Lectures on Event Generators

Stephen Mrenna

Computing Division
Generators and Detector Simulation Group
Fermilab

CTEQSS 06/04–05/07
Missing Energy Events are Confirmed

1) Events are real

2) Not conventional physics
   \[ W \rightarrow \tau, \, Q\bar{Q}, \, Z^0 \rightarrow \nu\bar{\nu} \]

3) Not \[ X \rightarrow Z^0 + \text{jet}(s) \]
   \[ \text{L} \rightarrow \nu\bar{\nu} \]

4) Not \[ Z^0 \rightarrow X_1X_2 \]
   \[ \text{L} \rightarrow \nu's \text{ or stable jet(s)} \]

[ORIGINAL TRANSPARENCY FROM 1986 UA1 'DISCOVERY' OF SUSY
ASPER CONF, 1986]
Table 1. Predicted rates for processes giving large missing transverse energy events passing all event selection cuts.

<table>
<thead>
<tr>
<th>Process</th>
<th>Events (total)</th>
<th>Events with $L_\tau &lt; 0$</th>
<th>Events with $L_\tau &lt; 0$ and $E_\tau^{jet} &lt; 40$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W \rightarrow e\nu$</td>
<td>3.6</td>
<td>2.0</td>
<td>1.4</td>
</tr>
<tr>
<td>$W \rightarrow \mu\nu$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu \rightarrow \nu\bar{\nu}$ + hadrons</td>
<td>36.7</td>
<td>8.0</td>
<td>7.1</td>
</tr>
<tr>
<td>$W \rightarrow c\bar{s}$</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>$Z^0 \rightarrow \tau^+\tau^-$</td>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$Z^0 \rightarrow \nu\bar{\nu}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3 neutrino species)</td>
<td>7.4</td>
<td>7.1</td>
<td>5.6</td>
</tr>
<tr>
<td>$Z^0 \rightarrow c\bar{c}$ and $b\bar{b}$</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>$c\bar{c}$ and $b\bar{b}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(direct production)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Jet fluctuations (fake missing energy)</td>
<td>3.8</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>TOTAL</td>
<td>52.2</td>
<td>20.8 ± 5.1 ± 1.0</td>
<td>17.8 ± 3.7 ± 1.0</td>
</tr>
</tbody>
</table>
Many “negligible” sources of background summed up to explain the data.

The mixing of Standard Model cocktails has become an important component of analyzing collider data:
- relies on a mixture of physics tools and measurements
- event generators are indispensable in this process

These lectures are focused on preparing you to do the same at the energy frontier.
How much does the $t\bar{t}$ cross section change from TeV to LHC?

- $10 \times$
- $100 \times$
- $500 \times$

[Kidonakis]
Warm-Up

How much does the $t\bar{t}$ cross section change from TeV to LHC?

- $10 \times$
- $100 \times$ ✓
- $500 \times$

[Kidonakis]

$q\bar{q} \rightarrow t\bar{t}$ vs $gg \rightarrow t\bar{t}$
Warm-Up

How much does the $\widetilde{\chi}^+ \widetilde{\chi}^- (m_\chi = 200 \text{ GeV})$ cross section change from TeV to LHC?

- $10 \times$
- $100 \times$
- $500 \times$

[Pythia]
Warm-Up

How much does the $\tilde{\chi}^+\tilde{\chi}^- (m_\chi = 200$ GeV) cross section change from TeV to LHC?

- 10× ✓
- 100×
- 500×

[Pythia]

No corresponding $gg$ process at LO
Warm-Up

How much does the $W^{jjjj}$ cross section change from TeV to LHC?

- 10×
- 100×
- 500×

[MadEvent parton level, $p_T, k_T > 20$ GeV]
How much does the \( W_{jjjj} \) cross section change from TeV to LHC?

- 10\( \times \)
- 100\( \times \)
- 500\( \times \) \( \checkmark \)

[MadEvent parton level, \( p_T, k_T > 20 \) GeV]

Many new topologies, lots of phase space
First Steps

- LHC phenomenology begins with re-orienting our Standard Model compass
  - recalibrating our Standard Model tools
- Understanding of the Standard Model relies on Event Generators
Event Generators

Predict multiparticle event configurations in HEP experiments

- \( P(x) \Rightarrow N \) performed using Monte Carlo methods
  - Estimate physical quantities (the total cross section)
  - Sample quantities (generate events) one at a time
- Relies on ability to generate (pseudo) random numbers
Lecture 1

- Defining Event Generators
  - Modularity of HEP Events

- Monte Carlo Techniques
  - Calculating Integrals
  - Sampling Distributions

- Matrix Element Calculations
  - Applications
  - Limitations

- Parton Shower
  - Sudakov Form Factor
  - Coherence
  - Dipoles

- Summary
Phases of High Energy Collisions

- **hard scattering**
- **initial/final state radiation**
- **partonic decays,** \( t \rightarrow bW \)
- **parton shower evolution**

- **nonperturbative phase**
- **colorless clusters**
- **cluster \( \rightarrow \) hadrons**
- **hadronic decays**
- **backward parton evolution**
- **underlying event**
Monte Carlo

What is it?

Numerical method for estimating integrals based on “random” evaluations of the integrand

Why do we use it?

- Large dimension of integration variables
- Limits of integration are complicated
- Integrand is a convolution of several functions
Some people use *Monte Carlo* to refer to event generators, because they exploit Monte Carlo methods. However, these days, NLO calculations often use the same methods. I will try to use *Monte Carlo* as a method, not a program.
Mean Value Theorem for Integration

\[ I = \int_{x_1}^{x_2} dx \, f(x) = (x_2 - x_1)\langle f(x) \rangle \quad \left\{ \langle \mathcal{O} \rangle = \int dx \, \frac{d\mathcal{O}}{dx} \right\} \]

\[ \simeq I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^{N} f(x_i) \]

\[ \simeq I_N \pm (x_2 - x_1) \sqrt{\frac{\langle f^2 \rangle - \langle f \rangle^2}{N}} \]

Randomly select \( N \) values of \( x_i \), evaluate \( f(x_i) \), and average
Non-uniform sampling can be more efficient:

\[
\int_{x_1}^{x_2} dx \ p(x) = 1 \Rightarrow I = \int_{x_1}^{x_2} \{ dx \ p(x) \} \frac{f(x)}{p(x)}
\]

\[
I = \left\langle \frac{f(x)}{p(x)} \right\rangle \pm \frac{1}{\sqrt{N}} \sqrt{\left( \left\langle \frac{f(x)^2}{p(x)^2} \right\rangle - \left\langle \frac{f(x)}{p(x)} \right\rangle^2 \right)}
\]

Sample according to \( p(x) \) and make \( f/p \) as flat as possible (reduce variance)

if \( f(x) \sim \frac{1}{x} \Rightarrow \text{sample according to} \ \frac{dx}{x} = d \ln(x) \)
- **Importance sampling:** choose $x_N$ based on prior knowledge of $I_{N-1}$
- **VEGAS** is an adaptive integrator that adjusts step functions to mimic integrand
- **VEGAS** is trying to find $p(x)$ (from previous example) numerically
- Over 30 years old, but still the primary engine in HEP
Vegas in Many Dimensions

(a) Vegas likes this function: it is aligned with the axes

(b) Vegas dislikes this function: but a transformation will align it with the axes

Need to input some information about the behavior of the integrand. For physical processes, you often will know singular behavior.
Multi-Channel Integration

- Full integrand is horrendous
- Consider as sum of several channels
- \( p(x) = \alpha_1 p_1(x) + (1 - \alpha_1) p_2(x) \)
Monte Carlo for Sampling distributions

- Up to this point, only considered MC as a numerical integration method
- If function being integrated is a probability density (positive definite), can convert it to a simulation of physical process = an event generator
- Monte Carlo can explore possible histories when there are many degrees of freedom
- Events selected with same frequency as in nature
Given \( f(x) > 0 \) over \( x_{\text{min}} \leq x \leq x_{\text{max}} \)

Prob in \((x + dx, x)\) is \( f(x)dx \)

\[
\int_{x_{\text{min}}}^{x} f(x) \, dx = R \int_{x_{\text{min}}}^{x_{\text{max}}} f(x) \, dx
\]

\( x = F^{-1}(F(x_{\text{min}}) + R(F(x_{\text{max}}) - F(x_{\text{min}}))) \)

