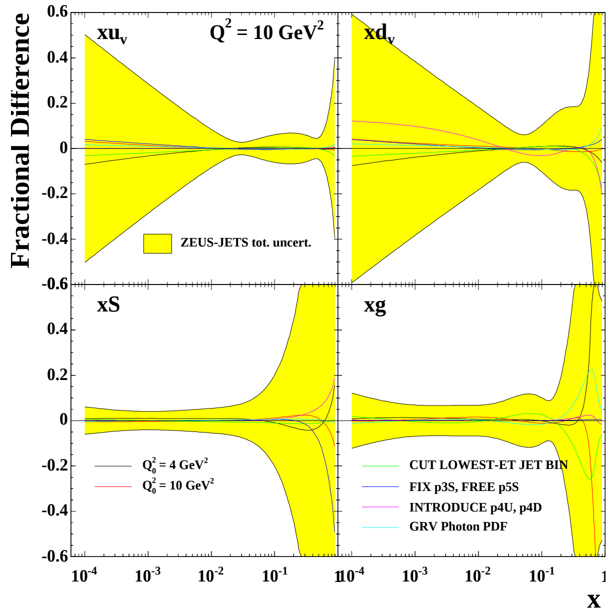
The sea is poorly known at high-x, but the valence quarks are more important in this region

The gluon is poorly known at high-x

And it can still be very important for physics e.g.– high ET jet xsecn

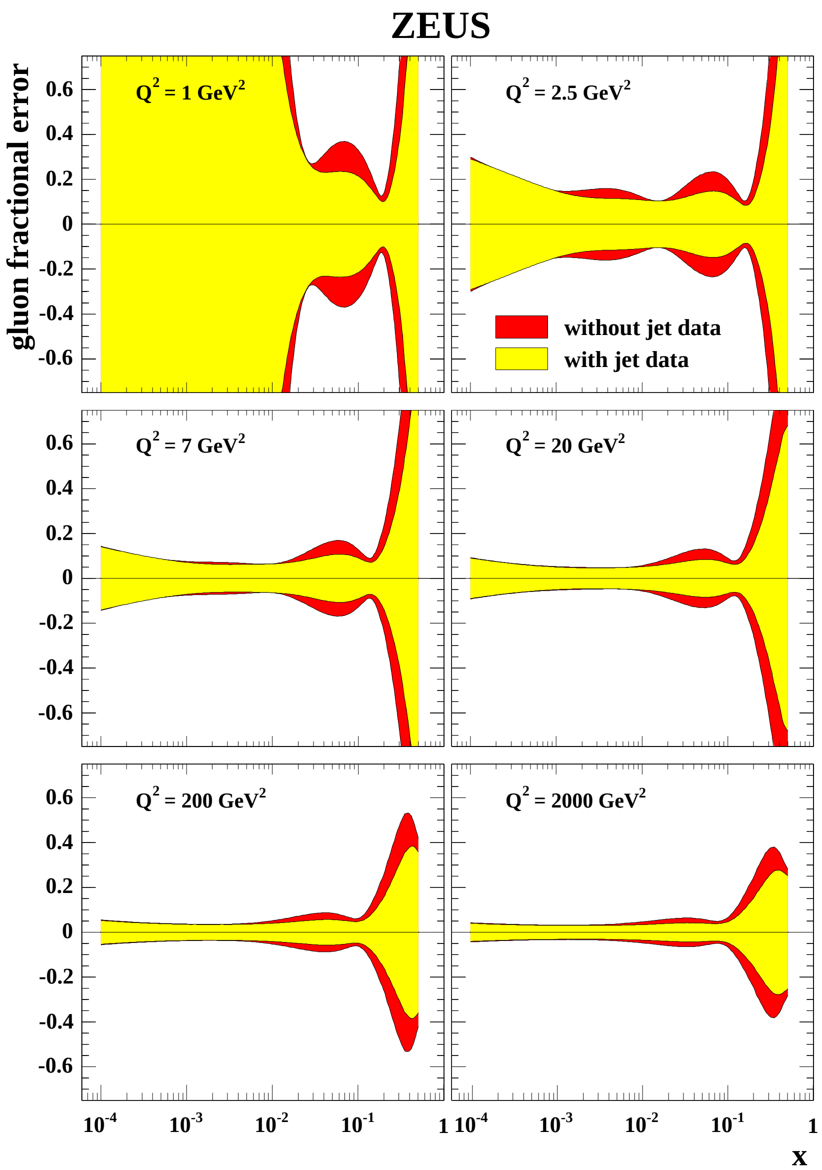
need to tie down the high-x gluon

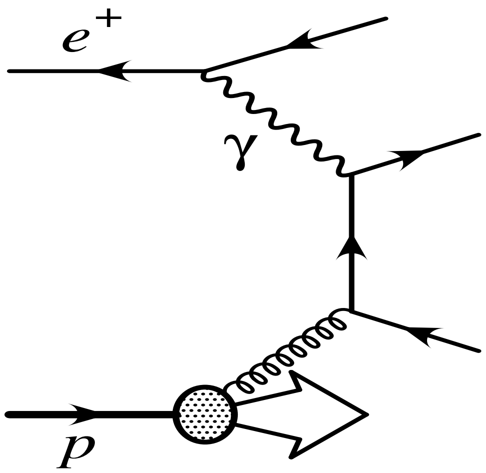




Good news: PDF uncertainties will decrease before LHC comes on line

HERA-II and Tevatron Run-II will improve our knowledge





Example- decrease in gluon PDF uncertainty from using ZEUS jet data in ZEUS PDF fit.

Direct* Measurement of the Gluon Distribution

ZEUS jet data much more accurate than Tevatron jet data- small energy scale uncertainties

Inputting data to a PDF fit needs a prediction for the cross-section which can be easily obtained analytically –true for DIS inclusive cross-section. But many NLO cross-sections can only be computed by MC and can take 1-2 CPU days to compute. This cannot be done for every iteration of a PDF fit.

Recently grid techniques have been developed to include DIS jet cross-sections in PDF fits (ZEUS-JETs fit)

Separating PDFs From The Integral

- A NLO Cross-Section for DIS is normally calculated using MC by:

$$W = \sum_{m=1}^{N} w_m \left(\frac{\alpha_s(Q_m^2)}{2\pi} \right)^{p_m} q(x_m, Q_m^2)$$

For events m=1....N, (w_m is an MC weight, $q(x,Q^2)$ a PDF).

- Can instead define a weight grid in (x,Q^2), which is updated for each event m:

$$W_{i,j}^{(p)} = W_{i,j}^{(p)} + w_m$$

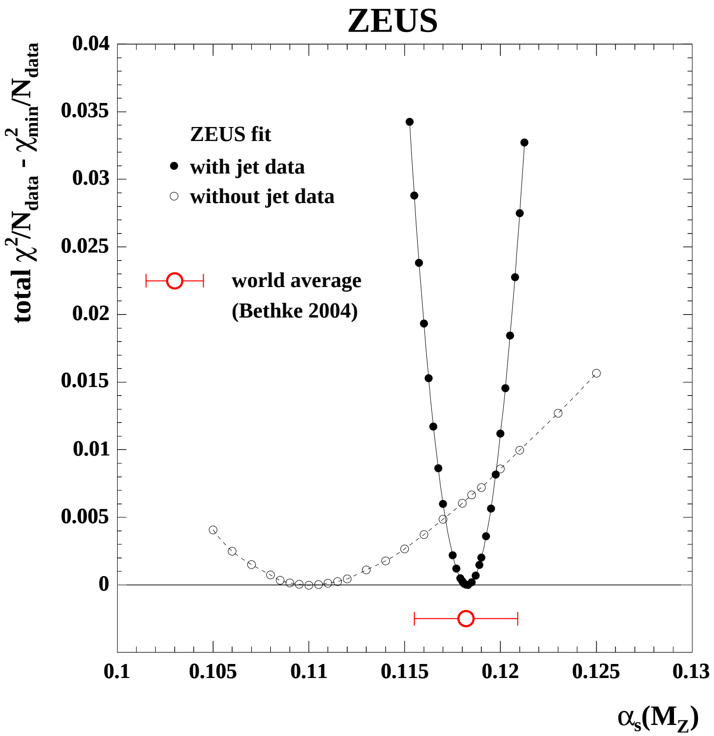
Where i, j define a discrete point in x,Q^2 space relating to the event.

- A PDF grid is also defined in x,Q^2 as $q_{i,j}$.

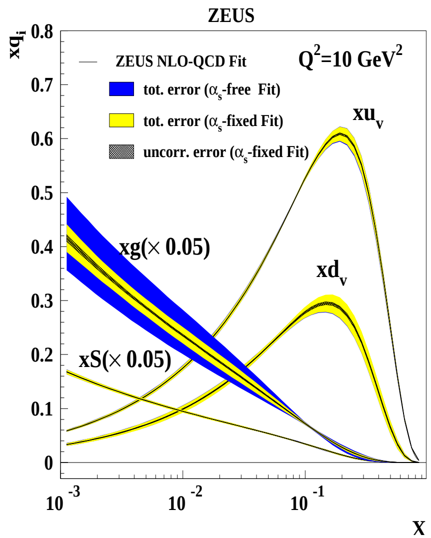
- Cross-Section can be reproduced by combining the PDF and weight grids *after* the Monte-Carlo run:

$$W = \sum_i \sum_j W_{i,j}^{(p)} \left(\frac{\alpha_s(Q^2)}{2\pi} \right)^{p} q_{i,j}$$

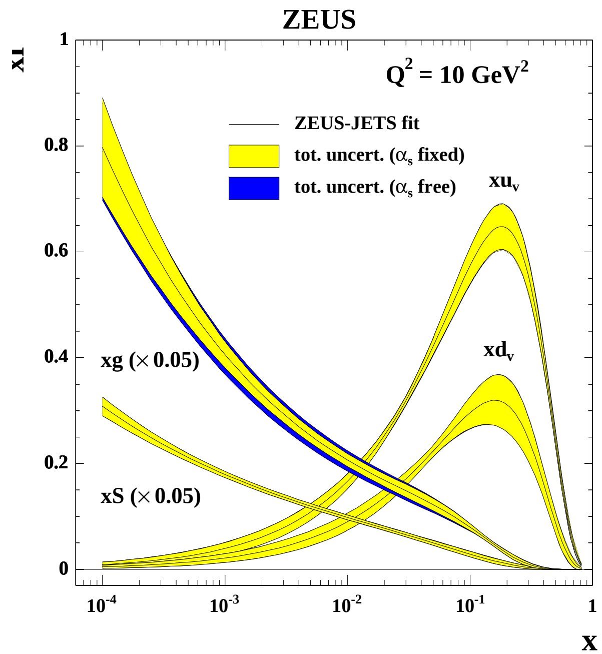
4



Before jets



After jets



Also gives a nice measurement of

$\alpha_s(M_Z)$ = 0.1183 ± 0.0028 (exp) ± 0.0008 (model)

From simultaneous fit of $\alpha_s(M_Z)$ & PDF parameters

And correspondingly the contribution of the uncertainty on $\alpha_s(M_Z)$ to the uncertainty on the PDFs is much reduced

•HERA now in second stage of operation (HERA-II)

substantial increase in luminosity

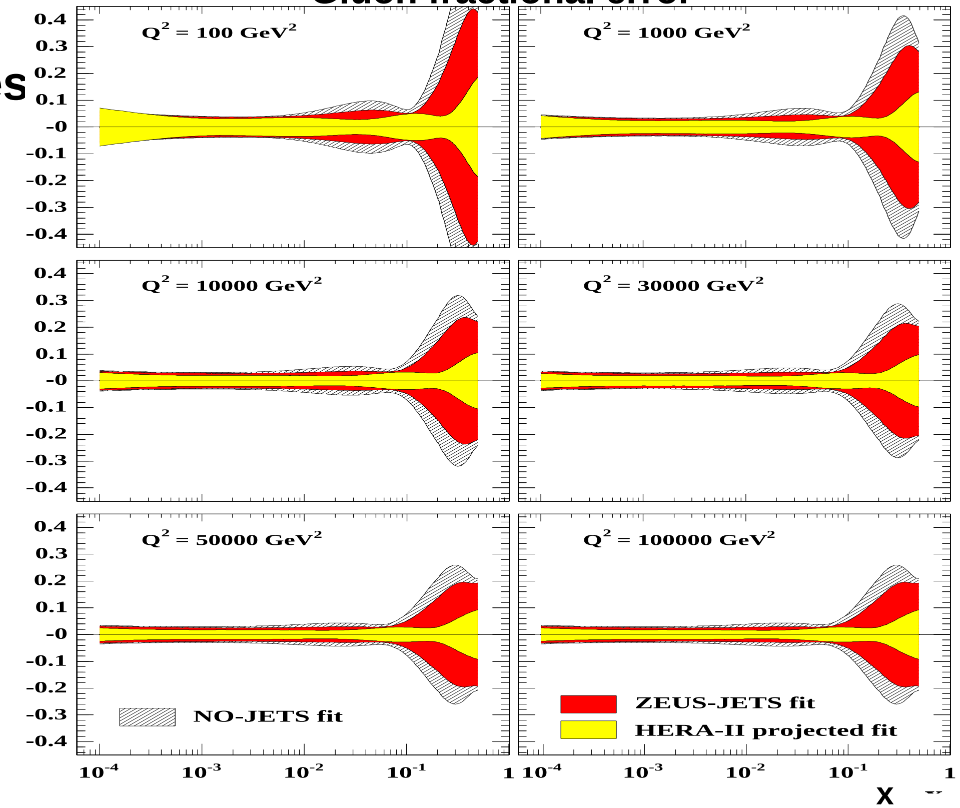
possibilities for new measurements

HERA-II projection shows significant improvement to high-x PDF uncertainties

⇒ relevant for high-scale physics at the LHC

→ where we expect new physics !!

Gluon fractional error



Why are PDF's important for the LHC?

At the LHC high precision (SM and BSM) cross section predictions require precision Parton Distribution Functions (PDFs)

PDF uncertainties affect discovery physics

- Higgs cross-sections
- high ET jets..contact interactions/extra dimensions

Precision PDFs are also needed for 'standard candle' SM processes which are insensitive to

- calibrate experiment
- measure machine luminosity?

HERA and the LHC- transporting PDFs to hadron-hadron cross-sections

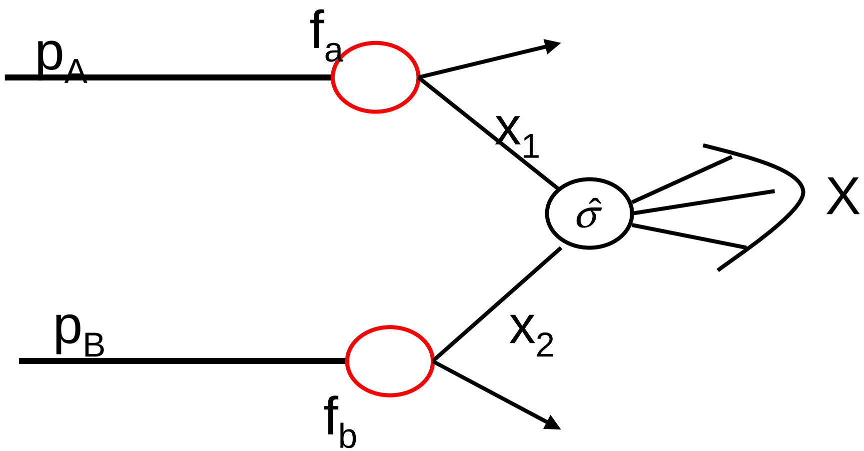
QCD factorization theorem for short-distance inclusive processes

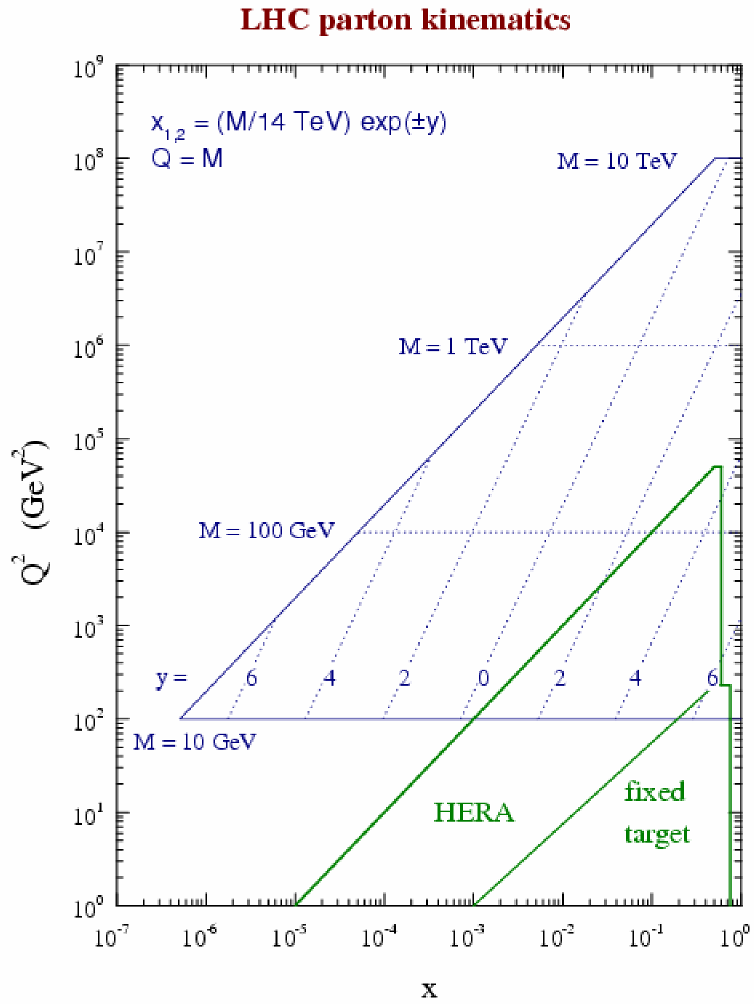
$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

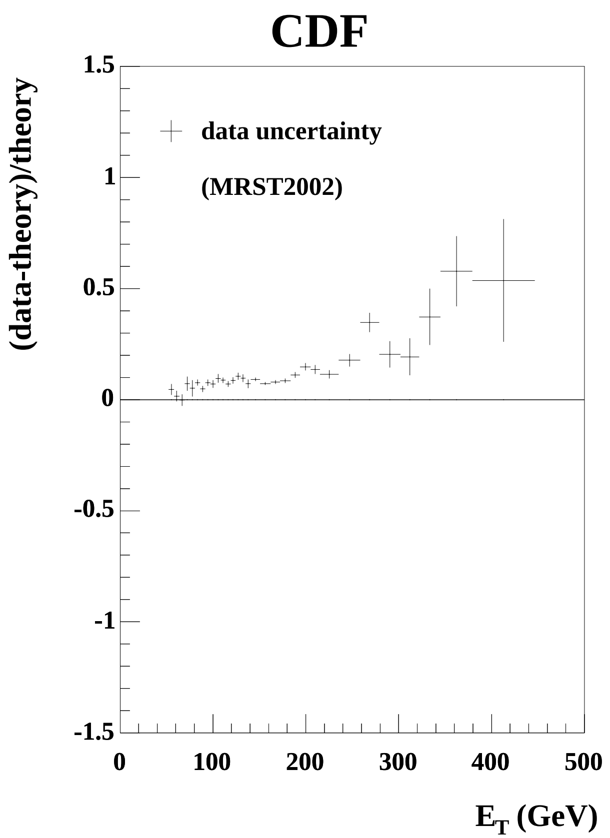
where X=W, Z, D-Y, H, high-E_T jets, prompt-γ

and $\hat{\sigma}$ is known

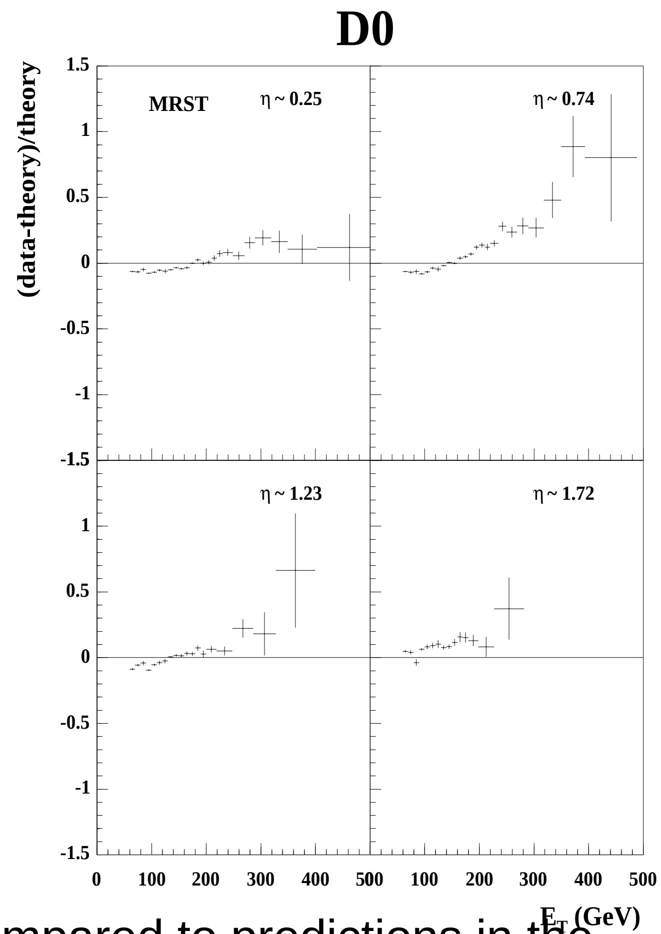
- to some fixed order in pQCD and EW
- in some leading logarithm approximation (LL, NLL, …) to all orders via resummation







