## The Standard Model of Electroweak Physics

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# Electromagnetic force

## Quark color force



#### **Standard Electroweak Model**

Weak Force:

#### SU(2) x U(1)



What gives rise to masses of W and Z boson?

SU(2)xU(1) is "Spontaneously broken Symmetry"

Higgs Field?

#### **Standard Electroweak Model**

Weak Force:





Based upon a nonabelian gauge symmetry: Yang-Mills Field Theory

> SU(2)xU(1) is "Spontaneously broken Symmetry"

> > Higgs Field?

## Symmetry is:

Invariance of a system or object under a transformation or collection of transformations.

Symmetry Groups

#### Symmetry of the Equilateral Triangle





Rotation about center through 120 degrees



Rotation about center through 240 degrees





Reflection about axis I



Reflection about axis II



Reflection about axis III

#### The Symmetry Operations of the Equilateral Triangle

1	do nothing	ABC
<b>R</b> <sub>120</sub>	rotate through 120 degrees	CAB
R <sub>240</sub>	rotate through 240 degrees	BCA
R <sub>I</sub>	reflect about axis I	ACB
R <sub>II</sub>	reflect about axis II	BAC
R <sub>III</sub>	reflect about axis III	CBA

How do we know we have them all? What principle is at work here?



**Evariste Galois** 

"Combine symmetry operations"

 $R_{240} \times R_I = ?$ 



#### **Symmetry Group** of the Equilateral Triangle: S<sub>3</sub>

	1	<b>R</b> <sub>(120)</sub>	<b>R</b> <sub>(240)</sub>	R <sub>I</sub>	R <sub>II</sub>	R <sub>III</sub>
1	1	<b>R</b> <sub>(120)</sub>	<b>R</b> <sub>(240)</sub>	R <sub>I</sub>	R <sub>II</sub>	R <sub>III</sub>
<b>R</b> <sub>(120)</sub>	<b>R</b> <sub>(120)</sub>	<b>R</b> <sub>(240)</sub>	1	R <sub>III</sub>	R <sub>I</sub>	R <sub>II</sub>
R <sub>(240)</sub>	<b>R</b> <sub>(240)</sub>	1	<b>R</b> <sub>(120)</sub>	R <sub>II</sub>	R <sub>III</sub>	R <sub>I</sub>
R <sub>I</sub>	R <sub>I</sub>	R <sub>II</sub>	R <sub>III</sub>	1	<b>R</b> <sub>(120)</sub>	<b>R</b> <sub>(240)</sub>
R <sub>II</sub>	R <sub>II</sub>	R <sub>III</sub>	R <sub>I</sub>	<b>R</b> <sub>(240)</sub>	1	<b>R</b> <sub>(120)</sub>
R <sub>III</sub>	R <sub>III</sub>	R <sub>I</sub>	R <sub>II</sub>	<b>R</b> <sub>(120)</sub>	<b>R</b> <sub>(240)</sub>	1

 $A \times B = C$ 

## Commutation generally doesn't hold in nature:

#### $A \times B \neq B \times A$



## Symmetry Groups

- A group G is a collection of elements  $\{r_i\}$
- G has a "multiplication" operation:  $r_j x r_k = r_k$ where  $r_k$  is in G
- There is a unique identity in G, 1, such that 1  $x r_k = r_k x 1 = r_k$
- Each element  $r_k$  has a unique inverse  $r_k^{-1}$  such that  $r_k^{-1} \ge r_k \ge r_k^{-1} = 1$
- Group multiplication is associative

## Continuous Symmetry Groups Cartan Classification

- Spheres in N dimensions:
- Complex Spheres in N dimensions: U(1), SU(2), ..., SU(N)
- N dimensional phase space
- Exceptional Groups:

O(2), O(3), ..., SO(N) U(1), SU(2), ..., SU(N) Sp(2N)  $G_2$ ,  $F_4$ ,  $E_6$ ,  $E_7$ ,  $E_8$ 

Continuous rotations are exponentiated angles x generators. Generators form a Lie Algebra, e.g. SU(N) has N<sup>2</sup>-1 generators.

