Resummation: Vector Bosons, the Higgs and Beyond (II)

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OUTLINE

Lecture 1

I. Review: how we get away with pQCD

II. The physical basis of factorization

III. Vector bosons: Q_T & and its factorization

Lecture 2

IV. Threshold resummation

V. Jet shapes and nonperturbative input

VI. Resummation and the Higgs

VII. Generalizations and limitations

IV. Threshold resummation

- **Back to the one-loop DY hard-scattering**

$$\frac{d\hat{\sigma}^{(1)}_{q\bar{q}\to\gamma^* g}}{dQ^2\, d^2\mathbf{Q}_T} = \sigma_0 \frac{\alpha_s C_F}{\pi^2} \left(1 - \frac{4\mathbf{Q}_T^2}{(1-z)^2 \xi_1 \xi_2 S}\right)^{-1/2} \times \left[\frac{1}{\mathbf{Q}_T^2}\frac{1+z^2}{1-z} - \frac{2z}{(1-z)Q^2}\right]$$

- **Factorized cross section at fixed $\mathbf{Q}_T$:**

$$\frac{d\sigma_{NN\to\mu^+\mu^-+X}(Q,p_1,p_2)}{dQ^2 d^2\mathbf{Q}_T} = \int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \frac{d\hat{\sigma}_{a\bar{a}\to\mu^+\mu^-(Q)+X}(Q,\mu,\xi_1 p_1,\xi_2 p_2,\mathbf{Q}_T)}{dQ^2 d^2\mathbf{Q}_T} \times f_{a/N}(\xi_1,\mu)\, f_{\bar{a}/N}(\xi_2,\mu)$$

Integrate over $\mathbf{Q}_T$: the NLO total DY cross section

Integrate over $\mathbf{Q}_T$ at fixed $z = \frac{Q^2}{\xi_1 \xi_2 S}$. $Q_T \to 0$ is singular.

Add diagrams with virtual gluons: *their* k_T **integrals are singular.**

Factorize low $\mathbf{k}_T = -\mathbf{Q}_T < \mu$ gluons as in DIS.

The remainder is now finite at fixed Q_T, $z \neq 1$.

- **The Q_T-integrated NLO partonic cross section**

$$
\frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}\to\gamma^* g}(z,Q^2,\mu^2)}{dQ^2}
= \sigma_0(Q^2)\ \left(\frac{\alpha_s(\mu)}{\pi}\right)\ \left\{2(1+z^2)\ \left[\frac{\ln(1-z)}{1-z}\right]_+
- \frac{(1+z^2)\ln\ z}{(1-z)} + \left(\frac{\pi^2}{3}-4\right)\ \delta(1-z)\right\}
+ \sigma_0(Q^2)\ C_F\ \frac{\alpha_s}{\pi}\ \left[\frac{1+z^2}{1-z}\right]_+\ \ln\left(\frac{Q^2}{\mu^2}\right)
$$

- **Plus distributions: “generalized functions” (c.f. delta function).**

- **What they are, how they work**

$$\int_0^1 dx\, \frac{f(x)}{(1-x)_+} \equiv \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)}$$

$$\int_0^1 dx\ f(x)\left(\frac{\ln(1-x)}{1-x}\right)_+ \equiv \int_0^1 dx\ \big(\, f(x) - f(1)\,\big)\ \frac{\ln(1-x)}{(1-x)}$$

and so on . . . where $f(x)$ will be parton distributions

- **$f(x)$ term: real gluon, with momentum fraction $1-x$.**
- **$f(1)$ term: virtual, with elastic kinematics.**
- **If $f(x)$ is changing rapidly, find a large correction.**

- **A Special Distribution is the**

- **DGLAP "evolution kernel" = "splitting function":**

$$P_{qq}(z) = C_F \, \frac{\alpha_s}{\pi} \, \left[\frac{1+z^2}{1-z}\right]_+ \rightarrow \frac{A(\alpha_s)}{1-z} + \ldots$$

- **Nonsinglet, leading order**

- **A neat bit of soft-gluon kinematics:** $p_q + p_{\bar{q}} = q + k \Rightarrow$:

$$z = \frac{Q^2}{\xi_1 \xi_2 S} = \frac{(p_q + p_{\bar{q}} - k)^2}{(p_q + p_{\bar{q}})^2}$$
$$z \sim 1 - \frac{2Q \cdot k}{Q^2}$$

And in the $\vec{Q} = 0$ (c.m.) frame,

$$1 - z = \frac{2k_0}{Q}$$

So one singularity in $\hat{\sigma}^{(1)}$ is from $\mathrm{k}_T = 0$, one from $k_0 = 0$, for any number of soft partons in the final state.