- assumes \( F(x), F^{-1}(x) \) are known
- fraction \( R \) of area under \( f(x) \) should be to the left of \( x \)
- *Realistic \( f(x) \) are rarely this nice*
If \( \text{max}[f(x)] \) is known, but not \( F^{-1}(x) \), use hit-or-miss

1. select \( x = x_{\text{min}} + R(x_{\text{max}} - x_{\text{min}}) \)
2. if \( f(x)/f_{\text{max}} \leq \text{(new)} R \), reject \( x \) and \( \Rightarrow 1 \)
3. otherwise, keep \( x \)

- Works because probability \( f(x)/f_{\text{max}} > R \propto f(x) \)
- Acceptable method if \( f(x) \) does not fluctuate too wildly
- Often guess at \( \text{max}[f(x)] \) and update if a larger estimate is found in a run
\( f(x) \) is complicated

Find \( g(x) \), with \( f(x) \leq g(x) \) over \( x \) range

- \( G(x) \) and its inverse \( G^{-1}(x) \) known
- e.g., \( \int_{\epsilon}^{z} dx \frac{1 + x^2}{1 - x} < \int_{\epsilon}^{z} dx \frac{2}{1 - x} = 2 \ln \left[ \frac{1 - \epsilon}{1 - z} \right] \)

1. select an \( x \) according to \( g(x) \), using Method 1
2. if \( f(x)/g(x) \leq \text{(new)} \ R \), reject \( x \) and \( \Rightarrow \) 1
3. otherwise, keep \( x \)

- first step selects \( x \) with a probability \( g(x) \)
- second step retains this choice with probability \( f(x)/g(x) \)
- total probability to pick a value \( x \) is then just the product of the two, i.e. \( f(x) \, dx \)
Radioactive Decay Problem

- Know probability $f(t)$ that ‘something will happen’ (a nucleus decay, a parton branch, a transistor fail) at time $t$
- *something happens at $t$ only if it did not happen at $t' < t$*

Equation for nothing $\mathcal{N}(t)$ to happen *up to time* $t$ is ($\mathcal{N}(0) = 1$):

$$-\frac{d\mathcal{N}}{dt} = f(t)\mathcal{N}(t) = \mathcal{P}(t)$$

$$\mathcal{N}(t) = \exp\left\{-\int_0^t f(t')\,dt'\right\}$$

$$\mathcal{P}(t) = f(t)\exp\left\{-\int_0^t f(t')\,dt'\right\}$$

- Naive answer $\mathcal{P}(t) = f(t)$ modified by exponential suppression
- In the parton shower, this is the *Sudakov form factor*
If $F(t)$ and $F^{-1}(t)$ exist:

\[
\int_0^t \mathcal{P}(t') \, dt' = \mathcal{N}(0) - \mathcal{N}(t) = 1 - \exp \left\{ - \int_0^t f(t') \, dt' \right\} = 1 - R
\]

\[
F(0) - F(t) = \ln R \quad \Rightarrow \quad t = F^{-1}(F(0) - \ln R)
\]

If not, use **veto algorithm** with a “nice” $g(t)$

1. start with $i = 0$ and $t_0 = 0$
2. increment $i$ and select $t_i = G^{-1}(G(t_{i-1}) - \ln R)$
3. if $f(t_i)/g(t_i) \leq \text{(new)} R$, \(\Rightarrow\) 2
4. otherwise, keep $t_i$
Unweighted Event Example

- I have 3 samples of MC events corresponding to different processes.
- Each individual sample has a uniform weight (they have been unweighted).
- How do I select $N$ (uniform weight) events for my cocktail?

<table>
<thead>
<tr>
<th>Sample</th>
<th>Events</th>
<th>$\sigma$ (pb)</th>
<th>Weight (pb/evt)</th>
<th>Hit-or-Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100k</td>
<td>100</td>
<td>$10^{-3}$</td>
<td>100k</td>
</tr>
<tr>
<td>2</td>
<td>300k</td>
<td>60</td>
<td>$.2 \times 10^{-3}$</td>
<td>60k</td>
</tr>
<tr>
<td>3</td>
<td>160k</td>
<td>40</td>
<td>$.25 \times 10^{-3}$</td>
<td>40k</td>
</tr>
<tr>
<td>Total</td>
<td>200</td>
<td></td>
<td>$200k$</td>
<td></td>
</tr>
</tbody>
</table>

- Select $N$ of these 200k randomly
- Note: the sample with highest weight/evt dominates
Use MC to perform integrals and sample distributions
  - Only need a few points to estimate $f$
  - Each additional point increases accuracy

Technique generalizes to many dimensions
  - Typical LHC phase space $\sim d^3\vec{p} \times 100\text{’s particles}$
  - Error scales as $1/\sqrt{N}$ vs $1/N^{2/d}$, $1/N^{4/d}$ (trap, Simp)

Suitable for complicated integration regions
  - Kinematic cuts or detector cracks

Can sample distributions where exact solutions cannot be found

Veto algorithm applied to parton shower
Hard Scattering

- Characterizes the rest of the event
- Sets a high energy scale $Q$
- Fixes a short time scale where partons are free objects
- Allows use of perturbation theory (focus on QCD)
- External partons can be treated as on the mass-shell
  - Valid to $\max[\Lambda, m]/Q$
  - Physics at scales below $Q$ absorbed into parton distribution and fragmentation functions (Factorization Theorem)
- Sets flow of Quantum numbers (Charge, Color)
  - Parton shower and hadronization models use $1/N_C$ expansion
  - Gluon replaced by color-anticolor lines
  - All color flows can be drawn on a piece of paper
Details of how to calculate in fixed-order perturbation theory have been provided by the other (expert) lecturers.

For the most part, event generators use lowest-order, hard-scattering calculations as their starting point.

When more detailed, tree-level calculations are performed, some care must be taken when adding on parton showers (later).
**NOT event generators**

- partonic jets: no substructure
- hard, wide-angle emissions only
- colored/fractionally charged states not suitable for detector simulation

Nonetheless, quite useful:

- can guide physics analyses by revealing gross kinematic features
  - Jacobian peak
- can estimate effect of higher-order corrections
- can modify the Lagrangian to implement new models
HEP Events are approximately modular:

- Events are transformations from $t = -\infty \rightarrow t = +\infty$
- Hard Interaction occurs over a short time scale
  \[ \Delta t \sim 10^{-2}\text{GeV}^{-1} \]
- Perturbation theory ($\alpha_s < \pi$) should work down to time
  \[ t = .1 - 1\text{GeV}^{-1} \]
- Hadronization on longer time scales
- Particle decays typically on longest time scales

Separation of time scales reduces the complex problem to manageable pieces (modules) which can be treated in series

- Previous step sets initial conditions for next one

Next step after hard scatter is the parton shower
Matrix Element to Parton Shower: $\gamma^* \rightarrow q\bar{q}g$

Write (cleverly) single gluon emission:

$$d\sigma(q\bar{q}g) = \sigma_0 \frac{\alpha_s}{2\pi} dz \left\{ \frac{ds_{qg}}{s_{qg}} \left[ P_{q\rightarrow q}(z) - \frac{s_{qg}}{Q^2} \right] + \frac{ds_{\bar{q}g}}{s_{\bar{q}g}} \left[ P_{q\rightarrow q}(z) - \frac{s_{\bar{q}g}}{Q^2} \right] \right\}$$

- $\sigma_0 = \sigma(\gamma^* \rightarrow q\bar{q})$
- $z = \frac{s_{q\bar{q}}}{Q^2}, P_{q\rightarrow q}(z) = \frac{4}{3} \frac{1 + z^2}{1 - z}$
- $s_{qg} = 2E_qE_g(1 - \cos \theta_{qg})$
- $s_{qg}, s_{\bar{q}g} \rightarrow 0$ when gluon is soft/collinear
- $z \rightarrow 1$ when gluon is soft ($E_g = (1 - z)E_{\text{mother}}$)
- In soft/collinear limit, independent radiation from $q$ and $\bar{q}$
|\mathcal{M}|^2 \text{ involving } q \rightarrow qg \text{ (or } g \rightarrow gg) \text{ strongly enhanced whenever emitted gluon is almost collinear.}

- Propagator factors (internal lines)

\[
\frac{1}{(p_q + p_g)^2} \approx \frac{1}{2E_q E_g (1 - \cos \theta_{qg})} \rightarrow \frac{1}{E_q E_g \theta^2_{qg}}
\]