Generators are in 1:1 correspondence with the gauge fields in a Yang-Mills threory.

#### Electroweak Theory: SU(2) X U(1) Yang-Mills Gauge Theory

two gauge coupling constants,  $g_2$  and  $g_1$ 

$$iD_{\mu} = i\partial_{\mu} - g_2 W^a_{\mu} Q^a - g_1 B_{\mu} \frac{Y}{2}$$

$$= i\partial_{\mu} - g_2 W^{+}_{\mu} Q^{-} - g_2 W^{-}_{\mu} Q^{+} - g_2 W^{3}_{\mu} Q^{3} - g_1 B_{\mu} \frac{Y}{2}$$

$$Q^{\pm} = (Q^1 \pm i Q^2) / \sqrt{2} \qquad \qquad W^{\pm}_{\mu} = (W^1_{\mu} \pm i W^2_{\mu}) / \sqrt{2}$$

 $Q^a$  are the SU(2) weak charges Y is the U(1) hypercharge

$$[Q^a, Q^b] = i\epsilon^{abc}Q^c$$
.  $\langle \Box \rangle$  SU(2) Lie Algebra

particular *representations* for the charges,

left-handed fermions, and Higgs boson,  $Q^a = \tau^a/2$ 

right-handed fermions are singlets, annihilated by the  $Q^a$ .

abstract operator Y  $\triangleleft$  eigenvalue,  $Y_r$ 

$$Q_{EM} = Q^3 + \frac{Y}{2}$$

$$\begin{pmatrix} u^{2/3} \\ d^{-1/3} \end{pmatrix}_{L}, Y_{r} = \frac{1}{3}; \qquad \begin{pmatrix} \nu^{0} \\ e^{-1} \end{pmatrix}_{L}, Y_{r} = -1; \qquad H = \begin{pmatrix} \phi^{0} \\ \phi^{-} \end{pmatrix} \qquad Y_{r} = -1$$

$$u_R^{2/3}$$
,  $Y_r = \frac{4}{3}$ ;  $d_R^{-1/3}$ ,  $Y_r = -\frac{2}{3}$ ;  $e_R^{-1}$ ,  $Y_r = -2$ ;  $\nu_R^0$ ,  $Y_r = 0$ .

A right-handed neutrino is sterile

$$W_{\mu}^{3} = Z_{\mu}^{0} \cos \theta + A_{\mu} \sin \theta$$

$$B_{\mu} = -Z_{\mu}^{0} \sin \theta + A_{\mu} \cos \theta$$

$$= i\partial_{\mu} - g_{2}W_{\mu}^{+}Q^{-} - g_{2}W_{\mu}^{-}Q^{+} - g_{2}W_{\mu}^{3}Q^{3} - g_{1}B_{\mu}\frac{Y}{2}$$

$$Q_{EM} = Q^{3} + \frac{Y}{2}$$

$$g_{2} \sin \theta = e; \qquad g_{1} \cos \theta = e; \qquad \tan \theta = \frac{g_{1}}{g_{2}}$$

The photon thus couples to  $eQ_{EM}$  with strength e where:

$$\frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1}{g_1^2}$$

The gauge covariant field strengths are defined by commutators of the covariant derivative:

$$F^{a}_{\mu\nu} = -\frac{i}{g_{2}^{2}} \operatorname{Tr} \left(\tau^{a} [D_{\mu}, D_{\nu}]\right) = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} + \epsilon^{abc} A^{b}_{\mu} A^{c}_{\nu}$$
$$F_{\mu\nu} = -\frac{i}{g_{1}^{2}} Y^{-1}_{r} \operatorname{Tr} \left( [D_{\mu}, D_{\nu}] \right) = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}$$

Then the gauge field kinetic terms are:

$$\mathcal{L}_{G.B.\ kinetic} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a}_{\mu\nu} - \frac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