$z \to 1$ **is called "partonic threshold".**

- **Threshold resummation is resummation for the plus distributions.**

- **Same method as for Q_T, but now fix $k_{\rm soft} \sim \frac{1}{2}(1-z)Q$.**

  Laplace or Mellin transform $e^{-N2k_0/Q} \sim z^N$ and $\overline{\rm MS}$ collinear subtraction gives (here NLL accuracy shown) $\exp[\, E_a^{\rm thr}(N,Q)\,]$:

$$E_a^{\rm thr}(N,Q) = \int_{Q^2/N^2}^{Q^2} \frac{du^2}{u^2}\, 2A_a\left(\alpha_s(u)\right)\, \ln\frac{Nu}{Q}$$

Threshold: small $1-z \sim 2k_0/Q$, large N: enhancement:

$$2C_i \frac{\alpha_s(Q)}{\pi} \int_{Q^2/N^2}^{Q^2} \frac{dk_T^2}{k_T^2} \Bigg[\left\{ 1 + \left(\frac{\alpha_s(Q)}{\pi} \right) \left(K - \frac{\beta_0}{4\pi} \right) \right\} \ln \left(\frac{Q^2}{k_T^2} \right) + 2\frac{\alpha_s(Q)}{\pi} \Bigg]$$

$$\sim \alpha_s \ln^2(N)(1 + \alpha_s \ln(N) + \dots) + \alpha_s \ln(N)(1 + \alpha_s \ln(N) + \dots) + \dots$$

- **As for Q_T, these are LL($A^{(1)}$), NLL ($B^{(1)}$, $A^{(2)}$), and so on.**

- **And again, NLL is good so long as $\alpha_s(Q) \ln N \leq 1$.**

In this case, the enhancement is entirely due to the subtraction of collinear singularities.

The $\overline{\mathrm{MS}}$ distributions decrease faster in N than the partonic cross section.

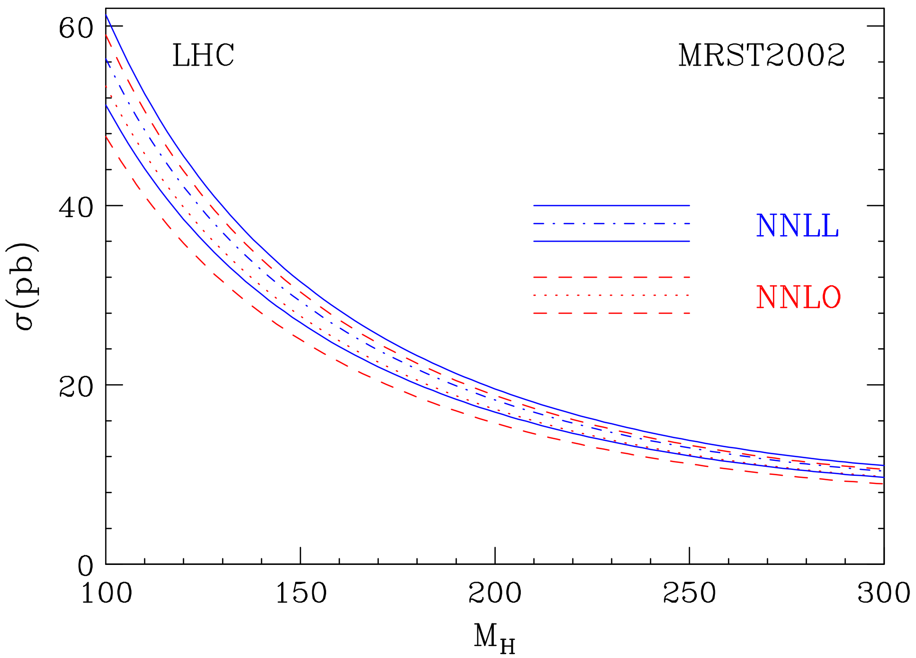
- **Inverse transform to the cross section:**

$$\frac{d\sigma_{NN}^{\rm res}}{dQ^2} = \sum_a \; \hat{\sigma}_a^{(0)}(Q,\mu) \int_{C_N} \frac{dN}{2\pi i} \left(\frac{Q^2}{S}\right)^{-N} \exp\left[\, E_a^{\rm thr}(N,Q,\mu)\,\right]$$
$$\times f_{a/N}(N,\mu)\, f_{\bar{a}/N}(N,\mu)$$

Formalism is similar for W, Z, H. “Electroweak annihilation”

Typical collider result . . .

- **Logs: threshold resummation vs. fixed order for H at LHC**



(from Catani, de Florian, Grazzini, Nason (2003))

(See also L. Reina lecture II.)

- **Modest change & decrease in μ-dependence**
  $\rightarrow$ increased confidence. But see Sec. VII.)