Example of how PDF uncertainties matter for BSM physics– Tevatron jet data were originally taken as evidence for new physics--

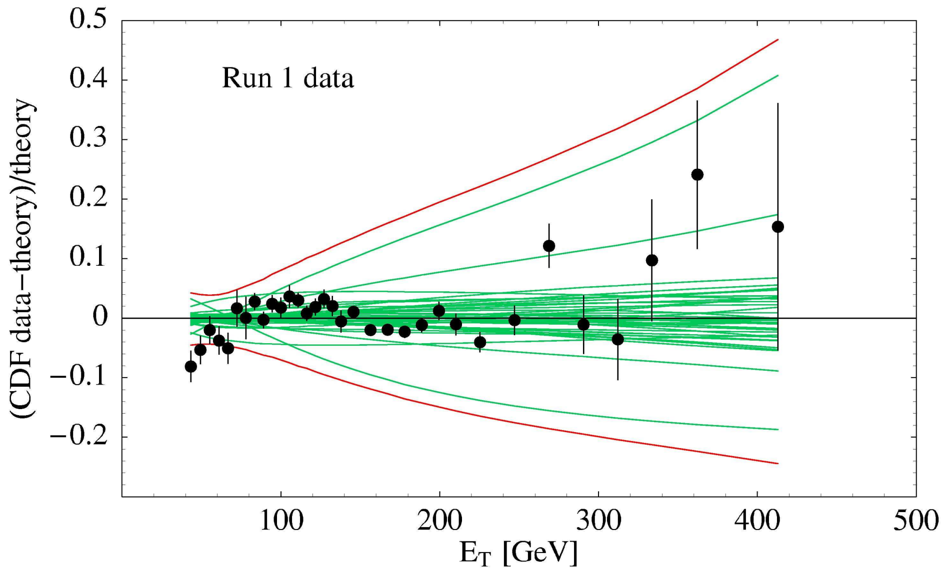


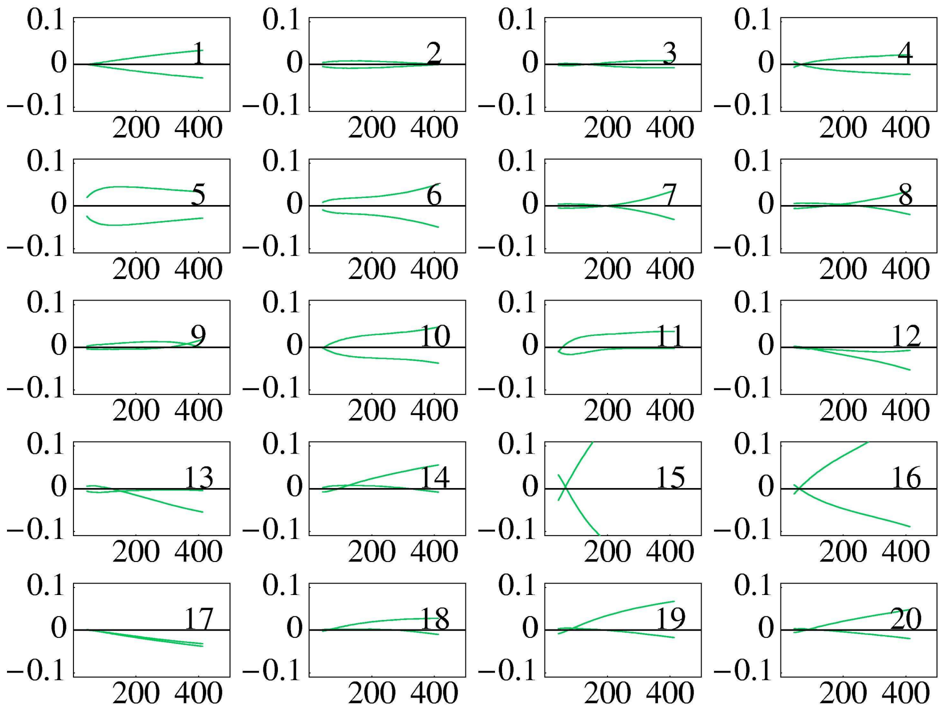
These figures show inclusive jet cross-sections compared to predictions in the form (data - theory)/ theory

Something seemed to be going on at the highest E_T

And special PDFs like CTEQ4/5HJ were tuned to describe it better- note the quality of the fits to the rest of the data deteriorated.

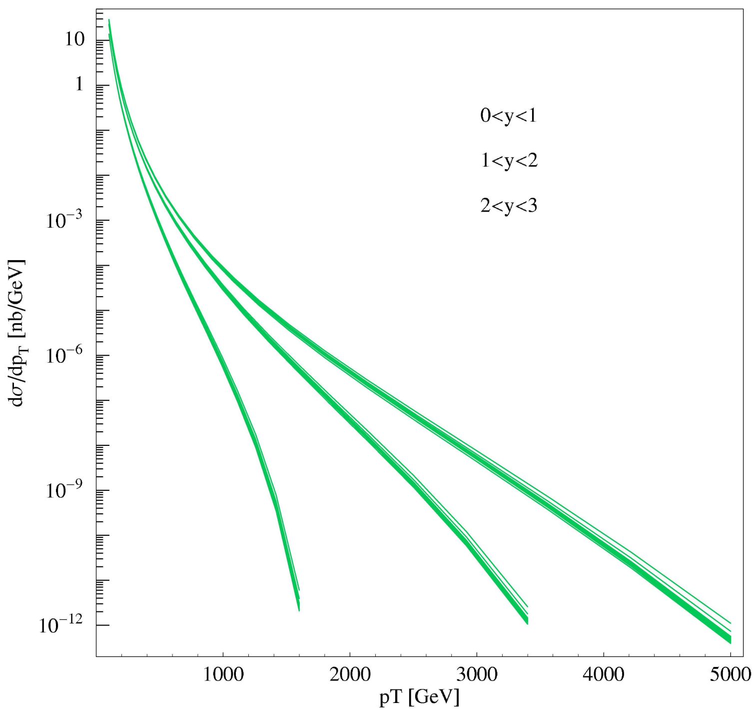
But this was before uncertainties on the PDFs were seriously considered

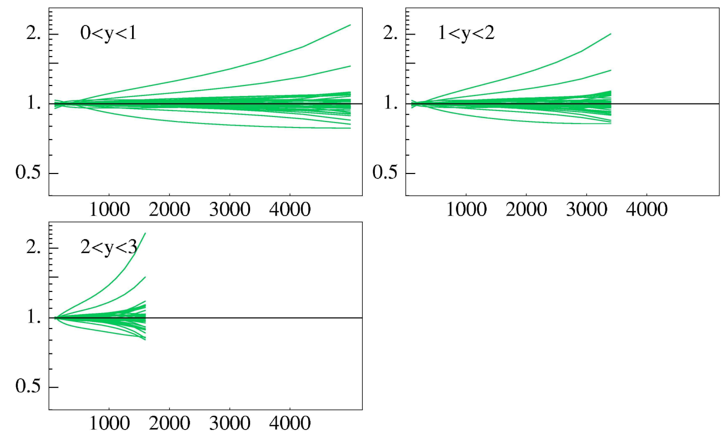




Today Tevatron jet data are considered to lie within PDF uncertainties. (Example from CTEQ hep-ph/0303013)

We can decompose the uncertainties into eigenvector combinations of the fit parameters-the largest uncertainty is along eigenvector 15 –which is dominated by the high x gluon uncertainty





And we can translate the current level of PDF uncertainty into the uncertainty on LHC jet cross-sections. This has consequences for any new BSM physics which can be described by a contact interaction-consider the case of extra dimensions

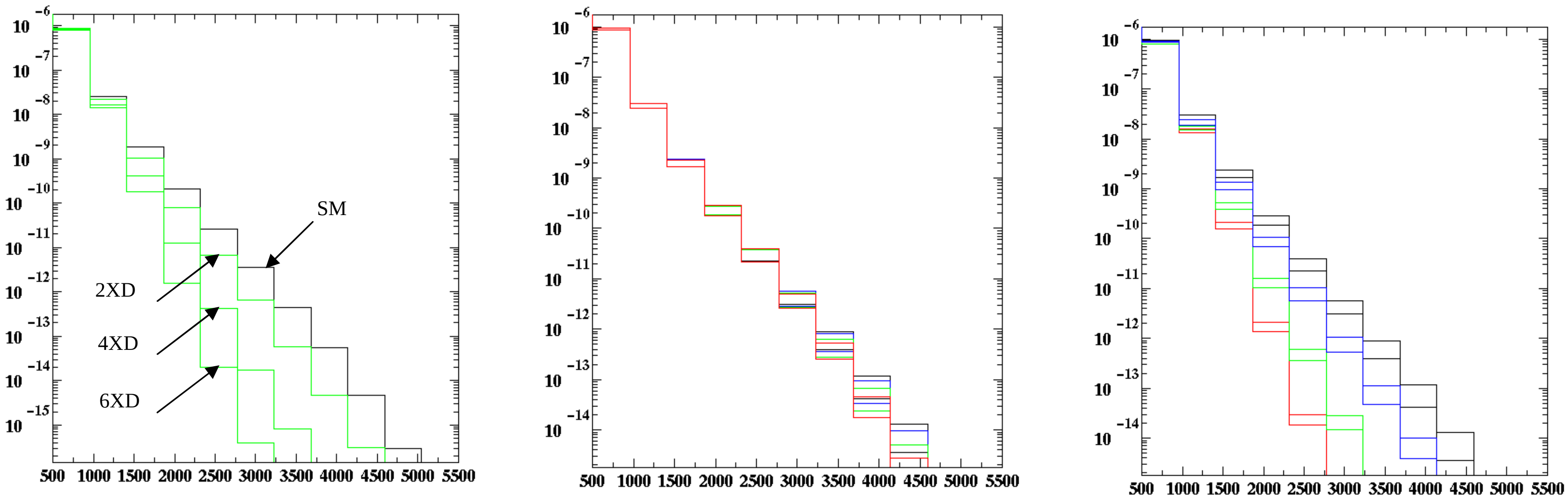
Such PDF uncertainties on the jet cross sections compromise the potential for discovery.

E.G. Dijet cross section potential sensitivity to compactification scale of extra dimensions (M_c) reduced from ~6 TeV to 2 TeV.

M_c = 2 TeV, no PDF error

M_c = 6 TeV, no PDF error

M_c = 2 TeV, with PDF error



Is there anything we could do about it?- we could use early ATLAS data to improve the gluon PDFs - **Use data at lower PT and higher η-where new physics is not expected**

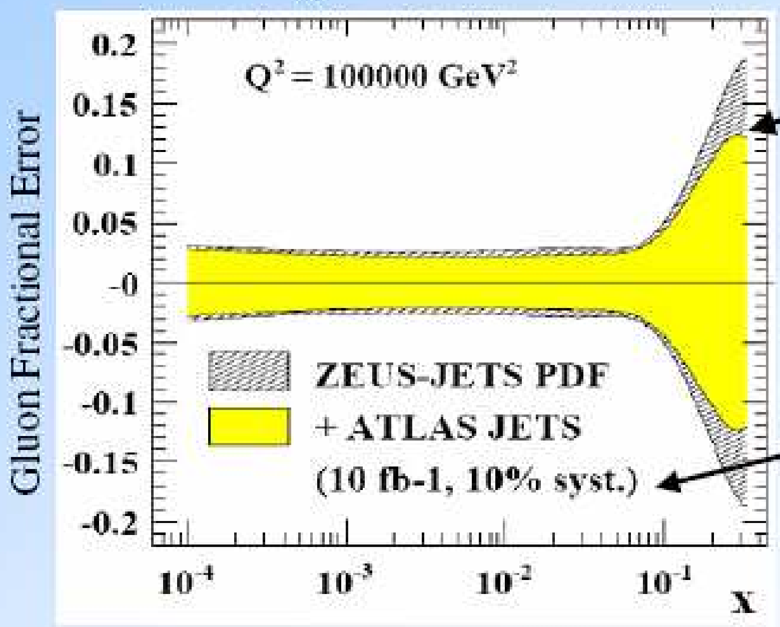
The grid technique has been extended to LHC high-ET jet cross-sections

PDF Fitting Using Pseudodata

•Grids were generated for the inclusive jet cross-section at ATLAS in the pseudorapidity ranges $0<\eta<1$, $1<\eta<2$, and $2<\eta<3$ up to pT=3TeV (NLOJET).

•In addition pseudodata for the same process was generated using JETRAD [4].

•The pseudo-data was then used in a global fit to assess the impact of ATLAS data on constraining PDFs:

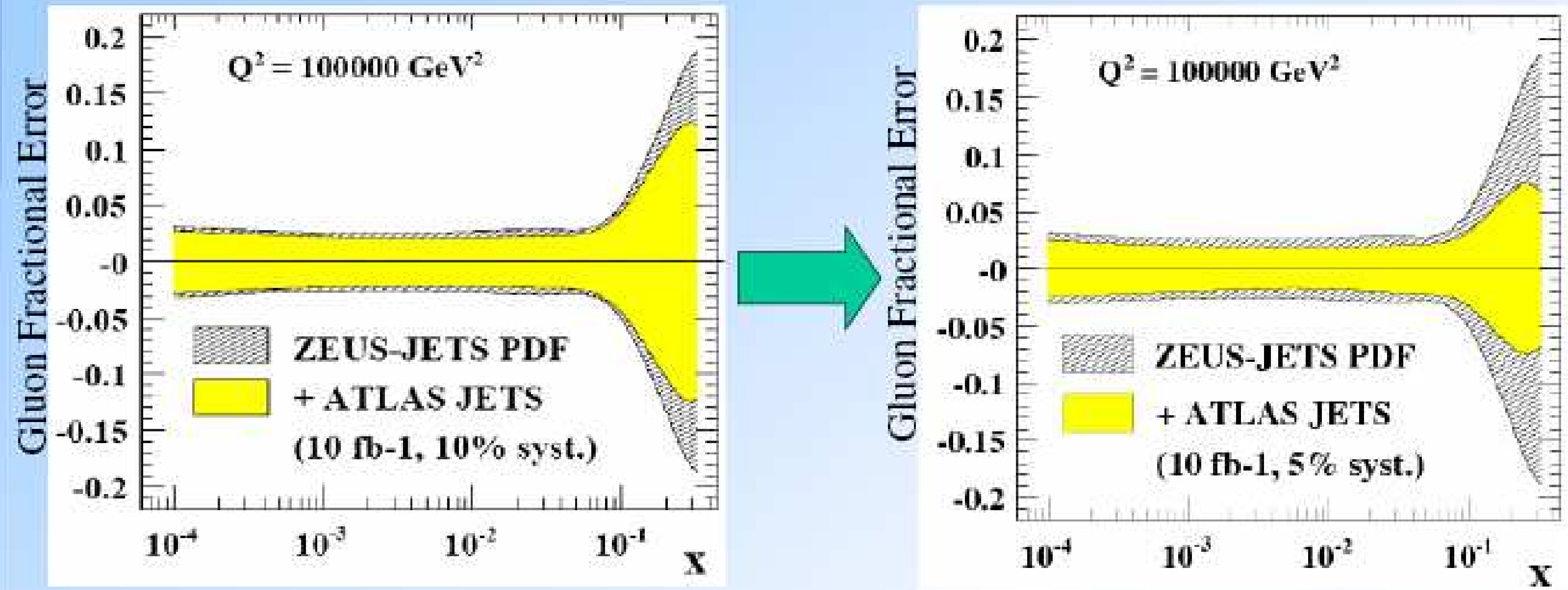


Preliminary indications suggest that ATLAS data can constrain the high x-gluon.

Systematic errors are uncorrelated, 10fb^{-1}=1 year of nominal data-taking at $10^{33}\text{cm}^{-2}\text{s}^{-1}$

Effect Of Decreased Systematic Errors On PDF Fits

Decrease (uncorrelated) Systematic errors
10%→5%

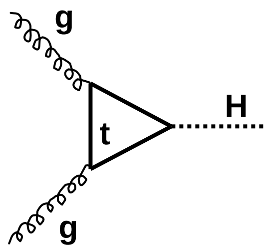


•Decreasing the systematic errors (on the ATLAS experiment) creates a significant improvement in constraining the PDFS.