### Spontaneous Symmetry Breaking

Consider the complex doublet scalar Higgs-boson with  $Y_r = -1$ :

$$H = \left( \begin{array}{c} \phi^0 \\ \phi^- \end{array} \right)$$



$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) - V(H)$$



$$V(H) = \frac{\lambda}{2} (H^{\dagger}H - v_{weak}^2)^2$$

#### Standard Model Symmetry Breaking

$$\langle H \rangle = \begin{pmatrix} v_{weak} \\ 0 \end{pmatrix} \qquad \qquad H = \exp\left(i\frac{\pi^a\tau^a}{\sqrt{2}v_{weak}}\right) \begin{pmatrix} v_{weak} + \frac{h}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$v_{weak} = 175 \text{ GeV}$$



Nambu-Goldstone Bosons are eaten

#### Massless W, Z



#### Higgs Boson Condenses in vacuum



A massive gauge boson oscillates through spacetime, colliding with the Higgs condensate and mixing with phase degrees of freedom:



#### Couple fermions to the Higgs





#### Searching for the Higgs (Vacuum Electroweak Superconductivity)



 $114 \text{ GeV} \le m_{\text{H}} \le 260 \text{ GeV}$ 

#### Mass Eigenstates

$$A_{\mu} = \sin \theta W_{\mu}^{3} + \cos \theta B_{\mu}$$
$$Z_{\mu} = \cos \theta W_{\mu}^{3} - \sin \theta B_{\mu} = \frac{(g_{2}W_{\mu}^{3} - g_{1}B_{\mu})}{\sqrt{g_{1}^{2} + g_{2}^{2}}}$$

$$iD_{\mu} = i\partial_{\mu} - g_2 W^{+}_{\mu} Q^{-} - g_2 W^{-}_{\mu} Q^{+} - eA_{\mu} \left(Q^3 + \frac{Y}{2}\right) - Z_{\mu} \tilde{Q}$$

$$\begin{split} \tilde{Q} &= \sqrt{g_1^2 + g_2^2} \left( \cos^2 \theta \; \frac{\tau^3}{2} - \sin^2 \theta \; \frac{Y}{2} \right) \\ &= e \left( \cot \theta \; \frac{\tau^3}{2} - \tan \theta \; \frac{Y}{2} \right) \end{split}$$

#### Mass Eigenstates

$$\begin{aligned} \mathcal{L}_{H} &= \frac{1}{2} (\partial h)^{2} + \frac{1}{2} M_{W}^{2} W_{\mu}^{+} W^{\mu-} + \frac{1}{2} M_{Z}^{2} Z_{\mu} Z^{\mu} - \frac{1}{2} M_{H}^{2} h^{2} - \frac{\sqrt{\lambda}}{2} M_{H} h^{3} - \frac{1}{8} \lambda h^{4} \\ &+ \frac{1}{2} \left( h^{2} + \frac{M_{H}}{\lambda} h \right) \left( g_{2}^{2} W_{\mu}^{+} W^{\mu-} + \left( g_{1}^{2} + g_{2}^{2} \right) Z_{\mu} Z^{\mu} \right) \end{aligned}$$

$$M_W^2 = \frac{1}{2}g_2^2 v_{weak}^2; \qquad M_Z^2 = \frac{1}{2}v_{weak}^2(g_1^2 + g_2^2).$$

$$\frac{M_W^2}{M_Z^2} = \cos^2 \theta_W \qquad \qquad v_{weak}^2 = \frac{1}{2\sqrt{2}G_F}$$

#### Fermions

$$\Psi_L = (t, b)_L$$
  $L = \frac{1}{2}(1 - \gamma^5)$ 

$$\begin{split} \bar{\Psi}_L i \not{\!D} \Psi_L &= \bar{\Psi}_L i \not{\!\partial} \Psi_L - \frac{1}{\sqrt{2}} \bar{t} \gamma_\mu L b W^{\mu \, +} - \frac{1}{\sqrt{2}} \bar{b} \gamma_\mu L t W^{\mu \, -} \\ &- \frac{2e}{3} \bar{t} \gamma_\mu L t A_\mu + \frac{e}{3} \bar{b} L b A_\mu - \bar{\Psi}_L \widetilde{Q} \gamma_\mu \Psi_L Z_\mu \end{split}$$

