

NEUTRINO PHYSICS

Neutrino Oscillations

Neutrino Masses and Mixing

Majorana or Dirac Nature of the Neutrino

[Neutrinos as new Messenger from the Universe]

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Rhodes
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Fermion Particles in the Standard Model

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad Y = -\frac{1}{2}$$

$$d_R \quad s_R \quad b_R \quad Y = -\frac{1}{3}$$

$$u_R \quad c_R \quad t_R \quad Y = +\frac{2}{3}$$

$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$$Y = +\frac{1}{2}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad Y = -\frac{1}{2}$$

$$e_R \quad \mu_R \quad \tau_R \quad Y = -1$$

$$(\nu_e)_R \quad (\nu_\mu)_R \quad (\nu_\tau)_R \quad Y = 0$$

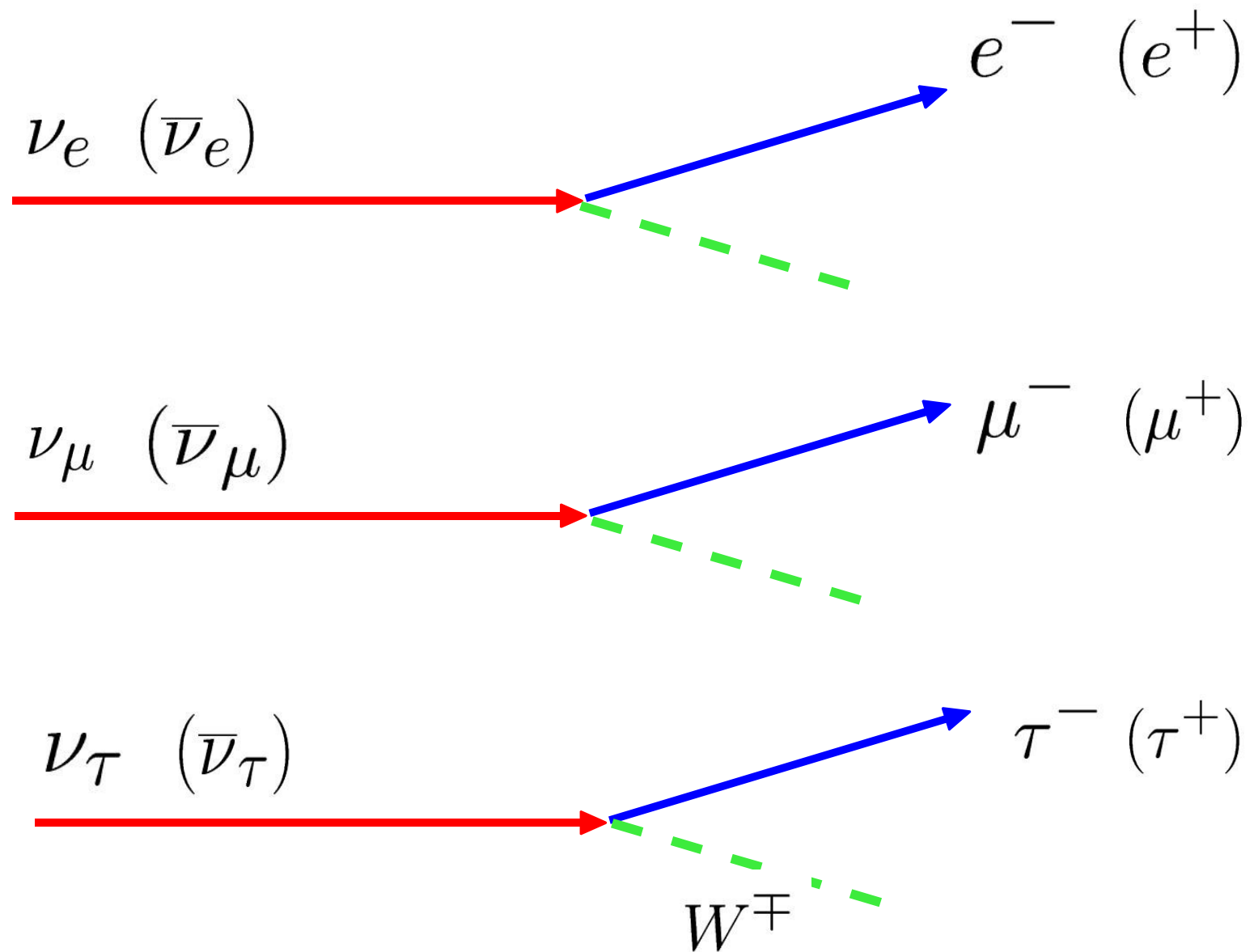
Minimal SM
 ν_R absent

3 type (FLAVORS) of Neutrinos

$$\nu_e \quad \nu_\mu \quad \nu_\tau$$

$$\bar{\nu}_e \quad \bar{\nu}_\mu \quad \bar{\nu}_\tau$$

FLAVOR



Dirac Particle

$$\begin{array}{cc} e_{\text{L}}^- & e_{\text{R}}^- \\ e_{\text{L}}^+ & e_{\text{R}}^+ \end{array}$$

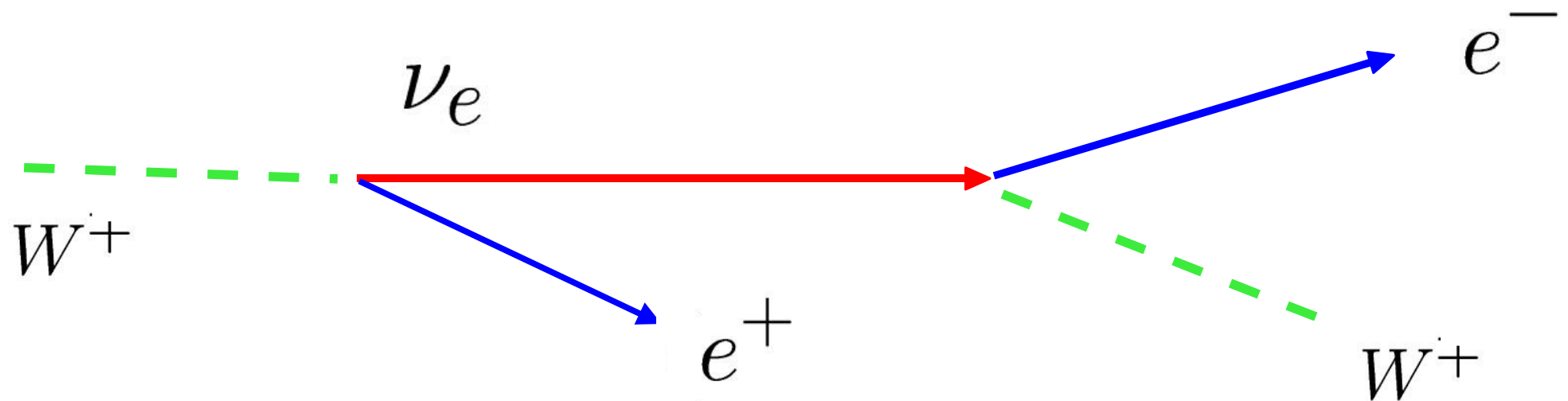
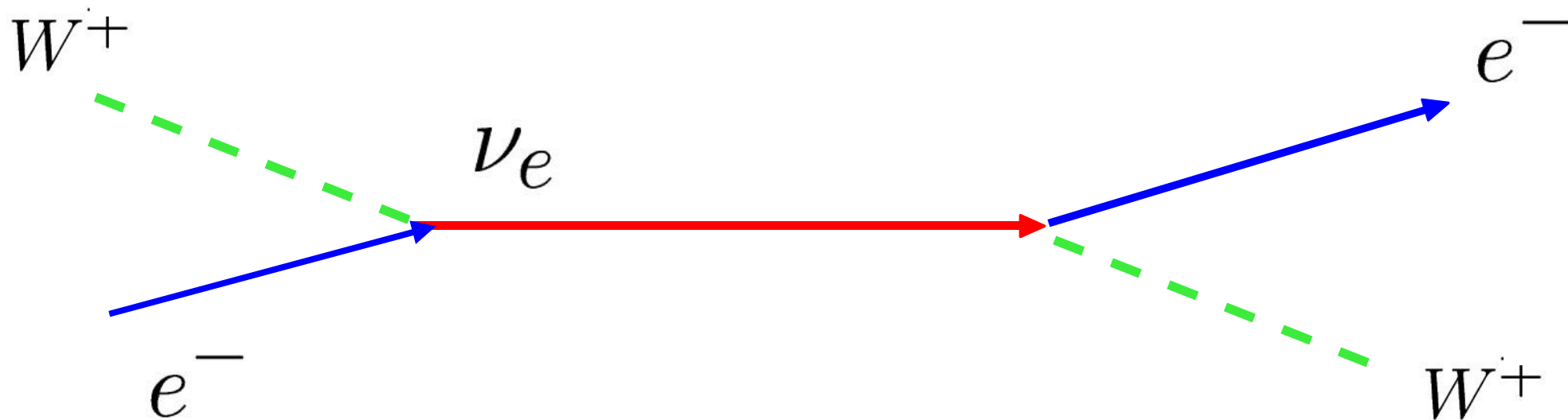
$$\begin{array}{cc} \nu_{\text{L}} & \nu_{\text{R}} \\ \bar{\nu}_{\text{L}} & \bar{\nu}_{\text{R}} \end{array}$$

Majorana Particle

$$\begin{array}{cc} \nu_{\text{L}} & \\ & \bar{\nu}_{\text{R}} \end{array}$$

FLAVOR

Electron-flavor



How Many Light Neutrinos Exist ?

Answer : **3**

$$Z^0 \rightarrow \nu_\alpha + \bar{\nu}_\alpha$$

$$\Gamma_{\nu\bar{\nu}} = 166.9 \text{ MeV}$$

$$\Gamma_{\text{invisible}} = N_\nu \Gamma_{\nu\bar{\nu}}$$

$$\Gamma_{\text{invisible}} = \Gamma_{\text{tot}} - \Gamma_{\text{vis}} = 498 \pm 4.2 \text{ MeV}$$

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu\bar{\nu}}} = 2.994 \pm 0.012$$

3 Neutrinos states: 3 masses m_1, m_2, m_3

States with definite masses
in general do **not** coincide with the "flavor" states

$$\{ |\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle \}$$

Flavor basis

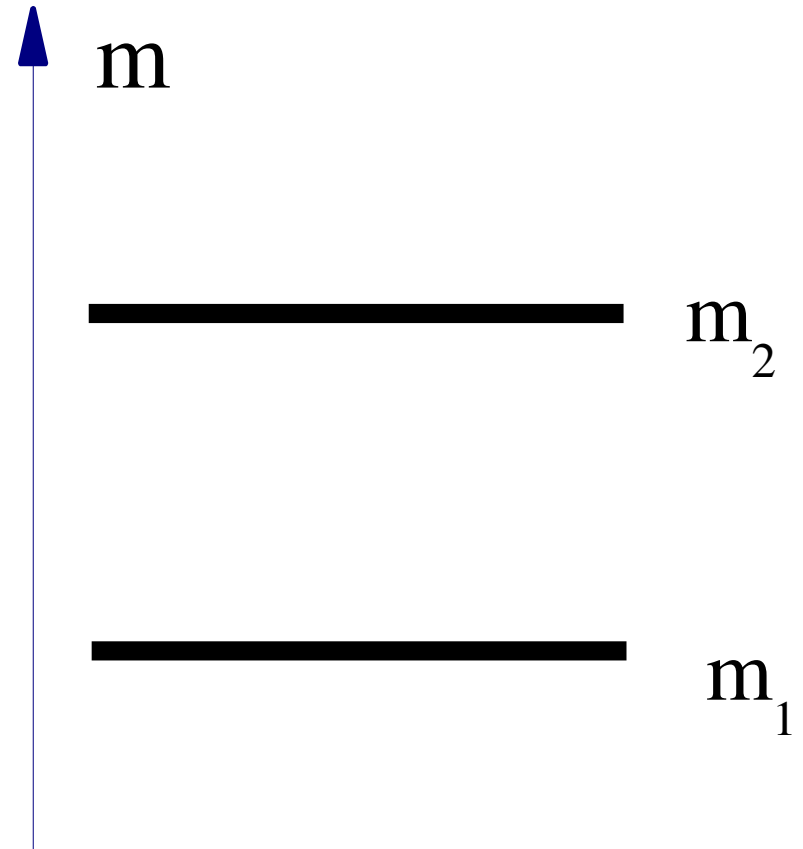
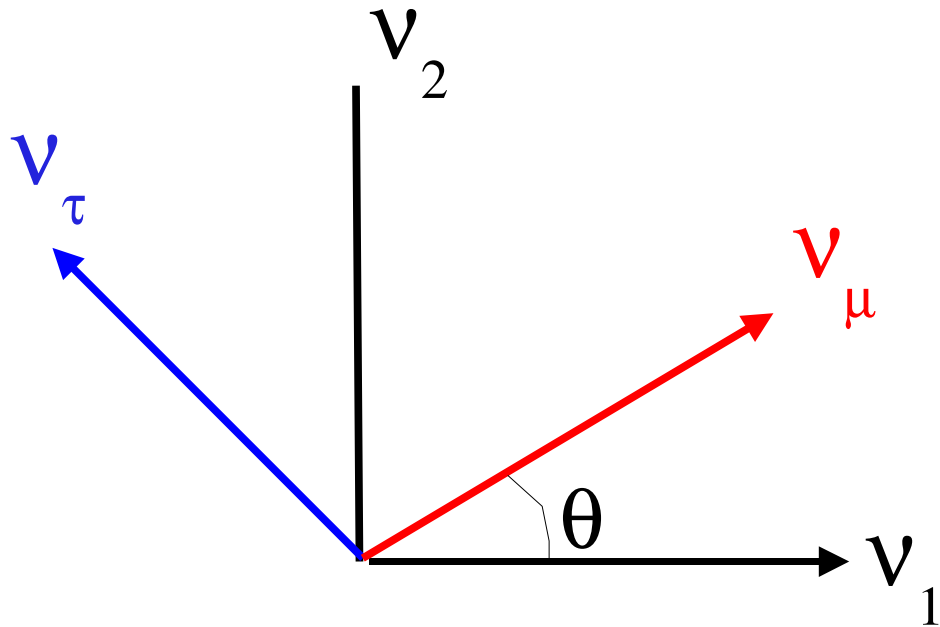
$$\{ |\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle \}$$

Mass basis

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V^{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

2 Flavor case



$$|\nu_\mu\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle$$

$$|\nu_\tau\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle$$

$$\Delta m^2 = m_2^2 - m_1^2$$

Neutrino Propagation

$$|\nu(0)\rangle = |\nu_\mu\rangle = \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle$$

ν_μ created at $t=0$
with momentum $\mathbf{p}$

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq E + \frac{m_i^2}{2E}$$

Different mass
components
have different energy

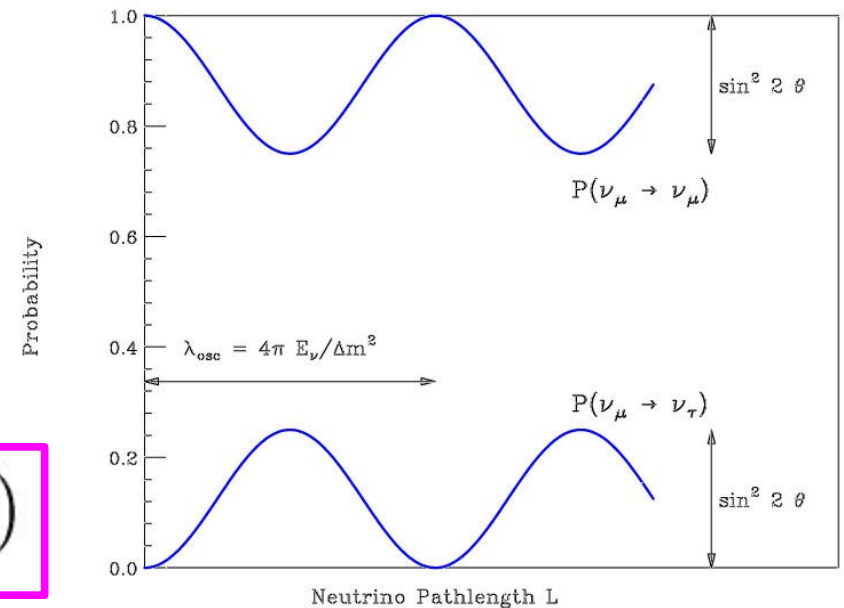
$$|\nu(t)\rangle = \cos\theta e^{-iE_1 t} |\nu_1\rangle + \sin\theta e^{-iE_2 t} |\nu_2\rangle$$

ν state at time t

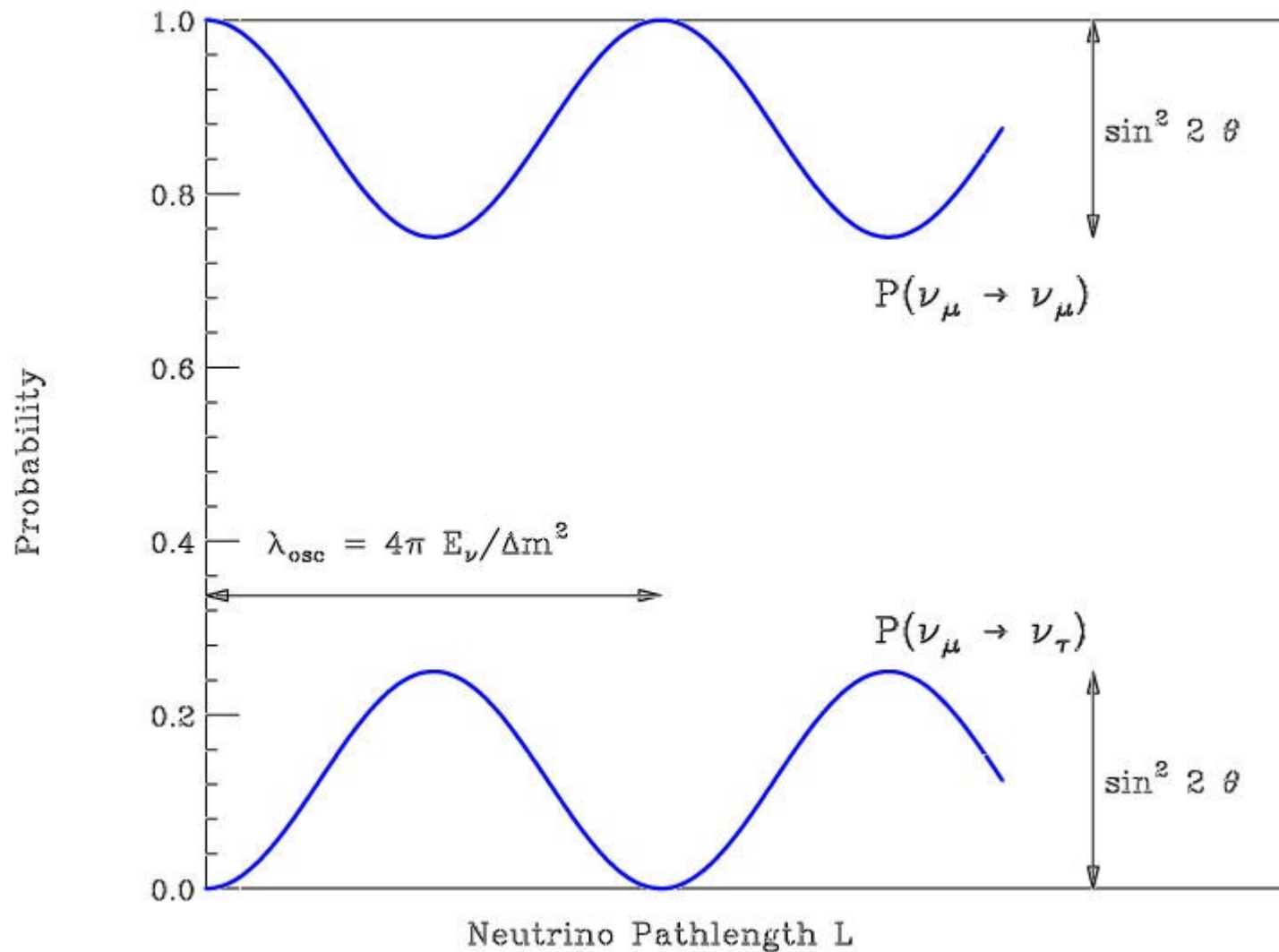
Oscillation Probability

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_\tau; t) &= \\
 &= |\langle \nu_\tau | \nu(t) \rangle|^2 \\
 &= |\{-\sin \theta \langle \nu_1 | + \cos \theta \langle \nu_2 | \} \{ \cos \theta e^{-iE_1 t} | \nu_1 \rangle + \sin \theta e^{-iE_2 t} | \nu_2 \rangle \}|^2 \\
 &= \cos^2 \theta \sin^2 \theta |e^{-iE_2 t} - e^{-iE_1 t}|^2 \\
 &= 2 \cos^2 \theta \sin^2 \theta \{1 - \cos[(E_2 - E_1)t]\} \\
 &= \sin^2 2\theta \sin^2 \left[\frac{\Delta m^2}{4E} t \right]
 \end{aligned}$$

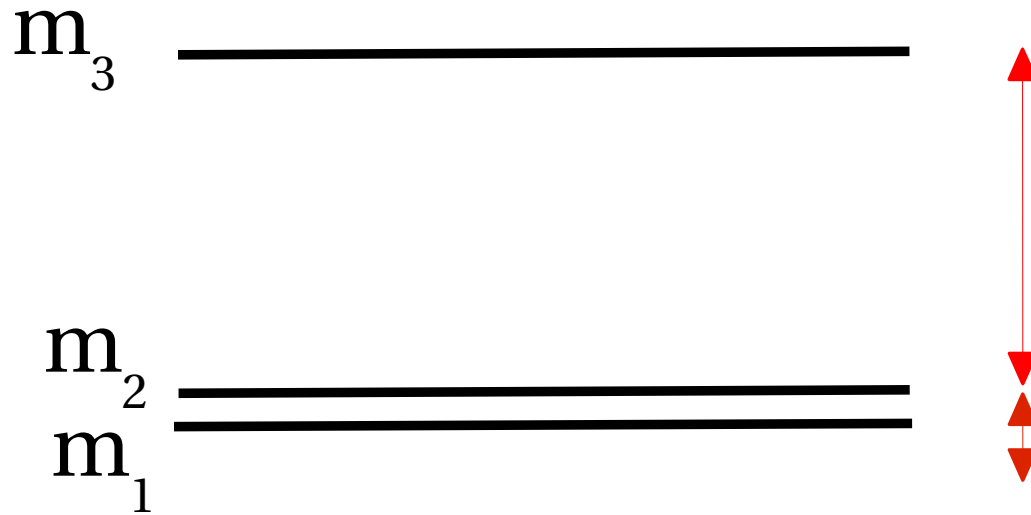
$$P(\nu_\mu \rightarrow \nu_\tau; t)$$



$$P(\nu_\mu \rightarrow \nu_\tau; L) = \sin^2 2\theta \sin^2 \left[1.27 \Delta m^2 (\text{eV}^2) \frac{L(\text{Km})}{E(\text{GeV})} \right]$$



3 Flavor Oscillations



$$|\nu_e\rangle = U_{e1}^* |\nu_1\rangle + U_{e2}^* |\nu_2\rangle + U_{e3}^* |\nu_3\rangle$$

$$|\nu_\mu\rangle = U_{\mu 1}^* |\nu_1\rangle + U_{\mu 2}^* |\nu_2\rangle + U_{\mu 3}^* |\nu_3\rangle$$

$$|\nu_\tau\rangle = U_{\tau 1}^* |\nu_1\rangle + U_{\tau 2}^* |\nu_2\rangle + U_{\tau 3}^* |\nu_3\rangle$$

Mixing Matrix: 3 angles, 1 phase

(relevant for neutrino oscillations)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

U^* : Mixing Matrix for Antineutrinos

More complex expressions
for the Oscillation Probabilities

3 - Flavor Transitions

$$|\nu(0)\rangle = |\nu_\alpha\rangle = \sum_j U_{\alpha j}^* |\nu_j\rangle$$

$$|\nu(t)\rangle = \sum_j U_{\alpha j}^* e^{-iE_j t} |\nu_j\rangle$$

$$\begin{aligned} A(\nu_\alpha \rightarrow \nu_\beta; t) &= \langle \nu_\beta | \nu(t) \rangle \\ &= \{U_{\beta k} \langle \nu_k | \} \left\{ e^{-iE_j t} U_{\alpha j}^* |\nu_j\rangle \right\} \\ &= U_{\beta k} U_{\alpha j}^* e^{-iE_j t} \langle \nu_k | \nu_j \rangle \\ &= U_{\beta j} U_{\alpha j}^* e^{-iE_j t} \end{aligned}$$

Oscillation Probability

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \left| \sum_j U_{\beta j} U_{\alpha j}^* e^{-i m_j^2 \frac{L}{2E_\nu}} \right|^2 \\ &= \sum_{j=1,3} |U_{\beta j}|^4 |U_{\alpha j}|^4 \\ &\quad + \sum_{j < k} 2 \operatorname{Re}[U_{\beta j} U_{\beta k}^* U_{\alpha j}^* U_{\alpha k}] \cos \left(\frac{\Delta m_{jk}^2 L}{2E} \right) \\ &\quad + \sum_{j < k} 2 \operatorname{Im}[U_{\beta j} U_{\beta k}^* U_{\alpha j}^* U_{\alpha k}] \sin \left(\frac{\Delta m_{jk}^2 L}{2E} \right) \end{aligned}$$

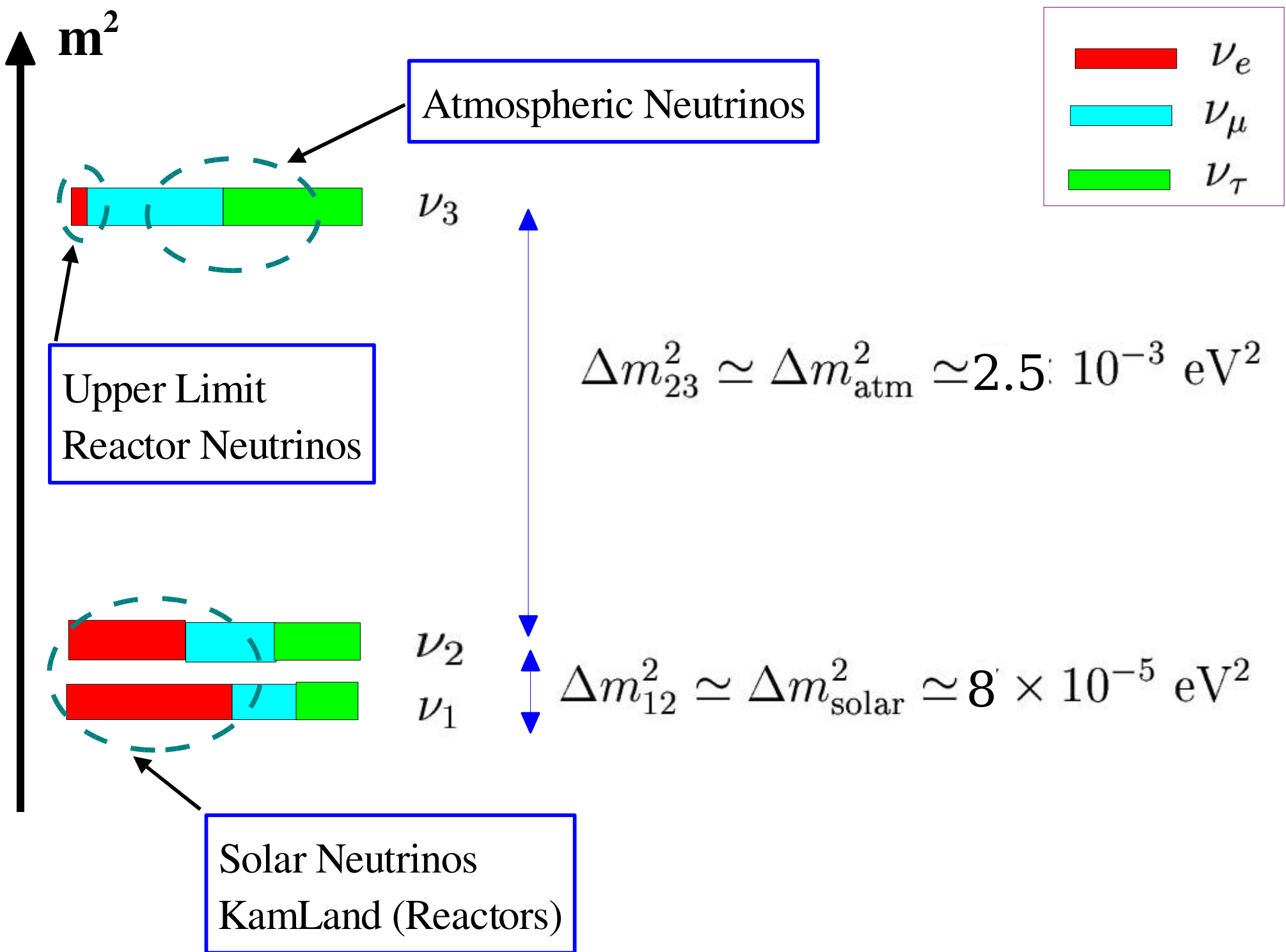
L, E_ν

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_j U_{\beta j} U_{\alpha j}^* e^{-i m_j^2 \frac{L}{2E_\nu}} \right|^2$$

$$P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \quad \mathbf{CP} \text{ violated}$$

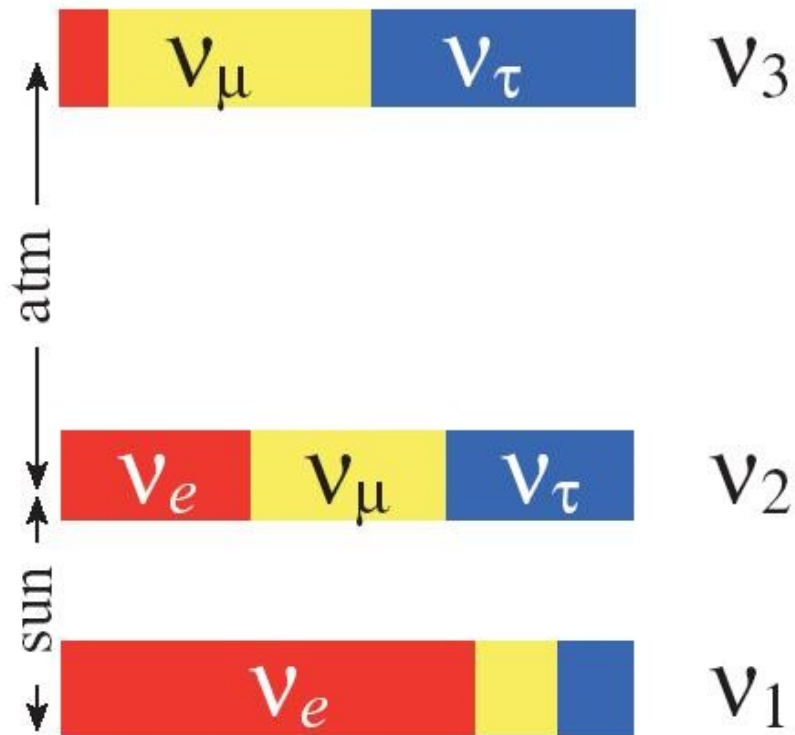
$$P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\nu_\beta \rightarrow \nu_\alpha) \quad \mathbf{T} \text{ violated}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) \quad \mathbf{CPT} \text{ conserved}$$

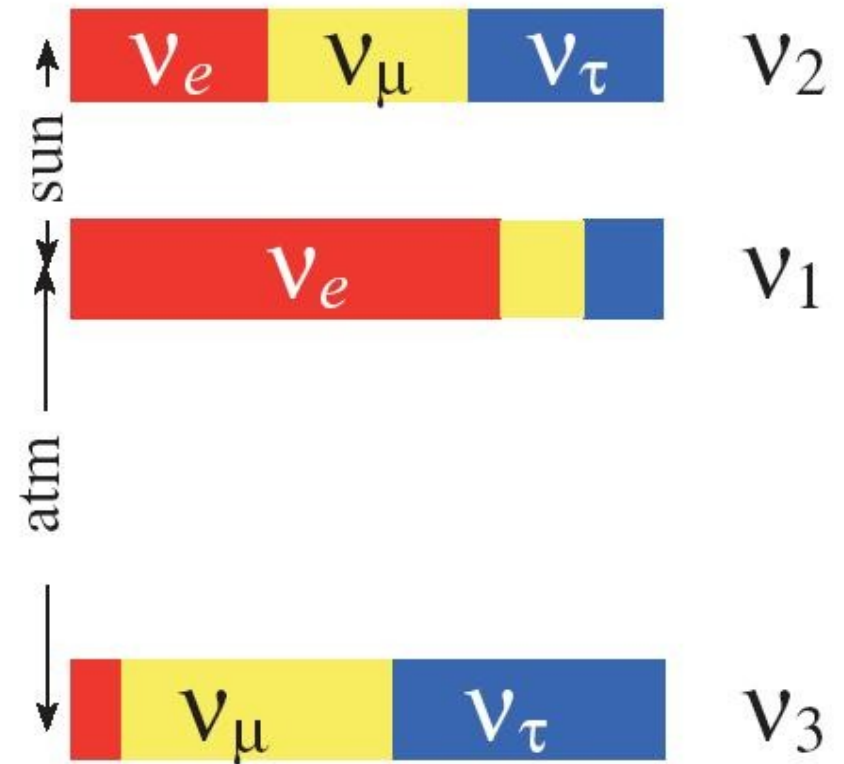


Neutrino MASS Hierarchy

Normal Hierarchy



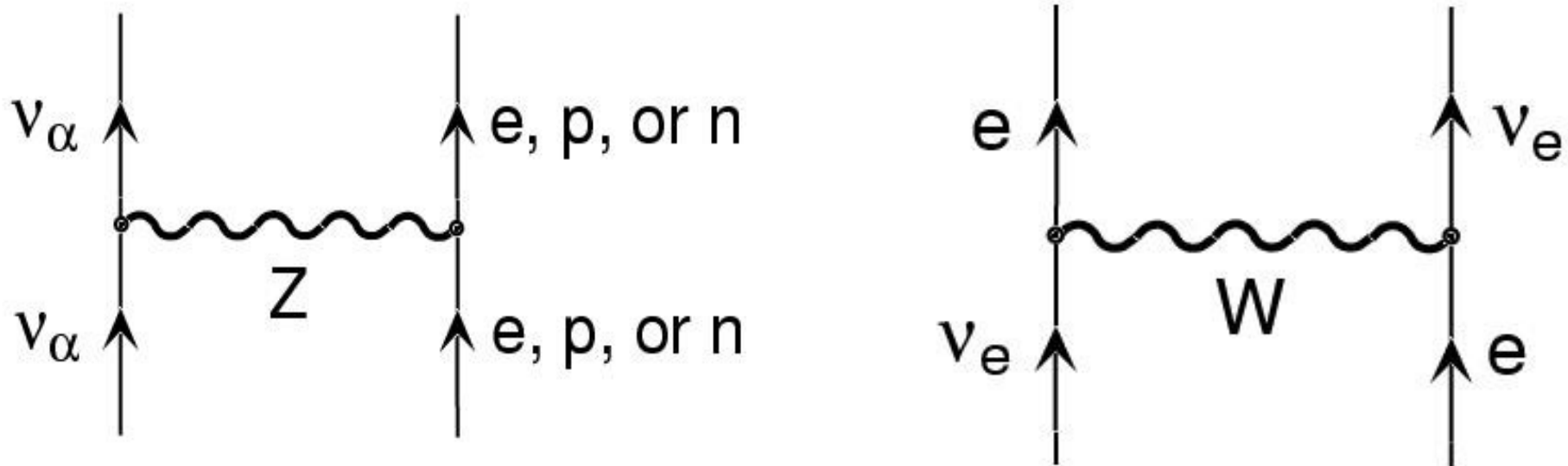
Inverted Hierarchy



Mass of the lightest Neutrino m_0

Matter Effects in Neutrino Oscillations

MATTER EFFECTS



Neutrino Propagation:

Amplitude =
Amplitude (No-Scattering) + Amplitude (forward Scattering)

EFFECTIVE POTENTIAL

$$V_{\nu_\mu e} = V_{\nu_\tau e} = V_{\nu_e e}^Z = -\frac{\sqrt{2}}{2} G_F N_e$$

$$V_{\nu_\mu p} = V_{\nu_\tau p} = V_{\nu_e p} = +\frac{\sqrt{2}}{2} G_F N_p$$

$$V_{\nu_\mu n} = V_{\nu_\tau n} = V_{\nu_e n} = -\frac{\sqrt{2}}{2} G_F N_n$$

$$V_{\bar{\nu}_\alpha} = -V_{\nu_\alpha}$$

$$V_{\nu_e e} = V_{\nu_e e}^Z + V_{\nu_e e}^W = -\frac{\sqrt{2}}{2} G_F N_e + \sqrt{2} G_F N_e$$

$$V \equiv V_{\nu_e} - V_{\nu_\mu} = V_{\nu_e} - V_{\nu_\tau} = +\sqrt{2} G_F N_e$$

$$3.8 \times 10^{-14} \left(\frac{\rho}{\text{g cm}^{-3}} \right) \left(\frac{Y_e}{0.5} \right) \text{ eV}$$

$$E - V = \sqrt{p^2 + m^2} \simeq p + \frac{m^2}{2p}$$

$$m^2 \rightarrow m^2 + 2EV.$$

$$i \frac{d}{dx} \nu_\alpha = \mathcal{H} \nu_\alpha$$

$$\mathcal{H}(\nu) = U \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{bmatrix} U^\dagger + \begin{bmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau \end{bmatrix}$$

NEUTRINO MASSES and MIXING

Fermion Masses : Yukawa Fermion-Higgs Coupling

$$\lambda_d (\overline{Q}_L \cdot H) d_R + \text{h.c.}$$

$$\lambda_u (\overline{Q}_{La} \epsilon_{ab} H_b^\dagger) u_R + \text{h.c.}$$

$$\lambda_d (\overline{u} \ \overline{d})_L \begin{pmatrix} H^+ \\ H_0 \end{pmatrix} d_R + \text{h.c.}$$

$$\lambda_d (\overline{u} \ \overline{d})_L \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} u_R + \text{h.c.}$$

$$\epsilon_{ab} = i(\tau^2)_{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\text{SU}(2)_L \otimes \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{em}}$$

$$\begin{pmatrix} H^+ \\ H_0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v \end{pmatrix}$$



$$m_u = \lambda_u v$$

$$m_d = \lambda_d v$$

QUARK MIXING

$$\lambda_{jk}^d (\overline{Q}_{Lj} \cdot H) (d_R)_k + \text{h.c.}$$

$$Y = -\frac{1}{6} \quad +\frac{1}{2} \quad -\frac{1}{3}$$

$$(M_{\text{down}})_{jk} = \lambda_{jk}^d v$$

Mass Matrix
for down quarks
{d,s,b}

$$\lambda_{jk}^u (\overline{Q}_{Lj} \epsilon H^\dagger) (u_R)_k + \text{h.c.}$$

$$Y = -\frac{1}{6} \quad -\frac{1}{2} \quad +\frac{2}{3}$$

$$(M_{\text{up}})_{jk} = \lambda_{jk}^u v$$

Mass Matrix
for up quarks
{u,c,t}

Linear Transformation of the fields

$$u'_L = U_u^{ij} u_L^j$$

$$u_L^i = \{u_L, c_L, t_L\}$$

$$d'_L = U_d^{ij} d_L^j$$

$$d_L^i = \{d_L, s_L, b_L\}$$

WEAK CURRENT

$$J_W^\alpha = \overline{u_L}^i \gamma_\alpha d_L^i$$

$$= \overline{u_L}^{'i} \gamma_\alpha (U_u^\dagger U_d)_{ij} d_L^j$$

$$= \overline{u_L}^{'i} \gamma_\alpha V_{ij} d_L^j$$

CKM Matrix

DIRAC , WEYL and MAJORANA SPINORS

DIRAC SPINOR

$$\psi = \psi_L + \psi_R$$

$$\psi_L = \left(\frac{1 - \gamma_5}{2} \right) \psi$$

$$\psi_R = \left(\frac{1 + \gamma_5}{2} \right) \psi$$

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

Weyl Representation

Charged Conjugation Operator

$$[i\gamma^\alpha (\partial_\alpha - ieA^\alpha) - m] \psi = 0$$

$$[i\gamma^\alpha (\partial_\alpha + ieA^\alpha) - m] \psi^c = 0$$

$$\{ [i\gamma^\alpha (\partial_\alpha - ieA^\alpha) - m] \psi = 0 \}^*$$

$$[-i\gamma^{\alpha*} (\partial_\alpha + ieA^\alpha) - m] \psi^* = 0$$

$$(C\gamma^0) \gamma^{\alpha*} (C\gamma^0)^{-1} = -\gamma^\alpha$$

Exchange
particle $\Leftrightarrow$ antiparticle

$$\psi^c = C \gamma_0 \psi^*$$

$$C = i\gamma^2 \gamma^0$$

$$(\psi^c)^c = \psi$$

The conjugate of
the (conjugate field)
goes back to the original field

$$(\psi_L)^c = (\psi^c)_R$$

$$(\psi_R)^c = (\psi^c)_L$$

The conjugate of a Left-field
is a Right-field

(and vice-versa)

$$(1 + \gamma_5) \psi = 0$$

$$\begin{aligned} (1 - \gamma_5) \psi^c &= (1 - \gamma_5) \gamma^2 \psi^* \\ &= \gamma^2 (1 + \gamma_5) \psi^* = 0 \end{aligned}$$