V. Jet shapes and $1/Q$ corrections

- **Angularity event shapes**

  (C.F. Berger, Kúcs, GS (2003), Berger, Magnea (2004))

$$\tau_a = \frac{1}{Q} \sum_{i \text{ in } N} E_i \, (\sin\theta_i)^a \, (1 - |\cos\theta_i|)^{1-a}$$

- θ_i **angle to thrust ($a = 0$) axis ($\hat{n}$ that gives $\tau_0^{\min}$).**

- **Jet "broadening": $a = 1$; total cross section: $a \to -\infty$.**

- **NLL resummed cross section is from an inverse transform:**

$$\sigma\,(\tau_a, Q, a) \;=\; \sigma_{\rm tot}\,\int_C d\nu\,{\rm e}^{\nu\,\tau_a}\,\left[\,J_i(\nu, p_{Ji})\,\right]^2$$

- **At NLL can define $U_{c\bar{c}} = 1$: indepenent jet "shower" evolution.** (Catani, Turnock, Trentadue, Webber (1990-92))

So we need the resummed jet function in transform space

$$J_i(\nu, p_{Ji}) = \int_0 d\tau_a \, \mathrm{e}^{-\nu\tau_{Ji}} \, J_i(\tau_{Ji}, p_{Ji}) = \mathrm{e}^{\frac{1}{2}E(\nu,Q,a)}$$

where the same reasoning as above gives:

$$E(\nu, Q, a) = 2 \int_0^1 \frac{du}{u} \left[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right) \left(\mathrm{e}^{-u^{1-a}\nu(p_T/Q)^a} - 1 \right) \right.$$
$$\left. + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right) \left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}} - 1 \right) \right]$$

Again, nonperturbative scales are implied by resummmed PT. But now, an expansion in powers of $1/Q$ . . .

Shape function approach for $\mathrm{e^+e^-}$ jets

- $p_T > \kappa$, **PT**
- $p_T < \kappa$, **expand exponentials: isolate "shape function".**
- **Low p_T ($< \kappa \leftrightarrow \mu_I$) replaced by f_{NP}**

$$E(\nu, Q, a) = E_{\mathrm{PT}}(\nu, Q, \kappa, a)$$

$$+ \frac{2}{1-a} \sum_{n=1}^{\infty} \frac{1}{n\, n!} \left(-\frac{\nu}{Q}\right)^n \int_0^{\kappa^2} \frac{dp_T^2}{p_T^2}\, p_T^n\, A\left(\alpha_s(p_T)\right) \left[1 - \left(\frac{p_T}{Q}\right)^{n(1-a)}\right]$$

$$+ \ldots$$

$$\equiv E_{\mathrm{PT}}(\nu, Q, \kappa, a) + \ln \tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q}, \kappa\right)$$

Shape function properties

- $f_{\rm NP}$ **factorizes under moments $\rightarrow$ convolution**

$$\sigma(\tau_a, Q) = \frac{1}{2\pi i}\int_C d\nu \tilde{f}_{a,{\rm NP}}\left(\frac{\nu}{Q}\right)\, \sigma_{\rm PT}(\nu, Q, a)$$

$$= \int d\xi f_{a,{\rm NP}}(\xi)\ \sigma(\tau_a - \xi, Q)$$

- $f_{\rm NP}$ **function of ν/Qonly**

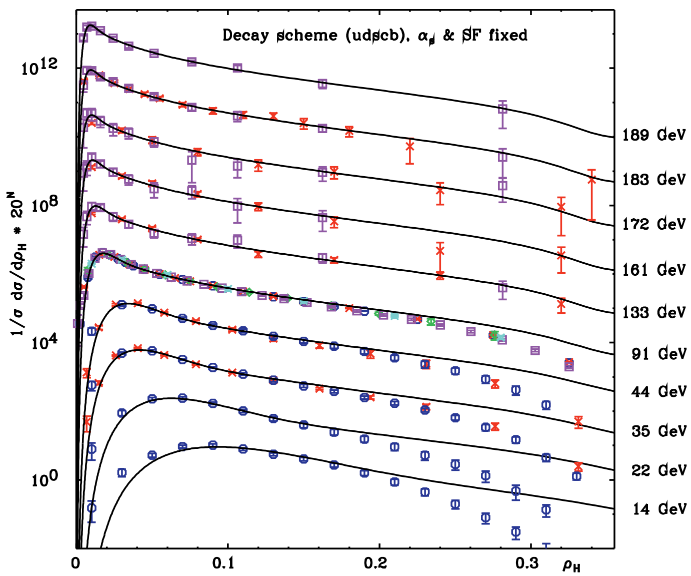
- **Linear in ν/Q: shift in PT distribution**

(Korchemsky & GS (1995), Dokshitzer & Webber (1997))

$$\tilde{f}_{a,{\rm NP}}\left(\frac{\nu}{Q}\right) \rightarrow {\rm e}^{\nu\left(\tau_a - \frac{1}{1-a}\frac{\lambda_1}{Q}\right)}$$

- **Shape function phenomenology for thrust**

(Korchemsky,GS, Belitsky; Gardi Rathsman,Magnea (1998 . . .))



