The Standard Model (and small extensions)

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- Introduction
- Strong interactions
- Electroweak interactions
- Mixing in the quark sector
- The neutrino sector
- The Standard Model of cosmology
- Conclusions

Introduction

- The Standard Model describes the interactions of elementary particles
- Four interactions:
  - electroweak (in principle two separate forces)
  - strong
  - gravitation

  however gravitation cannot be included in the model
- The model describes successfully basically all data
- However the model has many problems why we think it cannot be the final theory
- Every test of the Standard Model should thus be seen as an attempt to find its limits

Gauge theories

Elementary particle physics is successfully described by local gauge theories

- Take a gauge group $\mathcal{G}$
- The interactions (gauge bosons) are given by the generators of the group
- The fermions are arranged in multiplets on which the gauge bosons act
- The gauge group of the Standard Model: $SU(3) \times SU(2) \times U(1)$
  SU(3): strong interactions
  $SU(2) \times U(1)$: electroweak interaction
  Gravity is not included in the Standard Model
- In this scheme all particles have to be massless
- Masses can be generated breaking the symmetry

Fermions in the Standard Model

- Fermions exist in 3 families
- The families are identical apart from their masses
- Leptons have only electroweak interaction
- Quarks also have strong interactions

Leptons			Quarks		
Flavour	mass (GeV)	Q	Flavour	mass (GeV)	Q
ν_e	$< 1 \cdot 10^{-8}$	0	u	~ 0.003	$2/3$
e	0.000511	-1	d	~ 0.006	$-1/3$
ν_μ	< 0.0002	0	c	1.3	$2/3$
μ	0.106	-1	s	~ 0.1	$-1/3$
ν_τ	< 0.02	0	t	175	$2/3$
τ	1.78	-1	b	4.3	$-1/3$

Electroweak gauge bosons:

charged current: $W^{\pm}$ $m_{\mathrm{W}} \sim 80\,\mathrm{GeV}$

neutral current: Z $m_{\mathrm{Z}} \sim 90\,\mathrm{GeV}$

γ $m_{\gamma} = 0$ QED

Gauge group: $SU(2) \times U(1)$ with couplings $g,\ g'$

Fermions exist as left handed doublets and right handed singlets

$SU(2)$ $\begin{pmatrix} W^{+} \\ W^{0} \\ W^{-} \end{pmatrix}$ couple to left handed doublets only

$U(1)$ B couples to left and right-handed fermions

Up to here all particles are massless!

The Higgs mechanism

Complex Higgs doublet Φ with potential $V(\Phi) = \lambda(\Phi^*\Phi - v^2/2)^2$

- Minimum at $\Phi(0) = \begin{pmatrix} 0 \\ v \end{pmatrix}$
- $v = 246\,\mathrm{GeV}$ precisely known from muon decay

Gauge bosons acquire mass through coupling at Φ, absorbing 3 degrees of freedom in the longitudinal gauge boson components

Higgs mechanism requires one neutral scalar particle H^0,

Fermion masses are generated by ad hoc Yukawa couplings of the fermions to the Higgs field

The fermion mass term $m\overline{\Psi}_L\Psi_R$ couples left- and right handed particles

W^0 and B mix keeping photon massless:

$$\begin{aligned} Z &= W^0 \cos\theta_W - B \sin\theta_W \\ \gamma &= W^0 \sin\theta_W + B \cos\theta_W \end{aligned}$$

with $g \sin\theta_W = g' \cos\theta_W = e$

Resulting interactions:

$W^\pm$**:** stay purely left handed

γ**:** left-right symmetric vector coupling (Maxwell equations)

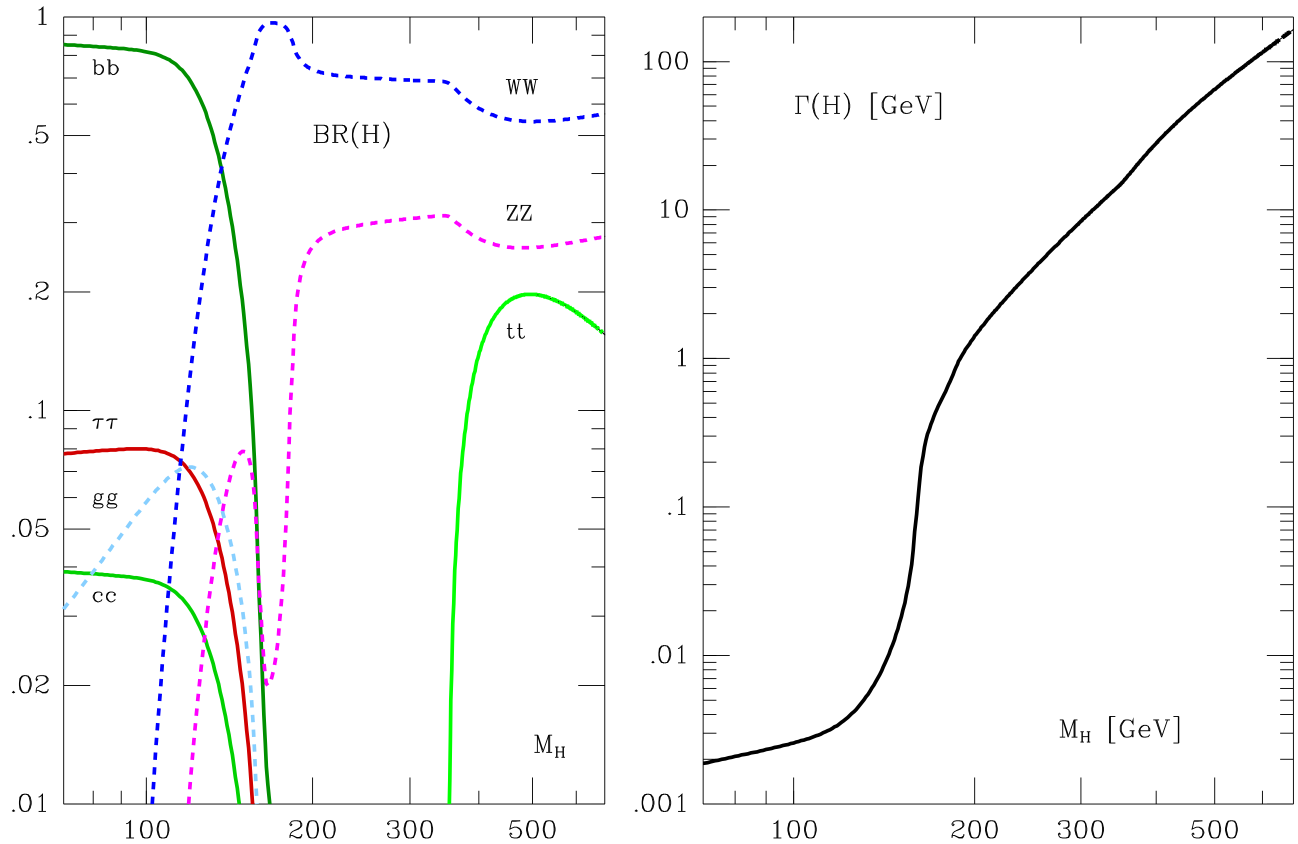
Z**:** complicated mixture of left- and right-handed coupling to restore the $SU(2) \times U(1)$ prediction

$$\begin{aligned} g_A &= \frac{g}{2} \\ g_V &= \frac{g}{2}(1 - 4|q| \sin^2\theta_W) \end{aligned}$$

(Neutrinos: electrically neutral ➠ Z coupling pure left-handed ➠ right handed neutrinos would be sterile)

What do we know about the Higgs?

- In the Standard Model only one free parameter left ($m_\mathrm{H}^2 = 2\lambda v^2$)
- LEP searches: $m_\mathrm{H} > 114\,\mathrm{GeV}$
- Higgs couples to mass $\Rightarrow$ partial widths proportional to particle masses
- Coupling to massless particles via loops

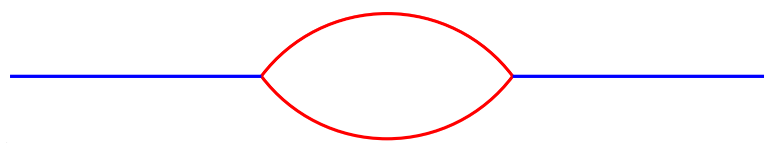


The hierarchy problem

A “final” theory should be valid up to the Planck scale:

$$m_{\rm Pl} = \sqrt{\hbar c/G_N} \approx 10^{19}\,\mathrm{GeV}$$

If the parameters (couplings, masses) are defined at the high scale, they receive radiative corrections running them down to the low scale



Radiative corrections in the SM: $\Delta m_{\rm H} \sim m_{\rm Pl}$

However Higgs mechanism works only if $m_{\rm H} < 1\,\mathrm{TeV}$
(e.g. WW $\rightarrow$ WW violates unitarity at $\sqrt{s} = 1.2\,\mathrm{TeV}$ without a light Higgs)

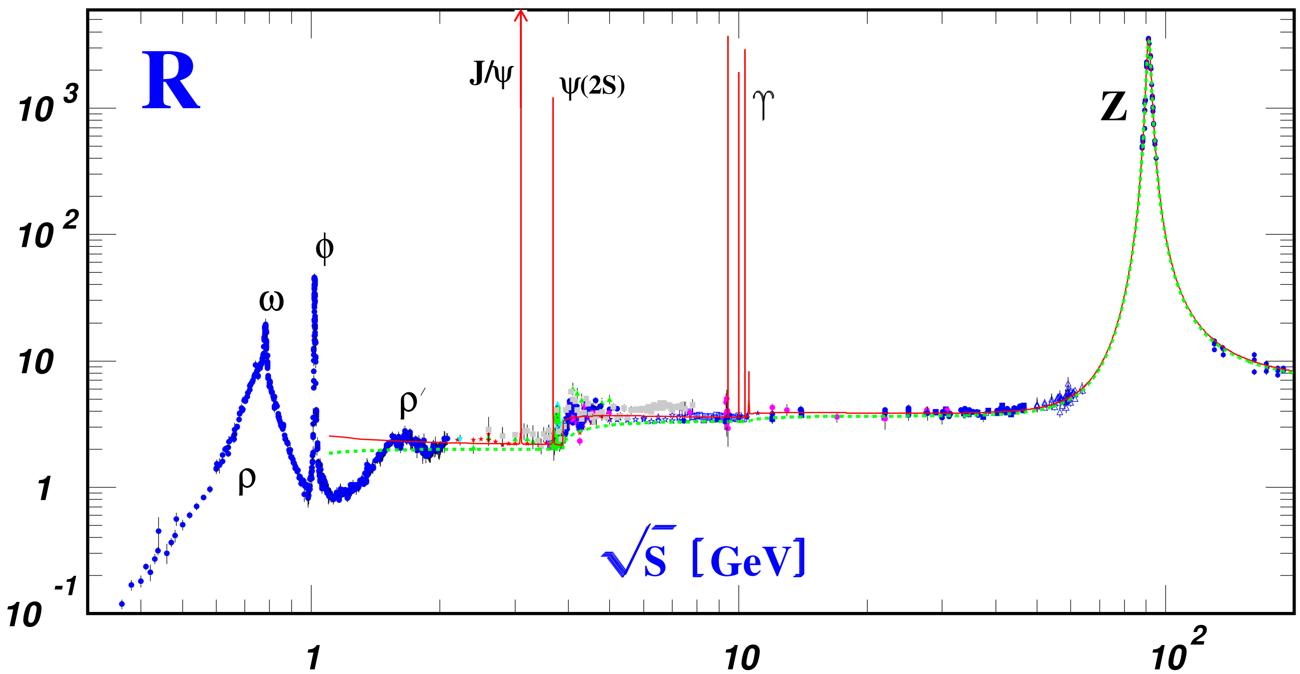
$\Rightarrow$ enormous fine-tuning required!