Weyl Representation

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\psi^c = C \gamma^0 \psi^* = \begin{pmatrix} +i\sigma^2 \psi_R^* \\ -i\sigma^2 \psi_L^* \end{pmatrix}$$

Majorana Field

Conjugate of itself

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\psi = \psi^c$$

$$\psi_M = \begin{pmatrix} \psi_L \\ -i\sigma^2 \psi_L^* \end{pmatrix} = \begin{pmatrix} \psi_L \\ (\psi_L)^c \end{pmatrix}$$

Mass Terms in the Lagrangian

$$-\mathcal{L}_{\text{mass}}^{\text{Dirac}} = m \bar{\psi} \psi = m (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)$$

$$-\mathcal{L}_{\text{mass}}^{\text{Majorana}} = m \left(\overline{(\psi_L)^c} \psi_L + \bar{\psi}_L (\psi_L)^c \right)$$

Majorana mass term
not invariant
for U(1) transformations

$$\begin{aligned} \psi &\rightarrow e^{i\alpha} \psi \\ \bar{\psi} &\rightarrow \bar{\psi} e^{-i\alpha} \end{aligned}$$

Include the 3-Right Neutrino Fields N_j

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{N}_i i \not{D} N_i$$

$$(\lambda_N^{ij} N^i L^j H + \frac{M_N^{ij}}{2} N_i N_j + \text{h.c.})$$

Dirac Masses

Majorana Masses

$$\begin{pmatrix} \nu_L & \nu_R \\ \nu_L & \nu_R \end{pmatrix} \begin{pmatrix} 0 & \boldsymbol{\lambda}_N^T v \\ \boldsymbol{\lambda}_N v & \mathbf{M}_N \end{pmatrix}$$

Terms bilinear
in the fields

Only One Family

$$\begin{array}{c} \nu_L \quad \nu_R \\ \nu_L \quad \left(\begin{array}{cc} 0 & m \\ m & M \end{array} \right) \\ \nu_R \end{array}$$

$$\begin{array}{c} \nu_L \quad \nu_R \\ \nu_L \quad \left(\begin{array}{cc} 0 & \lambda v \\ \lambda v & M \end{array} \right) \\ \nu_R \end{array}$$

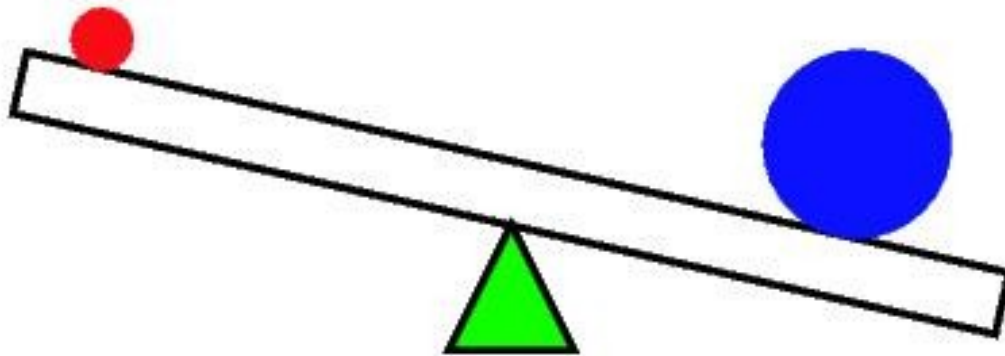
$$\frac{M}{2} \pm \frac{\sqrt{M^2 + 4m^2}}{2}$$

Eigenvalues

$$-\frac{m^2}{M} \quad \left\{ 1, -\frac{m}{M} \right\}$$

$$M + \frac{m^2}{M} \quad \left\{ \frac{m}{M}, 1 \right\}$$

The SEE-SAW Mechanism



$$m_v = m^2/M$$

What High-Mass scale is implied by the neutrino mass determinations ?

$$|\Delta m_{\text{atm}}^2| \simeq m_3^2 - m_1^2 \simeq m_3^2 \simeq \left(\frac{m_{\text{top}}^2}{M} \right)^2$$

$$M \sim \frac{m_{\text{top}}^2}{\sqrt{|\Delta m_{\text{atm}}^2|}} \sim 0.6 \times 10^{15} \text{ GeV}$$

Additional Phases in the Mixing Matrix (for Majorana neutrinos)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Majorana
Phases

Non detectable
in oscillation
experiments

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{i\varphi_2}{2}} & 0 \\ 0 & 0 & e^{\frac{i\varphi_3}{2}} \end{pmatrix}$$

DIRAC and Majorana phases

$$N_{(\text{physical phases})}^{\text{Dirac}} = \frac{n(n+1)}{2} - (2n-1)$$

$(n-1)$ “Majorana” phases

Phases can be eliminated
redefining the fermion fields

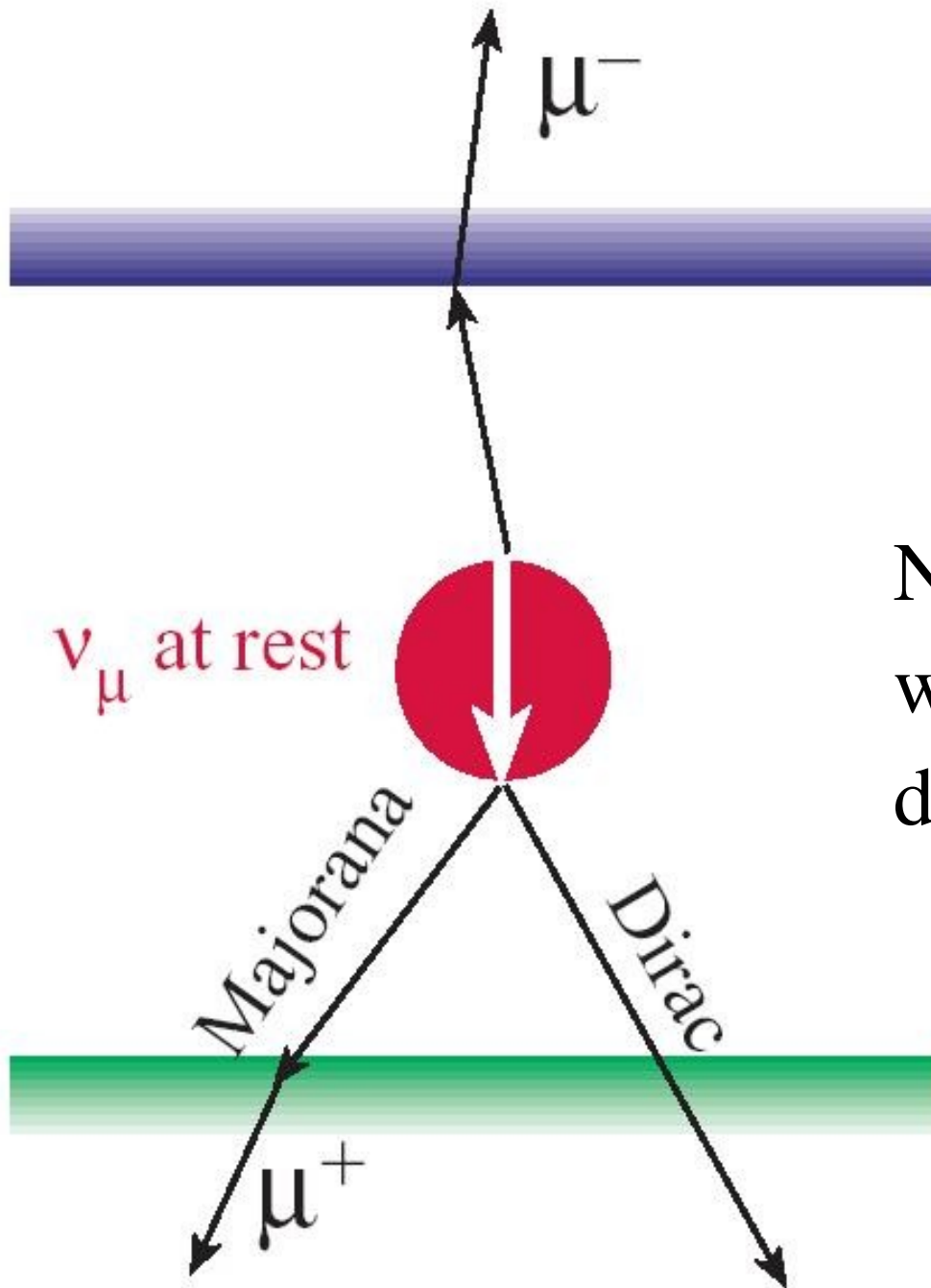
$$\lambda_{jk}^u \left(\overline{Q}_{Lj} \epsilon H^\dagger \right) (u_R)_k + \text{h.c.}$$

DIRAC or MAJORANA ?

$$\begin{array}{cc} \nu_L & \nu_R \\ \bar{\nu}_L & \bar{\nu}_R \end{array}$$

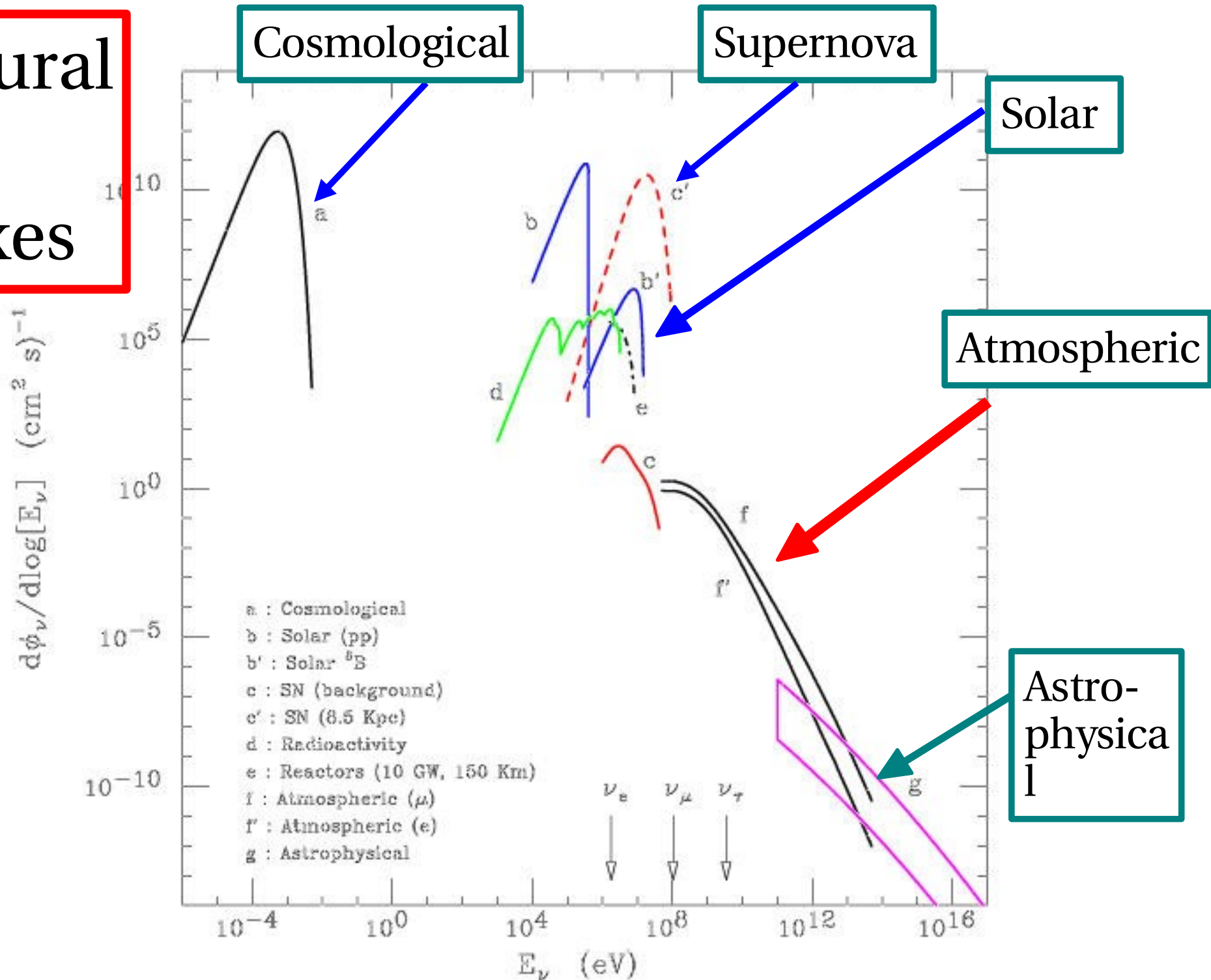
$$\begin{array}{cc} \nu_L & \\ & \bar{\nu}_R \end{array}$$

Gedanken Experiment



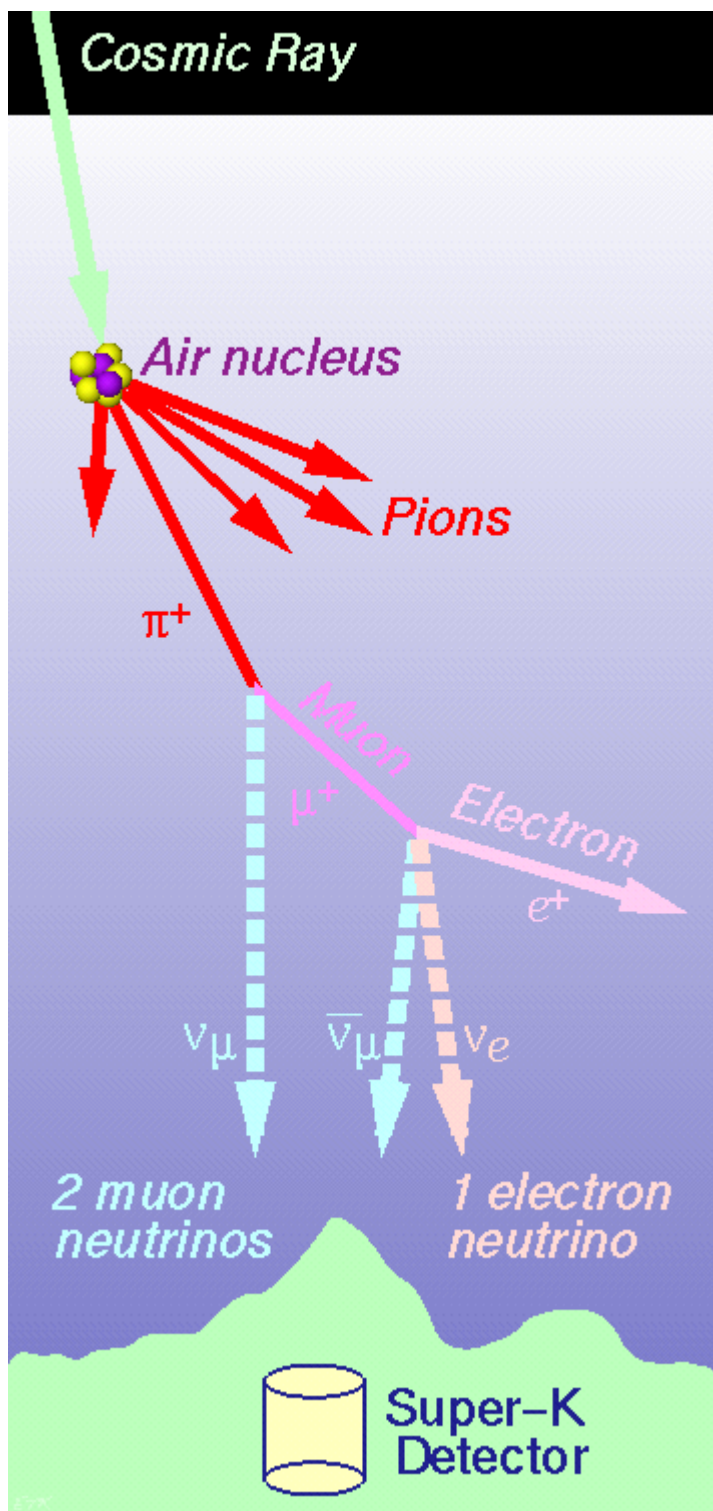
Neutrino at Rest
with spin pointing
downward.

Natural ν Fluxes

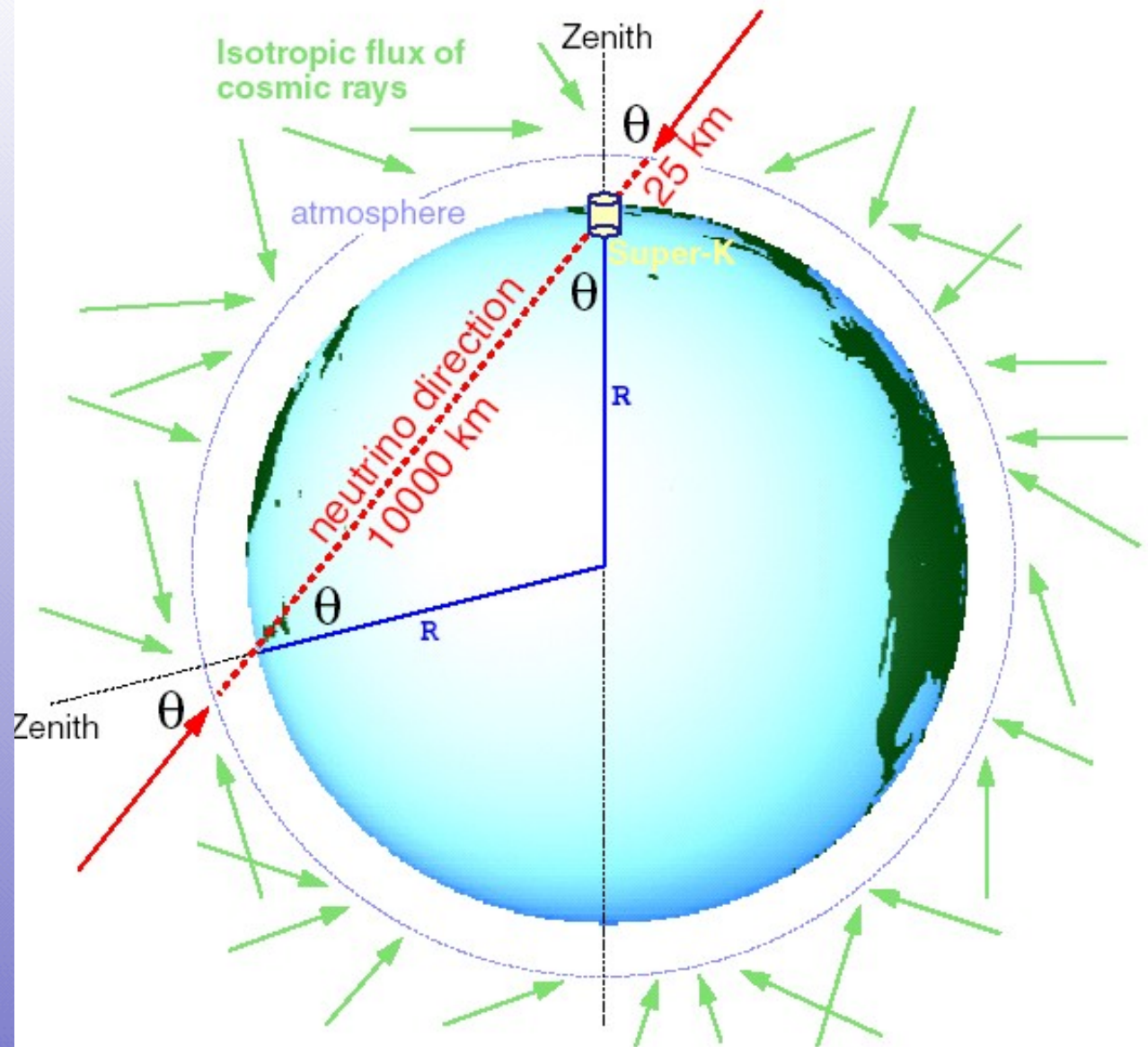


ATMOSPHERIC NEUTRINOS

Cosmic Ray

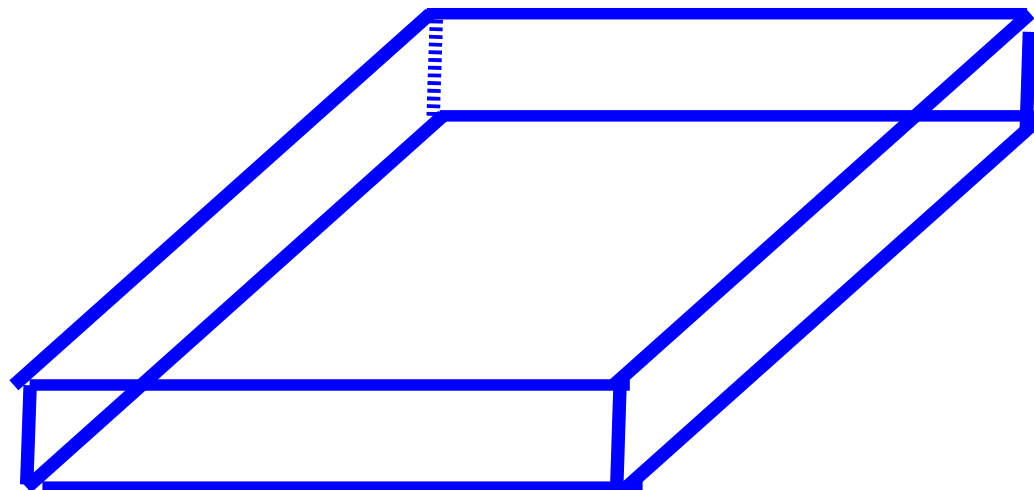
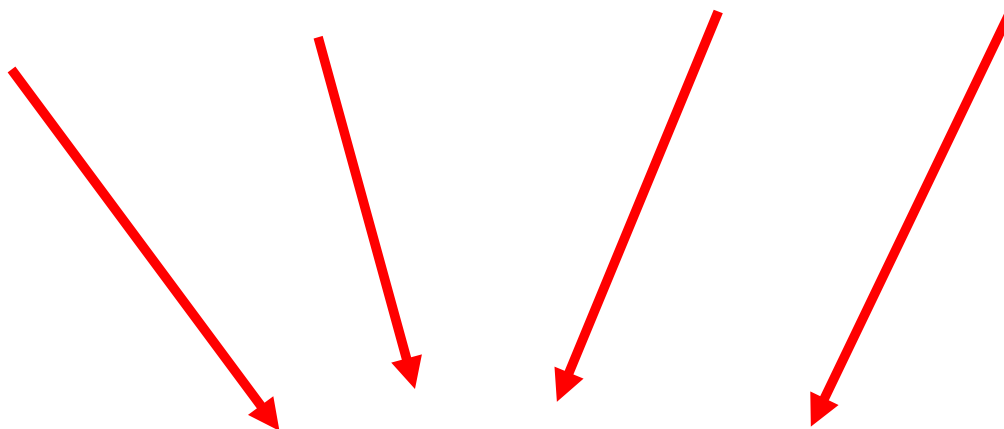


ATMOSPHERIC NEUTRINOS



Up-Down Symmetric Flux
(for $E_\nu > \text{few GeV}$)

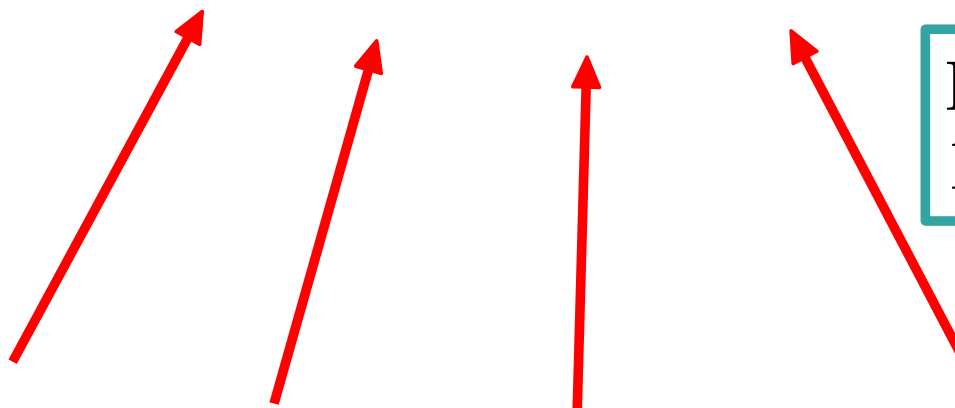
$\approx 3000 \nu_{\mu}$
 $\approx 3000 \bar{\nu}_{\mu}$
 $\approx 1600 \nu_e$
 $\approx 1400 \bar{\nu}_e$



1 m^2
 1 sec

Expected

$\approx 3000 \nu_{\mu}$
 $\approx 3000 \bar{\nu}_{\mu}$
 $\approx 1600 \nu_e$
 $\approx 1400 \bar{\nu}_e$



Event Rate
 $100 / (\text{Kton year})$

Detailed prediction:

Primary Cosmic Rays

Hadronic interactions

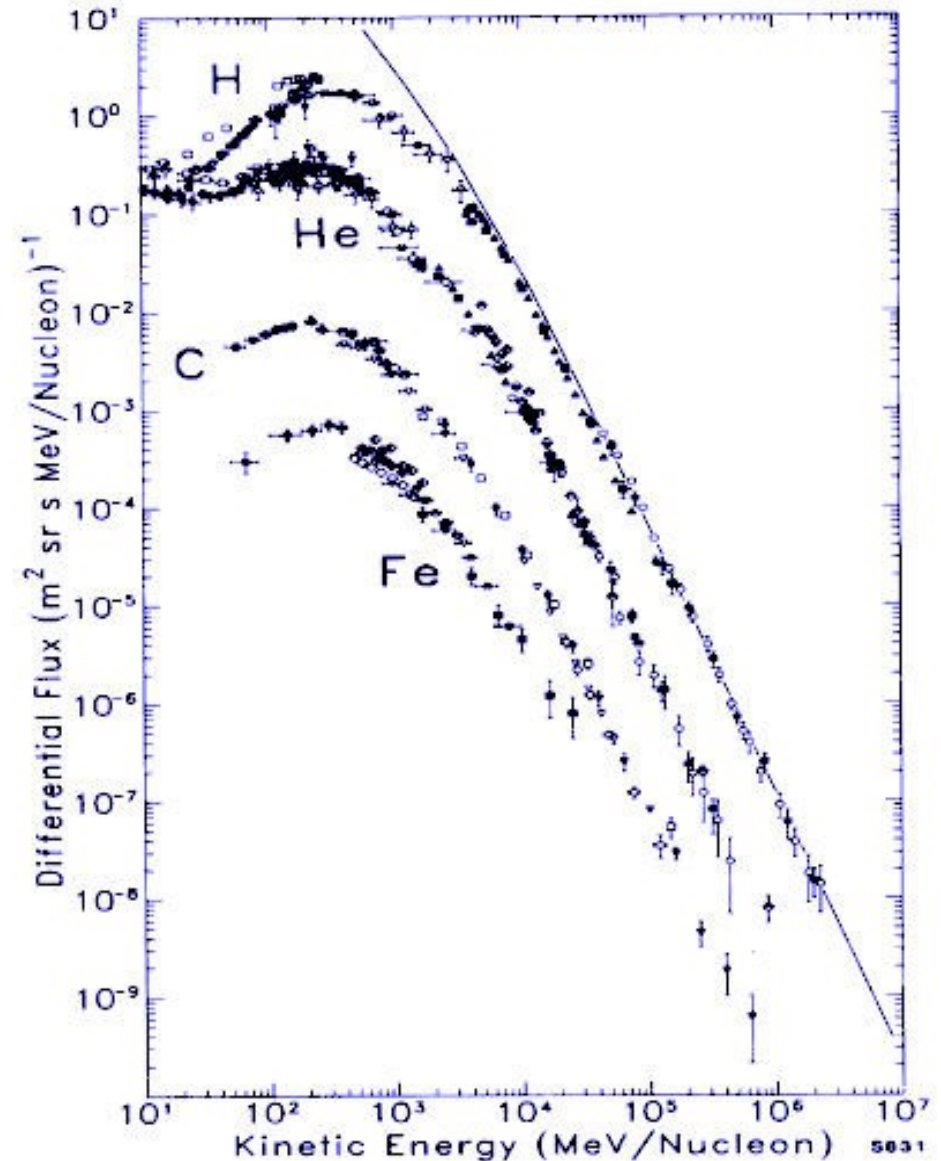
Neutrino Cross section

The Primary Cosmic Ray Fluxes

Interstellar Fluxes

Solar
Modulation

Geomagnetic
Effects



HADRONIC INTERACTIONS

$$p + A_{\text{Air}} \rightarrow p, n, \pi^0, \pi^\pm, K^\pm, K_L \dots$$

Leading nucleon
50% of energy

$\pi^0 \rightarrow \gamma\gamma$
Electromagnetic
Shower

Decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

↓

$$e^+ + \nu_e + \bar{\nu}_\mu$$

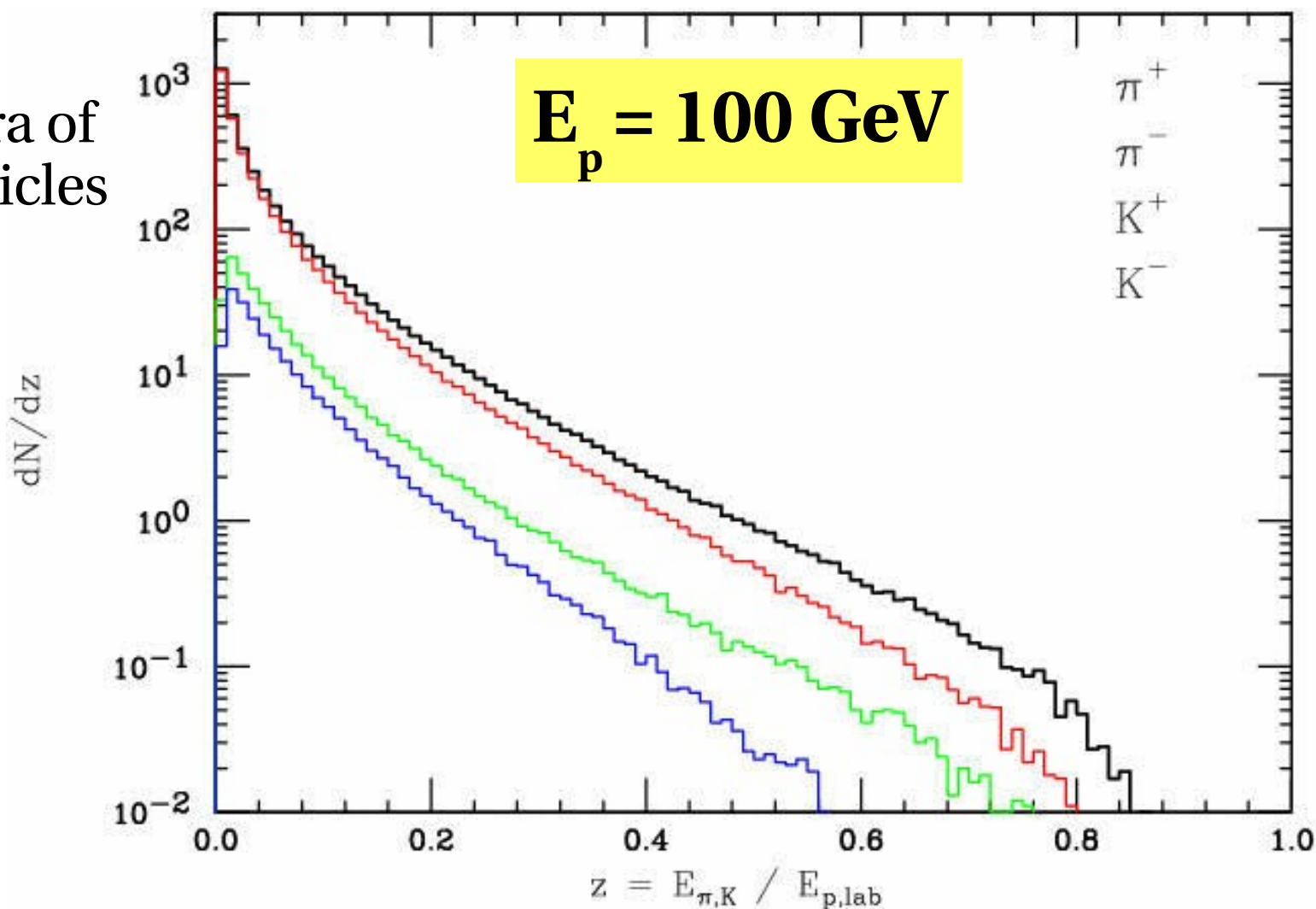
Interaction



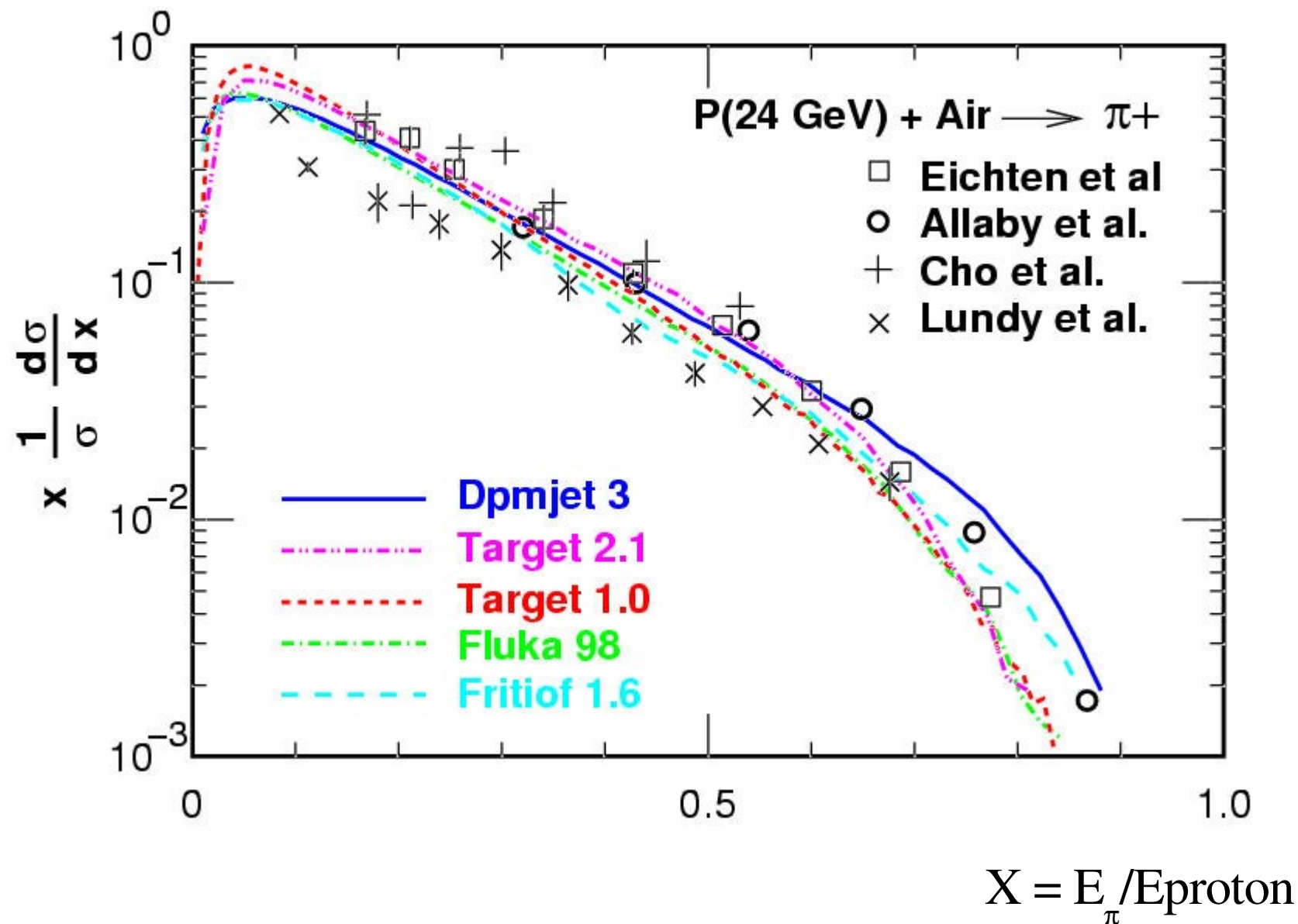
HADRONIC INTERACTIONS

A precise knowledge of hadronic interactions is essential for the calculation of the atmospheric neutrino fluxes.