- soft \( E_g \rightarrow 0 \) +collinear \( \theta_{qg} \rightarrow 0 \) divergences

- dominant contribution to the ME
  - the divergence can overcome smallness of \( \alpha_s \)
  - expansion parameter must be redefined
Collinear factorization

\[ |M_{p+1}|^2 d\Phi_{p+1} \approx |M_p|^2 d\Phi_p \frac{dQ^2}{Q^2} \frac{\alpha_s}{2\pi} P(z) dz d\phi \]

- DGLAP kernels:
  \[ P_{q\rightarrow q}(z) = C_F \frac{1 + z^2}{1 - z}, \quad P_{g\rightarrow g}(z) = N_C \frac{(1 - z(1 - z))^2}{z(1 - z)} \]

- Note the appearance of \( d \ln(Q^2) \alpha_s \sim \frac{d \ln(Q^2)}{\ln(Q^2)} \)

- The consideration of successive collinear emissions leads to the parton shower picture
Sudakov Form Factor

Variable $t = \ln\left(\frac{Q^2}{\Lambda^2}\right)$, $Q^2 \sim E_q E_g / \theta_{qg}^2$ is like a time-ordering

$$dP_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\rightarrow bc}(z) \, dt \, dz$$

$$\mathcal{I}_{a\rightarrow bc}(t) = \int_{z_-(t)}^{z_+(t)} dz \frac{\alpha_{abc}}{2\pi} P_{a\rightarrow bc}(z)$$

Probability for no emission in $(t, t + \delta t)$: $1 - \sum_{b,c} \mathcal{I}_{a\rightarrow bc}(t) \, \delta t$

Over a longer time period, product of no-emission prob's exponentiates:

$$P_{no}(t_0, t) = \exp \left\{ - \int_{t_0}^{t} dt' \sum_{b,c} \mathcal{I}_{a\rightarrow bc}(t') \right\} = S_a(t) = \frac{\Delta(t, t_c)}{\Delta(t_0, t_c)}$$
\[ P_{\text{no}}(t_0, t) = \exp \left\{ - \int_{t_0}^{t} \sum_{b, c} I_{a \rightarrow bc}(t') \right\} = S_a(t) = \frac{\Delta(t, t_c)}{\Delta(t_0, t_c)} \]

Notation: \( S_a(t) \) for Pythia, \( \Delta(t, t_c) \) for Herwig

- The exponentiation of emissions is common to resummation calculations
  - Arises when there are two very different scales in the problem (i.e. the scale of the hard collision vs. the scale of soft/collinear emissions)
- The parton shower includes the probability for many soft and collinear gluons to emitted
Actual probability that a branching of $a$ occurs at $t$ is:

$$\frac{dP_a}{dt} = -\frac{dP_{no}(t_0, t)}{dt} = \left( \sum_{b,c} I_{a\rightarrow bc}(t) \right) \exp \left\{ -\int_{t_0}^{t} dt' \sum_{b,c} I_{a\rightarrow bc}(t') \right\}$$

Like Radioactive Decay!

- Can be solved using veto algorithm

$S_a(t) = P_{no}(t_0, t)$ is referred to as the Sudakov form factor

- It is the prob. for nothing to happen
We can only observe emissions (red) above a certain resolution scale ($\Lambda_{QCD}$, calorimeter noise?)

Below resolution scale, singularities (blue) cancel, leaving a finite remnant

This cancellation occurs for an infinite tower of possible emissions as long as one considers the leading singularities
In analytics calculations, the tower is generalizable (NNLL, etc.)

In parton shower algorithms, a probabilistic interpretation is “easily” implementable for the leading logarithms (LL)

\[ \text{LL } \alpha_s \sim \frac{1}{\ln(Q^2)} \]

LL DGLAP kernels
Evolution of the parton shower

- Start parton shower by selecting $t$ from Sudakov FF
- Continue emissions with decreasing $t$ down to the cutoff scale $\sim \Lambda_{QCD}$

$\begin{align*}
  t_1 & > t_2 > t_3 > t_c \\
  (\text{note the ordering}) \\
  t_c & \to \Lambda_{QCD} \\
  \text{Make transition to a model of hadronization at } \Lambda_{QCD}
\end{align*}$
Movie of a Parton Shower
Movie of a Parton Shower
Movie of a Parton Shower
Movie of a Parton Shower

Stephen Mrenna  Event Generators
Movie of a Parton Shower
Movie of a Parton Shower
Movie of a Parton Shower
Movie of a Parton Shower
As this movie demonstrates, the topology generated by the parton shower can be quite complicated.

Such ‘event shapes’ are the forte of the parton shower:
- the bulk of the data cannot be described well by fixed-order calculations.

The total cross section is still given by the hard scattering calculation:
- usually LO
- experiments will often normalize to data, ignoring higher-order calculations.
Color Coherence

Up to here, interference effects between emitters were ignored

Add a soft gluon to a shower of $N$ almost collinear gluons
- incoherent emission: couple to all color
  \[ |\mathcal{M}_{N+1}|^2 \sim N \times \alpha_s \times N_C \]
- coherent emission: soft (long wavelength) resolves only overall color charge (that of initial object)
  \[ |\mathcal{M}_{N+1}|^2 \sim 1 \times \alpha_s \times N_C \]
Color Coherence as Angular Ordering

- Nature chooses coherent emissions
- Choose $Q^2 \rightarrow E^2 \zeta$
- $\zeta = \frac{p_i \cdot p_j}{E_i E_j} = (1 - \cos \theta_{ij}) \sim \theta_{ij}^2 / 2$

Soft radiation off color lines $i$, $j$

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_s}{2\pi} C_{ij} W_{ij}$$

$$W_{ij} = \frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{iq})(1 - \cos \theta_{jq})}$$

$$W_{ij} = W^{[i]} + W^{[j]}$$
Color Coherence: Derivation

\[ W_{ij}^{[i]} = \frac{1}{2} \left( W_{ij} + \frac{1}{1 - \cos \theta_{iq}} - \frac{1}{1 - \cos \theta_{jq}} \right) \]
\[ = \frac{1}{2(1 - \cos \theta_{iq})} \left( 1 + \frac{\cos \theta_{iq} - \cos \theta_{ij}}{1 - \cos \theta_{jq}} \right) \]

Average over azimuthal angle. Choose:

\[ \hat{i} = \hat{z}, \quad \hat{j} = \sin \theta_{ij} \hat{x} + \cos \theta_{ij} \hat{z} \]
\[ \hat{q} = \sin \theta_{iq} (\cos \phi_{iq} \hat{x} + \sin \phi_{iq} \hat{y}) + \cos \theta_{iq} \hat{z} \]
\[ \cos \theta_{jq} = \hat{j} \cdot \hat{q} = \sin \theta_{ij} \sin \theta_{iq} \cos \phi_{iq} + \cos \theta_{ij} \cos \theta_{iq} \]

\[ \left\langle \frac{1}{1 - \cos \theta_{jq}} \right\rangle = \frac{1}{|\cos \theta_{iq} - \cos \theta_{ij}|} \]

\[ \left\langle W_{ij}^{[i]} \right\rangle = \frac{1}{1 - \cos \theta_{iq}} \theta(\cos \theta_{iq} - \cos \theta_{ij}) \]
On average, emissions have decreasing angles w.r.t. emitters

A strict implementation of this leads to a dead-zone where no radiation occurs ($\Lambda_{QCD} \sim E_{\text{cut}} \theta_{\text{cut}}$) (Herwig)

- Can be corrected case-by-case, but is complicated

Decreasing angles can also be enforced with other evolution variables (Pythia-mass)

Another approach is to consider dipole radiation (Ariadne, Pythia-new)
Generalised Dipoles

- Color charges form dipoles, which beget other dipoles

\[ dn_{\text{dipole}} = \alpha_{\text{eff}} \frac{dk_{\perp}^2}{k_{\perp}^2} dy = \alpha_{\text{eff}} d\ln(k_{\perp}^2) dy \]

- \( E = k_{\perp} \cosh(y) \leq \frac{\sqrt{s}}{2} \) (\( \sqrt{s} \) is dipole mass)

- Rapidity range \( \Delta y \approx \ln\left(\frac{s}{k_{\perp}^2}\right) \)