Strong interactions

Strong interactions act only on quarks

Gauge group: SU(3)

- Quarks have to come in triplets of 3 “colours”



- Exchange particles: 8 massless gluons

Running of coupling constants

Due to vacuum polarisation effects the coupling "constants" depend on the momentum transfer

Gauge-boson-fermion interaction: screening The coupling constants fall with falling energy

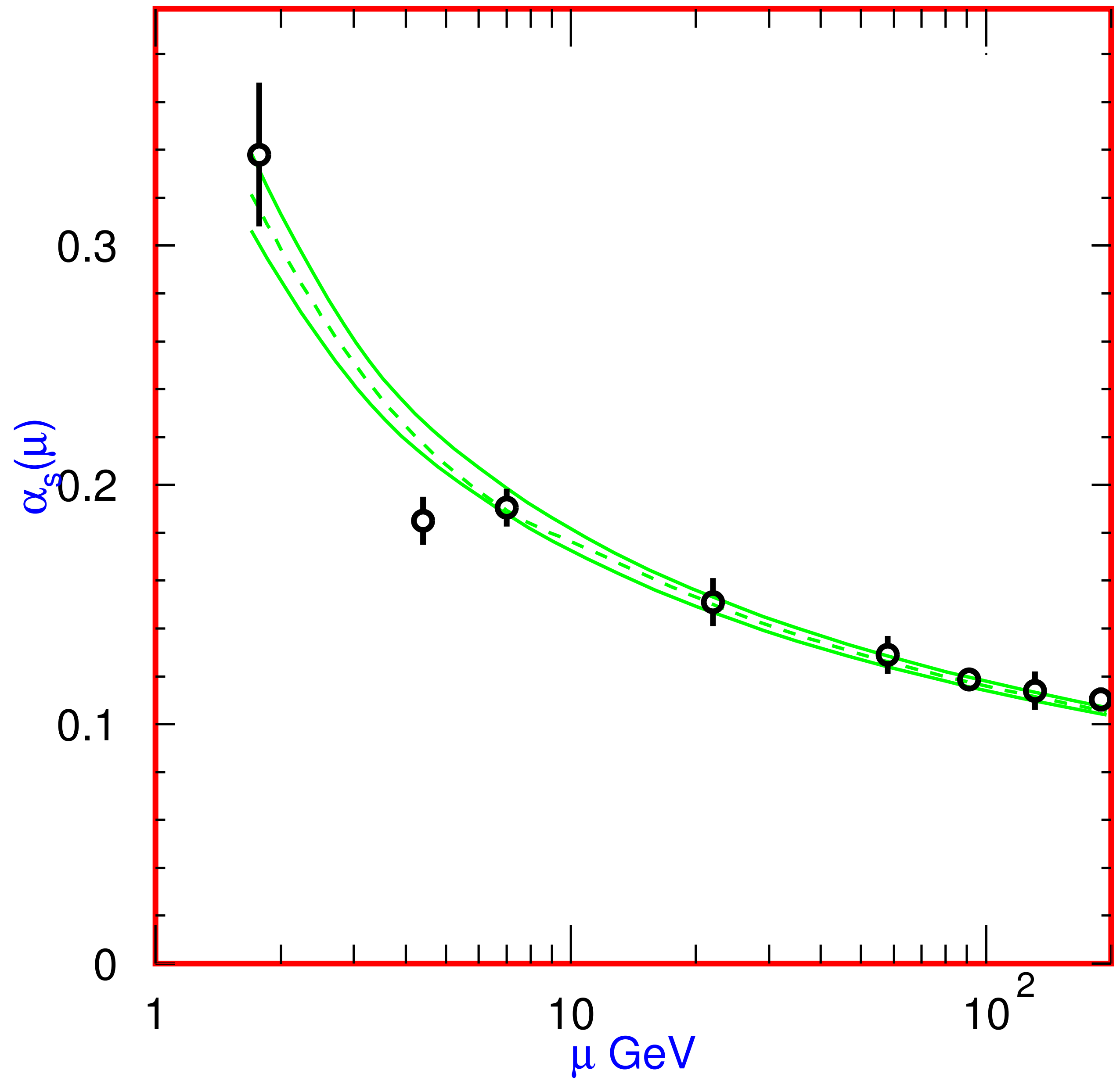
Gauge-boson self-interactions: enhancement The coupling constants rise with falling energy

Electroweak interactions: well behaved at $Q^2 \to 0$ with $\mathcal{O}(10\%)$ changes between 0 and 100 GeV

Strong interactions: coupling diverges for $Q^2 \to 0$

- quarks exist only in colour neutral states: quark-antiquark (mesons), 3 (anti)quarks (baryons) (confinement)
- "free" quarks and gluons are visible at high energy (asymptotic freedom)

Verification of running is a strong test of the gauge structure of QCD



Experimental tests of electroweak interactions

Gauge sector fully determined by three parameters: g, g', v

(In practise the three best measured parameters are used: $\alpha(0)$, G_{F}, m_{Z})
Can test the model if more than three observables are measured

Expect one-loop correction to be of order $\alpha \sim 1\%$

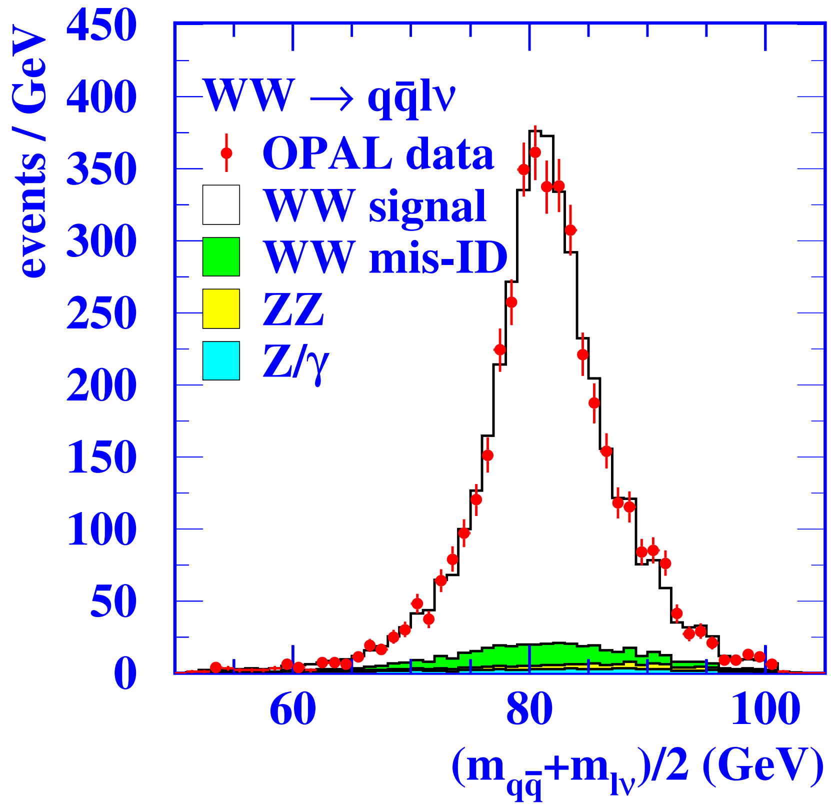
➠ have to be taken into account of precision better than that

- quantities get sensitive to other parameters (m_{t}, m_{H}...)
- model is tested at the quantum level
- sensitivity to physics at higher scales

Electroweak observables

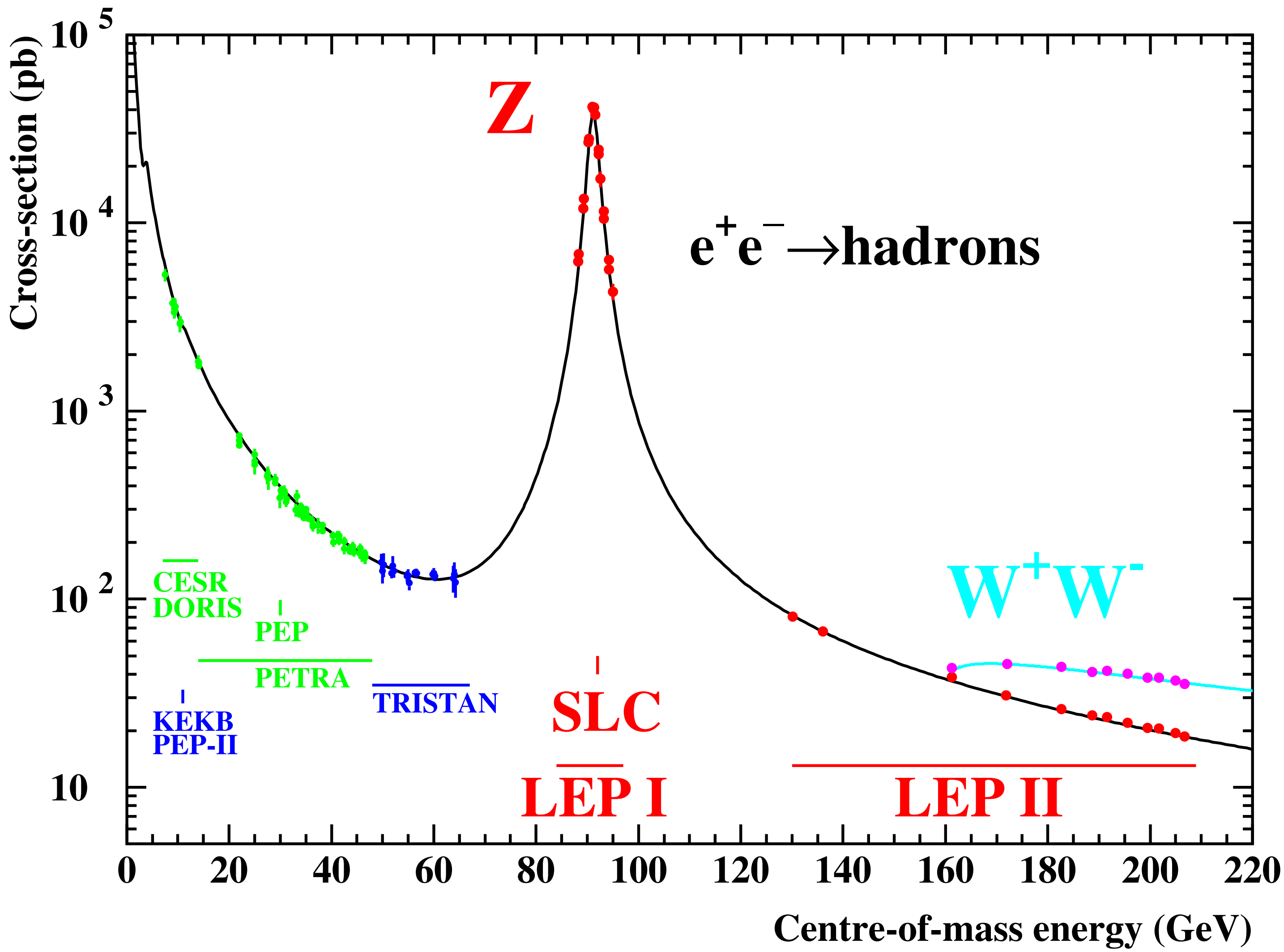
- The fermion sector is completely known (neutrino masses are irrelevant in this context)
- α and G_{F} are known with very good precision
- Many observables can be measured on the Z resonance