Dirac mass terms would be of the form  $\overline{\Psi}_L \psi_R$  isospin $-\frac{1}{2}$ 

$$g_t \bar{\Psi}_L \cdot H t_R + g_b \bar{\Psi}_L \cdot H^c b_R$$

$$m_t = g_t v_{weak}$$

e.g., Top and Bottom

 $m_b = g_b v_{weak}$ 



leptonic doublet  $\Psi_L = (\nu, \ell)_L$ 

$$\frac{g_{\nu}}{M}(\bar{\Psi}_L H)(H^C \Psi_L^C)$$

Majorana mass for 
$$\nu_L = g_{\nu} v_{weak}^2 / M$$

The interplay between gauge symmetries, and chiral symmetries, both of which are broken spontaneously, is fundamental to the Standard Model. The left-handed fermions carry the electroweak SU(2) quantum numbers, while the right-handed do not. All of the mathematical features of the symmetric Lagrangian remain intact, but the spectrum of the theory does not retain the original obvious symmetry properties. When a massive gauge boson was discovered, such as the  $W^{\pm}$  or Z of the Standard Model, we also discovered an extra piece of physics: the longitudinal component, i.e., the NGB which comes from the symmetry breaking sector. I apologize for not giving a review of CKM physics, rare electroweak processes, CP-violation and all that.

One Lecture on the Standard Model or Five Lectures is sufficient; Two is not.

#### Lightning Review of Radiative Corrections to Standard Model

generalize slightly our definition of  $v_{weak}$ 

$$M_W^2 = \frac{1}{2} v_W^2 g_2^2; \qquad M_Z^2 = \frac{1}{2} v_Z^2 (g_1^2 + g_2^2)$$

 $v_W(v_Z)$  is the Higgs VEV "as seen by" the W-boson (Z=boson)

tree level 
$$v_W^2 = v_Z^2 = v_{weak}^2$$

radiative corrections  $g_1^2(q^2), g_2^2(q^2) = v_W^2(q^2)$  and  $v_Z^2(q^2)$ .

to a good approximation, we really only need to know  $\alpha(M_Z) \approx \alpha(M_W)$ 

$$F.T. < 0|T\tilde{A}^{A}_{\mu}(0)\tilde{A}^{B}_{\mu}(x)|0> = g_{\mu\nu}\Pi_{AB} - q_{\mu}q_{\nu}\Pi^{T}_{AB}$$

$$\frac{1}{2}v_Z^2 = \frac{1}{2}v_{weak}^2 - \Pi_{3B}$$
$$= \frac{1}{2}v_{weak}^2 - \Pi_{3Q} + \Pi_{33}$$
$$\frac{1}{2}v_W^2 = \frac{1}{2}v_{weak}^2 + \Pi_{WW} - \Pi_{3Q}$$

$$\begin{aligned} \frac{1}{g_2^2} &= \frac{1}{g_{2un}^2} - \Pi_{33}^T - \Pi_{3B}^T \\ &= \frac{1}{g_{2un}^2} - \Pi_{3Q}^T \\ \frac{1}{g_1^2} &= \frac{1}{g_{1un}^2} - \Pi_{BB}^T - \Pi_{3B}^T \\ &= \frac{1}{g_{1un}^2} + \Pi_{3Q}^T - \Pi_{QQ}^T \end{aligned}$$

-1

$$\begin{aligned} v_W^2 - v_Z^2 &= \frac{N_c}{32\pi^2} \left[ (m_t^2 + m_b^2) - \frac{2m_t^2 m_b^2}{(m_t^2 - m_b^2)} \log(m_t^2/m_b^2) \right. \\ &+ \frac{M_W^2 m_H^2}{m_H^2 - M_W^2} \ln(m_H^2/M_W^2) - \frac{M_Z^2 m_H^2}{m_H^2 - M_Z^2} \ln(m_H^2/M_Z^2) \right] \end{aligned}$$