The emission of the first gluon splits the original color dipole into two dipoles which radiate independently.
- emission of a photon leaves the electromagnetic current unchanged except for small recoil effects
- emission of a gluon changes the current, however:

$$dn(q, g_1, g_2, \bar{q}) = dn(q, g_1, \bar{q}) \left[ dn(q, g_2, g_1) + dn(g_1, g_2, \bar{q}) - \epsilon \right]$$

Shower can be traced in origami diagram (triangular phase space):

1. Before emission
2. 1st emission at $\kappa_1$
3. After several emissions
4. Bottom view

$$\kappa = \ln(k_T^2)$$
$p_T$ Ordered Shower

- Retain parton shower evolution
  - $g \rightarrow q\bar{q}$ is natural (not so in dipole evolution)
  - easy to generalize to initial state radiation
- Evolution variable $p_T^2 = z(1 - z)m^2$
- *Coherence* from choosing dipole frame to determine kinematics
  - Effectively, the boost from the dipole to lab frame “orders” the emissions
Neglecting Sudakovs, rate of one emission is:

\[ P_{q \rightarrow qg} \approx \int \frac{dQ^2}{Q^2} \int dz \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{1 + z^2}{1 - z} \]

\[ \approx \alpha_s \ln \left( \frac{Q_{\text{max}}^2}{Q_{\text{min}}^2} \right) \frac{8}{3} \ln \left( \frac{1 - z_{\text{min}}}{1 - z_{\text{max}}} \right) \sim \alpha_s \ln^2 \left( \frac{Q_{\text{max}}^2}{Q_{\text{min}}^2} \right) \]

Rate for \( n \) emissions is of form:

\[ P_{q \rightarrow qng} \sim (P_{q \rightarrow qg})^n \sim \alpha_s^n \ln^{2n} \]

Next-to-leading log (NLL): include \( \alpha_s^n \ln^{2n-1} \)
No completely NLL generator, but

- energy-momentum conservation (and “recoil” effects)
- coherence
- scale choice $\alpha_s(p_\perp^2)$
  - absorbs singular terms $\propto \ln z, \ln(1 - z)$ in $\mathcal{O}(\alpha_s^2)$ splitting kernels $P_{q\rightarrow qg}$ and $P_{g\rightarrow gg}$
- ...

$\Rightarrow$ far better than naive, analytical LL
So far, have considered final state radiation (FSR)
- the evolution of the fragmentation functions $D_{h/i}(z, Q^2)$

The initial state partons of a hard collision can also radiate (ISR)
- the evolution of the parton distribution functions $f_{i/h}(x, Q^2)$
Parton Distribution Functions

Hadrons are composite, with time-dependent structure:

\[ f_i(x, Q^2) = \text{number density of partons } i \]
\[ \text{at momentum fraction } x \text{ and probing scale } Q^2 \]

\[ \frac{df_b(x, Q^2)}{d(\ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} P_{a \rightarrow bc} \left( z = \frac{x}{x'} \right) \]
Parton cascades in hadron are continuously born and recombined.

A hard scattering probes fluctuations up to $Q^2$.

Hard scattering inhibits recombination of the cascade.

Event generation could be addressed by **forwards evolution**:
pick a complete partonic set at low $Q_0$ and evolve, see what happens.

**Inefficient**

1. have to evolve and check for *all* potential collisions
2. difficult to steer the production e.g. of a narrow resonance
Backwards evolution

Start at hard interaction and trace what happened “before”

Recast:

\[
\frac{df_b(x, Q^2)}{dt} = \sum_a \int_x^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} P_{a\to bc}(z)
\]

with \( t = \ln(Q^2/\Lambda^2) \) and \( z = x/x' \)

To:

\[
d\mathcal{P}_b = \frac{df_b}{f_b} = |dt| \sum_a \int dz \frac{x'f_a(x', t)}{xf_b(x, t)} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)
\]

- solve for decreasing \( t \), i.e. backwards in time
- high \( Q^2 \) moving towards lower \( Q^2 \)
- Sudakov form factor \( \exp(-\int d\mathcal{P}_b) \)
\[ p_1 \rightarrow p_2 + k, \quad p_1^2 = p_2^2 = 0 \Rightarrow k^2 = (p_1 - p_2)^2 = -2p_1 \cdot p_2 < 0 \]

- Backwards (from hard scatter) evolution of partons with virtualities increasing \( \rightarrow 0 \)
- Since backwards, must normalize to the incoming flux of partons

- Hard scattering is characterized by large \( Q^2 \), small \( x \)
- Valence quarks characterized by large \( x \), small virtualities \( Q_0 \sim \Lambda_{QCD} \)

\[ Q^2 \]

\[ Q^2_0 \]
By the end of the parton shower, we have nearly exhausted our ability to apply perturbation theory

- Have a description of jet structure
- Can ask questions about energy flow and isolation
- See if kinematic features survive

This is still not enough

- Don’t know response of detector to a soft quark/gluon
- Cannot tag a $b$ quark
- Can’t ask about charged tracks or neutrals

Next step is into the Brown Muck
Modern PS models are very sophisticated implementations of perturbative QCD
- Derived from factorization theorems of full gauge theory
- Accelerated electric and color charges radiate
- Parton Shower development encoded in Sudakov FF
- Performed to LL and some sub-LL accuracy with exact kinematics
- Color coherence leads to angular ordering of shower
- Still need hadronization models to connect with data
- Shower evolves virtualities of partons to a low enough values where this connection is possible
Lecture 2

- Hadronization
  - string
  - cluster
- Underlying Event
  - parametrizations
  - multiple-interactions
- The Event Generator Programs
- New Developments
QCD partons are free only on a very short time scale.

Hadrons are the physical states of the strong interaction.

Need a description of how partons are confined.

Lacking a theory, we need a model.

- **Enough** variables to fit data.
- **Few enough** that there is some predictability.
- Start related to the end of the parton shower.
- Use **basic** understanding of QCD.
QCD is a confining theory

- Linear potential $V_{QCD}(r) \sim kr$
  - Confirmed by Lattice, Spectroscopy, Regge Trajectories
- Gluons are self-coupling
  - Field lines contract into Flux-tubes
  - Analogy with field behavior inside of superconductors
- Over time, 2 phenomenological models have survived
  - cluster
  - Lund string
- Not exactly Orthogonal, Exhaustive
Independent Fragmentation

- FF = Feynman-R. Field
- pure phenomenological model
- imagine $q\bar{q}$ pairs tunnel from the vacuum to dress bare quark
- $f_{q\rightarrow h}(z)$ is probability $q \rightarrow h$ with fraction $z$ of some $E/p$ variable
- $f_{g\rightarrow h}(z)$? $g \rightarrow q\bar{q}$?
- Lorentz invariant? ($E_q$)
- Useful for its time

**FF:** $f(z) = 1 - a + 3a(1 - z)^2$
Massless partons become massive jets. Energy and momentum cannot be conserved independently.
Perturbative evolution of quarks and gluons organizes them into clumps of color-singlet clusters.

In PS, color-singlet pairs end up close in phase space.

Cluster model takes this view to the extreme.

Color connections induce correlations to conserve $E, p$. 
Nonperturbative $g \rightarrow q\bar{q}$ splitting ($q = uds$) isotropically.

Here, $m_g \approx 750 \text{ MeV} > 2m_q$.