Inclusive spectra of secondary particles



COMPARISON with DATA

 π^+ 

Constraint from Atmospheric Muon measurements

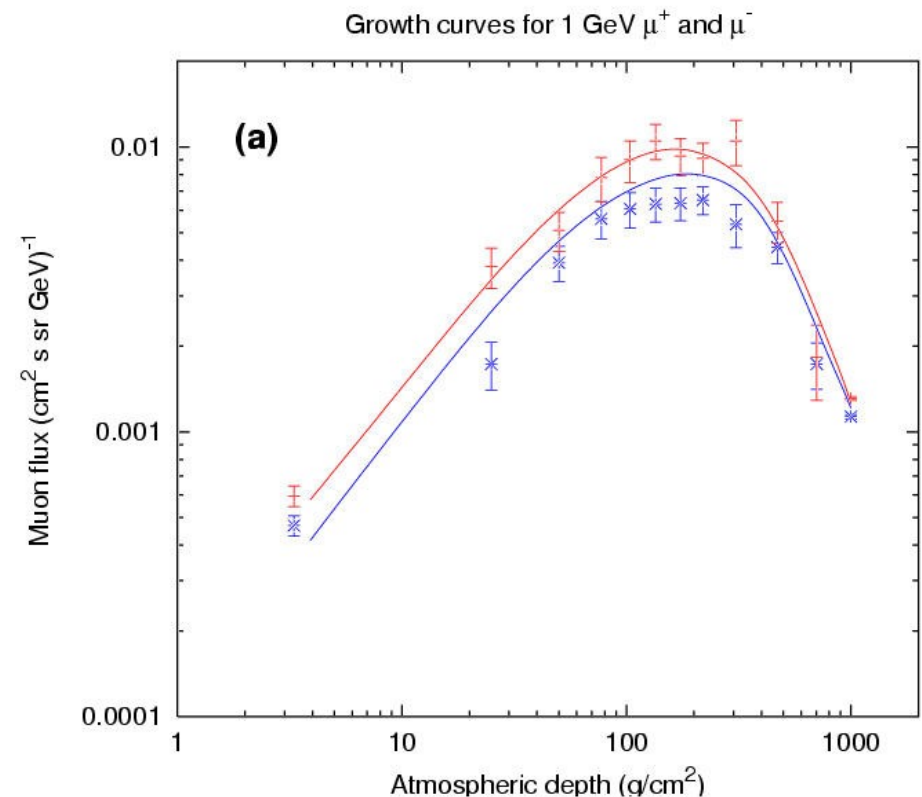
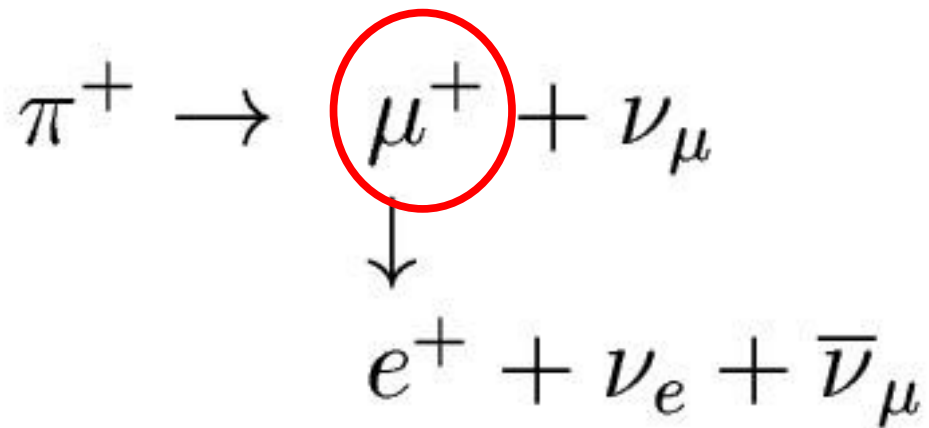
Measurements

Ground level

Balloon Float

Balloon Ascent

$$p + A_{\text{Air}} \rightarrow p, n, \pi^{\pm}, \pi^0, K^{\pm}, \dots$$



2 Very Robust Properties of the Atmospheric Neutrino Fluxes

Strict Relation between the fluxes of different flavor

$$\frac{\nu_{\mu} + \bar{\nu}_{\mu}}{\nu_e + \bar{\nu}_e} \simeq 2$$

Up-Down Symmetry of the Fluxes

$$\phi_{\nu_{\alpha}}(E, \theta) = \phi_{\nu_{\alpha}}(E, \pi - \theta)$$

$$\pi^+ \rightarrow \nu_\mu \mu^+ \rightarrow \nu_\mu \bar{\nu}_\mu \nu_e e^+$$

$$\pi^- \rightarrow \bar{\nu}_\mu \mu^- \rightarrow \bar{\nu}_\mu \nu_\mu \bar{\nu}_e e^-$$

$$\frac{\nu_e}{\bar{\nu}_e} \simeq \frac{\pi^+}{\pi^-} \simeq 1.2$$

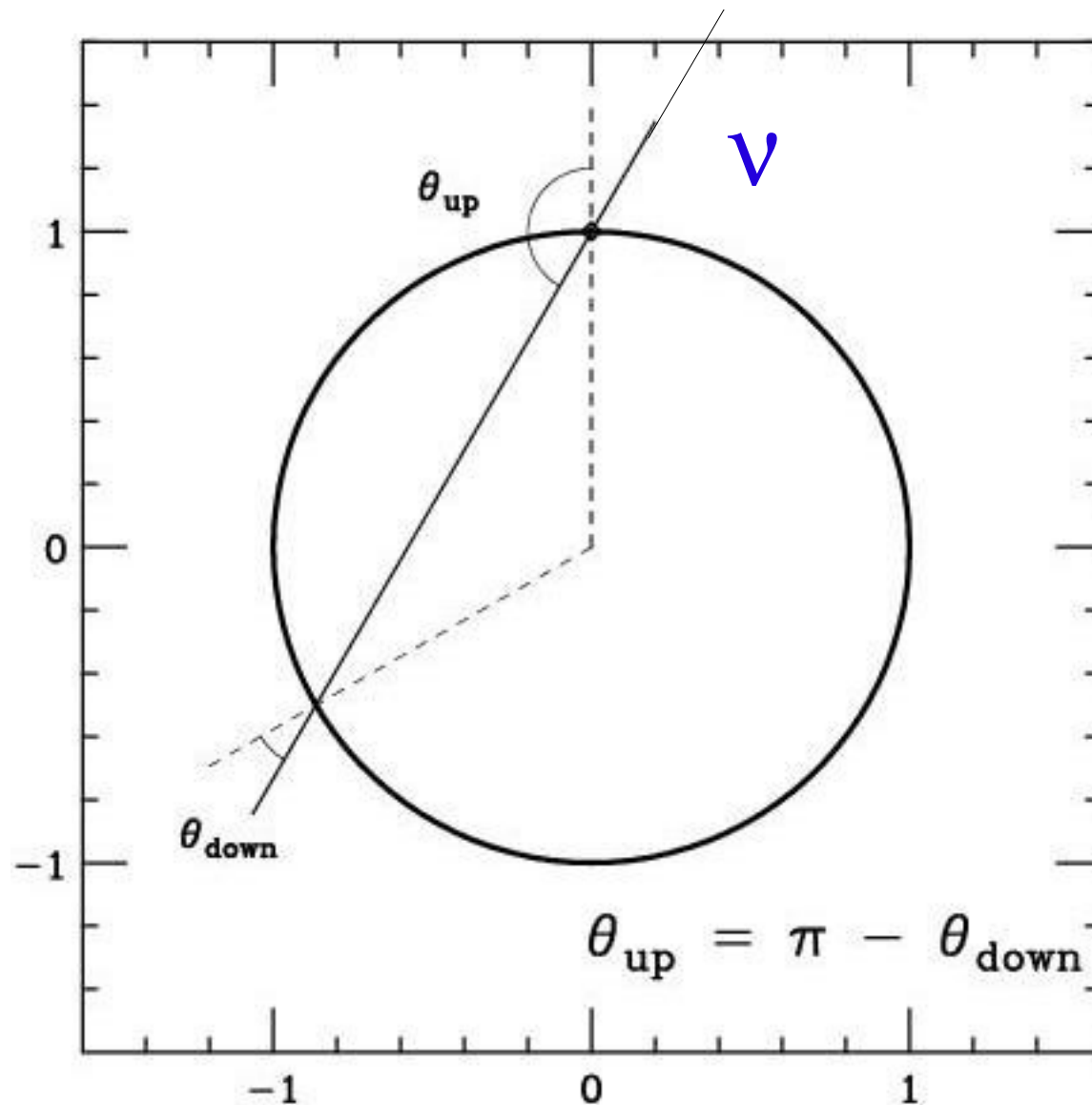
$$\frac{\nu_\mu}{\bar{\nu}_\mu} \simeq 1$$

$$\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} \simeq 2$$

Assume all muons decay
AND
 an important kinematical fact.
 All 3 neutrinos in decay
 have approximately the same
 energy

GEOMETRY THEOREM

$$\phi_{\nu_\alpha}(E, \theta) = \phi_{\nu_\alpha}(E, \pi - \theta)$$



If:
the Earth is spherical
Cosmic Rays are Isotropic

Then:

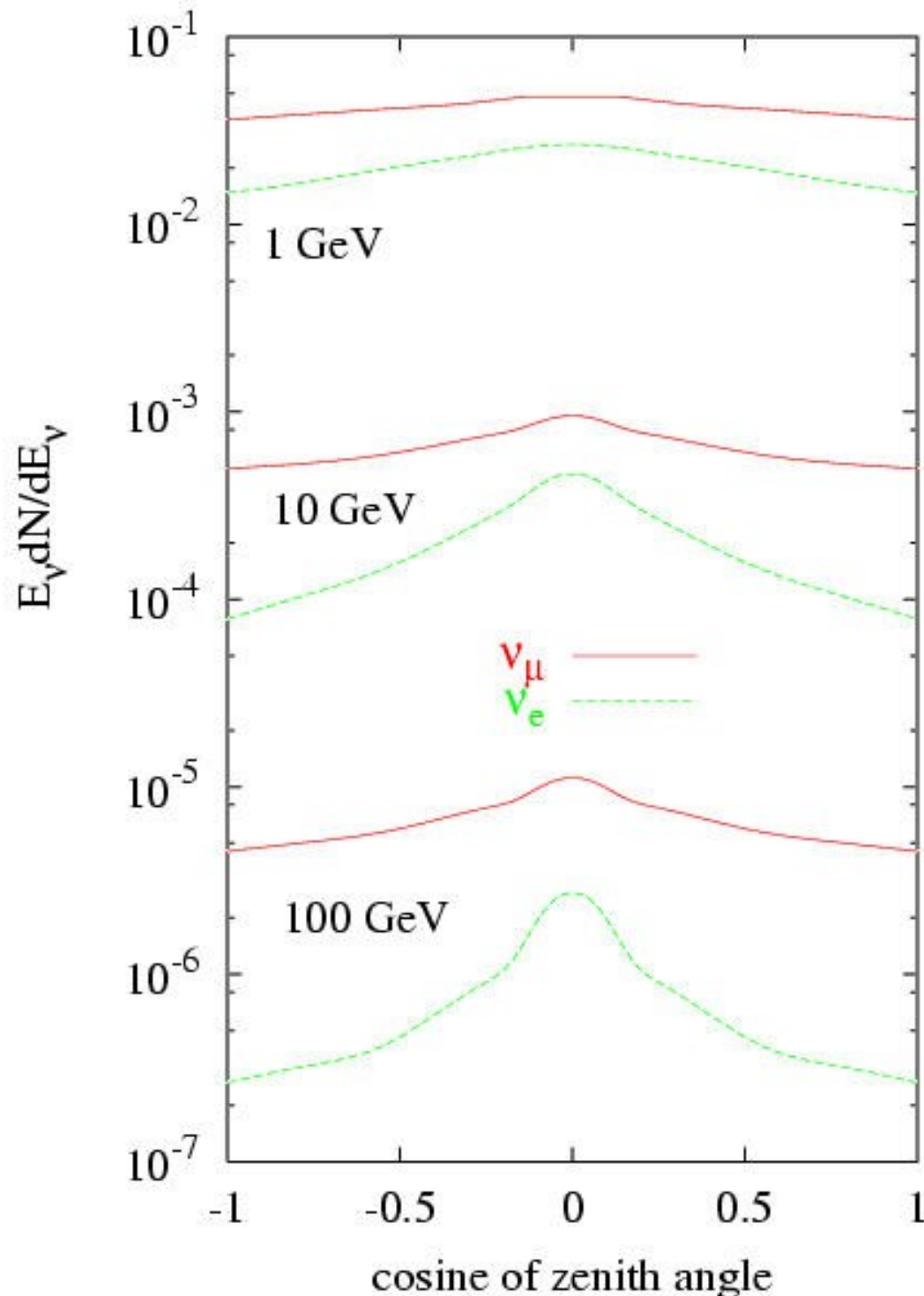
Neutrino Fluxes are
Up-Down Symmetric

Each neutrino
"enters" the Earth as
downgoing

with zenith angle θ_{down}

"exits" the Earth as upgoing
with zenith angle

$$\theta_{\text{up}} = \pi - \theta_{\text{down}}$$



Zenith angle distribution
is Up-Down symmetric

Zenith angle distribution
maximum $|\cos \theta| = 0$
minimum $|\cos \theta| = 1$
(μ and π decay effect)

Ratio ν_μ / ν_e
is energy dependent
(grows with increasing energy)

Ratio ν_μ / ν_e
is zenith angle dependent
(grows with $|\cos \theta|$)

The Neutrino Cross section

$$\nu_\ell + \text{Nucleus} \rightarrow \ell^- + X$$

Charged
Current

$$\nu_\ell + \text{Nucleus} \rightarrow \nu_\ell + X$$

Neutral
Current

$$\nu_\ell + n \rightarrow \ell^- + p$$

$$\bar{\nu}_\ell + p \rightarrow \ell^+ + n$$

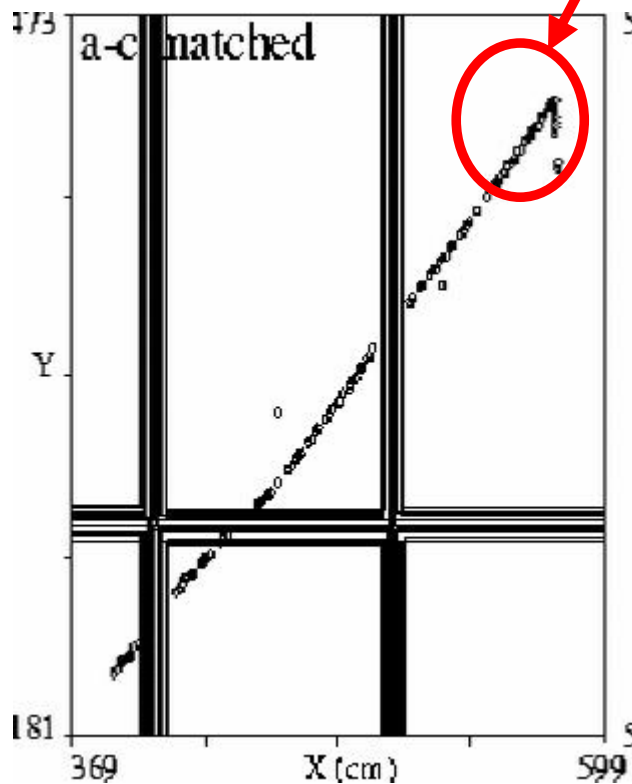
Quasi Elastic Scattering

Atmospheric Neutrino events

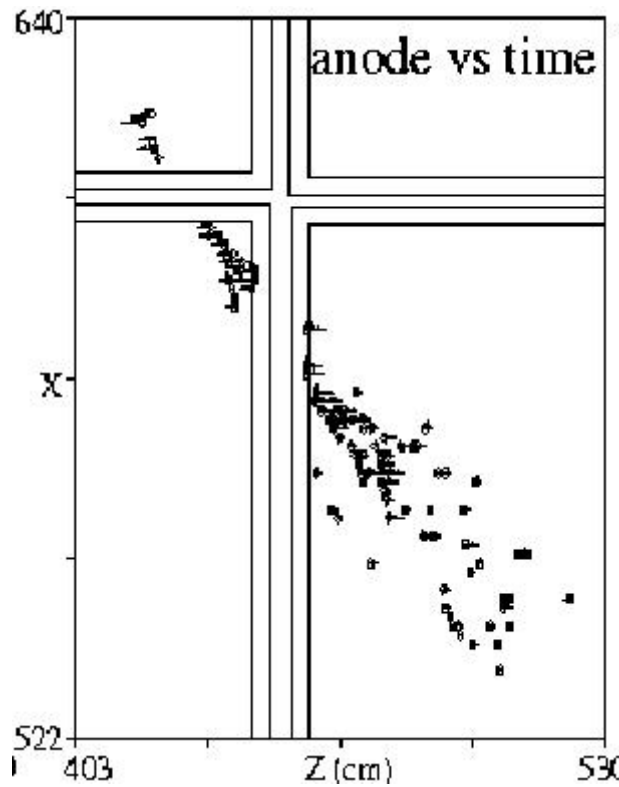
Soudan-2 detector

ν interaction
vertex

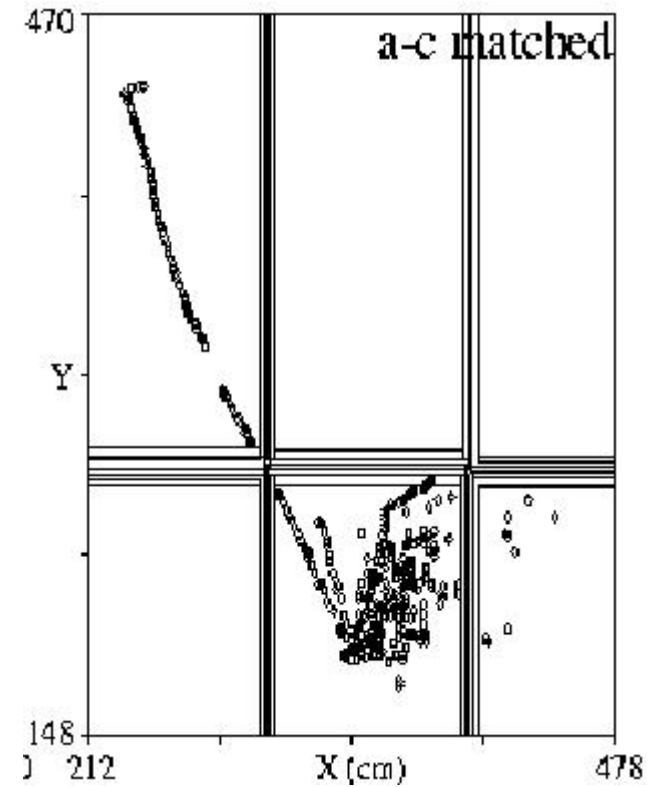
$$\nu_{\mu} + n \rightarrow \mu^{-} + p$$



$$\nu_e + N \rightarrow e^{\pm} + N'$$



$$\nu_{\mu} + N \rightarrow \mu^{\pm} + X$$



IMB detector

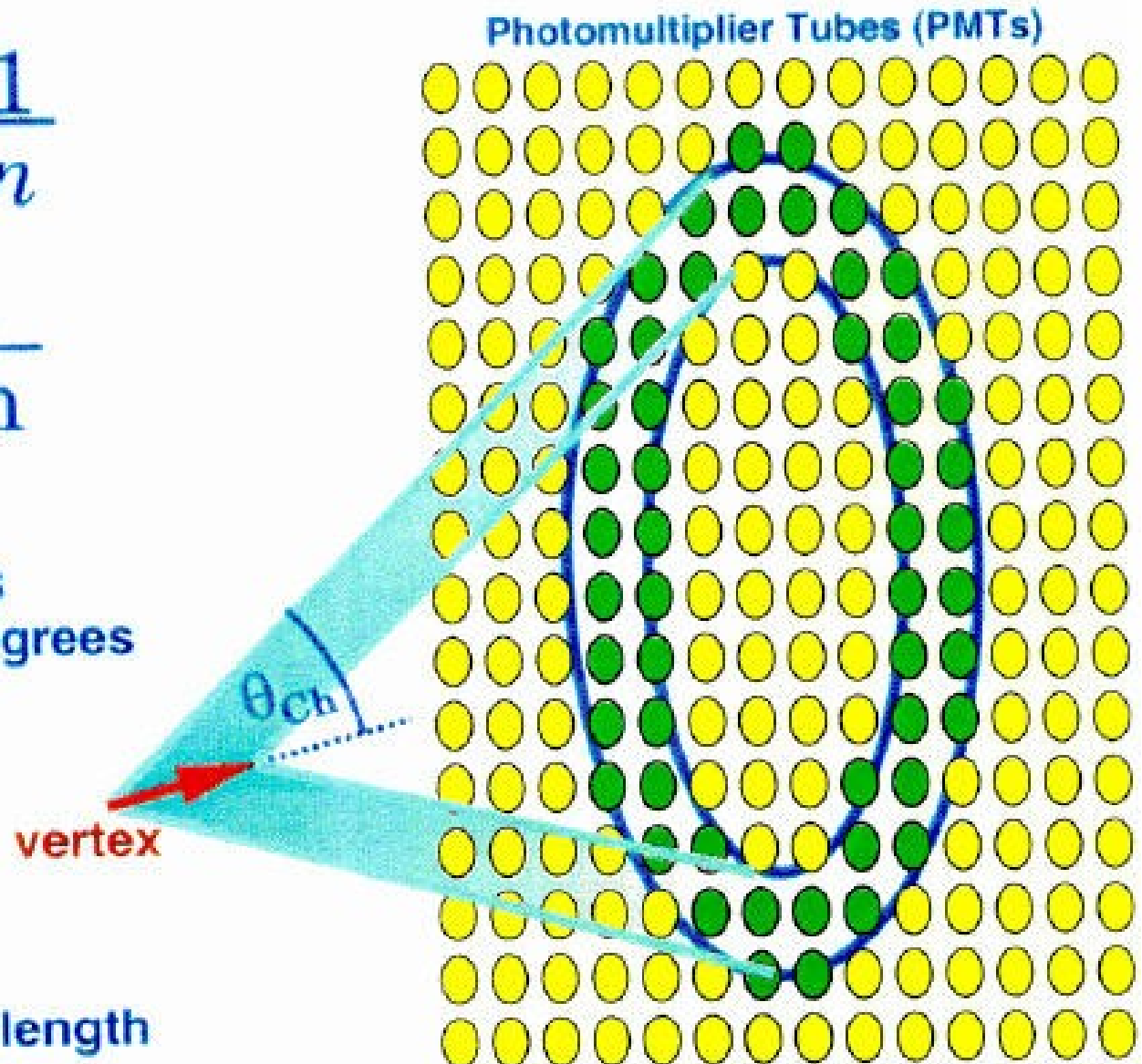


Cherenkov Radiation

$$\beta \left(= \frac{v}{c} \right) > \frac{1}{n}$$

$$\cos \theta_{\text{Ch}} = \frac{1}{\beta n}$$

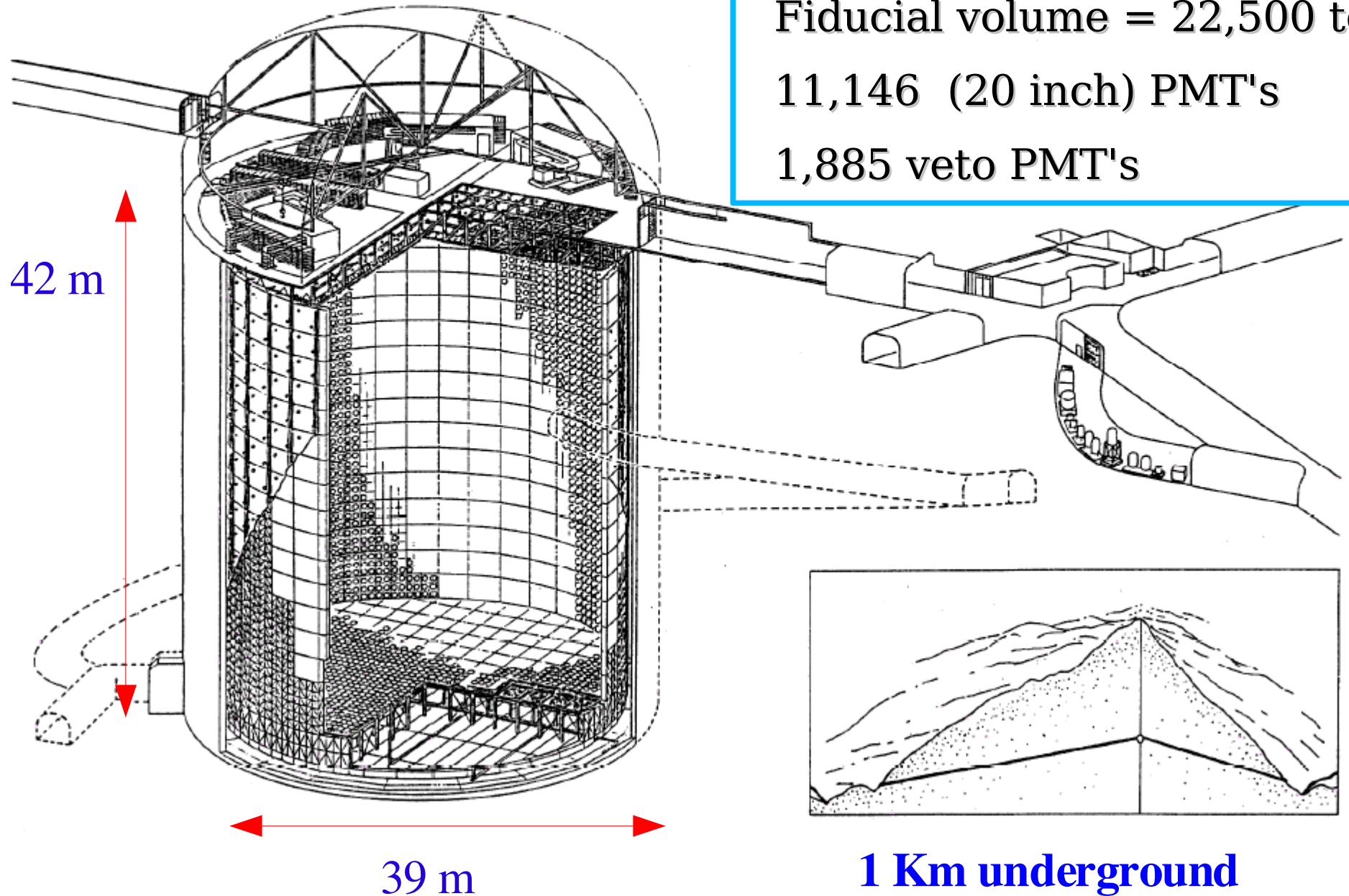
in water, $n = 1.33$
as $\beta \rightarrow 1$, $\theta_{\text{Ch}} \rightarrow 41$ degrees



~340 photons/cm pathlength
 $300 \text{ nm} < \lambda < 600 \text{ nm}$

SuperKamiokande detector

50,000 tons of ultrapure water
2 m of water = veto counter
Fiducial volume = 22,500 tons
11,146 (20 inch) PMT's
1,885 veto PMT's

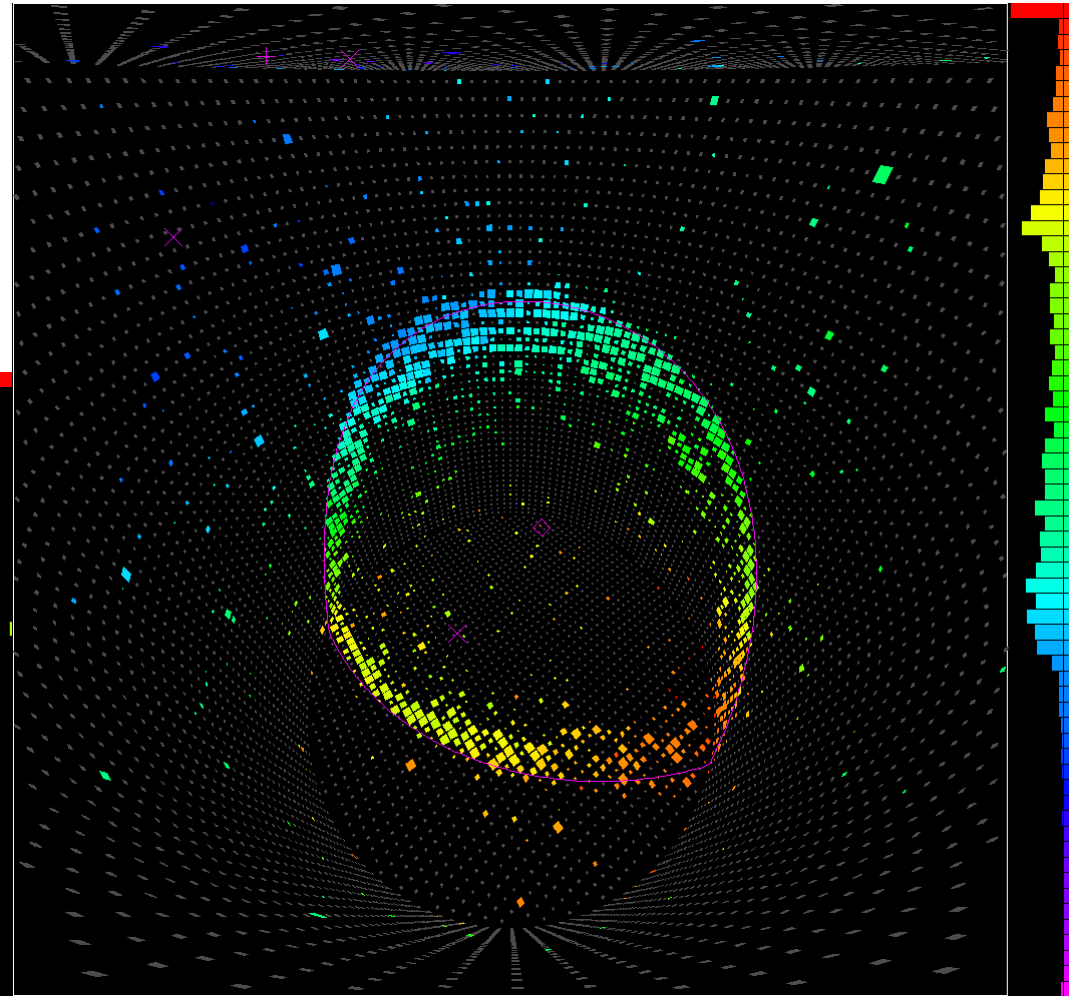
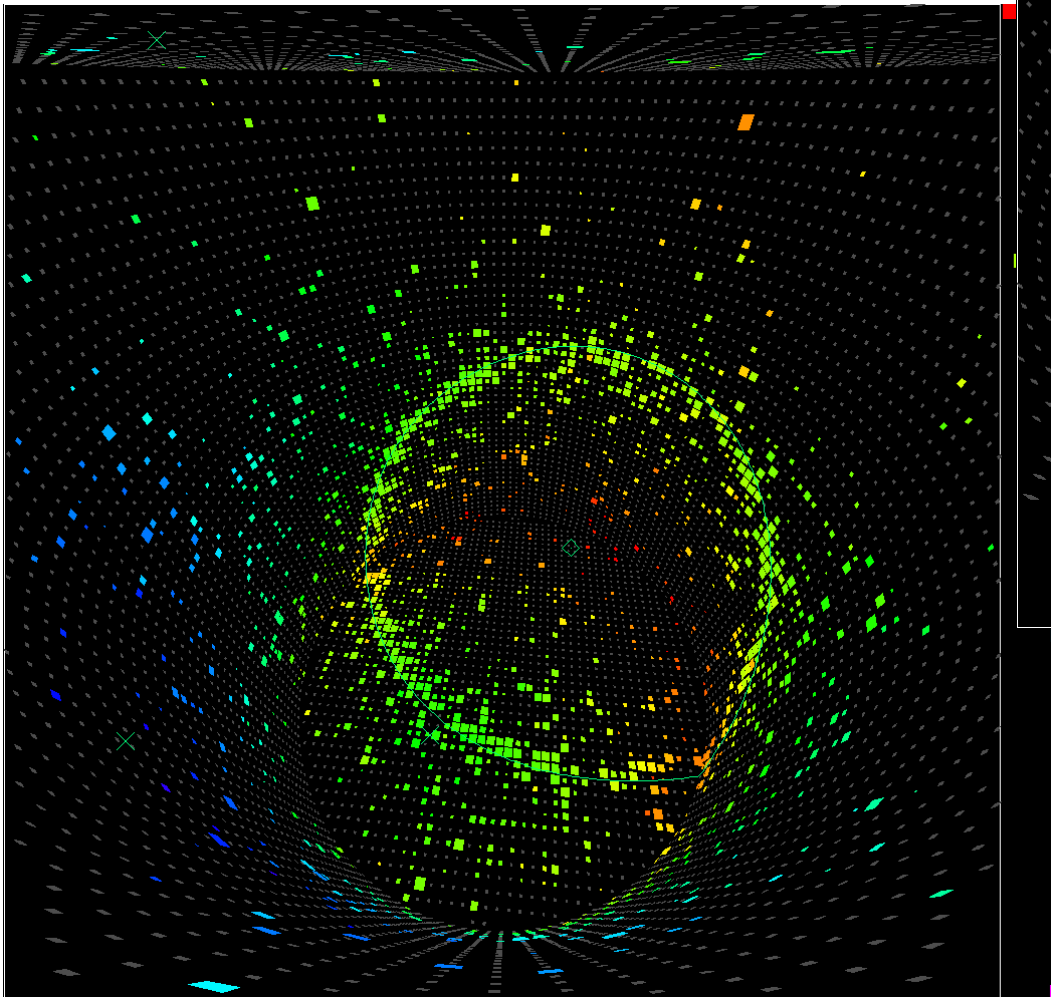


11,146 20 inch Photomultipliers (PMT's)
(40 % of surface is sensitive)





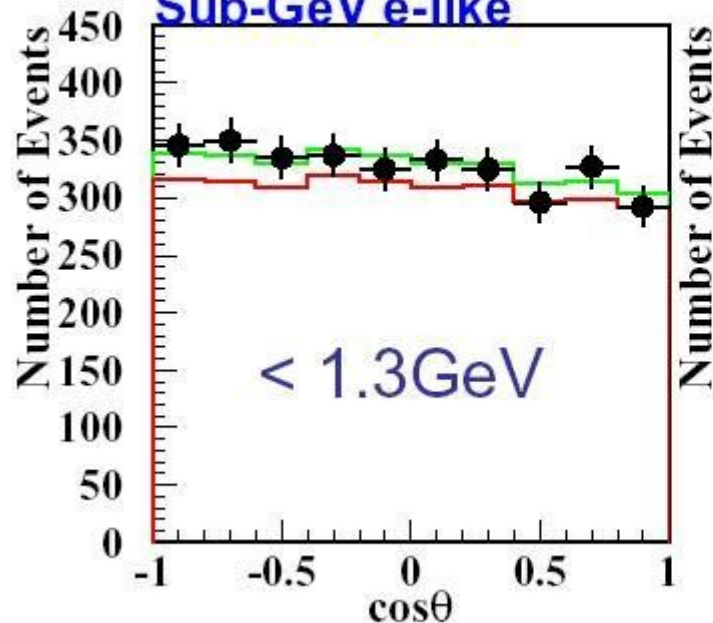
e-like



μ -like

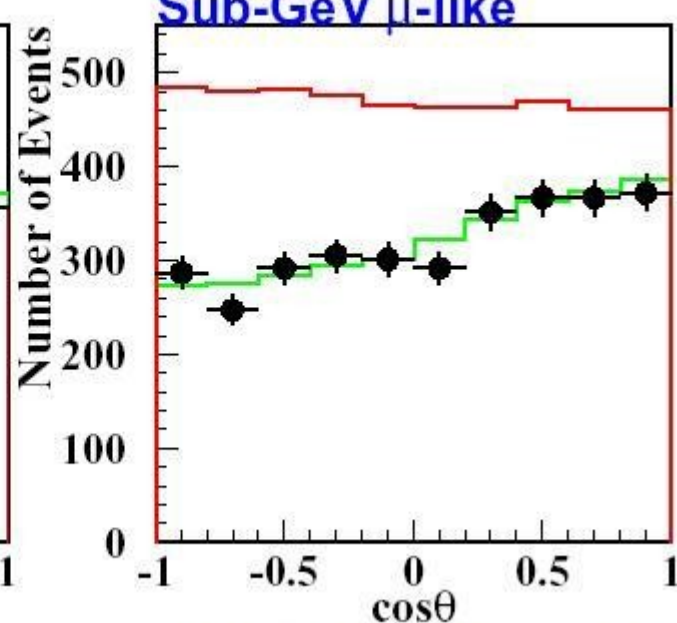
1-ring e -like

Sub-GeV e -like

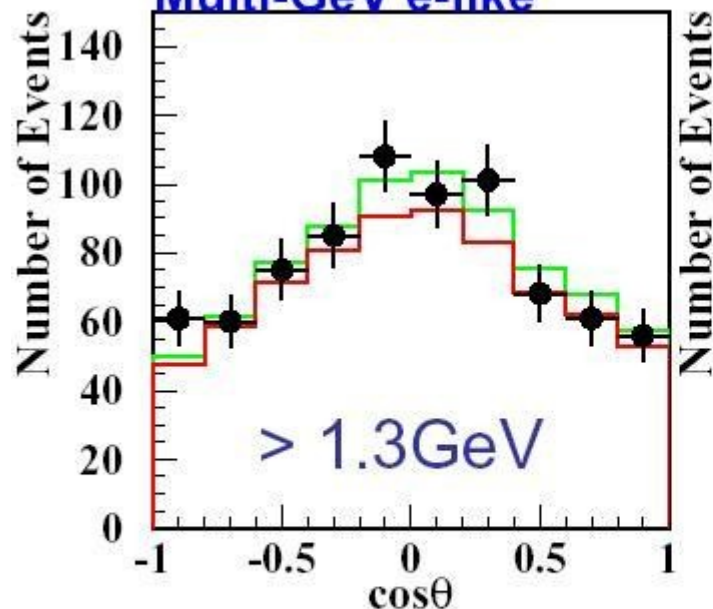


1-ring μ -like

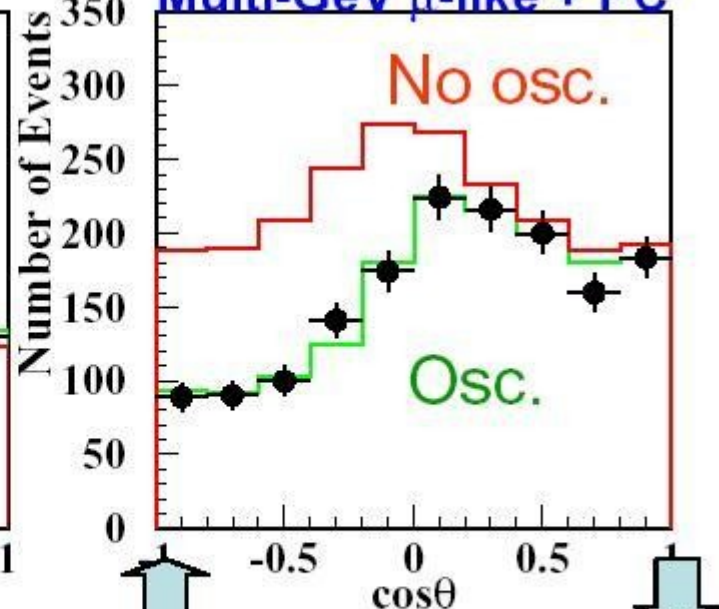
Sub-GeV μ -like



Multi-GeV e -like

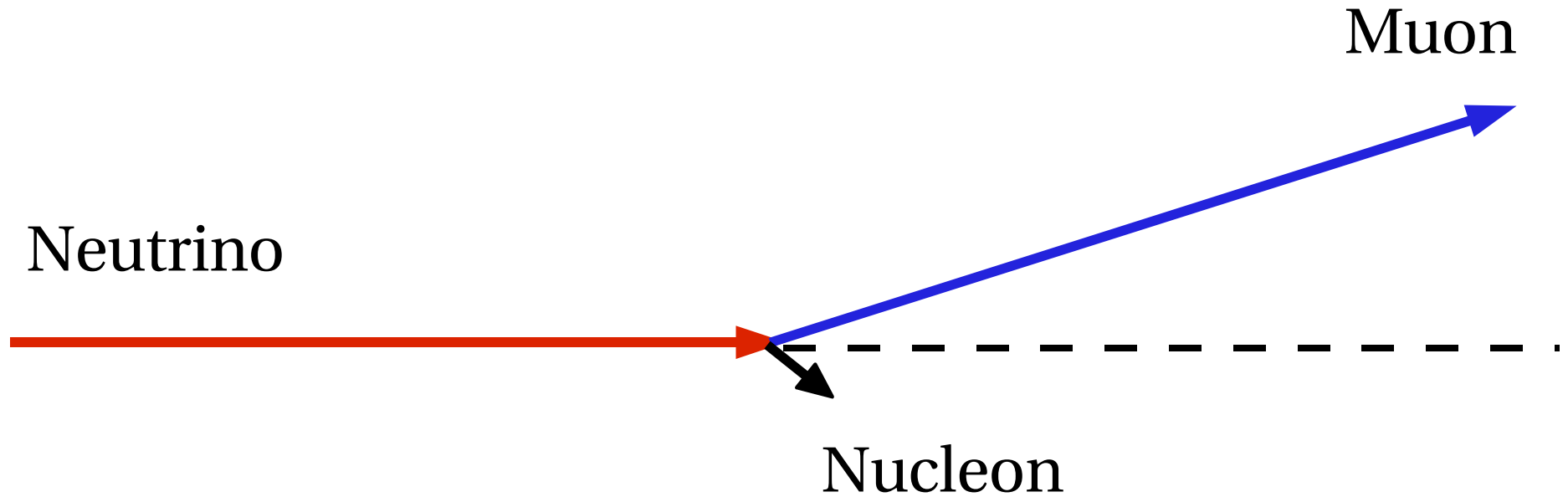


Multi-GeV μ -like + PC



Up-going

Down-going



Angle (Muon-Neutrino)
decreases with the Neutrino Energy

Angle 60 degrees for subGeV
 10-15 degrees for multiGeV

Interpretation in terms of:

$\nu_\mu \rightarrow \nu_\tau$ Oscillations

$$P_{\nu_\mu \rightarrow \nu_\tau} = \sin^2 2\theta \sin^2 \left[\frac{\Delta m^2 L}{4 E_\nu} \right]$$

$$\begin{aligned} \phi_{\nu_e} &\rightarrow \phi_{\nu_e} \\ \phi_{\nu_\mu} &\rightarrow [1 - \langle P_{\nu_\mu \rightarrow \nu_\tau} \rangle] \phi_{\nu_\mu} \\ \phi_{\nu_\tau} \simeq 0 &\rightarrow \langle P_{\nu_\mu \rightarrow \nu_\tau} \rangle \phi_{\nu_\mu} \end{aligned}$$

Disappearance

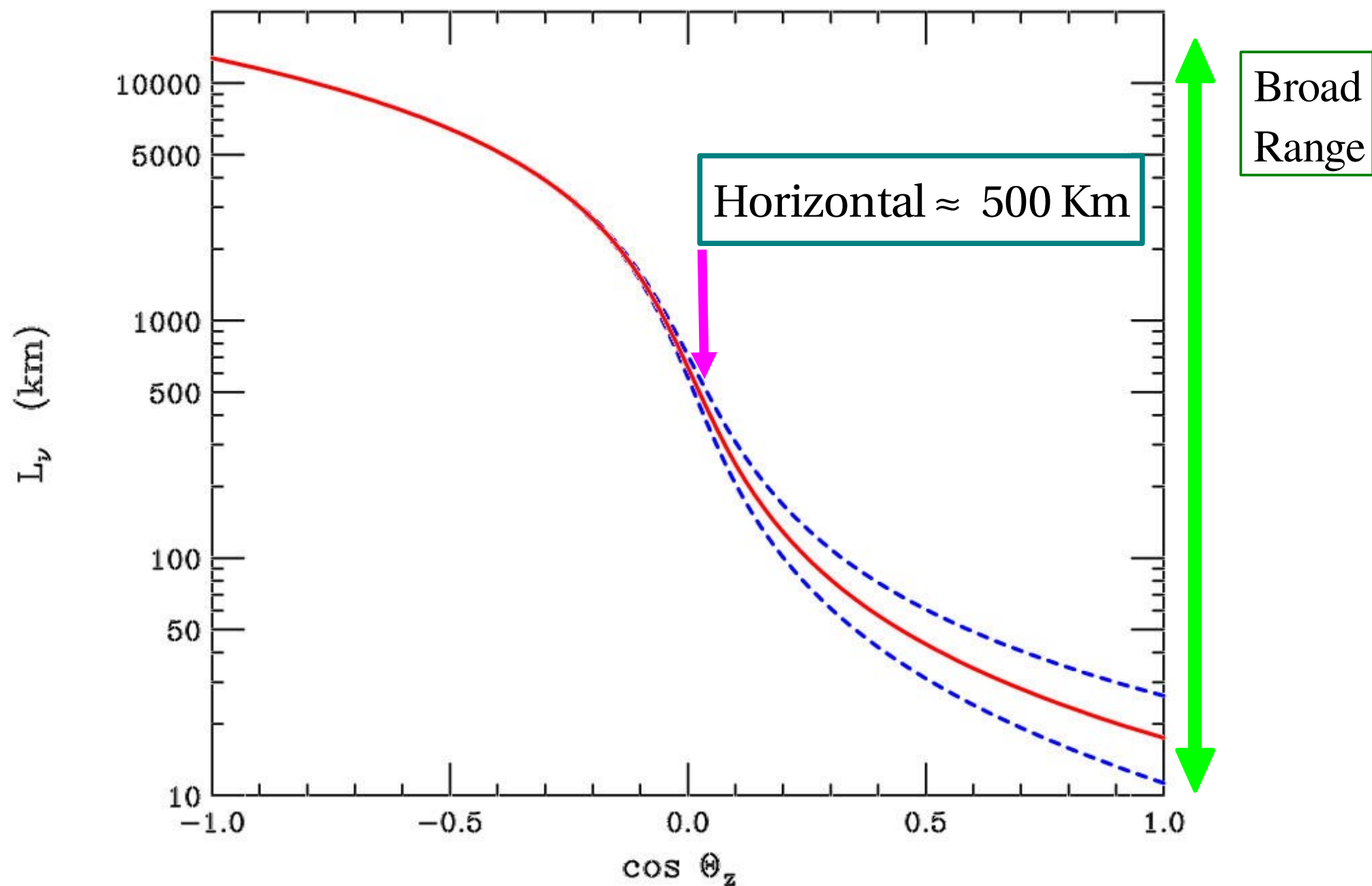
Appearance

Energy Threshold for CC interactions of ν_τ

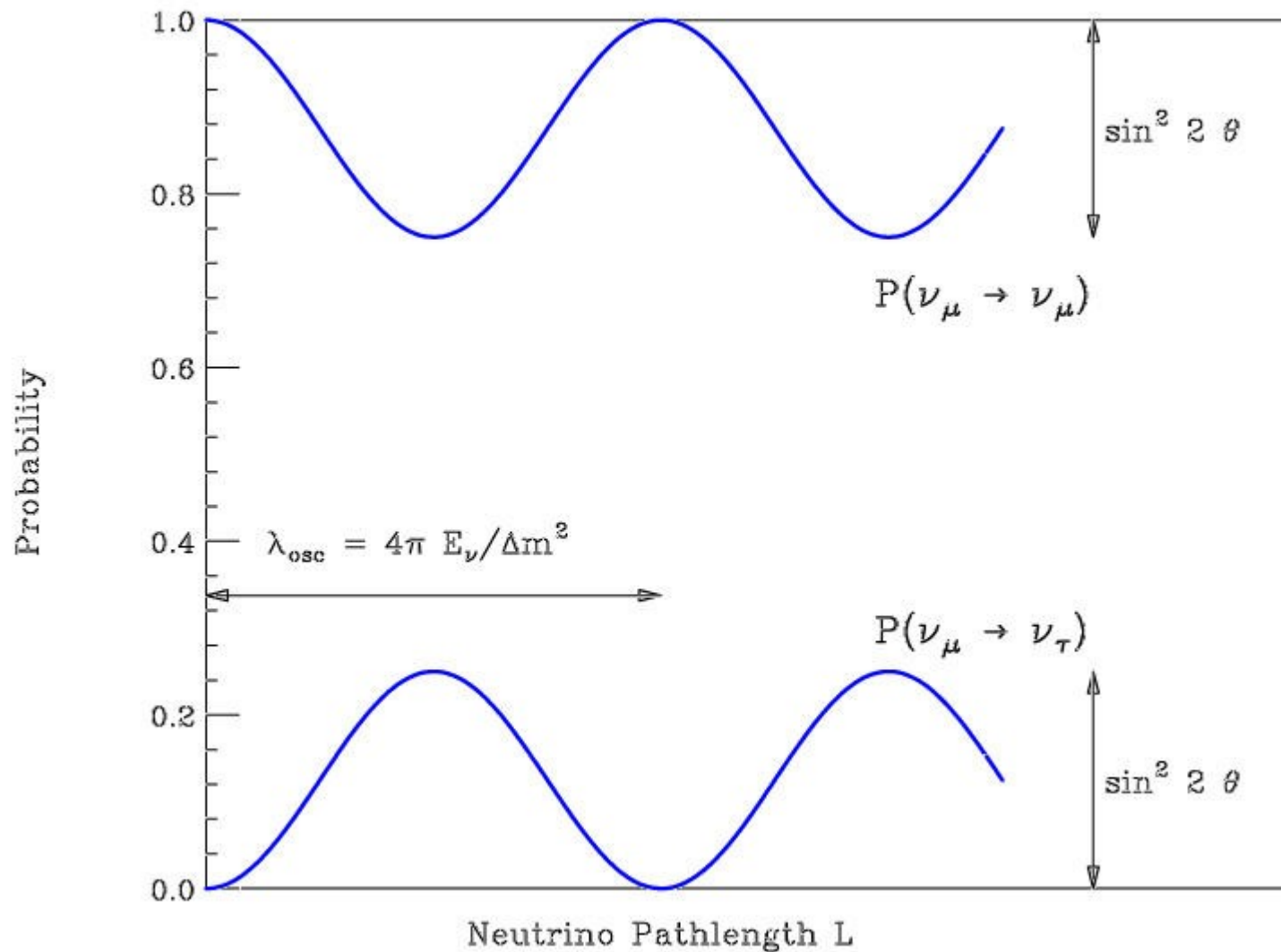
$$E(\nu_\tau) \geq m_\tau + m_\tau^2 / 2m_p \approx 3.5 \text{ GeV}$$

In atmospheric neutrinos most ν_τ are below threshold for CC interactions and therefore simply "disappear".