- The W-mass is measured at LEP and the TEVATRON
- Some other observables like atomic parity violation or low energy Moller scattering give additional information



Observables on the Z resonance

The Z-mass is given by the peak of the resonance curve



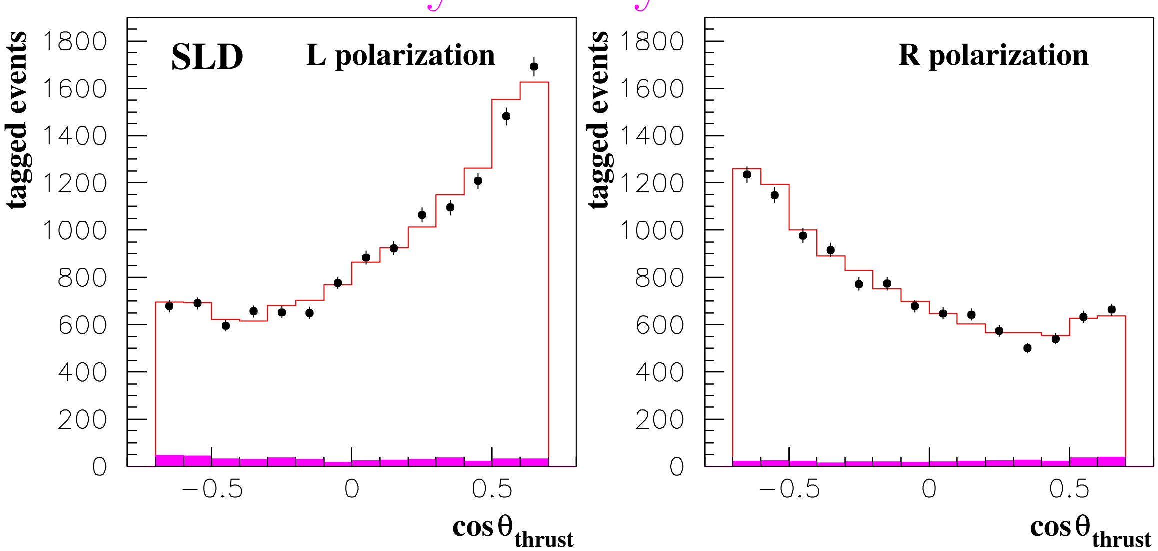
- All other observables (partial, total widths, asymmetries) can be expressed in terms of the vector and axial vector couplings of the Z to fermions
  - axial vector coupling measures the total normalisation of the SU(2) coupling constant
  - vector coupling is mainly sensitive to the Z-γ mixing, i.e. the weak mixing angle ($v/a = 1 - 4Q\sin^2\theta$)

- Z partial widths: for numerical reasons basically mainly sensitive to normalisation

- Asymmetries measure the interference of vector and axial vector coupling

➡ $\propto \mathcal{A} = \frac{2va}{v^2+a^2}$

$b\bar{b}$ asymmetry at SLD



The structure of radiative corrections

Most Z-observables and m_{W} can be described with three parameters:

- $\Delta\rho$: total normalisation of the Z-fermion coupling
- $\sin^2\theta^l_{eff}$: effective weak mixing angle
- Δr: Radiative corrections for m_{W}
- (Only $Z\mathrm{b\bar{b}}$ couplings are slightly special because of the heavy top)

In the SM there are two independent contributions:

- A large term from isospin violation $\propto m_{\mathrm{t}}^2 - \mathrm{m}_b^2$
- A much smaller contribution $\propto \log m_{\mathrm{H}}/m_{\mathrm{W}}$

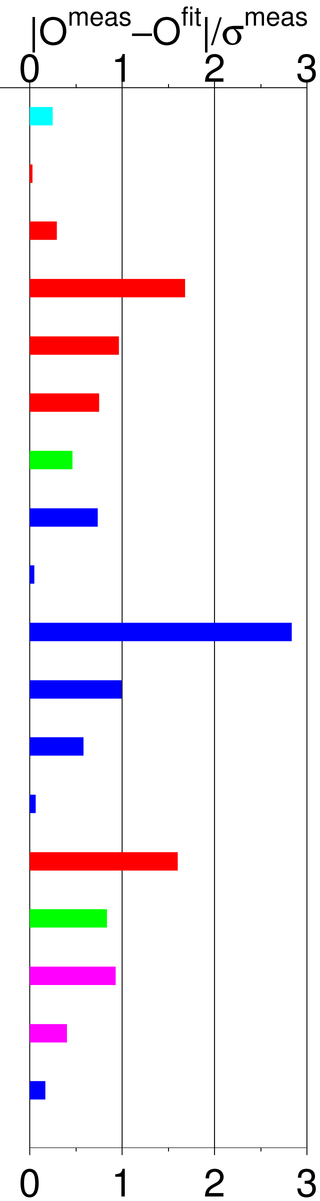
Reparameterisation:

- $\varepsilon_1 = \Delta\rho$ (or S): absorbs the isospin violating corrections
- ε_3 (or T): only sensitive to the logarithmic corrections
- ε_2 (or U): constant in the SM and most extensions (only m_{W})

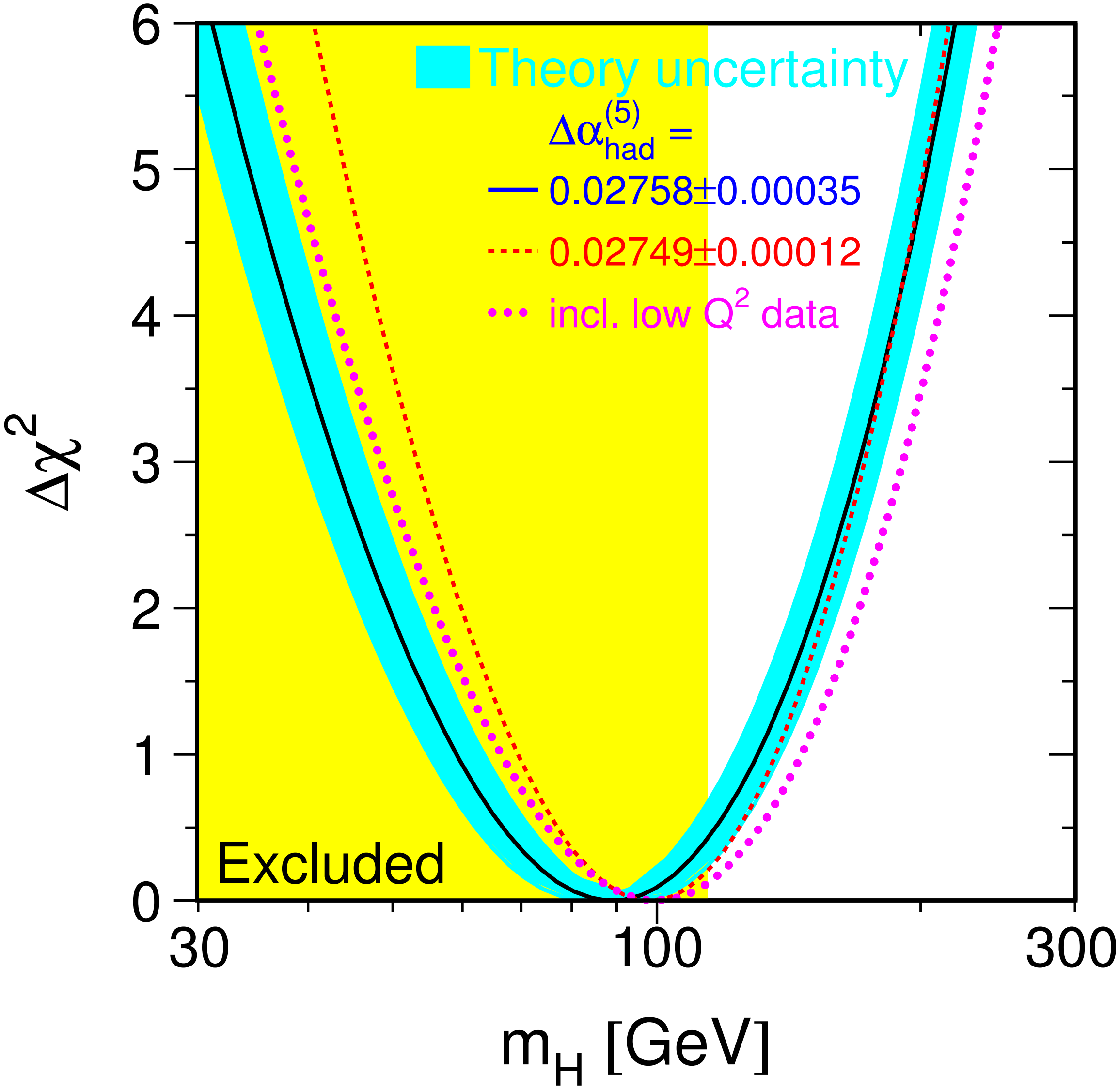
Electroweak fits

- All data are fitted simultaneously leaving $m_{\rm H}$ $(+\ldots)$ as free parameter
- The overall fit quality is good: $\chi^2/\mathrm{ndf} = 17.8/13$
- All observables agree individually with the SM prediction after the fit