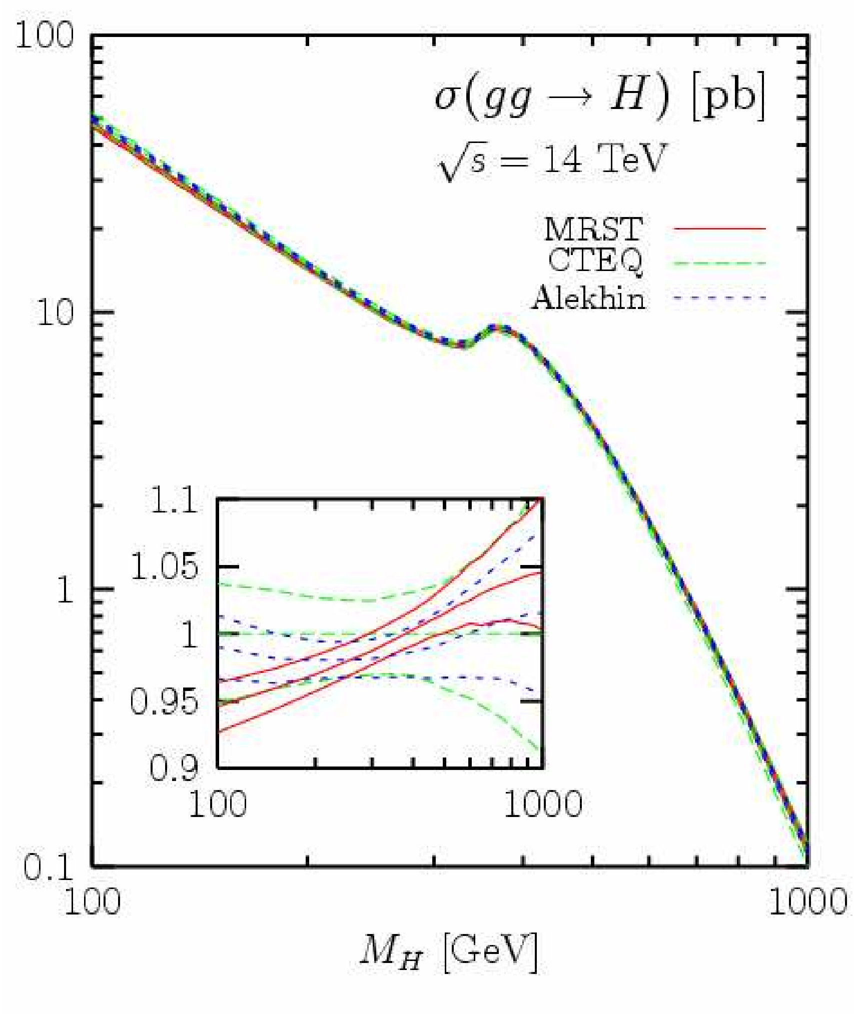
The reduced gluon uncertainties can then be used in background calculations for new physics signals

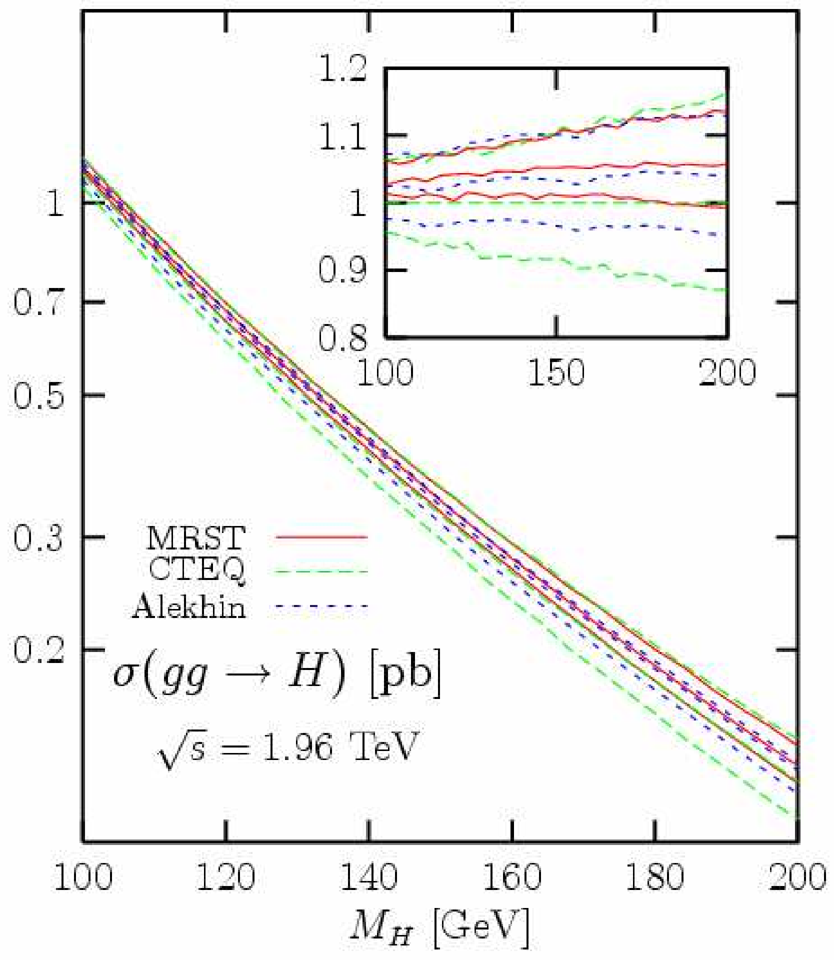
And how do PDF uncertainties affect the Higgs discovery potential?

Higgs at LHC

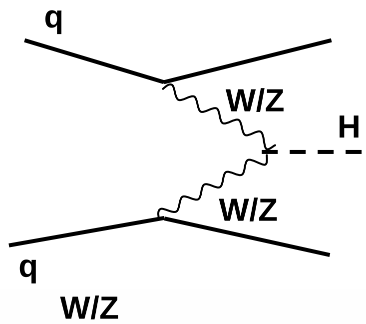


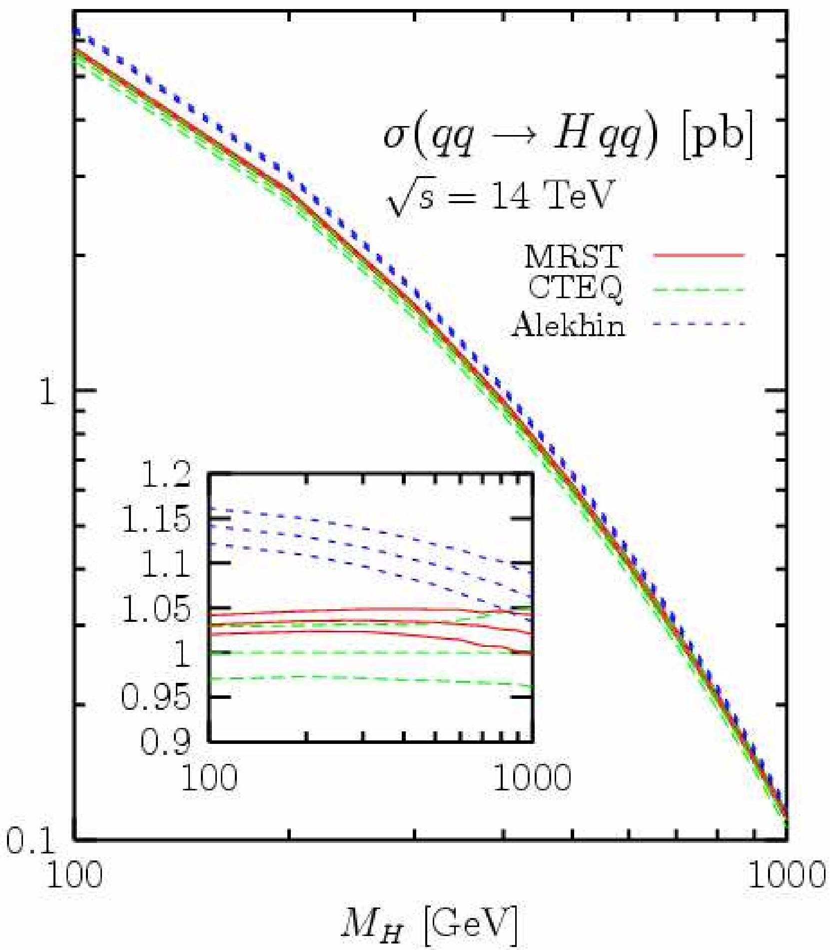
Higgs at Tevatron

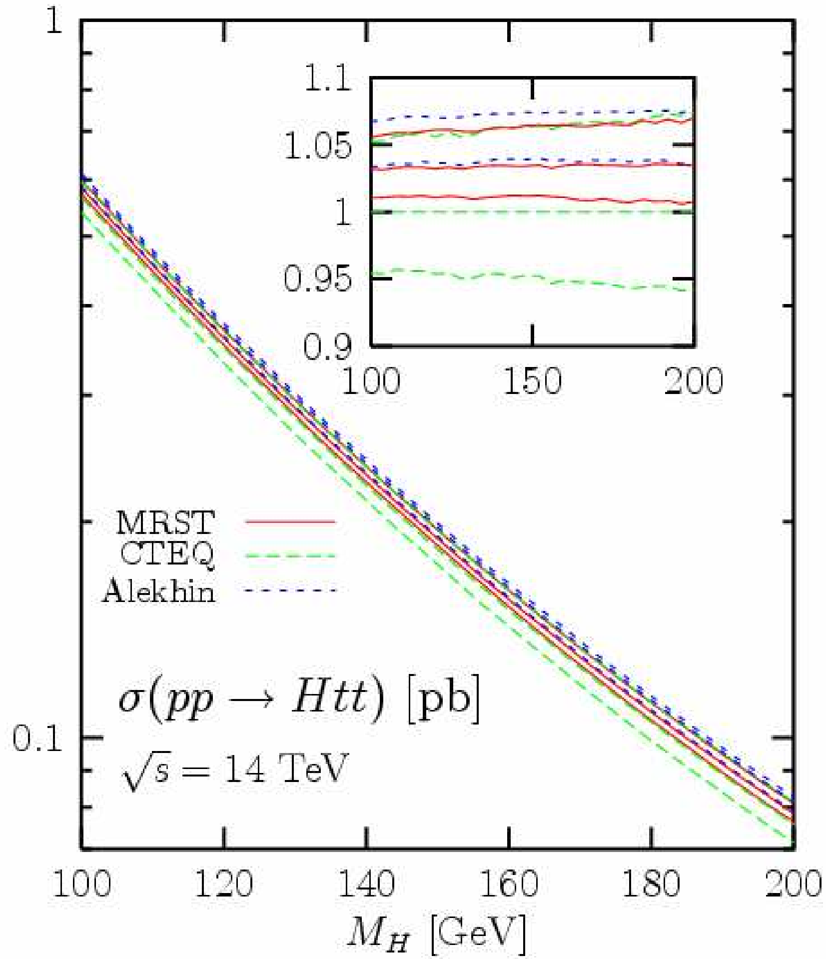




Higgs from qq at LHC

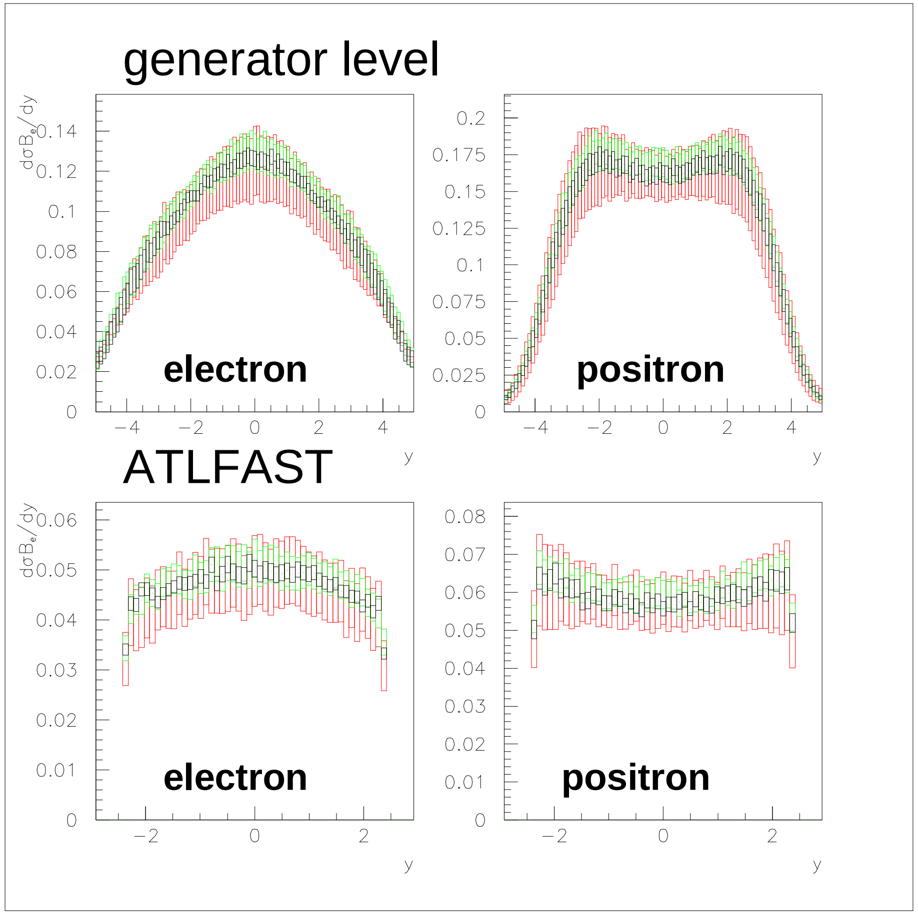






How about PDF uncertainties on SM processes?

W/Z production have been considered as good standard candle processes insensitive to PDF uncertainties……? This is true WITHIN a PDFset



But how about comparing PDFsets?

We actually measure the decay lepton spectra

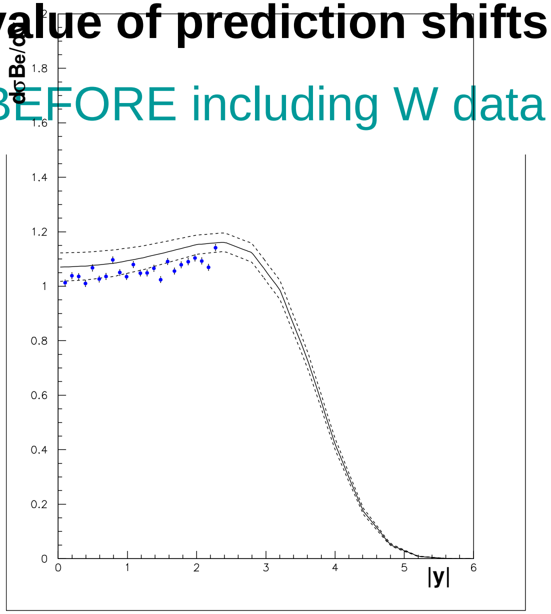
Generate with HERWIG+k-factors (checked against MC@NLO) using
CTEQ6.1M ZEUS_S MRST2001
PDFs with full uncertainties from LHAPDF eigenvectors
At y=0 the total uncertainty is
~ ±6% from ZEUS
~ ±4% from MRST01E
~ ±8% from CTEQ6.1
ZEUS to MRST01 central value difference ~5%
To improve the situation we NEED to be more accurate than this:**~4%**

Study of the effect of including the LHC W Rapidity distributions in global PDF fits

by how much can we reduce the PDF errors with early LHC data?

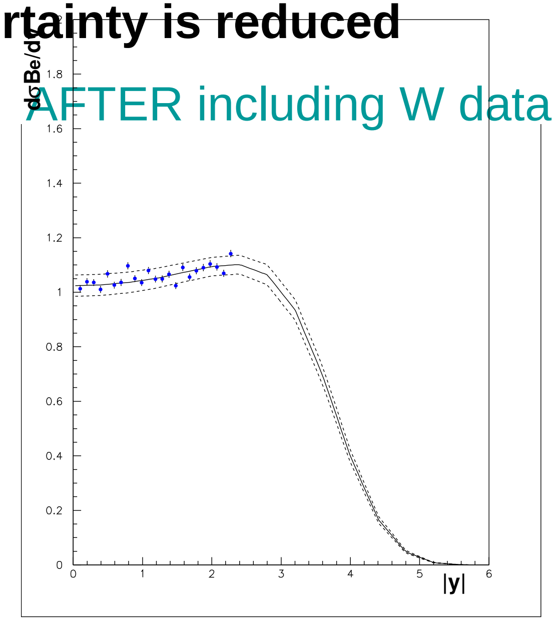
Generate data with 4% error using CTEQ6.1 PDF, pass through ATLFAST detector simulation and then include this pseudo-data in the global ZEUS PDF fit **Central value of prediction shifts and uncertainty is reduced**

BEFORE including W data



Lepton+ rapidity spectrum data generated with CTEQ6.1 PDF compared to predictions from ZEUS PDF

AFTER including W data



Lepton+ rapidity spectrum data generated with CTEQ6.1 PDF compared to predictions from ZEUS PDF AFTER these data are included in the fit

Specifically the low-x gluon shape parameter λ, $xg(x) = x^{-\lambda}$, was

λ = -.199 ± .046 for the ZEUS PDF before including this pseudo-data

It becomes λ = -.181 ± .030 after including the pseudodata

The **uncertainty on the W/Z rapidity distributions** is dominated by –- **gluon PDF** dominated eigenvectors and there is cancellation in the ratios

$A_W = (W^+ - W^-)/(W^+ + W^-)$ $Z_W = Z/(W^+ + W^-)$

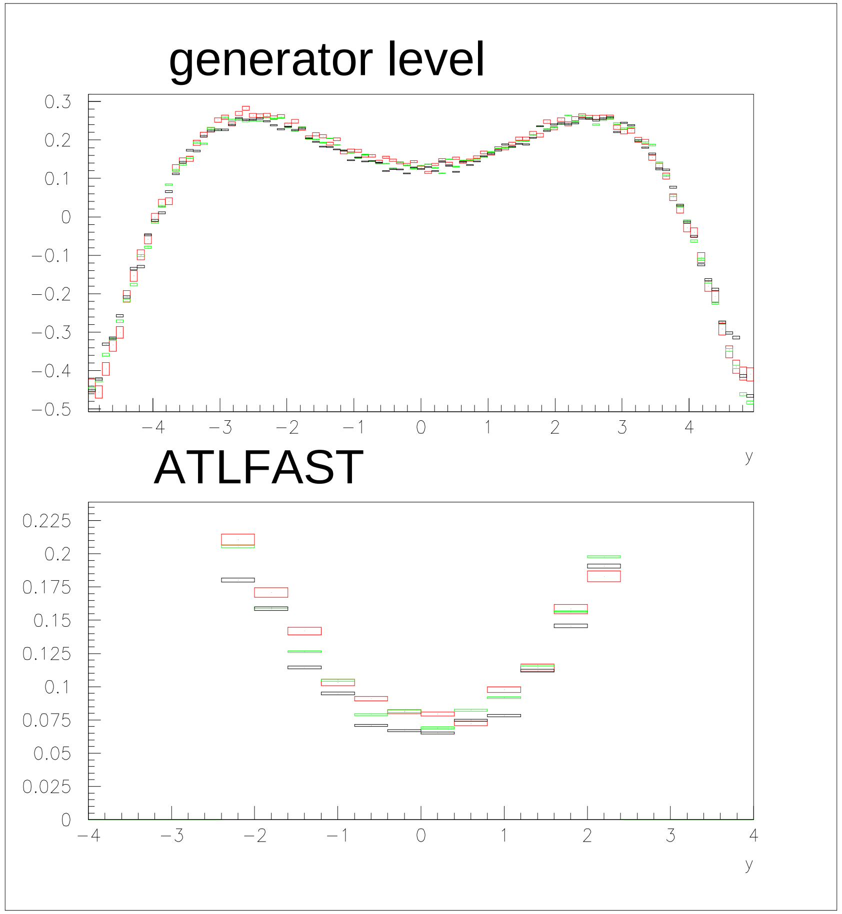
Remaining uncertainty comes from **valence PDF** related eigenvectors **Well Known? Gold plated?**

We will measure the lepton asymmetry

Within each PDF set uncertainty in the lepton asymmetry IS LESS than in the lepton rapidity spectra, e.g about 2% for the asymmetry at y=0, as opposed to about 4% for the lepton rapidity spectra themselves (using MRST2001 PDFS)

However the PDF sets differ from each other more strikingly- MRST01and CTEQ6.1 differ by about 13% at y=0!

But this is an opportunity to use ATLAS measurements to increase knowledge of the valence PDFs at x~0.005



How could you calculate the PDF uncertainty yourself for a cross-section of interest?

-use the eigenvector PDF sets from LHAPDFv5.
http://hepforge.cedar.ac.uk/lhapdf

A PDF fit results in a set of parameters which fix the form of the PDF parametrisations at the starting scale for evolution $Q^2=Q^2_0 \sim$ few GeV2 and an error matrix V_{jk} describing the correlations between these parameters- deriving from the experimental statistical and systematic errors.