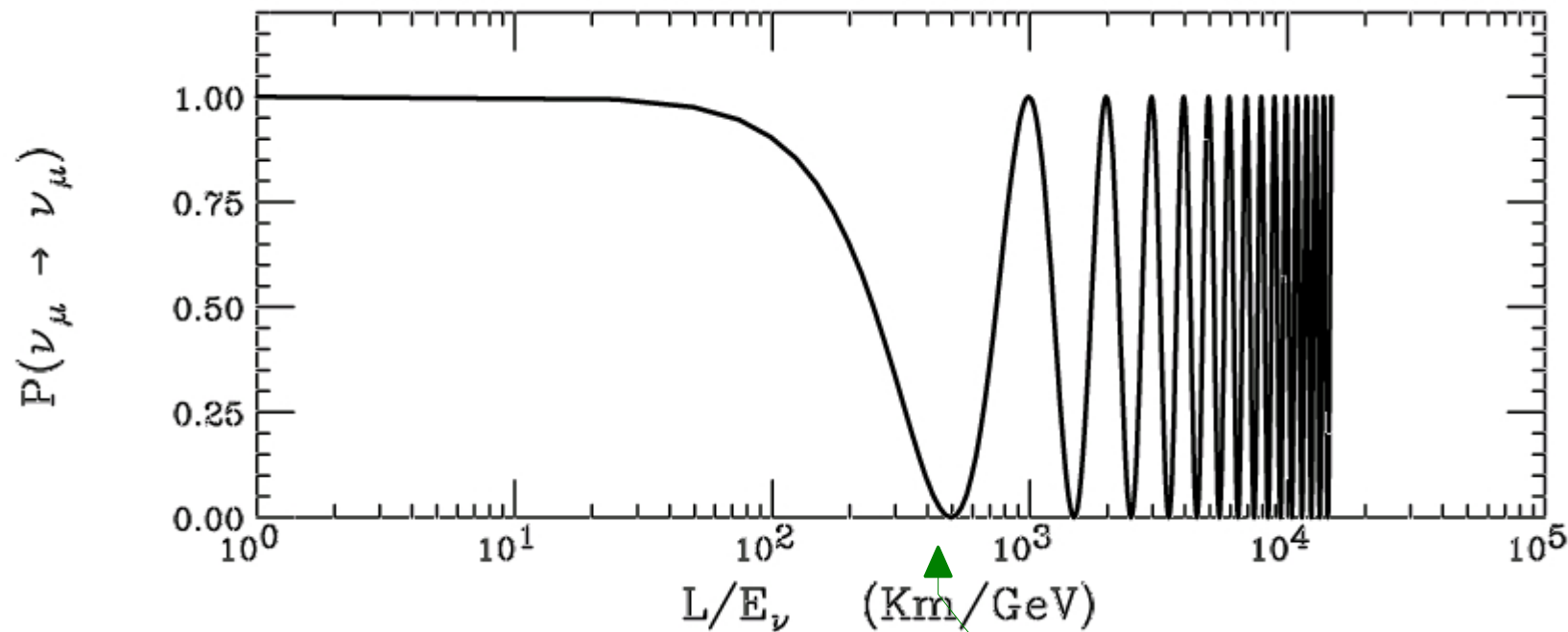
Distance from the Neutrino creation point



$$P(\nu_\mu \rightarrow \nu_\tau; L) = \sin^2 2\theta \sin^2 \left[1.27 \Delta m^2 (\text{eV}^2) \frac{L(\text{Km})}{E(\text{GeV})} \right]$$

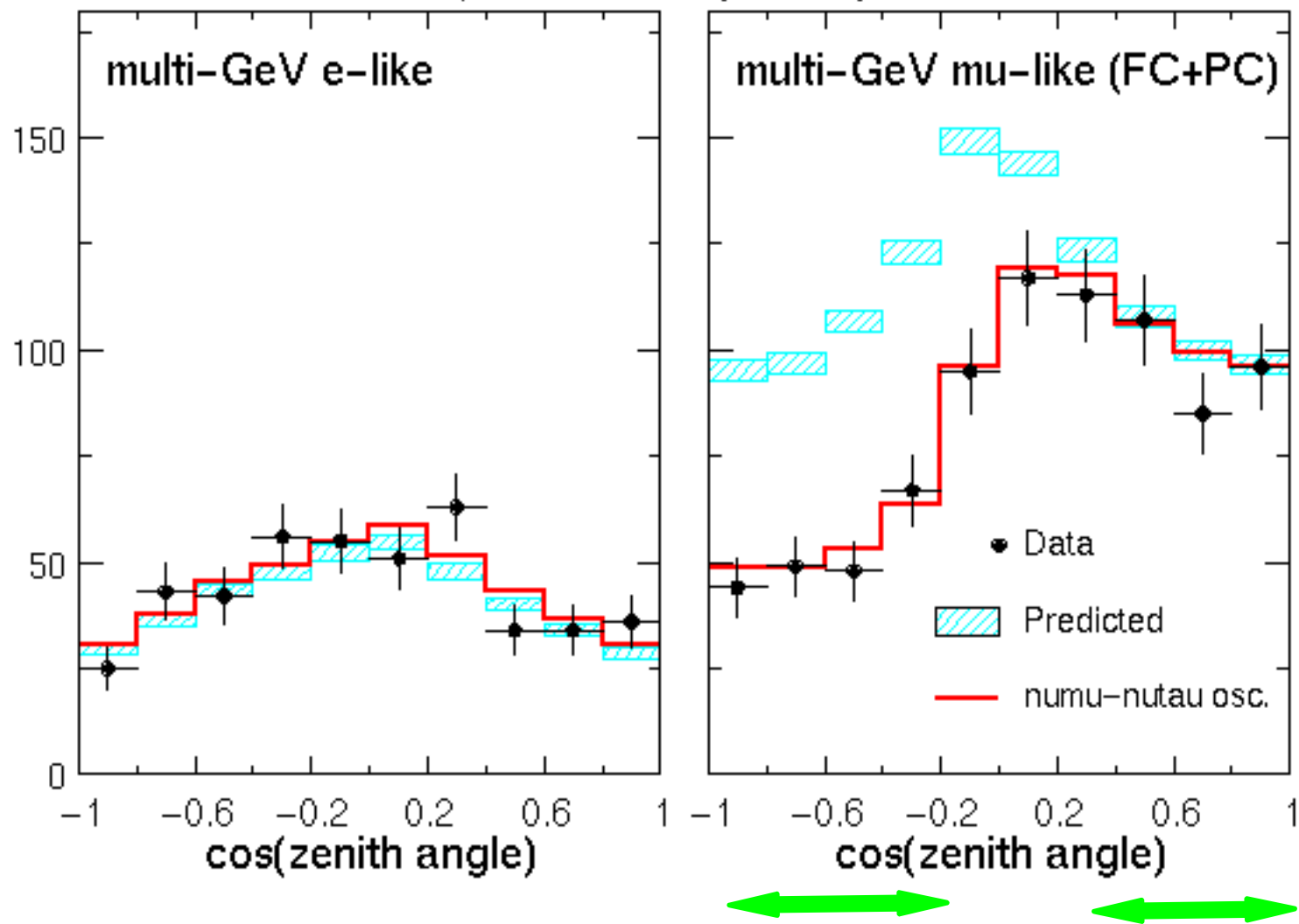


$$P_{\nu_\mu \rightarrow \nu_\mu}(L, E_\nu) = 1 - \sin^2 2\theta \sin^2 \left[\frac{\Delta m^2 L}{4E_\nu} \right]$$



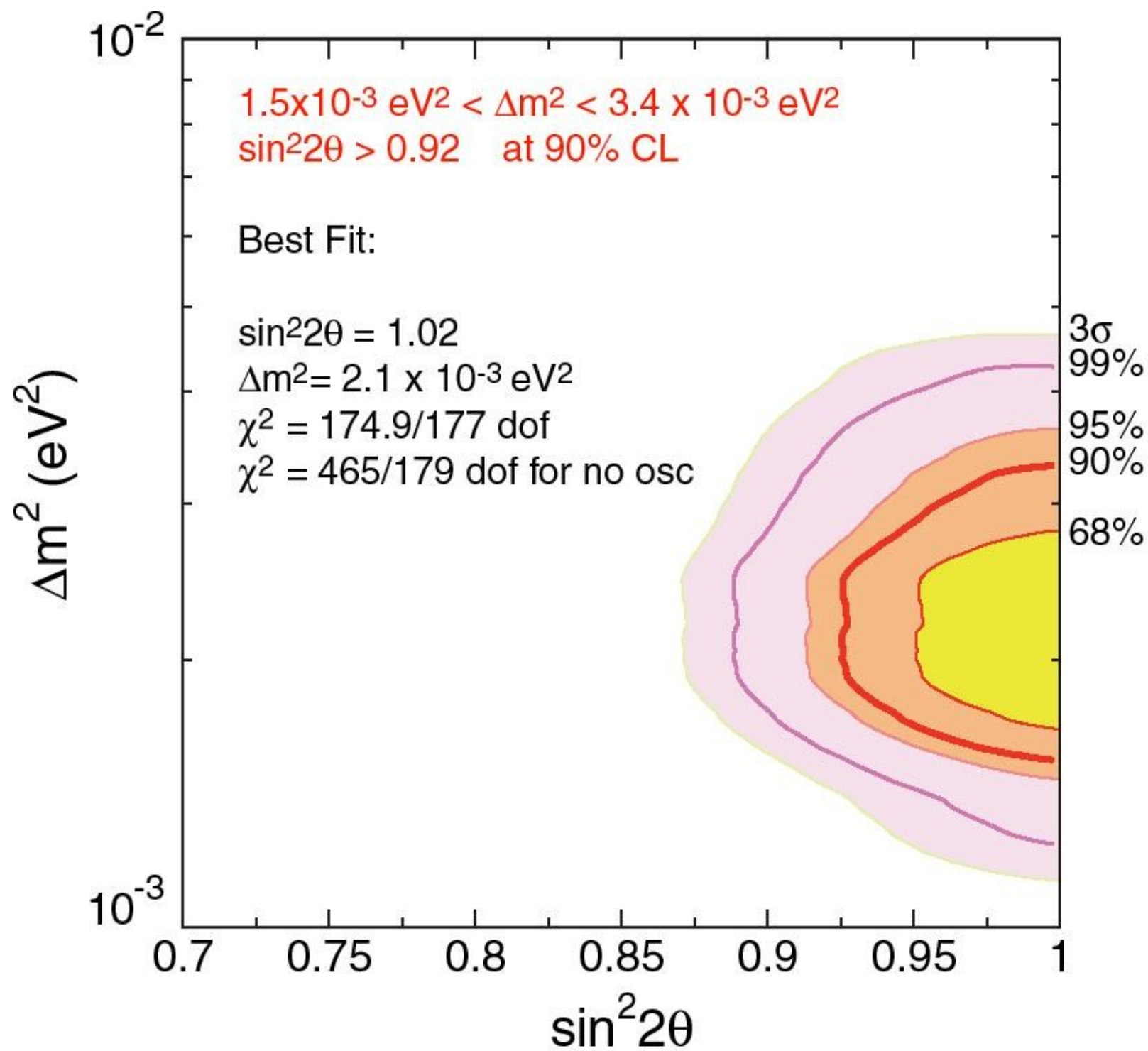
$$P_{\nu_\mu \rightarrow \nu_\mu} = \begin{cases} 1 & \text{for } L \text{ small,} \\ 1 - \frac{\sin^2 2\theta}{2} & \text{for } L \text{ large.} \end{cases}$$

$$\simeq \frac{\lambda_{\text{osc}}^*}{2} \simeq \frac{2\pi \langle E_\nu \rangle}{|\Delta m^2|}$$

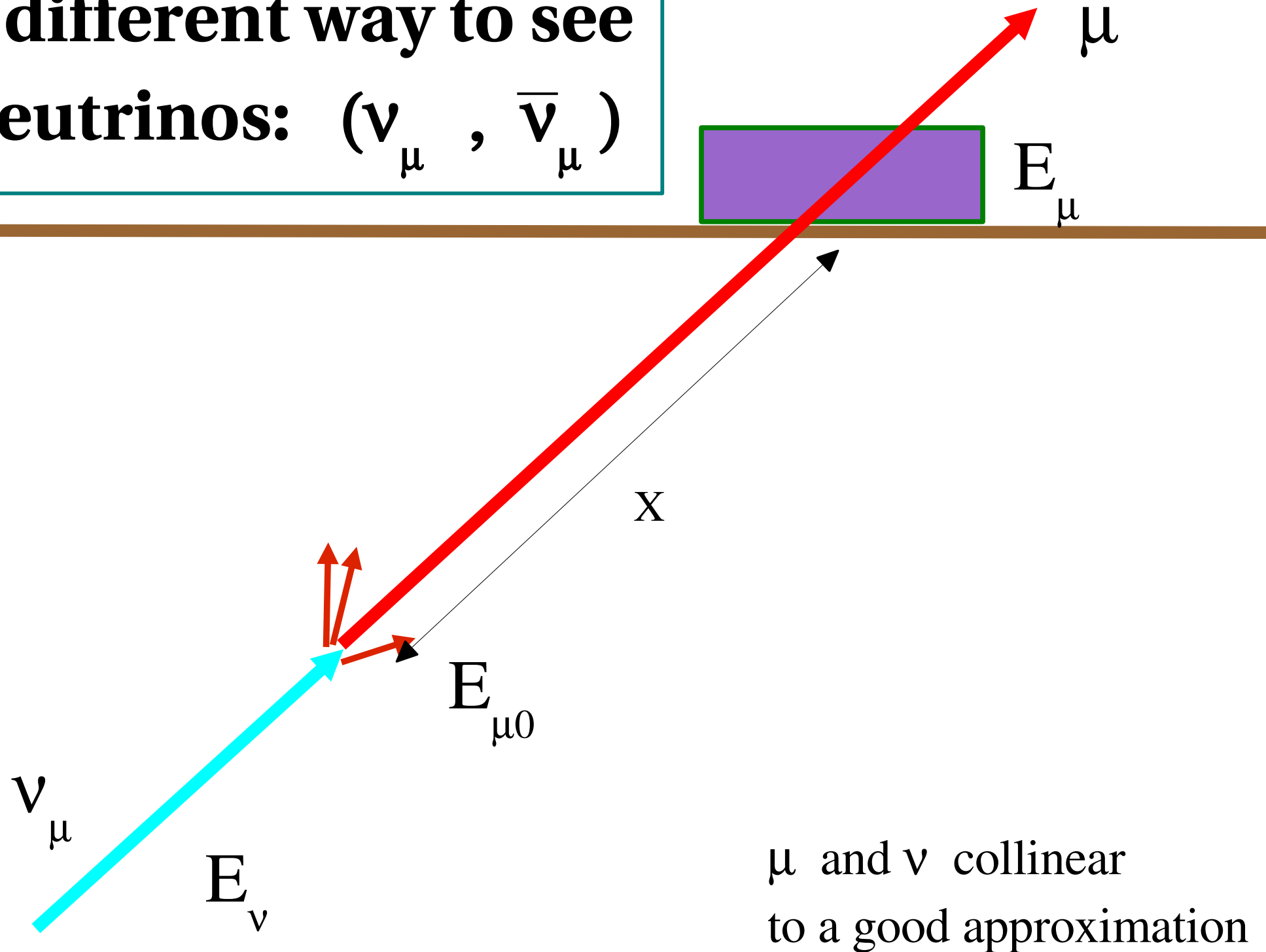


Measurement
of $\sin^2 2\theta$

$$\left(\frac{\text{Up}}{\text{Down}} \right)_{\text{multiGeV}} = 0.54 \pm 0.04 \pm 0.01$$



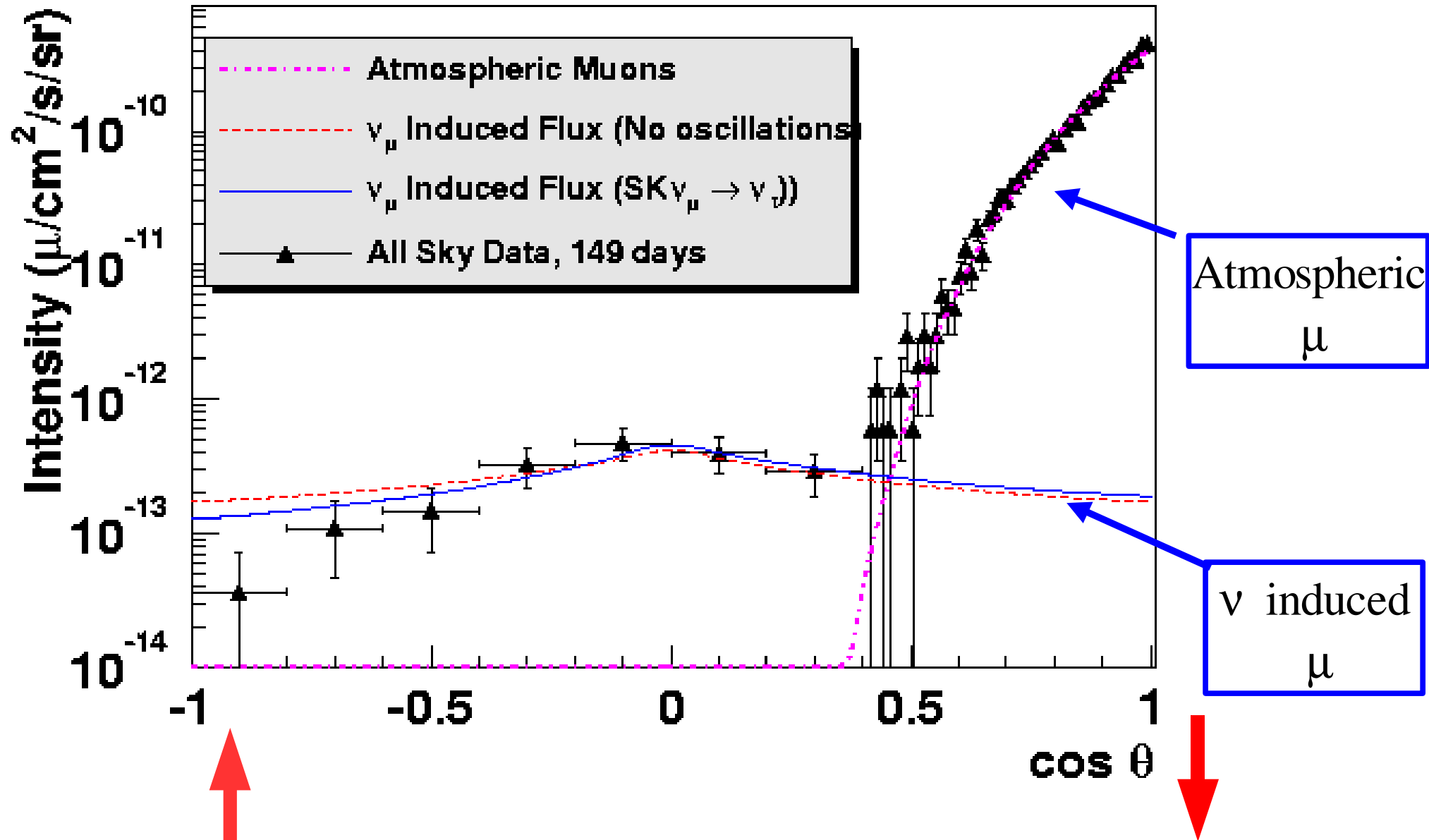
**A different way to see
Neutrinos: $(\nu_\mu, \bar{\nu}_\mu)$**



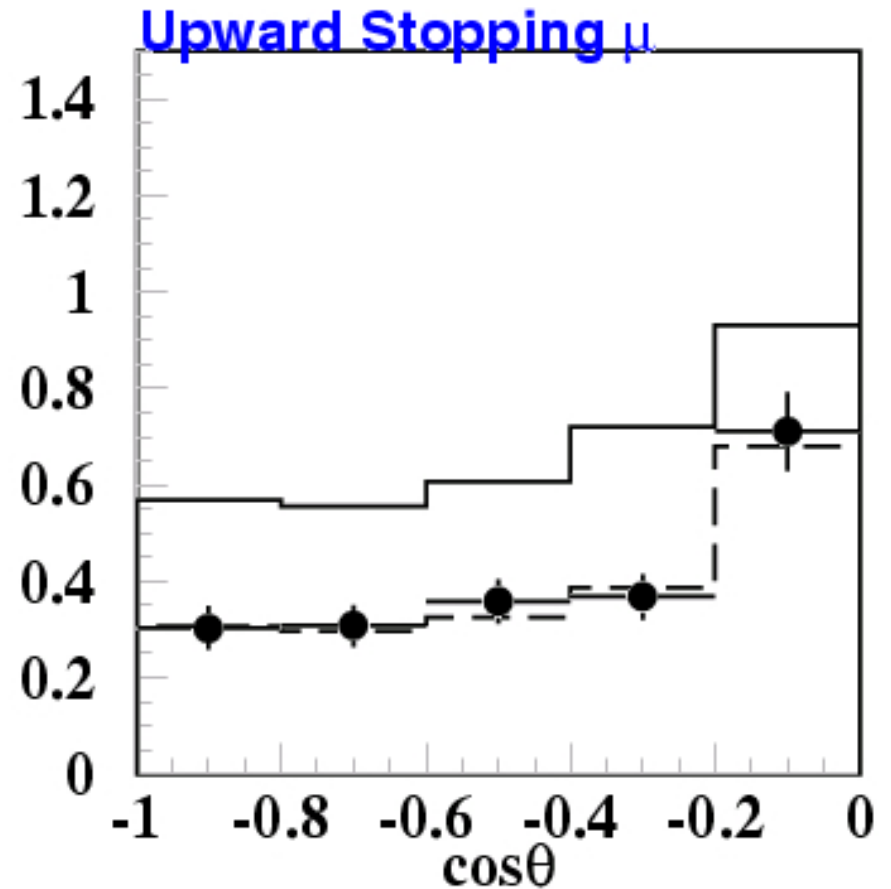
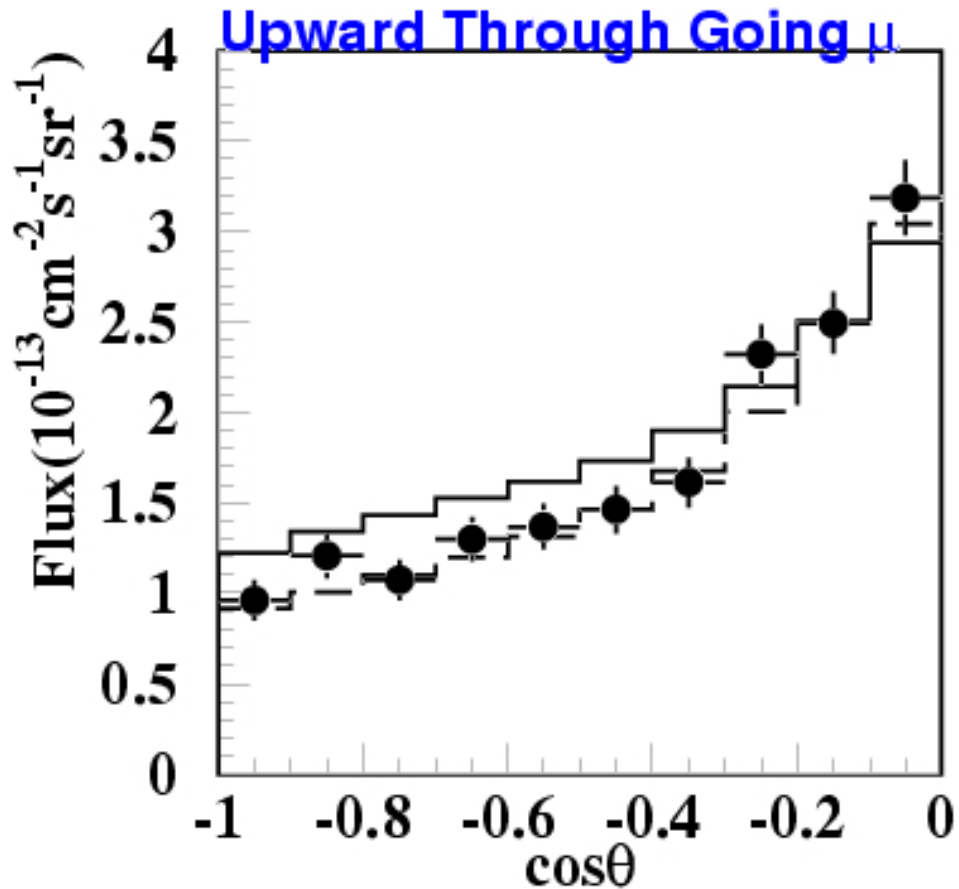
SNO detector

μ angular distribution

Through-Going Muon Zenith Angle Distribution (PRELIMINARY)

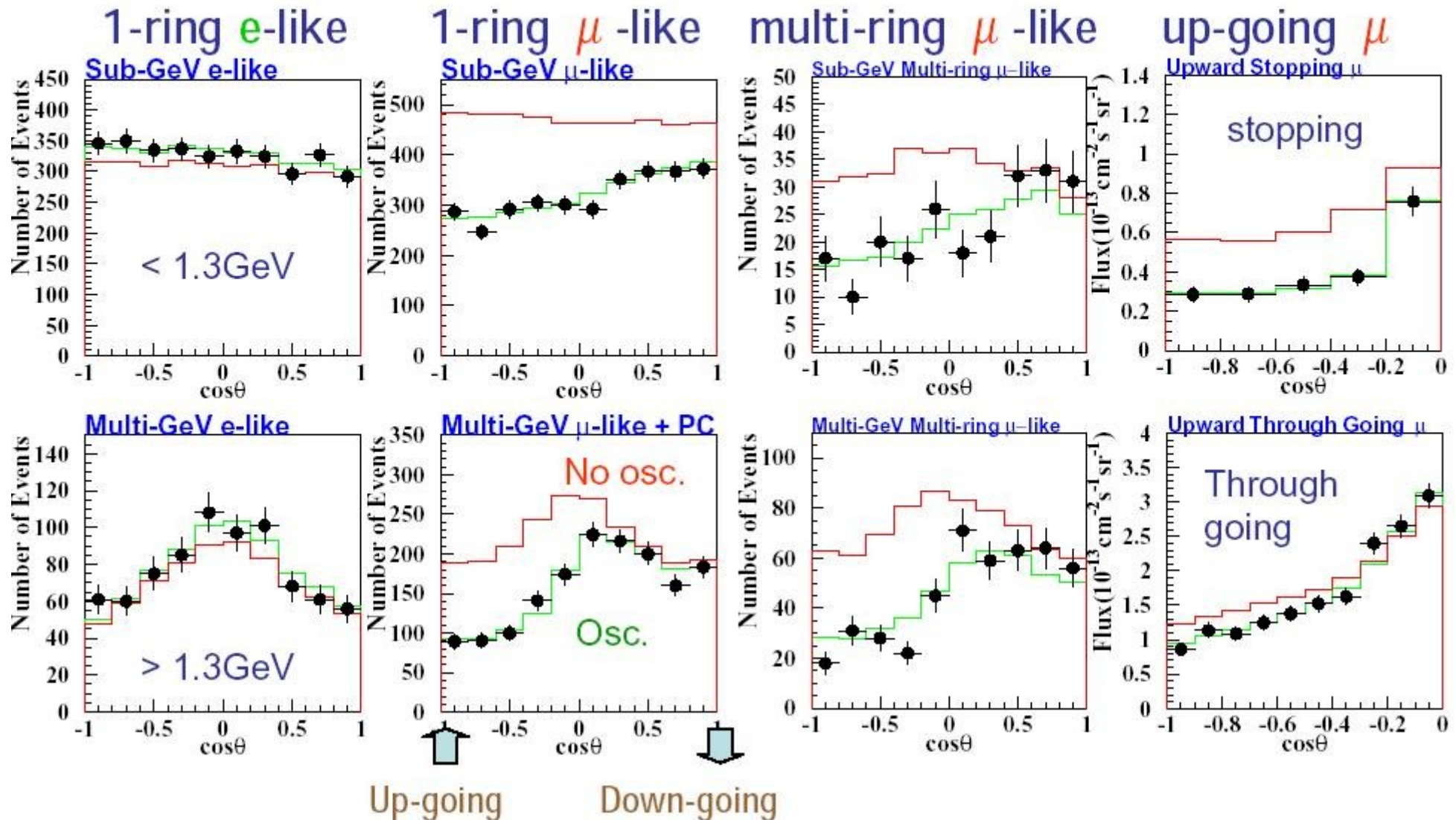


Super-Kamiokande

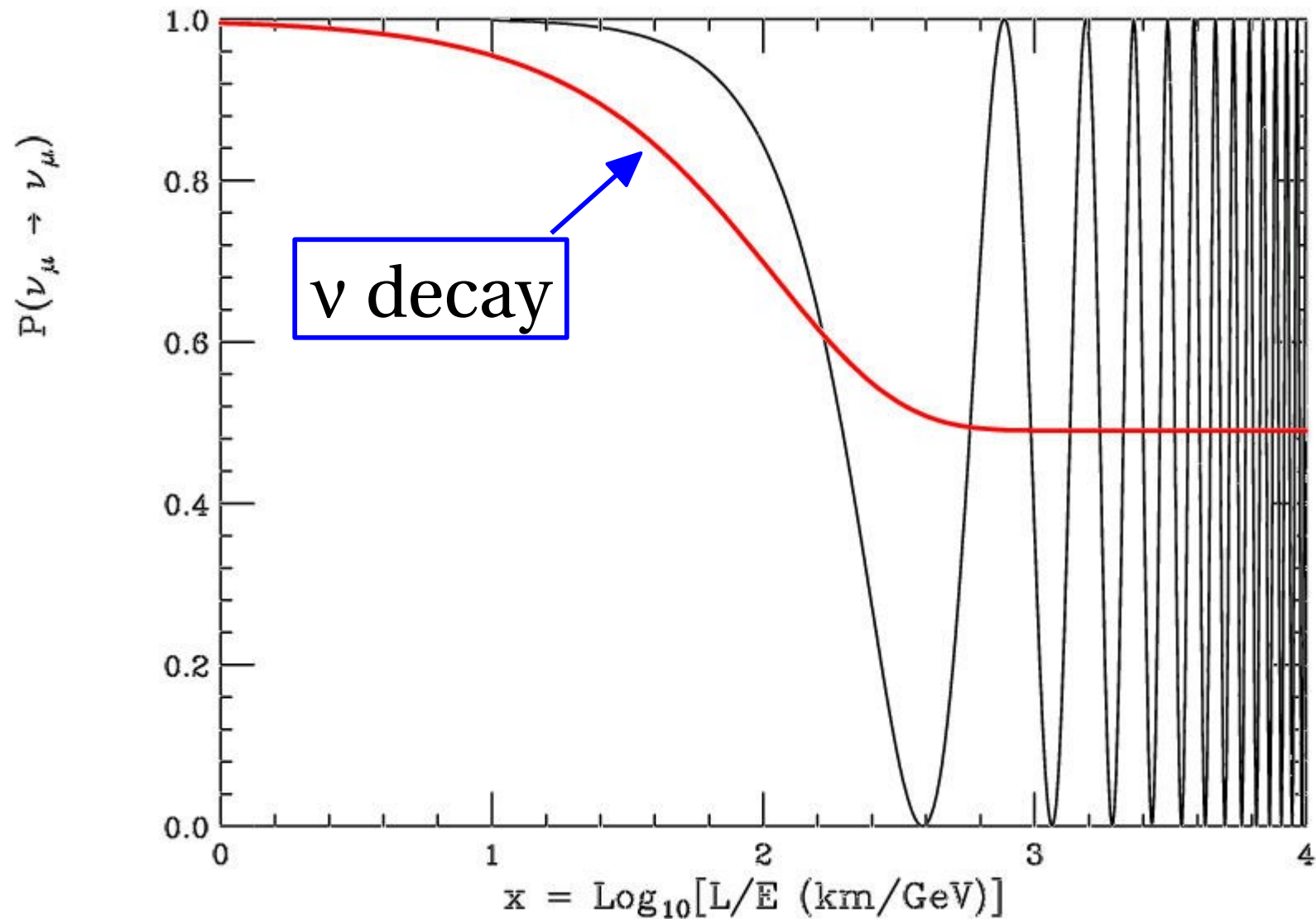


Super-Kamiokande data

1489day FC+PC data + 1678day upward going muon data



Alternative Models for the "disappearance" of ν_τ



$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_{\text{ster}} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_2 \\ \nu_3 \\ \nu_1 \end{bmatrix}$$