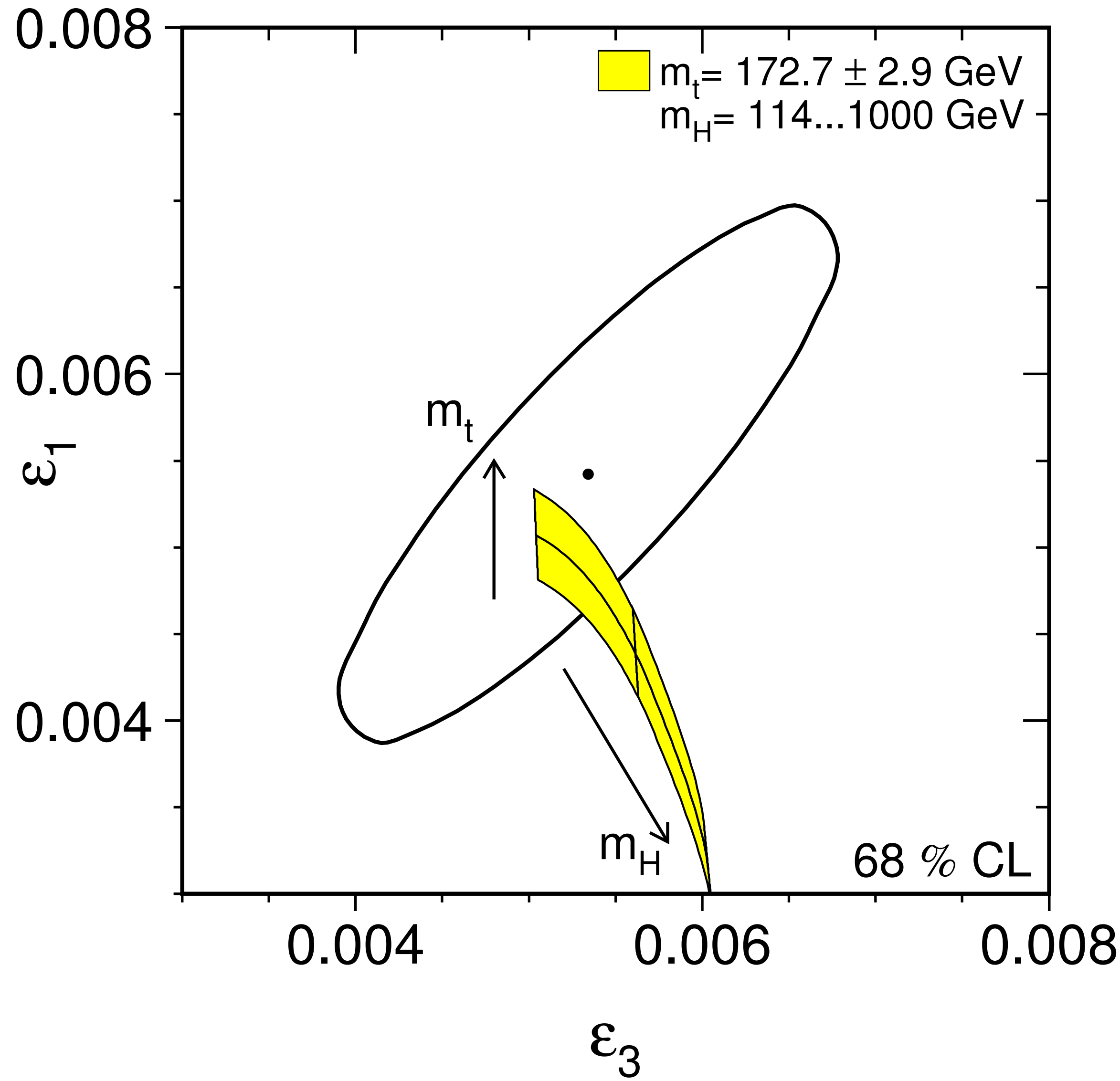
	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.743
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21581
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
$A_l(SLD)$	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.404 ± 0.030	80.376
Γ_W [GeV]	2.115 ± 0.058	2.092
m_t [GeV]	172.5 ± 2.3	172.9



- Within the Standard Model the Higgs is predicted to be light
- A one sided 90% c.l. is around 200 GeV
- (This is perfectly consistent with SUSY)
- (Of course the limit is only valid within the SM)

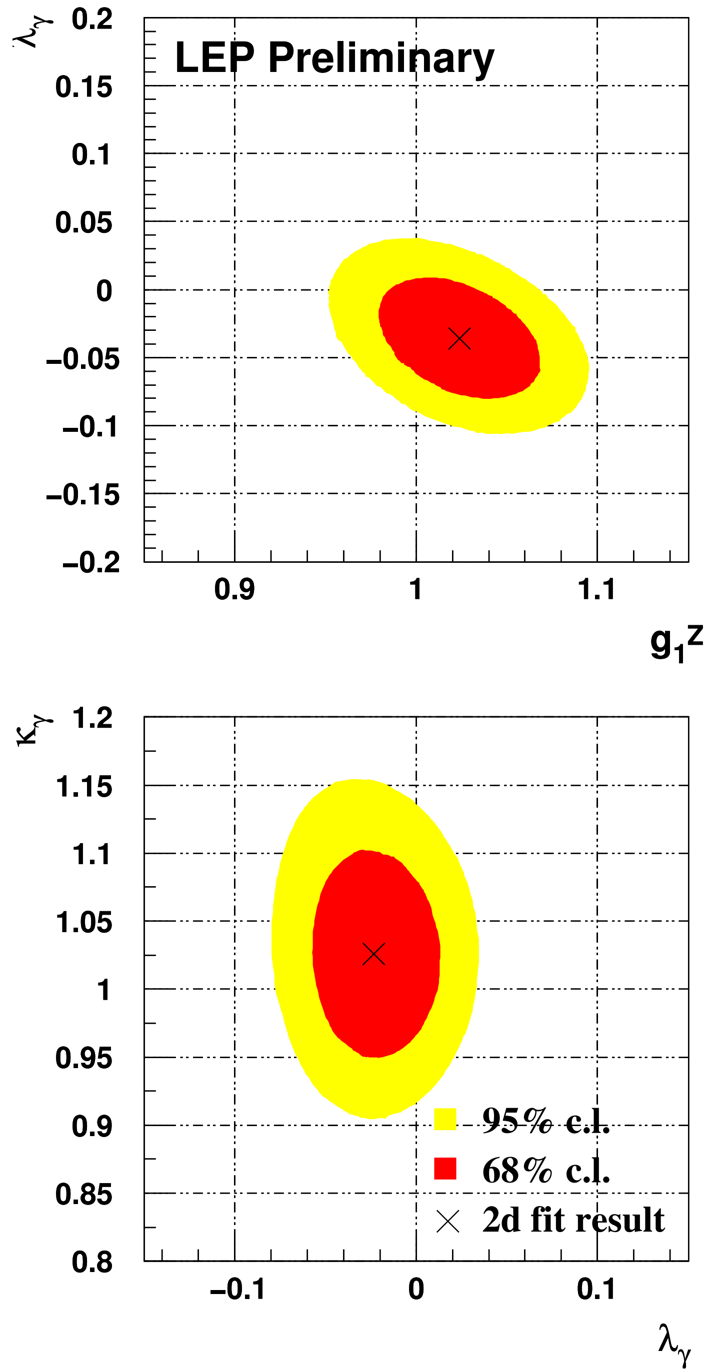


- The data can also be fitted with $\varepsilon_{1,2,3}$ (STU) as free parameters
- This shows again the agreement of the data with the SM
- However this allows also the interpretation of the beyond beyond the SM
- E.g. QCD-like technicolour and a large part of the parameter space for little Higgs models can be excluded this way



Gauge boson self couplings

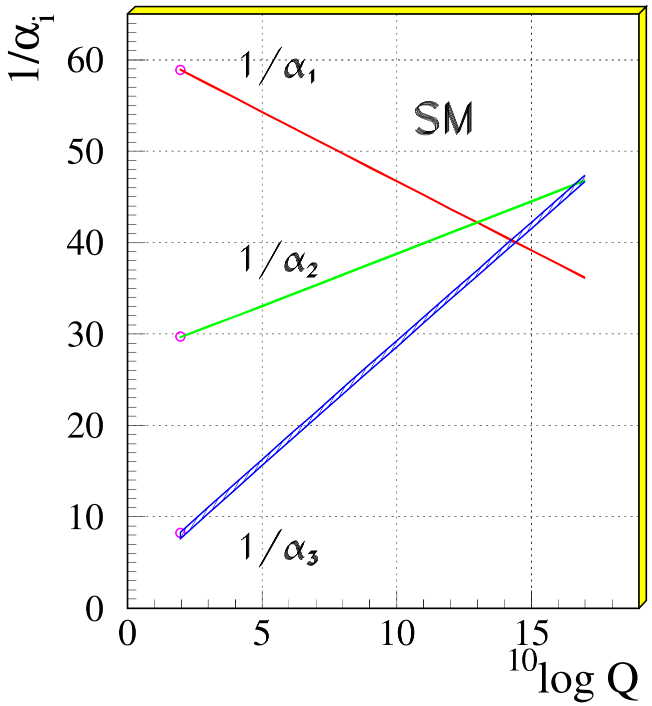
- The gauge boson couplings are uniquely defined by the structure of the gauge group
- A measurement of these couplings thus probes the gauge structure itself
- The WWZ and WWγ are usually described and terms of 5 parameters where only 3 can be measured independently at LEP (no Z-γ separation)
- The measurements agree with the SM on the few % level
- Hadron colliders can separate the Z and the γ using WZ and Wγ final states, however the precision is at present not competitive

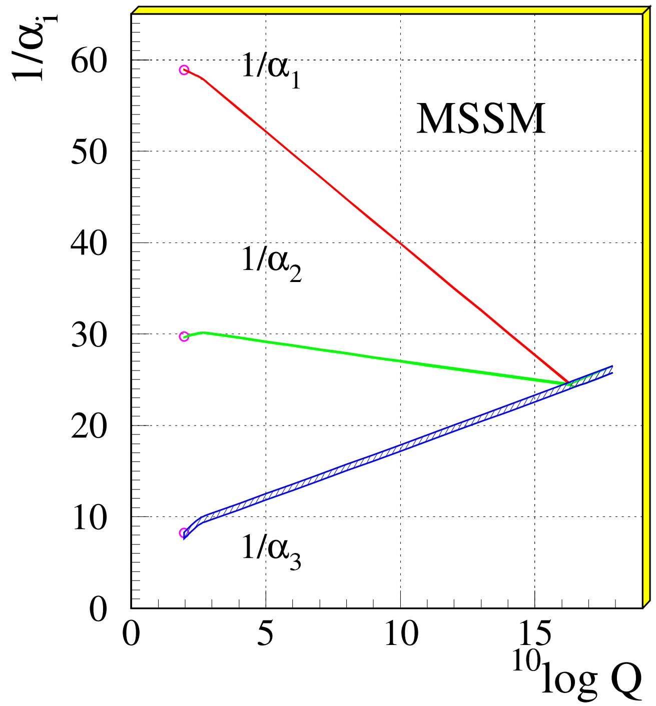


Unification of Gauge Couplings

Why do we have 3 (+1) forces in nature and not one?