Cluster formation, universal spectrum

Cluster fission until

$$M^p < M_{\text{fiss}}^p = M_{\text{max}}^p + (m_{q1} + m_{q2})^p$$

where masses are chosen from

$$M_i = \left[ \left( M^p - (m_{qi} + m_{q3})^p \right) r_i + (m_{qi} + m_{q3})^p \right]^{1/P}$$

with additional phase space constraints

Cluster decay

- isotropically into pairs of hadrons
- simple rules for spin, species
Cluster Fission

- Mass spectrum of color-singlet pairs asymptotically independent of energy, production mechanism
- Peaked at low mass
- Broad tail at large mass

- Small fraction of clusters heavier than typical
  - ⇒ Cluster fission (string-like)
- Fission threshold becomes crucial parameter
  - 15% of primary clusters split
  - produces 50% of hadrons
String Model

String = color flux tube is stretched between $q$ and $\bar{q}$

- Classical string will oscillate in space-time
- Endpoints $q$, $\bar{q}$ exchange momentum with the string

**Quantum Mechanics:** string energy can be converted to $q\bar{q}$ pairs (tension $\kappa \sim 1$ GeV/fm)

\[
d\text{Prob}/dx/dt = (\text{constant})\exp(-\pi m^2/\kappa) \quad \text{[WKB]}
\]

- $u : d : s : qq = 1 : 1 : 0.35 : 0.1$

\[
dP_n(\{p_j\}; P_{tot}) = \prod_{j=1}^{n} N_j d^2 p_j \delta(p_j^2 - m_j^2) \delta(\sum_{j=1}^{n} p_j - P_{tot}) \exp(-bA)
\]
String Break-Up

The derivation of the tunnelling probability is the same as Schwinger’s for $e^+e^-$ pair production in a static field, but $V(z) = \kappa z$ (QCD potential is linear)

$$\Psi(\ell = p_T/\kappa) = \Psi(0) \exp \left( - \int_0^\ell dz \sqrt{p_T^2 - (\kappa z)^2} \right)$$

$$= \Psi(0) \exp \left( - \frac{p_T^2}{\kappa} \int_0^\pi d\theta \sin^2 \theta \right)$$

$$= \Psi(0) \exp \left( - \frac{\pi p_T^2}{2\kappa} \right)$$

Tunnelling Prob

$$\propto \Psi^* \Psi \Rightarrow \frac{1}{\pi} \exp \left( - \frac{\pi p_T^2}{\kappa} \right)$$

$p_T^2 \rightarrow p_T^2 + m^2$
Adjacent breaks form a hadron

- $m_{\text{had}}^2 \propto \text{area swept out by string}$
Iterative Solution

- String breaking and hadron formation can be treated as an iterative process
- Use light-cone coordinates $x^\pm = x \pm t$
- Boundary Conditions:
  \[ x_0^+ = \frac{2E_0}{\kappa}, \quad x_{n+1}^- = \frac{2\bar{E}_0}{\kappa}, \quad x_n^- = x_{n+1}^+ = 0 \]
  1. select $z_i$ according to $f(z)dz$
     - $f^h(z, p_T) \sim \frac{1}{z}(1 - z)^a \exp\left[-\frac{b(m_h^2 + p_T^2)}{z}\right]$
  2. $\Delta x^+ = (x_{i-1}^+ - x_i^+) = z_i x_{i-1}^+$
  3. $\Delta x^- = (x_{i-1}^- - x_i^-) = \frac{-m_i^2}{\kappa^2 \Delta x^+}$
     - mass$^2$ of hadron $\propto \Delta x^+ \Delta x^-$
  4. Continue until string is consumed
Inclusion of Gluon Radiation

- Perturbative Parton Shower generates gluons
- Gluon = kink on string, i.e. some motion to system
- String effect $\Rightarrow$ particles move in direction of kink

![Diagram showing the process of gluon radiation and string effects.](image)

Stephen Mrenna  
Event Generators
Clusters (Herwig)

- perturbation theory can be applied down to low scales if the coherence is treated correctly
- There must be non-perturbative physics, but it should be very simple
- Improving data has meant successively making non-pert phase more string-like

Strings (Pythia, Ariadne)

- dynamics of the non-perturbative phase must be treated correctly
- Model includes some non-perturbative aspect of color (interjet) coherence (string effect)
- Improving data has meant successively making non-pert phase more cluster-like
Underlying Event

- Hadrons (protons) are extended objects
- Remnant remains after hard partons scatter
- Need a description of how partonic remnants are confined, similar to the way quarks and gluons from radiation are confined

Historically, Two Approaches

1. Soft parton-parton collisions dominate \((\text{parametrize})\)
2. Semi-Hard parton-parton cross section can be applied even at low \(p_T\)
UA5 Monte Carlo

- hadron-hadron scattering produces two leading clusters and several central ones
- parametrize $N_{\text{ch}}$ and sample
- clusters given $p_T$ and $y$ from an *ad hoc* distribution
  \[
  \frac{dN}{dp_T^2} \sim e^{-bp_T}, \quad \frac{1}{(p_T + p_0)^n}
  \]
- $y \sim$ flat with Gaussian tails
- $p_L = m \sinh(y)$

- Herwig adds in their cluster model
- UE model is a mechanism for producing the objects used in description of hadronization
Multiple Interaction Model

- Soft model does not agree well with data
- Multi-interaction dynamics observed by AFS, UA1, CDF
- Implied by the width of the multiplicity distribution in UA5
- Forward-backward correlations: UA5
- Pedestal effect: UA1, H1, CDF
What are multiple interactions?

QCD $2 \rightarrow 2$ interactions dominated by $t$-channel gluon exchange, so diverges like $d\sigma/dp_{\perp}^2 \approx 1/p_{\perp}^4$ for $p_{\perp} \rightarrow 0$.

Integrate QCD $2 \rightarrow 2$

$qq' \rightarrow qq'$  $q\bar{q} \rightarrow q'\bar{q}'$

$q\bar{q} \rightarrow gg$  $qg \rightarrow qq$

$gg \rightarrow gg$  $gg \rightarrow q\bar{q}$

with CTEQ 5L PDF’s
\[ \sigma_{\text{int}}(p_{\perp \text{min}}) > \sigma_{\text{tot}} \text{ for } p_{\perp \text{min}} \lesssim 5 \text{ GeV} \]

What does this mean?

Half a solution: many interactions per event

\[
\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n
\]

\[
\sigma_{\text{int}} = \sum_{n=0}^{\infty} n \sigma_n
\]

\[
\sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1
\]

If interactions occur independently then Polssonian statistics

\[
P_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}
\]

but energy–momentum conservation

\[ \Rightarrow \text{large } n \text{ suppressed} \]
Other half of solution:
perturbative QCD not valid at small $p_{\perp}$ since $q, g$ not asymptotic states (confinement!).

Naively breakdown at

$$p_{\perp\text{min}} \simeq \frac{\hbar}{r_p} \approx \frac{0.2 \text{ GeV} \cdot \text{fm}}{0.7 \text{ fm}} \approx 0.3 \text{ GeV} \simeq \Lambda_{\text{QCD}}$$

… but better replace $r_p$ by (unknown) colour screening length $d$ in hadron

![Diagram of resolved and screened states with $\lambda \sim 1/p_{\perp}$]
\[ \bar{n} = \sigma_{\text{hard}}(p_{\perp \text{min}})/\sigma_{\text{nd}}(s) > 1 \]

- Not a violation of unitarity! \( \sigma_{\text{hard}} \) is inclusive
- On average, \( \bar{n} \) semi-hard interactions in one hard collision
- Collisions ranked in \( x_\perp = 2p_\perp/E_{\text{cm}} \), produced with prob
  \[ f(x_\perp) = \frac{1}{\sigma_{\text{nd}}(s)} \frac{d\sigma}{dx_\perp} \]
- The probability that the hardest interaction is at \( x_{\perp 1} \):
  \[ f(x_{\perp 1}) \exp \left\{ - \int_{x_{\perp 1}}^{1} f(x'_\perp) \, dx'_\perp \right\} \]
  - like radioactive decay
- generate a chain of scatterings \( 1 > x_{\perp 1} > x_{\perp 2} > \cdots > x_{\perp i} \)
  using \( x_{\perp i} = F^{-1}(F(x_{\perp i-1}) - \ln R_i) \)
  \[ F(x_\perp) = \int_{x_\perp}^{1} f(x'_\perp) \, dx'_\perp = \frac{1}{\sigma_{\text{nd}}(s)} \int_{s/4}^{s/4} \frac{d\sigma}{dp^2_\perp} \, dp^2_\perp \]
Strings and the UE

- Each additional interaction adds more color flow
  - Color information encoded in strings
  - The way subsequent interactions color-connect is a parameter of the model
  - Fits prefer a minimization of total string length
MSTP(82) :
(D=1) structure of multiple interactions. For QCD processes, used down to values below, it also affects the choice of structure for the one hard/semi-hard interaction.