Strategy: $f_{\rm NP}(\epsilon)$ at Z pole; predict other Q (viz. PDFs)

- **Scaling property for τ_a event shapes**

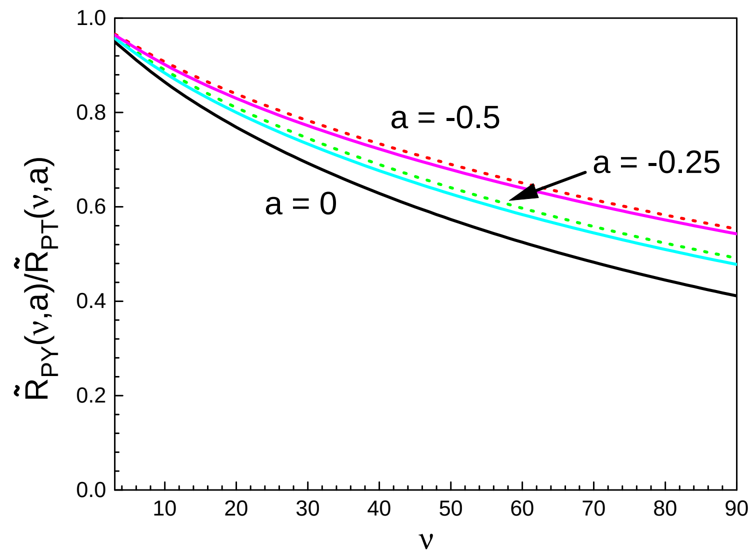
- **(Approximate rapidity-independence of NP dynamics)**

$$\ln \tilde{f}_{a,\mathrm{NP}} \left(\frac{\nu}{Q}, \kappa \right) = \frac{1}{1-a} \sum_{n=1}^{\infty} \lambda_n(\kappa) \left(-\frac{\nu}{Q} \right)^n$$

$$\tilde{f}_a \left(\frac{\nu}{Q}, \kappa \right) = \left[\tilde{f}_0 \left(\frac{\nu}{Q}, \kappa \right) \right]^{\frac{1}{1-a}}$$

- **All a-dependence is in the exponent.**

• **What PYTHIA gives**



• **Intriguing, but untested as yet.**

VI. Resummation and the Higgs

- See also L. Reina, lecture II.

- Main differences from W and Z are because Higgs is produced primarily from gluon fusion: $gg \rightarrow t \text{ loop} \rightarrow H$.

  $C_F = 4/3 \rightarrow C_A = 3$ in the exponent for Q_T resummation.

  State of the art: Bozzi, Catani, de Florian, Grazzini (2006).

  Includes all of NLO at large Q_T ("matching") and NNLL.

  $A^{(3)}$: from NNLO splitting function

  (Moch, Vermaseren and Vogt, 2004-6).

Directly from Bozzi et al., 2006:

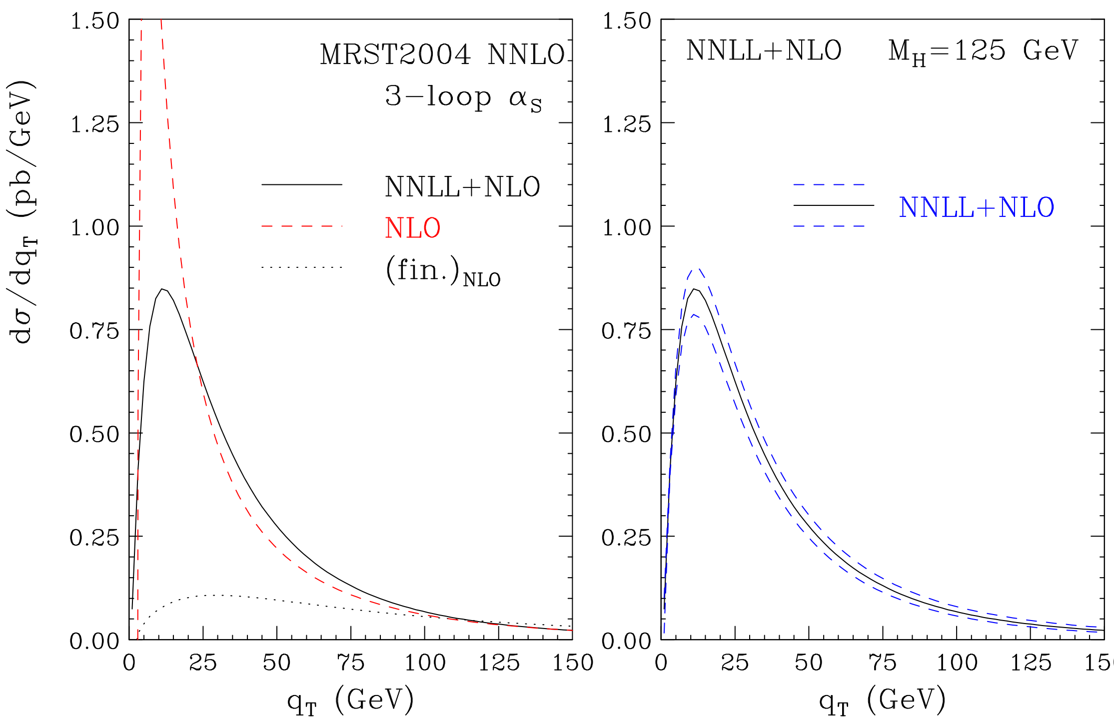


Figure 1: ***The q_T spectrum at the LHC with $M_H = 125$ GeV: (left) setting $\mu_R = \mu_F = Q = M_H$, the results at NNLL+NLO accuracy are compared with the NLO spectrum and the finite component of the NLO spectrum; (right) the uncertainty band from variations of the scales μ_R and μ_F at NNLL+NLO accuracy.***

VII. Generalizations and limitations
A) Factorization with no hard scattering: BFKL

(Sen (1980) Balitsky (1996) Kúcs (2003))

- $q^2 = -t \ll s$; **Regge limit in PT**

$$A(t,s) = \sum_{m,\ell} \int \left(\prod_{i=1}^{m-1} \mathrm{d}^{D-2} k_{i\perp} \right) \left(\prod_{j=1}^{\ell-1} \mathrm{d}^{D-2} p_{j\perp} \right)$$
$$\times \Gamma_A^{(m)\, a_1 \dots\, a_m}(p_A, q, n, k_{1\perp}, \dots, k_{m\perp})$$
$$\times S'^{\,(n,\ell)}_{a_1 \dots\, a_n,\, b_1 \dots\, b_ell}(q, n; k_{1\perp}, \dots, k_{n\perp}; p_{1\perp}, \dots, p_{m\perp})$$
$$\times \Gamma_B^{(\ell)\, b_1 \dots\, b_m}(p_B, q, n; p_{1\perp}, \dots, p_{\ell\perp})$$

- **Factorization at fixed rapidity separation:**
  Jets, Γ **&** **soft,** S; **no** H. **Introduce vector** n^μ **as above.**

- **Evolution equations (in $\ln s \sim$ rapidity) give**
- **generically m convolutions at $N^m LL$**

$$\left(p_A \cdot n \frac{\partial}{\partial p_A \cdot n} - 1\right) \Gamma_A^{(\ell)\ a_1 \dots a_\ell}(p_A, q, n; k_{1\perp}, \dots, k_{\ell\perp}) =$$

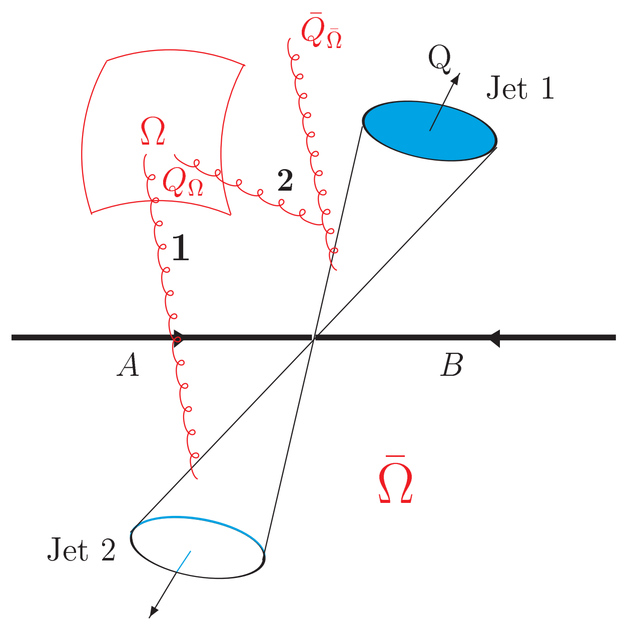
$$\sum_m \int \prod_{j=1}^{m} \mathrm{d}^{D-2} l_{j\perp}\, \mathcal{K}^{(\ell,m)}_{a_1 \dots a_n;\, b_1 \dots b_m}(k_{1\perp}, l_{1\perp}, \dots ; q, n)$$

$$\times\, \Gamma_A^{(m)\ b_1 \dots b_m}(p_A, q, n; l_{1\perp} \dots)$$

- **Can project onto different color exchange:**
  octet, $m = 0$ LL reggeized gluon
  singlet, $m = 1$, BFKL LL pomeron . . .