The errors on the parameters can be propagated to the PDFs which are functions of these parameters (albeit very complex functions calculated through QCD evolution for any $Q^2 > Q^2_0$) and obviously they can also be propagated to any function of the PDFs- such as your cross-section

$$\langle \Delta F^2 \rangle = \Sigma_j \Sigma_k \frac{\partial F}{\partial p_j} V_{jk} \frac{\partial F}{\partial p_k}$$

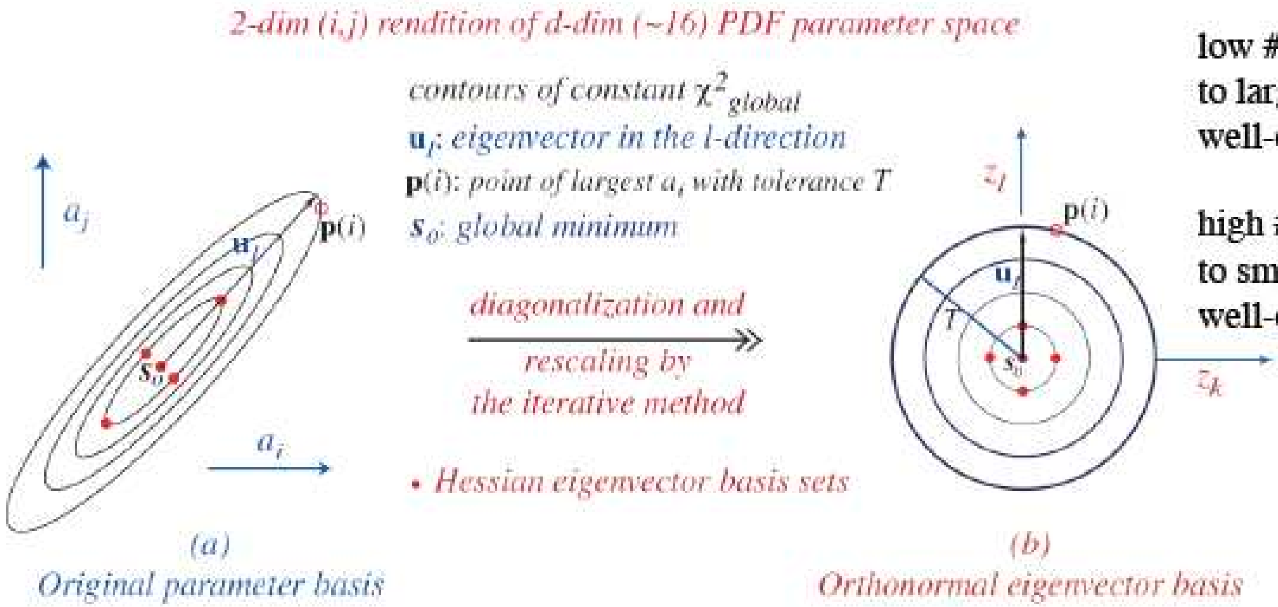
---- this would be easier if V_{jk} were diagonal

So diagonalise it and determine the eigenvalues and eigenvectors
Clearly the eigenvectors are linear combinations of the original parameters and the eigenvalues are the squared errors on these combinations. Some eigenvectors are dominated by one parameter –e.g. CTEQ eigenvector 15 by a high-x gluon parameter

- The results of the fit are then summarised in one central PDF set and 2 * N_{pdf} parameter sets for the errors, where N_{pdf} is the number of PDF parameters

These parameter sets are obtained by moving up(+) or down(-) along the i=1,Npdf eigenvector directions by the corresponding error. These moves are propagated back to the original PDF parameters to create new PDF sets- **(Si+) (Si-).** The error on a derived quantity is then obtained from

$$\Delta F^2 = \tfrac{1}{2}\Sigma_I \left(F(Si+) - F(Si-) \right)$$



low # eigenvectors correspond to large eigenvalues, well-determined directions

high # eigenvectors correspond to small eigenvalues, less well-determined directions

But what if evaluating your cross-section involved running a Monte-Carlo?- not many calculations can be done purely analytically

The **full PDF uncertainty** for **CTEQ61** involves 40 sub-sets (20 eigenvectors)
40 event samples would have to be generated to evaluate the PDF uncertainties

TOO LONG

The PDF re-weighting technique, is a useful tool to quickly evaluate the full PDF uncertainties for many PDF sets, saving generation time.

Generate an MC event with one specific PDF set, say PDF set n.1
and one hard process scale (e.g Q=MW)
And two primary partons with flavours(flav1,flav2) and momentum fractions x1, x2
(calculated at the Hard Process, before the PS in the backward evolution is applied in the MC) according to the probabilities (i.e. *xf*) appropriate for PDF set n.1.

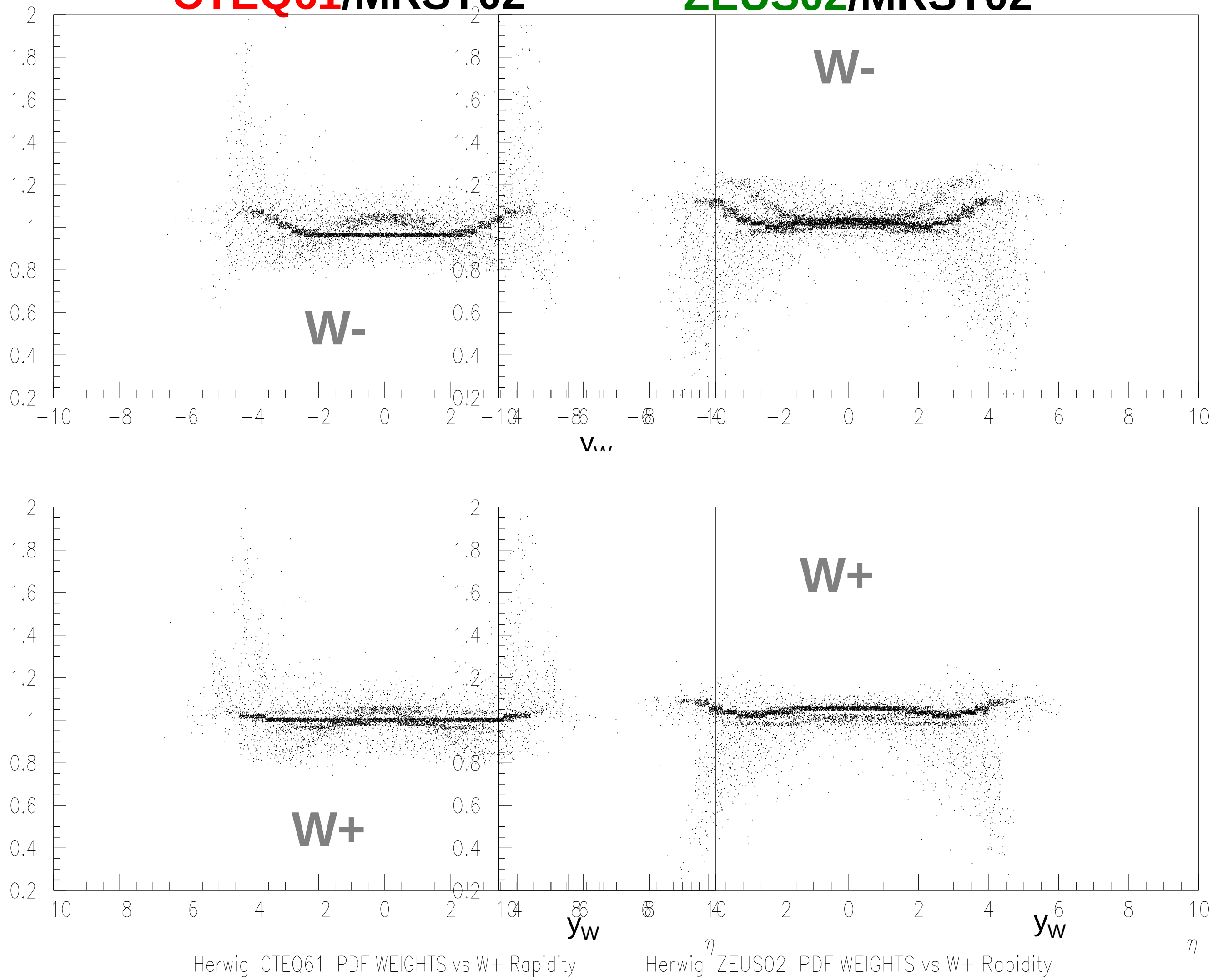
Evaluate the probability, i.e. *xf*, of picking up the same flavoured partons with the same momentum fractions x1,x2, according to the probabilities appropriate for PDF set n.2, at the same energy scale, i.e.Q.
Then take the Ratio:

$$EventWeight = \frac{f_{PDFn.2}(x_1, flav_1, Qscale)}{f_{PDFn.1}(x_1, flav_1, Qscale)} \bullet \frac{f_{PDFn.2}(x_2, flav_2, Qscale)}{f_{PDFn.1}(x_2, flav_2, Qscale)}$$

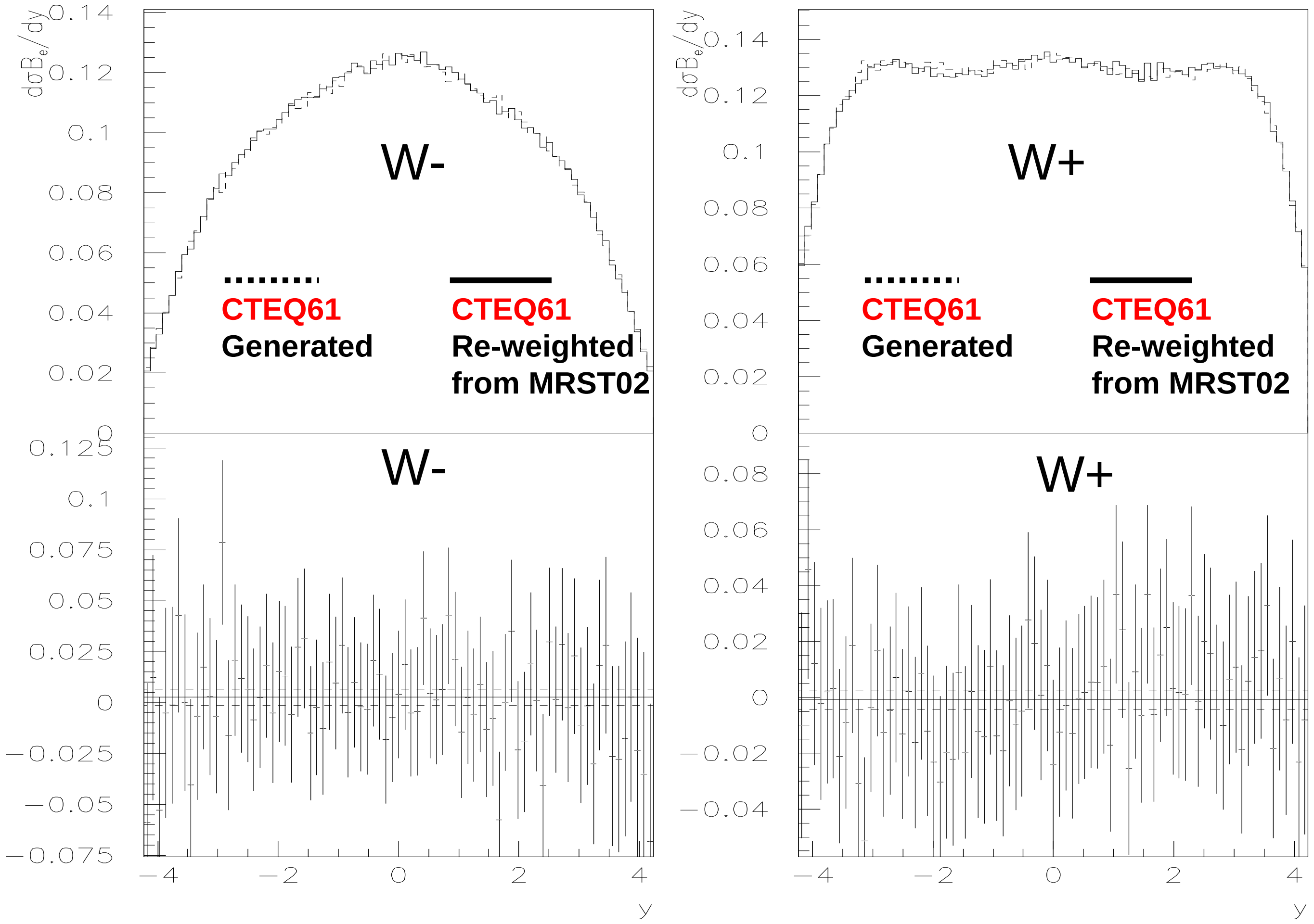
PDF Weights for CTEQ61, ZEUS02 from MRST02

(calculated using LHAPDFv3)

CTEQ61/MRST02 **ZEUS02/MRST02**



Does it work?- seems OK for RAPIDITY distributions



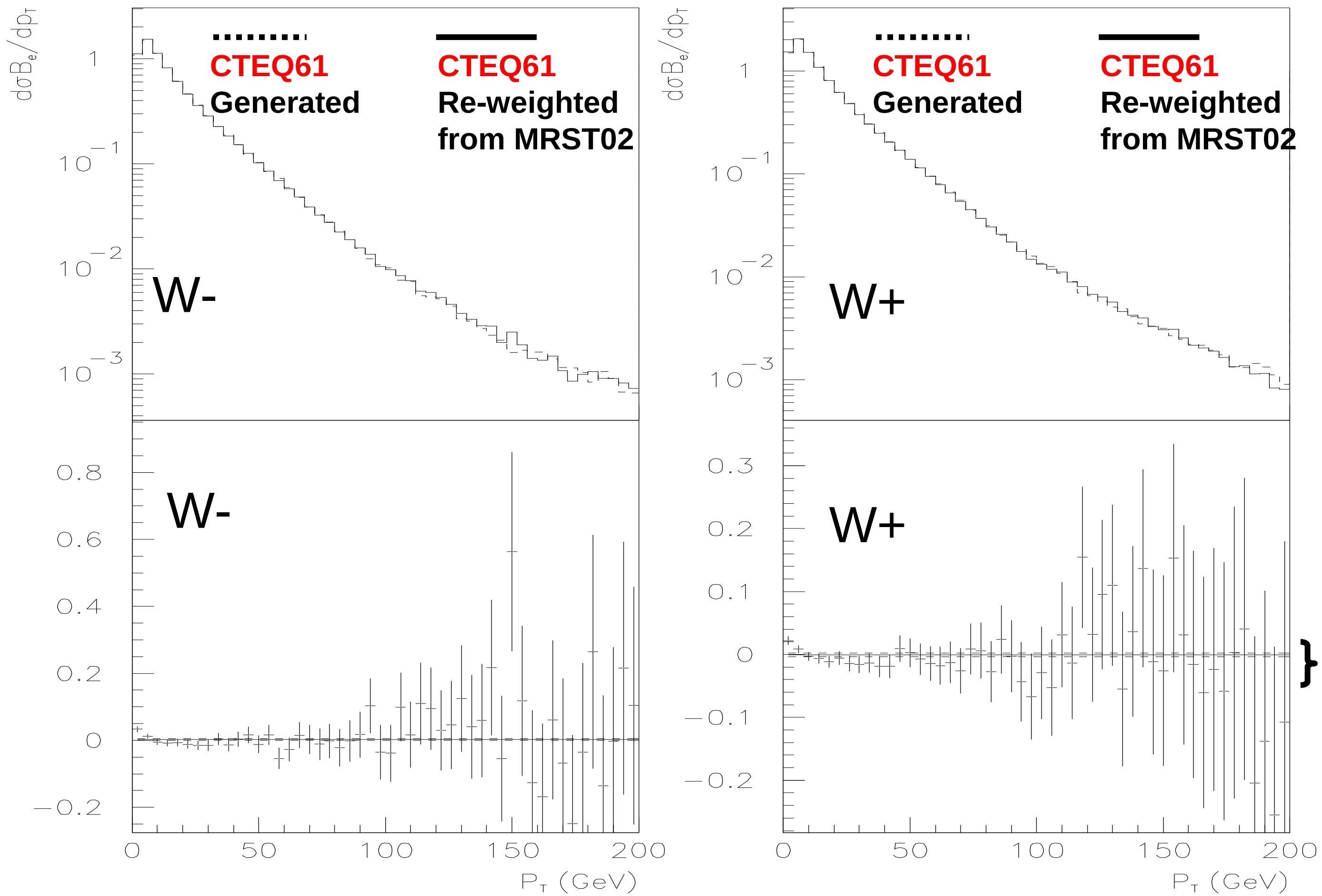
Events *generated* with HERWIG+**MRST02** and *re-weighted* with **CTEQ61** are compared to Events *generated* with HERWIG+**CTEQ61**

Relative difference between *Re-weighted* and *Generated* distributions

Weighted mean on all range

PDF Re-weighting: for rapidity Distributions good to ~0.5% and no evidence of a y-dependent bias.

Also seems to work for Pt distributions



Events *generated* with HERWIG+**MRST02** and *re-weighted* with **CTEQ61** are compared to Events *generated* with HERWIG+**CTEQ61**

Relative difference between *Re-weighted* and *Generated* distributions

Weighted mean on all range

PDF Re-weighting: for rapidity Distributions overall good to better than 1% but evidence of a slight Pt-dependent bias.

LHC is a low-x machine (at least for the early years of running)

Low-x information comes from evolving the HERA data

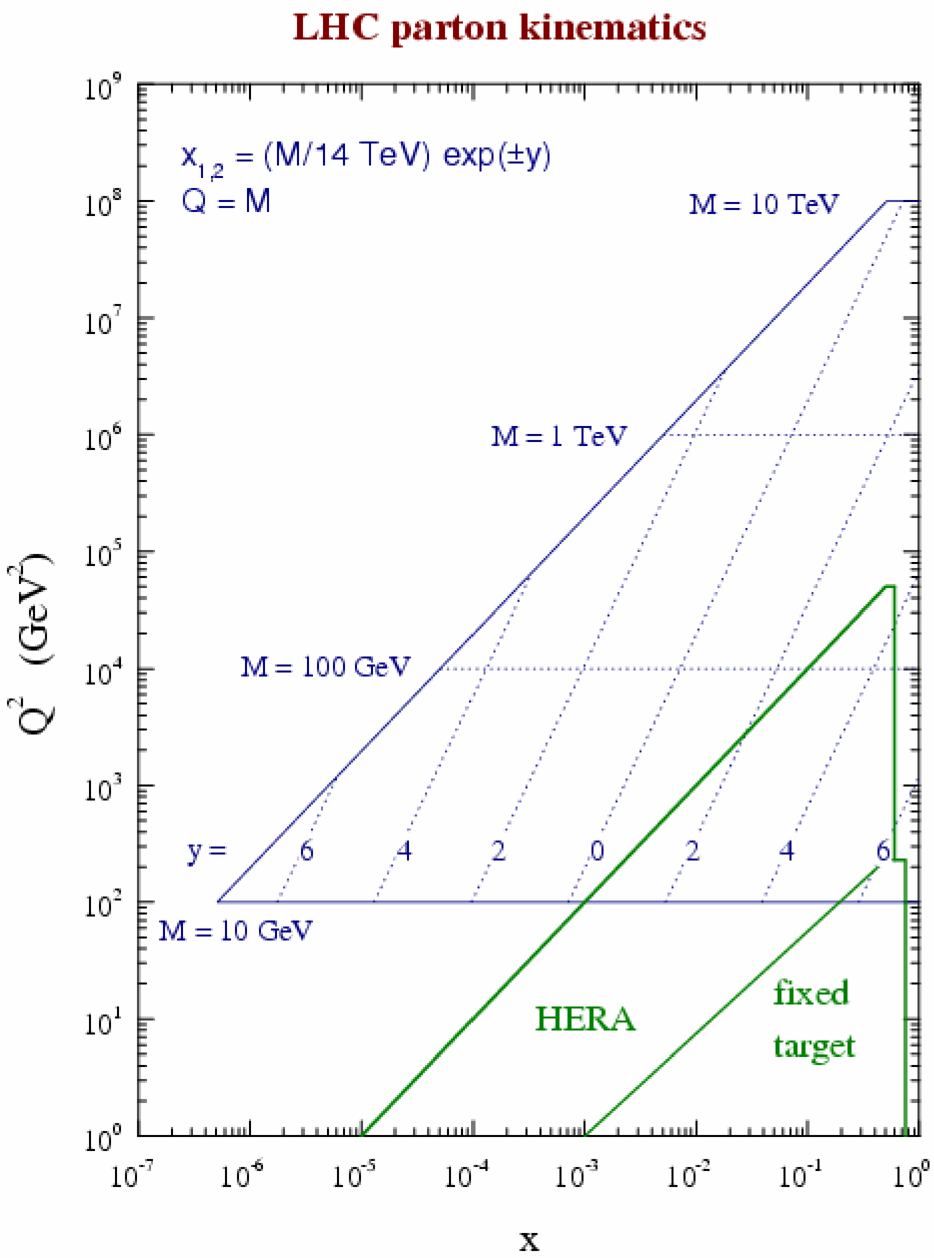
Is NLO (or even NNLO) DGLAP good enough?

