The Standard Model of Electroweak Physics

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Lecture II: Structure of the Electroweak Theory
Electromagnetic force

Quark color force
Standard Electroweak Model

Weak Force:

\[ \text{SU}(2) \times \text{U}(1) \]

What gives rise to masses of \( W \) and \( Z \) boson?

\[ \text{SU}(2) \times \text{U}(1) \text{ is “Spontaneously broken Symmetry”} \]

Higgs Field?
Standard Electroweak Model

Weak Force:

SU(2) x U(1)

Based upon a nonabelian gauge symmetry: Yang-Mills Field Theory

SU(2)xU(1) is “Spontaneously broken Symmetry”

Higgs Field?
Symmetry is:

**Invariance** of a system or object under a transformation or collection of transformations.

*Symmetry Groups*
Symmetry of the Equilateral Triangle

1

identity
Rotation about center through 120 degrees
Rotation about center through 240 degrees
Reflection about axis I

A

B

C

I

II

III

$R_I$
Reflection about axis II

$\text{R}_{\text{II}}$
Reflection about axis III
The Symmetry Operations of the Equilateral Triangle

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
<th>Result</th>
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</thead>
<tbody>
<tr>
<td>$1$</td>
<td>do nothing</td>
<td>$ABC$</td>
</tr>
<tr>
<td>$R_{120}$</td>
<td>rotate through 120 degrees</td>
<td>$CAB$</td>
</tr>
<tr>
<td>$R_{240}$</td>
<td>rotate through 240 degrees</td>
<td>$BCA$</td>
</tr>
<tr>
<td>$R_I$</td>
<td>reflect about axis I</td>
<td>$ACB$</td>
</tr>
<tr>
<td>$R_{II}$</td>
<td>reflect about axis II</td>
<td>$BAC$</td>
</tr>
<tr>
<td>$R_{III}$</td>
<td>reflect about axis III</td>
<td>$CBA$</td>
</tr>
</tbody>
</table>
How do we know we have them all?
What principle is at work here?

“Combine symmetry operations”

\[ R_{240} \times R_1 = ? \]
$R_{240} \times R_I = R_{II}$
Symmetry Group of the Equilateral Triangle: $S_3$

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<thead>
<tr>
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<th>1</th>
<th>$R_{(120)}$</th>
<th>$R_{(240)}$</th>
<th>$R_I$</th>
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<td>$R_{II}$</td>
<td>$R_{(120)}$</td>
<td>$R_{(240)}$</td>
<td>1</td>
</tr>
</tbody>
</table>

$A \times B = C$
Commutation generally doesn’t hold in nature: \[ A \times B \neq B \times A \]
Symmetry Groups

- A group $G$ is a collection of elements \{ $r_j$ \}
- $G$ has a “multiplication” operation: $r_j \times r_k = r_k$
  where $r_k$ is in $G$
- There is a unique identity in $G$, $1$, such that $1 \times r_k = r_k \times 1 = r_k$
- Each element $r_k$ has a unique inverse $r_k^{-1}$ such that $r_k^{-1} \times r_k = r_k \times r_k^{-1} = 1$
- Group multiplication is associative
Continuous Symmetry Groups
Cartan Classification

- Spheres in N dimensions: \( O(2), O(3), ..., \) SO(N)
- Complex Spheres in N dimensions: \( U(1), SU(2), ..., \) SU(N)
- N dimensional phase space: \( Sp(2N) \)
- Exceptional Groups: \( G_2, F_4, E_6, E_7, E_8 \)

Continuous rotations are exponentiated angles \( x \) generators. Generators form a Lie Algebra, e.g. \( SU(N) \) has \( N^2-1 \) generators.