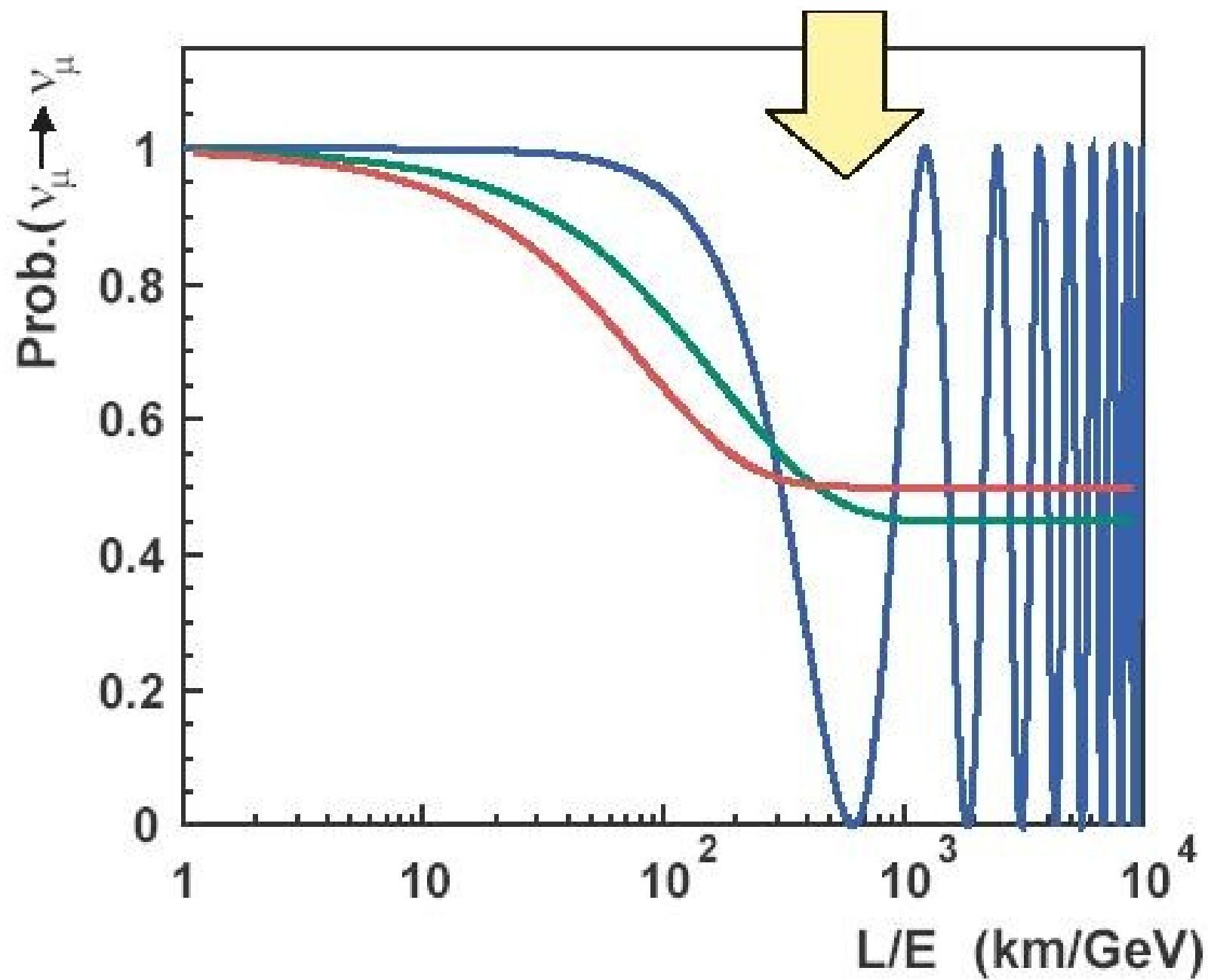
$$\nu_2 \rightarrow \bar{\nu}_1 + J$$

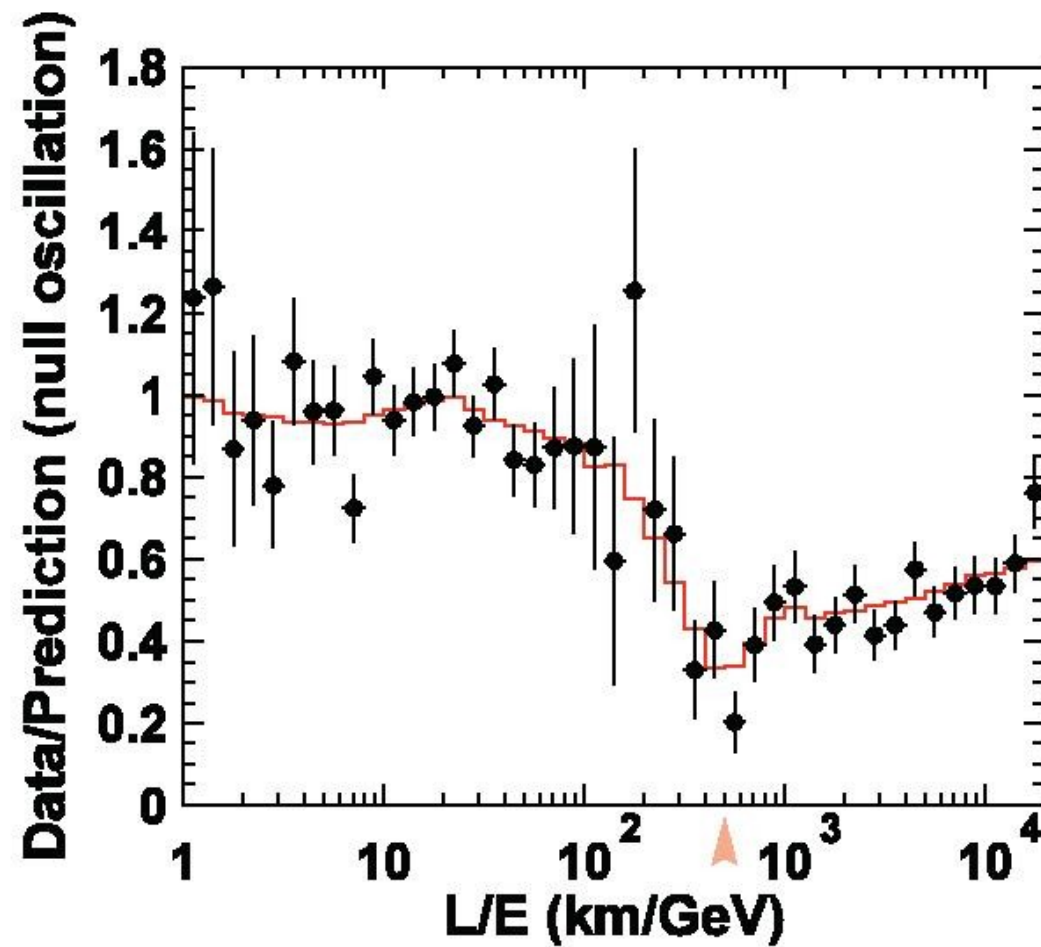
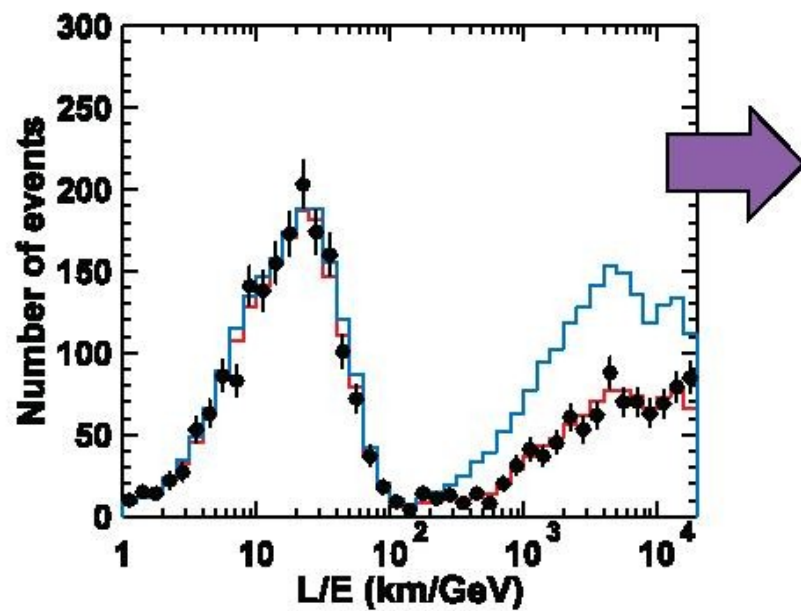
Neutrino Decay
(Barger et al)

$$P_{\text{decay}} = \left\{ \sin^2 2\theta + \cos^2 2\theta \exp \left[-\frac{m}{\tau_\nu} \frac{L}{E_\nu} \right] \right\}$$

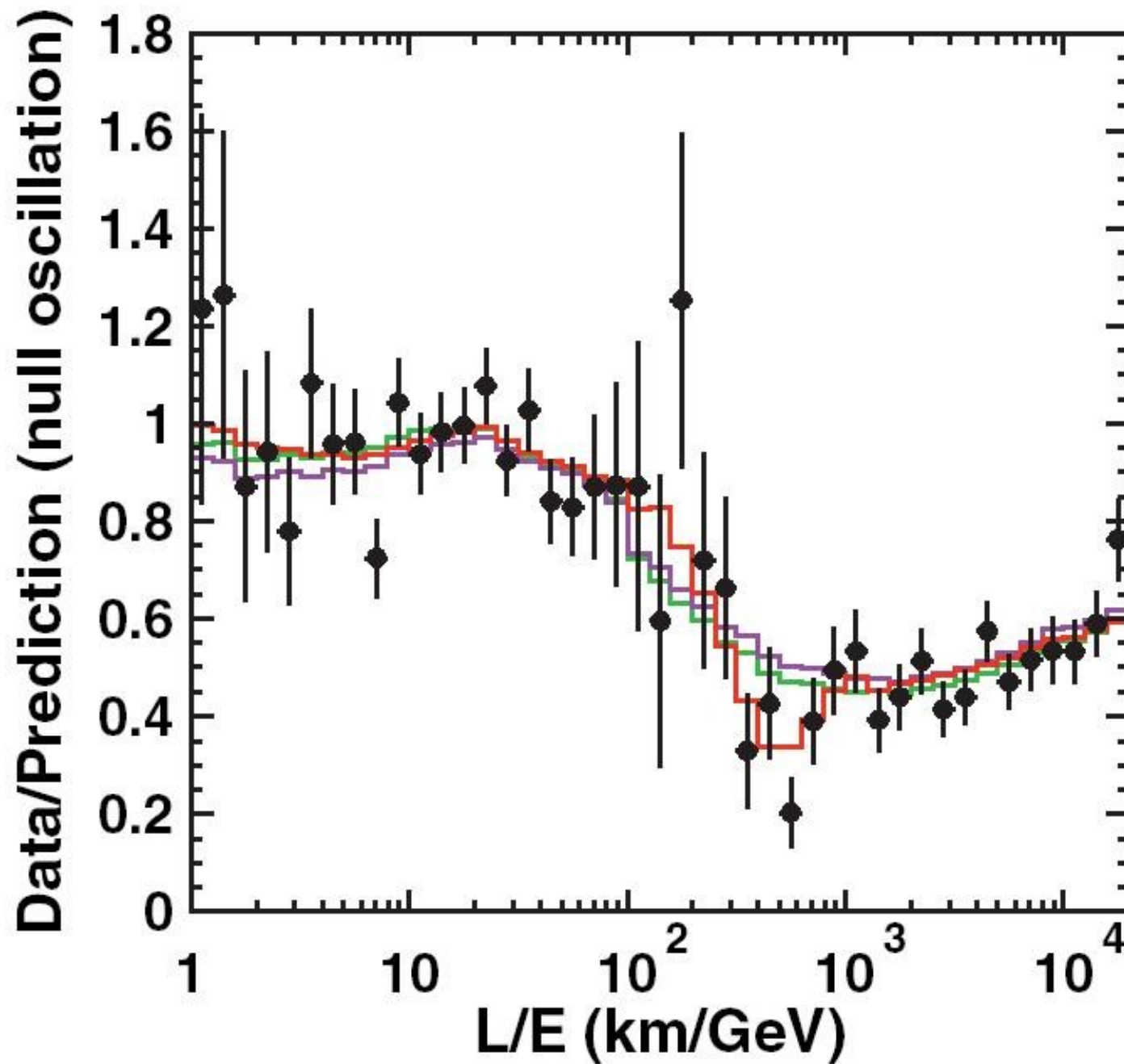
$$P_{\text{decoherence}} = \frac{1}{2} \left[1 + \exp \left(-\beta \frac{L}{E_\nu} \right) \right]$$

Decoherence : Lisi, Marrone e Montanino





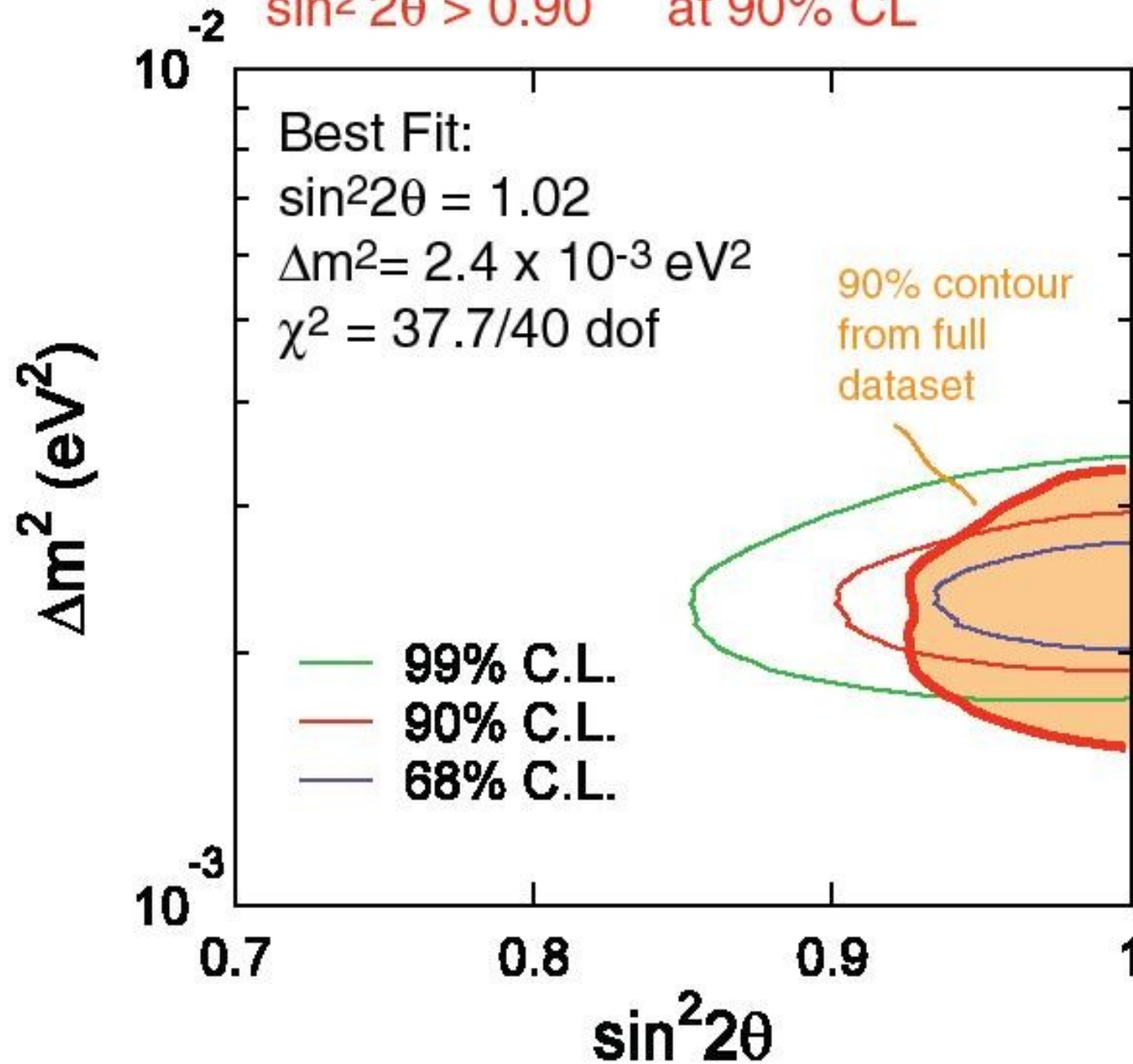
*oscillation dip seen
at $\sim 500 \text{ km/GeV}$*



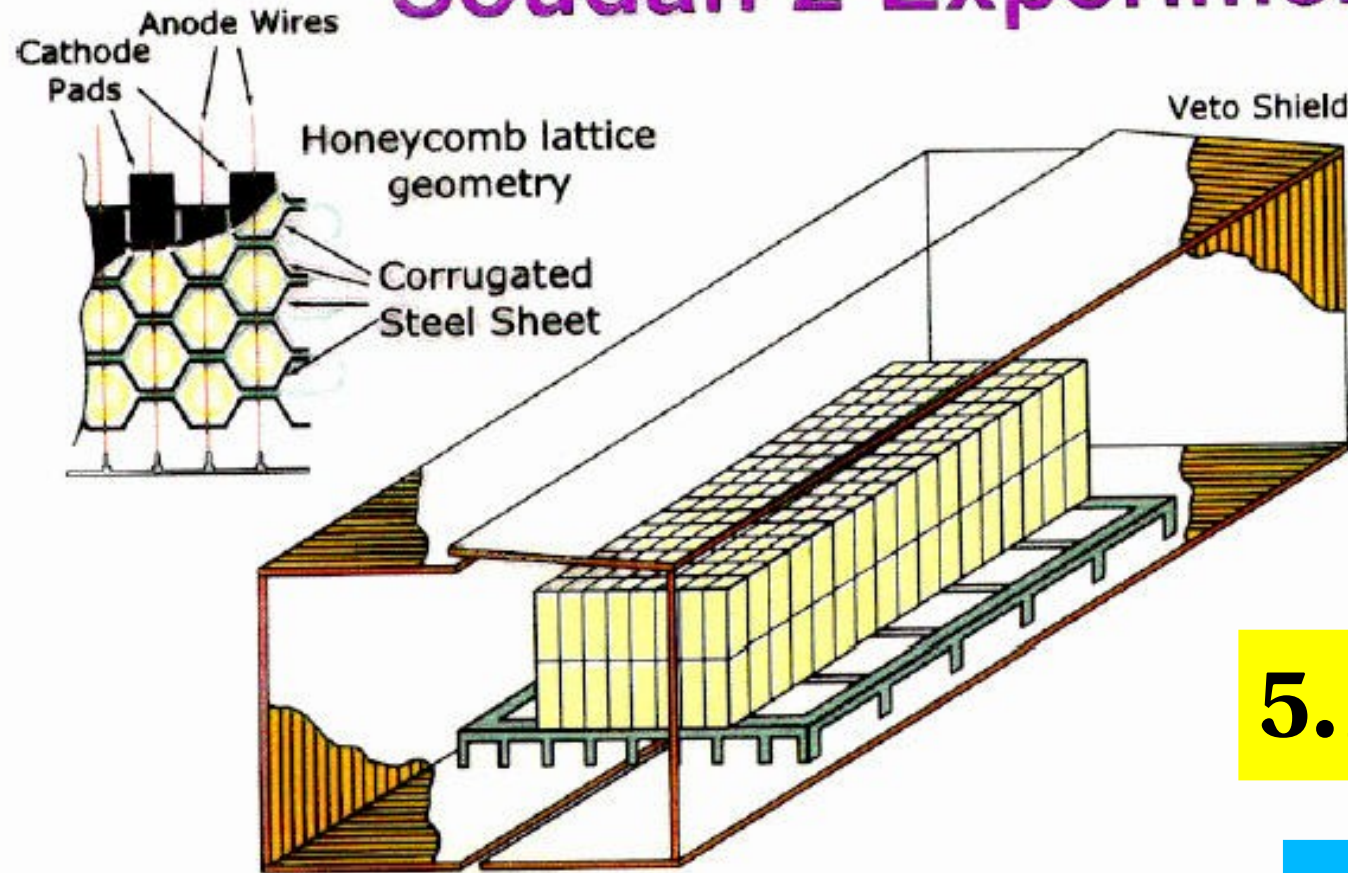
Decay rejected at 3.4σ

Decoherence rejected at 3.8σ

$$1.9 \times 10^{-3} \text{ eV}^2 < \Delta m^2 < 3.0 \times 10^{-3} \text{ eV}^2$$
$$\sin^2 2\theta > 0.90 \quad \text{at 90\% CL}$$



Soudan 2 Experiment



5.1 (Kton yr)

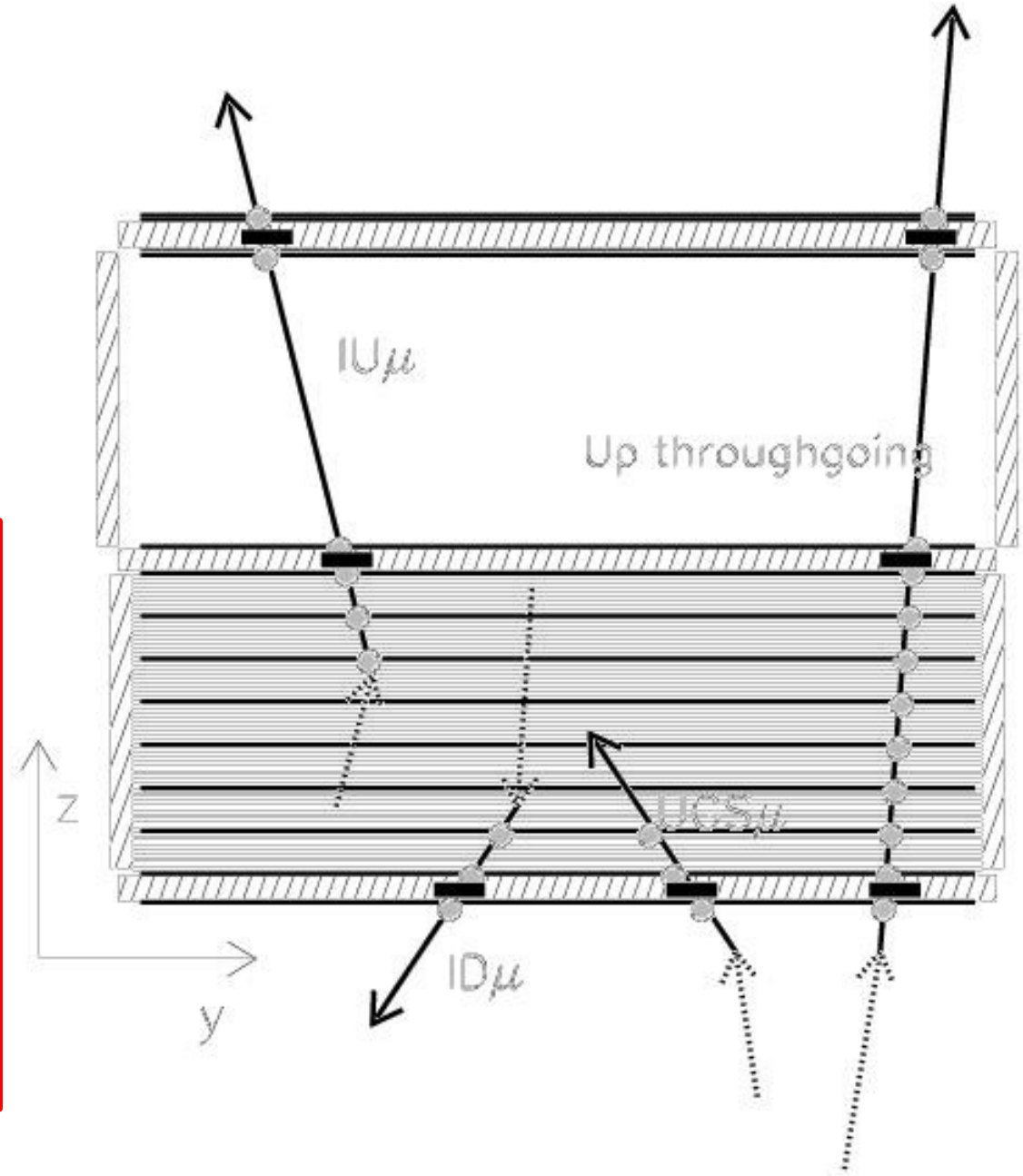
561 ν events

Soudan Mine (Minnesota), 2100 m.w.e.
iron tracking calorimeter
high geomagnetic latitude
5.1 kt-yr, 326 contained interactions
~24% background subtraction using veto events

MACRO detector Gran Sasso laboratory



- [a] Through-going muons
- [b] Internally produced up-going events
- [c] Up-going stopping + Down-going internal interactions



Comparison of:

$$\nu_{\mu} \leftrightarrow \nu_{\tau}$$

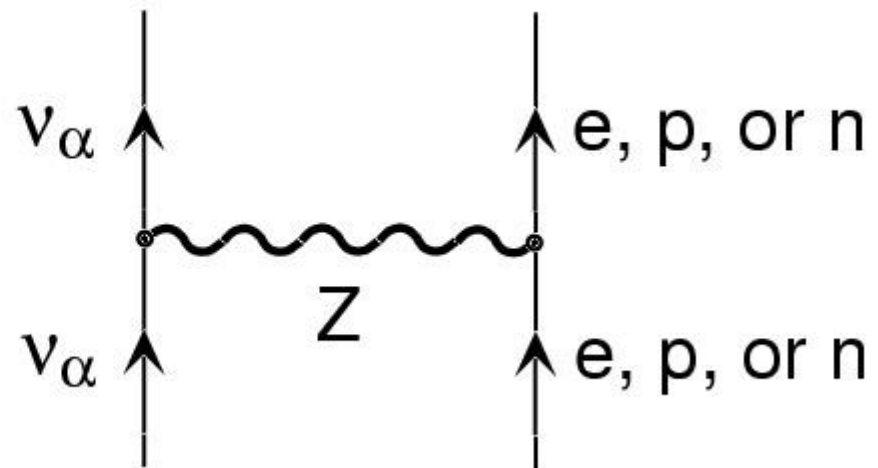
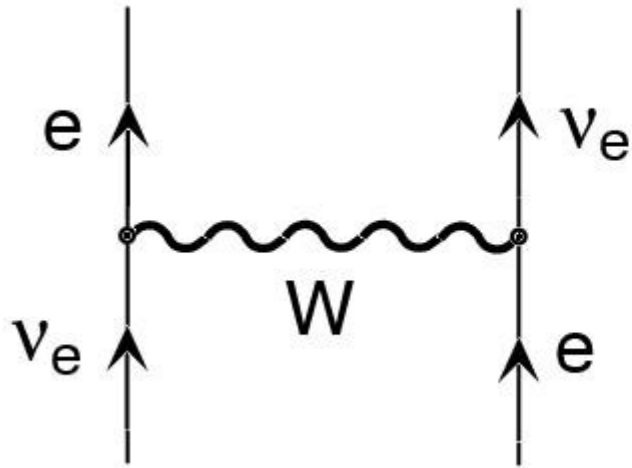
$$\nu_{\mu} \leftrightarrow \nu_{\text{sterile}}$$

Absence of Neutral Currents

Matter Effects

Tau Appearance events

Effective Potential in Ordinary Matter



$$V_{\mu S} = -\sqrt{2} G_F \frac{N_n}{2} \simeq -10^{-13} \text{ eV} \left(\frac{\rho}{5 \text{ g cm}^{-3}} \right)$$

$$V_{\nu_\mu e} = V_{\nu_\tau e} = V_{\nu_e e}^Z = -\frac{\sqrt{2}}{2} G_F N_e$$

$$V_{\nu_\mu p} = V_{\nu_\tau p} = V_{\nu_e p} = +\frac{\sqrt{2}}{2} G_F N_p$$

$$V_{\nu_\mu n} = V_{\nu_\tau n} = V_{\nu_e n} = -\frac{\sqrt{2}}{2} G_F N_n$$

$$V_{\nu_{ee}} = V_{\nu_{ee}}^Z + V_{\nu_{ee}}^W = -\frac{\sqrt{2}}{2} G_F N_e + \sqrt{2} G_F N_e$$

$$V_{\text{sterile}} = 0$$

$$A = \mp \frac{2\sqrt{2} G_F N_e E_\nu}{\Delta m^2}$$

$$\sin^2 2\theta \Rightarrow \frac{\sin^2 2\theta}{(A - \cos 2\theta)^2 + \sin^2 2\theta}$$

$$\Delta m^2 \Rightarrow \Delta m^2 \sqrt{(A - \cos 2\theta)^2 + \sin^2 2\theta}$$

small E_ν , $|A| \ll 1$ no matter effects $\rightarrow$ vacuum oscillation

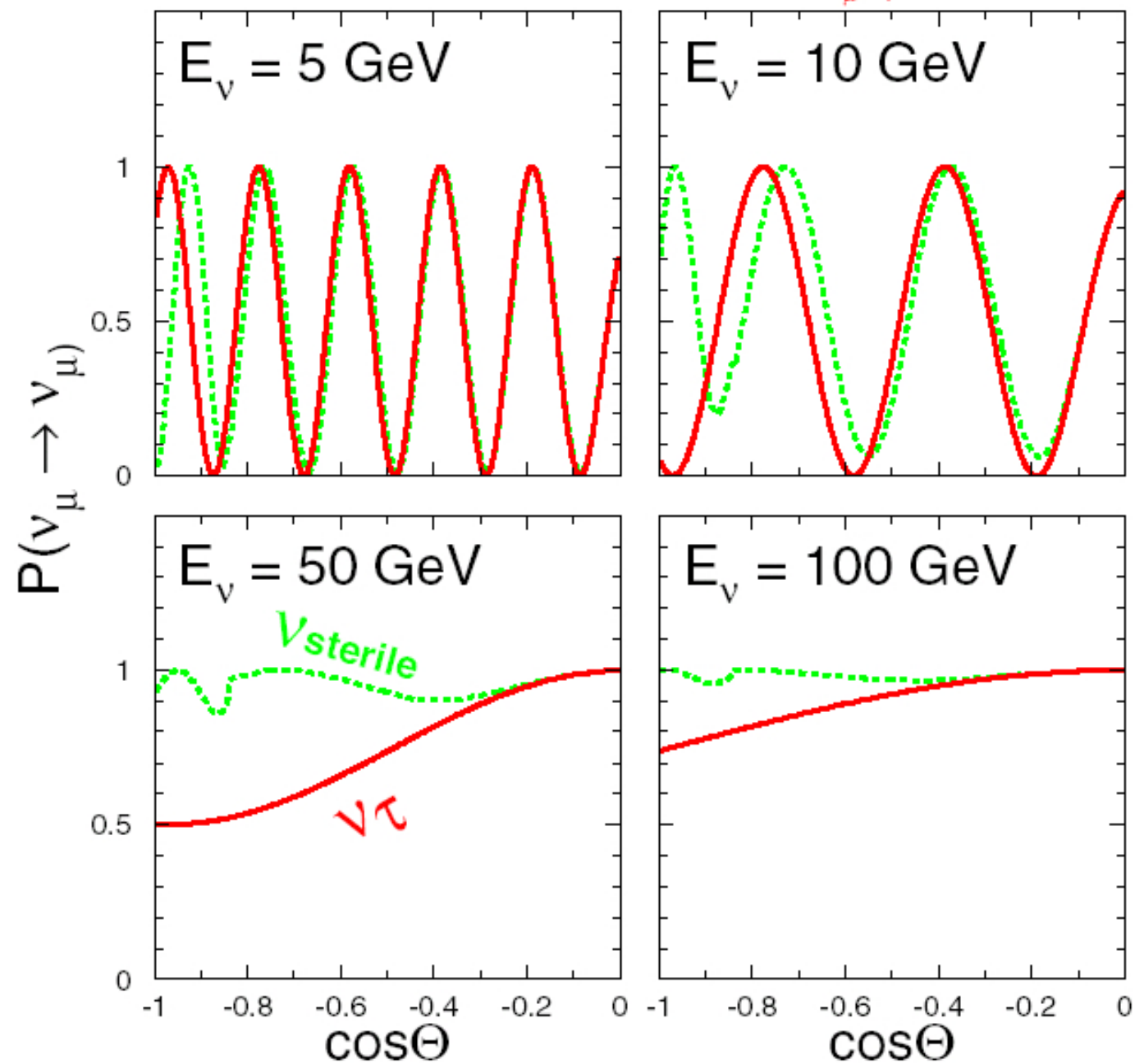
large E_ν , $|A| \gg 1$ oscillation is suppressed

$$|A| \sim 1 \text{ in earth for } E_\nu = 5 \text{ GeV} \times \Delta m^2 (10^{-3} \text{ eV}^2)$$

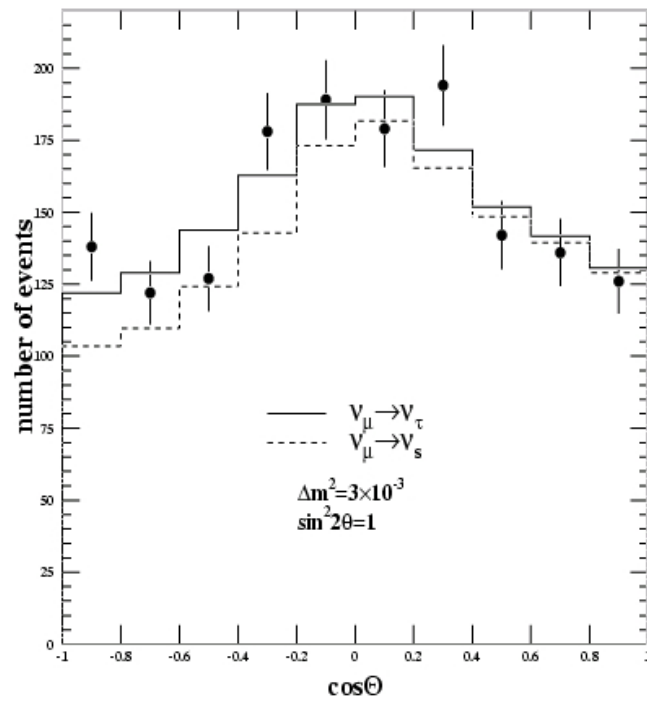
$\nu_\mu - \nu_{\text{sterile}}$ Matter Effects

$$\sin^2 2\theta = 1, \Delta m^2 = 5 \times 10^{-3} \text{ eV}^2$$

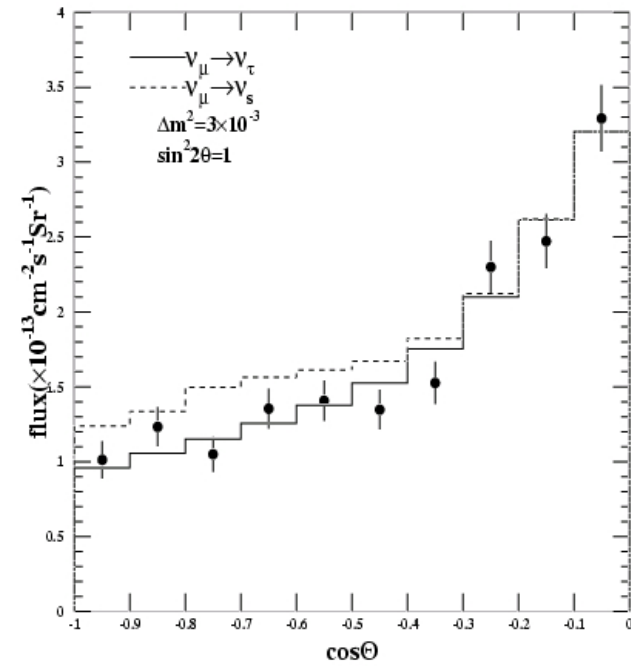
$$\begin{aligned} \text{---} & \nu_\mu - \nu_{\text{sterile}} \\ \text{---} & \nu_\mu - \nu_\tau \end{aligned}$$



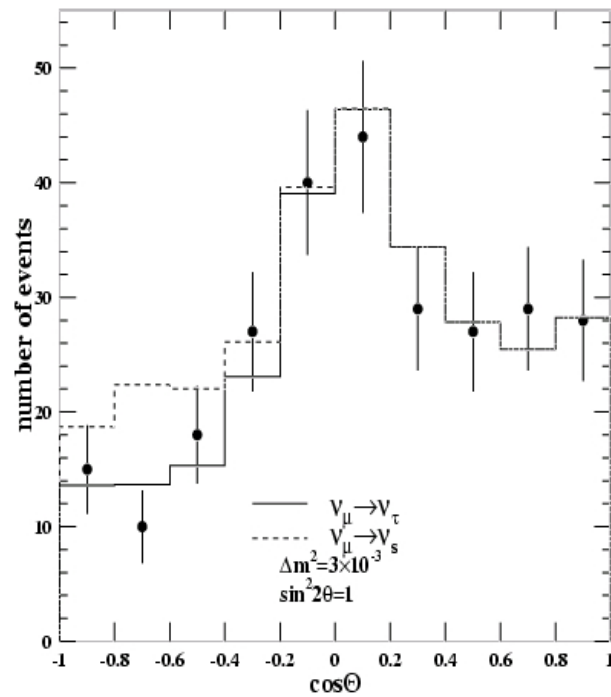
N.C. enriched FC sample (1140days)



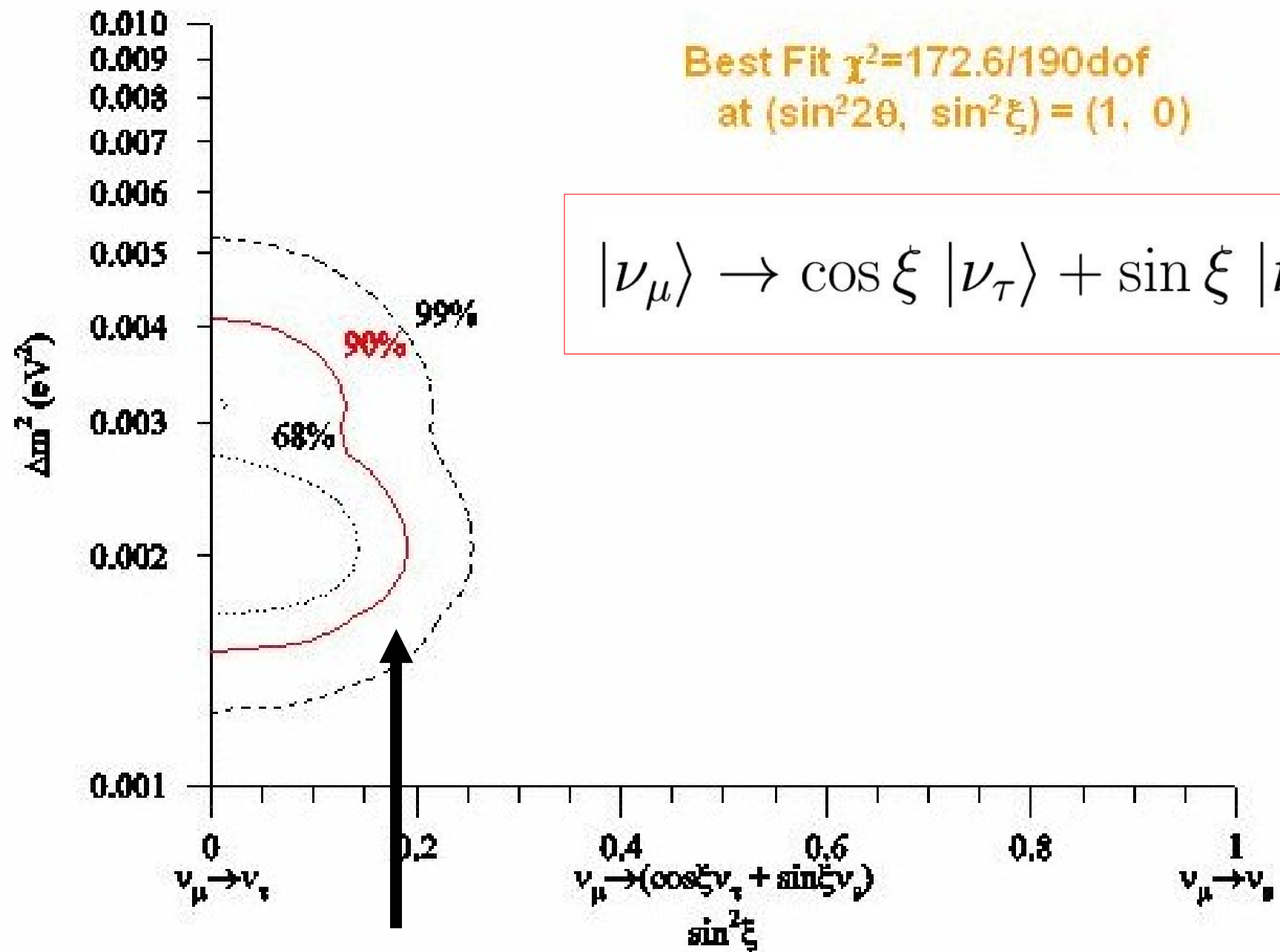
up-through-going muons (1140days)



high-energy PC sample (1140days)

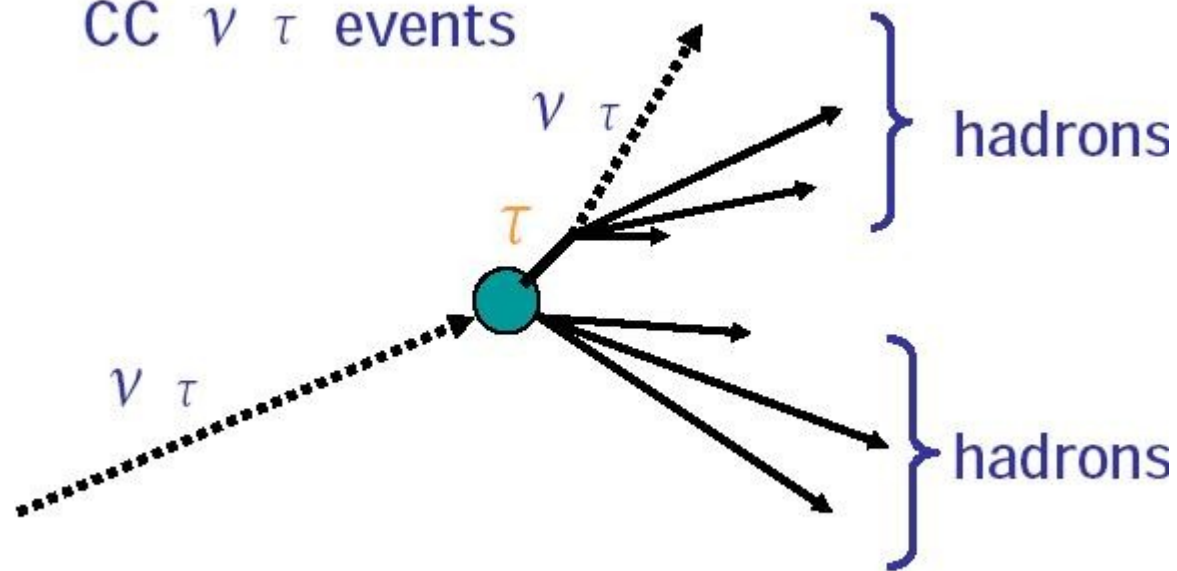


3 samples of data to study
the $\nu_\mu \rightleftharpoons \nu_s$ hypothesis



$$\sin^2 \xi < 0.19$$

CC ν_τ events

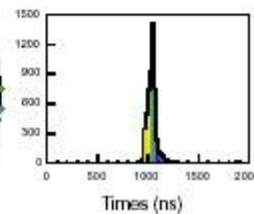
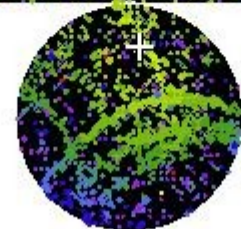
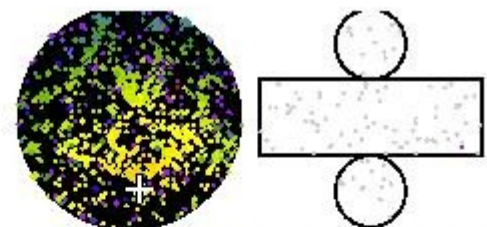