The QCD formalism may need extending at small-x

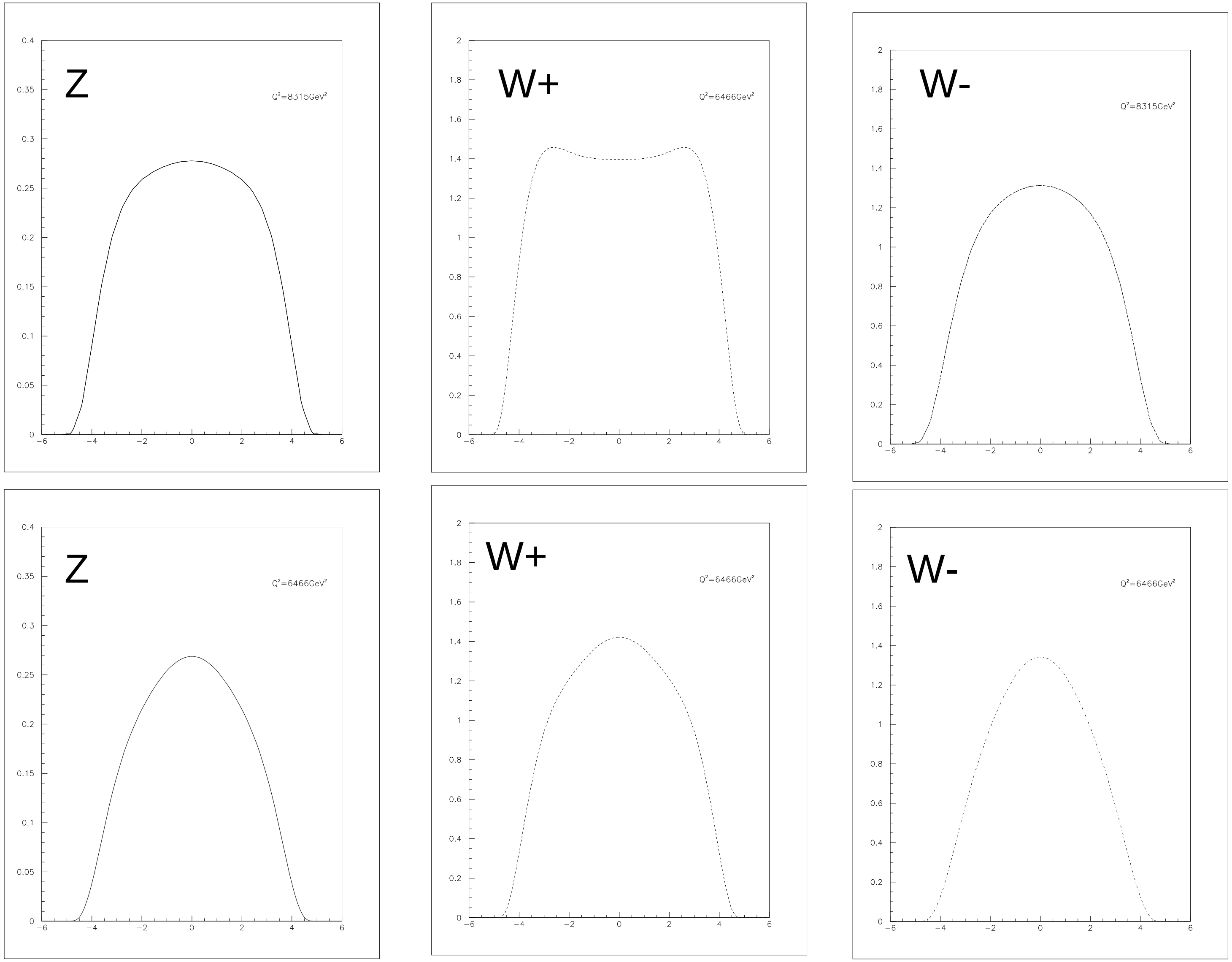
BFKL ln(1/x) resummation

High density non-linear effects etc.

(Devenish and Cooper-Sarkar, ‘Deep Inelastic Scattering’, OUP 2004, Section 6.6.6 and Chapter 9 for details!)



MRST have produced a set of PDFs derived from a fit without low-x data –ie do not use the DGLAP formalism at low-x- called MRST03 ‘conservative partons’. These give VERY different predictions for W/Z production to those of the ‘standard’ PDFs.



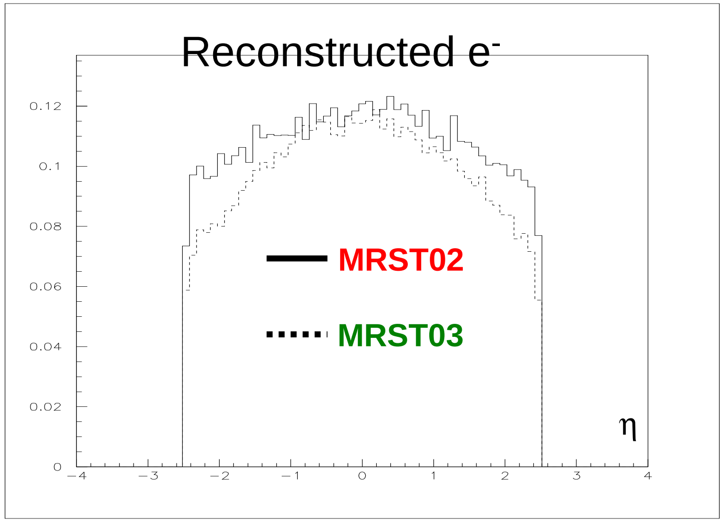
MRST02

MRST03

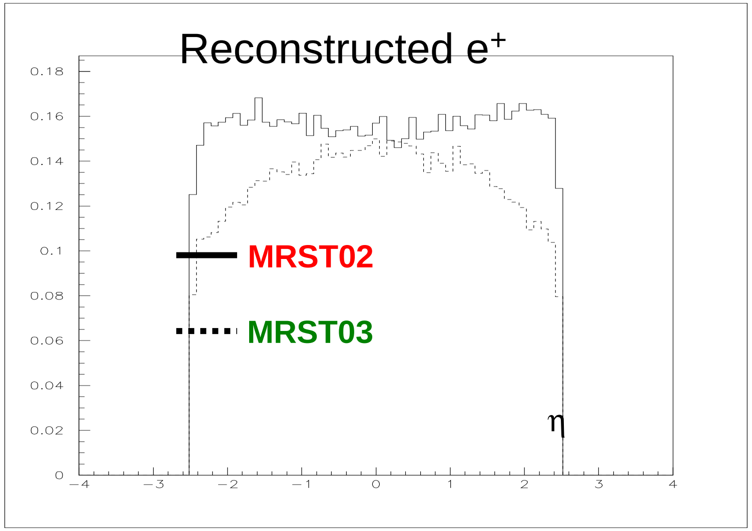
Differences persist in the decay lepton spectra and even in their ratio and asymmetry distributions

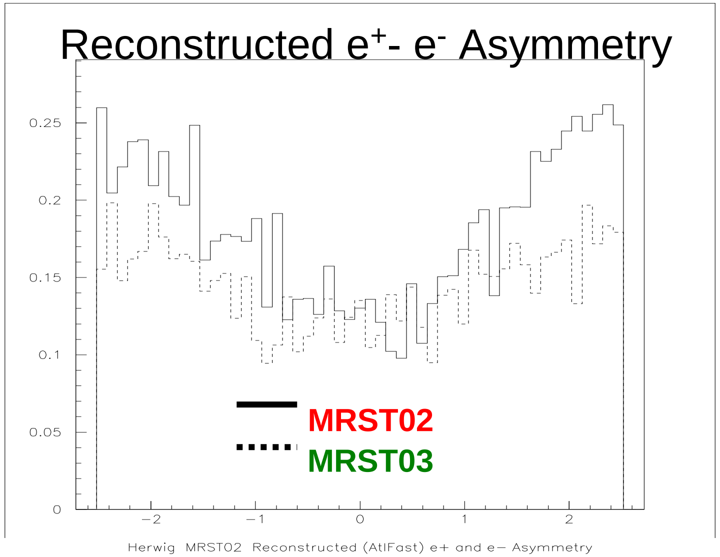
Reconstructed Electron Pseudo-Rapidity Distributions (ATLAS fast simulation)

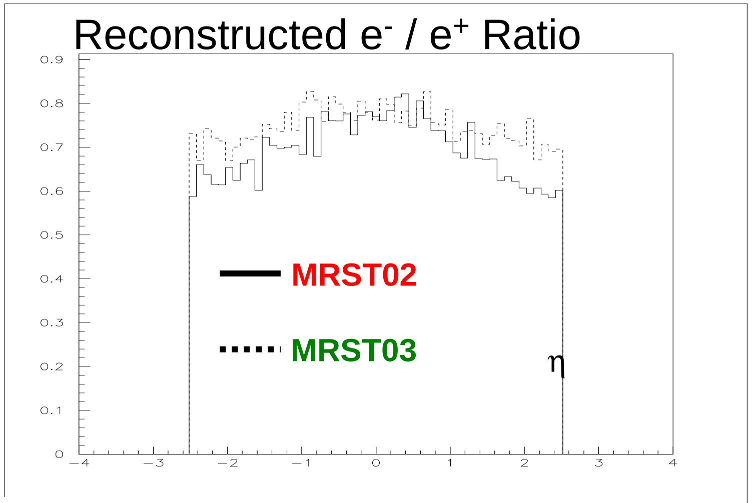
200k events of **W^{+-} -> e^{+-}** generated with HERWIG 6.505 + NLO K factors



6 hours running







Note of caution. MRST03 conservative partons DO NOT describe the HERA data for x< 5 10^{-3} which is not included in the fit which produces them. So there is no reason why they should correctly predict LHC data at non-central y, which probe such low x regions.

What is really required is an alternative theoretical treatment of low-x evolution which would describe HERA data at low-x, and could then predict LHC W/Z rapidity distributions reliably – also has consequences for pt distributions.

The point of the MRST03 partons is to illustrate that this prediction COULD be very different from the current 'standard' PDF predictions. When older standard predictions for HERA data were made in the early 90's they did not predict the striking rise of HERA data at low-x. This is a warning against believing that a current theoretical paradigm for the behaviour of QCD at low-x can be extrapolated across decades in Q2 with full confidence.

→ **The LHC measurements may also tell us something new about QCD**

Summary

Parton distributions are extracted from NLOQCD fits to DIS data- But they are needed for predictions of all cross-sections involving hadrons.

I have introduced you to the history of this in order to illustrate that it's not all cut and dried- our knowledge evolves continually as new data come in to confirm or confound our input assumptions

You need to appreciate the sources of uncertainties on PDFs – experimental, model and theoretical- in order to appreciate how reliable predictions for interesting collider cross-sections are.

At the LHC high precision (SM and BSM) cross section predictions require precision Parton Distribution Functions

We will improve our current knowledge from the HERA data, and the Tevatron data, before the LHC turns on

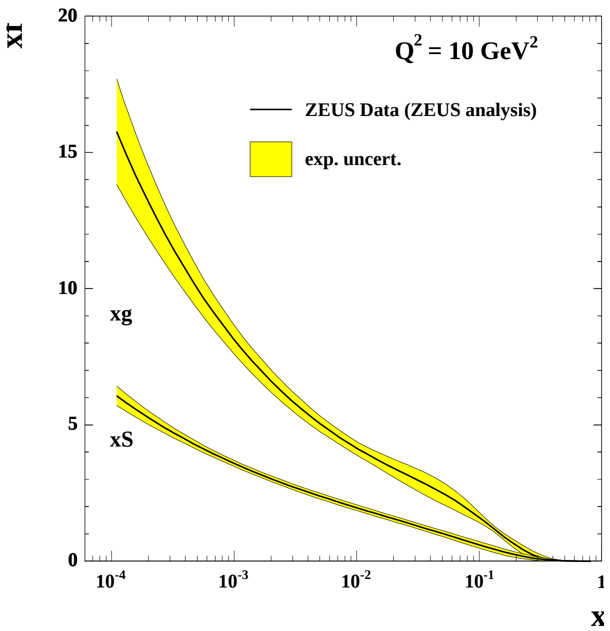
We can begin LHC physics by measuring 'standard candle' processes which are insensitive to PDF uncertainties

We can even use early LHC measurements, at low scales where BSM physics is not expected, to increase precision on PDFs and thus improve limits for discovery physics

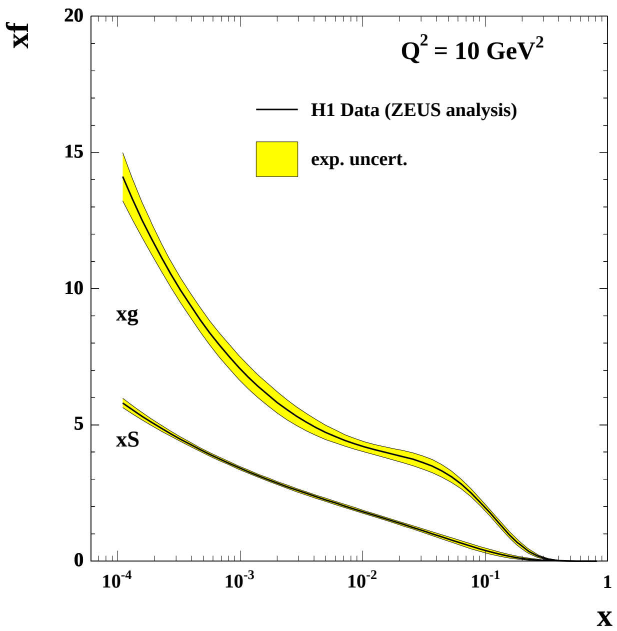
But there is some possibility that the Standard Model is wrong not due to exciting exotic physics, but because the standard QCD framework is not fully developed at small-x, hence we may first learn more about QCD!

End lecture-2

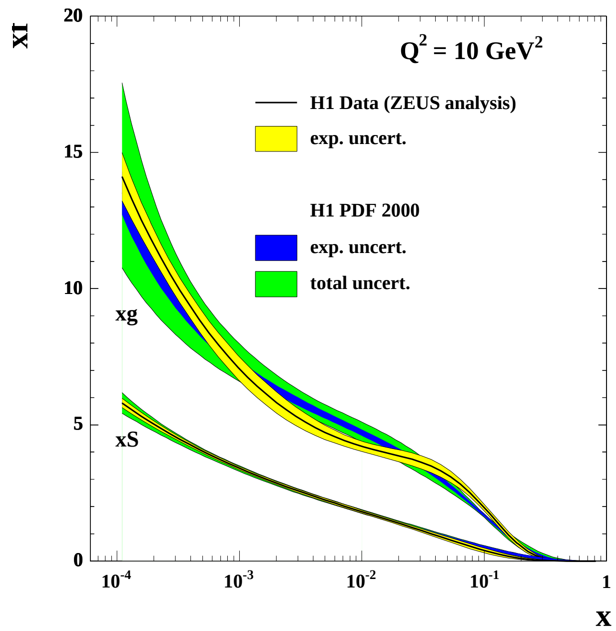
- Extras after here
- Details on ZEUS-H1 ‘theory free’ combination
- Details on LHAPDFv5
- More on low-x physics, what does it all mean etc…



ZEUS analysis/ZEUS data



ZEUS analysis/H1 data



ZEUS analysis/H1 data compared to

H1 analysis/H1 data

Here we see the effect of differences in the data, recall that the gluon is not directly measured (no jets)

The data differences are most notable in the large 96/97 NC samples at low-Q2 The data are marginally incompatible

Here we see the effect of differences of analysis choice - form of parametrization at Q2_0 etc

Lessons from comparing ZEUS and H1- they're supposed to measure the same thing!

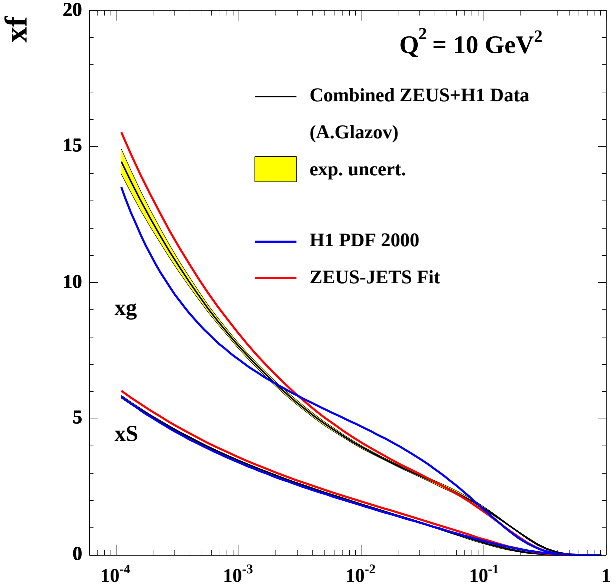
Combining ZEUS and H1 data sets

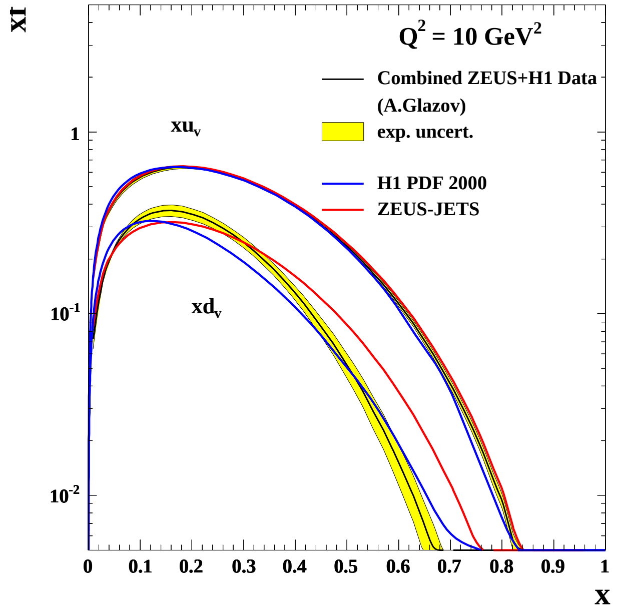
Combining the data sets could bring real advantages in decreasing the PDF errors, if the differences in the data sets can be resolved.