Generators are in 1:1 correspondence with the gauge fields in a Yang-Mills theory.
Electroweak Theory:
SU(2) X U(1) Yang-Mills Gauge Theory

two gauge coupling constants, $g_2$ and $g_1$

\[ iD_\mu = i\partial_\mu - g_2 W^a_\mu Q^a - g_1 B_\mu \frac{Y}{2} \]

\[ = i\partial_\mu - g_2 W^+_\mu Q^- - g_2 W^-_\mu Q^+ - g_2 W^3_\mu Q^3 - g_1 B_\mu \frac{Y}{2} \]

\[ Q^\pm = (Q^1 \pm iQ^2)/\sqrt{2} \quad \quad W^\pm_\mu = (W^1_\mu \pm iW^2_\mu)/\sqrt{2} \]

$Q^a$ are the $SU(2)$ weak charges \quad $Y$ is the $U(1)$ hypercharge

\[ [Q^a, Q^b] = i\epsilon^{abc} Q^c. \]
particular representations for the charges,

left-handed fermions, and Higgs boson, \[ Q^a = \tau^a / 2 \]

right-handed fermions are singlets, annihilated by the \( Q^a \).

abstract operator \( Y \) \[ \leftrightarrow \] eigenvalue, \( Y_r \)

\[ Q_{EM} = Q^3 + \frac{Y}{2} \]

\[
\begin{pmatrix}
  u^{2/3} \\
  d^{-1/3}
\end{pmatrix}_L, \quad Y_r = \frac{1}{3}; \quad
\begin{pmatrix}
  \nu^0 \\
  e^{-1}
\end{pmatrix}_L, \quad Y_r = -1; \quad
H = \begin{pmatrix}
  \phi^0 \\
  \phi^-
\end{pmatrix}, \quad Y_r = -1
\]

\[ u_R^{2/3}, \quad Y_r = \frac{4}{3}; \quad d_R^{-1/3}, \quad Y_r = -\frac{2}{3}; \quad e_R^{-1}, \quad Y_r = -2; \quad \nu_R^0, \quad Y_r = 0. \]

A right–handed neutrino is sterile
\[ W^3_\mu = Z^0_\mu \cos \theta + A_\mu \sin \theta \]
\[ B_\mu = -Z^0_\mu \sin \theta + A_\mu \cos \theta \]

\[ = i \partial_\mu - g_2 W^+_\mu Q^- - g_2 W^-_\mu Q^+ - g_2 W^3_\mu Q^3 - g_1 B_\mu \frac{Y}{2} \]

\[ Q_{EM} = Q^3 + \frac{Y}{2} \]

\[ g_2 \sin \theta = e; \quad g_1 \cos \theta = e; \quad \tan \theta = \frac{g_1}{g_2} \]

The photon thus couples to \( eQ_{EM} \) with strength \( e \) where:

\[ \frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1}{g_1^2} \]
The gauge covariant field strengths are defined by commutators of the covariant derivative:

\[ F^{a}_{\mu\nu} = -\frac{i}{g^{2}_{2}} \text{Tr} (\tau^{a}[D_{\mu}, D_{\nu}]) = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} + \epsilon^{abc} A^{b}_{\mu} A^{c}_{\nu} \]

\[ F^{\mu\nu} = -\frac{i}{g^{2}_{1}} Y^{-1} \text{Tr} ([D_{\mu}, D_{\nu}]) = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} \]

Then the gauge field kinetic terms are:

\[ \mathcal{L}_{G.B.\ kinetic} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a}_{\mu\nu} - \frac{1}{4} F^{\mu\nu} F^{\mu\nu} \]
Spontaneous Symmetry Breaking
Consider the complex doublet scalar Higgs-boson with $Y_r = -1$:

$$H = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$$

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) - V(H)$$

$$V(H) = \frac{\lambda}{2} (H^\dagger H - v_{weak}^2)^2$$
Standard Model Symmetry Breaking

\[ \langle H \rangle = \begin{pmatrix} v_{weak} \\ 0 \end{pmatrix} \]

\[
H = \exp \left( i \frac{\pi^a \tau^a}{\sqrt{2} v_{weak}} \right) \begin{pmatrix} v_{weak} + \frac{h}{\sqrt{2}} \\ 0 \end{pmatrix}
\]

\[ v_{weak} = 175 \text{ GeV} \]
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Nambu-Goldstone Bosons are eaten
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"Higgs" Boson is physical
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Nambu-Goldstone Bosons are eaten
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Massless W, Z