Only ~ 1.0 CC ν_τ
FC events/kton yr



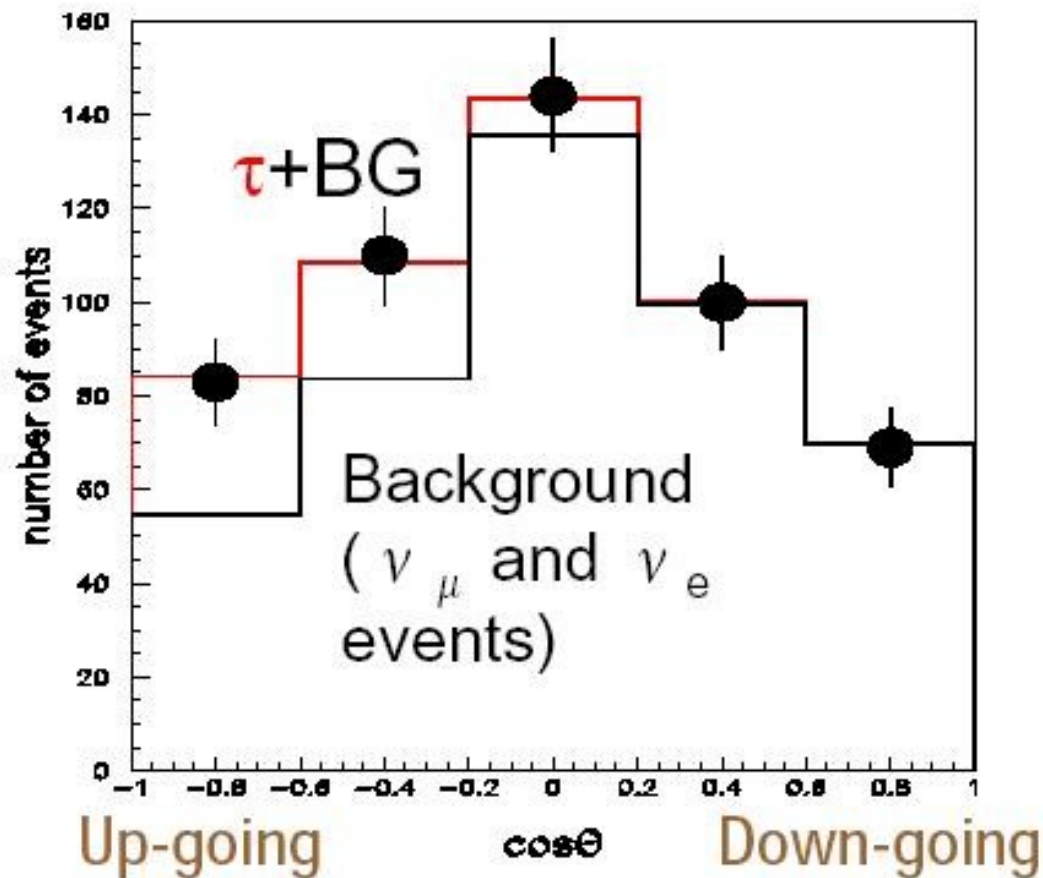
(BG (other ν events)
 ~ 130 ev./kton yr)

Tau
MC



τ "candidates" Zenith Angle Distribution

Max. likelihood analysis



$$N_{\tau}^{\text{FC}} = \alpha N_{\text{MC}}^{\tau} / (\text{eff.} = 0.44) \\ = 145 \pm 44 (\text{stat.}) \\ + 11 / -16 (\text{sys.})$$

$$N_{\text{exp}} = 86$$

$$N_{\tau} = 145 \pm 44^{+11}_{-16}$$

Evidence for New Physics
in Atmospheric Neutrinos
is SOLID

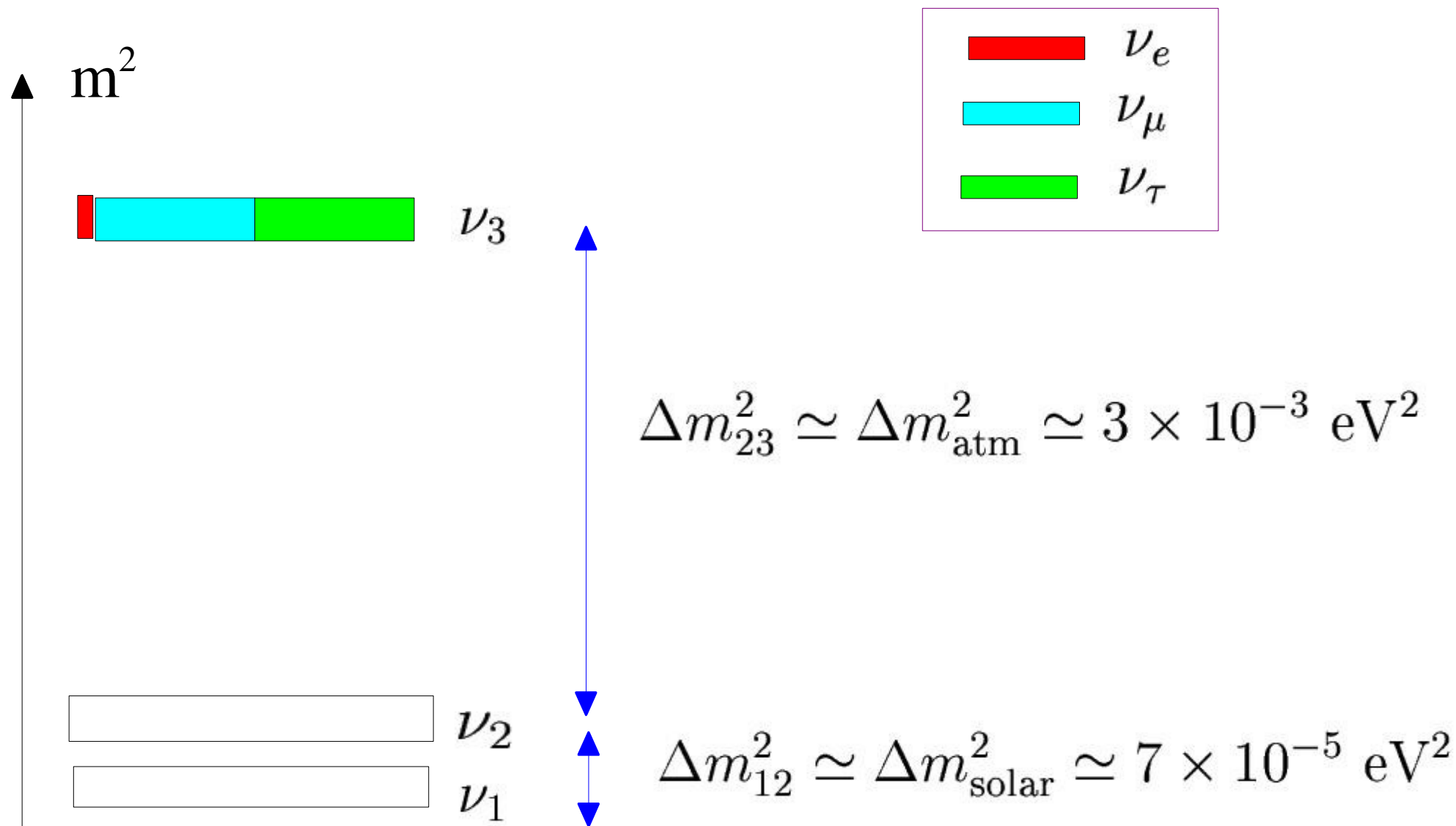
The Oscillation interpretation
is very satisfactory

Parameter determination:

$$|\Delta m_{23}^2| = (2.1 \div 3.1) 10^{-3} \text{ eV}^2$$

$$36^\circ < \theta_{23} < 54^\circ$$

90% CL



$$|\nu_j\rangle = U_{ej} |\nu_e\rangle + U_{\mu j} |\nu_\mu\rangle + U_{\tau j} |\nu_\tau\rangle \quad |U_{ej}|^2 + |U_{\mu j}|^2 + |U_{\tau j}|^2 = 1$$

Confirmation of the
Atmospheric Neutrino Results

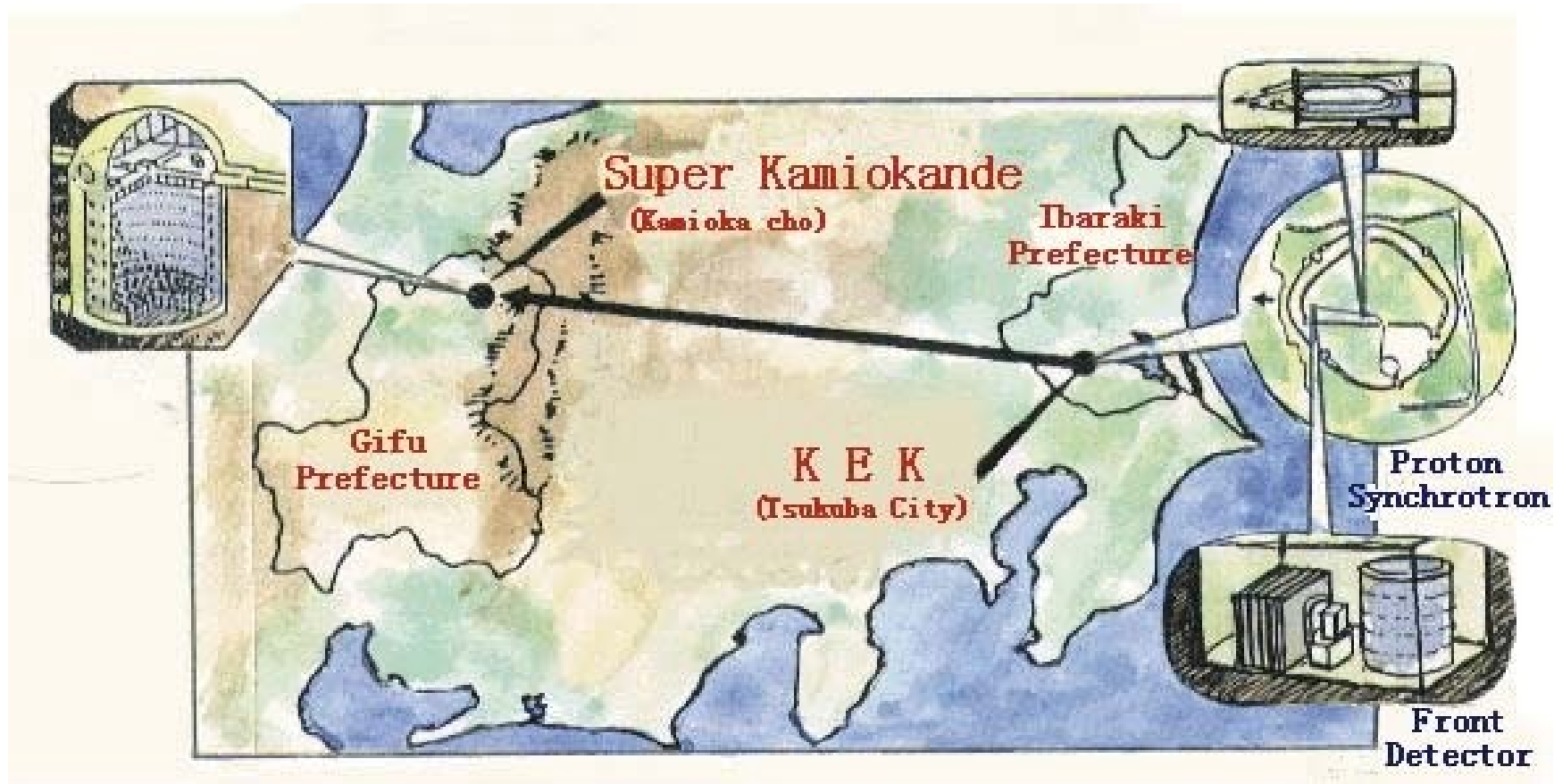
with

ACCELERATOR NEUTRINOS

LONG BASELINE NEUTRINO BEAMS

KEK to Kamioka (K2K) project

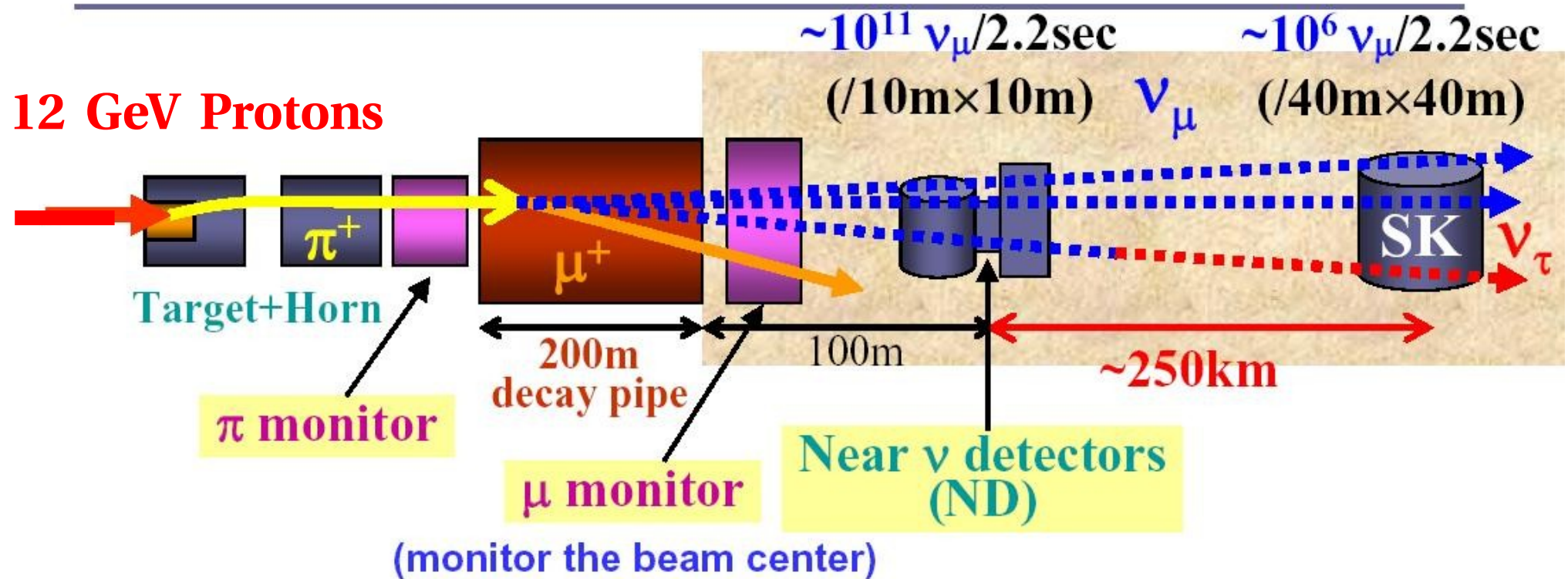
Fermilab to MINOS
CERN to Gran Sasso

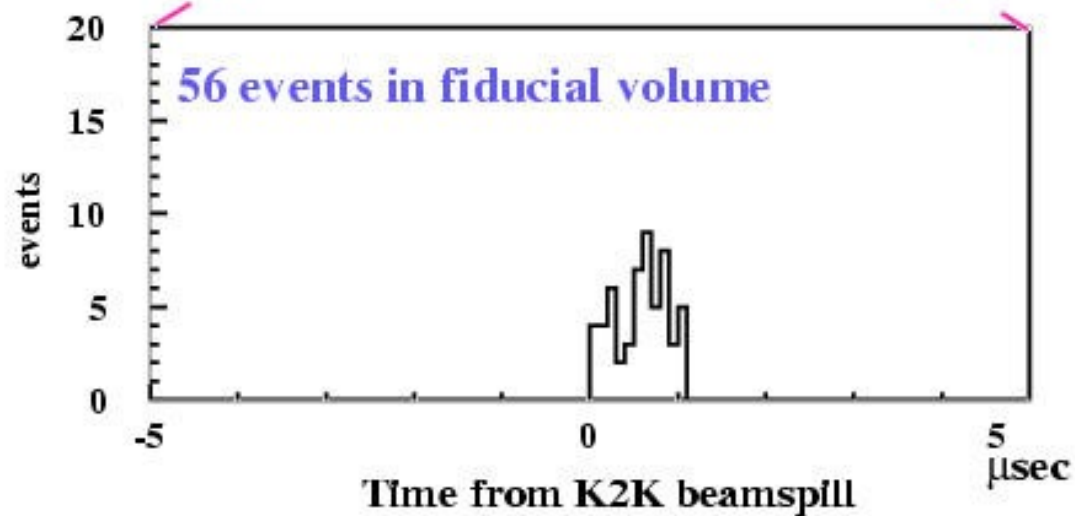
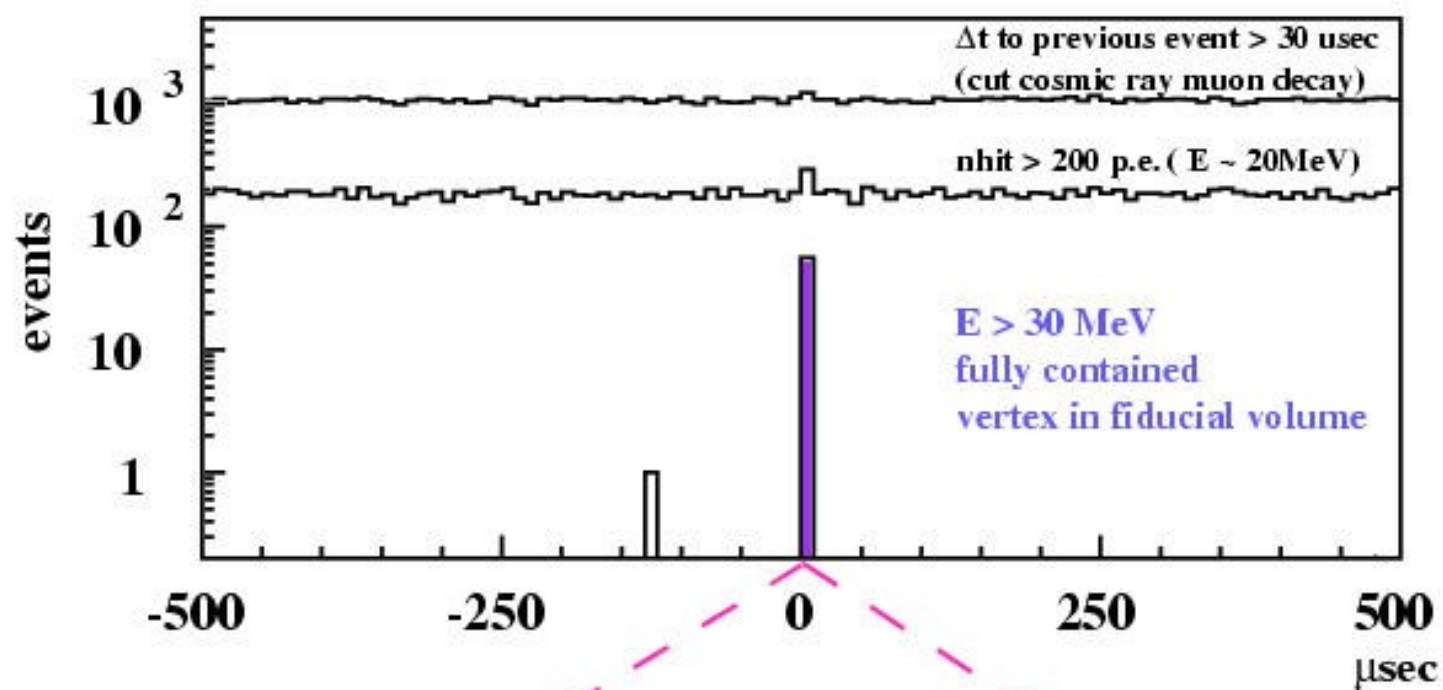


K2K experiment

~1 event/2days

12 GeV Protons





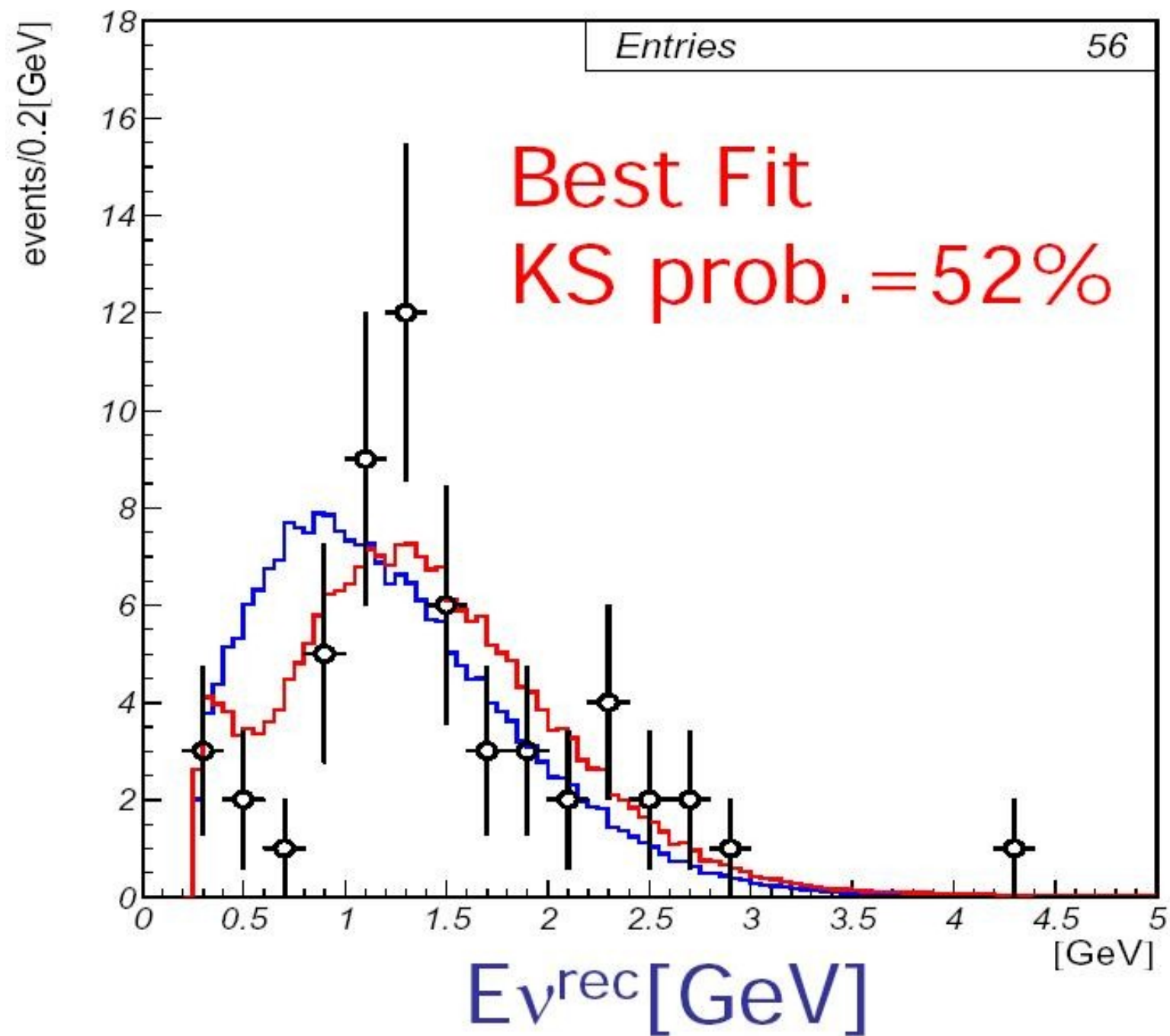
K2K-SK events

preliminary

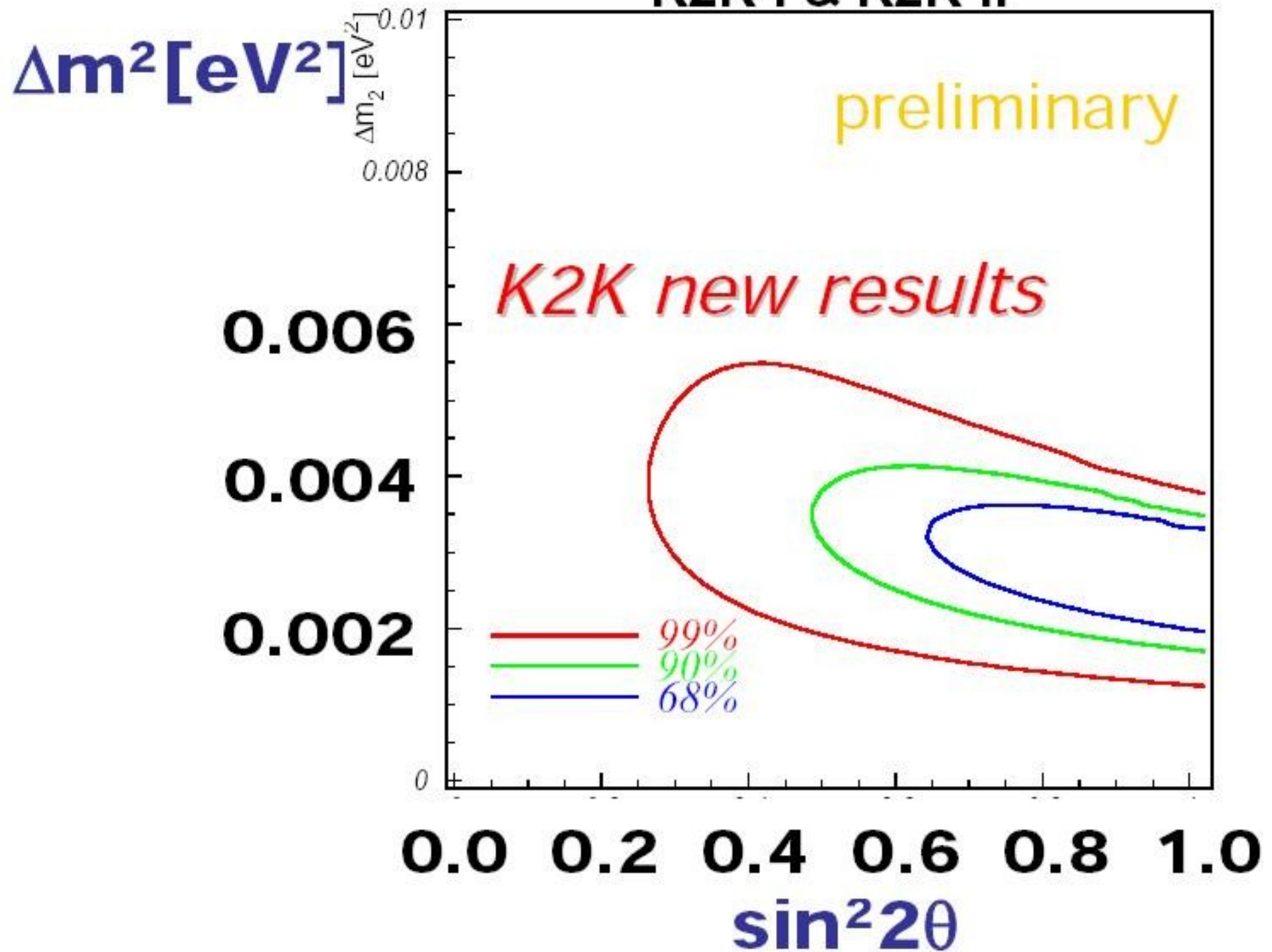
K2K-all (K2K-I, K2K-II)	DATA (K2K-I, K2K-II)	MC (K2K-I, K2K-II)
FC 22.5kt	108 (56, 52)	150.9 (79.1*, 71.8)
1ring	66 (32, 34)	93.7 (48.6, 45.1)
μ-like for E_{ν}^{rec}	57 (56) (30, 27)	84.8 (44.3, 40.5)
e-like	9 (2, 7)	8.8 (4.3, 4.5)
Multi Ring	42 (24, 18)	57.2 (30.5, 26.7)

Ref; K2K-I(47.9×10^{18} POT), K2K-II(41.2×10^{18} POT)

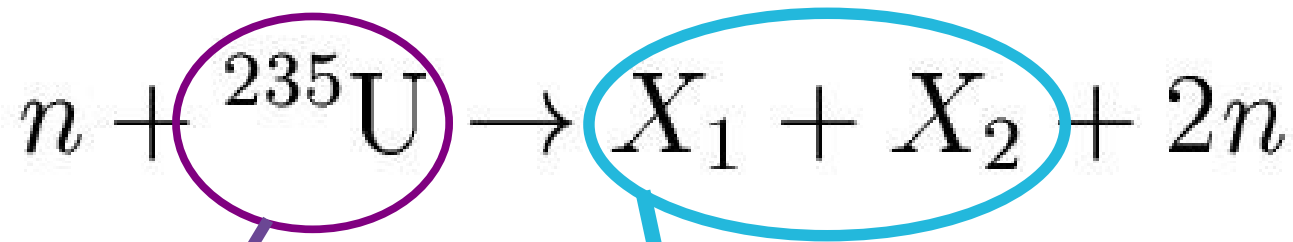
*: The number is changed from the previous one.



K2K-I & K2K-II



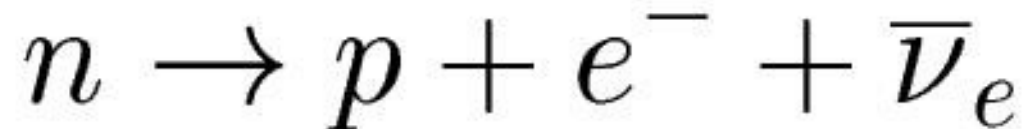
Neutrinos from NUCLEAR REACTORS

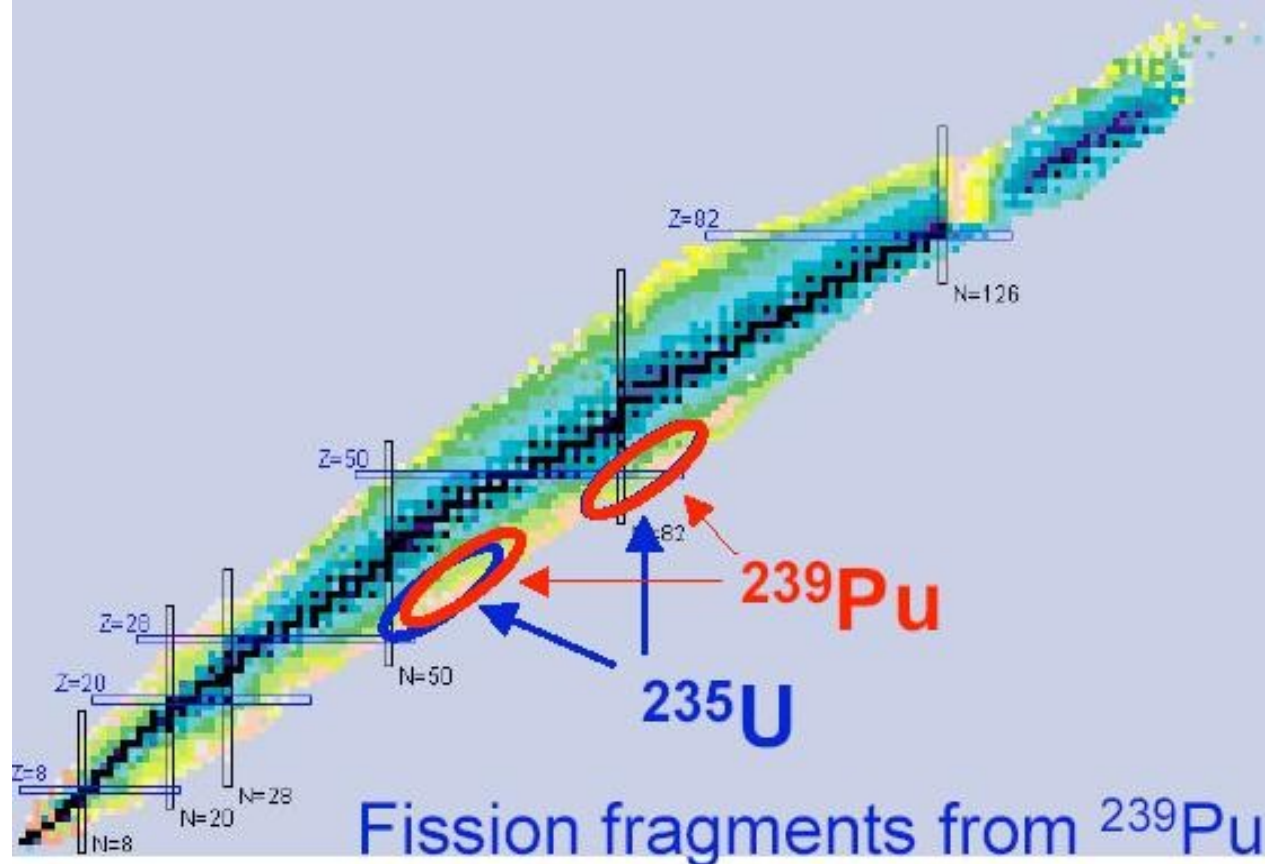


92 p, 143 n

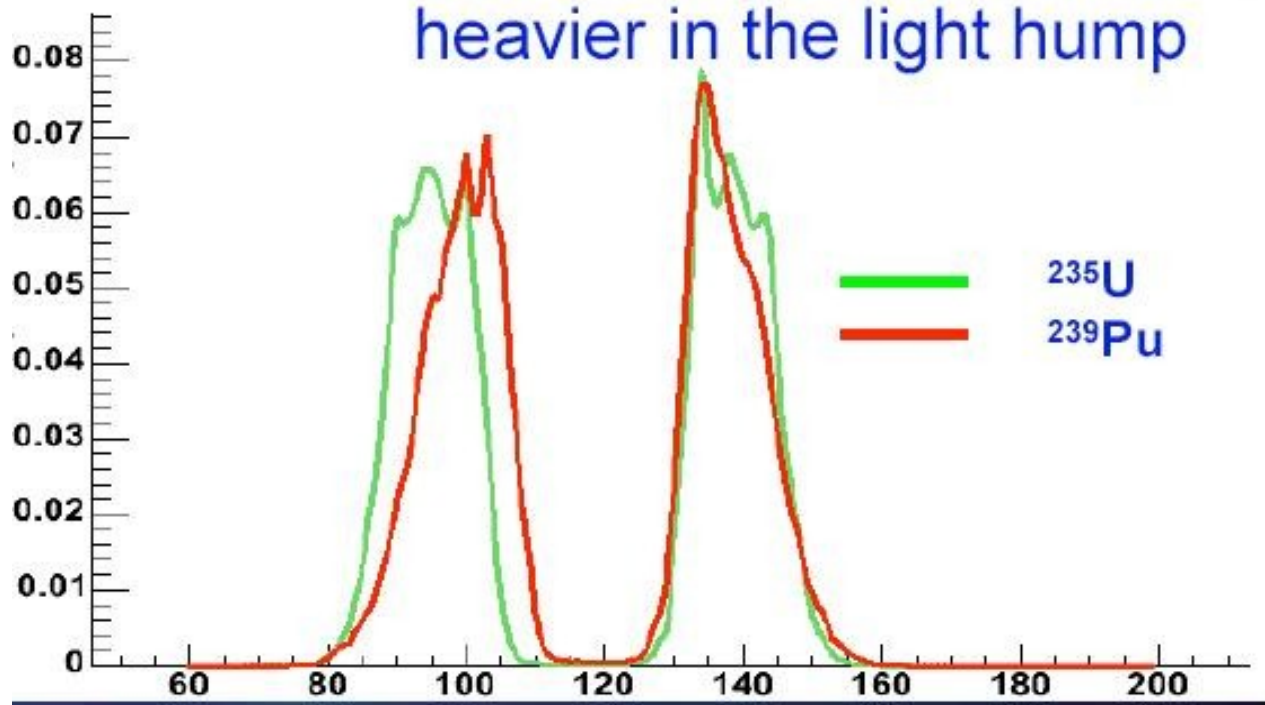
Neutron Rich Nuclei

Beta decay





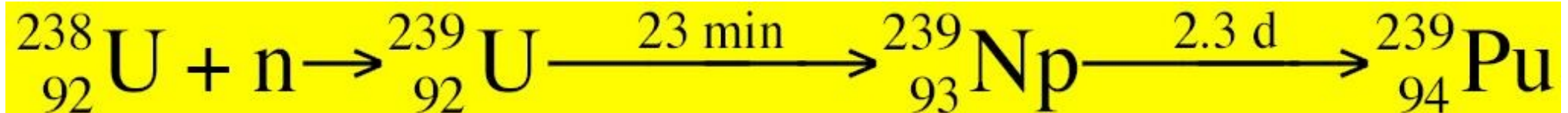
heavier in the light hump



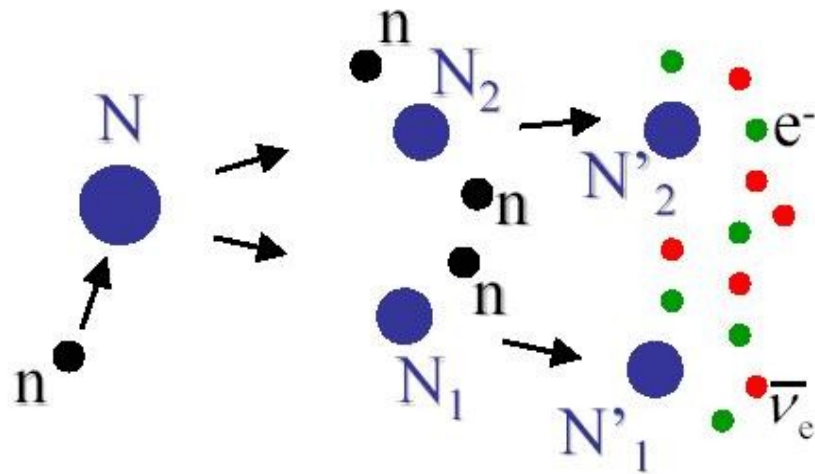
$$\text{Yield} \simeq \frac{6 \bar{\nu}_e}{200 \text{ MeV}}$$

$$\text{Yield} \simeq \frac{1.8 \times 10^{20} \bar{\nu}_e/\text{sec}}{1 \text{ GWatt}}$$

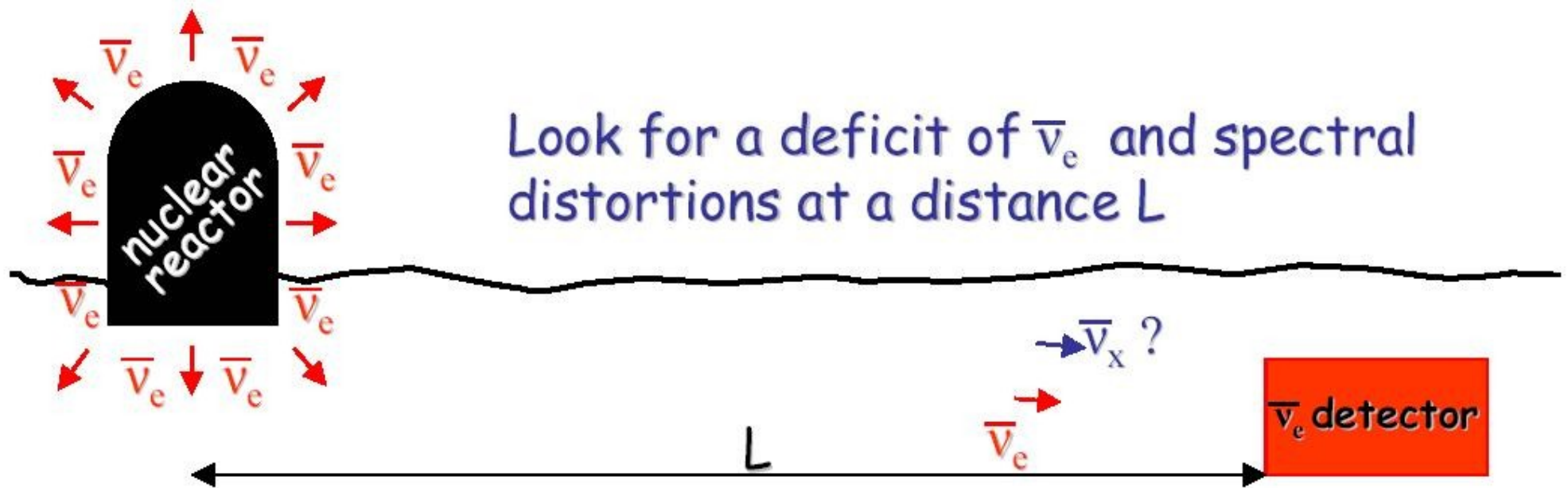
For precision estimate of the flux need to follow the chemical evolution of the nuclear fuel



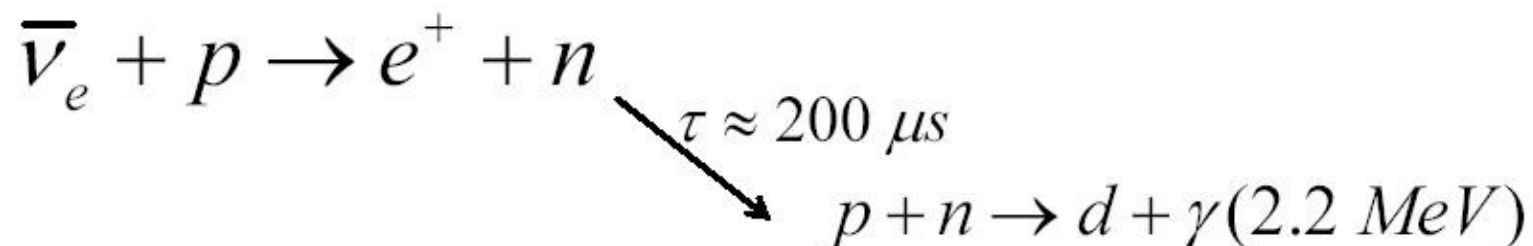
Nuclear reactors are very intense sources of $\bar{\nu}_e$ deriving from beta-decay of the neutron-rich fission fragments



Yield :
 $200 \text{ MeV} / \text{fission}$
 $6 \bar{\nu}_e / \text{fission}$



A specific signature is provided by the inverse- β reaction



Event tagging by coincidence in **time**,
space and **energy** of the neutron capture

$E_{\bar{\nu}}$ measurement

$$E_{\bar{\nu}} \cong T_{e^+} + T_n + \underbrace{(M_n - M_p)}_{10-40 \text{ keV}} + \underbrace{m_{e^+}}_{1.8 \text{ MeV}}$$

Threshold: $E_{\bar{\nu}} > 1.8 \text{ MeV}$

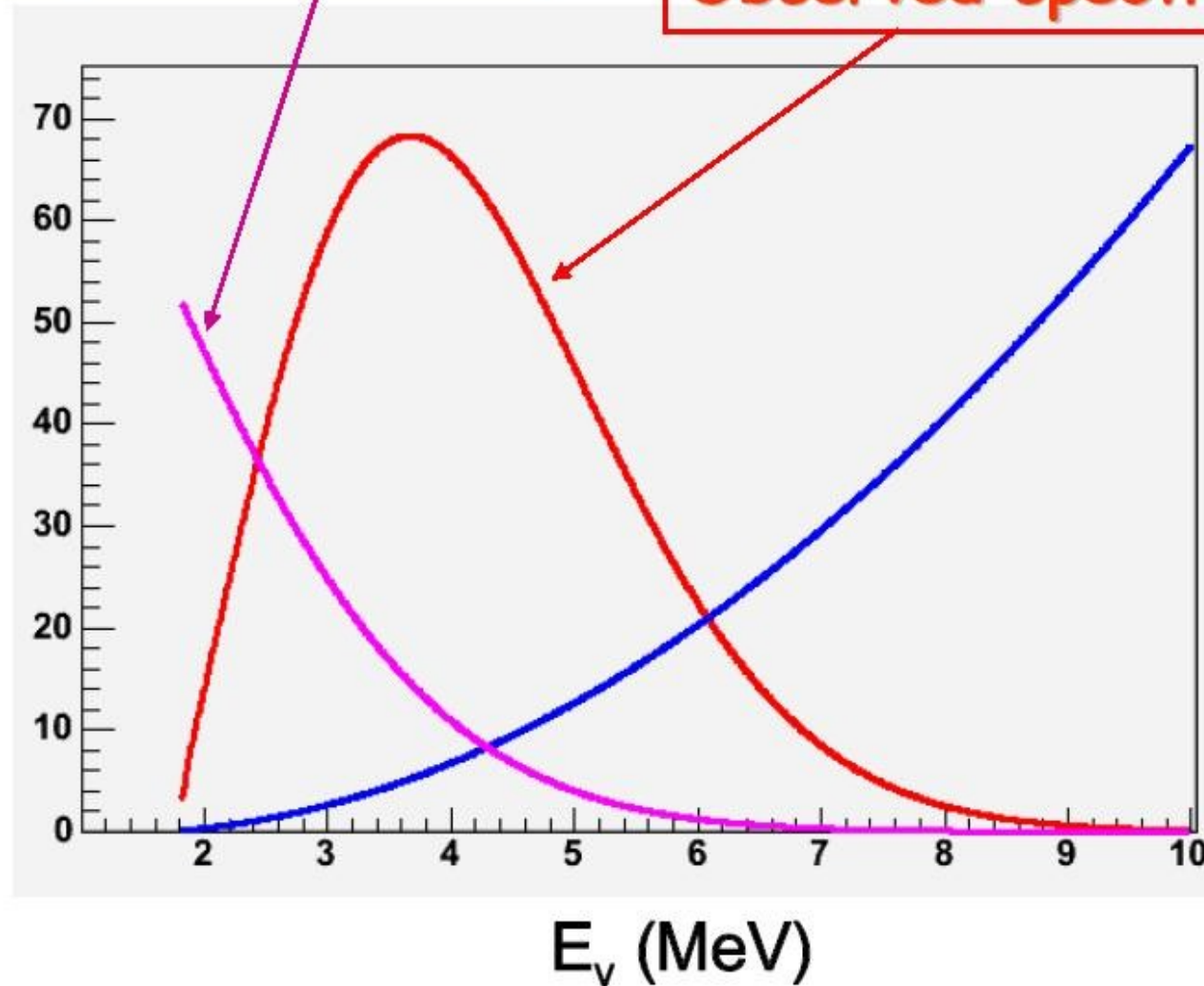
→ only ~1.5 antineutrinos/fission can be detected

The $\bar{\nu}_e$ energy spectrum

Reactor ν_e spectrum (a.u.)