Combine using a Hessian fit which is'theory free' assuming only that each experiment is measuring the same 'truth'

e.g. if each experiment measures ~300 data points for the same cross-section then there are 600 data points and 300 free parameters for the true values plus~20 more free systematic uncertainty parameters sλ for both experiments

- The technique amounts to using each experiment to calibrate the other since they have rather different sources of experimental systematics
- Once the fit is done the systematic uncertainties of the combined data points (set by $\Delta\chi 2 = 1$ for the averaging fit) are a lot smaller than the statistical errors-
- one can try a simple PDF fit to this combined data for which statistical and systematic errors are combined in quadrature





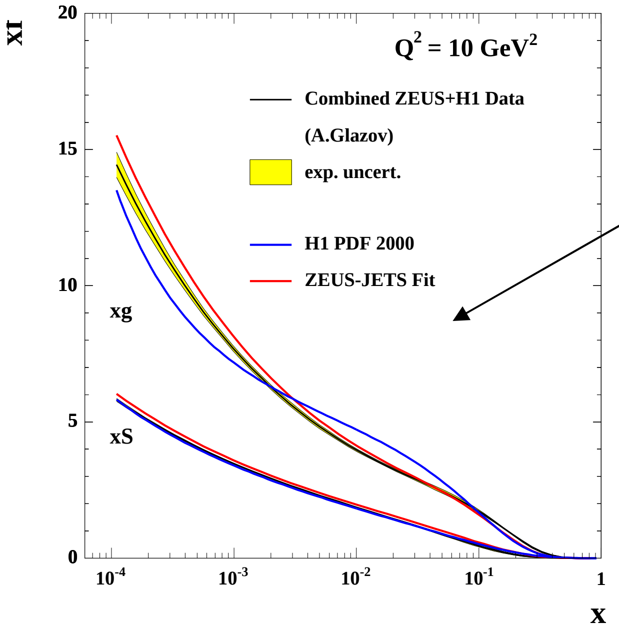
Fit to the ZEUS + H1 averaged inclusive cross section data set

And this simple fit results in very small experimental uncertainties on the PDFs

Compare to the published PDF shapes for H1 PDF 2000 and ZEUS-JETS-

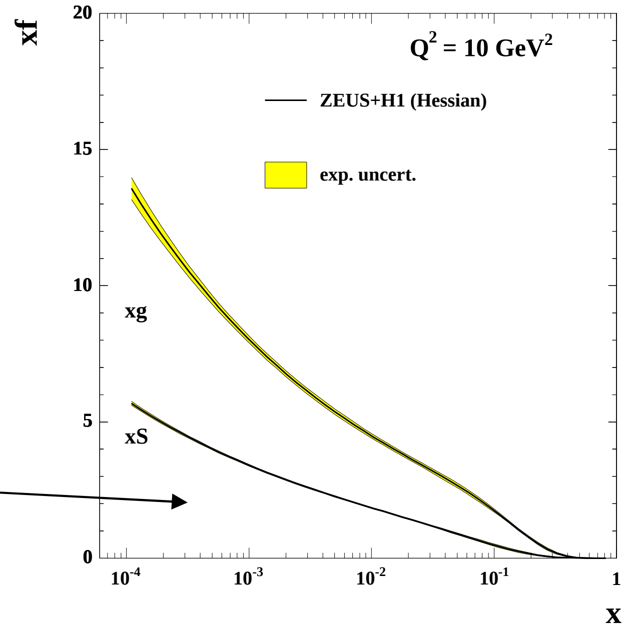
Gluon is more ‘ZEUS-like’

d valence is not really like either

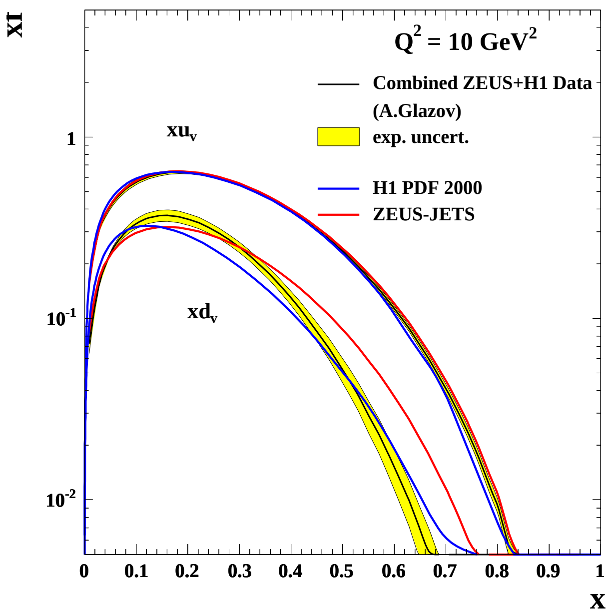


Compare this PDF fit to the H1 and ZEUS averaged inclusive cross-section data

To a PDF fit to H1 and ZEUS published inclusive cross-section data NOT averaged –done by the HESSIAN method

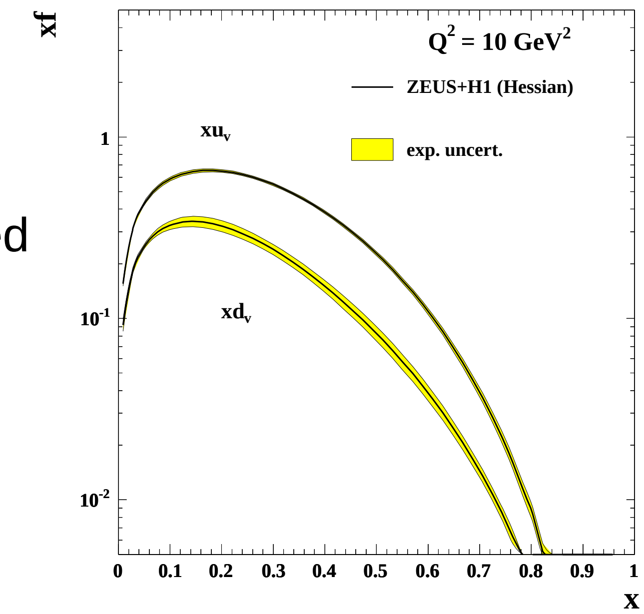


The errors are comparable



But the central values are rather different

This is because the systematic shifts determined by these fits are different



systematic shift s_{λ}	QCD ZEUS+H 1fit	Theory free ZEUS+H1fit
zd1_e_eff	1.65	0.31
zd2_e_theta_a	-0.56	0.38
zd3_e_theta_b	-1.26	-0.11
zd4_e_escale	-1.04	0.97
zd5_had1	-0.40	0.33
zd6_had2	-0.85	0.39
zd7_had3	1.05	-0.58
zd8_had_flow	-0.28	0.83
zd9_bg	-0.23	-0.42
zd10_had_flow_b	0.27	-0.26
h2_Ee_Spacal	-0.51	0.61
h4_ThetaE_sp	-0.19	-0.28
h5_ThetaE_94	0.39	-0.18
h7_H_Scale_S	0.13	0.35
h8_H_Scale_L	-0.26	-0.98
h9_Noise_Hca	1.00	-0.63
h10_GP_BG_Sp	0.16	-0.38
h11_GP_BG_LA	-0.36	0.97

A very boring slide- but the point is that it may be dangerous to let the QCD fit determine the optimal values for the systematic shift parameters.

And using $\Delta\chi 2=1$ on such a fit gives beautiful small PDF uncertainties but a central value which is far from that of the theory free combination.. So what are the real uncertainties? – Conclusion: an increased tolerance $\boldsymbol{\Delta\chi 2 = T^2}$**, T ~ 10,** is a good idea!

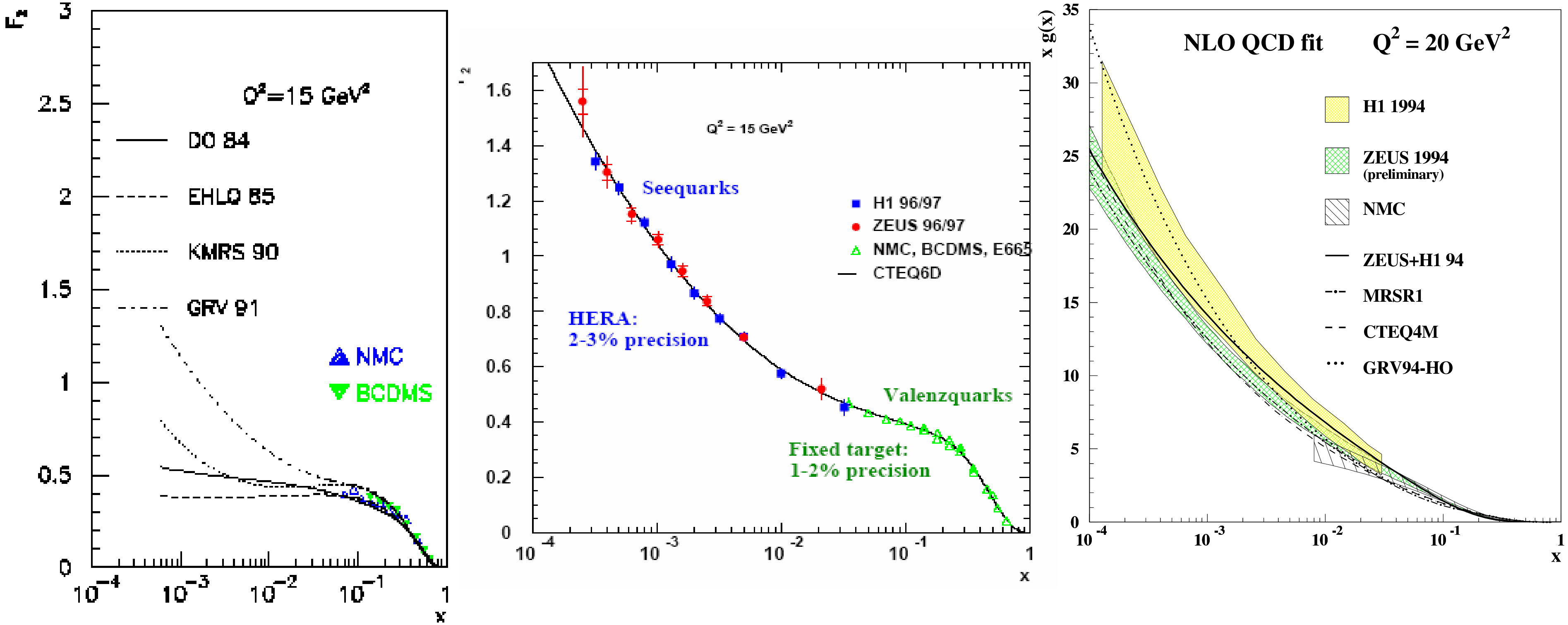
EXTras after here

Very easy to download the library and all PDFsets
Successor to PDFLIB- even has an interface LHAGLUE to make it look alike
User manual AND examples and a C++ Wrapper
What makes it different from PDFLIB?
It also has information on the uncertainties on the PDFs- Eigenvector PDF sets.

call InitPDFset(name)

call InitPDF(imem)……….where imem=0 is the central PDF set
then Call evolvePDF(x,q,xf) returns the PDFs for input to your calculation at x and q=sqrt(Q^2) (where xf(1,…6) gives d,u,s,c,b,t and xf(-1,..-6) gives qbar)

Then call NumberPDF(Nmem), where Nmem=2*Npdf
And do imem=1,Nmem
Call InitPDF(Imem) to repeat the calculation for each eigenvector set
Where imem=1,2 gives up(+) and down(-) along eigenvector 1
imem=3,4 gives up(+) and down(-) along eigenvector 2 etc…..



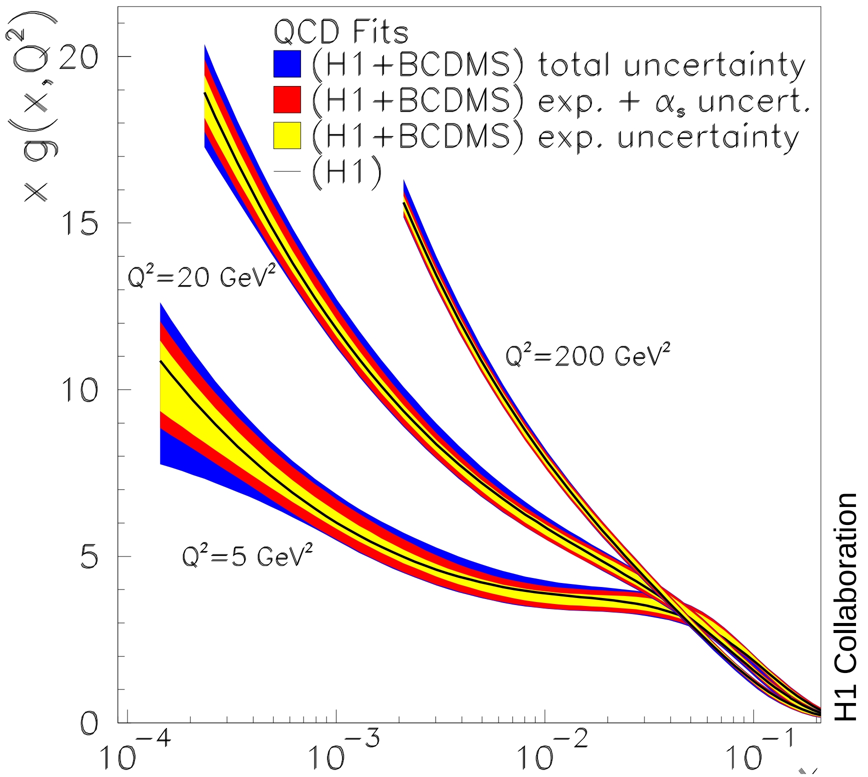
Before the HERA measurements most of the predictions for low-x behaviour of the structure functions and the gluon PDF were wrong

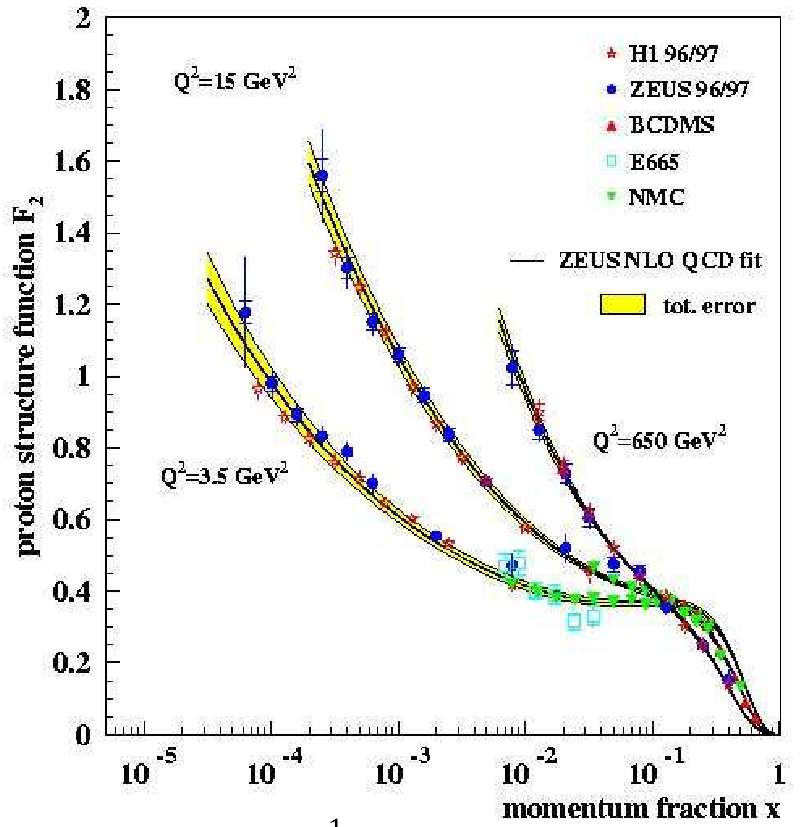
Now it seems that the conventional NLO DGLAP formalism works TOO WELL _

there **should be** ln(1/x) corrections and/or non-linear high density corrections for

$x < 5\ 10^{-3}$

Low-x





$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\left[\Sigma_q P_{gq}(z)q(y,Q^2) + P_{gg}(z)g(y,Q^2)\right]$$

At small x,

small z=x/y

$$P_{gq} \to \frac{C_F}{z}, \quad P_{gg} \to \frac{2C_A}{z}$$

Gluon splitting functions become singular

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\frac{6}{z}g(y,Q^2)$$

$xg(x,Q^2) \sim x^{-\lambda g}$

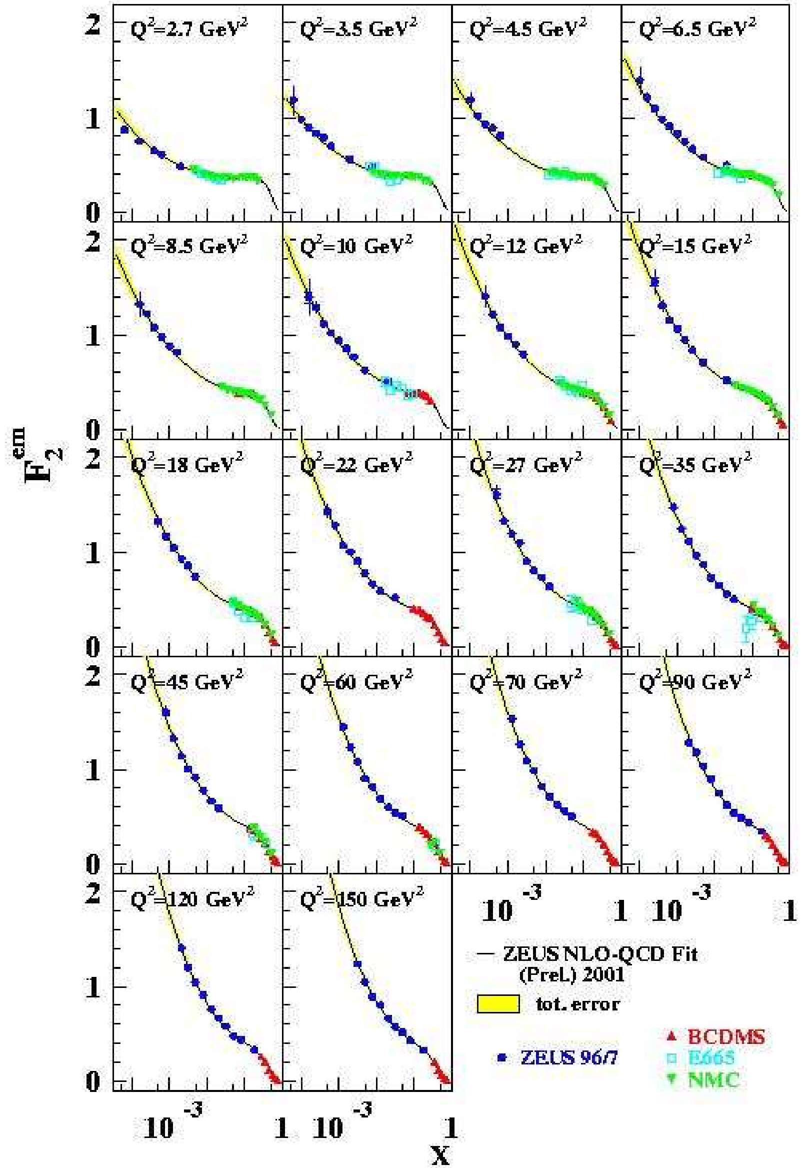
$$\lambda_g = \left(\frac{12\ln(t/t_0)}{\beta_0 \ln(1/x)}\right)^{\frac{1}{2}},$$

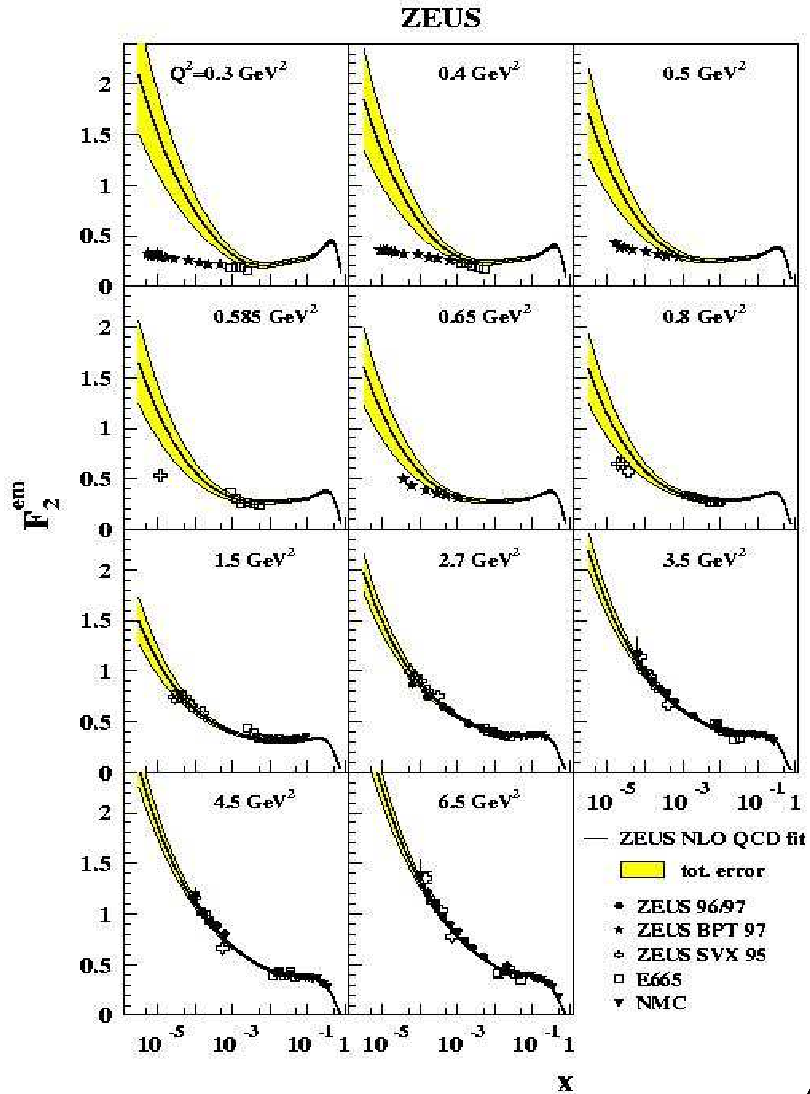
$t = \ln Q^2/\Lambda^2$

$\alpha_s \sim 1/\ln Q^2/\Lambda^2$

A flat gluon at low Q^2 becomes very steep AFTER Q^2 evolution AND F_2 becomes gluon dominated

$F_2(x,Q^2) \sim x^{-\lambda s}$, $\lambda s=\lambda g - \varepsilon$





So it was a surprise to see F_2 steep at small x - for low Q^2, $Q^2 \sim 1$ GeV^2

Should perturbative QCD work? α_s is becoming large - α_s at $Q^2 \sim 1$ GeV^2 is ~ 0.4

Need to extend formalism at small x?