- GUT theories assume one force at high scales which is broken down to three forces at the GUT scale
- This requires the three coupling constants to meet at some point
- The precision measurements of the couplings show that this is not possible within in the SM
- Only when new thresholds are introduced (Supersymmetry) gauge coupling unification is possible





The quark sector

- Quarks "mix", i.e. the mass eigenstates are not equal to the weak eigenstates
- Per construction the mixing is only in the down-quark sector
- Mixing matrix:

$$\begin{aligned} \vec{d}\,' &= M_{\mathrm{CKM}}\,\vec{d} \\ M_{\mathrm{CKM}} &= \left(V_{ij}\right)\ i = u, c, t\ \ j = d, s, b \\ &\approx \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix} \end{aligned}$$

(Wolfenstein parametrisation)

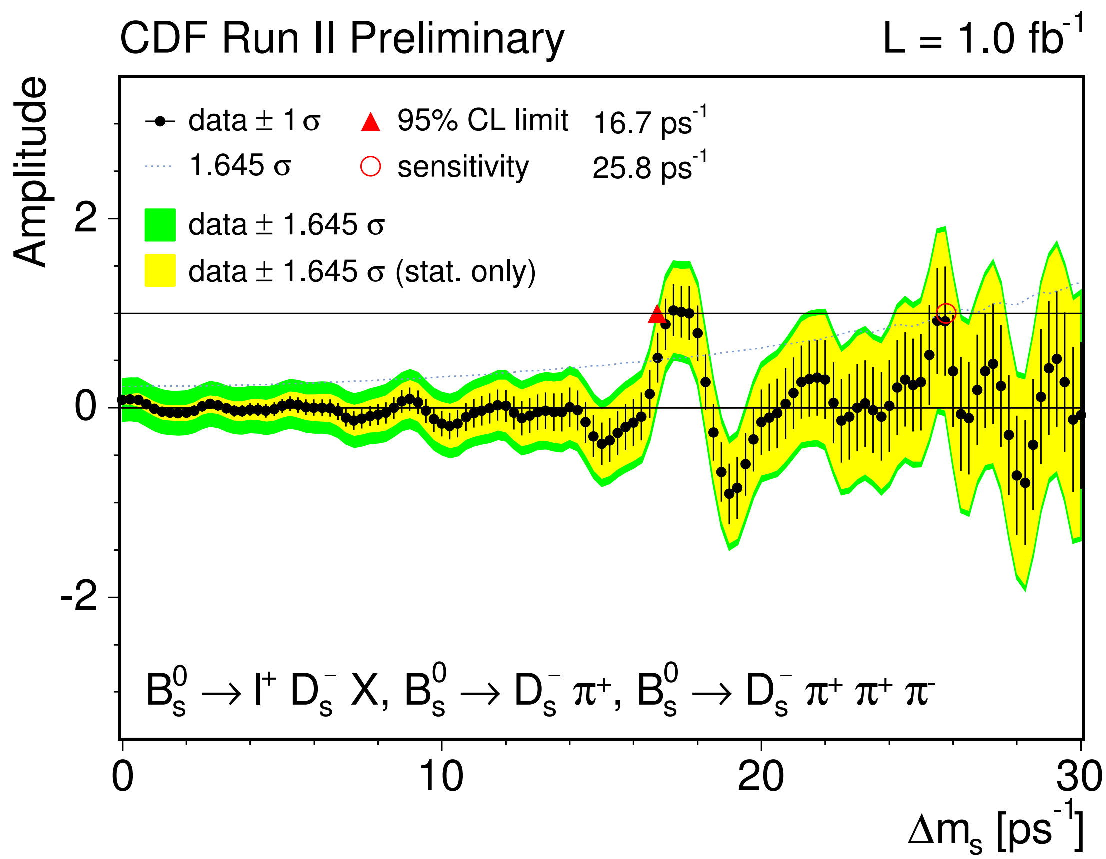
- The matrix contains one non-trivial complex phase
  ➠ CP violation

The importance of CP violation

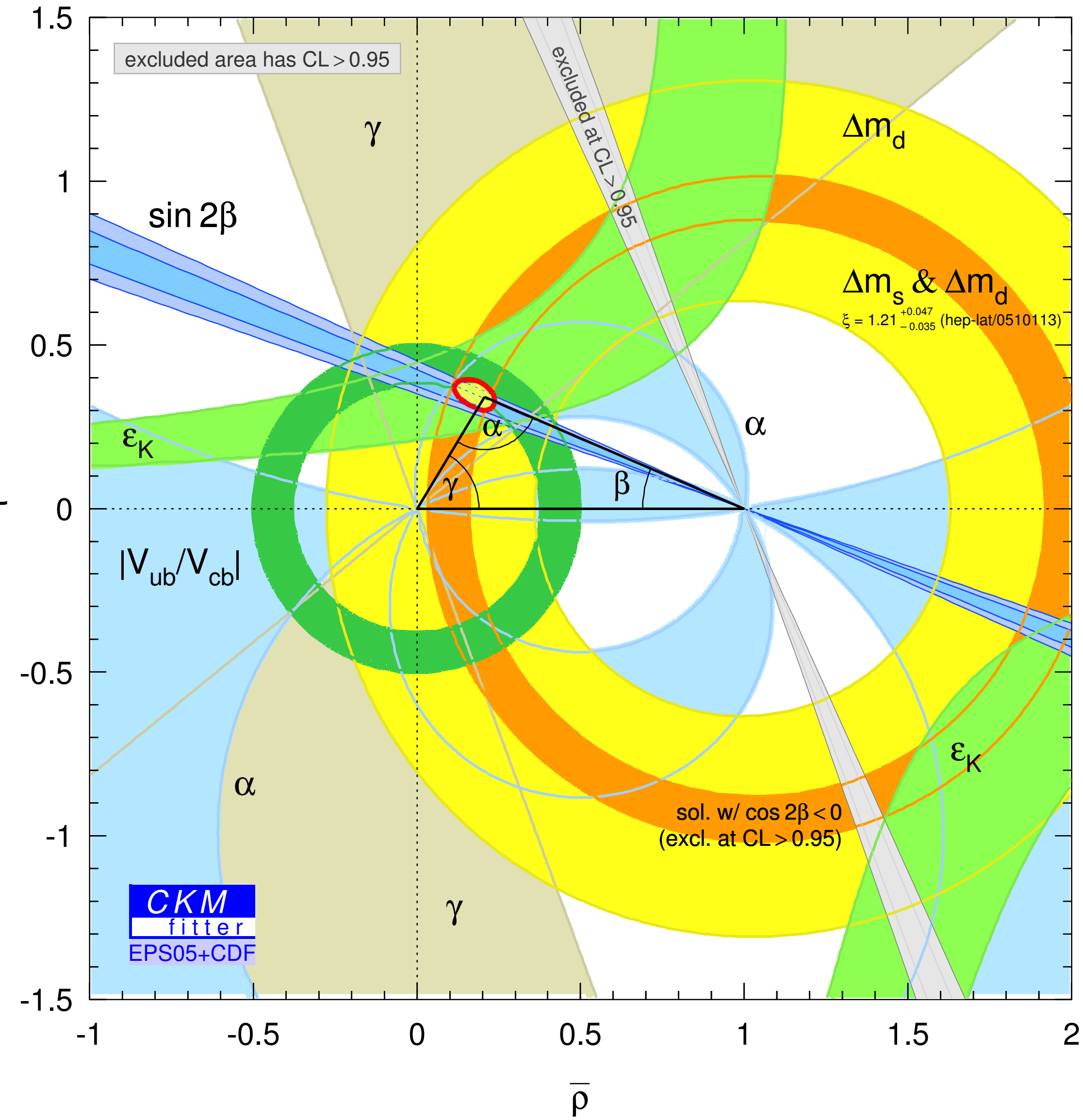
- CP violation is a prerequisite to transform a symmetric universe into a matter dominated universe (no antimatter!)
- We need at least three families to have CP violation in the SM
- However the CP violation in the SM is not sufficient to create the observed baryon-antibaryon asymmetry
- CP violation has been discovered in the K^0 system long ago
- In recent years also many CP violating effects in the B system have been measured

The matrix has to be unitary

- No FCNC
- 6 unitarity triangles: $\sum_i V_{im} V^*_{in} = 0 \quad m \neq n$
- Most interesting one: $V_{ud} V^*_{ub} + V_{cd} V^*_{cb} + V_{td} V^*_{tb} = 0$
  (measures ρ, η)
- The area of the triangle defines CP violation
- Many complementary measurements:
  B decay rates, $B\overline{B}$-mixing, CP violation in K-system, several CP violating observables in B-system



- Each measurement defines a region in the $\rho - \eta$ plane
- The combination gives an accurate measurement of the parameters
- More important: each measurement is individually consistent with the combination
  ➠ the CKM description describes the quark sector without the need for new physics contributions



The neutrino sector

Mass terms in the SM: $m\overline{\Psi}_R\Psi_L$

SM: neutrinos are massless ➠ right handed neutrinos do not exist

Neutrino oscillations:

- atmospheric+accelerator neutrinos: $\nu_\mu - \nu_\tau$ mix
- solar+reactor neutrinos: $\nu_e - \nu_\mu$ mix
- mixing frequency and amplitude:

$$P(\nu_1 \to \nu_2) = \sin^2\theta \sin^2 \frac{\Delta m^2}{4E}$$

$\Rightarrow$ if νs oscillate, they have mass

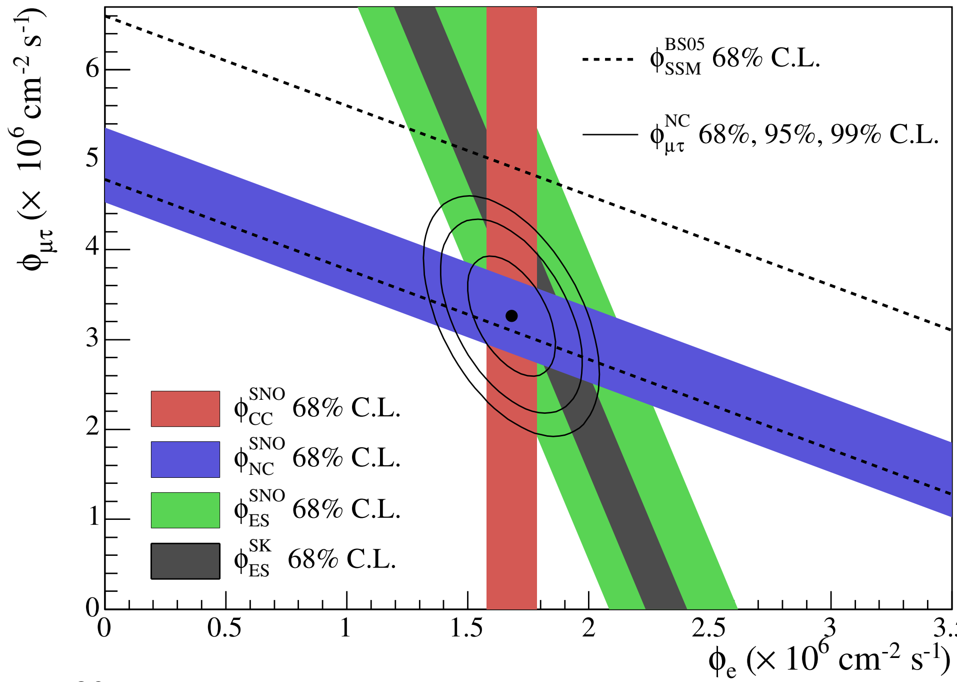
$\Rightarrow$ oscillation experiments only sensitive to mass differences