The  $\rho$  parameter of Veltman is:

$$\begin{split} \rho &= \frac{v_W^2}{v_Z^2} \\ &= 1 + \frac{N_c}{32v_0^2\pi^2} \left[ (m_t^2 + m_b^2) - \frac{2m_t^2m_b^2}{(m_t^2 - m_b^2)} \log(m_t^2/m_b^2) \right. \\ &+ \frac{M_W^2m_H^2}{m_H^2 - M_W^2} \ln(m_H^2/M_W^2) - \frac{M_Z^2m_H^2}{m_H^2 - M_Z^2} \ln(m_H^2/M_Z^2) \right] \end{split}$$

A simple set of parameters

$$\begin{aligned} v_W^2(q^2) &= v_{weak}^2 + \sigma q^2 + \tau v_r^2 + \omega q^2 \\ v_Z^2(q^2) &= v_{weak}^2 + \sigma q^2 - \tau v_r^2 - \omega q^2 \end{aligned}$$

Peskin and Takeuchi define:

$$S = 16\pi \left[ \frac{\partial}{\partial q^2} \Pi_{33} |_{q^2=0} - \frac{\partial}{\partial q^2} \Pi_{3Q} |_{q^2=0} \right]$$
$$T = \frac{4\pi}{\sin^2 \theta \cos^2 \theta M_Z^2} \left[ \Pi_{WW} |_{q^2=0} - \Pi_{33} |_{q^2=0} \right]$$
$$U = 16\pi \left[ \frac{\partial}{\partial q^2} \Pi_{WW} |_{q^2=0} - \frac{\partial}{\partial q^2} \Pi_{33} |_{q^2=0} \right]$$

we see that:

$$\sigma = \frac{2S+U}{16\pi}; \qquad \omega = \frac{U}{16\pi}; \qquad \tau = \frac{\sin^2 \theta \cos^2 \theta M_Z^2 T}{4\pi v_r^2} = \frac{\alpha}{2}T;$$

input parameters:

$$G_F = 1.16639 \ (1) \ \times 10^{-5} \ {\rm GeV^{-2}}$$
  
 $m_Z^2 = 8315.18(38) \ {\rm GeV^{-2}}$   
 $\alpha^{-1}(m_Z) = :$ 

$$\alpha^{-1}(m_Z) = \alpha^{-1}(0) \left[ 1 - \Delta \alpha_\ell - \Delta \alpha_h - \Delta \alpha_{t,W} \right]$$
  
0.03150 ? < 0.0001

$\Delta \alpha_h = 0.02761 \ (36)$	Burkhardt–Pietrzyk $(01)$
= 0.02757 (36)	Jegerlehner $(03)$
= 0.02737 (20)	Jegerlehner (Euclidean) $(03)$
= 0.02763 (16)	Davier–Höcker $(\tau, \text{QCD})$ (98)

$$G_F = 1/2\sqrt{2}v_{weak}(0)^2$$
$$M_Z^2 = \frac{1}{2}(g_1^2(M_Z^2) + g_2^2(M_Z^2))v_{weak}^2(M_Z^2)$$
$$4\pi\alpha(0) = \frac{g_1^2g_2^2}{g_1^2 + g_2^2}\Big|_{\mu^2 = 0}$$

$$\begin{aligned} \sin^2 \theta_{Z-pole} &= g_1(M_Z^2) / (g_1(M_Z^2) + g_2(M_Z^2)) \\ M_W^2 &= \frac{1}{2} g_2^2(M_W^2) v_{weak}^2(M_W^2) \\ T &= \frac{N_c}{4\pi \sin^2 \theta \cos^2 \theta M_Z^2} \left[ (m_t^2 + m_b^2) - \frac{2m_t^2 m_b^2}{(m_t^2 - m_b^2)} \log(m_t^2/m_b^2) \right. \\ &+ \frac{M_W^2 m_H^2}{m_H^2 - M_W^2} \ln(m_H^2/M_W^2) - \frac{M_Z^2 m_H^2}{m_H^2 - M_Z^2} \ln(m_H^2/M_Z^2) \right] \\ S &= \frac{N_c}{6\pi} \left[ 1 - Y \log \left\{ m_b^2/m_t^2 \right\} \right] \end{aligned}$$