B) Non-global logs: color and energy flow

(Dasgupta & Salam (2001))



- **Simplest cases: 2 jets. Measure distribution $\Sigma_\Omega(E)$**
- **Very interesting case: energy flow between jets in WW fusion to H.**

- **Choices for Cross Section:**

- **a) Inclusive in $\bar{\Omega}$ $\longrightarrow$ Number of jets not fixed!**

- **b) Correlation with event shape τ_a . . . :**
  fixes number of jets $\longrightarrow$ factorization

  (Berger, Kúcs, GS (2003), Dokshitzer, Marchesini (2003), Banfi, Salam, Zanderighi (2004,5))

- **for a): Number of jets not fixed: nonlinear evolution**
  (Banfi, Marchesini, Smye (2002)) LL in E/Q, large-N_c,
  approximate evolution equation for distribution Σ is non-linear!

$$\partial_\Delta \Sigma_{ab}(E) = -\partial_\Delta R_{ab}\, \Sigma_{ab}(E) + \int_{k \text{ not in } \Omega} dN_{ab\to k}\, \left(\Sigma_{ak}\Sigma_{kb} - \Sigma_{ab}\right)$$

$$dN_{ab\to k} = \frac{d\Omega_k}{4\pi}\, \frac{\beta_a \cdot \beta_b}{\beta_k \cdot \beta_b\, \beta_k \cdot \beta_a} \qquad R_{ab} = \int_E^Q \frac{dE'}{E'} \int_\Omega dN_{ab\to k}$$

- $\partial_\Delta = E\partial_E$

- **Origin of the nonlinearity**
  - ∂_E **requires a "hard" gluon** k**.**
  - **New hard gluon acts as new, recoil-less source.**
  - **Large-**N **limit:** $\bar{q}(a)G(k)q(b)$ **sources** $\rightarrow \bar{q}(a)q(k) \oplus \bar{q}(k)q(a)$**.**
  - **"Global" event shape eliminates extra hard gluon.**
  - **But fixing an event shape limits the number of events.**
  - **We are far from a full understanding.**

C) Large threshold effects in observed hadrons

- **Pions at fixed target and RHIC** (Vogelsang and de Florian, 2004)

$$\frac{p_T^3\, d\sigma(x_T)}{dp_T} = \sum_{a,b,c} \int_0^1 dx_1\, f_{a/H_1}\left(x_1, \mu_F^2\right) \int_0^1 dx_2\, f_{b/H_2}\left(x_2, \mu_F^2\right)$$

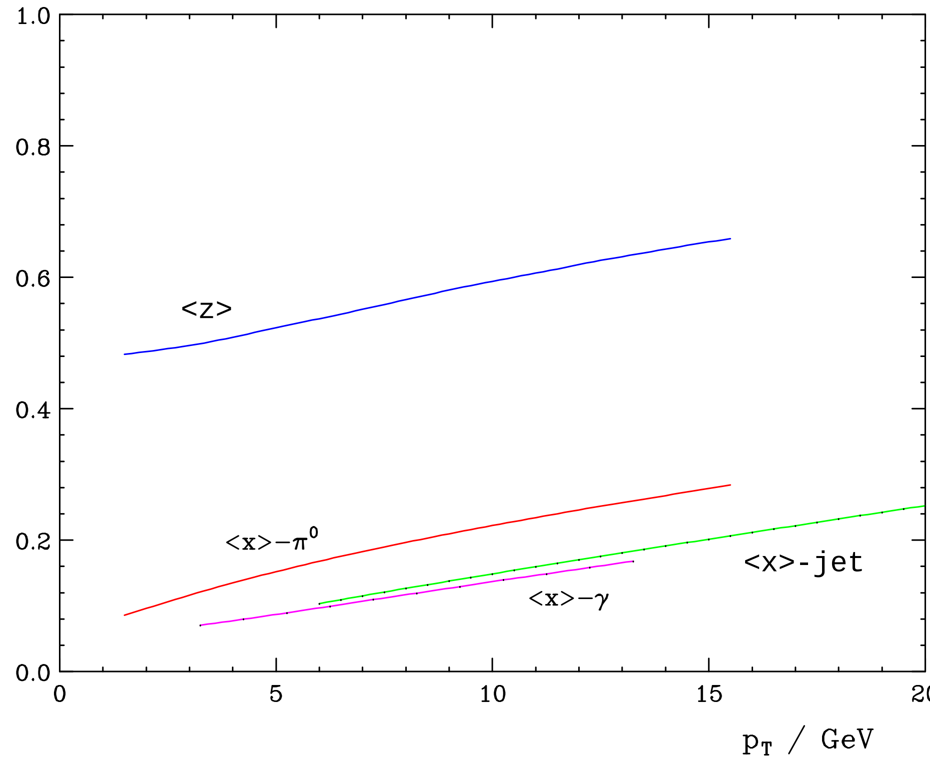
$$\times \int_0^1 dz\, z^2\, D_{h/c}\left(z, \mu_F^2\right)$$

$$\times \int_0^1 d\hat{x}_T\, \delta\left(\hat{x}_T - \frac{x_T}{z\sqrt{x_1 x_2}}\right) \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta}\, \frac{\hat{x}_T^4\, \hat{s}}{2} \frac{d\hat{\sigma}_{ab\to cX}(\hat{x}_T^2, \hat{\eta})}{d\hat{x}_T^2 d\hat{\eta}}$$

$\hat{\eta}$: **pseudorapidity at parton level**

$$\hat{\eta}_+ = -\hat{\eta}_- = \ln\left[(1+\sqrt{1-\hat{x}_T^2})/\hat{x}_T\right]$$

- **Averages for distribution x and fragmentation z's**



RHIC 200 GeV midrapidity average z for pions, and average x for pions, photons, jets at (NLO). Thanks to Werner Vogelsang.