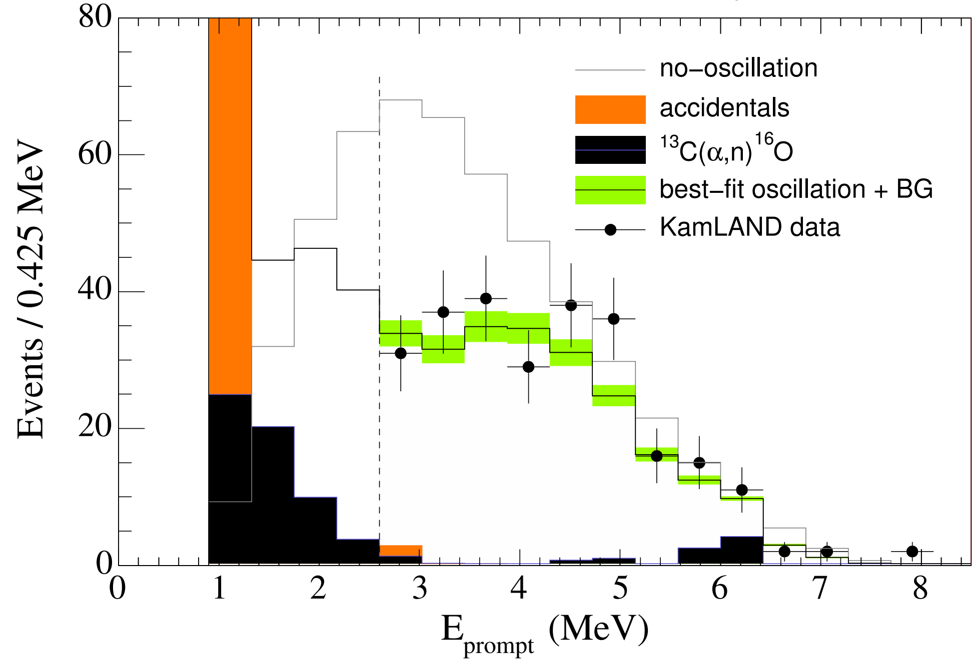
- neutrinos must have mass!

Quantitative results:

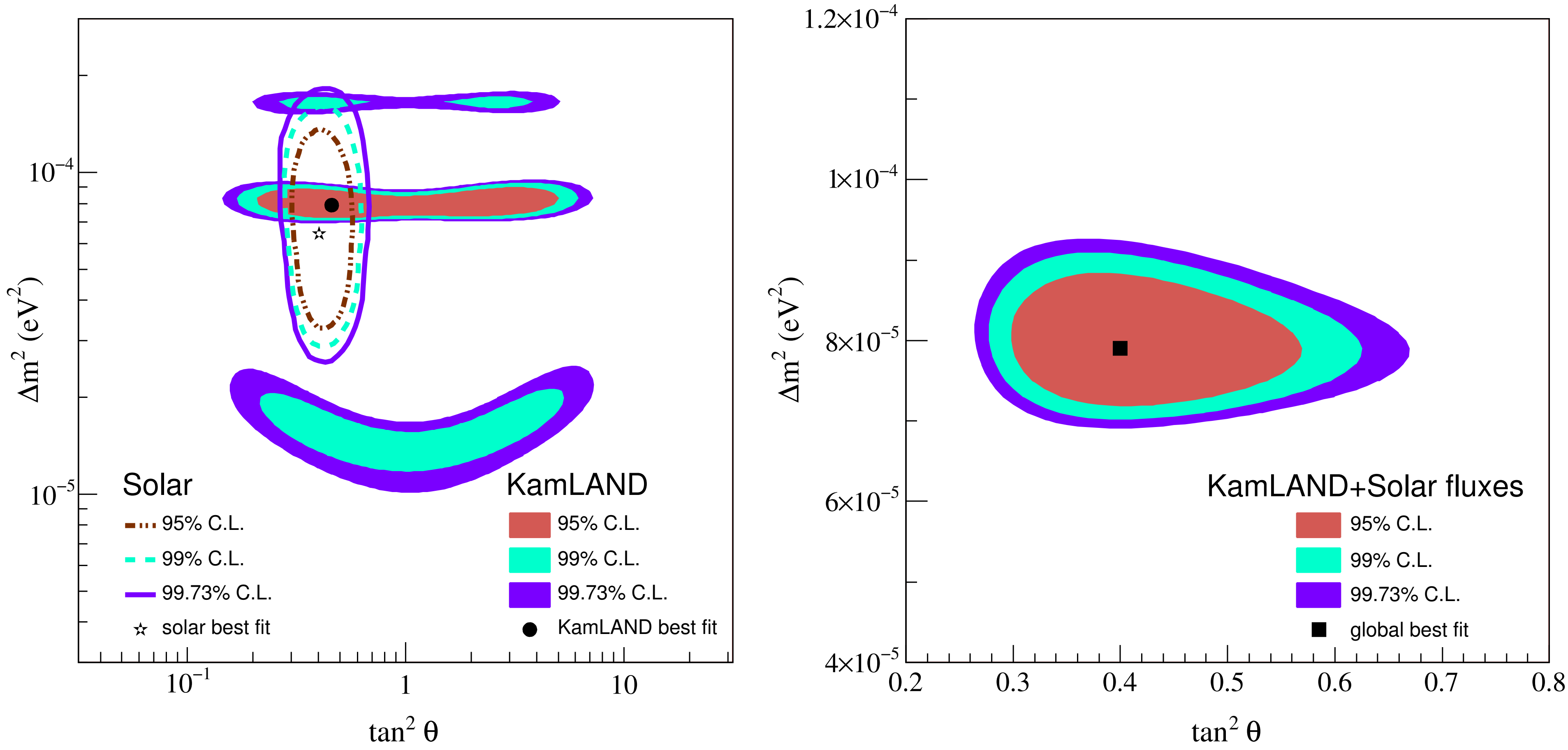
$\nu_e - \nu_\mu$ mixing

- Solar neutrino experiments show that neutrinos transform away from ν_e on their way to the earth
  ➟ mixing angle
- Reactor experiment show μ_e disappearance at shorter distances
  ➟ mass difference



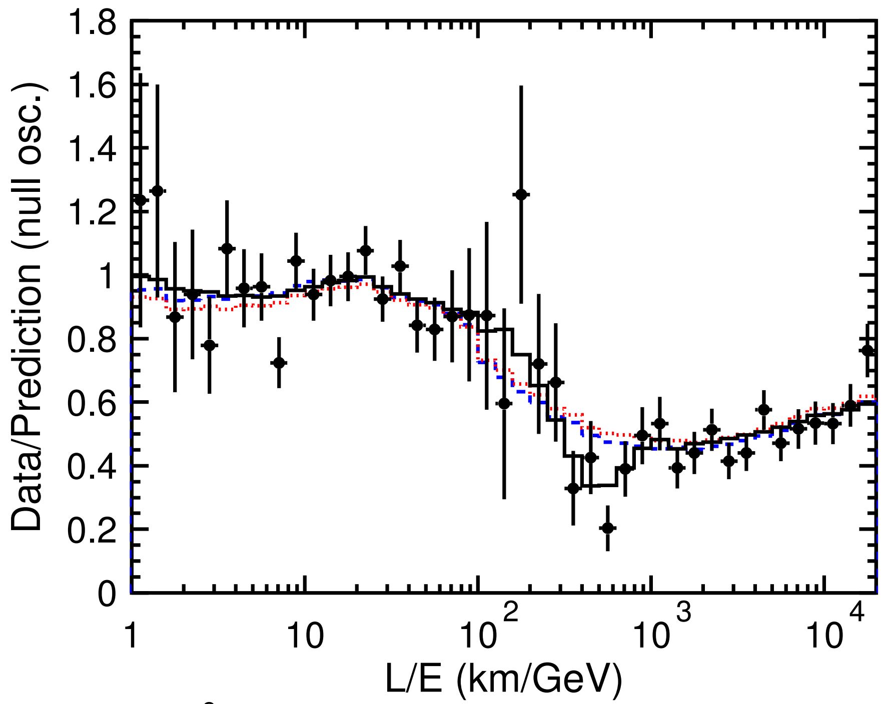


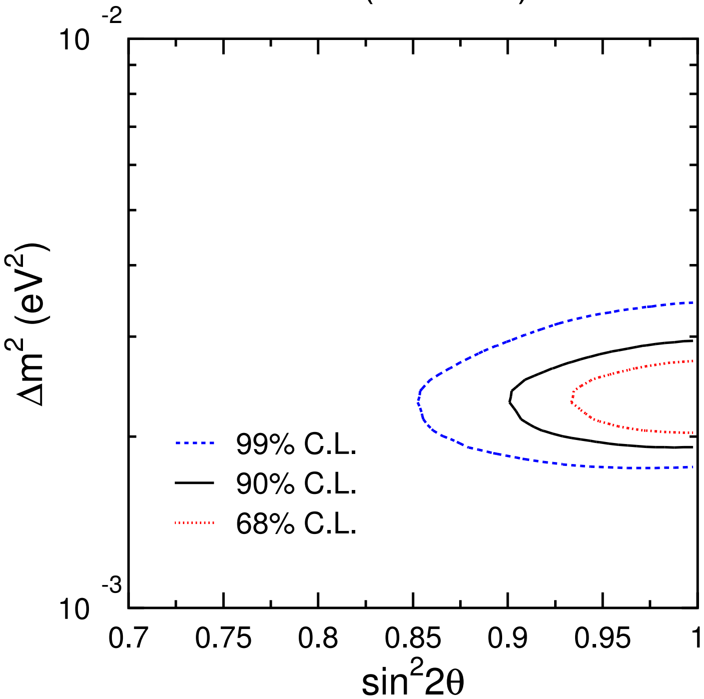
Combination gives precise measurement of both quantities