Observed spectrum (a.u.)

$\nu_e + p \rightarrow n + e^+$ cross
section (10^{-43} cm^2)



Study of

$$\begin{array}{l} \nu_e \rightarrow \nu_\mu \\ \nu_e \rightarrow \nu_\tau \end{array}$$

Transitions

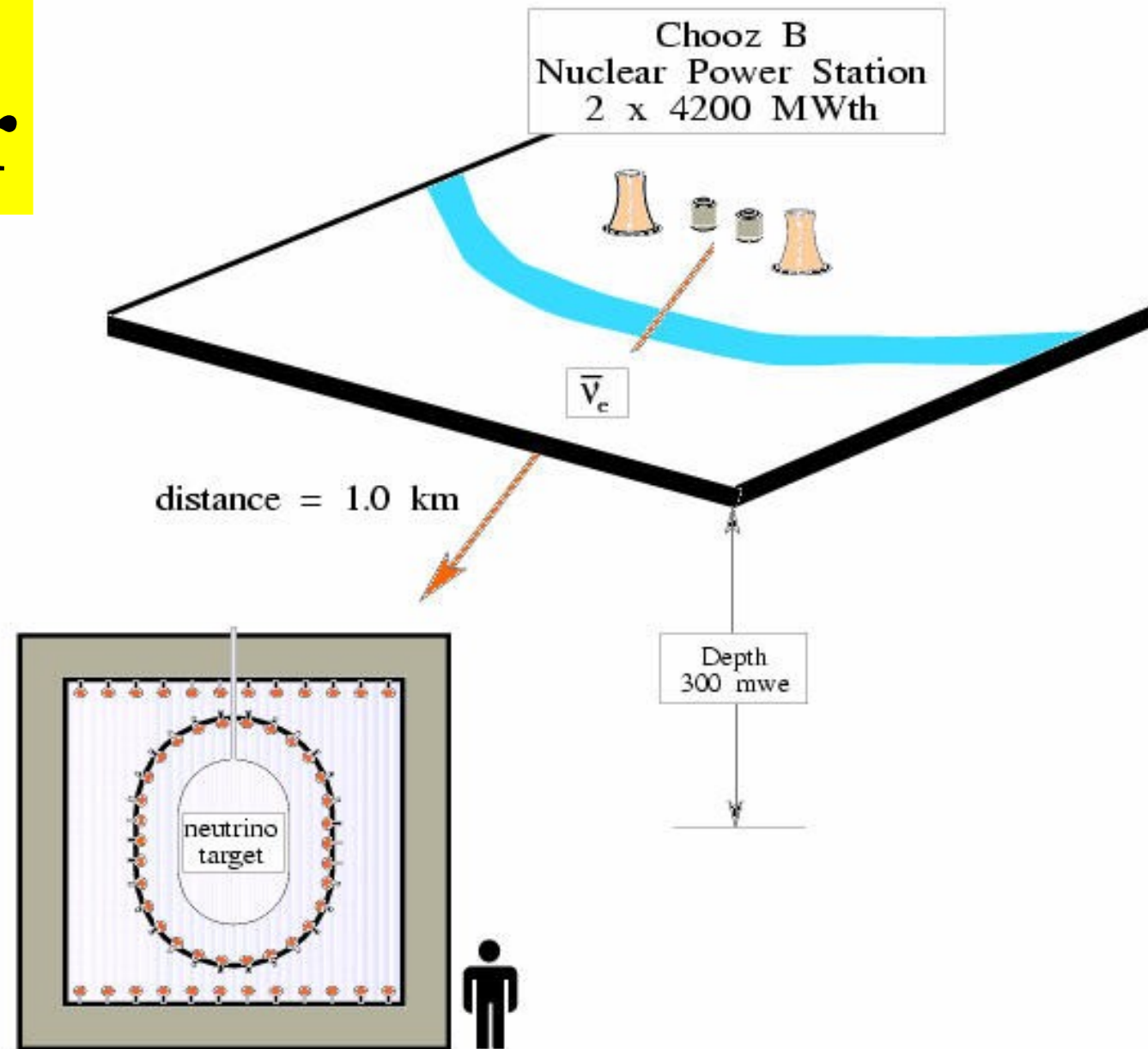
Chooz Experiment
(Palo Verde)

(L = 1 Km)

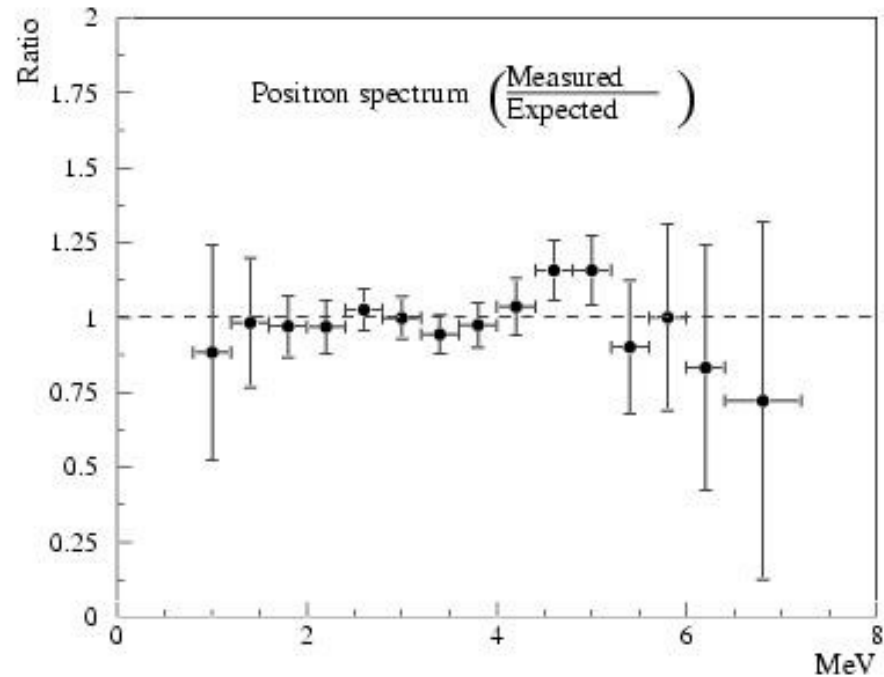
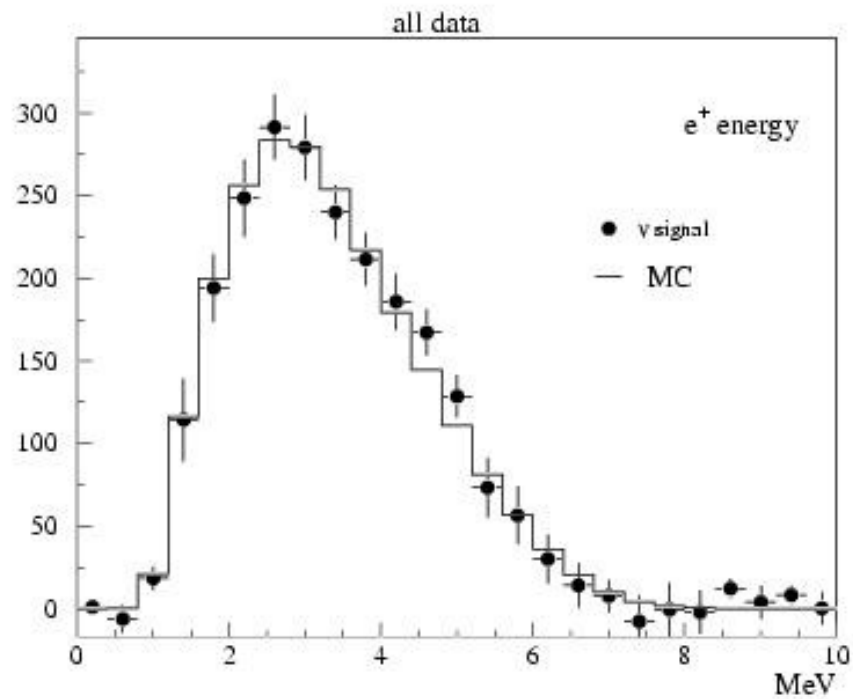
KamLand Experiment

(L 150 Km)

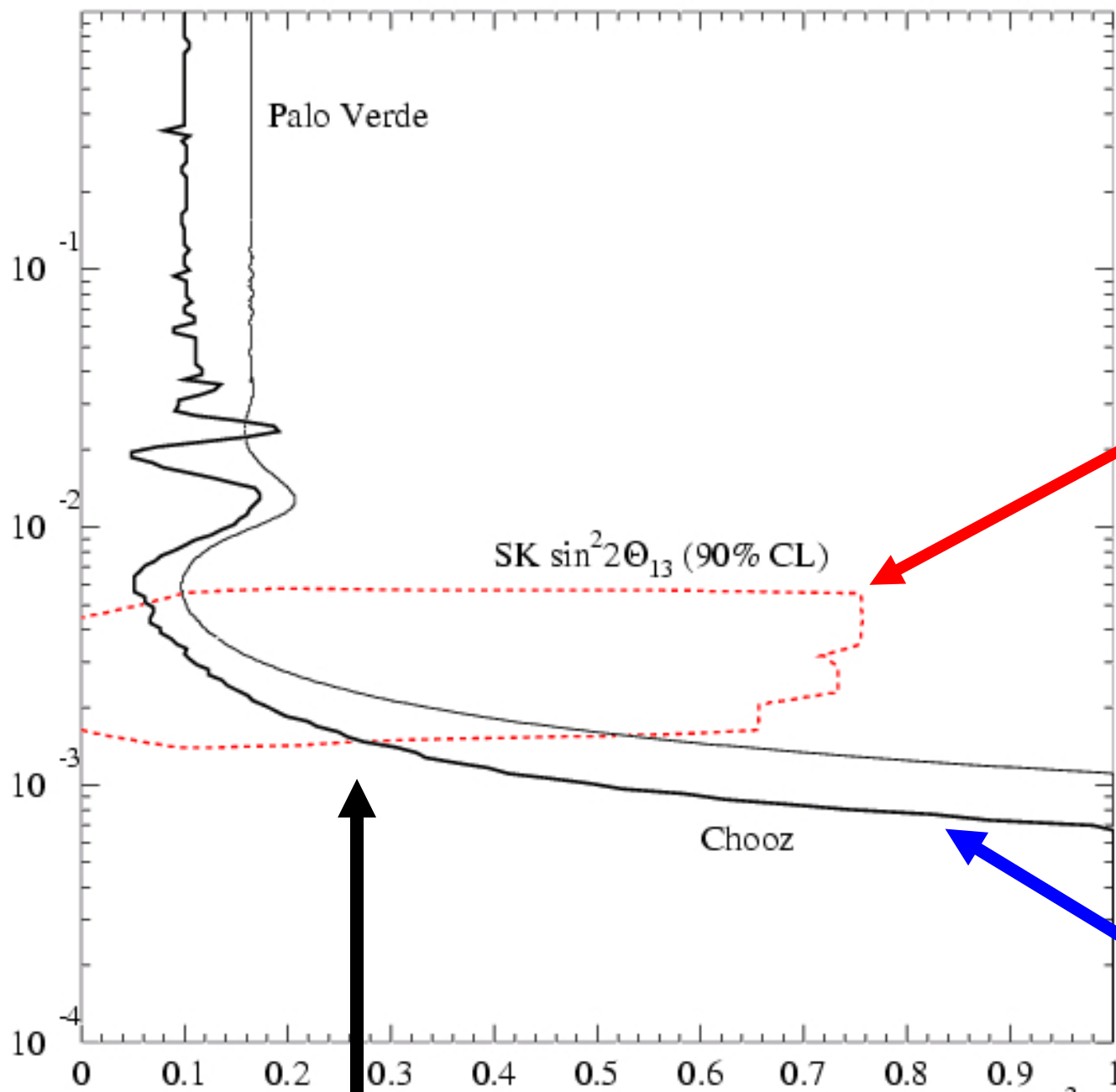
Chooz Detector



Chooz Underground Neutrino Laboratory
Ardennes, France



Data/Prediction =
 $1.01 \pm 2.8\% \text{ (stat)} \pm 2.7\% \text{ (sys)}$

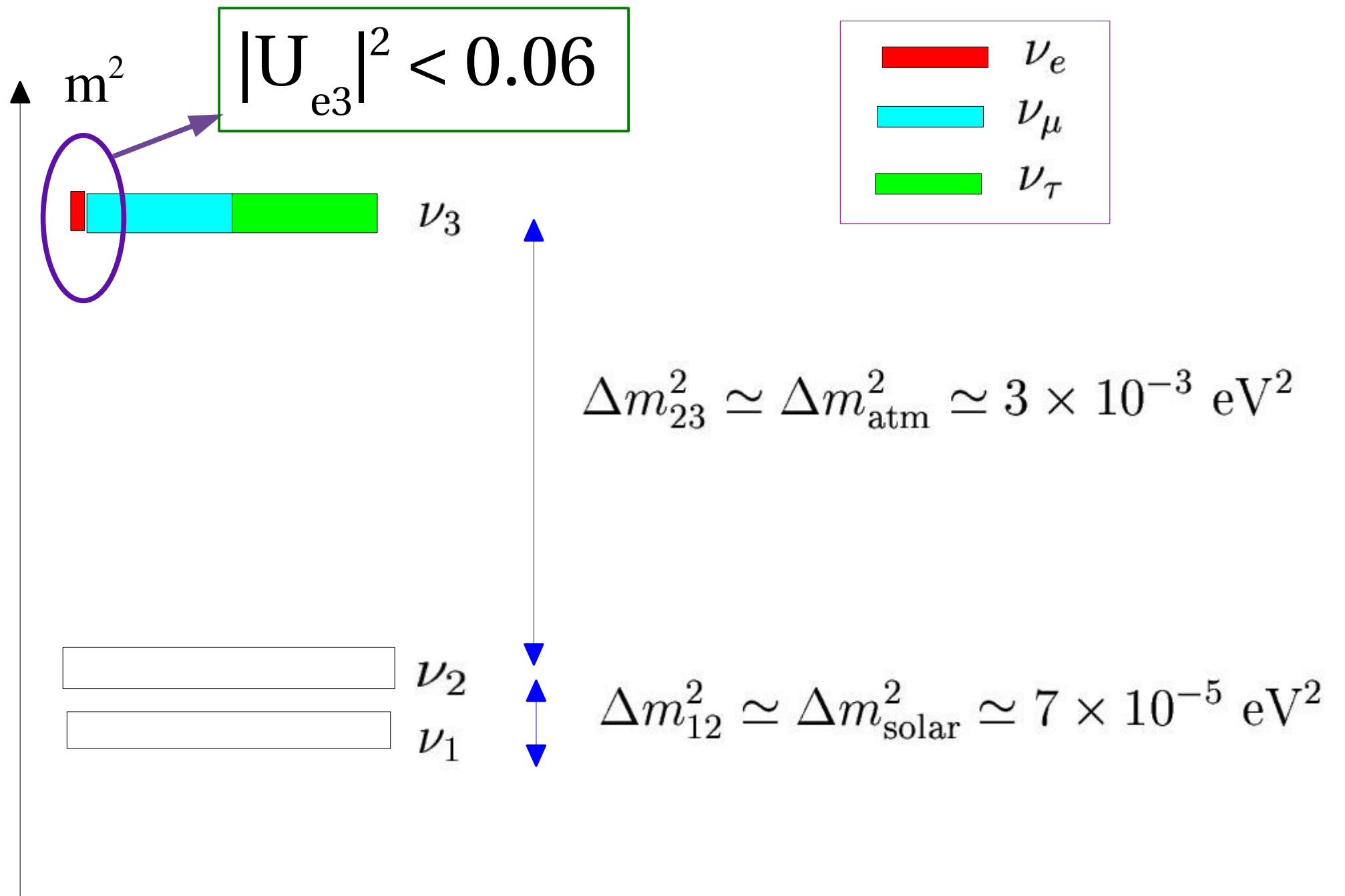
Δm^2 

SK

Chooz

$$|U_{e3}|^2 < 0.06$$

 $\sin^2 2\theta_{13}$

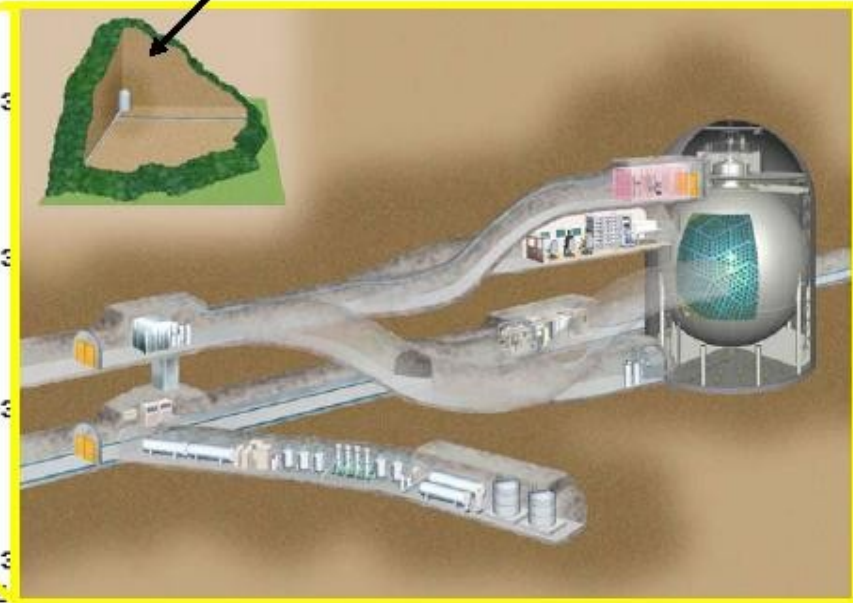


$$|\nu_j\rangle = U_{ej} |\nu_e\rangle + U_{\mu j} |\nu_\mu\rangle + U_{\tau j} |\nu_\tau\rangle \quad |U_{ej}|^2 + |U_{\mu j}|^2 + |U_{\tau j}|^2 = 1$$

KAMLAND Experiment



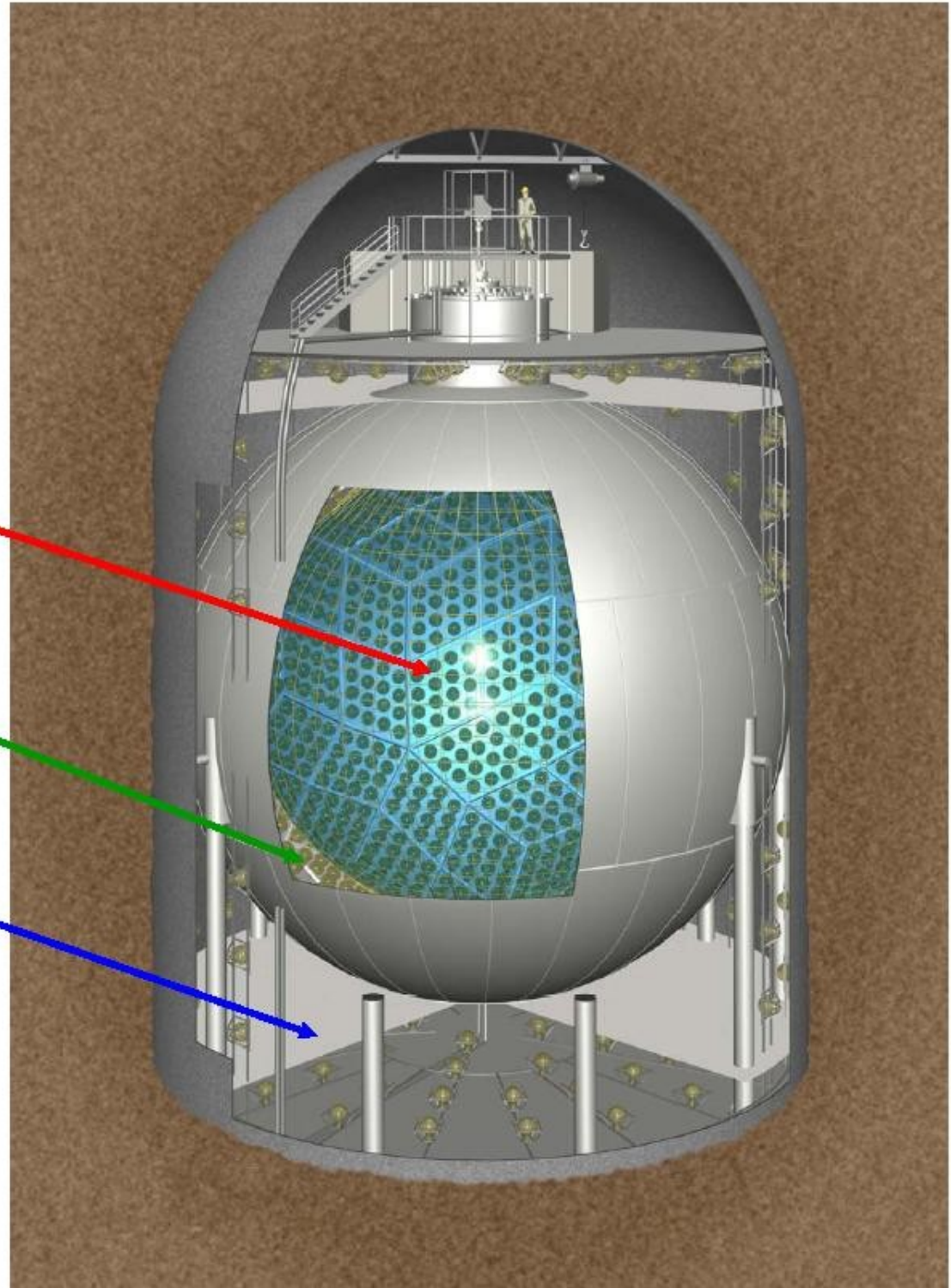
~1 km high
Mt Ikenoyama



KamLAND:

Kamioka Liquid scintillator AntiNeutrino Detector

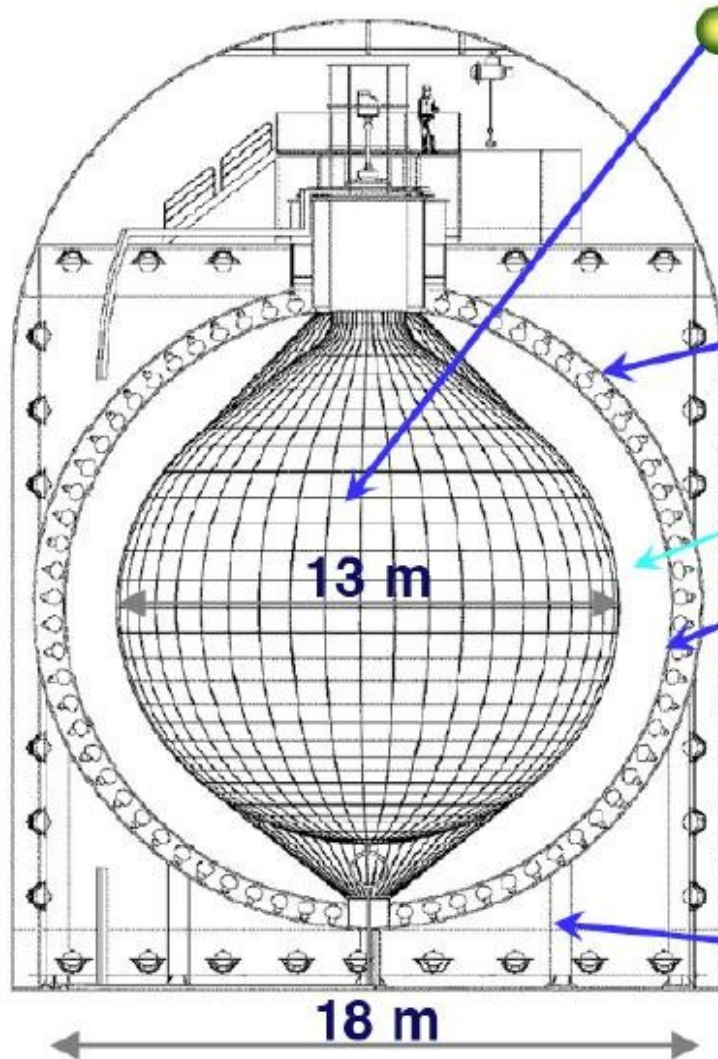
- 1 kton liq. Scint. Detector
in the Kamiokande cavern
- 1325 17" fast PMTs
- 554 20" large area PMTs
- 34% photocathode coverage
- H_2O Cerenkov veto counter



KamLAND Detector

- detector location: old Kamiokande site

: 2700 m.w.e.



- 1000 ton liquid scintillator

: 80% (dodecane) + 20% (pseudocumene)
+ 1.52 g/l PPO

: housed in spherical plastic balloon

- 3000 m³ stainless steel vessel

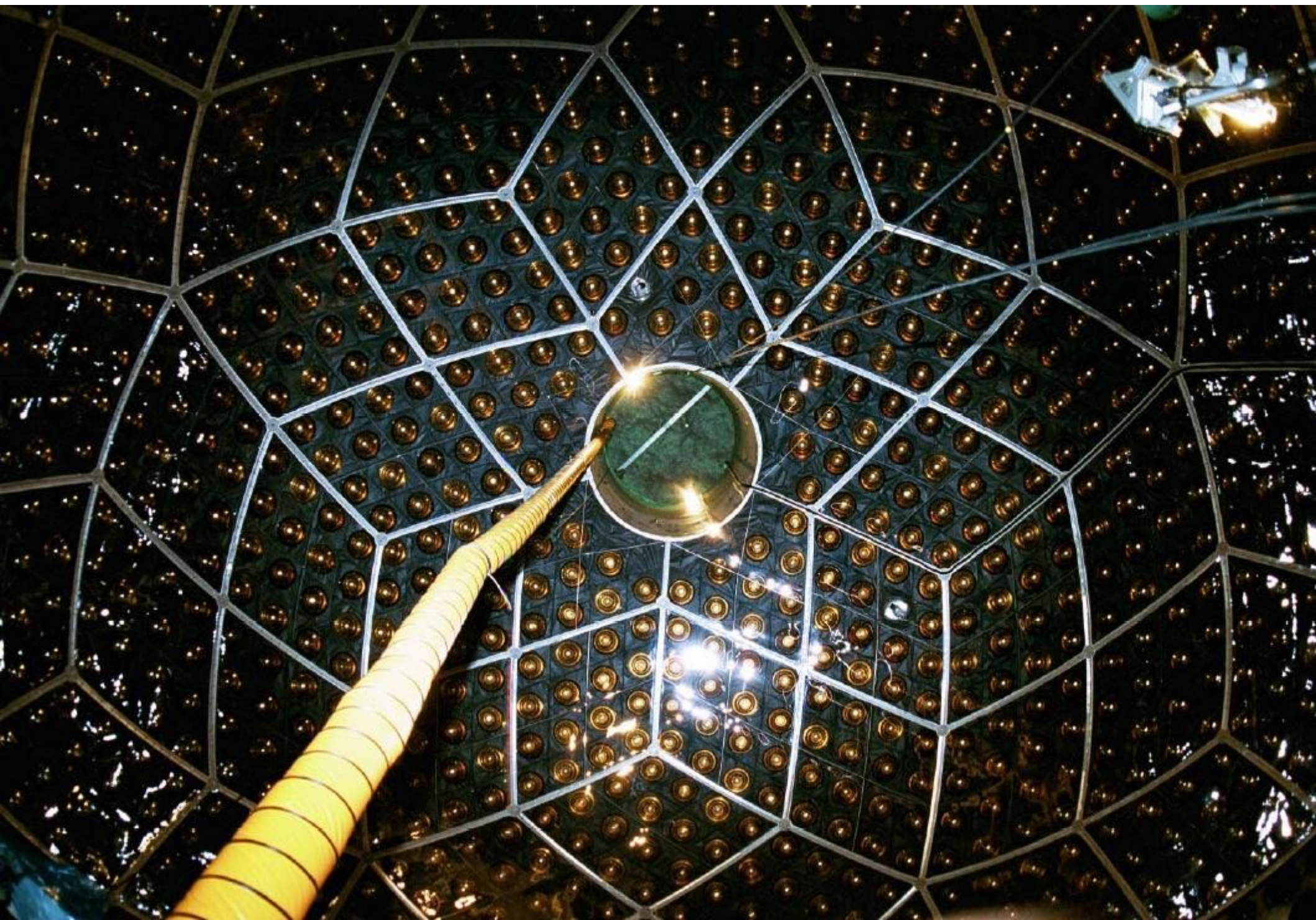
: filled with a mixture of paraffin oil
and dodecane ($\Delta\rho = 0.04\%$)

- 1325 17-inch + 554 20-inch PMT's

commissioned in February, 2003

photocathode coverage : 22% $\rightarrow$ 34%
energy resolution at 1 MeV : 7.3% $\rightarrow$ 6.3%

- water Cerenkov outer detector



Many reactors contribute to the antineutrino flux at KamLAND

Site		Dist (km)	Cores (#)	P _{therm} (GW)	Flux (cm ⁻² s ⁻¹)	Rate noosc* (yr ⁻¹ kt ⁻¹)
Japan	Kashiwazaki	160	7	24.3	4.1·10 ⁵	254.0
	Ohi	179	4	13.7	1.9·10 ⁵	114.3
	Takahama	191	4	10.2	1.2·10 ⁵	74.3
	Tsuruga	138	2	4.5	1.0·10 ⁵	62.5
	Hamaoka	214	4	10.6	1.0·10 ⁵	62.0
	Mihama	146	3	4.9	1.0·10 ⁵	62.0
	Sika	88	1	1.6	9.0·10 ⁴	55.2
	Fukushima1	349	6	14.2	5.1·10 ⁴	31.1
	Fukushima2	345	4	13.2	4.8·10 ⁴	29.5
	Tokai2	295	1	3.3	1.6·10 ⁴	10.1
	Onagawa	431	3	6.5	1.5·10 ⁴	9.3
	Simane	401	2	3.8	1.0·10 ⁴	6.3
	Ikata	561	3	6.0	8.3·10 ³	5.1
	Genkai	755	4	10.1	7.8·10 ³	4.8
	Sendai	830	2	5.3	3.4·10 ³	2.1
Tomari	783	2	3.3	2.3·10 ³	1.4	
South Korea	Ulchin	712	4	11.5	9.9·10 ³	6.1
	Yonggwang	986	6	17.4	7.8·10 ³	4.8
	Kori	735	4	9.2	7.5·10 ³	4.6
	Wolsong	709	4	8.2	7.1·10 ³	4.3
Total Nominal		-	70	181.7	1.3 · 10 ⁶	803.8

*E_v > 3.4 MeV
(E_{prompt} > 2.6 MeV)

↑ Detailed power and fuel composition calculation used

↓ From electrical power
Japanese average fuel used

Results

(766.3 ton·yr,
~4.7× the statistics of the first paper)

Observed events 258
No osc. expected $365 \pm 24(\text{syst})$
Background 7.5 ± 1.3

Background	Events
Accidentals	2.69 ± 0.02
$^8\text{He}/^9\text{Li}$	4.8 ± 0.9
μ -induced n	< 0.89
Total	7.5 ± 1.3

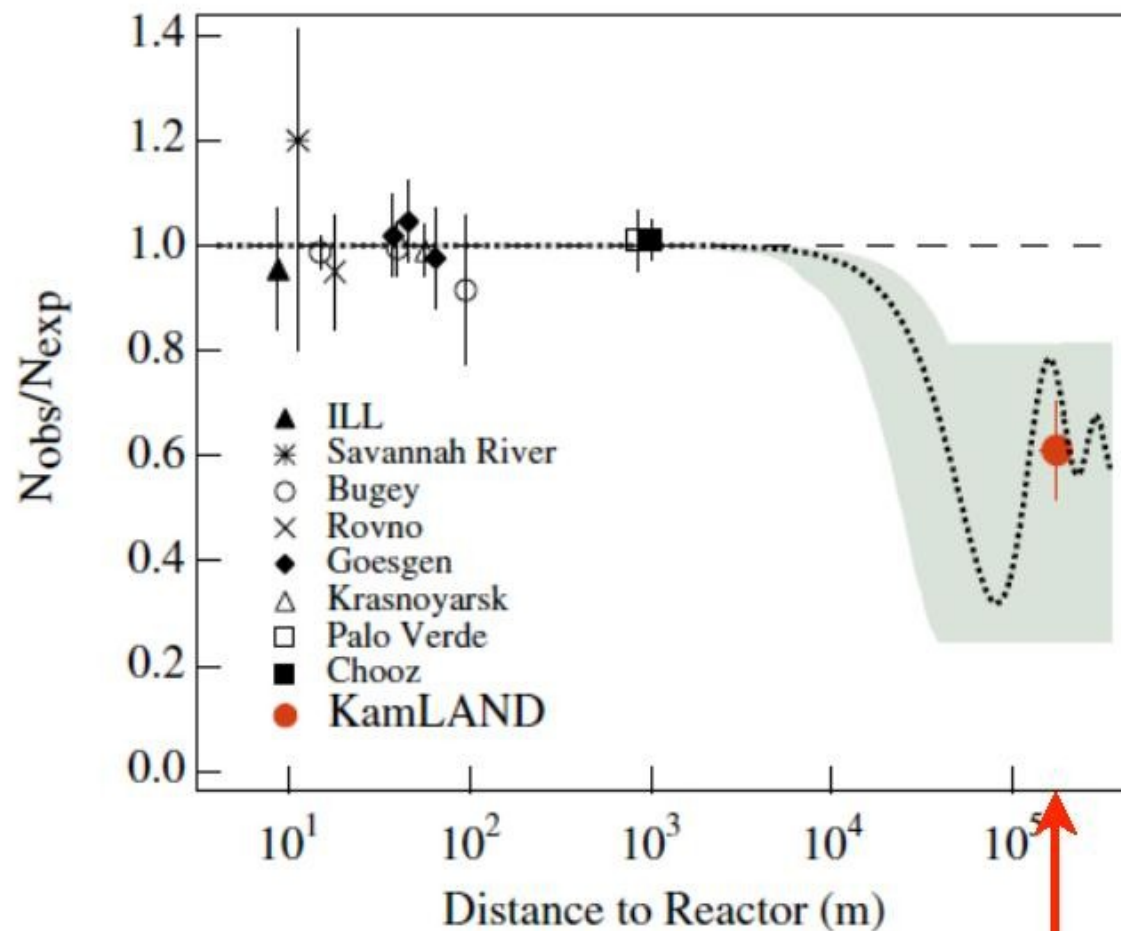
Inconsistent with simple $1/R^2$ propagation
at 99.995% CL

$$(\text{Observed} - \text{Background}) / \text{Expected} = 0.686 \pm 0.044(\text{stat}) \pm 0.045(\text{syst})$$

*Caveat: this specific number does not have an absolute meaning in KamLAND,
since, with oscillations, it depends on which reactors are on/off*