Time

Light cone

+Z axis
Higgs Boson Condenses in vacuum

Higgs boson vacuum expectation value = 175 GeV

Weak charge is hidden in vacuum!!
A massive gauge boson oscillates through spacetime, colliding with the Higgs condensate and mixing with phase degrees of freedom:
Couple fermions to the Higgs

Weak charge is conserved
Fermion Masses in Electroweak Theory

Fermion Mass requires Higgs to maintain Electroweak Gauge Symmetry!!!
Searching for the Higgs
(Vacuum Electroweak Superconductivity)

114 GeV < m_H < 260 GeV
Mass Eigenstates

\[ A_\mu = \sin \theta \, W_\mu^3 + \cos \theta \, B_\mu \]
\[ Z_\mu = \cos \theta \, W_\mu^3 - \sin \theta \, B_\mu = \frac{(g_2 W_\mu^3 - g_1 B_\mu)}{\sqrt{g_1^2 + g_2^2}} \]

\[ iD_\mu = i\partial_\mu - g_2 W_\mu^+ Q^- - g_2 W_\mu^- Q^+ - eA_\mu \left( Q^3 + \frac{Y}{2} \right) - Z_\mu \tilde{Q} \]

\[ \tilde{Q} = \sqrt{g_1^2 + g_2^2} \left( \cos^2 \theta \, \frac{\tau^3}{2} - \sin^2 \theta \, \frac{Y}{2} \right) \]
\[ = e \left( \cot \theta \, \frac{\tau^3}{2} - \tan \theta \, \frac{Y}{2} \right) \]
Mass Eigenstates

\[ \mathcal{L}_H = \frac{1}{2} (\partial h)^2 + \frac{1}{2} M_W^2 W_\mu^+ W^{\mu -} + \frac{1}{2} M_Z^2 Z_\mu Z^{\mu} - \frac{1}{2} M_H^2 h^2 - \frac{\sqrt{\lambda}}{2} M_H h^3 - \frac{1}{8} \lambda h^4 \\
+ \frac{1}{2} \left( h^2 + \frac{M_H}{\lambda} h \right) \left( g_2^2 W_\mu^+ W^{\mu -} + (g_1^2 + g_2^2) Z_\mu Z^{\mu} \right) \]

\[ M_W^2 = \frac{1}{2} g_2^2 v_{weak}^2; \quad M_Z^2 = \frac{1}{2} v_{weak}^2 (g_1^2 + g_2^2). \]

\[ \frac{M_W^2}{M_Z^2} = \cos^2 \theta_W \]

\[ v_{weak}^2 = \frac{1}{2\sqrt{2}G_F} \]
Fermions

e.g., Top and Bottom

$$\Psi_L = (t, b)_L$$

$$L = \frac{1}{2}(1 - \gamma^5)$$

$$\bar{\Psi}_L \not D \Psi_L = \bar{\Psi}_L i\not D \Psi_L - \frac{1}{\sqrt{2}} \bar{t} \gamma_\mu L b W^\mu + \frac{1}{\sqrt{2}} \bar{b} \gamma_\mu L t W^\mu -$$

$$- \frac{2e}{3} \bar{t} \gamma_\mu L t A_\mu + e \frac{1}{3} \bar{b} L b A_\mu - \bar{\Psi}_L \bar{Q} \gamma_\mu \Psi_L Z_\mu$$

Dirac mass terms would be of the form $\bar{\Psi}_L \psi_R$

isospin $- \frac{1}{2}$

$$g_t \bar{\Psi}_L \cdot H t_R + g_b \bar{\Psi}_L \cdot H^c b_R$$

$$m_t = g_t v_{\text{weak}}$$

$$m_b = g_b v_{\text{weak}}$$
apply to muon decay

\[ \frac{ig_2^2}{(1/2)g_2^2\nu_{weak}^2} \overline{\nu}_\mu \gamma^\mu \mu_L \overline{\nu} \gamma^\mu \nu_{eL} \]

\[ = (G_F/\sqrt{2}) \overline{\nu}_\mu \gamma^\mu \mu_L \overline{\epsilon} \gamma^\mu \nu_{eL} \]