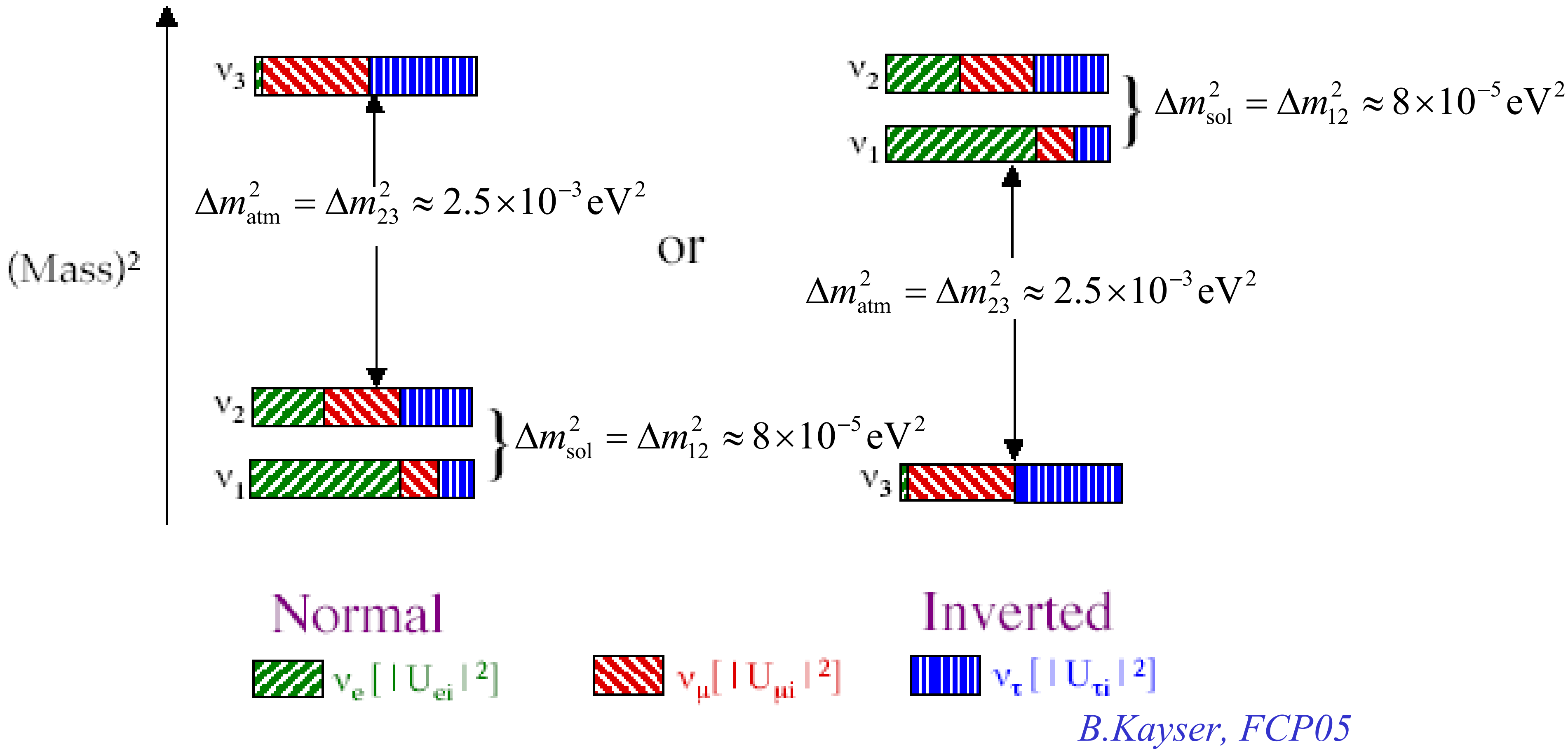


$\nu_\mu - \nu_\tau$ mixing

- Precise measurements from atmospheric neutrinos (up/down ν_μ/ν_e ratio
  - mixing angle maximal
  - mass difference^2 factor 100 larger than $\nu_e - \nu_\mu$
- Confirmed by accelerator experiment







- Mixing matrix pretty different from quark sector
- Still two possibilities for the hierarchy
- $\nu_2 > \nu_1$ known from matter effects in the sun

The nature of neutrinos

- In principle neutrinos can be normal Dirac particles in the SM
- Tritium endpoints measurements: $m_\nu < 2\,\text{eV}$
- Difficult to explain the large mass difference in the SM (but we don't understand masses anyway)
- Alternative: neutrinos are Majorana particles
  small masses follow naturally from the seesaw mechanism

The seesaw mechanism

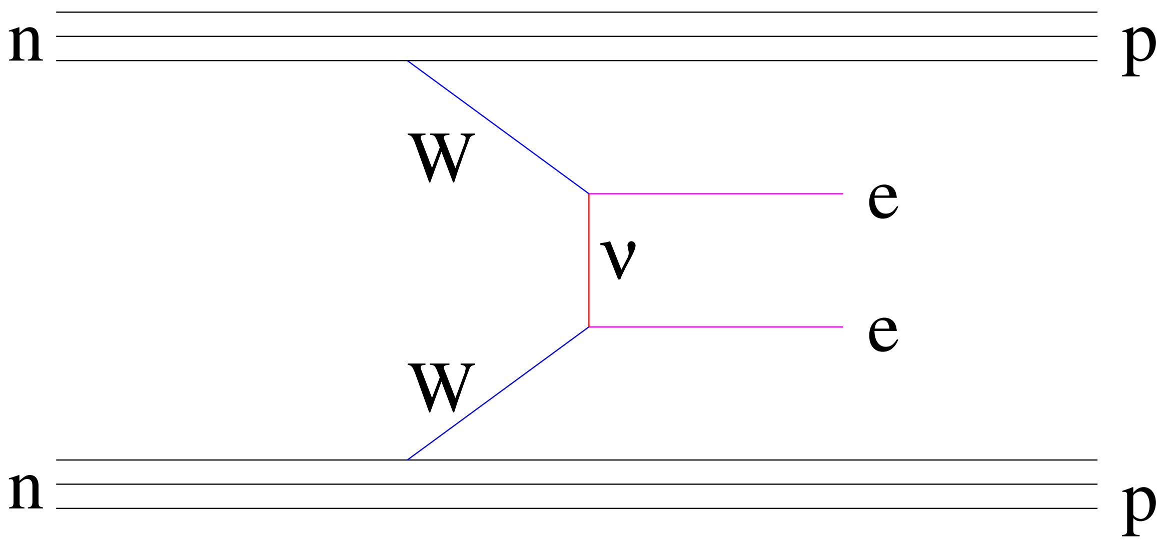
- Neutrinos are mixture of a Dirac particle with mass $m_D = \mathcal{O}(v)$ and a Majorana particle with mass $M = \mathcal{O}(m_{\text{GUT}})$
- Mass matrix

$$M = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$$

  Smaller eigenvalue: $m_\nu \approx m_D^2/M$: right size

How can the Majorana nature be proven?

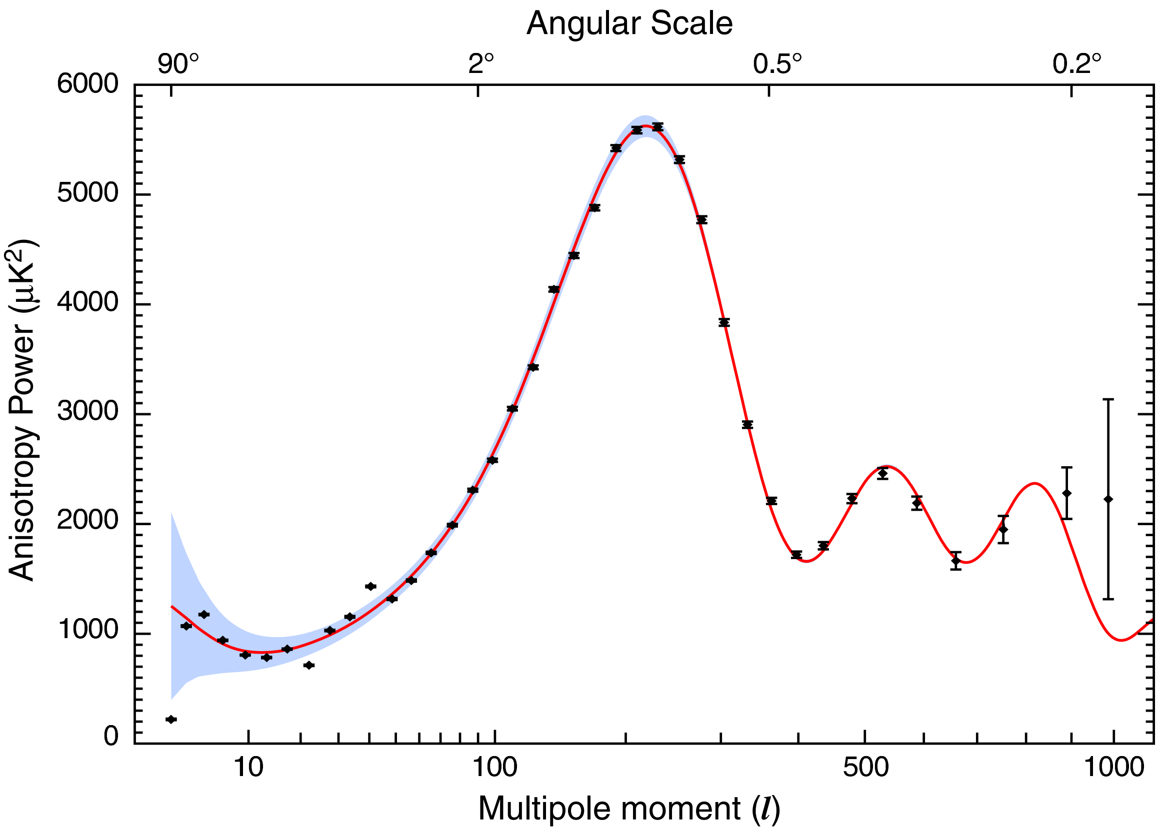
Neutrinoless double β decay:



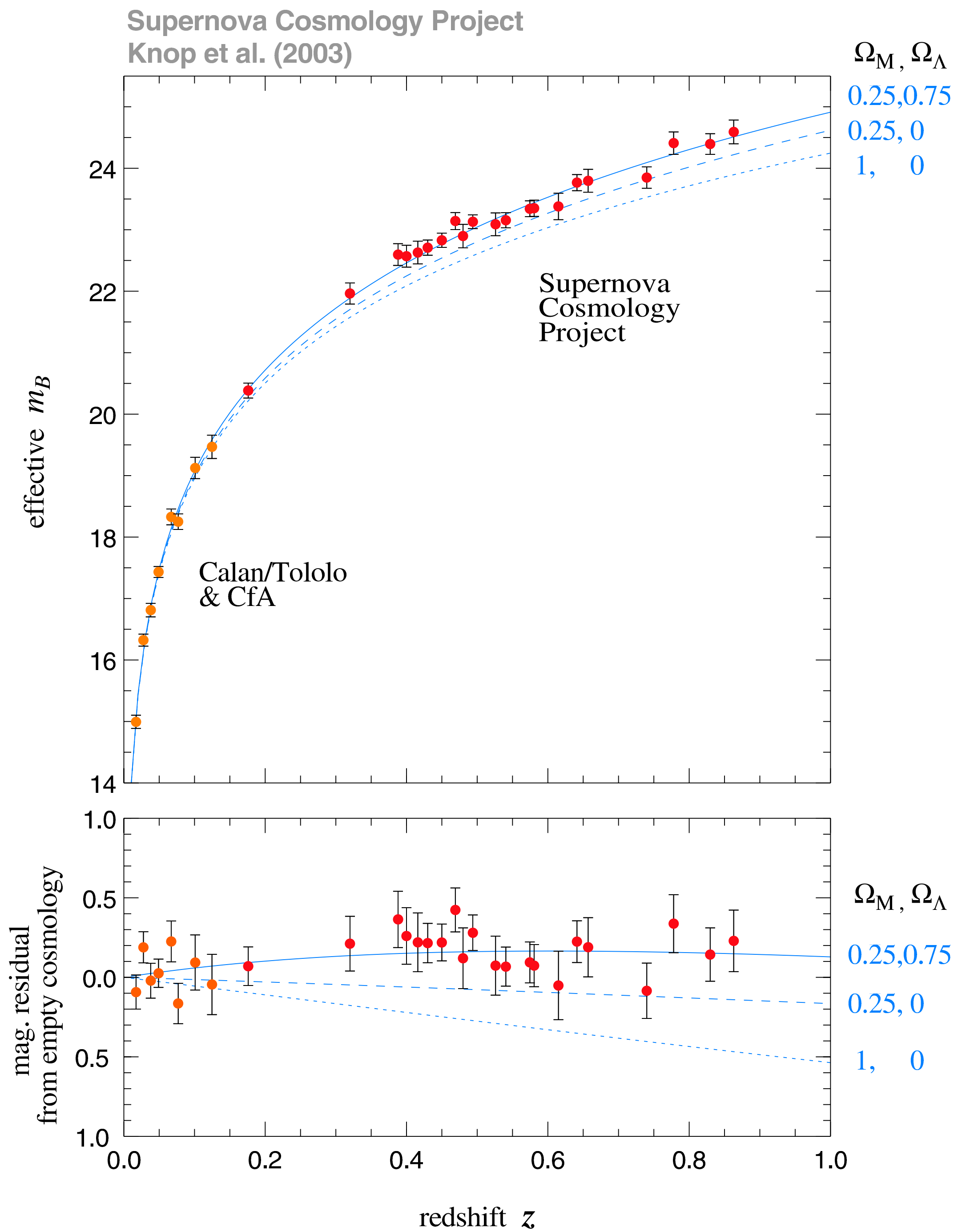
- Lepton number violation requires Majorana nature
- Helicity flip requires neutrino mass
- Current limits from ^{76}Ge $\rightarrow^{76}$ Se ee: $m_\nu < 0.36$ eV if Majorana particle

The SM of cosmology or why the SM cannot be all

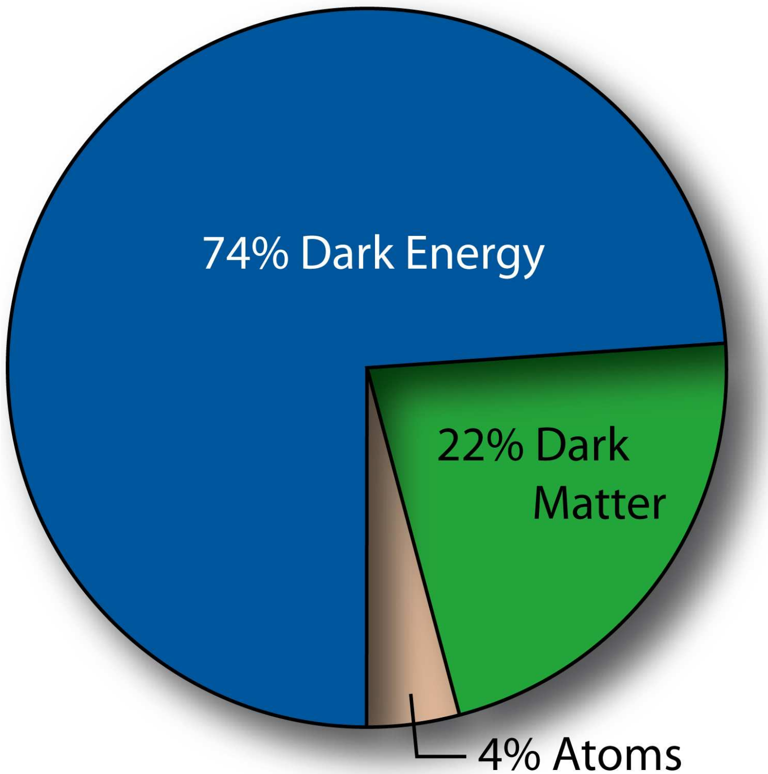
- The universe originates from an initial singularity ≈ 15 billion years ago
- Since then it cools and expands
- Many features like light element production are described very well by this model
- However there are some features that don't fit together in the SM
  - from the anisotropy of the cosmic microwave background it follows that the geometry is flat

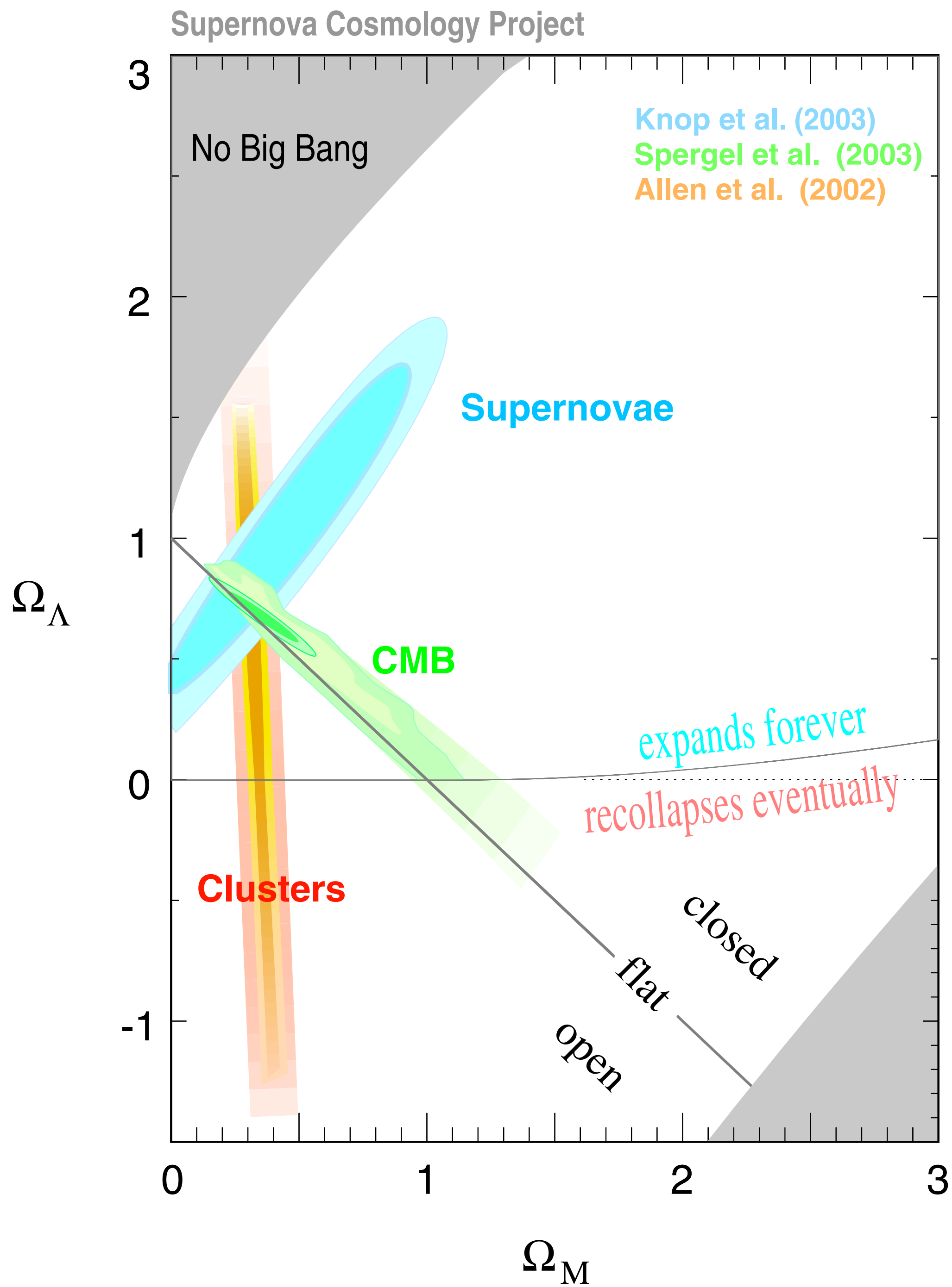


- The matter needed for this cannot be all baryonic (dark matter)
- There are many more pieces of evidence for dark matter: rotation curves around galaxies, patterns of galaxy clusters, further features of cosmic microwave spectrum
- From far supernova explosions one sees that the expansion of the universe is accelerating
- This requires that there exists a dark energy or cosmological constant



Composition of the universe:





What do we know about dark matter?

- Weakly interacting particles of mass $m = \mathcal{O}(100\,\mathrm{GeV})$
- Dark matter cannot be accommodated in the Standard Model
- However several extensions contain a credible dark matter candidate (Supersymmetry, universal extra dimensions, little Higgs)

What do we know about dark energy?

- No way to put dark energy into the SM
- Agreement at the loop level shows that the SM is more than an effective theory
- Factor 10^{62} smaller than naive calculations (Higgs)
- No idea how to put dark energy into any future theory

Conclusions

- The Standard Model describes a vast amount of data with very good precision
- (The neutrino masses may be the first experimental hint beyond the SM)
- However the SM is unnatural (hierarchy problem)
- Dark matter definitely requires extensions of the SM
- Any new theory has to contain the SM as a low energy approximation
- Dark energy gives a hint that something is fundamentally ununderstood in our description of subatomic physics