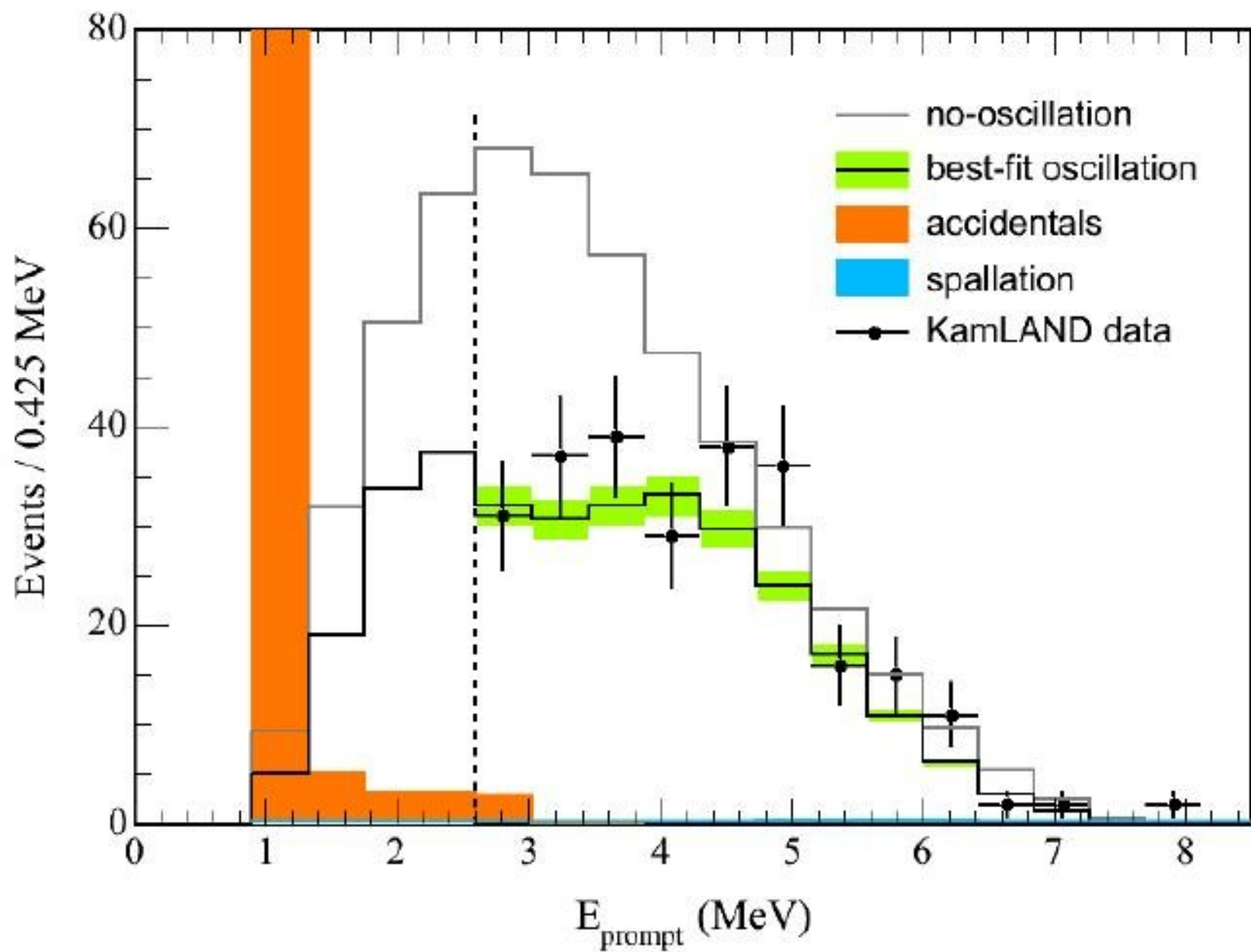


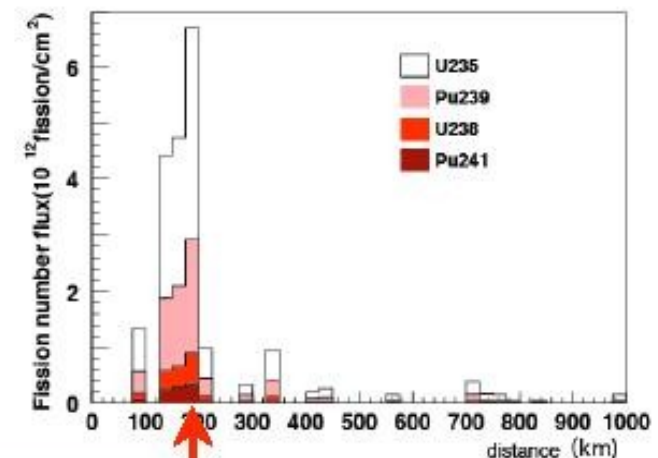
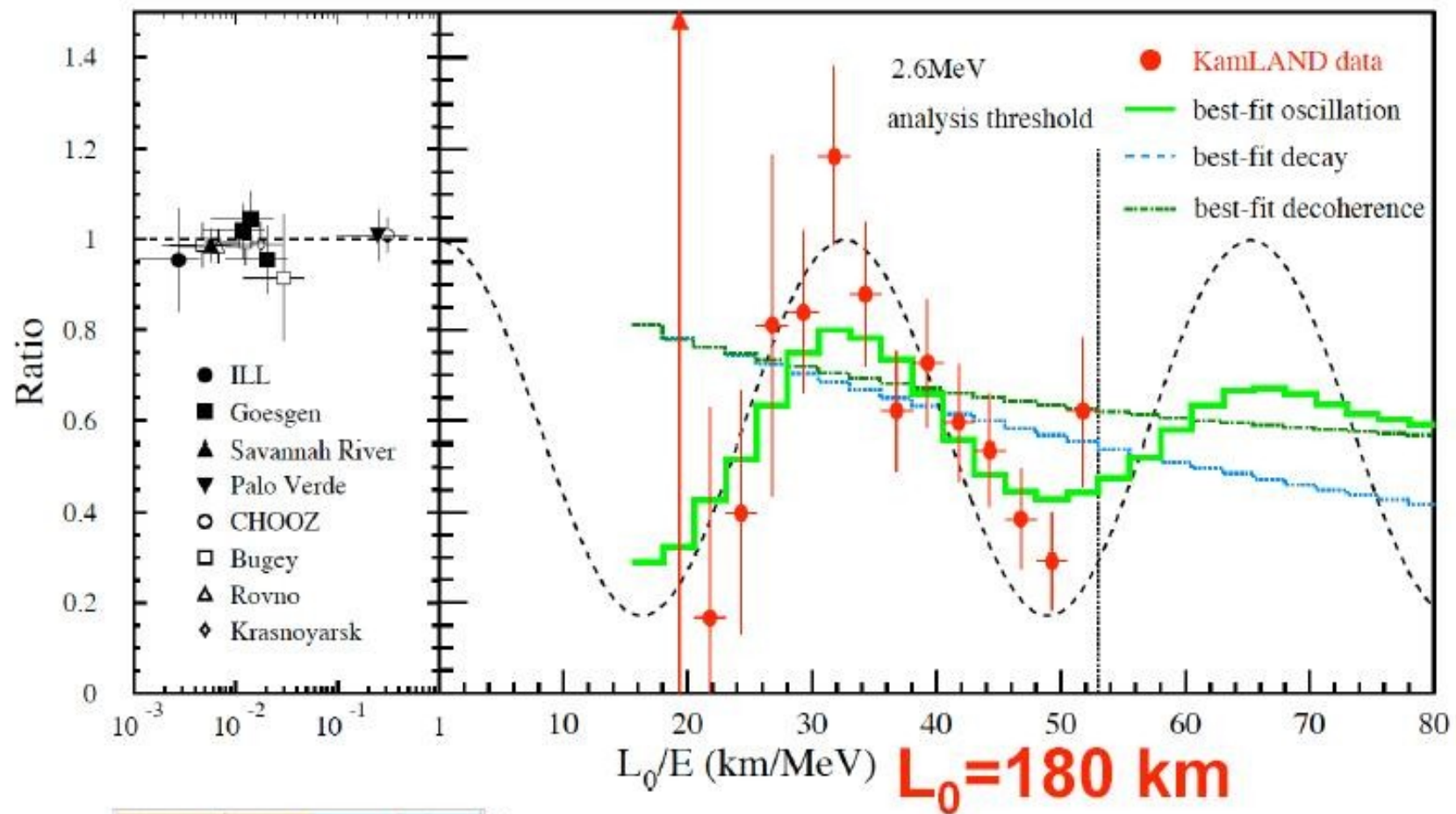
180 km



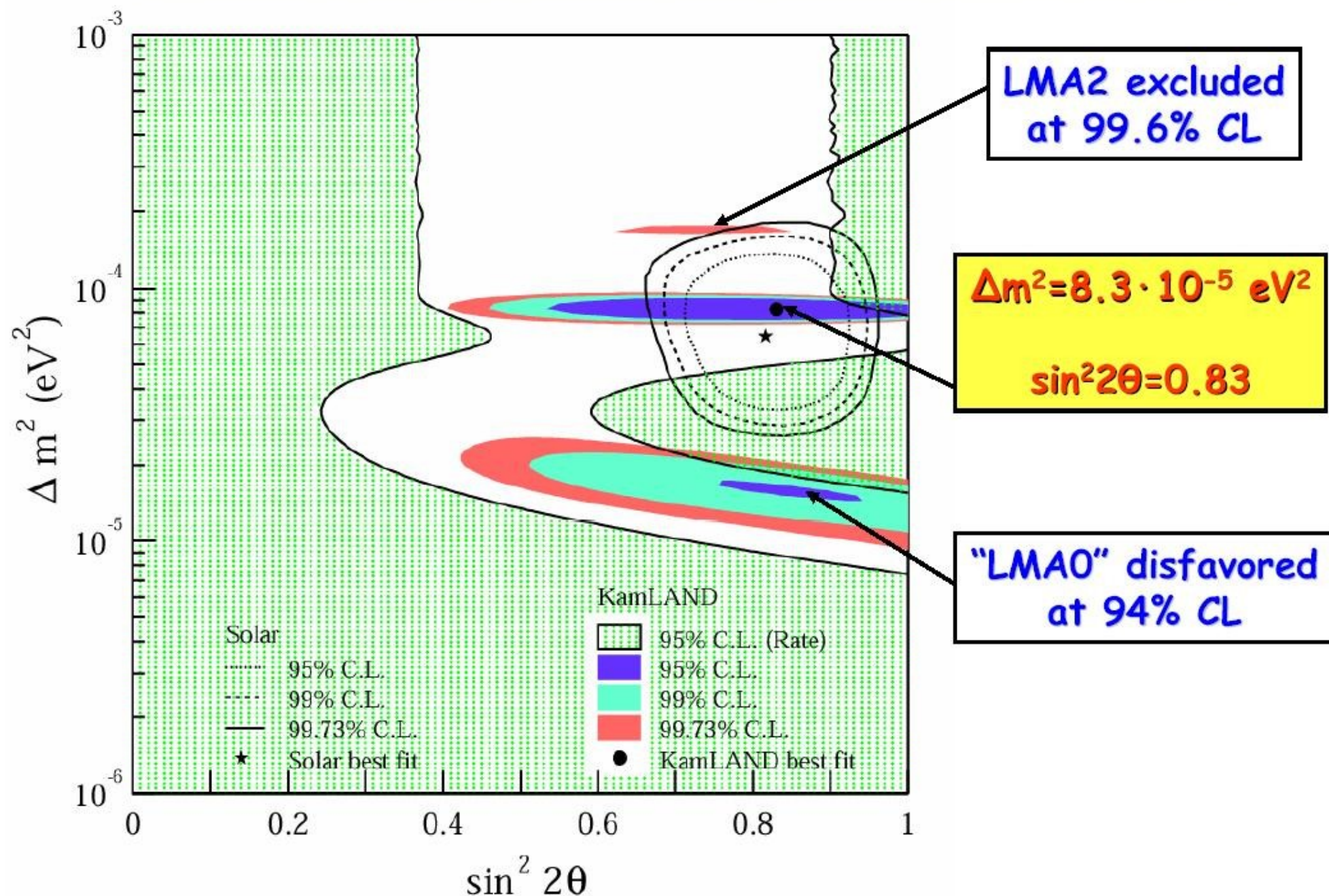
180 km

$$\frac{N_{obs} - N_{BG}}{N_{expected}} = 0.611 \pm 0.085(\text{stat}) \pm 0.041(\text{syst})$$

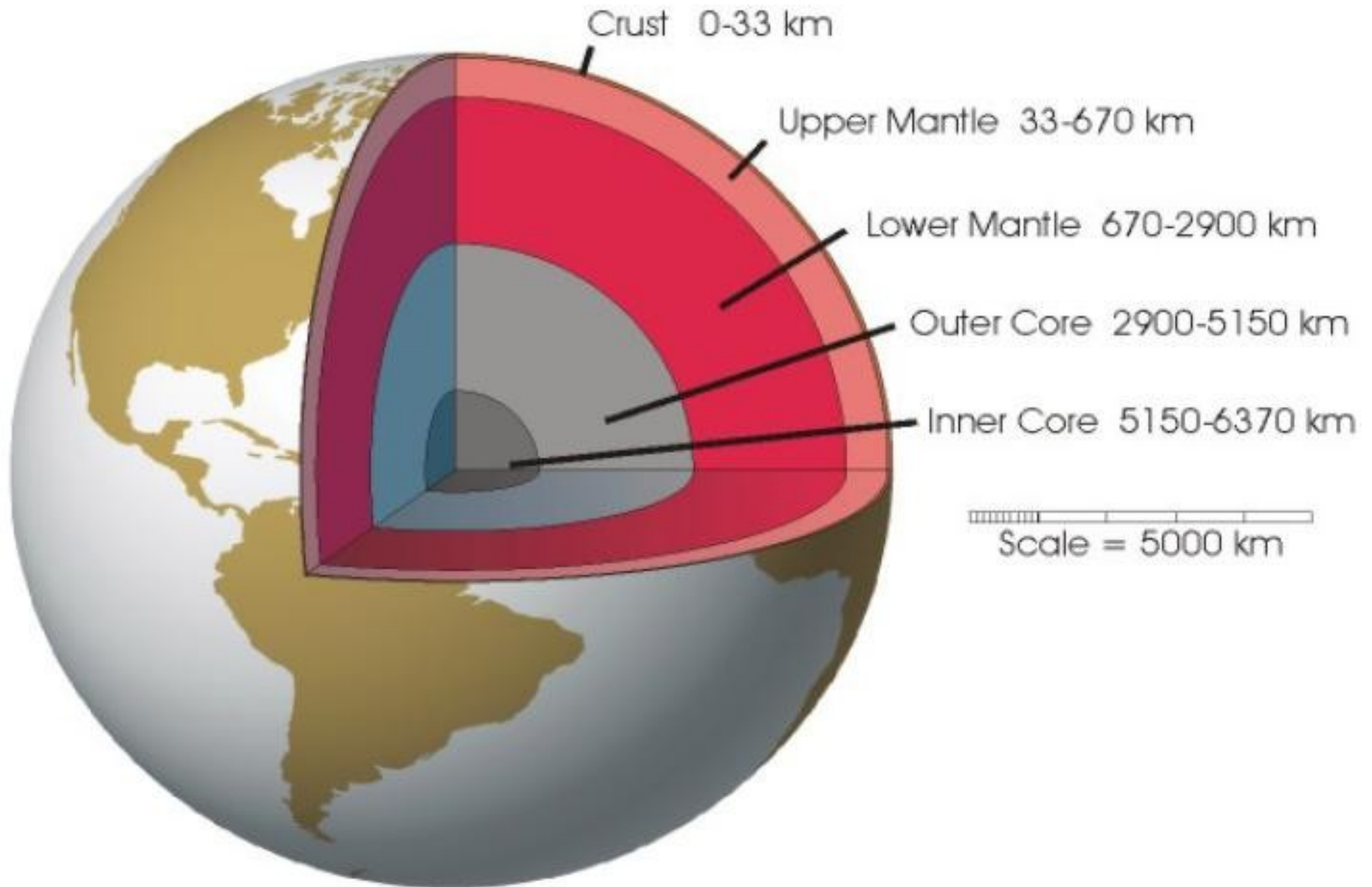




Un-binned likelihood fit to 2-flavor oscillations

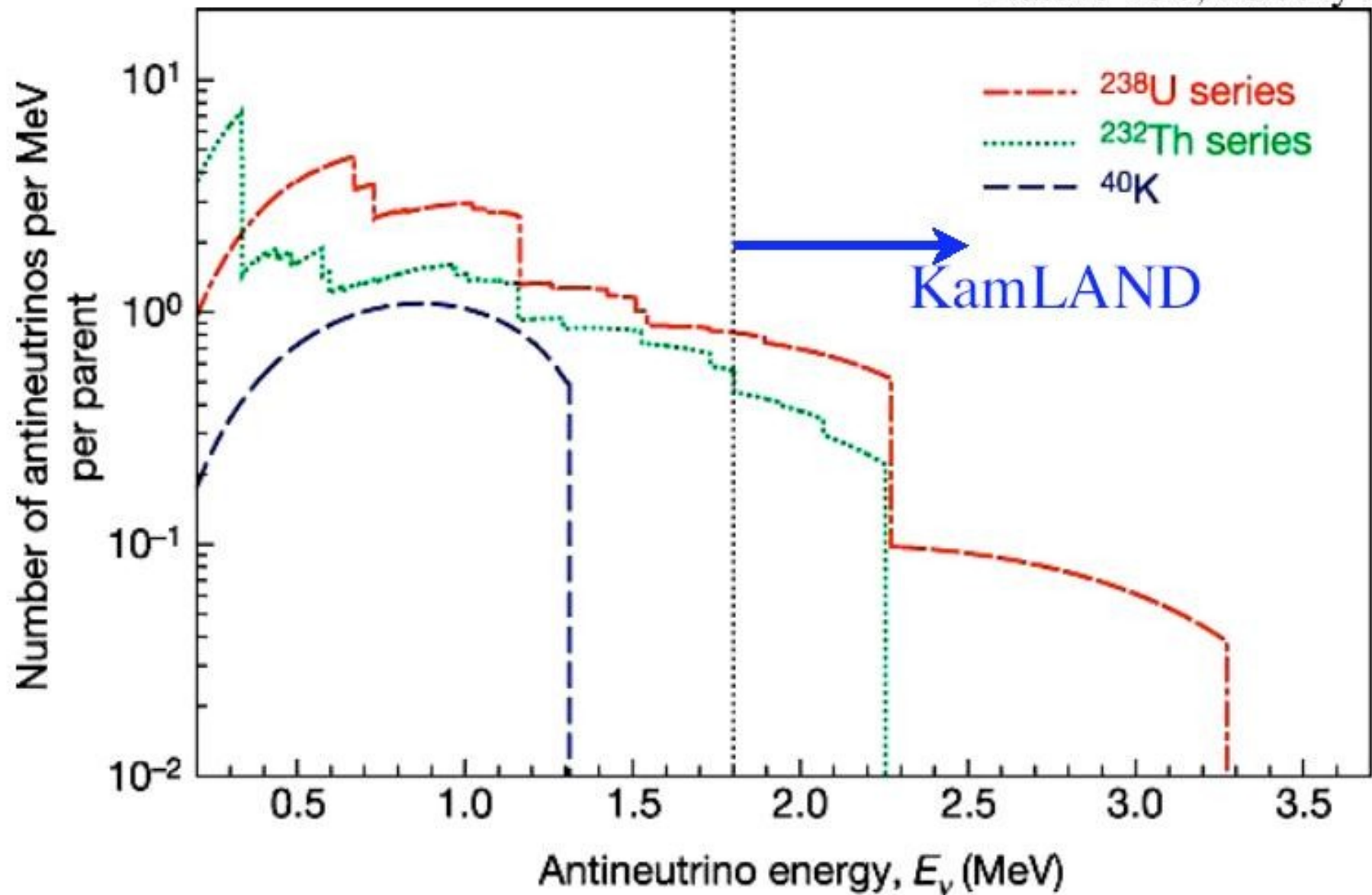


Geophysical Neutrinos



Geophysical Neutrinos

Decay	Q [MeV]	$\tau_{1/2}$ [10^9 yr]	E_{max} [MeV]	ε_H [W/kg]	$\varepsilon_{\bar{\nu}}$ [kg ⁻¹ s ⁻¹]
$^{238}\text{U} \rightarrow ^{206}\text{Pb} + 8^4\text{He} + 6e + 6\bar{\nu}$	51.7	4.47	3.26	$0.95 \cdot 10^{-4}$	$7.41 \cdot 10^7$
$^{232}\text{Th} \rightarrow ^{208}\text{Pb} + 6^4\text{He} + 4e + 4\bar{\nu}$	42.8	14.0	2.25	$0.27 \cdot 10^{-4}$	$1.63 \cdot 10^7$
$^{40}\text{K} \rightarrow ^{40}\text{Ca} + e + \bar{\nu}$	1.32	1.28	1.31	$0.36 \cdot 10^{-8}$	$2.69 \cdot 10^4$

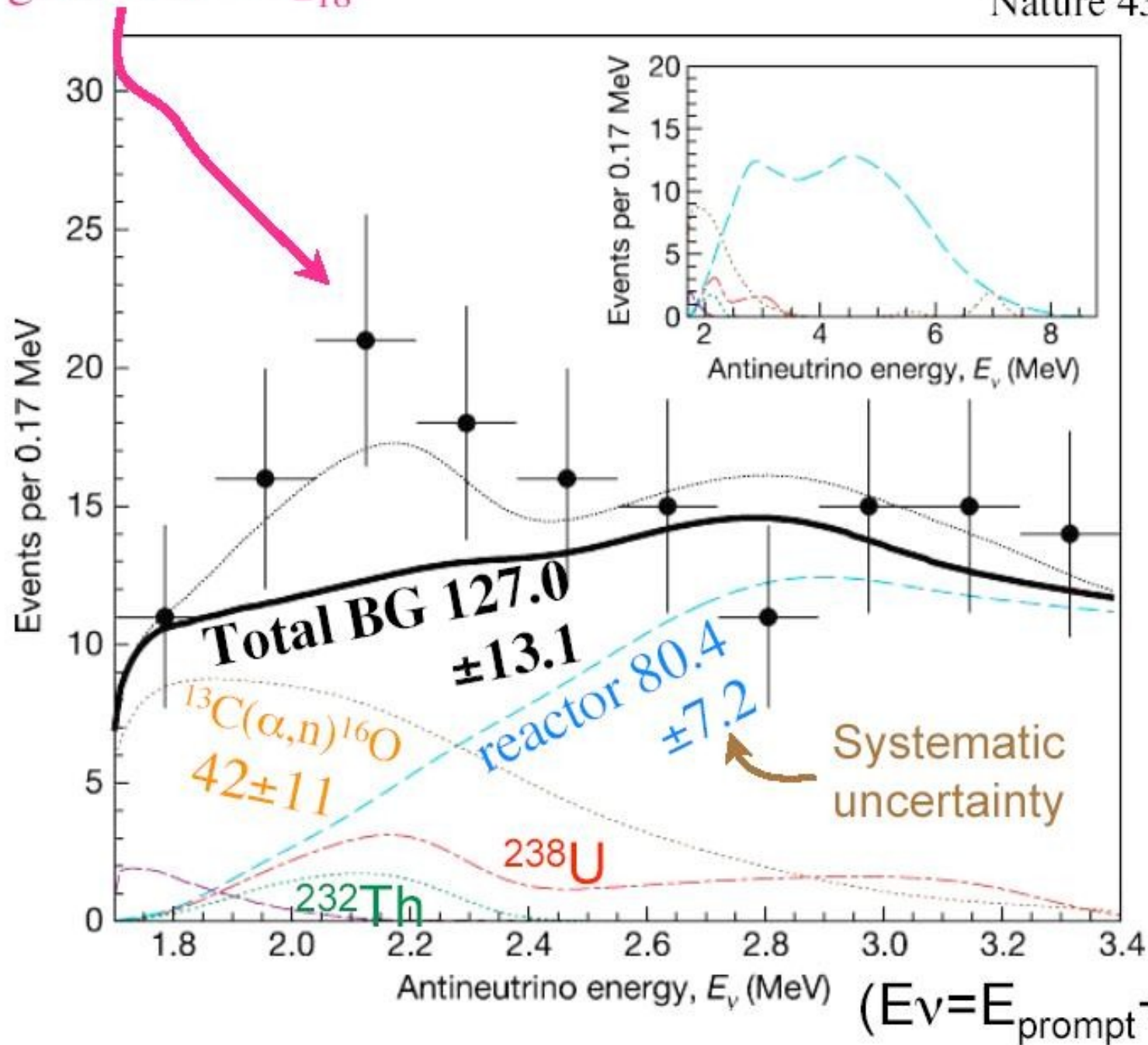


The expected ^{238}U , ^{232}Th , and ^{40}K decay chain electron anti-neutrino energy distribution. KamLAND can only detect electron antineutrinos to the right of the vertical dotted black line; hence it is insensitive to ^{40}K electron antineutrinos.

152 events observed
“signal” 25^{+19}_{-18}

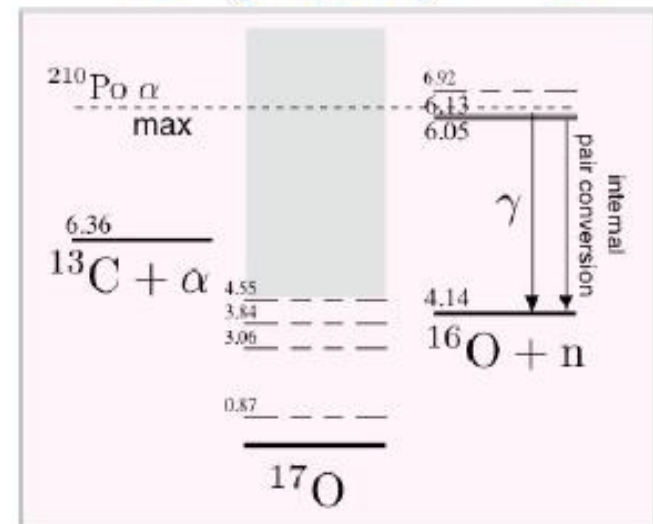
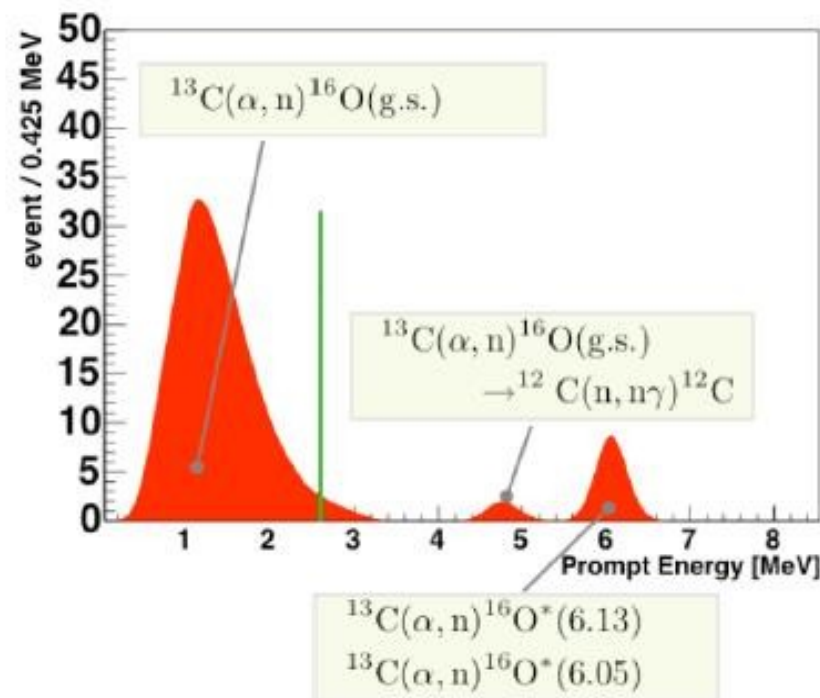
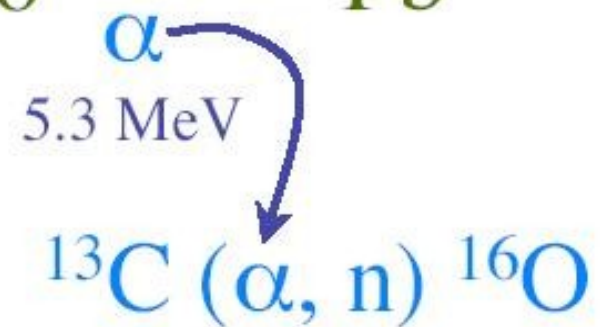
Geoneutrino results

Nature 436, 28 July 2005



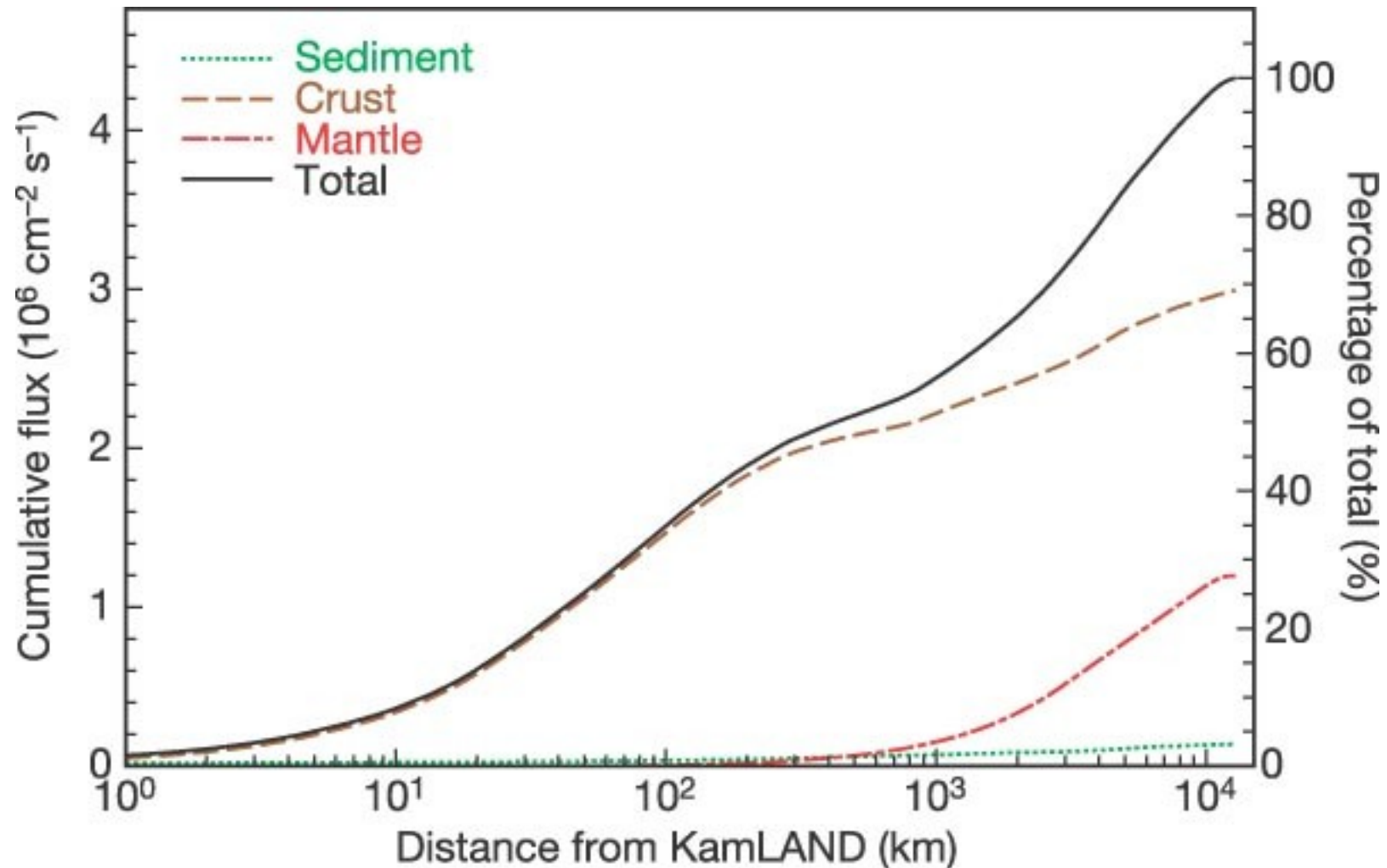
Data-set:
749.1 days
(Mar. 9, 2002
-Oct. 30, 2004)
Fiducial:
5 m radius

Geoneutrino future: $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reduction



- Reduction of systematic error: currently going on
- Reduction of ^{210}Pb : near future (see “solar neutrino future” next)

Where are the Geo-neutrinos coming from ?



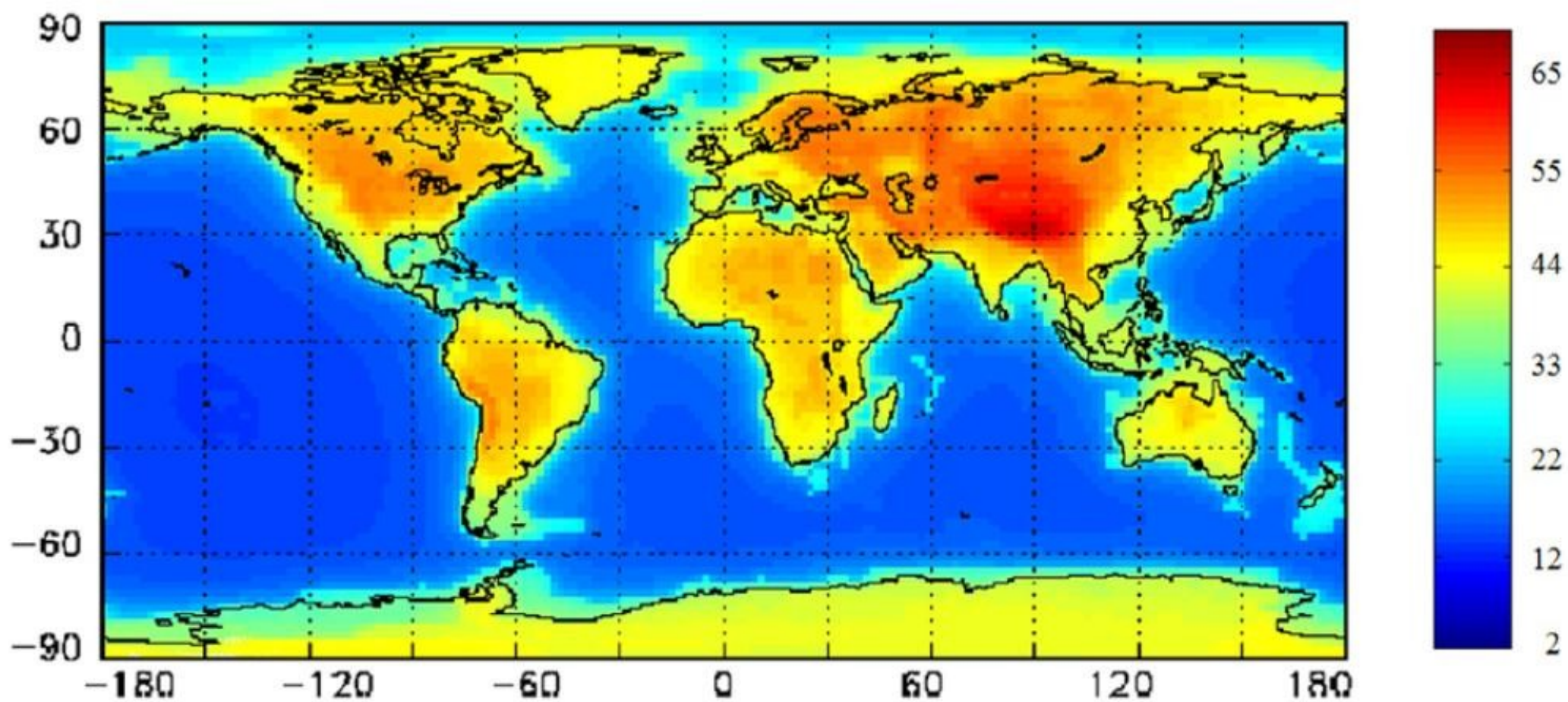
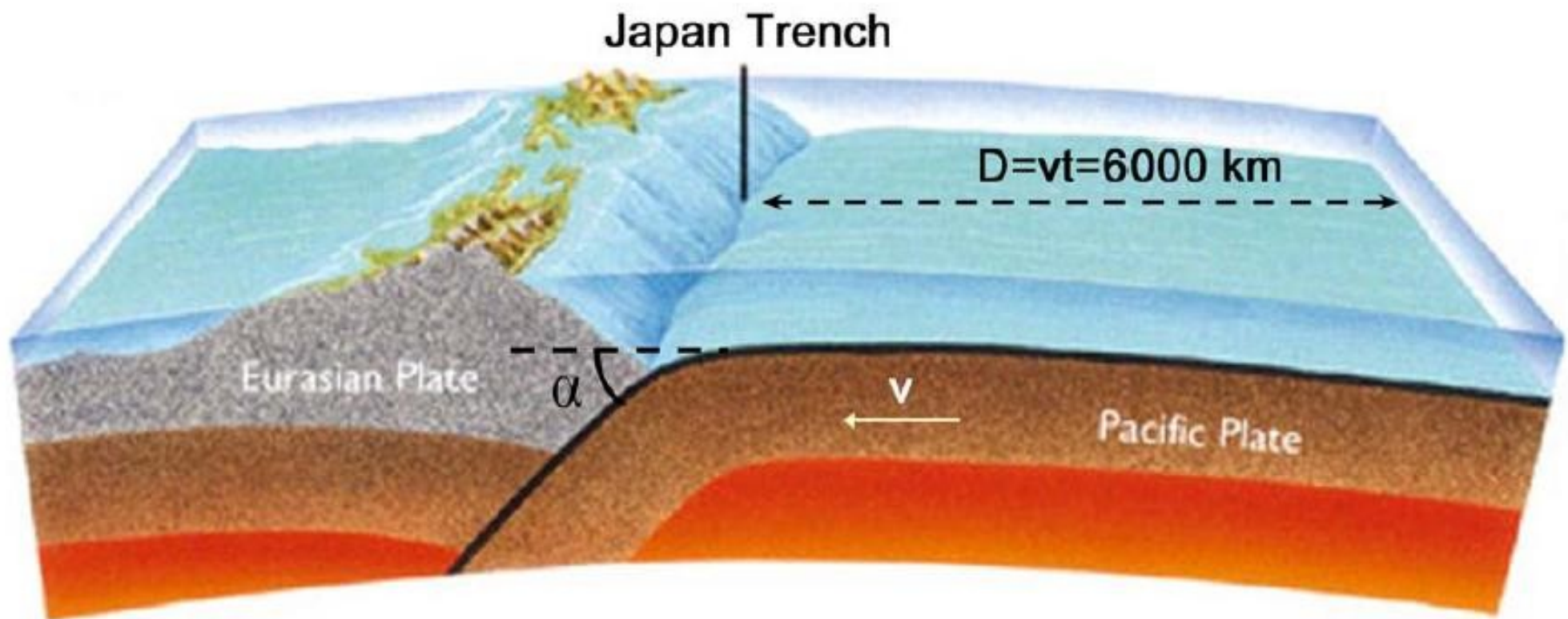
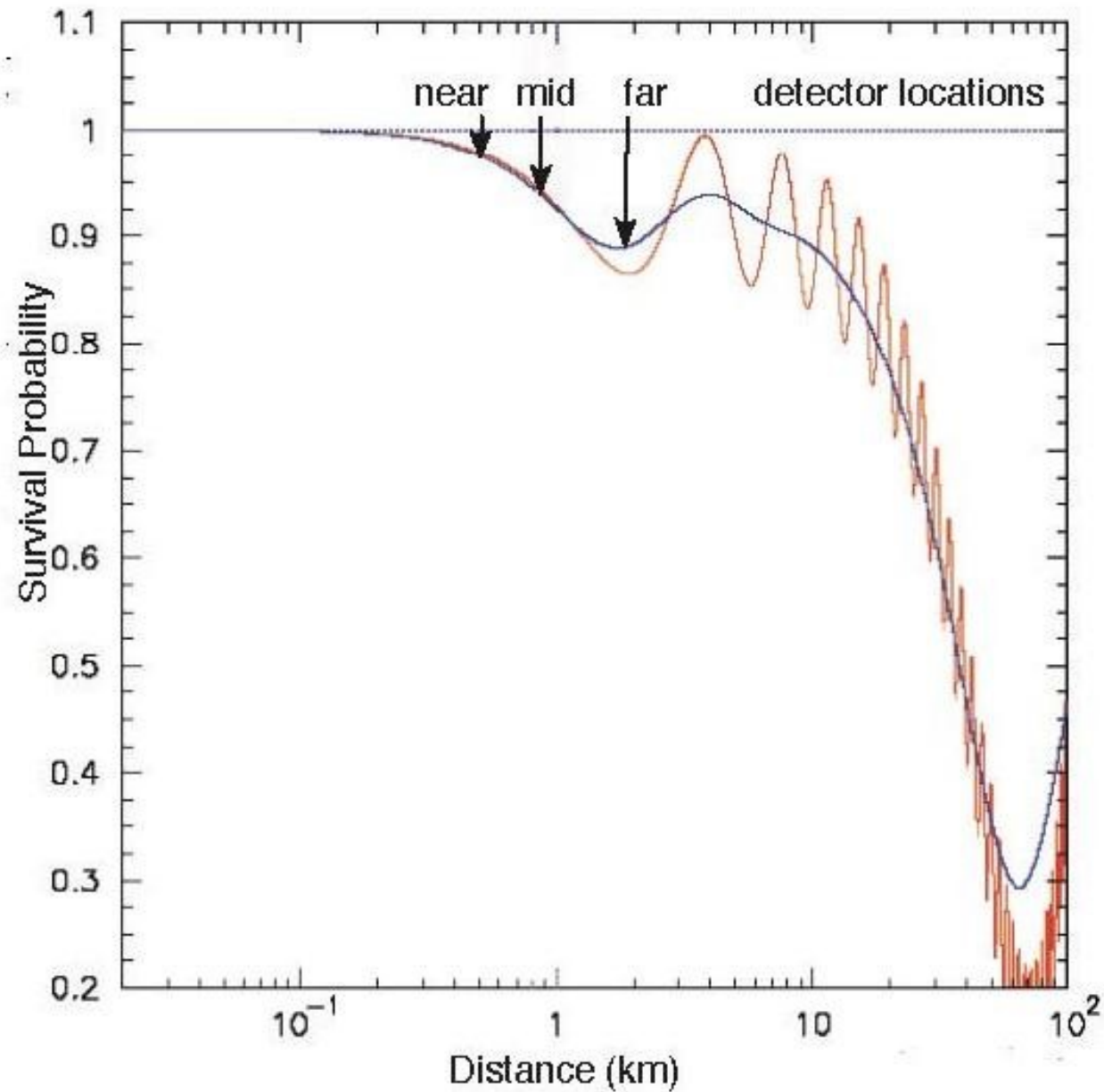


Fig. 2. Predicted geo-neutrino events from Uranium and Thorium decay chains, normalized to 10^{32} protons yr and 100% efficiency.



From FIORENTINI

Measurement of θ_{13}



Within

DOUBLE



$> 500 \bar{\nu}_e / d$

280 m



1051 m

2 x 4270 MWth





Systematic Errors

		Chooz	Double Chooz	
Reactor-induced	ν flux and σ	1.9 %	<0.1 %	Two "identical" detectors, Low bkg
	Reactor power	0.7 %	<0.1 %	
	Energy per fission	0.6 %	<0.1 %	
Detector-induced	Solid angle	0.3 %	<0.1 %	Distance measured @ 10 cm + monitor core barycenter
	Volume	0.3 %	0.2 %	Same weight sensor for both det.
	Density	0.3 %	<0.1 %	Accurate T control (near/far)
	H/C ratio & Gd concentration	1.2 %	<0.1 %	Same scintillator batch + Stability
	Spatial effects	1.0 %	<0.1 %	"identical" Target geometry & LS
	Live time	few %	0.25 %	Measured with several methods
Analysis	From 7 to 3 cuts	1.5 %	0.2 - 0.3 %	
Total		2.7 %	< 0.6 %	



Remote survey



- ❖ Movable submarine "KamLAND"
 - only 2 - 5 bigger
- ❖ Count at 3 positions :
 - $\text{Signal} \approx (P_{\text{th}} ???) / R^2$
 - Triangulation
- ❖ Detection of underground clandestine reactor

- ❖ Global survey
 - 10 Mtons units
 - ≈ 1000 units in ocean
 - *J. Learned at Neutrino'04*