- **Large z enhances threshold singularities.**

- **Singularities at one loop:**

$$\frac{\hat{s}\, d\hat{\sigma}^{(1)}_{ab\to cX}(v,w)}{dv\, dw} \approx \frac{\hat{s}\, d\hat{\tilde{\sigma}}^{(0)}_{ab\to cd}(v)}{dv} \left[A'\, \delta(1-w) + B' \left(\frac{\ln(1-w)}{1-w}\right)_+ \right.$$
$$\left. + C' \left(\frac{1}{1-w}\right)_+ \right]$$

- **For resummation, take x_T^{2N} moments $\rightarrow$ factorization:**

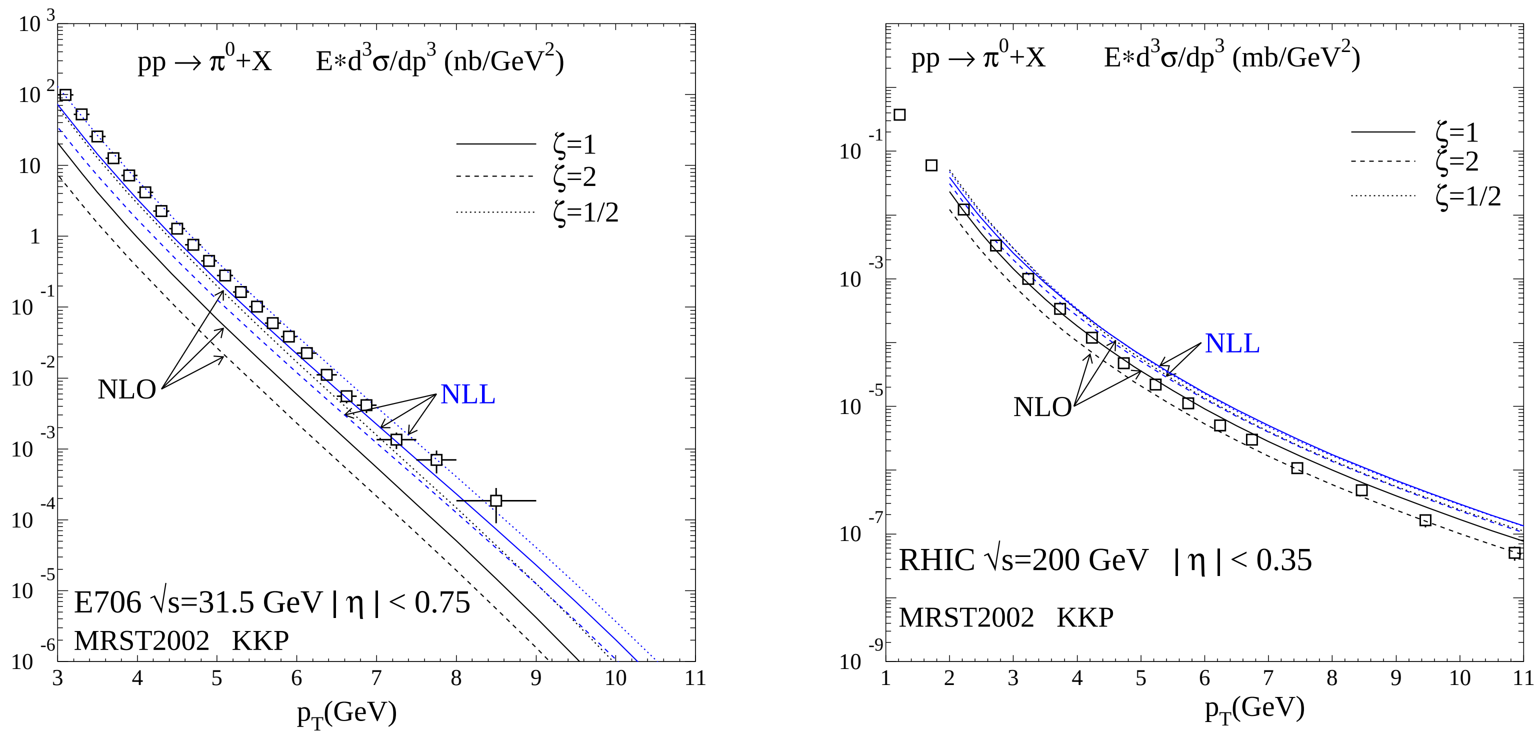
$$\hat{\sigma}^{(\mathrm{res})}_{ab\to cd}(N) = C_{ab\to cd}\, \Delta^a_N\, \Delta^b_N\, \Delta^c_N\, J^d_N \left[\sum_I G^I_{ab\to cd}\, \Delta^{(\mathrm{int})ab\to cd}_{I\;N} \right] \hat{\sigma}^{(\mathrm{Born})}_{ab\to cd}(N)$$