The splitting functions $P^n(x)$, n= 0,1,2……for LO, NLO, NNLO etc

Have contributions $P^n(x) = 1/x\,[\, a_n \ln^n (1/x) + b_n \ln^{n-1} (1/x)$ ….

These splitting functions are used in evolution $dq/d\ln Q^2 \sim \alpha_s \int dy/y\, P(z)\, q(y,Q^2)$

And thus **give rise to contributions to the PDF** $\boldsymbol{\alpha_s^{\,p} (Q^2) (\ln Q^2)^q (\ln 1/x)^{\,r}}$

DGLAP sums- LL(Q^2) and NLL(Q2) etc STRONGLY ordered in pt.

But if ln(1/x) is large we should consider Leading Log 1/x (LL(1/x))

and Next to Leading Log (NLL(1/x)) - **BFKL summations**

LL(1/x) is STRONGLY ordered in ln(1/x) and can be disordered in pt

BFKL summation at LL(1/x) $\Rightarrow$ $xg(x) \sim x^{-\lambda}$

$$\lambda = \frac{\alpha_s C_A \ln 2}{\pi} \sim 0.5$$

$\Rightarrow$ **steep gluon even at moderate Q^2**

$\Rightarrow$ Disordered gluon ladders

But NLL(1/x) softens this somewhat

The steep behaviour of the gluon is deduced from the DGLAP QCD formalism –

BUT the steep behaviour of the low-x Sea can be measured from

$$F_2 \sim x^{-\lambda s}, \quad \lambda s = \frac{d \ln F_2}{d \ln 1/x}$$

Small x is high W^2, $x=Q^2/2p.q$ Q^2/W^2. At small x

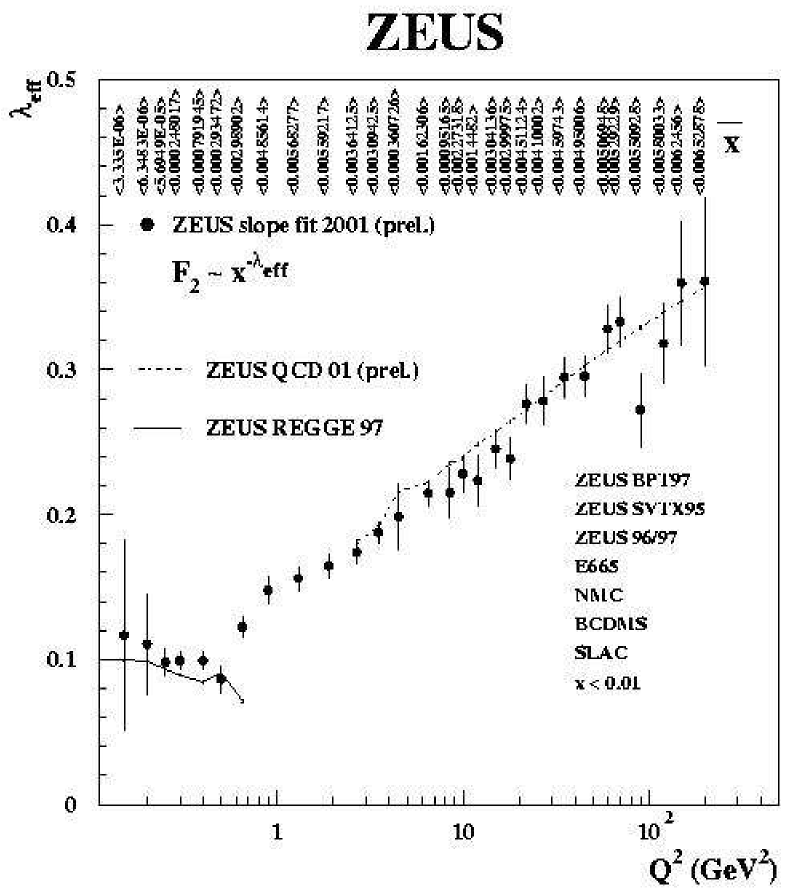
$$\sigma(\gamma^* p) = 4\pi^2\alpha\, F_2/Q^2$$

$F_2 \sim x^{-\lambda s} \rightarrow \sigma(\gamma^* p) \sim (W^2)^{\lambda s}$

But $\sigma(\gamma^* p) \sim (W^2)^{\alpha-1}$ – is the Regge prediction for high energy cross-sections

α is the intercept of the Regge trajectory $\alpha = 1.08$ for the SOFT POMERON

Such energy dependence is well established from the SLOW RISE of all hadron-hadron cross-sections - including $\sigma(\gamma p) \sim (W^2)^{0.08}$ for real photon- proton scattering



Does the steeper rise of $\sigma(\gamma^* p)$ require a hard Pomeron?

What about the Froissart bound?

Furthermore if the **gluon density becomes large** there maybe **non-linear effects**

Gluon recombination g g → g

$\sigma \sim \alpha_s^2 \rho^2/Q^2$

may compete with gluon evolution g → g g

$\sigma \sim \alpha_s \rho$

where ρ is the gluon density

$$\sim \frac{xg(x,Q2)}{\pi R^2} \text{ – no. } \frac{\text{of gluons per } \ln(1/x)}{\text{nucleon size}}$$

Non-linear evolution equations – GLR

$$\frac{d^2 xg(x,Q2)}{d\ln Q2\, d\ln 1/x} = \frac{3\alpha_s}{\pi}\, xg(x,Q2) - \frac{\alpha_s^2\, 81}{16Q^2R^2}\, [xg(x,Q2)]^2$$

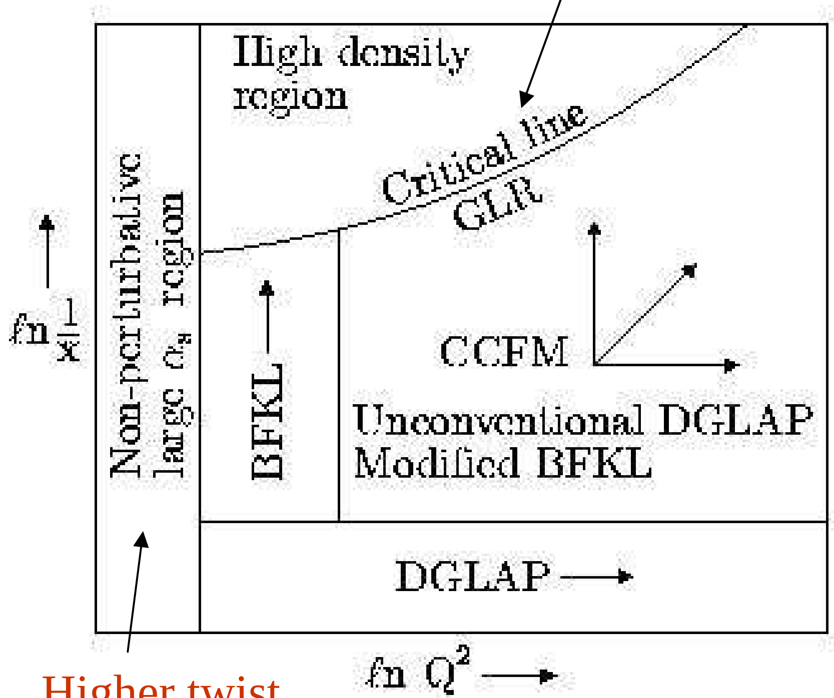
$\alpha_s \rho$ $\qquad$ $\alpha_s^2\, \rho^2/Q^2$

The non-linear term slows down the evolution of $xg(x,Q^2)$ and thus tames the rise at small x

The gluon density may even saturate

(-respecting the Froissart bound)

Colour Glass Condensate, JIMWLK, BK



Higher twist

Extending the conventional DGLAP equations across the x, Q2 plane

Plenty of debate about the positions of these lines!

Do the data NEED unconventional explanations ?

In practice the NLO DGLAP formalism works well down to $Q^2 \sim 1\ \mathrm{GeV}^2$

BUT below $Q^2 \sim 5\ \mathrm{GeV}^2$ the gluon is no longer steep at small x – in fact its becoming negative!

$$xS(x) \sim x^{-\lambda s},\ xg(x) \sim x^{-\lambda g}$$
$$\lambda g < \lambda s \text{ at low Q2, low x}$$

We only measure $F_2 \sim xq$

$$dF_2/d\ln Q^2 \sim Pqg\ xg$$

Unusual behaviour of $dF_2/d\ln Q^2$ may come from

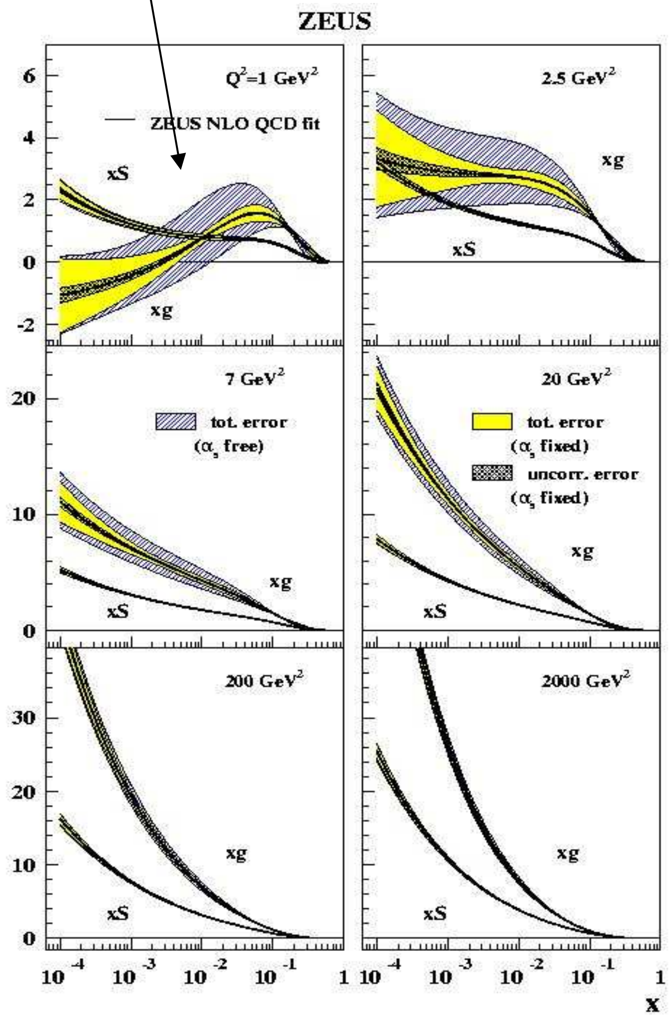
unusual gluon or from unusual Pqg- alternative evolution?. Non-linear effects?

We need other gluon sensitive measurements at low x

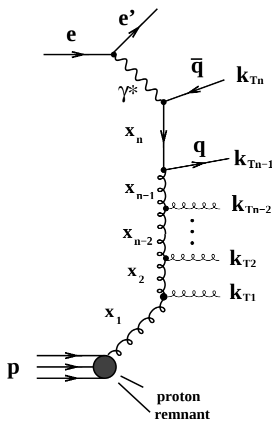
Like F_L- but a fully model independent measurement involves changing the beam energy

There are now plans to do this at the end of HERA-II running

`Valence-like' gluon shape

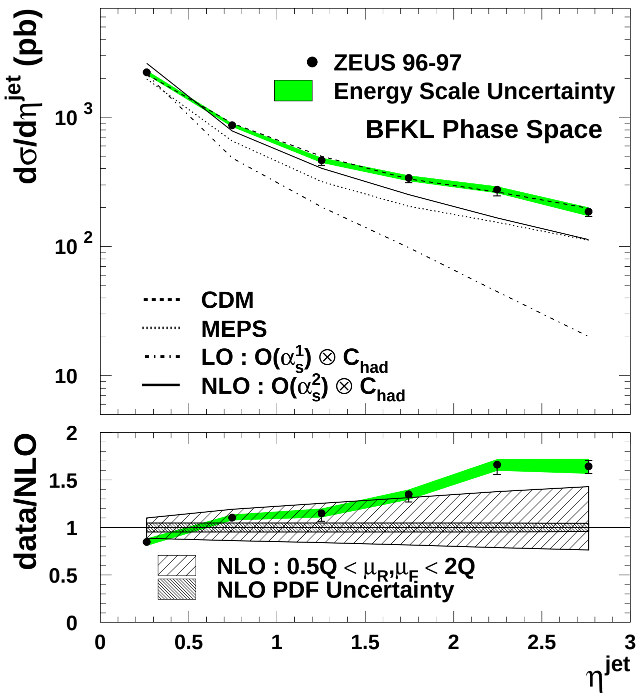


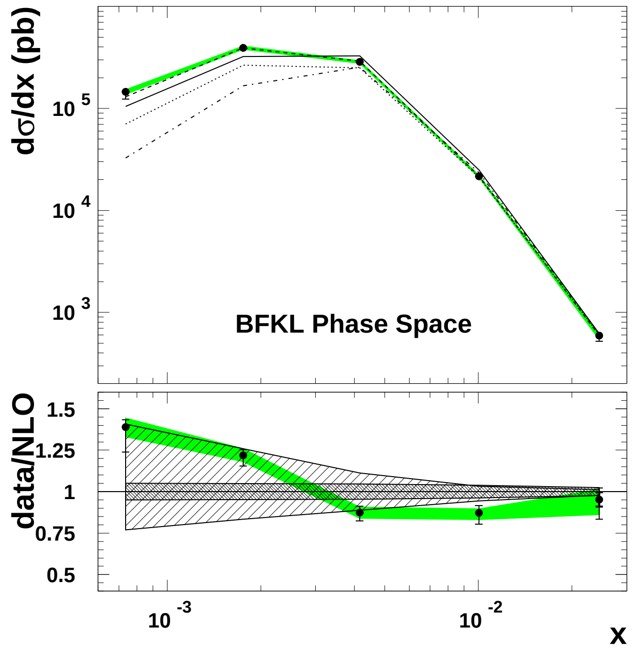
Look at the hadron final states..lack of pt ordering has its consequences. But this has only served to highlight the fact that the conventional calculations of jet production were not very well developed. There has been much progress on MC@NLO rather than ad-hoc calculations (MEPS, ARIADNE CDM …) e.g.



Forward jets with $x_j \gg x$ and $k_{tj}{}^2 \sim Q^2$ are suppressed for DGLAP evolution but not for kt disordered BFKL evolution

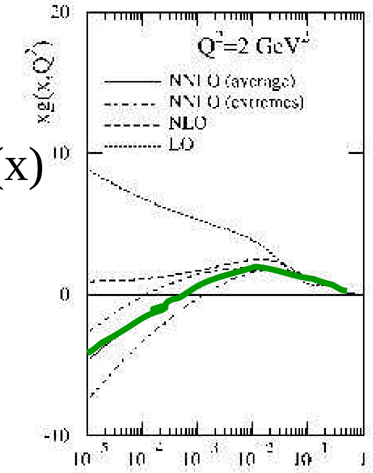
Data do not agree with DGLAP at LO or NLO, or with MEPS..but agree with CDM (part of ARIADNE). This is not kt ordered but it is not a convincing BFKL calculation either.