= 0 :
  simple two-string model without any hard interactions. Toy model only!
= 1 :
  multiple interactions assuming the same probability in all events, with an abrupt cut-off at PARP(81). (With a slow energy dependence given by PARP(89) and PARP(90).)
= 2 :
  multiple interactions assuming the same probability in all events, with a continuous turn-off of the cross section at PARP(82). (With a slow energy dependence given by PARP(89) and PARP(90).)
= 3 :
  multiple interactions assuming a varying impact parameter and a hadronic matter overlap consistent with a Gaussian matter distribution, with a continuous turn-off of the cross section at PARP(82). (With a slow energy dependence given by PARP(89) and PARP(90).)
= 4 :
  multiple interactions assuming a varying impact parameter and a hadronic matter overlap consistent with a double Gaussian matter distribution given by PARP(83) and PARP(84), with a continuous turn-off of the cross section at PARP(82). (With a slow energy dependence given by PARP(89) and PARP(90).)
Pythia at Run2: Underlying Event

**Table:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default</th>
<th>Tune</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARP(67)</td>
<td>1.0</td>
<td>4.0</td>
<td>Scale factor for ISR</td>
</tr>
<tr>
<td>MSTP(82)</td>
<td>1.0</td>
<td>4</td>
<td>Double Gaussian matter distribution</td>
</tr>
<tr>
<td>PARP(82)</td>
<td>1.9</td>
<td>2.0</td>
<td>Cutoff (GeV) for MPIs</td>
</tr>
<tr>
<td>PARP(83)</td>
<td>0.5</td>
<td>0.5</td>
<td>Warm Core with % of matter</td>
</tr>
<tr>
<td>PARP(84)</td>
<td>0.2</td>
<td>0.4</td>
<td>within a given radius</td>
</tr>
<tr>
<td>PARP(85)</td>
<td>0.33</td>
<td>0.9</td>
<td>Prob. that two gluons have NNC</td>
</tr>
<tr>
<td>PARP(86)</td>
<td>0.66</td>
<td>0.95</td>
<td>gg versus q̄q</td>
</tr>
<tr>
<td>PARP(89)</td>
<td>1000.0</td>
<td>1800.0</td>
<td>Reference energy (GeV)</td>
</tr>
<tr>
<td>PARP(90)</td>
<td>0.16</td>
<td>0.25</td>
<td>Power of Energy scaling for cutoff</td>
</tr>
</tbody>
</table>
## PYTHIA 6.2 Tunes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tune DW</th>
<th>Tune DWT</th>
<th>ATLAS</th>
<th>Tune QW</th>
<th>Tune QWT</th>
<th>Tune QK</th>
<th>Tune QKT</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDF</td>
<td>CTEQ5L</td>
<td>CTEQ5L</td>
<td>CTEQ5L</td>
<td>CTEQ6.1</td>
<td>CTEQ6.1</td>
<td>CTEQ6.1</td>
<td>CTEQ6.1</td>
</tr>
<tr>
<td>MSTP(2)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MSTP(33)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PARP(31)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>MSTP(81)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MSTP(82)</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>PARP(82)</td>
<td>1.9 GeV</td>
<td>1.9409 GeV</td>
<td>1.8 GeV</td>
<td>1.1 GeV</td>
<td>1.1237 GeV</td>
<td>1.9 GeV</td>
<td>1.9409 GeV</td>
</tr>
<tr>
<td>PARP(83)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>PARP(84)</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>PARP(85)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.33</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>PARP(86)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.66</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>PARP(89)</td>
<td>1.8 TeV</td>
<td>1.96 TeV</td>
<td>1.0 TeV</td>
<td>1.8 TeV</td>
<td>1.96 TeV</td>
<td>1.8 TeV</td>
<td>1.96 TeV</td>
</tr>
<tr>
<td>PARP(90)</td>
<td>0.25</td>
<td>0.16</td>
<td>0.16</td>
<td>0.25</td>
<td>0.16</td>
<td>0.25</td>
<td>0.16</td>
</tr>
<tr>
<td>PARP(62)</td>
<td>1.25</td>
<td>1.25</td>
<td>1.0</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>PARP(64)</td>
<td>0.2</td>
<td>0.2</td>
<td>1.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>PARP(67)</td>
<td>2.5</td>
<td>2.5</td>
<td>1.0</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>MSTP(91)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PARP(91)</td>
<td>2.1</td>
<td>2.1</td>
<td>1.0</td>
<td>2.1</td>
<td>2.1</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>PARP(93)</td>
<td>15.0</td>
<td>15.0</td>
<td>5.0</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
</tr>
</tbody>
</table>

**Use LO $\alpha_s$ with $\Lambda = 192$ MeV!**

**K-factor (Sjöstrand)**

**UE Parameters**

**ISR Parameter**

**Intrinsic KT**

---

Stephen Mrenna | Event Generators
Remember the $p_T$ cut-off, $P_{T0}^0$, of the MPI cross section is energy dependent and given by

$$P_{T0}(E_{cm}) = PARP(82) \times \left( \frac{E_{cm}}{E_0} \right)^\varepsilon$$

with $\varepsilon = PARP(90)$ and $E_0 = PARP(89)$.  

Average charged particle density and PTsum density in the “transverse” region ($p_T > 0.5$ GeV/c, $|\eta| < 1$) versus $P_T(\text{jet#1})$ at 1.96 TeV for PY Tune DW, Tune QW, and Tune QK.
The $p_T$ ordered shower in Pythia was developed to have a consistent description of ISR and UE, and to allow for fiddling of the color connections.
Tune parameters affect much more than just the charged track properties

These are full “Event” tunes
DØ Dijet Azimuthal Correlation

\[ \frac{1}{\sigma_{\text{dijet}}} \frac{d\sigma_{\text{dijet}}}{d\Delta\phi_{\text{dijet}}} \]

- $p_T^{\text{max}} > 180$ GeV ($\times 8000$)
- $130 < p_T^{\text{max}} < 180$ GeV ($\times 400$)
- $100 < p_T^{\text{max}} < 130$ GeV ($\times 20$)
- $75 < p_T^{\text{max}} < 100$ GeV

\[ \Delta\phi_{\text{dijet}} \]

- HERWIG 6.505
- PYTHIA 6.225
- PYTHIA increased ISR (CTEQ6L)
Even resummation calculations need non-pert. $k_T$

Catalysis for “-W"\(^1\) tunes

\(^1\)W=Willis Sakumoto
High-$p_T$ is sensitive to UE

Should allow FSR for multiple parton interactions
Tune A gives too much ISR

Don’t increase starting scale for ISR
## The f77 Parton Shower Programs

<table>
<thead>
<tr>
<th></th>
<th>Pythia</th>
<th>Herwig</th>
<th>Ariadne</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS Ordering</td>
<td>Mass ((\theta \text{ veto}))</td>
<td>Angle</td>
<td>(k_T)</td>
</tr>
<tr>
<td></td>
<td>(p_T)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hadronization</td>
<td>String</td>
<td>Cluster</td>
<td>String</td>
</tr>
<tr>
<td>Underlying Event</td>
<td>Mult. Int</td>
<td>UA6/(Jimmy)</td>
<td>LDCM</td>
</tr>
</tbody>
</table>

Finding them:

- [http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html](http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html)
- [http://hepwww.rl.ac.uk/theory/seymour/herwig/](http://hepwww.rl.ac.uk/theory/seymour/herwig/)
- [http://www.thep.lu.se/~leif/ariadne/](http://www.thep.lu.se/~leif/ariadne/)

Fortran codes

- [http://www.ibiblio.org/pub/languages/fortran/ch1-1.html](http://www.ibiblio.org/pub/languages/fortran/ch1-1.html)

- Herwig-f77 frozen, Pythia-f77 evolving: primary tools at Tevatron
Why so many programs?

- Need to resum large logarithms, because there are two scales in the program
- The large scale is $M_W, M_Z, m_t, \ldots$
- Which small scale? The mass of jets? $p_T$? $E_0\theta_{qg}$?
- How are they related?

\[ m^2 = 2E_iE_j(1 - \cos \theta_{ij}) \]

\[ E_i = zE_0, \quad E_j = (1 - z)E_0; \quad 2(1 - \cos \theta_{ij}) = 4\sin^2(\theta_{ij}/2) \rightarrow \theta_{ij}^2 \]

\[ q_{Py-old}^2 = m^2 \times \theta(\theta_{old} - \theta_{new}) \]

\[ q_{Hw}^2 = E_0^2\theta_{ij}^2 = \frac{m^2}{z(1 - z)} \]

\[ q_{Ar}^2 = z(1 - z)m^2 = q_{Py-new}^2 \]
The cpp programs

- **Pythia & Herwig being rewritten**
  - QCD FSR, QCD ISR, particle decays, etc.
  - Improvements to showers, accounting of particle properties, couplings

- **Herwig++ “will be ready for LHC”; Pythia8 likely same**

**Sherpa** is also C++ event generator in a different framework
Includes some new ideas with and some older models

- overlap with some Pythia physics assumptions
  - hadronization is the Lund string model
  - parton shower is virtuality ordered with some modifications
  - underlying event is of the multiple-interaction kind

- “automatic” inclusion of higher-order (tree level) matrix elements
For all new generators, there is a long road of tuning and validation ahead.
The parton showers were developed using the soft and collinear approximations.