\[ \nu_{weak}^2 = \frac{G_F}{2\sqrt{2}} = (175 \text{ GeV})^2 \]
leptonic doublet $\Psi_L = (\nu, \ell)_L$

$$\frac{g_\nu}{M} (\bar{\Psi}_L H)(H^C \Psi^C_L)$$

Majorana mass for $\nu_L$ \quad $g_\nu v_{weak}^2 / M$

The interplay between *gauge symmetries*, and *chiral symmetries*, both of which are broken spontaneously, is fundamental to the Standard Model. The left-handed fermions carry the electroweak $SU(2)$ quantum numbers, while the right-handed do not. All of the mathematical features of the symmetric Lagrangian remain intact, but the spectrum of the theory does not retain the original obvious symmetry properties. When a massive gauge boson was discovered, such as the $W^\pm$ or $Z$ of the Standard Model, we also discovered an extra piece of physics: the longitudinal component, i.e., the NGB which comes from the symmetry breaking sector.
I apologize for not giving a review of CKM physics, rare electroweak processes, CP-violation and all that.

One Lecture on the Standard Model or Five Lectures is sufficient; Two is not.
Lightning Review of Radiative Corrections to Standard Model

generalize slightly our definition of $v_{weak}$

$$M_W^2 = \frac{1}{2} v_W^2 g_2^2; \quad M_Z^2 = \frac{1}{2} v_Z^2 (g_1^2 + g_2^2)$$

$v_W$ ($v_Z$) is the Higgs VEV “as seen by” the W-boson (Z=bozon)

<table>
<thead>
<tr>
<th>tree level</th>
<th>$v_W^2 = v_Z^2 = v_{weak}^2$</th>
</tr>
</thead>
</table>

radiative corrections

$g_1^2(q^2), g_2^2(q^2)$

$v_W^2(q^2)$ and $v_Z^2(q^2)$

to a good approximation, we really only need to know $\alpha(M_Z) \approx \alpha(M_W)$
\[ F.T. \left< 0 | T \tilde{A}_\mu^A(0) \tilde{A}_\mu^B(x) | 0 \right> = g_{\mu\nu} \Pi_{AB} - q_\mu q_\nu \Pi_{AB}^T \]

\[ \frac{1}{2} v_Z^2 = \frac{1}{2} v_{weak}^2 - \Pi_{3B} \]
\[ = \frac{1}{2} v_{weak}^2 - \Pi_{3Q} + \Pi_{33} \]

\[ \frac{1}{2} v_W^2 = \frac{1}{2} v_{weak}^2 + \Pi_{WW} - \Pi_{3Q} \]

\[ \frac{1}{g_2^2} = \frac{1}{g_{2un}^2} - \Pi_{33}^T - \Pi_{3B}^T \]
\[ = \frac{1}{g_{2un}^2} - \Pi_{3Q}^T \]

\[ \frac{1}{g_1^2} = \frac{1}{g_{1un}^2} - \Pi_{BB}^T - \Pi_{3B}^T \]
\[ = \frac{1}{g_{1un}^2} + \Pi_{3Q}^T - \Pi_{QQ}^T \]

\( T \) stands for “transverse”
\[ v_W^2 - v_Z^2 = \frac{N_c}{32 \pi^2} \left[ (m_t^2 + m_b^2) - \frac{2m_t^2 m_b^2}{(m_t^2 - m_b^2)} \log(m_t^2/m_b^2) \right. \\
+ \frac{M_W^2 m_H^2}{m_H^2 - M_W^2} \ln(m_H^2/M_W^2) - \frac{M_Z^2 m_H^2}{m_H^2 - M_Z^2} \ln(m_H^2/M_Z^2) \left. \right] \]