#### **Application: Top Seesaw Model**

Mass matrix for  $t - \chi$  system is,

$$-\left(\overline{t_L} \ \overline{\chi_L}\right) \left(\begin{array}{cc} 0 & \mu \approx 600 \ GeV \\ m \approx 1 \ TeV & M \approx 4 \ TeV \end{array}\right) \left(\begin{array}{c} t_R \\ \chi_R \end{array}\right)$$

Diagonalized:

$$m_t \approx \frac{\mu m}{M}$$
  
 $m_\chi \approx M$ 

1998: Top Seesaw DOA (outside of the S-T ellipse  $\sim 4\sigma$ . (Chivukula, Dobrescu, Georgi, Hill)

1999: S-T error ellipse shifts along major axis towa upper right (predicted by the theory!).

2001: Inconsistencies in data; keep only leptons  $\rightarrow$  Top Seesaw consistent and SM ruled out at  $\sim 2\sigma!!!$ 

Theory consistent for natural values of its parameters at the  $2\sigma$  level (He, CTH, Tait)



What is the Higgs Boson?

The mysterious role of Scale Symmetry

- We live in 1+3 dimensions
- The big cosmological constant conundrum
- The Higgs Boson mass scale
- QCD solves its own problem of hierarchy
- New Strong Dynamics?

Origin of Mass in QCD

$$\frac{dg}{d\ln\mu} = \beta(g)$$

Gell-Mann and Low:

Gross, Politzer and Wilczek:  

$$\beta(g) = \hbar \beta_0 g^3 \quad \text{where} \quad \beta_0 = -\frac{1}{16\pi^2} \left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)$$

$$\frac{\Lambda_{QCD}}{M_0} = \exp\left(\frac{1}{2\hbar\beta_0 g_0^2}\right)$$



#### A Puzzle: Murray Gell-Mann lecture ca 1975

$$S_{\mu} = x^{\nu} T_{\mu\nu}$$

$$\partial_{\mu}S^{\mu} = T^{\mu}_{\mu}$$

$$T_{\mu\nu} = \operatorname{Tr}(G_{\mu\rho}G^{\rho}_{\nu}) - \frac{1}{4}g_{\mu\nu}\operatorname{Tr}(G_{\rho\sigma}G^{\rho\sigma})$$

$$\partial_{\mu}S^{\mu} = T^{\mu}_{\mu} = \operatorname{Tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{4}{4}\operatorname{Tr}(G_{\mu\nu}G^{\mu\nu}) = 0$$
 !???

QCD is scale invariant!!!???

## Resolution: The Scale Anomaly

$$\partial_{\mu}S^{\mu} = \frac{\beta(g)}{g} \operatorname{Tr} G_{\mu\nu}G^{\mu\nu} = \mathcal{O}(\hbar)$$

Origin of Mass in QCD = Quantum Mechanics

## A heretical Conjecture:

All mass scales in physics are intrinsically quantum mechanical and associated with scale anomalies. The  $\hbar \rightarrow 0$  limit of nature is exactly scale invariant.



#### "Predictions" of the Conjecture:

We live in D=4! 
$$T^{\mu}_{\mu} = \operatorname{Tr} G_{\mu\nu} G^{\mu\nu} - \frac{D}{4} \operatorname{Tr} G_{\mu\nu} G^{\mu\nu}$$

Cosmological constant is zero in classical limit

QCD scale is generated in this way; Hierarchy is naturally generated

Testable in the Weak Interactions?

Weyl Gravity in D=4 is QCD-like:

Is the Higgs technically natural?

 $\frac{1}{h^2}\sqrt{-g}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2)$ 

On naturalness in the standard model. <u>William A. Bardeen (Fermilab)</u>. FERMILAB-CONF-95-391-T, Aug 1995. 5pp.

Conjecture on the physical implications of the scale anomaly. Christopher T. Hill (Fermilab) . hep-th/0510177



#### Symmetry Principles Define Modern Physics







Beauty

Physics