We would like to control this approximation and make systematic improvements.

How can we include more hard jets in the “hard scattering”? Can we include NLO normalization?
How to do Tree Level Calculations

- Read Feynman rules from $i\mathcal{L}_{int}$ from a textbook
- Use Wave Functions from Relativistic QM
  - Propagators (Green functions) for internal lines
- Specify initial and final states
  - Track spins/colors/etc. if desired
- Draw all valid graphs connecting them
  - Tedious, but straight-forward
- Calculate $(\text{Matrix Element})^2$
  - Evaluate Amplitudes, Add and Square
  - Symbolically Square, Evaluate
  - ALPHA (numerical functional evaluation with no Feynman graphs)
- Integrate over Phase Space
Complications:

- $|\mathcal{M}|^2$: Number of graphs grows quickly with number of external partons
- $d\Phi_n$: Efficiency decreases with number of internal lines

Programs:

- MadEvent, CompHep, Alpgen, Amegic++
- Differ in methods of attack
- Most rely on VEGAS for MC integration

Limitations:

- Fixed number of partons
- No control of large logarithms as $E_g, \theta_{qg}, \theta_{gg} \to 0$
Automatically calculate code needed for a given HEP process and generate events

List of those actively supporting hadron colliders

- Alpgen@ http://m.home.cern.ch/m/mlm/www/alpgen/
- CompHep@ http://theory.sinp.msu.ru/comphep
- Grace@ http://atlas.kek.jp/physics/nlo-wg/grappa.html
- MadEvent@ http://madgraph.hep.uiuc.edu/index.html
- Sherpa/Amegic++@ http://141.30.17.181/

Advantages and disadvantages of each
An impressive improvement from several years ago
MadGraph HomePage

by Fabio Maltoni and Tim Stelzer

Generate Process  Calculated Cross Sections  Source Codes  FAQ Developments  Other approaches  Citations

Generate Process Code On-Line

Quarks: d u s c b t d u s c b t

Leptons: e- mu- ta- ve vm vt e+ mu+ ta+ ve vm vt

Bosons: A Z W+ W- h g

Special: Pj (sums over d u s c d u s c)

Process: PP > W+ > e+ ve jjj

Max QCD Order: 4
Max QED Order: 2

To improve our web services we now request that you register. Registration is quick and free. You may register for a password by clicking here
Interfacing with PS Tools: Les Houches Accord

## Initialization

```
INTEGER MAXPUP
PARAMETER (MAXPUP=100)
INTEGER IDBMUP,PDFGUP,PDFSUP,IDWTUP,NPRUP,LPRUP
DOUBLE PRECISION EBMUP,XSECUP,XERRUP,XMAXUP
COMMON/HEPRUP//IDBMUP(2),EBMUP(2),PDFGUP(2),PDFSUP(2),IDWTUP,
&NPRUP,XSECUP(MAXPUP),XERRUP(MAXPUP),XMAXUP(MAXPUP),LPRUP(MAXPUP)
```

- **IDBMUP**: incoming beam particles (PDG codes, $p = 2212$, $\bar{p} = -2212$)
- **EBMUP**: incoming beam energies (GeV)
- **PDFGUP, PDFSUP**: PDFLIB parton distributions (not used by PYTHIA)
- **IDWTUP**: weighting strategy
  - $1$: PYTHIA mixes and unweights events, according to known $d\sigma_{\text{max}}$
  - $2$: PYTHIA mixes and unweights events, according to known $\sigma_{\text{tot}}$
  - $3$: unit-weight events, given by user, always to be kept
  - $4$: weighted events, given by user, always to be kept
  - $-1$, $-2$, $-3$, $-4$: also allow negative $d\sigma$
- **NPRUP**: number of separate user processes
- **XSECUP(i)**: $\sigma_{\text{tot}}$ for each user process
- **XERRUP(i)**: error on $\sigma_{\text{tot}}$ for each user process
- **XMAXUP(i)**: $d\sigma_{\text{max}}$ for each user process
- **LPRUP(i)**: integer identifier for each user process
Sufficiently Describe the Hard Scattering

The event

```
INTEGER MAXNUP
PARAMETER (MAXNUP=500)
INTEGER NUP,IDPRUP,IDUP,ISTUP,MOTHUP,ICOLUP
DOUBLE PRECISION XWGTUP,SCALUP,AQEDUP,AQCDUP,PUP,VTIMUP,SPINUP
COMMON/HEPEUP/NUP,IDPRUP,XWGTUP,SCALUP,AQEDUP,AQCDUP,
&IDUP(MAXNUP),ISTUP(MAXNUP),MOTHUP(2,MAXNUP),ICOLUP(2,MAXNUP),
&PUP(5,MAXNUP),VTIMUP(MAXNUP),SPINUP(MAXNUP)
```

**IDPRUP**: identity of current process

**XWGTUP**: event weight (meaning depends on IDWTUP weighting strategy)

**SCALUP**: scale $Q$ of parton distributions etc.

**AQEDUP**: $\alpha_{\text{em}}$ used in event

**AQCDUP**: $\alpha_s$ used in event

**NUP**: number of particles in event

**IDUP(i)**: PDG identity code for particle $i$

**ISTUP(i)**: status code ($-1 =$ incoming parton, $1 =$ final-state parton, $2 =$ intermediate resonance with preserved $m$)

**MOTHUP(j,i)**: position of one or two mothers

**PUP(j,i)**: $(p_x, p_y, p_z, E, m)$

**VTIMUP(i)**: invariant lifetime $c\tau$

**SPINUP(i)**: spin (helicity) information
Examples of colour flows and indices

\textsc{icolup}(j,i): colour and anticolour indices

= colour line tags, in the \( N_C \to \infty \) limit, starting e.g. with number 501.

Example 1: hadronic \( t\bar{t} \) production

Example 2: baryon number violation

User-process BNV not (yet) implemented in PYTHIA
(but part of internal PYTHIA SUSY machinery)
Want to use these matrix-element tools with parton showers

Each topology (e.g. $W + 0, 1, 2, 3, 4$ partons) has no soft/collinear approximation

How do I rigorously add a parton shower to each topology with no double counting of hard emissions?

Solution (CKKW):

1. Make the $|\mathcal{M}|^2$ result “look” like a parton shower down to a reasonable cutoff scale ($k_T^{\text{cut}}/Q_{\text{hard}} \sim .1$)

2. Add on ordinary parton shower below $K_T^{\text{cut}}$

$$k_T^2 = 2\min(E_i, E_j)^2(1 - \cos \theta_{ij})$$
Pseudo-Shower Method

1. Generate \( W + N \) parton events, applying a cut \( p_T^{2\text{cut}} \) on shower \( p_T^2 \) (\( p_T^2 \) for ISR, \( z(1-z)m^2 \) for FSR)

2. Form a \( p_T^2 \)-ordered parton shower history

3. Reweight with \( \alpha_s(p_T^2) \) for each emission

4. Add parton shower and keep if no emission harder than \( p_T^{2\text{cut}} \):
   (save this event)

5. Remove softest of \( N \) partons, fix up kinematics, add parton shower and keep if no emission harder than \( p_T^{2\text{softest}} \)

6. Continue until no partons remain, or an emission is too hard

7. If not rejected, use the saved event
\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{\text{cut}} \]
ISR Parton Shower–Matrix Element Movie

\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut} \]

\[ k_T^3 \]

\[ k_T^1 \]

\[ k_T^{PS} < k_T^{cut} \]

\[ k_T^2 \]

\[ k_T^4 \]
ISR Parton Shower–Matrix Element Movie

\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{\text{cut}} \]

\[ k_T^3 \]

\[ k_T^1 \]

\[ k_T^2 \]

\[ W \]
\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{\text{cut}} \]
\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut} \]
\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut} \]
\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut} \]
\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{\text{cut}} \]
\[ k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut} \]
$k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{\text{cut}}$