The \( \rho \) parameter of Veltman is:

\[ \rho = \frac{v_W^2}{v_Z^2} \]

\[ = 1 + \frac{N_c}{32 v_0^2 \pi^2} \left[ (m_t^2 + m_b^2) - \frac{2m_t^2 m_b^2}{(m_t^2 - m_b^2)} \log(m_t^2/m_b^2) \right. \\
+ \frac{M_W^2 m_H^2}{m_H^2 - M_W^2} \ln(m_H^2/M_W^2) - \frac{M_Z^2 m_H^2}{m_H^2 - M_Z^2} \ln(m_H^2/M_Z^2) \left. \right] \]
A simple set of parameters

\[ v_{W}^{2}(q^{2}) = v_{\text{weak}}^{2} + \sigma q^{2} + \tau v_{r}^{2} + \omega q^{2} \]

\[ v_{Z}^{2}(q^{2}) = v_{\text{weak}}^{2} + \sigma q^{2} - \tau v_{r}^{2} - \omega q^{2} \]

Peskin and Takeuchi define:

\[ S = 16\pi \left[ \frac{\partial}{\partial q^{2}} \Pi_{33}|_{q^{2}=0} - \frac{\partial}{\partial q^{2}} \Pi_{33}|_{q^{2}=0} \right] \]

\[ T = \frac{4\pi}{\sin^{2} \theta \cos^{2} \theta M_{Z}^{2}} \left( \Pi_{WW}|_{q^{2}=0} - \Pi_{33}|_{q^{2}=0} \right) \]

\[ U = 16\pi \left[ \frac{\partial}{\partial q^{2}} \Pi_{WW}|_{q^{2}=0} - \frac{\partial}{\partial q^{2}} \Pi_{33}|_{q^{2}=0} \right] \]

we see that:

\[ \sigma = \frac{2S + U}{16\pi} ; \quad \omega = \frac{U}{16\pi} ; \quad \tau = \frac{\sin^{2} \theta \cos^{2} \theta M_{Z}^{2} T}{4\pi v_{r}^{2}} = \frac{\alpha}{2} T ; \]
input parameters:

\[ G_F = 1.16639 \ (1) \times 10^{-5} \ \text{GeV}^{-2} \]
\[ m_Z^2 = 8315.18(38) \ \text{GeV}^{-2} \]
\[ \alpha^{-1}(m_Z) = \text{?} < 0.0001 \]

\[ \alpha^{-1}(m_Z) = \alpha^{-1}(0) \left[ 1 - \Delta \alpha_L - \Delta \alpha_h - \Delta \alpha_{t,W} \right] \]

\[ \Delta \alpha_h = 0.02761 \ (36) \quad \text{Burkhardt–Pietrzyk} \ (01) \]
\[ = 0.02757 \ (36) \quad \text{Jegerlehner} \ (03) \]
\[ = 0.02737 \ (20) \quad \text{Jegerlehner (Euclidean)} \ (03) \]
\[ = 0.02763 \ (16) \quad \text{Davier–Höcker (}\tau, \text{QCD}) \ (98) \]
\[ G_F = \frac{1}{2} \sqrt{2} v_{\text{weak}}(0)^2 \]

\[ M_Z^2 = \frac{1}{2} (g_1^2(M_Z^2) + g_2^2(M_Z^2)) v_{\text{weak}}^2(M_Z^2) \]

\[ 4\pi \alpha(0) = \left. \frac{g_1^2 g_2^2}{g_1^2 + g_2^2} \right|_{\mu^2=0} \]

\[ \sin^2 \theta_{Z-\text{pole}} = \frac{g_1(M_Z^2)}{(g_1(M_Z^2) + g_2(M_Z^2))} \]

\[ M_W^2 = \frac{1}{2} g_2^2(M_W^2) v_{\text{weak}}^2(M_W^2) \]

\[ T = \frac{N_c}{4\pi \sin^2 \theta \cos^2 \theta M_Z^2} \left[ (m^2 + m_b^2) - \frac{2m_t^2 m_b^2}{(m_t^2 - m_b^2)} \log(m_t^2/m_b^2) \right. \]
\[ + \left. \frac{M_W^2 m_Z^2}{m_H^2 - M_W^2} \ln(m_H^2/M_W^2) - \frac{M_Z^2 m_H^2}{m_H^2 - M_Z^2} \ln(m_H^2/M_Z^2) \right] \]

\[ S = \frac{N_c}{6\pi} \left[ 1 - Y \log \left\{ \frac{m_b^2}{m_t^2} \right\} \right] \]
Application: Top Seesaw Model