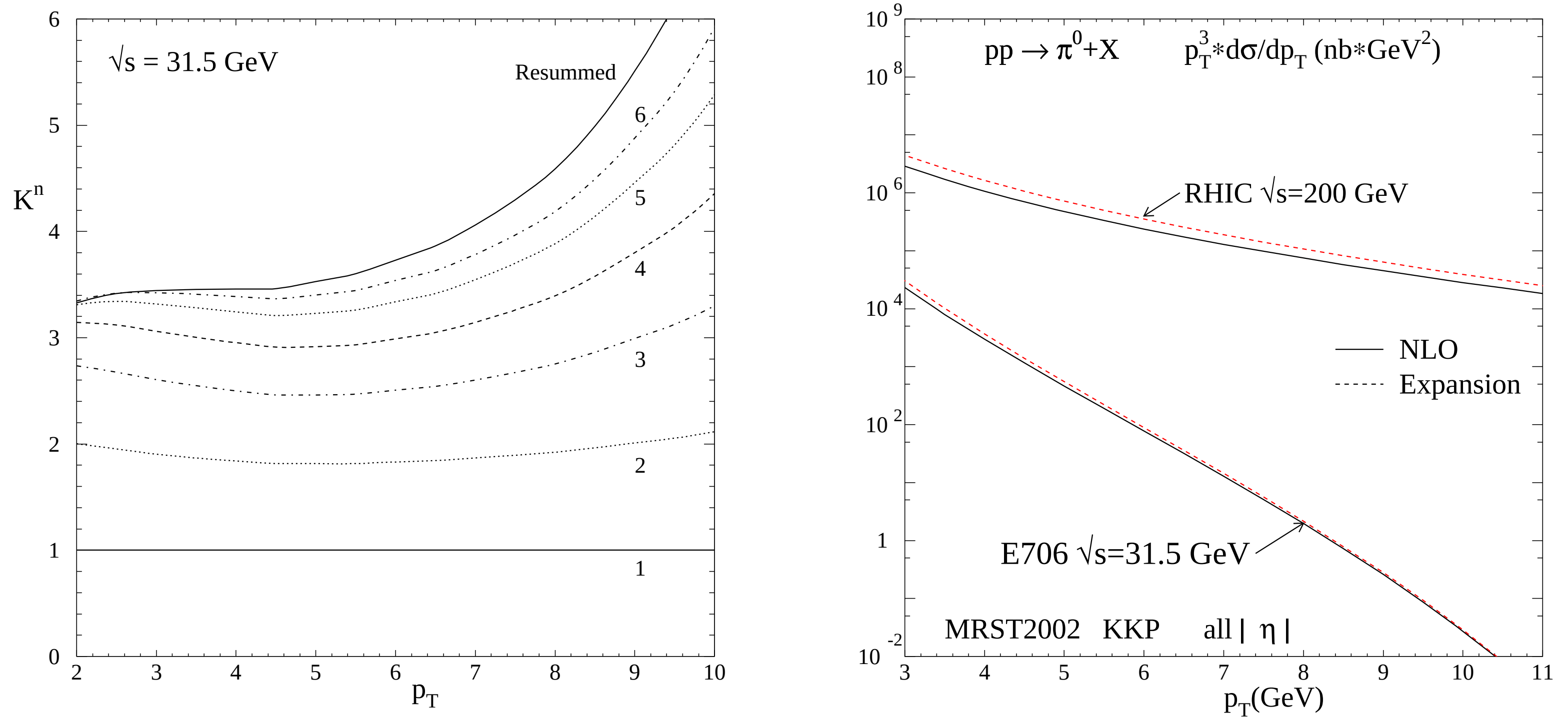
- **A typical NLL resummed factor:**

$$\Delta^a_N = \exp\left[\int_0^1 \frac{z^{N-1}-1}{1-z} \int_{\mu^2_{FI}}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_a(\alpha_s(q^2)) \right]$$

$$A = C_F(\alpha_s/\pi)(1 + K(\alpha_s/\pi)\,) \; + \; \ldots$$

- **Invert the moments: resolve a long-standing fixed-target vs. collider puzzle.**





- **Left: expansion of resummed cross section to fixed orders.**
- **Right: exact NLO vs. NLO expansion.**
- **Shows in π^0 1PI cross sections threshold resummation is more accurate and more important in fixed target range.**

Conclusion

- **Time's up for a sample of a large subject.**

- **Resummation just scratches the surface of QCD.**
  But it makes a mark.