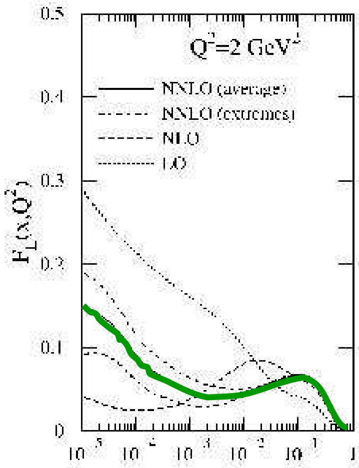


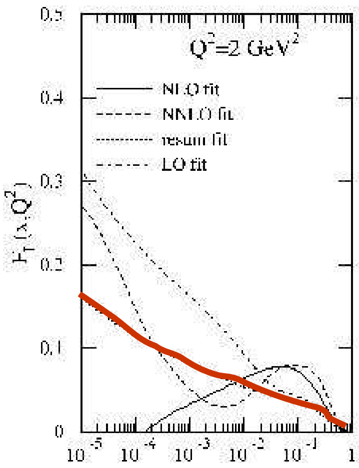


$Q^2 = 2GeV^2$

xg(x)







The negative gluon predicted at low x, low Q^2 from NLO DGLAP remains at NNLO (worse)

The corresponding F_L is NOT negative at $Q^2 \sim 2$ GeV^2 – but has peculiar shape

Including ln(1/x) resummation in the calculation of the splitting functions (BFKL `inspired') can improve the shape - and the χ^2 of the global fit improves

The use of non-linear evolution equations can also *improve* the shape of the gluon at low x, Q^2

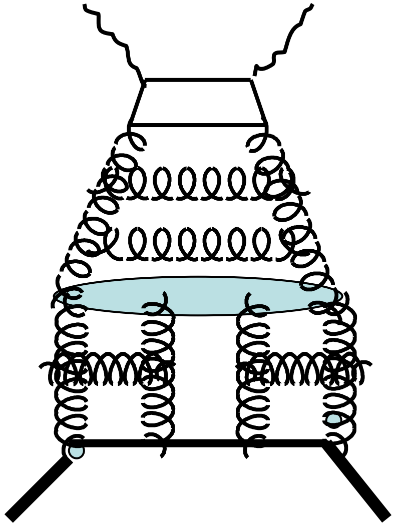
The gluon becomes steeper (high density) and the sea quarks less steep

But this doesn't really prove anything

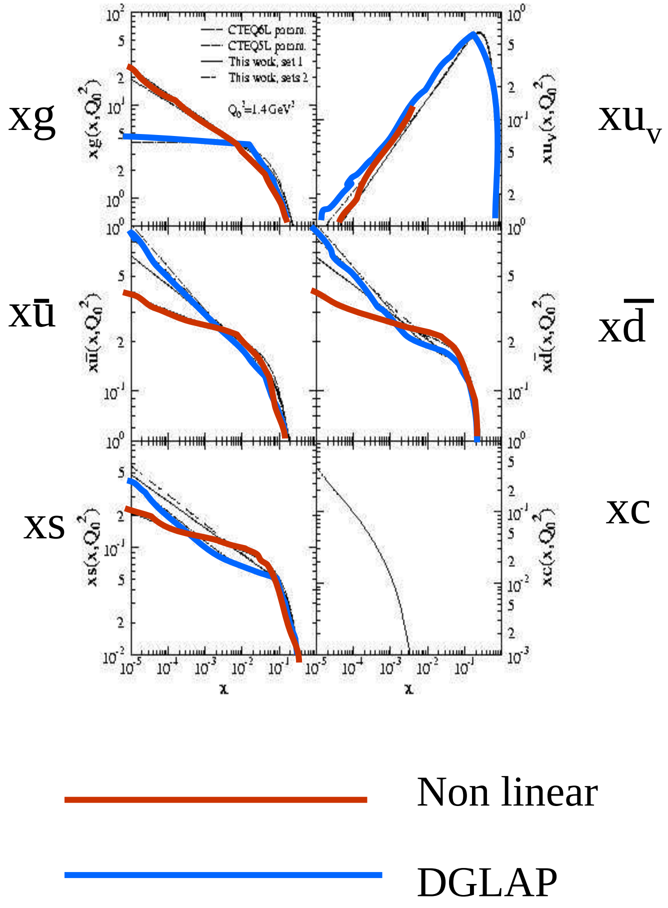
The use of non-linear evolution equations also *improves* the shape of the gluon at low x, Q^2

The gluon becomes steeper (high density) and the sea quarks less steep

Non-linear effects $gg \to g$ involve the summation of FAN diagrams – higher twist

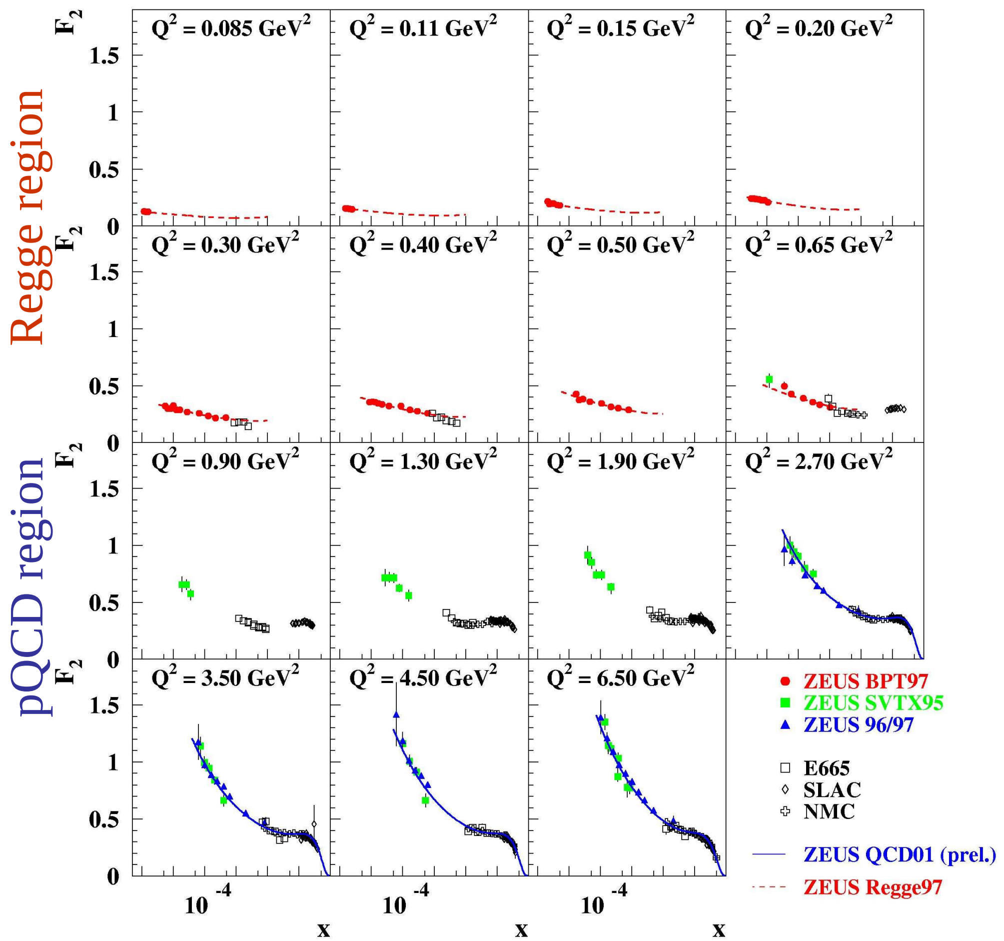


$Q^2 = 1.4\ \mathrm{GeV}^2$

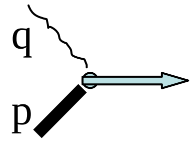


But this doesn't really prove anything

Linear DGLAP evolution doesn't work for
$Q^2 < 1$ GeV2, WHAT does? – REGGE ideas?



Small x is high W^2, $x=Q^2/2p.q$ Q^2/W^2



$p_x^{\,2} = W^2$

$\sigma(\gamma^* p) \sim (W^2)^{\alpha-1}$ – Regge prediction for high energy cross-sections

α is the intercept of the Regge trajectory
α=1.08 for the SOFT POMERON

Such energy dependence is well established from the SLOW RISE of all hadron-hadron cross-sections - including
$\sigma(\gamma p) \sim (W^2)^{0.08}$
for real photon- proton scattering

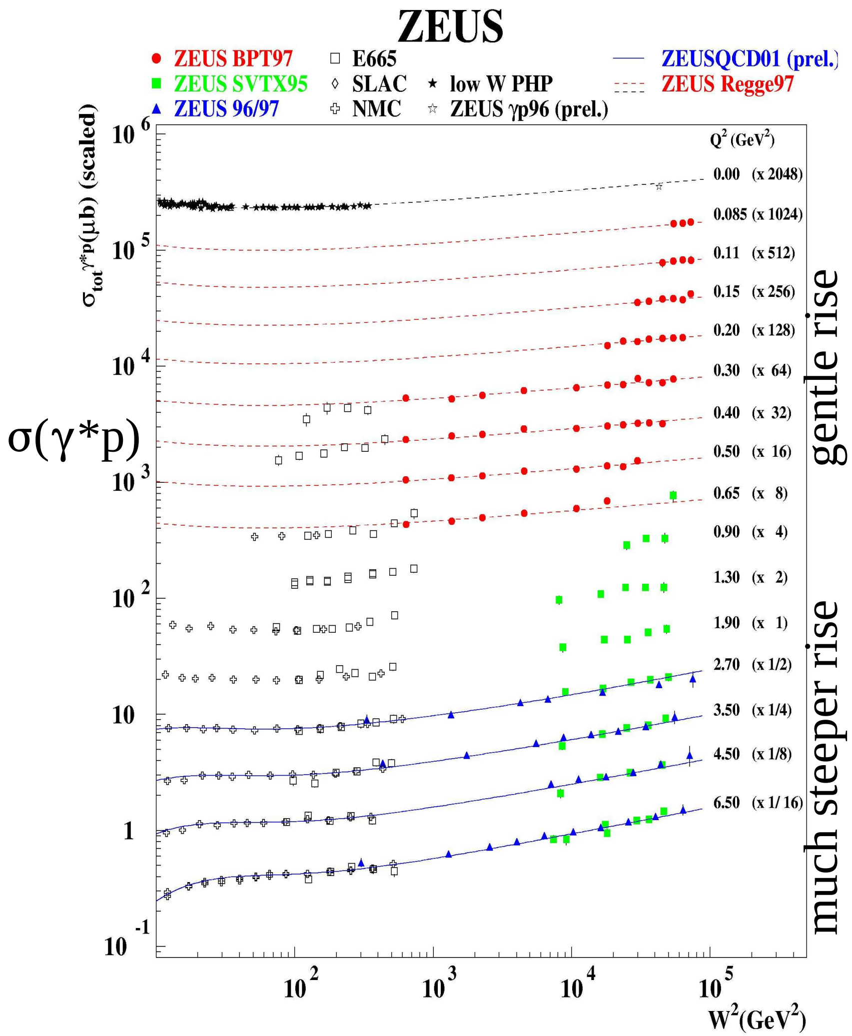
For virtual photons, at small x

$$\sigma(\gamma^* p) = \frac{4\pi^2\alpha}{Q^2} F_2$$

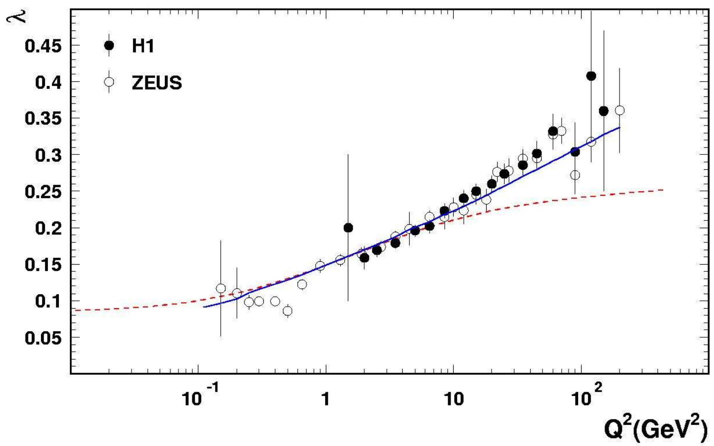
$$\Rightarrow \sigma \sim (W^2)^{\alpha-1} \Rightarrow F_2 \sim x^{1-\alpha} = x^{-\lambda}$$

so a SOFT POMERON would imply
$\lambda = 0.08$ gives only a very gentle rise of F_2 at small x

For $Q^2 > 1$ GeV^2 we have observed a much stronger rise…..



The slope of F_2 at small x , $F_2 \sim x^{-\lambda}$, is equivalent to a rise of $\sigma(\gamma^*p) \sim (W^2)^\lambda$ which is only gentle for $Q^2 < 1$ GeV^2



QCD improved dipole

GBW dipole

Regge region pQCD generated slope

So is there a HARD POMERON corresponding to this steep rise?

A QCD POMERON, $\alpha(Q^2) - 1 = \lambda(Q^2)$

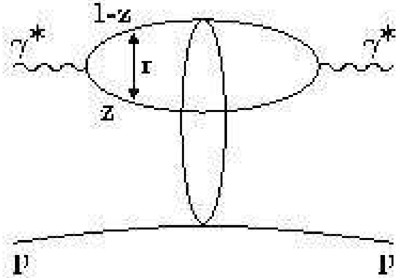
A BFKL POMERON, $\alpha - 1 = \lambda = 0.5$

A mixture of HARD and SOFT Pomerons to explain the transition $Q^2 = 0$ to high Q^2?

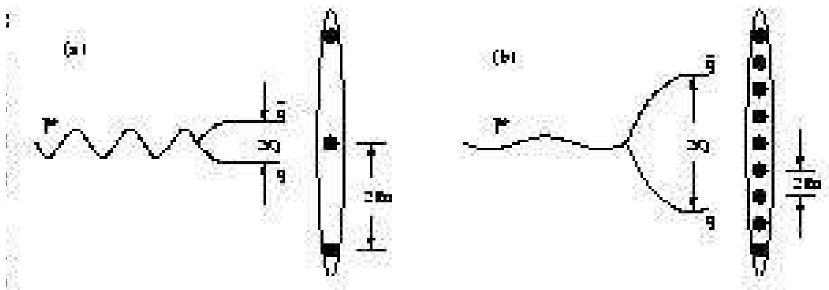
What about the Froissart bound ? – the rise MUST be tamed – non-linear effects?

Dipole models provide a way to model the transition $Q^2=0$ to high Q^2

At low x, $\gamma^* \Rightarrow q\bar{q}$ and the LONG LIVED $(q\bar{q})$ dipole scatters from the proton



The dipole-proton cross section depends on the relative size of the dipole $r \sim 1/Q$ to the separation of gluons in the target R_0



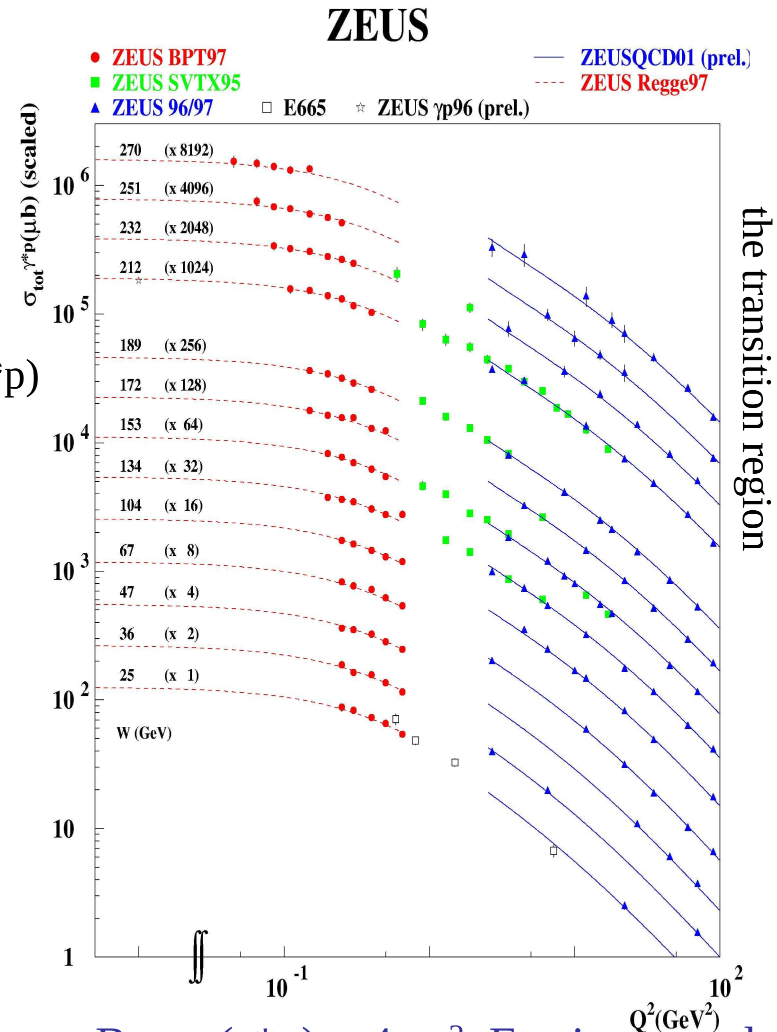
$\sigma = \sigma_0(1 - \exp(-r^2/2R_0(x)^2))$, $R_0(x)^2 \sim (x/x_0)^{\lambda} \sim 1/xg(x)$

r/R_0 small $\Rightarrow$ large Q^2, x
$\sigma \sim r^2 \sim 1/Q^2$

r/R_0 large $\Rightarrow$ small Q2, x
$\sigma \sim \sigma_0 \Rightarrow$ saturation of the dipole cross-section

GBW dipole model

$\sigma(\gamma^*p)$



Now there is HERA data right across the transition region

But $\sigma(\gamma^*p) = \frac{4\pi\alpha^2}{Q^2} F_2$ is general (at small x)

$\sigma(\gamma p)$ is finite for real photons, $Q^2=0$. At high Q^2, F_2 ~flat (weak $\ln Q^2$ breaking) and $\sigma(\gamma^*p) \sim 1/Q^2$

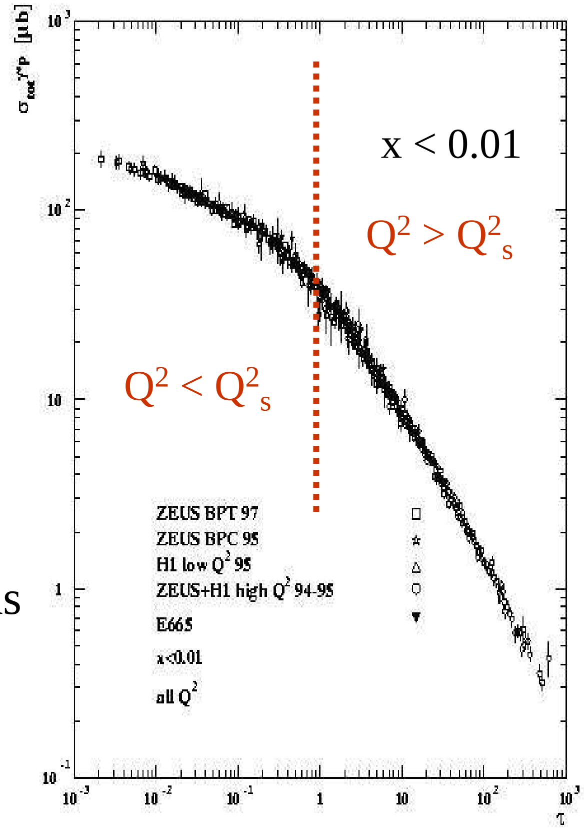
$\sigma = \sigma_0 \, (1 - \exp(-1/\tau))$

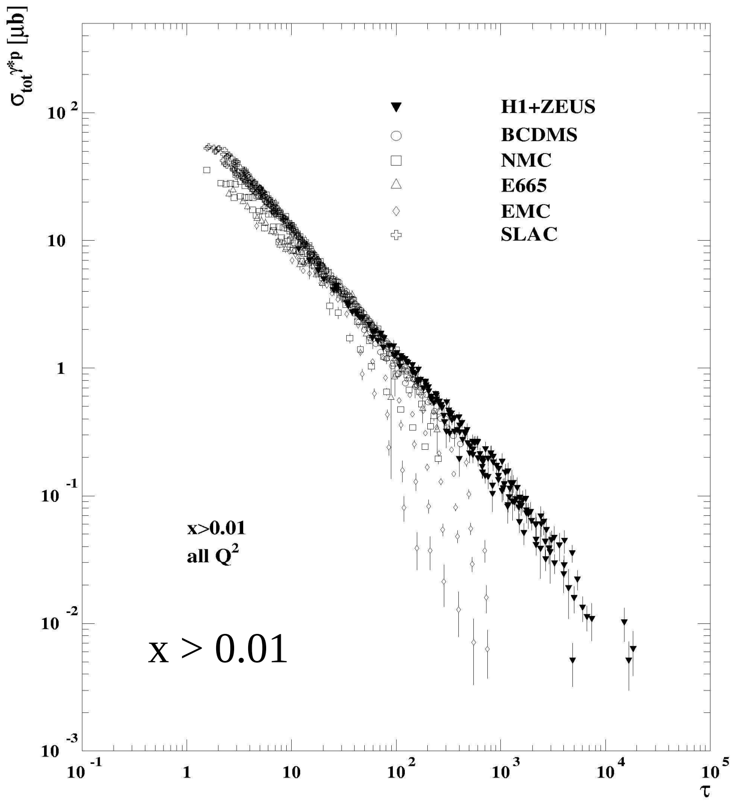
Involves only

$\tau = Q^2 R_0^2(x)$

$\tau = Q^2/Q_0^2 \, (x/x_0)^{\lambda}$

And INDEED, for $x<0.01$, $\sigma(\gamma^* p)$ depends only on τ, not on x, Q^2 separately





τ is a new scaling variable, applicable at small x

It can be used to define a `saturation scale' , $Q^2_s = 1/R_0^2(x) \approx x^{-\lambda} \sim x\,g(x)$, gluon density

- such that saturation extends to higher Q^2 as x decreases

Some understanding of this scaling, of saturation and of dipole models is coming from work on non-linear evolution equations applicable at high density– Colour Glass Condensate, JIMWLK, Balitsky-Kovchegov. There can be very significant consequences for high energy cross-sections e.g. neutrino cross-sections – also predictions for heavy ions- RHIC, diffractive interactions – Tevatron and HERA, even some understanding of soft hadronic physics