Mass matrix for $t - \chi$ system is,

$-
\begin{pmatrix}
0 & \mu \approx 600 \text{ GeV} \\
m \approx 1 \text{ TeV} & M \approx 4 \text{ TeV}
\end{pmatrix}
\begin{pmatrix}
t_R \\
\chi_R
\end{pmatrix}$

Diagonalized:

$m_t \approx \frac{\mu m}{M}$

$m_\chi \approx \frac{M}{M}$

1998: Top Seesaw DOA (outside of the $S$-$T$ ellipse $\sim 4\sigma$. (Chivukula, Dobrescu, Georgi, Hill)

1999: $S$-$T$ error ellipse shifts along major axis toward upper right (predicted by the theory!).

2001: Inconsistencies in data; keep only leptons $\rightarrow$ Top Seesaw consistent and SM ruled out at $\sim 2\sigma$!!!

Theory consistent for natural values of its parameters at the $2\sigma$ level (He, CTH, Tait)
What is the Higgs Boson?
The mysterious role of Scale Symmetry

- We live in 1+3 dimensions
- The big cosmological constant conundrum
- The Higgs Boson mass scale
- QCD solves its own problem of hierarchy
- New Strong Dynamics?

Origin of Mass in QCD
Gell-Mann and Low:
\[
\frac{dg}{d \ln \mu} = \beta(g)
\]

Gross, Politzer and Wilczek:
\[
\beta(g) = \bar{\hbar} \beta_0 g^3 \quad \text{where} \quad \beta_0 = \frac{1}{16\pi^2} \left( \frac{11}{3} N_c - \frac{2}{3} n_f \right)
\]
\[
\frac{\Lambda_{QCD}}{M_0} = \exp \left( \frac{1}{2\bar{\hbar} \beta_0 g^2_0} \right)
\]
$\Lambda_{MS} = 217^{+25}_{-23}$ MeV

$\beta(g) = \mu \frac{\partial g}{\partial \mu}.$

$\alpha_s(k^2) \equiv \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{\beta_0 \ln(k^2/\Lambda^2)}$
A Puzzle: Murray Gell-Mann lecture ca 1975

\[ S_\mu = x^\nu T_{\mu\nu} \]

\[ \partial_\mu S^\mu = T^\mu_\mu \]

\[ T_{\mu\nu} = \text{Tr}(G_{\mu\rho}G^\rho_\nu) - \frac{1}{4} g_{\mu\nu} \text{Tr}(G_{\rho\sigma}G^{\rho\sigma}) \]

\[ \partial_\mu S^\mu = T^\mu_\mu = \text{Tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{4}{4} \text{Tr}(G_{\mu\nu}G^{\mu\nu}) = 0 \]

QCD is scale invariant!!!???
Resolution: The Scale Anomaly

\[ \partial_\mu S^\mu = \frac{\beta(g)}{g} \text{Tr} \ G_{\mu\nu} G^{\mu\nu} = \mathcal{O}(\hbar) \]

Origin of Mass in QCD = Quantum Mechanics
A heretical Conjecture:

All mass scales in physics are intrinsically quantum mechanical and associated with scale anomalies. The $\hbar \to 0$ limit of nature is exactly scale invariant.
“Predictions” of the Conjecture:

We live in D=4!

\[
T^\mu_\mu = \text{Tr} G_{\mu\nu}G^{\mu\nu} - \frac{D}{4} \text{Tr} G_{\mu\nu}G^{\mu\nu}
\]

Cosmological constant is zero in classical limit

QCD scale is generated in this way; Hierarchy is naturally generated

Testable in the Weak Interactions?

Weyl Gravity in D=4 is QCD-like:

\[
\frac{1}{\hbar^2} \sqrt{-g} \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right)
\]

Is the Higgs technically natural?

On naturalness in the standard model.

Conjecture on the physical implications of the scale anomaly.
Christopher T. Hill (Fermilab) . hep-th/0510177
Symmetry Principles Define Modern Physics