$k_T^{\text{PS}} < k_T^1$
Why it works

- For each $N$, PS does not add any jet harder than $p_T^{2\text{cut}}$
- Can safely add different $N$ samples with no double-counting
  - Apply looser rejection on highest $N$
- Pseudo-showers assure correct PS limit, while retaining hard emissions
  - Interpolates between limits

Why it is necessary

- Suppress unphysical enhancements in tree level calculations from
  \[ \alpha_s^n(p_T) \ln^{(2n,2n-1)} \left( \frac{Q}{p_T} \right) \]
- Account for many topologies in physical observables, e.g.
  \[ H_T = \sum p_T(\text{hard object}) \]
- Tames hard emissions from PS
\[ W + 0 \oplus \cdots \oplus W + 4 \text{ hard partons} \]

**Dashed** is Pythia with default (ME) correction

**Solid** is Pseudoshower result

Combines ME contributions (0, 1, 2, 3, 4 partons)
Other methods for performing such matching are “MLM” and “CKKW”

There is no attempt to account for individual “K”-factors for different topologies

Such calculations are currently included in CDF and DØ Standard Model cocktails

Theoretical uncertainty on such methods is beginning to limit Run2 prospects for extracting top properties
NLO Calculations give an improved description of the hard kinematics and cross sections, but are inclusive, i.e. not (exclusive) event generators.

Solution (MC@NLO): Remove divergences by adding and subtracting the Monte Carlo result for one emission.
Consider a system that can emit a number of quanta (photons) with energy $z_0 < x < x_{\text{max}}(x)$, $x_{\text{max}}(1) = 1$

$$0 \leq Q(z) \leq 1, \quad \lim_{z \to 0} Q(z) = 1,$$

IF the prob. of one emission is $a \frac{Q(x)}{x} dx$

THEN the *Sudakov form factor* is

$$\Delta(x_2, x_1) = \exp \left[ -a \int_{x_1}^{x_2} dz \frac{Q(z)}{z} \right],$$

<table>
<thead>
<tr>
<th>Limit</th>
<th>Sudakov</th>
<th># of Quanta</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \ll 1$</td>
<td>$\Delta \sim 1 - a \int \frac{Q(x)}{x} dx$</td>
<td>few</td>
</tr>
<tr>
<td>$a \gg 1$</td>
<td>$\Delta \sim 0$</td>
<td>many</td>
</tr>
</tbody>
</table>
Constructing an “Event” Generator

Event ≡ original system + emissions down to scale $x_0$

Take $Q(x) = 1$

To solve for the shower evolution:

1. Pick $r = \exp \left( -a \int_{x}^{x_2} \frac{dx}{x} \right) = \left(\frac{x}{x_2}\right)^a$

2. Solve $x = x_2 r^{(1/a)}$

3. Calculate remaining energy $x_2$

4. Continue until $x < x_0$

This generates an energy-ordered shower with multiple photon emissions
## Example Event Record

### Event listing (summary)

<table>
<thead>
<tr>
<th>I</th>
<th>particle/jet</th>
<th>KS</th>
<th>KF</th>
<th>orig</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>e-</td>
<td>1</td>
<td>11</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>nu_e</td>
<td>1</td>
<td>12</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>(e-)</td>
<td>11</td>
<td>11</td>
<td>0</td>
<td>0.296</td>
</tr>
<tr>
<td>4</td>
<td>gamma</td>
<td>1</td>
<td>22</td>
<td>3</td>
<td>0.704</td>
</tr>
<tr>
<td>5</td>
<td>(e-)</td>
<td>11</td>
<td>11</td>
<td>3</td>
<td>0.285</td>
</tr>
<tr>
<td>6</td>
<td>gamma</td>
<td>1</td>
<td>22</td>
<td>3</td>
<td>0.011</td>
</tr>
<tr>
<td>7</td>
<td>(e-)</td>
<td>11</td>
<td>11</td>
<td>5</td>
<td>0.283</td>
</tr>
<tr>
<td>8</td>
<td>gamma</td>
<td>1</td>
<td>22</td>
<td>5</td>
<td>0.002</td>
</tr>
<tr>
<td>9</td>
<td>e-</td>
<td>1</td>
<td>11</td>
<td>7</td>
<td>0.282</td>
</tr>
<tr>
<td>10</td>
<td>gamma</td>
<td>1</td>
<td>22</td>
<td>7</td>
<td>0.001</td>
</tr>
</tbody>
</table>

sum: -1.00 1.000
Spectra for Toy Model

Real (NLO) spectrum = \[ \frac{d\sigma}{dx} = a \frac{R(x)}{x} \]

\[ R(x) \rightarrow Q(x) \text{ as } x \rightarrow 0 \]

Here: \[ R(x) = (1 + x/10)^2 \]

Enlo = energy at NLO
Einc = summed energy from PS
Emax = max[E] from PS

Parton shower underestimates high energy emissions
NLO Computation for Toy Model

\[
\begin{align*}
\left(\frac{d\sigma}{dx}\right)_B &= B\delta(x), \\
\left(\frac{d\sigma}{dx}\right)_V &= a\left(\frac{B}{2\epsilon} + V\right)\delta(x), \\
\left(\frac{d\sigma}{dx}\right)_R &= a\frac{R(x)}{x},
\end{align*}
\]

\[
\lim_{x \to 0} R(x) = B.
\]

infrared-safe observable \( O \)

\[
\langle O \rangle = \lim_{\epsilon \to 0} \int_0^1 dx \, x^{-2\epsilon} O(x) \left[ \left(\frac{d\sigma}{dx}\right)_B + \left(\frac{d\sigma}{dx}\right)_V + \left(\frac{d\sigma}{dx}\right)_R \right],
\]
Write the real contribution as:

\[
\langle O \rangle_R = aBO(0) \int_0^1 dx \frac{x^{-2\epsilon}}{x} + a \int_0^1 dx \frac{O(x)R(x) - BO(0)}{x^{1+2\epsilon}}.
\]

Set \( \epsilon = 0 \) in the second term

\[
\langle O \rangle_R = -a \frac{B}{2\epsilon} O(0) + a \int_0^1 dx \frac{O(x)R(x) - BO(0)}{x}.
\]

NLO prediction:

\[
\langle O \rangle_{\text{sub}} = \int_0^1 dx \left[ O(x) \frac{aR(x)}{x} + O(0) \left( B + aV - \frac{aB}{x} \right) \right].
\]
\[
\langle O \rangle_{\text{sub}} = \int_{0}^{1} dx \left[ O(x) \frac{aR(x)}{x} + O(0) \left( B + aV - \frac{aB}{x} \right) \right]
\]

Adding a parton shower makes it difficult to cancel singularities $O(0)$ and $O(x)$ observables both contribute to order $a$:

\[
Ba \frac{Q(x)}{x} + a \frac{R(x)}{x} \] (double counting problem)
Showering with full NLO corrections

**Modified Subtraction Method** (Frixione and Webber: MC@NLO)

\[
\left( \frac{d\sigma}{dO} \right)_{\text{msub}} = \int_0^1 dx \left[ I_{\text{MC}}(O, x_M(x)) \frac{a[R(x) - BQ(x)]}{x} + I_{\text{MC}}(O, 1) \left( B + aV + \frac{aB[Q(x) - 1]}{x} \right) \right]
\]

Singular terms cancel among themselves

\(O(0)\) and \(O(x)\) observables still *both* contribute to \(O(a)\)

They cancel to yield \(a \frac{R(x)}{x}\)

Assignment: read (Soper and Kraemer: Beowulf + PS)
Matrix Element Correction to Parton Shower

Assume the parton shower samples all of phase space and gives the hardest emission first.

For the 1st emission, weight according to \( \frac{R(x)}{Q(x)} \)

Here: \( (1 + x/10)^2 < 2 \)

Parton shower gets correct limit for large \( x \) and includes multiple photon emission.
Event Generators accumulate our understanding of the Standard Model into one package

- Apply perturbation theory whenever possible
  - hard scattering, parton showering, decays
- Rely on models or parametrizations when present calculational methods fail
  - hadronization, underlying event, beam remnants
Out of the box, they give reliable estimates of the full, complicated structure of HEP events

Attentive users will find more flexibility & applications

Understanding the output can lead to a broader understanding of the Standard Model (and physics beyond)

Many new developments
  (more difficult questions ⇒ better tools)